# Chapter 21

## 21.1

No. It is desirable that the minimum value of the output signal be greater than zero, in order to readily detect instrument failures. Thus, for a conventional electronic instrument, an output signal of 0 mA indicates that a malfunction has occurred such as a power failure. If the instrument range were 0-20 mA, instead of 4-20 mA, the output signal could be zero during normal operation. Thus, instrument failures would be more difficult to detect.

## 21.2

The difference between a measurement of 5.9 and the sample mean, 5.75, is 0.15 pH units. Because the standard deviation is $s$ = 0.05 pH units, this measurement is three standard deviations from the mean. If the pH measurement is normally distributed (an assumption), then Fig. 21.3 indicates that the probability that the measurement is less than or equal to three standard deviations from the mean is 0.997. Thus, the probability $p$ of a measurement being greater than three standard deviations from the mean is only $p$ =1-0.997=0.003.

## 21.3

Make the usual SPC assumption that the temperature measurement is normally distributed. According to Figure 21.3, the probability that the measurement is within two standard deviations from the mean is 0.95. Thus, the probability that a measurement is beyond these limits, during routine operation is $p$=1-0.95=0.05. From (21-19), the average run length *ARL* between false positives is,

$$ARL = \frac{1}{p} = 20 \text{ samples}$$

Thus, for a sampling period of one minute, on average we would expect a false positive every 20 min. Consequently, we would expect an average of 3 false alarms per hour or 24 false alarms over an eight hour period.

**21.4**

The CUSUM and EWMA control charts for the sample means are shown in Figure S21.4. The Shewhart chart is also shown for the sake of comparison. The CUSUM and EWMA charts do not show any chart violations. For the Shewhart chart, a violation occurs for sample #5, as is also evident from Fig. 21.5 in the textbook.

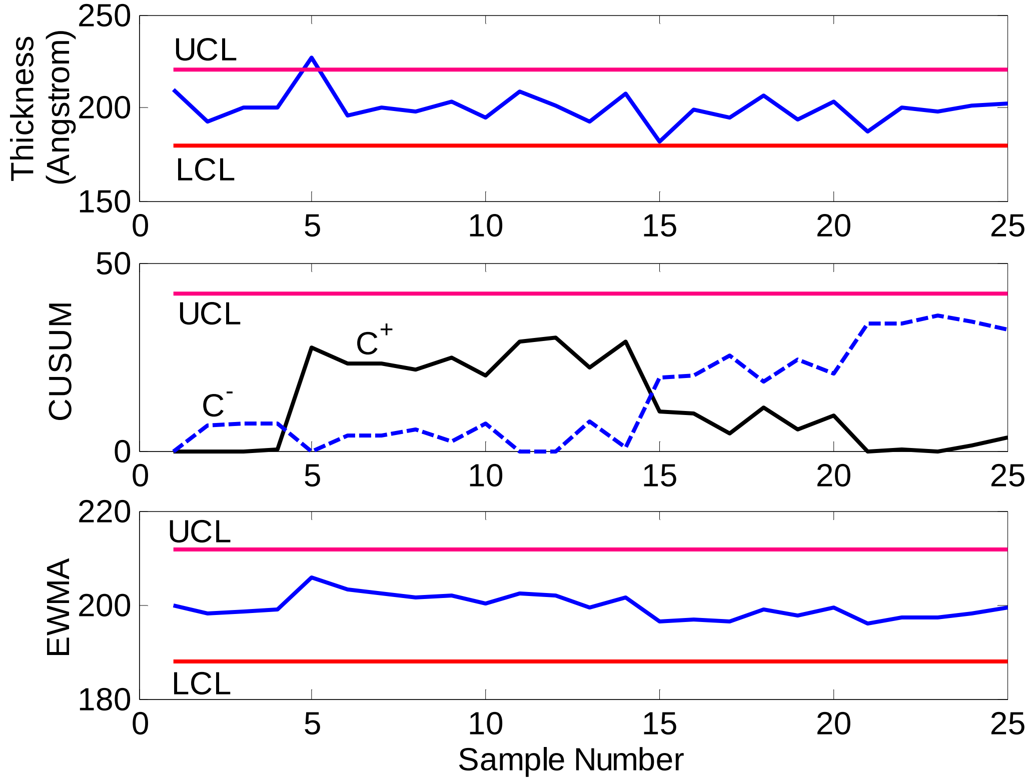


**Figure S21.4**. *Control charts for the thickness data of Example 21.2.*

## 21.5

A plot of the data in Figure S21.5 does not indicate any abnormal behavior.

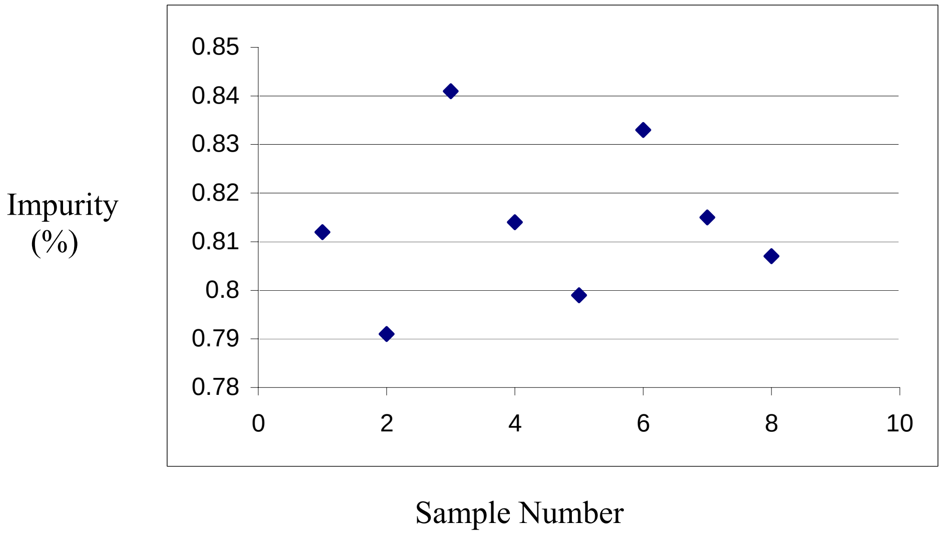


**Figure S21.5**. *Impurity data for Exercise 21.5.*

The following statistics and chart limits can be calculated from the data:

$\overline{x} = 0.814$ %  $UCL = 0.863$ %

$s = 0.016$ %  $LCL = 0.737$ %

Because the sample mean, 0.815, is less than one standard deviation larger than the normal value of 0.8, it does not appear that the mean impurity level has shifted. Furthermore, Fig. S21.5 indicates that all eight data points are within the Shewhart chart limits. Thus, there is no statistical evidence that the mean impurity level has shifted.

## 21.6

(a) The Shewhart chart for the rainfall data is shown in Fig. S21.6a. The following items were calculated from the data for 1870-1919:

$s =$ 7.74 in.  $UCL$ = 41.9 in.

$\overline{x}$ =18.6 in.  $LCL$ = - 4.71 in. (actually zero)

The rainfall exceeded a chart limit for only one year, 1941.

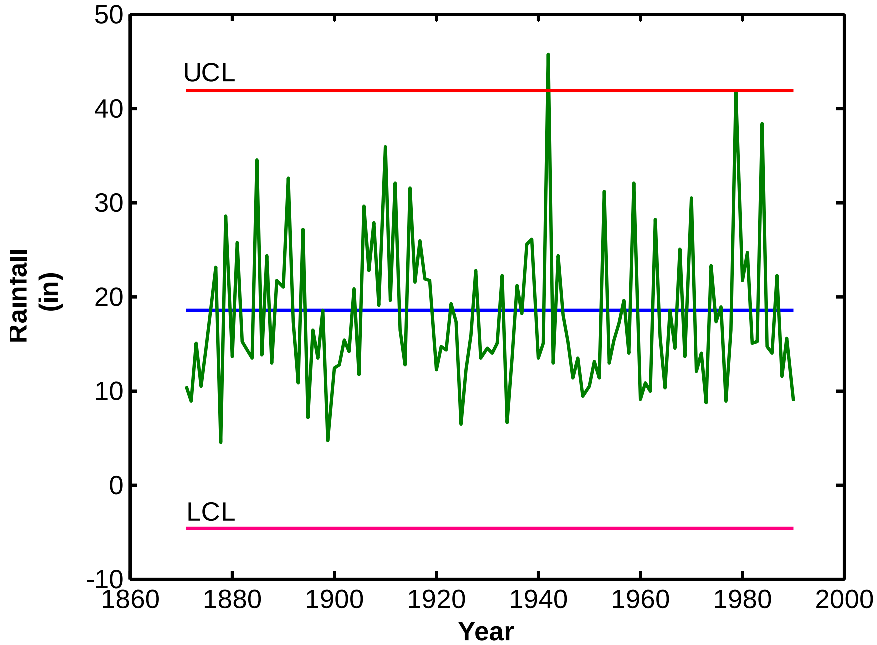


**Figure S21.6a**. *Shewhart chart for rainfall data.*

(b) The control chart for the standard deviation of the subgroup data (for each decae) is shown in Fig. S21.6b. The following items were calculated for the sub-group data prior to 1940:

$$\bar{s} = 6.87 \text{ in.}$$

$$UCL = B_4 \bar{s} = (1.716)(6.87 \text{ in}) = 11.8 \text{ in.}$$

$$LCL = B_3 \bar{s} = (0.284)(6.87 \text{ in}) = 1.95 \text{ in.}$$

The sub-group data does not violate the chart limits for 1940 90.

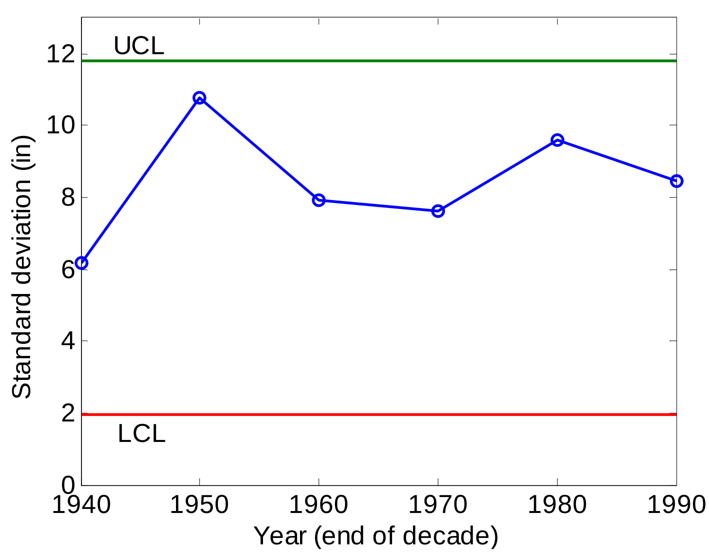


**Figure S21.6b**. *Standard deviations for sub-groups.*

**21.7**

The CUSUM and EWMA control charts for the period 1900-1960 are shown in Figure S21.7. The Shewhart and the data are also shown in the top portion, for the sake of comparison. The following statistics and chart limits were calculated from the data for 1900 through 1929:

$s =$ 7.02 in. $\bar{x}$ =19.2 in.

| Control Chart | UCL (in.) | LCL (in.) |
|---|---|---|
| Shewhart | 40.2 | - 1.9 (actually zero) |
| CUSUM | 35.1 | 0 |
| EWMA | 27.1 | 11.2 |

The rainfall exceeded a Shewhart chart limit for only one year, 1941. The CUSUM chart has both high ($C^+$) and low ($C^-$) chart violations during the initial period, 1900-1929. Two subsequent low limit violations occurred after 1930. After each CUSUM violation, the corresponding sum was reset to zero. No chart violations occur for the EWMA chart.

The CUSUM and EWMA charts indicate that the period from 1930 to 1950 had two dry spells and one wet spell. The Shewhart chart violation is for the wettest year in the entire dataset, 1941. The rainfall during the 1950s was quite normal.

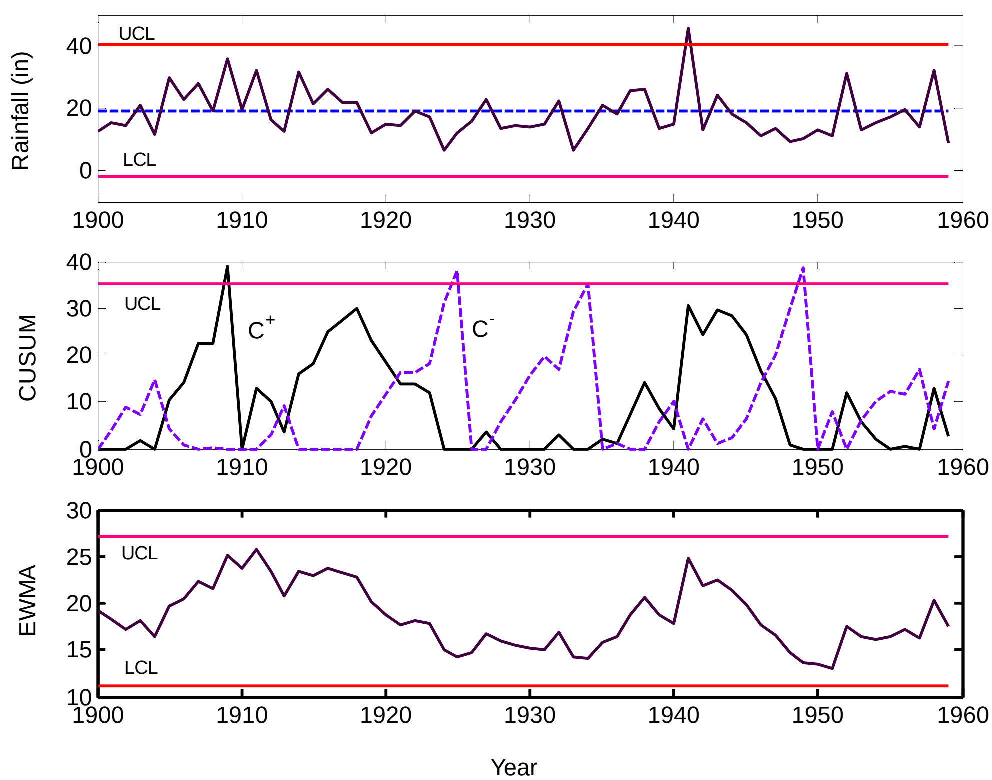


**Fig. S21.7.** *Control charts for Rainfall Data.*

**21.8**

In general, it is preferable to plot unfiltered measurements because they contain the most information. However, it is important to be consistent. Thus, if the control chart limits were calculated based on unfiltered data, unfiltered measurements should be plotted for subsequent monitoring. Conversely, if the chart limit calculations were based on filtered data, filtered measurements should be plotted.

**21.9**

The control charts in Fig. S21.9 do not exhibit any control chart violations. Thus, the process performance is considered to be normal. The CUSUM chart was designed using the default values of $K$=0.5 and $H=5\hat{\sigma}=5s$ where $s$ is the sample standard deviation. The EWMA chart was designed using $\lambda$=0.25.

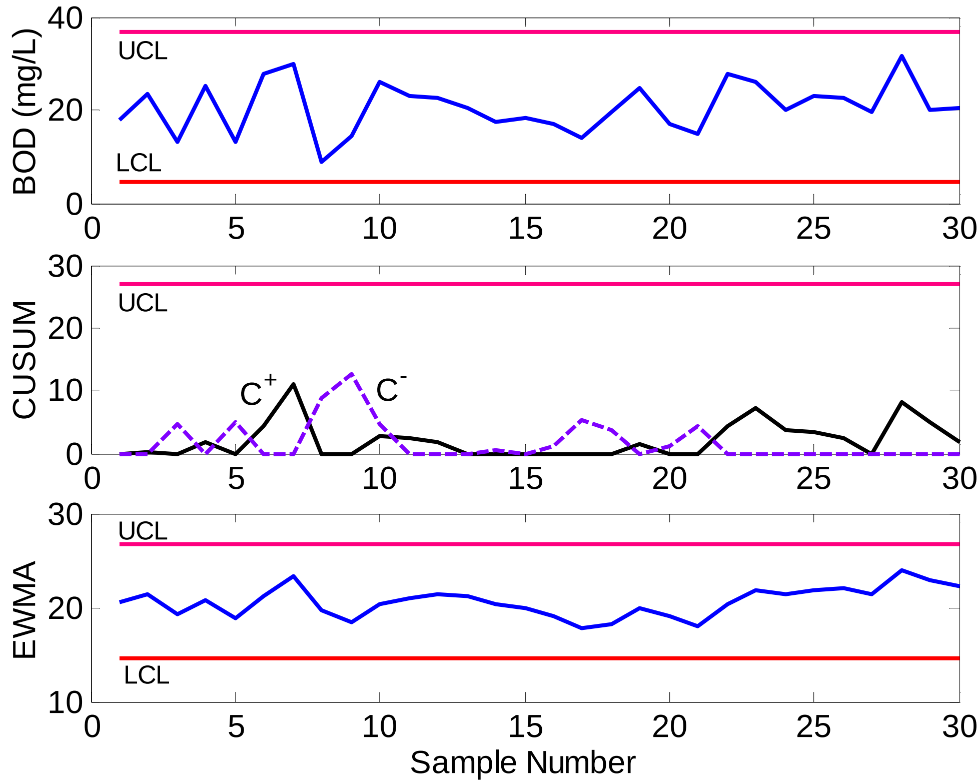


**Figure S21.9**. *Control charts for the BOD data of Example 21.5.*

**21.10**

**Correction:** USL and LSL values for this exercise should be switched to give USL=35 mg/L and LSL= 5 mg/L.

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By definition,

$$C_p \triangleq \frac{USL - LSL}{6\sigma} \qquad (21-25)$$

Because the population standard deviation σ is not known, it must be replaced by an estimate, $\hat{\sigma}$. Let $\hat{\sigma} = s$ where s is the sample standard deviation. The standard deviation of the BOD data is $s$ =5.41 mg/L. Substitution gives,

$$C_p = \frac{35-5}{6(5.41)} = \boxed{0.924}$$

Capability index $C_{pk}$ is defined as:

$$C_{pk} \triangleq \frac{\min\,[\bar{x} - LSL, USL - \bar{x}]}{3\sigma} \qquad (21-26)$$

The sample mean for the BOD data is $\bar{x}$ = 20.6 mg/L. Substituting numerical values into (21-26) gives:

$$C_{pk} = \frac{\min\,[20.6-5, 35-20.6]}{3(5.41)} = \boxed{0.887}$$

Because both capability indices are below 1.0, the process is considered to be performing very well.

**21.11**

The control charts in Fig. S21.11 do not exhibit any chart violations. Thus, the process performance is considered to be normal. The CUSUM chart was designed using the default values of $K$=0.5 and $H = 5\hat{\sigma} = 5s$ where $s$ is the sample standard deviation. The EWMA chart was designed using $\lambda$=0.25.

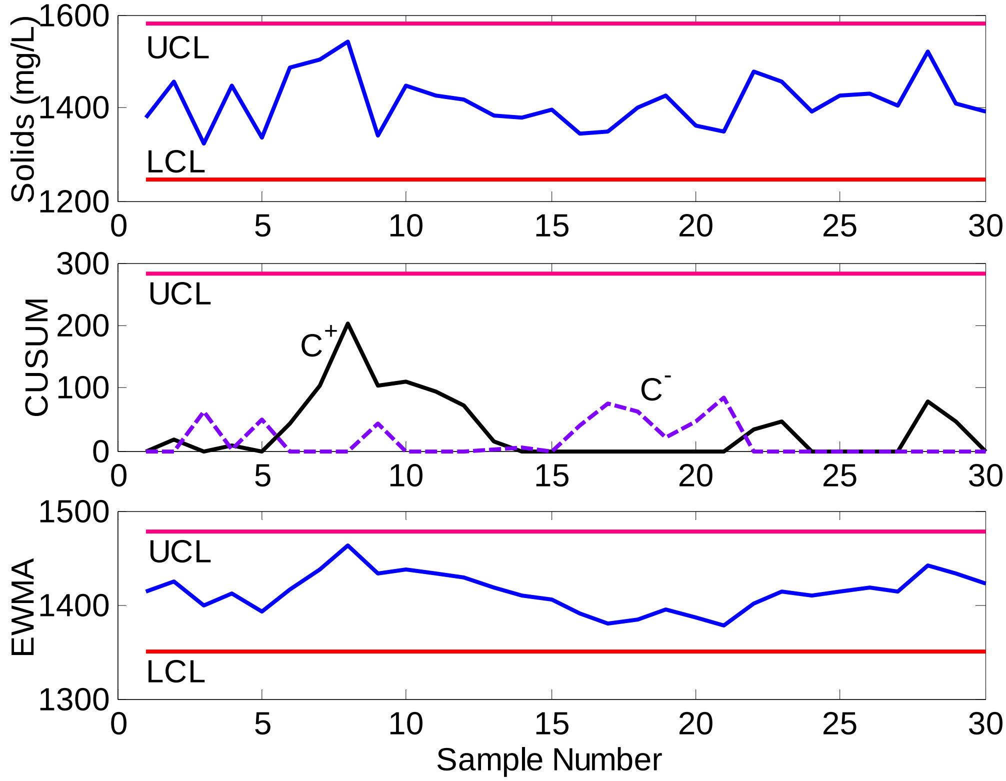


**Figure S21.11**. *Control charts for the solids data of Example 21.5.*

**21.12**

The new data are plotted on a $T^2$ chart in Fig. S21.12. A chart violation occurs for the second data point. Because one of the six measurements is beyond the chart limit, it appears that the process behavior could be abnormal. However, this measurement may be an "outlier" and thus a further investigation is advisable. Also, additional data should be collected before concluding that the process operation is abnormal.

Note that the previous control chart limit of 11.63 from Example 21.6 is also used in this exercise.

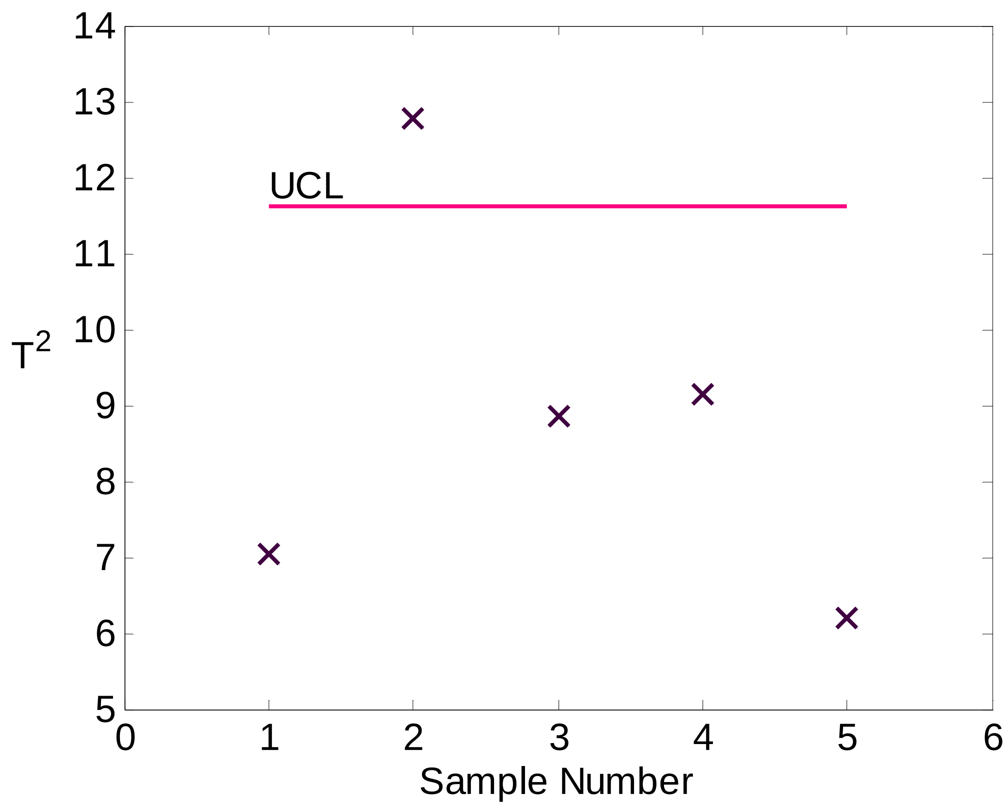


**Figure S21.12**. *$T^2$ Control chart and new wastewater data.*