Chapter 24

24.1

a) **i. First model (full compositions model):**

Number of variables: $N_V = 22$

w_1	$x_{R,A}$	x_{2B}
w_2	$x_{R,B}$	x_{2D}
w_3	$x_{R,C}$	x_{4C}
w_4	$x_{R,D}$	x_{5D}
w_5	$x_{T,D}$	x_{6D}
w_6	V_T	x_{7D}
w_7	H_T	x_{8D}
w_8		

Number of Equations: $N_E = 17$
Eqs. 2-8, 9, 10, 12, 13, 15, 16, 18, 20(3X), 21, 22, 27, 28, 29, 31

Number of Parameters: $N_P = 4$

V_R, k, α, ρ

Degrees of freedom: $N_F = 22 - 17 = 5$

Number of manipulated variables: $N_{MV} = 4$

w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$

x_{2D}

Number of controlled variables: $N_{CV} = 4$

x_{4A}, w_4, H_T, x_{8D}

ii. Second model (simplified compositions model):

Number of variables: $N_V = 14$

w_1	$x_{R,A}$	w_4
w_2	$x_{R,B}$	x_{4A}
w_6	$x_{R,D}$	x_{8D}
w_8	$x_{T,D}$	H_T
x_{2D}	V_T	

Number of Equations: $N_E = 9$

Eq. 2-33 through Eq. 2-41

Number of Parameters: $N_P = 4$
V_R, k, α, ρ

Degrees of freedom: $N_F = 14 - 9 = 5$

Number of manipulated variables: $N_{MV} = 4$
w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$
x_{2D}

Number of controlled variables: $N_{CV} = 4$
x_{4A}, w_4, H_T, x_{8D}

iii.Third model (simplified holdups model):

Number of variables: $N_V = 14$

w_1	$H_{R,A}$	w_4
w_2	$H_{R,B}$	x_{4A}
w_6	$H_{R,D}$	x_{8D}
w_8	$H_{T,B}$	H_T
x_{2D}	$H_{T,D}$	

Number of Equations: $N_E = 9$

Eq. 2-48 through Eq. 2-56

Number of Parameters: $N_P = 3$

V_R, k, α

Degrees of freedom: $N_F = 14 – 9 = 5$

Number of manipulated variables: $N_{MV} = 4$

w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$

x_{2D}

Number of controlled variables: $N_{CV} = 4$

x_{4A}, w_4, V_T, x_{8D}

b) Model 1:
The first model is left in an intermediate form, i.e., not fully reduced, so the key equations for the units are more clearly identifiable. Also, such a model is easier to develop using traditional balance methods because not as much algebraic effort is expended in simplification.

Models 2 and 3:
Both of the reduced models are easier to simulate (fewer equations), yet contain all of the dynamic relations needed to simulate the plant.

Model 3:
The "holdups model" has the further advantage of being easier to analyze using a symbolic equation manipulator because of its more symmetric organization. Also, it requires one less parameter for its specification.

c) Each model can be simulated using the equations given in Appendix E of the text. Models 2 and 3 are simulated using the differential equation editor (dee) in Matlab. An example can be found by typing *dee* at the command prompt. Step changes are made in the manipulated variables w_1, w_2, w_6 and w_8 and in disturbance variable x_{2D} to illustrate the dynamics of the entire plant.

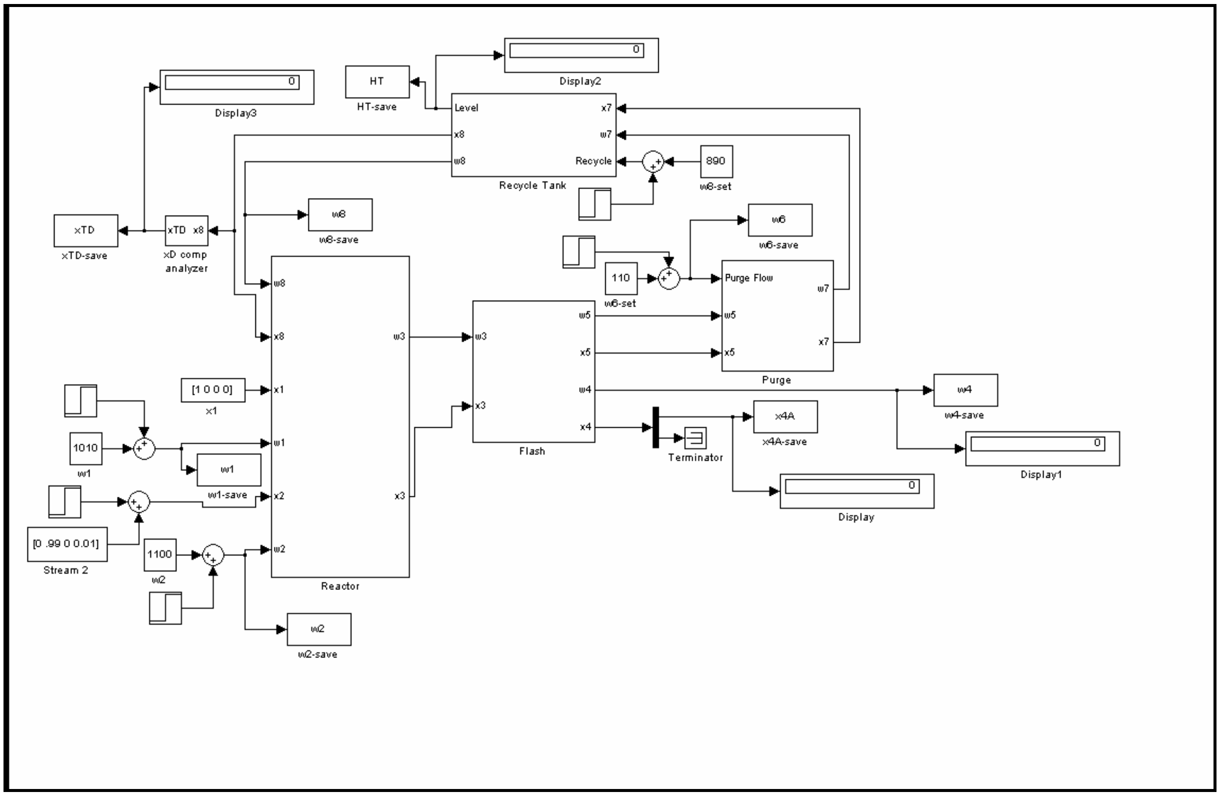

Figure S24.1a. *Simulink-MATLAB block diagram for first model*

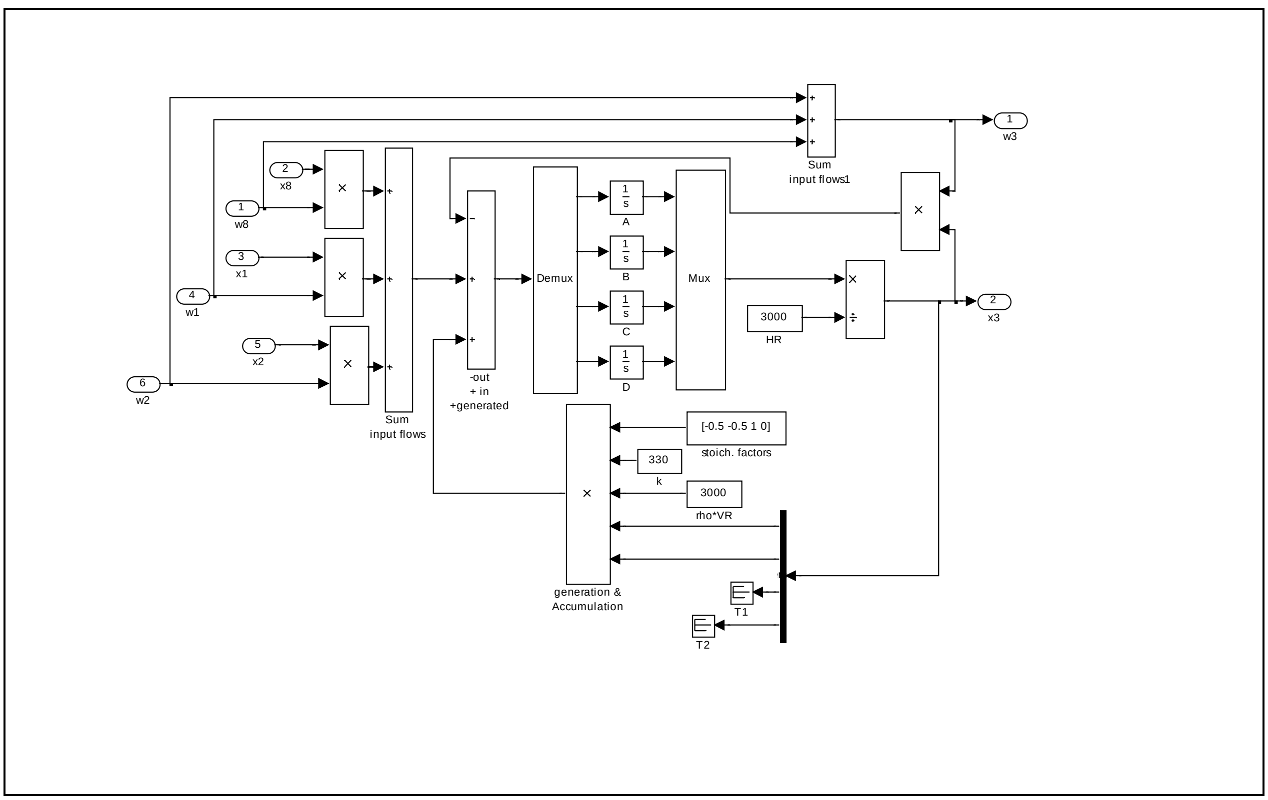

Figure S24.1b. *Simulink-MATLAB block diagram for the reactor block*

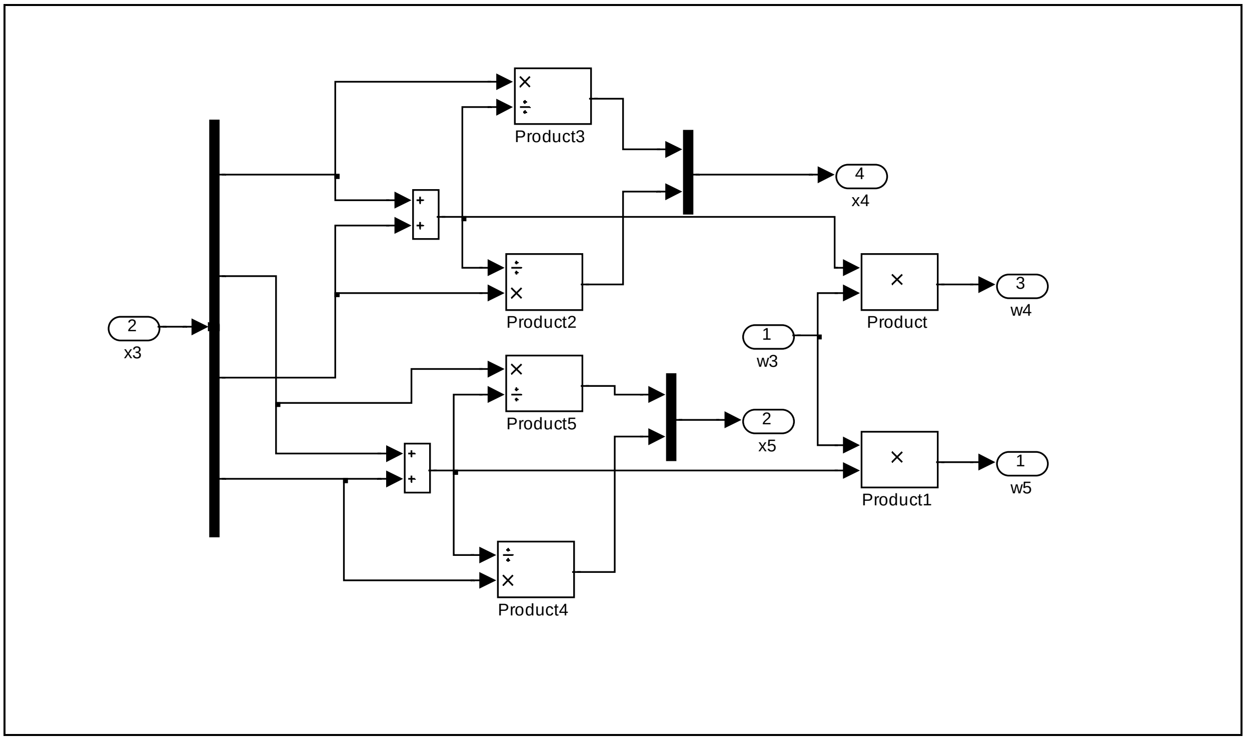

Figure S24.1c. *Simulink-MATLAB block diagram for the flash block*

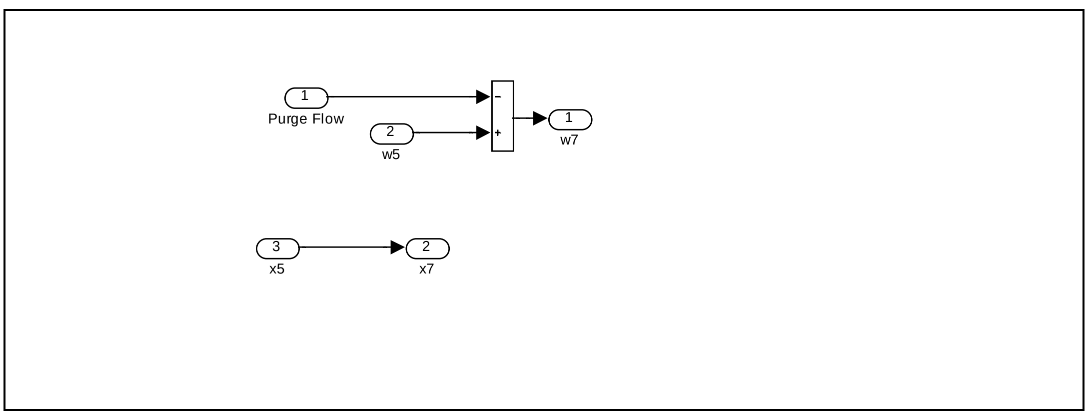

Figure S24.1d. *Simulink-MATLAB block diagram for purge block*

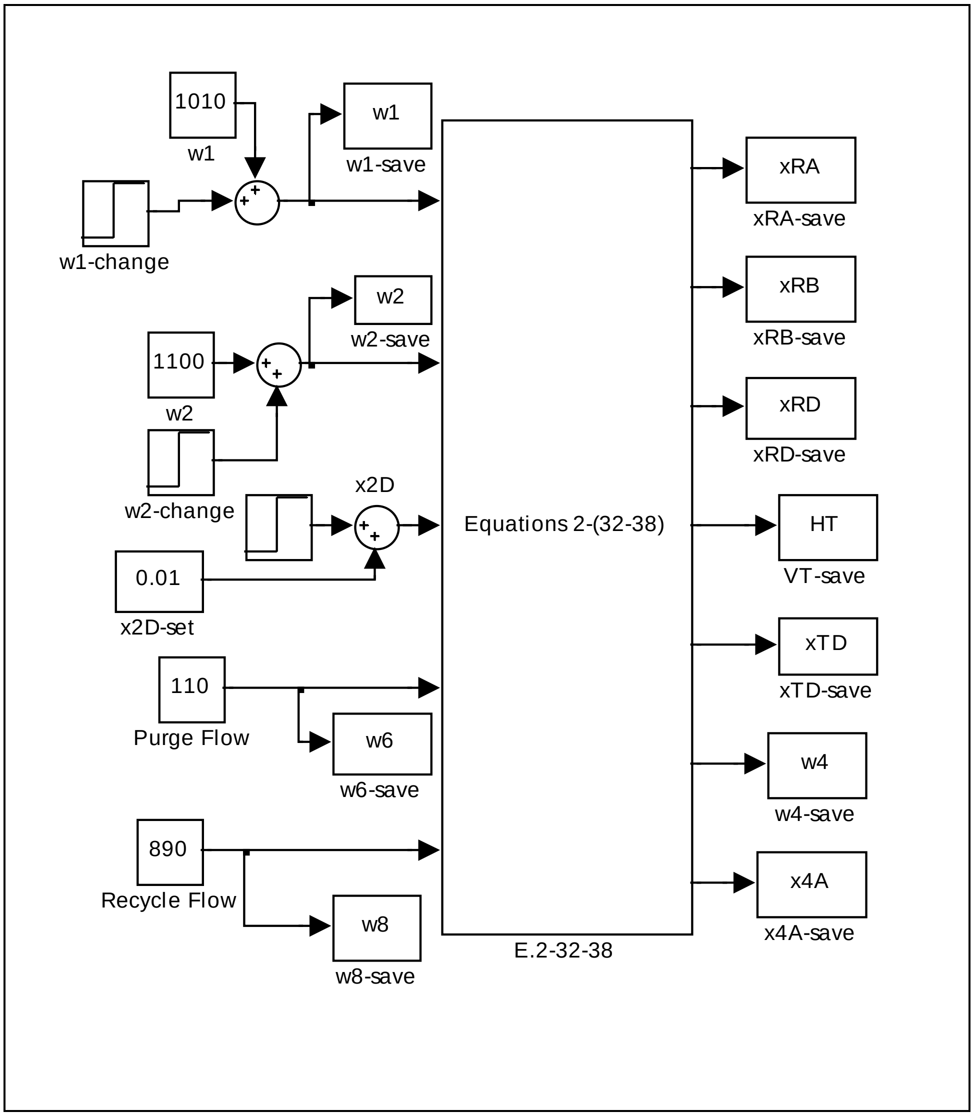

Figure S24.1e. *Simulink-MATLAB block diagram for second model*

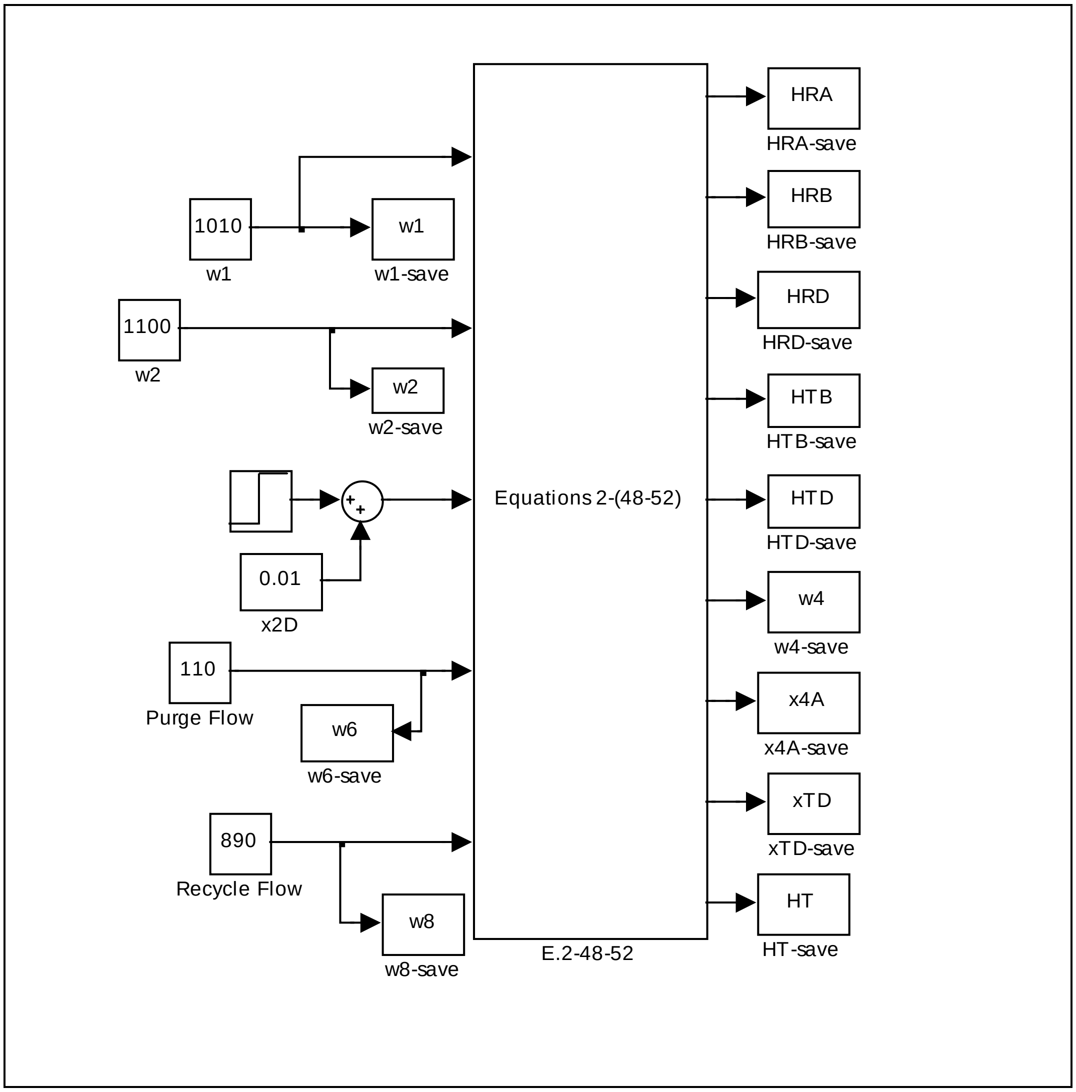

Figure S24.1f. *Simulink-MATLAB block diagram for third model*

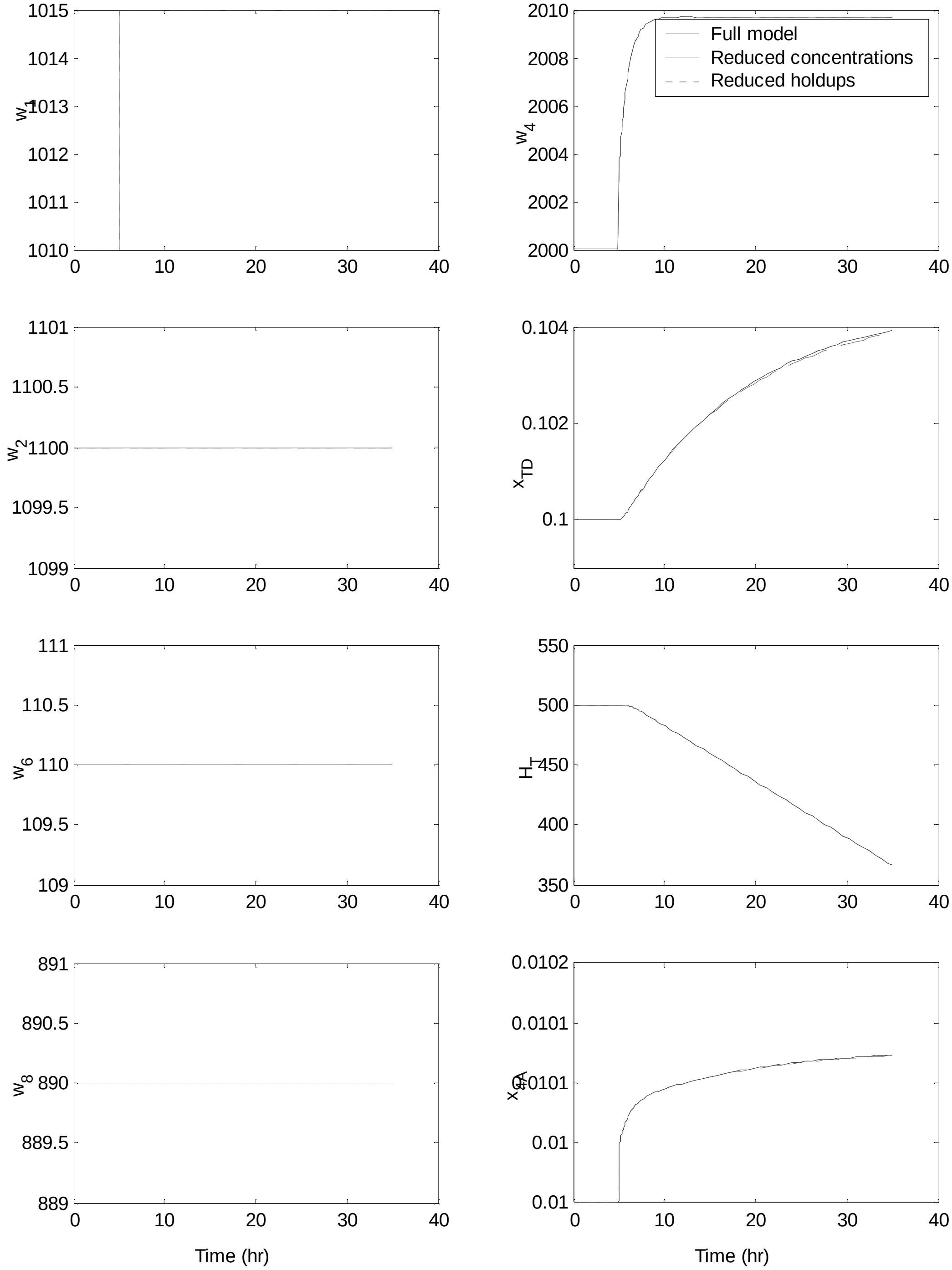

Figure S24.1g. *Step change in w_1 (+5) at t=5*

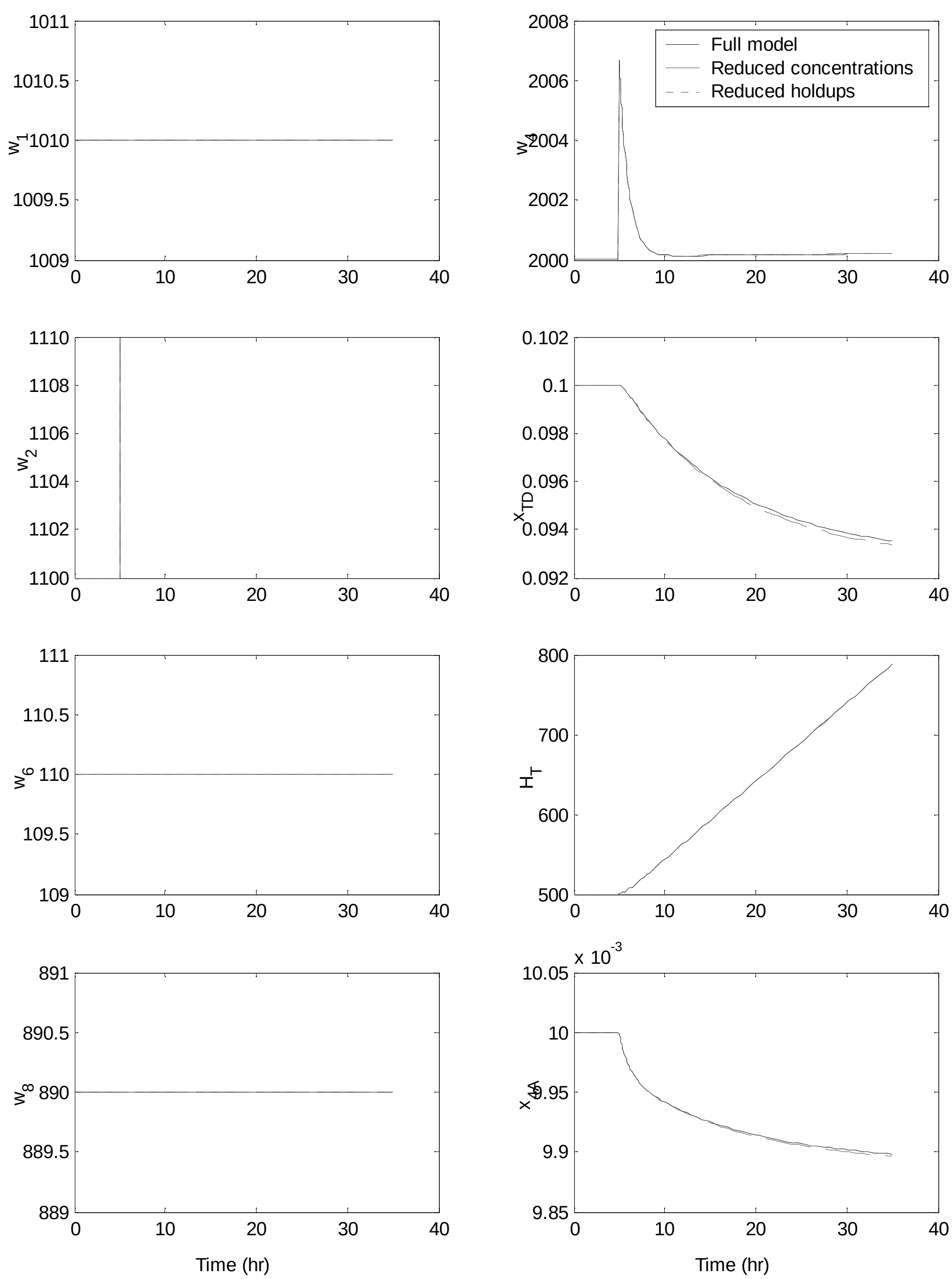

Figure S24.1h. *Step change in w_2 (+10) at t=5*

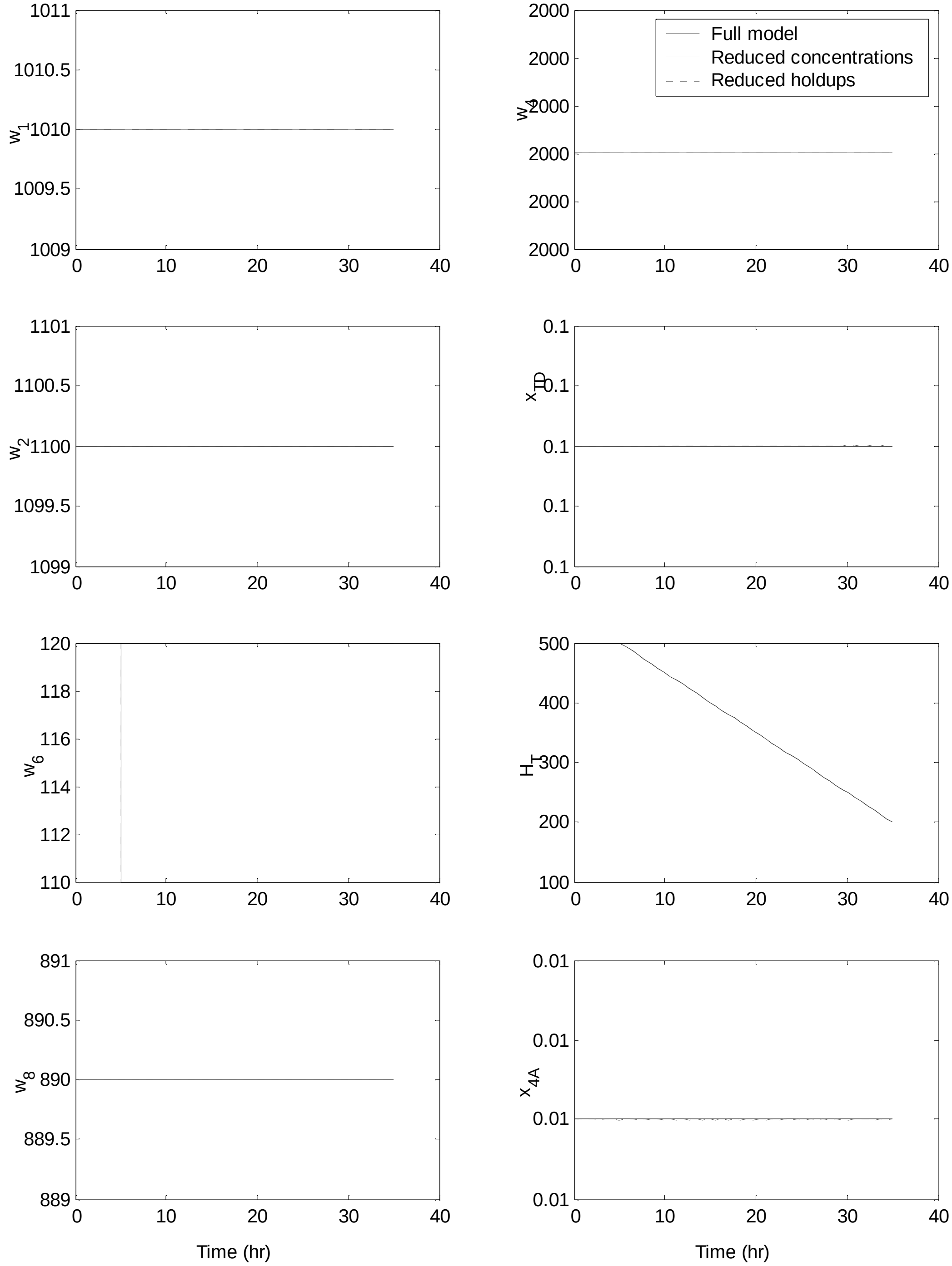

Figure S24.1i. *Step change in w_6 (+10) at t=5*

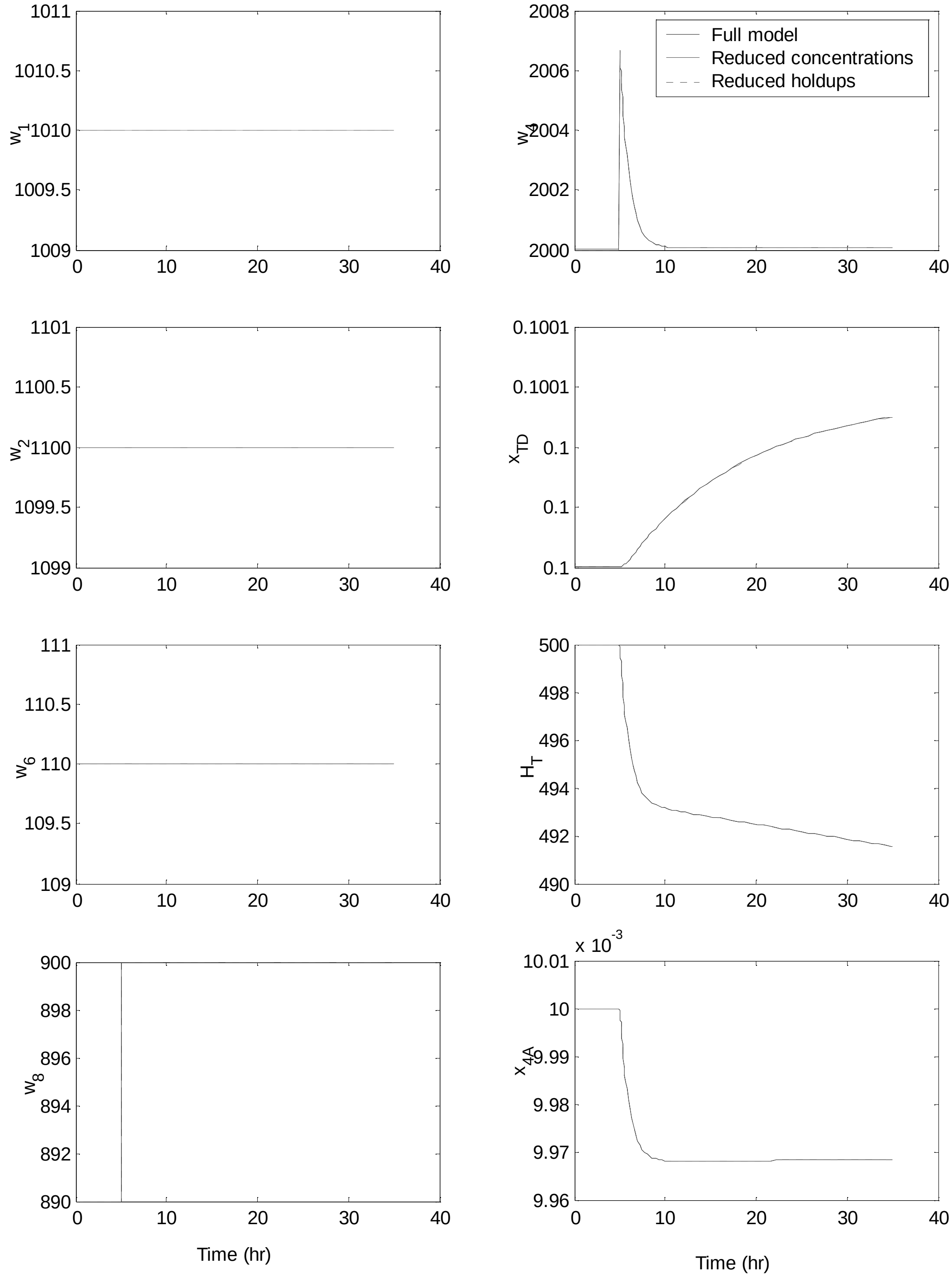

Figure S24.1j. *Step change in w_8 (+10) at t=5*

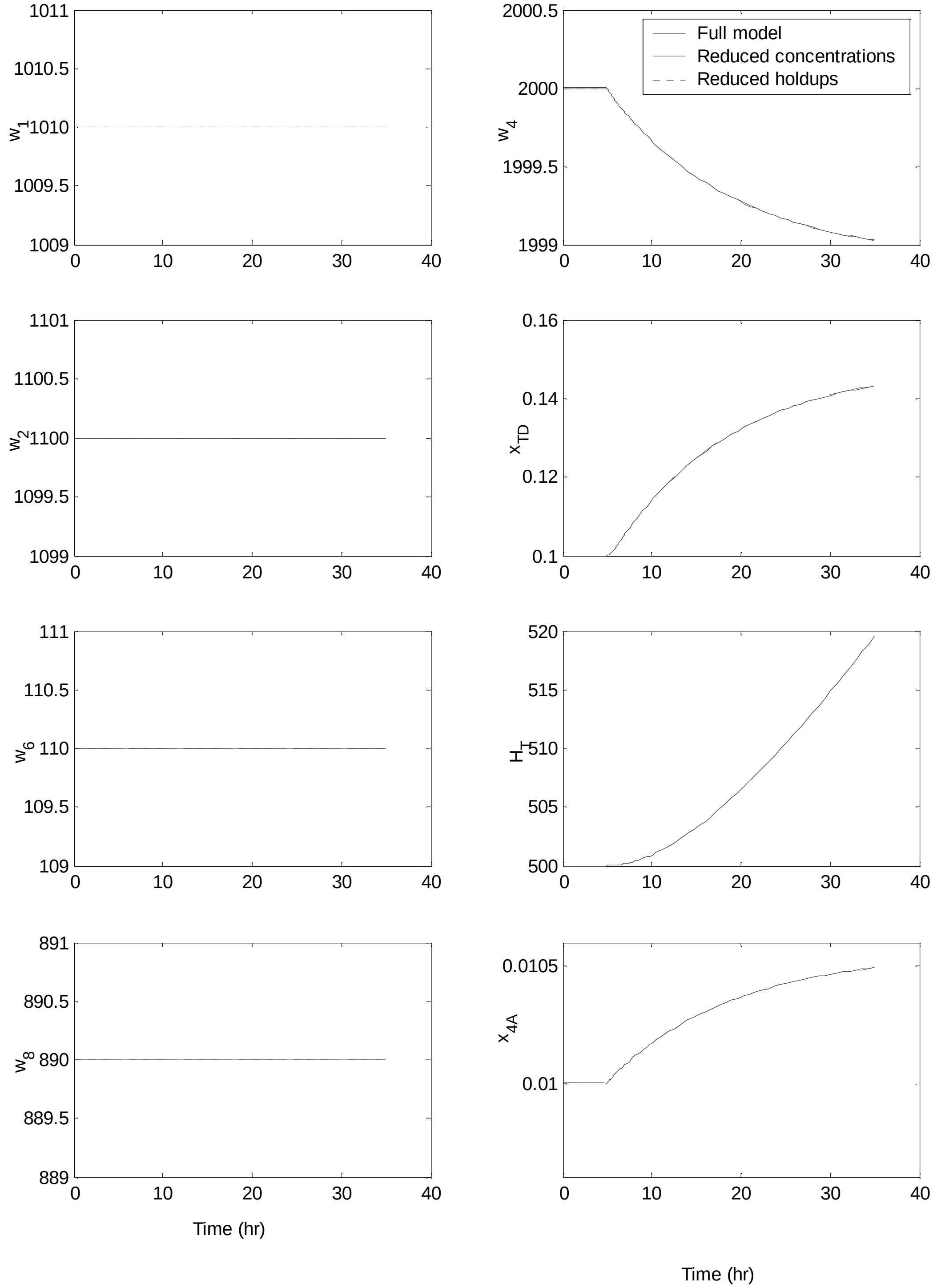

Figure S24.1k. *Step change in x_{2D} (+0.005) at t=5*

24.2

To obtain a steady state (SS) gain matrix through the use of simulation, step changes in the manipulated variables are made. The resulting matrix should compare closely with that found in Eq. 24.1 of the text (or the table below). The values calculated are:

Gain Matrix	w_1	w_2	w_6	w_8
w_4	1.93	2.26 E-2	0	6.25 E-3
x_{8D}	8.8 E-4	-7.62 E-4	0	5.68 E-6
x_{4A}	2.57 E-5	-1.14 E-5	0	-3.15 E-6
H_T	-0.918*	0.973*	-1*	-6.25 E-3*

* For integrating variables: "Gain" = the slope of the variable vs. time divided by the magnitude of the step change.

RGA	w_1	w_2	w_6	w_8
w_4	**0.9743**	0.0135	0	0.0122
x_{8D}	0	**0.9737**	0	0.0263
x_{4A}	0.0257	0.0128	0	**0.9615**
H_T	0	0	**1**	0

24.3

Controller parameters are given in Tables E.2.7 and E.2.8 in Appendix E of the text. A transfer function block is placed inside each control loop to slow down the fast algebraic equations, which otherwise yield large "output spikes". These blocks are of the form of a first-order filter.:

$$G_f(s) = \frac{1}{0.001s + 1}$$

In principle, ratio control can provide tighter control of all variables. However, it is clear from the x_{2D} results that it offers no advantage for this disturbance variable. For a step change in production rate, w_4, one would anticipate a different situation because a change in manipulated variable w_1 is induced. Using ratio control, w_2 does change along with w_1 to maintain a satisfactory ratio of the two feed streams. Thus, ratio control does provide enhanced control for the recycle tank level, H_T, and composition, x_{TD}, but not for the key performance variables, w_4 and x_{4A}. This characteristic is likely a result of the particular features of the recycle plant, namely the use of a splitter (instead of a flash unit) and the lack of holdup in that vessel.

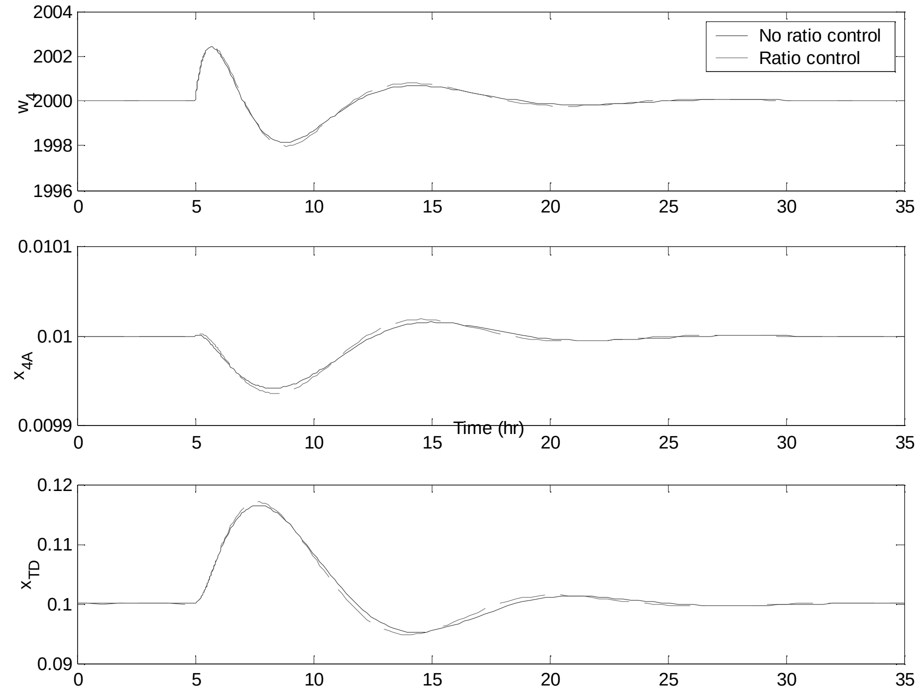

Figure S24.3a. *Step change in x_{2D} (+0.02) at t=5 (Corresponds to Fig.24.7)*

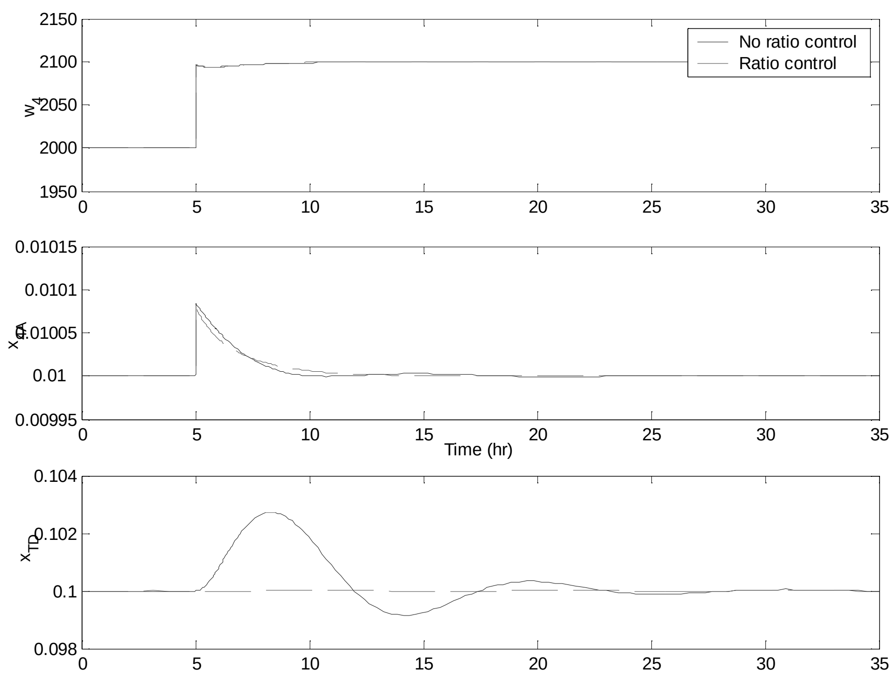

Figure S24.3b. *Step change in w_4 (+100) at t=5 (Corresponds to Fig.24.8)*

24.4

a) A simple modification of the controller pairing is needed.
The settings for the modified controller setup are:

Loop	Gain	Integral Time (hr)
x_{TD}-w_6	-5000	1
H_T-R	0.002	1
w_4-w_1	2	1
x_{4A}-w_8	-500000	1

(See Figure S24.4a)

b) The RGA shows that flowrate w_6 will not directly affect the composition of D in the recycle tank, x_{TD}, but the x_{TD}-w_6 loop will cause unwanted interaction with the other control loops. The system can be controlled, however, if the other three loops are tuned more conservatively and "assist" the x_{TD}-w_6 loop.

c) The manipulated variable, w_6, is the rate of purge flow. Purging a stream does not affect the compositions of its constituent species, only the total flowrate. Therefore, purging the stream before the recycle tank will only affect the level in the tank and not its compositions. The resulting RGA yields a zero gain between x_{TD} and w_6.

d) The RGA structure handles a positive 5% step change in the production rate well, as it maintains the plant within the specified limits. The setup with one open feedback loop defined by this exercise, however, goes out of control. The x_{TD}-w_6 loop requires the interaction of the other loops to maintain stability. When the x_{4A}-w_8 loop is broken, the system will no longer remain stable.

(See Figure S24.4b)

e) With a set point change of 10%, the controllers must be detuned to keep variables within operating constraints. The H_T-w_6 loop in the RGA structure must be more conservative (gain reduced to -1) to keep the purge flow, w_6, from hitting its lower constraint, zero. A 20% change will create a problem within the system that these control structures cannot handle. The new set point for w_4 does not allow a steady-state value of 0.01 for x_{4A}. This will make the x_{4A}-w_8 control loop become unstable. This outcome results for production rate step changes larger than roughly 12% (for this system).

(See Figure S24.4c)

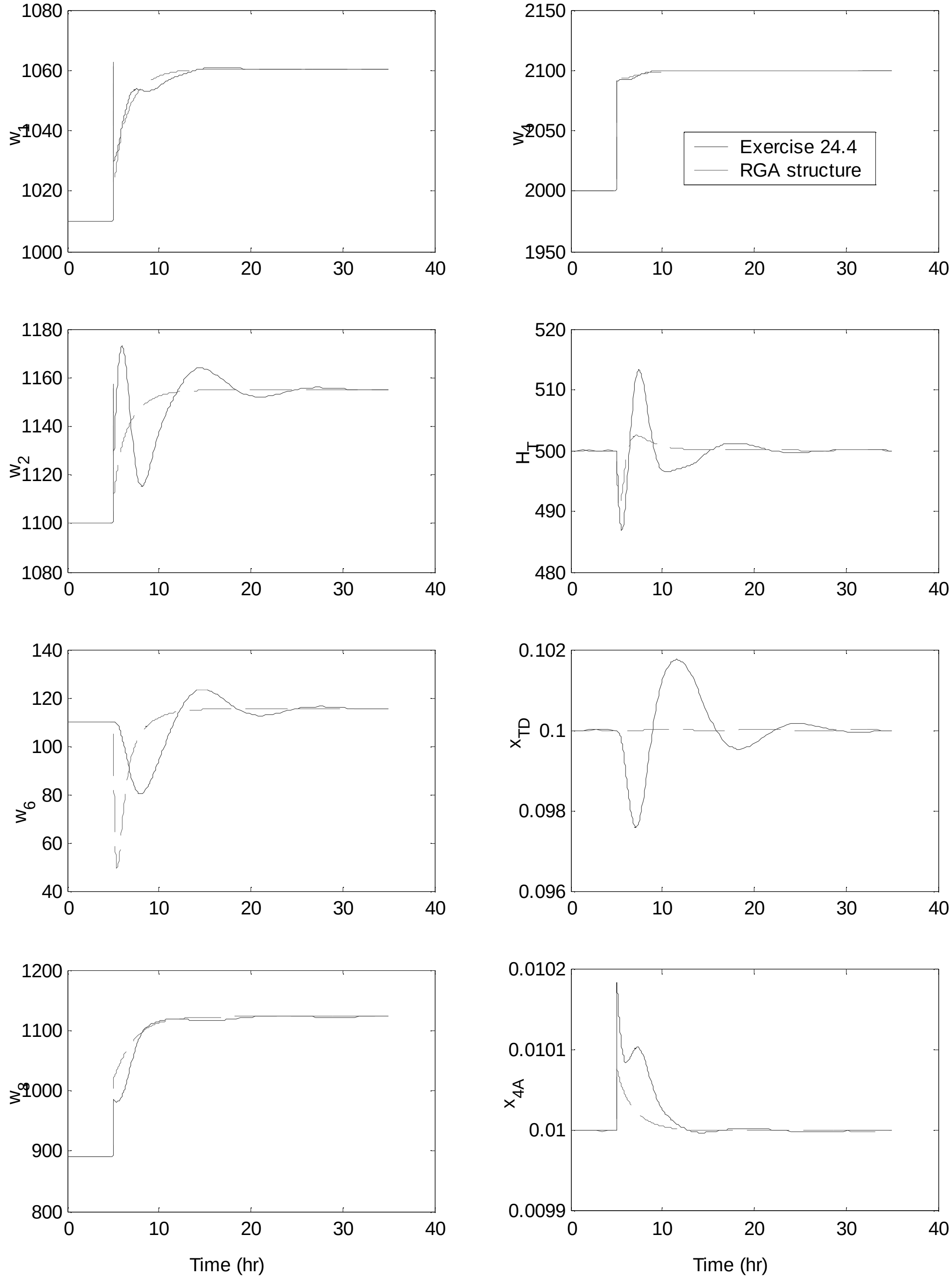

Figure S24.4a. *Step change in w_4 set point(+5%) at t=5*

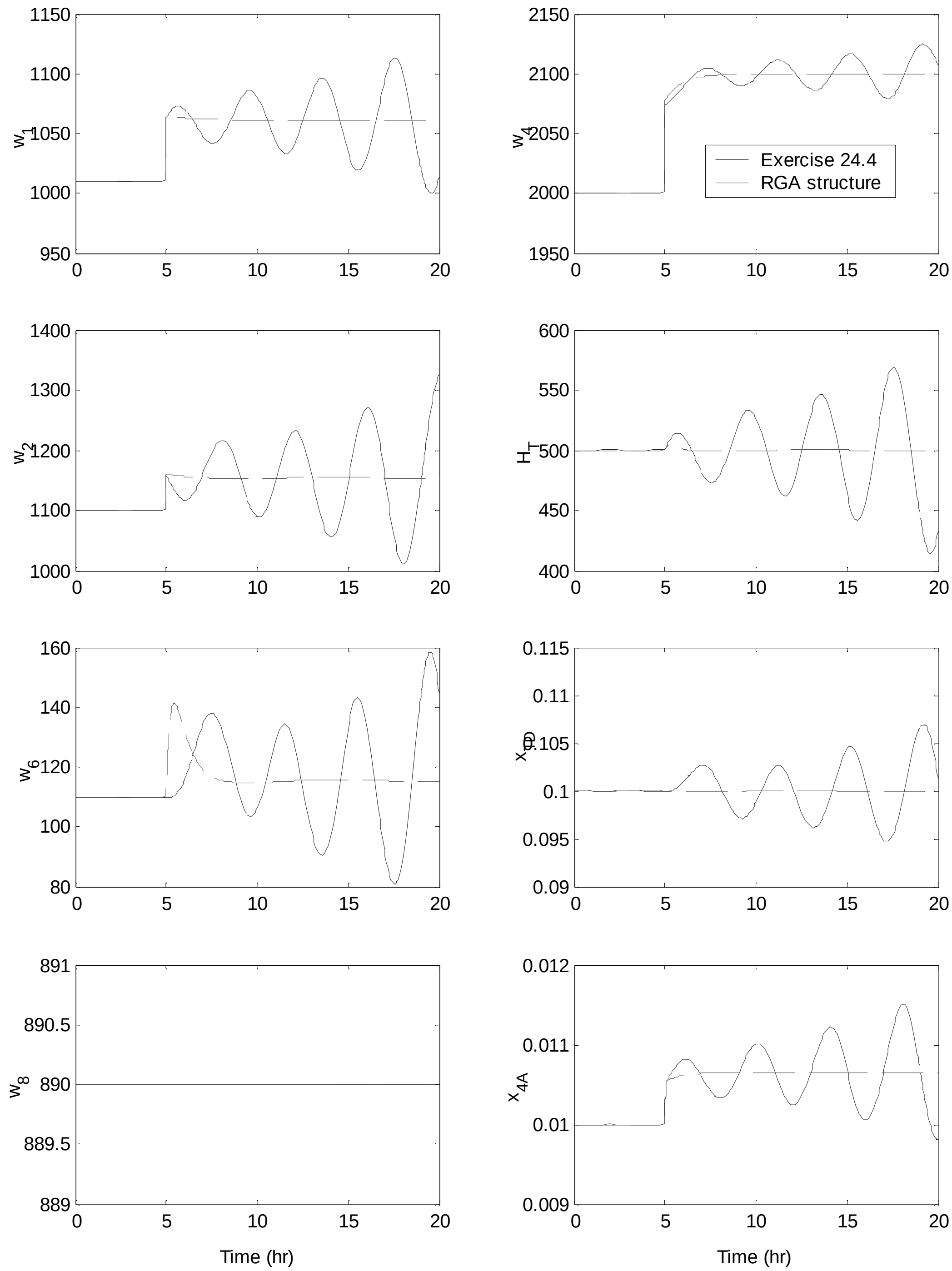

Figure S24.4b. *Step change in w_4 set point(+5%) at t=5 with one loop open.*

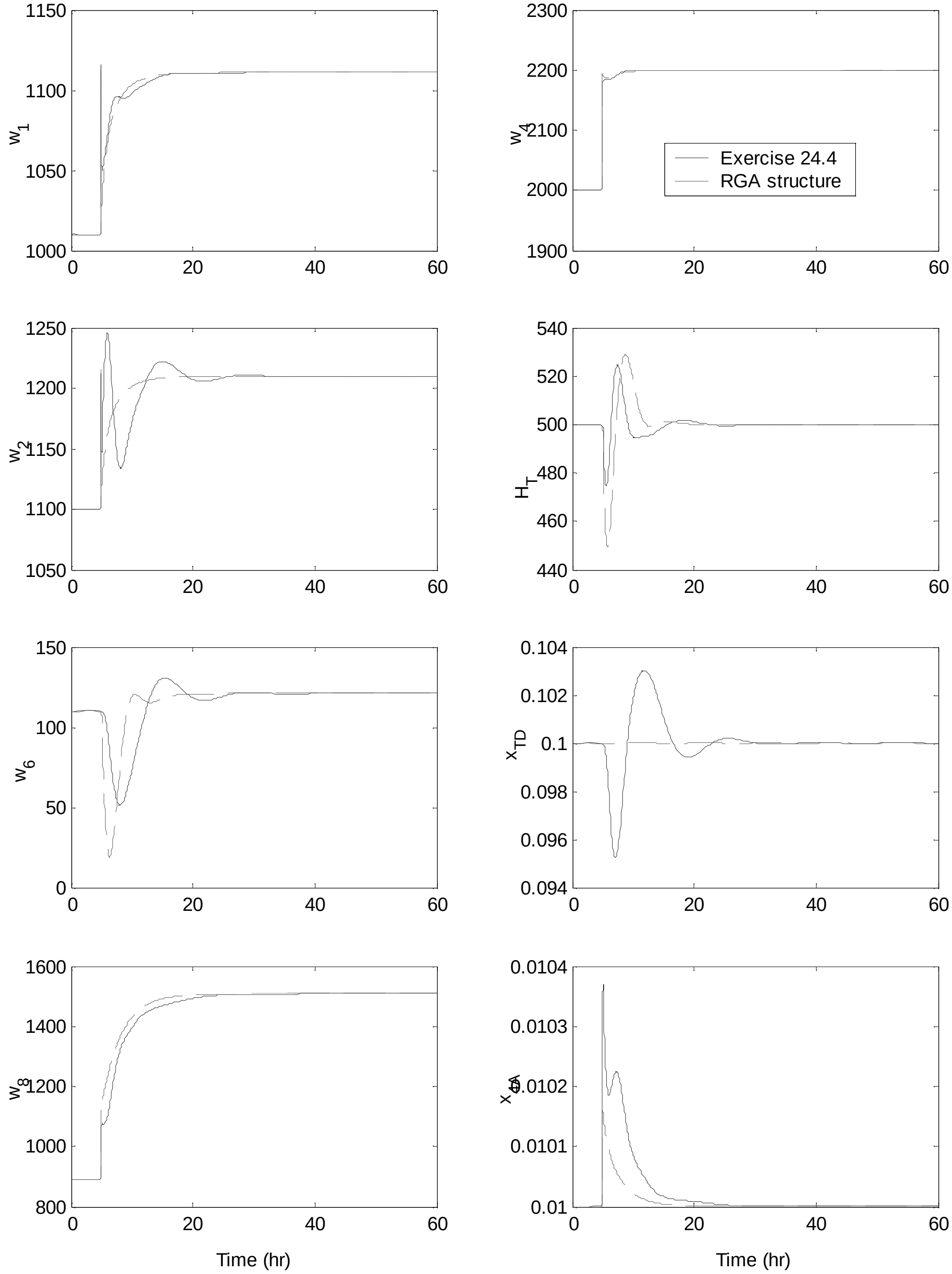

Figure S24.4c. *Step change in w_4 set point(+10%) at t=5 with H_T controller detuned*

24.5

a) Using the same methods as described in solution 24.3, the resulting gain matrix is:

Gain Matrix	w_1	w_2	w_6	w_8	w_3
w_4	5.762E-3	4.760E-3	0	5.831E-3	-1.285E-2
x_{8D}	5.554E-6	4.558E-6	0	5.445E-6	-9.130E-6
x_{4A}	-2.905E-6	-2.398E-6	0	-2.944E-6	6.542E-6
H_T	-2.137	-1.927	-1	-10.12	8.829
H_R	1	1	0	1	-1

All variables are integrating

The resulting RGA does not provide useful insight for the preferred controller pairing due to the nature of these integrating variables.

b) Results similar to those obtained in Exercise 24.3 can be obtained with an added loop for reactor level using the w_3 flow rate as the manipulated variable. Both P and PI controllers yield relatively constant reactor level. The quality variable, x_{4A}, cannot be controlled as tightly however. The responses with P-only control are only slightly different as compared to PI control, which means that zero-offset control on the reactor volume is not necessary for reliable plant operation.

Controller parameters used for variable reactor holdup simulation:

Loop	Gain (K_c)	Integral Time (τ_I)
w_4-w_1	1	1
x_{TD}-w_2	-6300	1
x_{4A}-w_8	-200000	1
H_T-w_6	-3.5	1
H_R-w_3	-10	1*

* For PI control

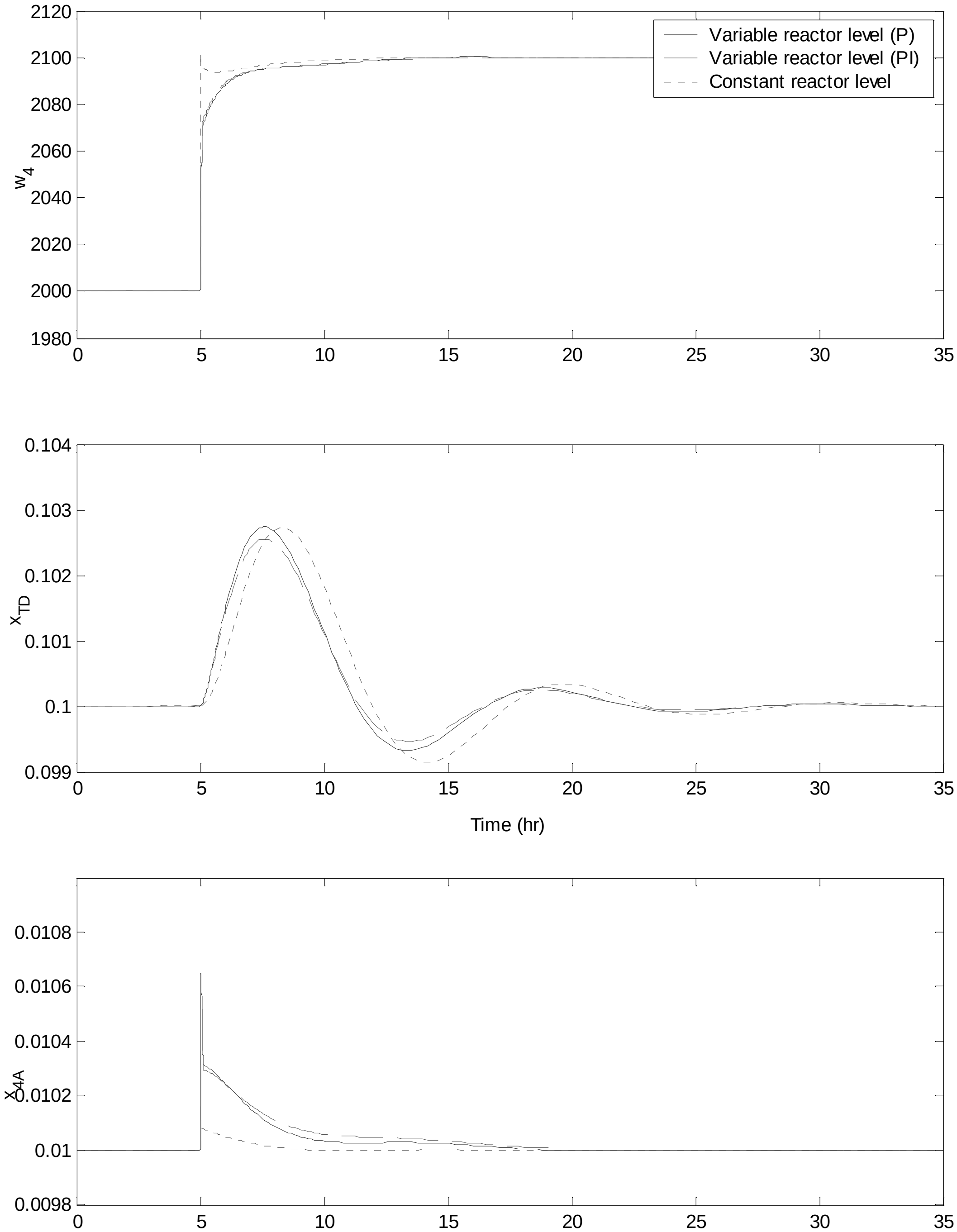

Figure S24.5a. *Step change in w_4 set point (+100) at t=5*

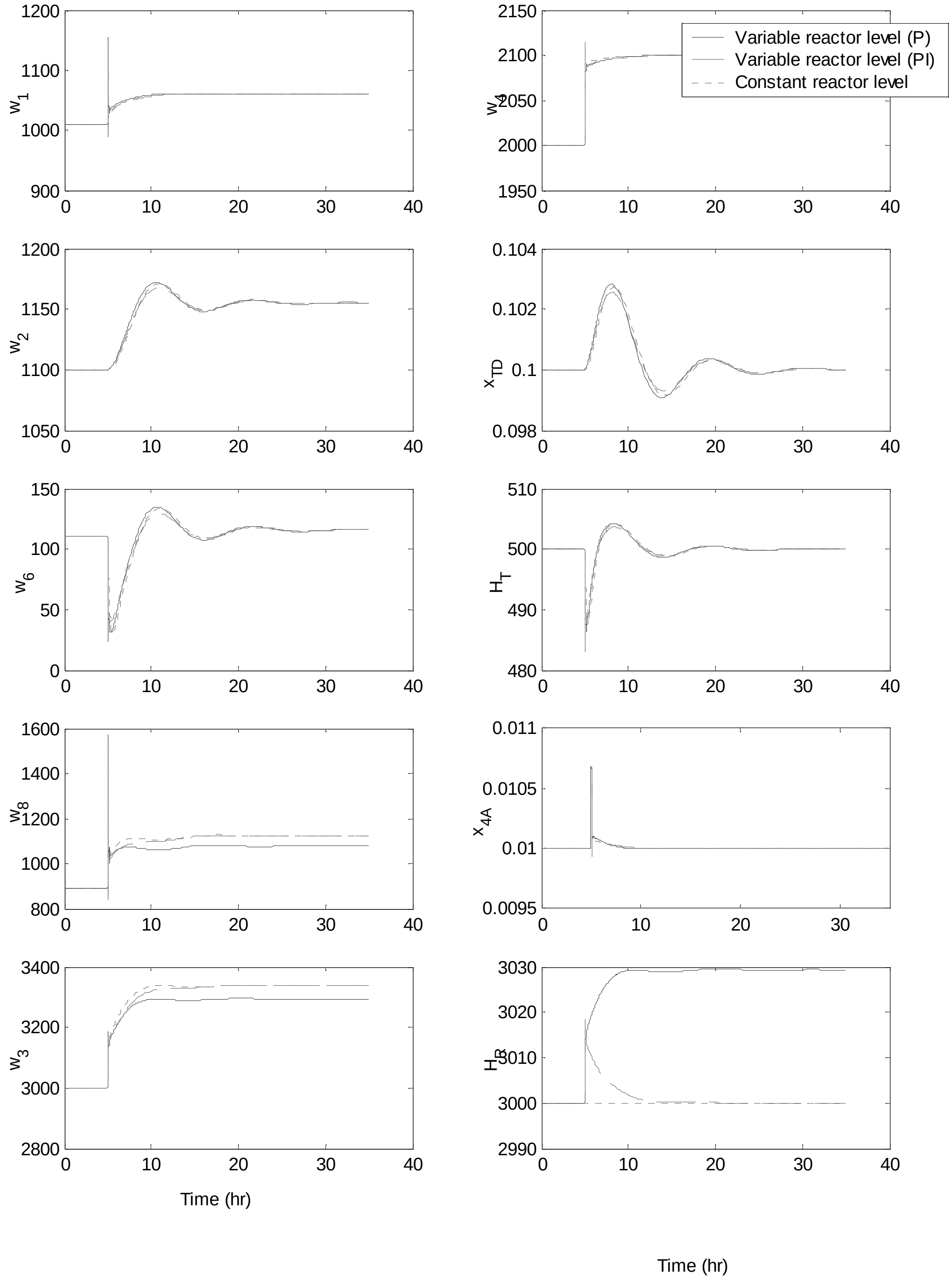

Figure S24.5b. *Step change in w_4 setpoint (+100) at t=5*

24.6

To simulate the flash/splitter with a non-negligible holdup, derive a mass balance around the unit. Assume that components A and C are well mixed and are held up in the flash for an average H_F/w_4 amount of time. Also assume that the vapor components B and D are passed through the splitter instantaneously.

$$\frac{dH_F}{dt} = w_3 - w_4 - w_5 = 0$$

$$\frac{d(H_F x_{FA})}{dt} = w_3 x_{3A} - w_4 x_{4A}$$

$$\frac{d(H_F x_C)}{dt} = w_3 x_{3C} - w_4 x_{4C}$$

Since the holdup is constant, the flows out of the splitter can be modeled as:

$$w_5 = w_3(x_{3B} + x_{3D})$$

$$w_4 = w_3 - w_5$$

Use the component balances and output flow equations to simulate the flash/splitter unit. This will add a dynamic lag to the unit which slows down the control loops that have the splitter in between the manipulated variable and the controlled variable. However, a 1000kg holdup only creates a residence time of 0.5 hr. Considering the time scale of the entire plant, this is very small and confirms the assumption of modeling the flash/splitter as having a negligible holdup.

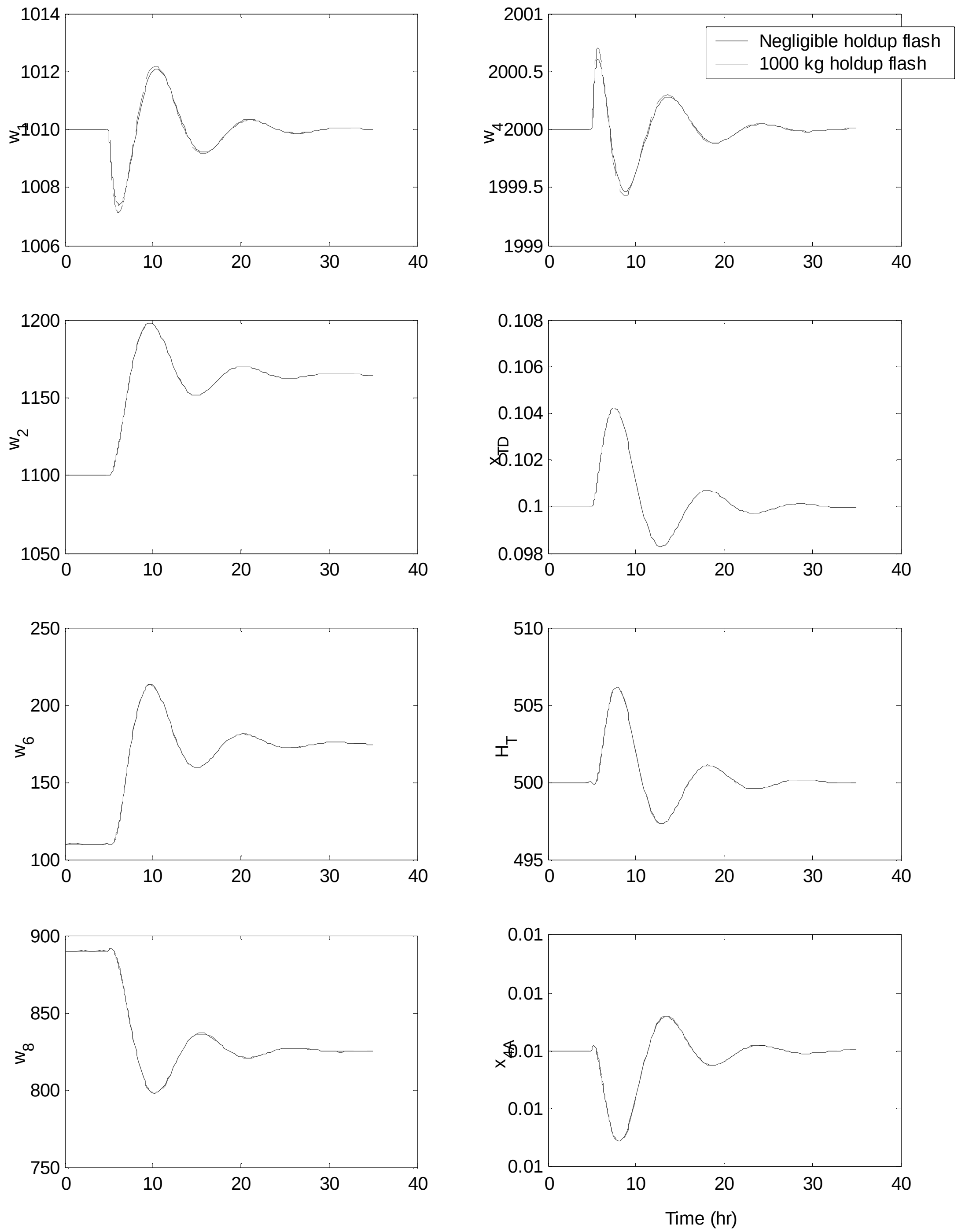

Figure S24.6. *Step change in x_{2D} (+0.005) at t=5*

24.7

The MPC controller achieves satisfactory results for step changes made within the plant. The production rate can easily be maintained within desirable limits and large set-point changes (20%) do not cause a breakdown in the quality of this stream. A change in the kinetic coefficient (*k*), occurring simultaneously with a 50% disturbance change will, however, initially draw the product quality (composition of the production stream) out of the required limits.

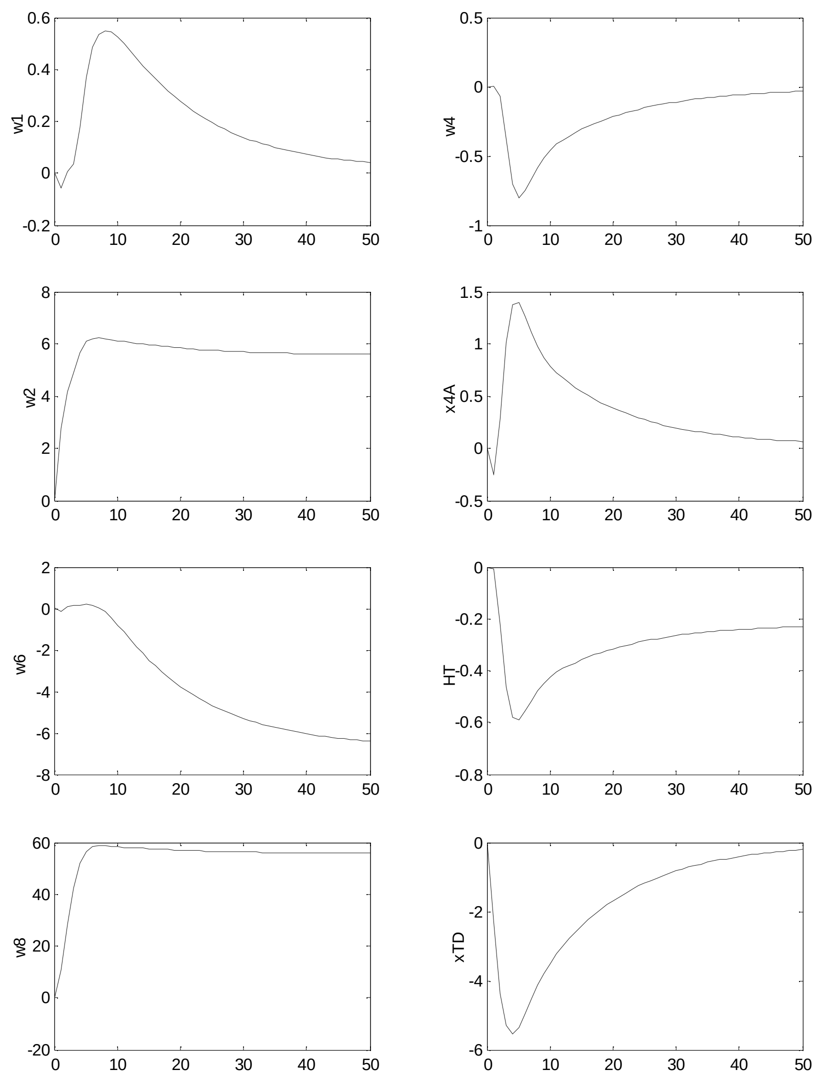

Figure S24.7a. *Step change in* x_{2D} *(+50%). All variables are recorded in percent deviation.*

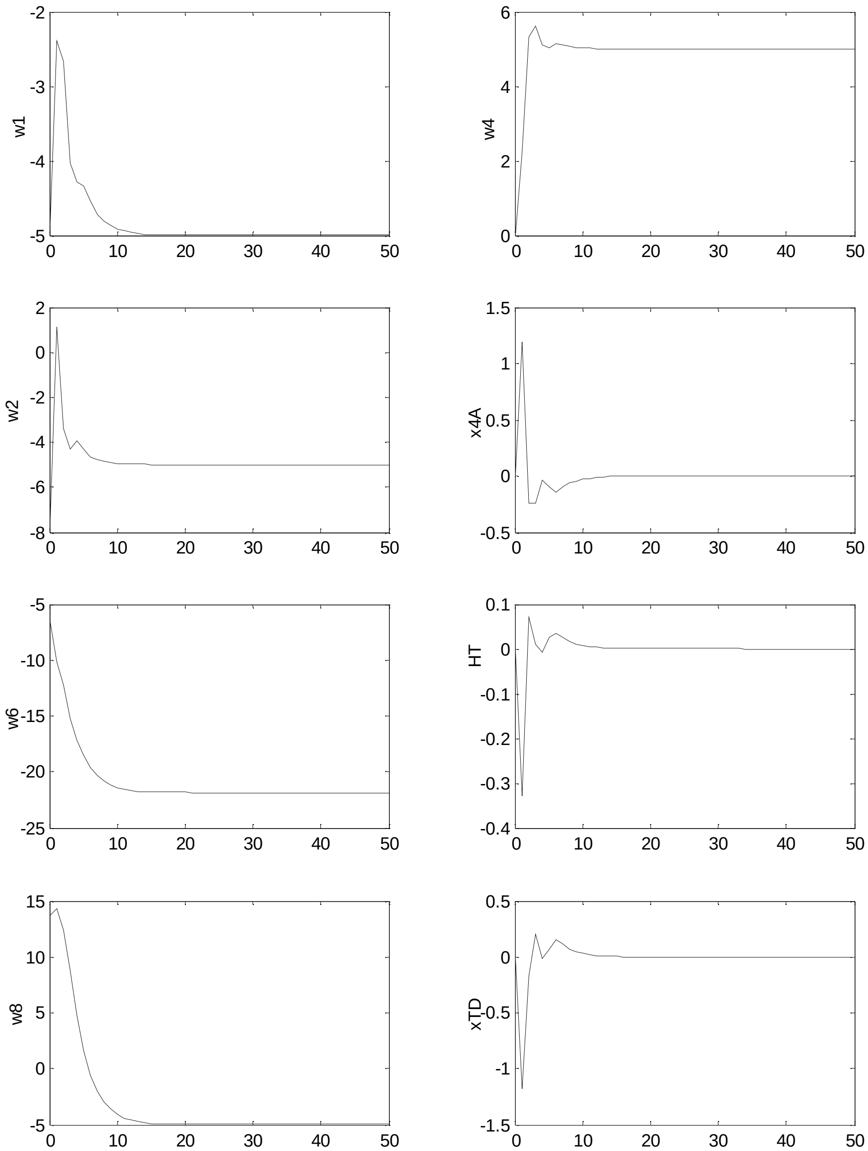

Figure S24.7b. *Step change in production rate w_4 (+5%). All variables are recorded in percent deviation.*

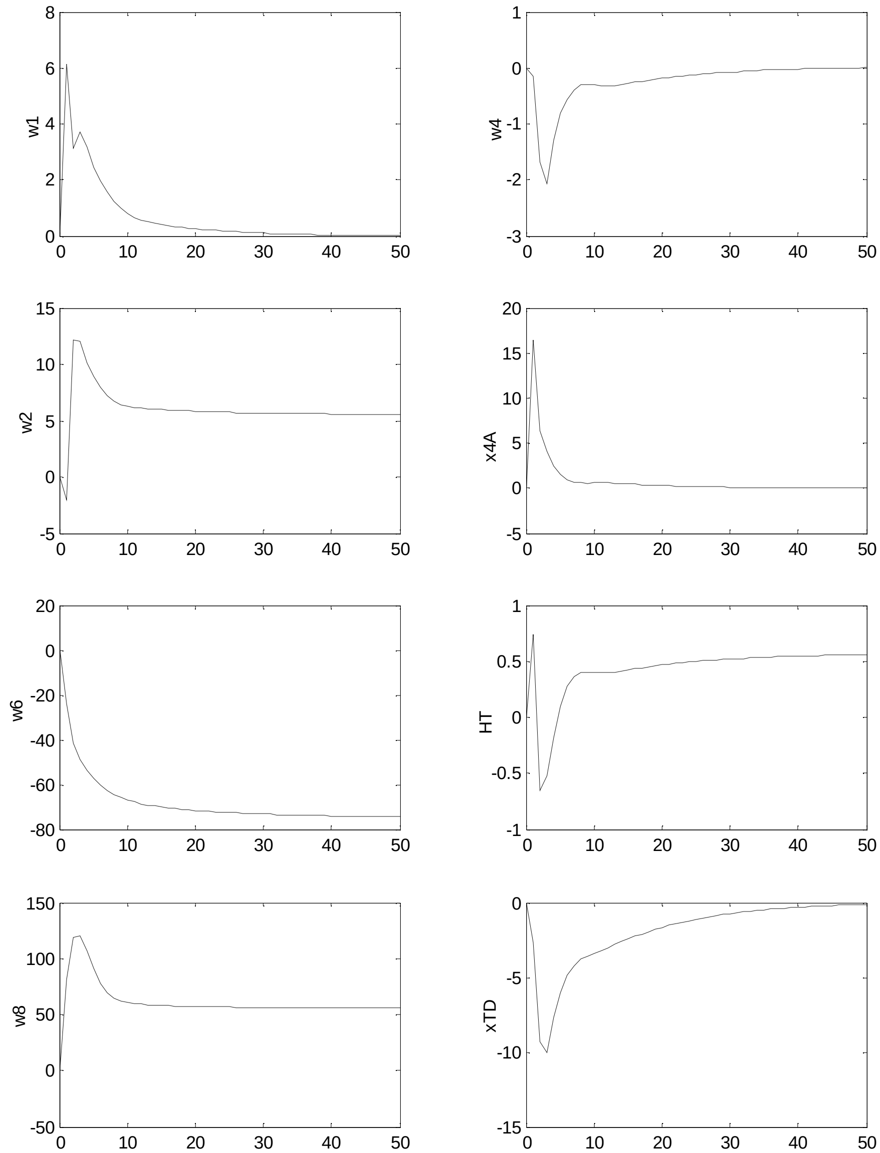

Figure S24.7c. *Simultaneous step changes in* x_{2D} *(+50%) and k(+20%). All variables are recorded in percent deviation.*

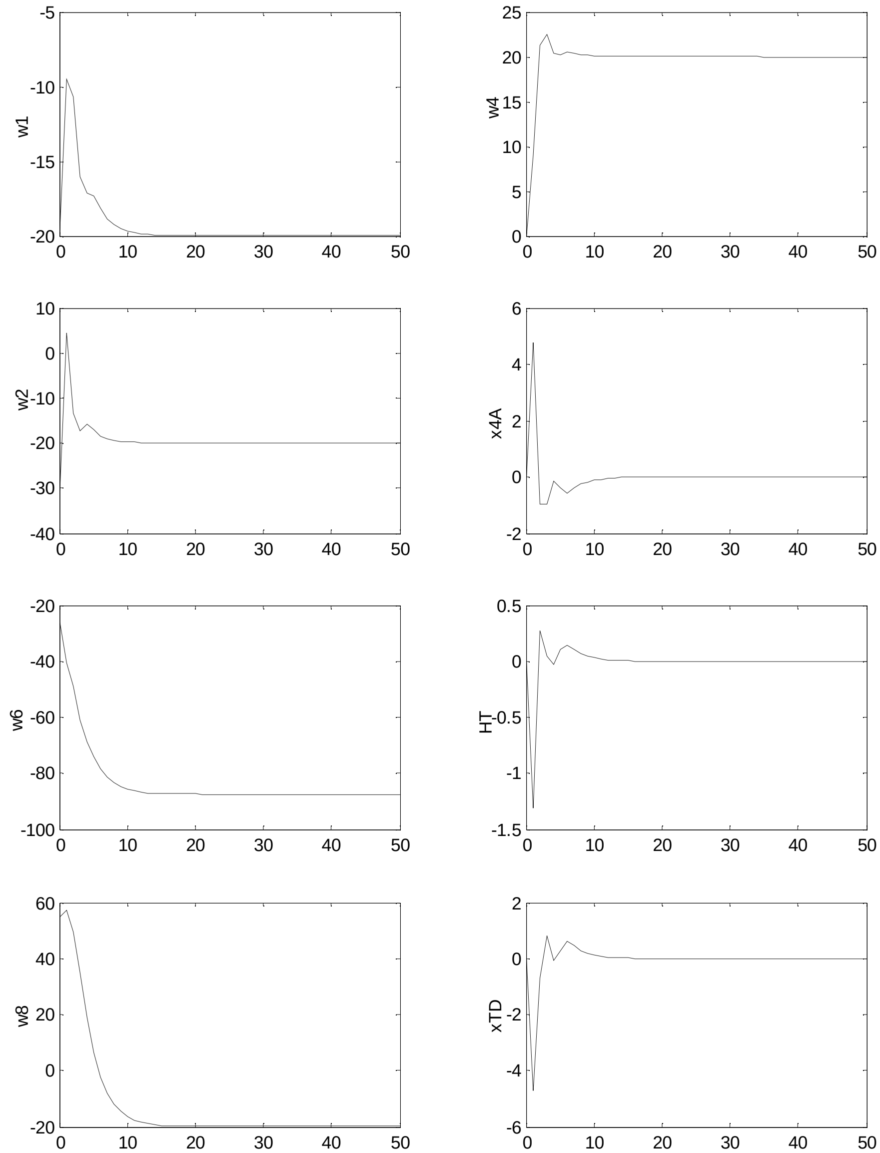

Figure S24.7d. *Step change in production rate w_4 (+20%). All variables are recorded in percent deviation.*