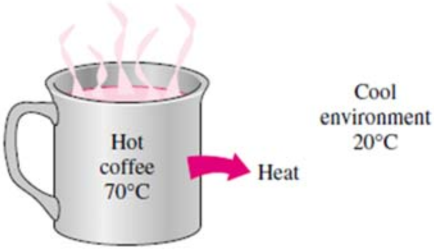
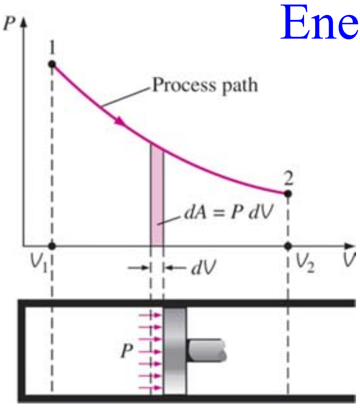
$$\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}$$

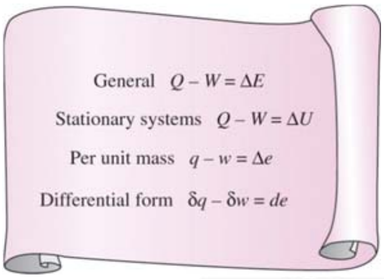
Thermodynamics I



Energy Analysis for Closed Systems



Dr.-Eng. Zayed Al-Hamamre


General $Q - W = \Delta E$
Stationary systems $Q - W = \Delta U$
Per unit mass $q - w = \Delta e$
Differential form $\delta q - \delta w = de$

Content

- Energy balance for Closed Systems
- Moving boundary work
  - ✓ W_b for an isothermal process
  - ✓ W_b for a constant-pressure process
  - ✓ W_b for a polytropic process
- Energy balance for closed systems
  - ✓ Energy balance for a constant-pressure expansion or compression process
- Specific heats
  - ✓ Constant-pressure specific heat, c_p
  - ✓ Constant-volume specific heat, c_v
- Internal energy, enthalpy, and specific heats of ideal gases
  - ✓ Specific heat relations of ideal gases
- Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)

The General Energy Balance for A System

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}} \quad (\text{kW})$$

Energy balance in the rate form

- For constant rate, the total quantities are related to the quantities per unit time is

$$Q = \dot{Q}\,\Delta t, \quad W = \dot{W}\,\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\,\Delta t \qquad (\text{kJ})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \qquad (\text{kJ/kg})$$

Energy balance per unit mass basis

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$$

Energy balance in differential form

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$$

Energy balance for a cycle

Energy Balance: Closed Systems

Change in amount of energy contained within the system during some time interval	=	Net amount of energy transferred ***in*** across the system boundary by heat transfer during the time interval	-	Net amount of energy transferred ***out*** across the system boundary by work during the time interval

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

$$\left\{ \begin{matrix} \textit{Initial} \\ \textit{Energy} \\ E_1 \end{matrix} \right\} \begin{matrix} + \\ - \end{matrix} \left\{ \begin{matrix} \textit{Energy gain } W_{1-2} \\ \textit{Energy loss } Q_{1-2} \end{matrix} \right\} = \left\{ \begin{matrix} \textit{Final} \\ \textit{Energy} \\ E_2 \end{matrix} \right\}$$

$$\boxed{E_1 + W_{1-2} - Q_{1-2} = E_2}$$

Energy Balance: Closed Systems

$$\Delta \text{KE} + \Delta \text{PE} + \Delta U = Q - W$$

Differential Form: $dE = \delta Q - \delta W$

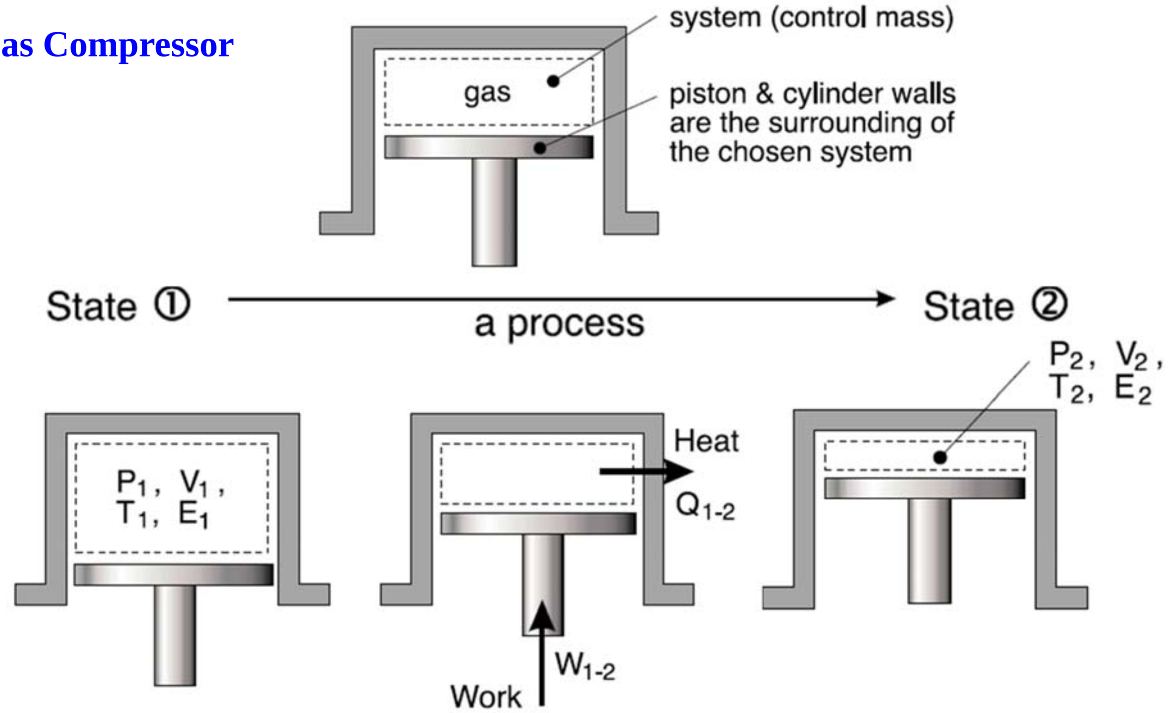
Time Rate Form: $\dfrac{dE}{dt} = \dot{Q} - \dot{W}$

$$\begin{bmatrix}\text{time } \textit{rate of change}\\ \text{of the energy}\\ \text{contained within}\\ \text{the system } \textit{at}\\ \textit{time } t\end{bmatrix} = \begin{bmatrix}\text{net } \textit{rate} \text{ at which}\\ \text{energy is being}\\ \text{transferred in}\\ \text{by heat transfer}\\ \textit{at time } t\end{bmatrix} - \begin{bmatrix}\text{net } \textit{rate} \text{ at which}\\ \text{energy is being}\\ \text{transferred out}\\ \text{by work } \textit{at}\\ \textit{time } t\end{bmatrix}$$

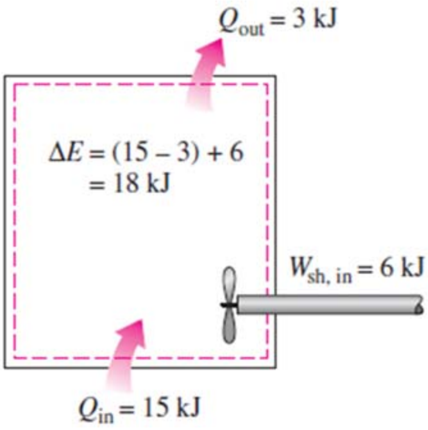
$$\longrightarrow \quad \frac{d\text{KE}}{dt} + \frac{d\text{PE}}{dt} + \frac{dU}{dt} = \dot{Q} - \dot{W}$$

Energy Balance: Closed Systems

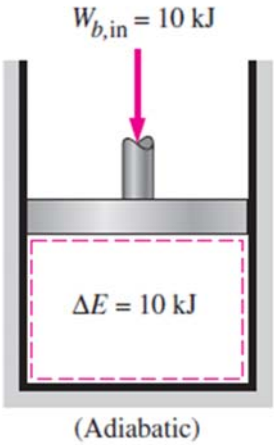
Gas Compressor



Energy Balance: Closed Systems



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings



The work (boundary) done on an adiabatic system is equal to the increase in the energy of the system

Cycle Energy Balance

- The energy balance for any system undergoing a thermodynamic cycle takes the form

$$\Delta E_{cycle} = Q_{cycle} - W_{cycle}$$

where Q_{cycle} and W_{cycle} represent *net* amounts of energy transfer by heat and work,

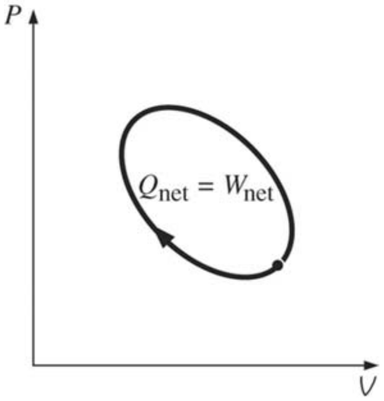
- For the cycle, since the system is returned to its initial state after the cycle, there is no *net* change in its energy

$$\Delta E = 0$$

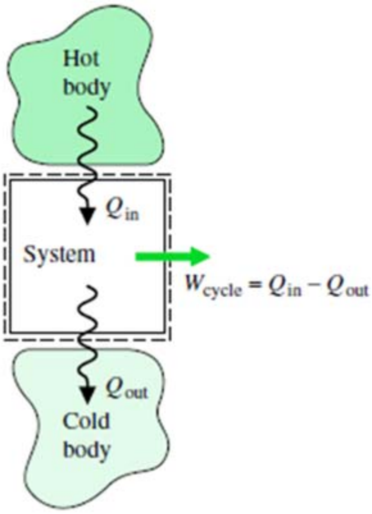
$$\longrightarrow \quad W_{cycle} = Q_{cycle}$$

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in} \qquad \text{(for a cycle)}$$

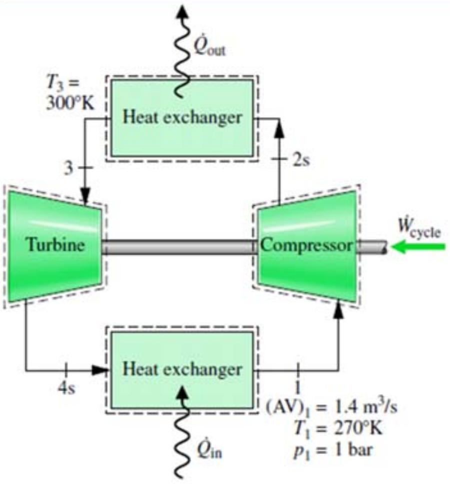
the net work output during a cycle is equal to net heat input

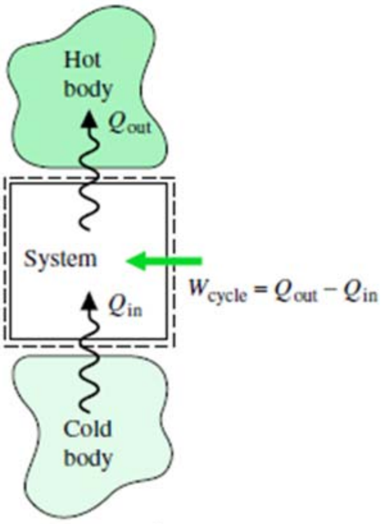


Cycle Energy Balance



Power cycles.





Refrigeration and heat pump cycles.

$W_{cycle} = Q_{in} - Q_{out}$ (power cycle)

$W_{cycle} = Q_{out} - Q_{in}$

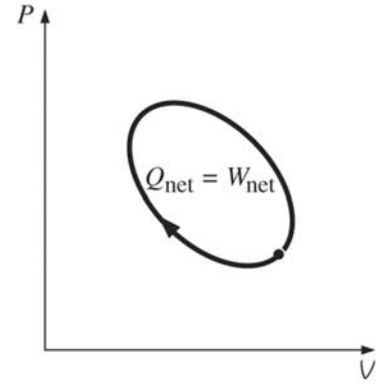
The General Energy Balance for A System

$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E$$

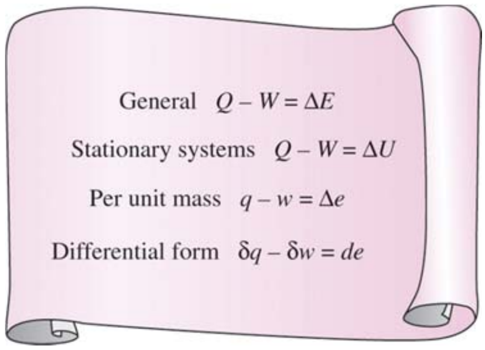
$$Q = Q_{net,in} = Q_{in} - Q_{out}$$

$$W = W_{net,out} = W_{out} - W_{in}$$

- Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).



For a cycle $\Delta E = 0$, thus $Q = W$.



General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

Various forms of the first-law relation for closed systems when sign convention is

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

Moving Boundary Work

- **Moving boundary work ($P\,dV$ work)**: The expansion and compression work in a piston-cylinder device.

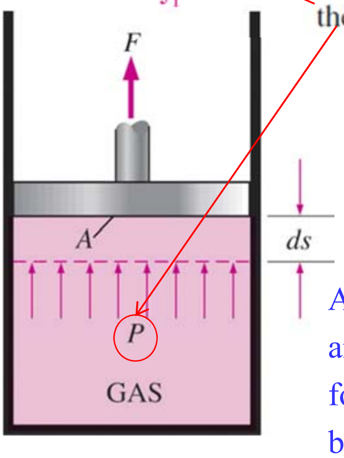
$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

$$W_b = \int_1^2 P\,dV \qquad (\text{kJ})$$

the pressure at the inner face of the piston.

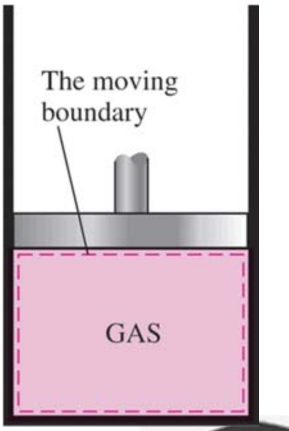
- **Quasi-equilibrium process**: A process during which the system remains nearly in equilibrium at all times.

- W_b is positive → for expansion
- W_b is negative → for compression

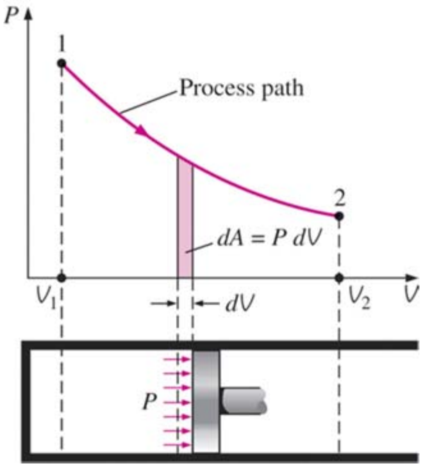


The work associated with a moving boundary is called *boundary work.*

A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount *ds*.



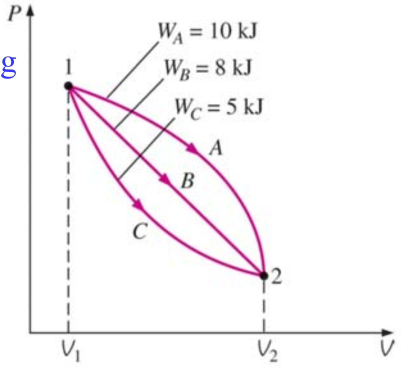
Moving Boundary Work



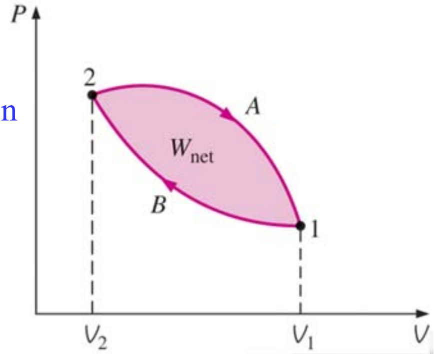
The area under the process curve on a P- V diagram represents the boundary

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV$$

The boundary work done during a process depends on the path followed as well as the end states.



The net work done during a cycle is the difference between the work done by the system and the work done on the system.



Polytropic, Isothermal, and Isobaric processes

$P = CV^{-n}$ Polytropic process: *C*, *n* (polytropic exponent) constants

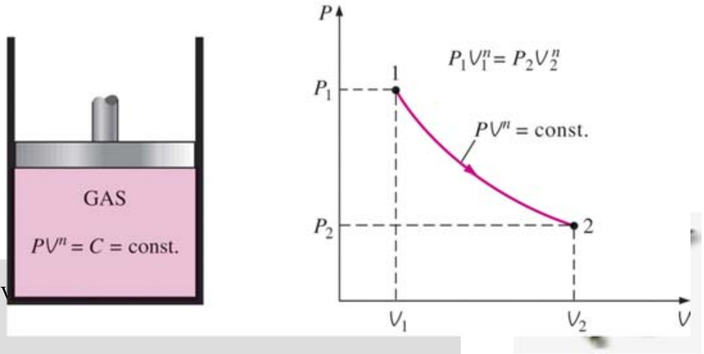
$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$ Polytropic process

$$W_b = \frac{mR(T_2 - T_1)}{1-n}$$ Polytropic and for ideal gas $n \neq 1$

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$ When $n = 1$ (isothermal process)

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$ Constant pressure process

What is the boundary work for a constant-volume process?



Schematic and *P*-*V* diagram for a polytropic process.

Example

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

Example

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320F. Heat is now transferred to the steam until the temperature reaches 400F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

Example

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

Example

A gas in a piston–cylinder assembly undergoes an expansion process for which the relationship between pressure and volume is given by

$$pV^n = constant$$

The initial pressure is 3 bar, the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. Determine the work for the process, in kJ, if (a) $n = 1.5$, (b) $n = 1.0$, and (c) $n = 0$.

Example Cont.

Example

Summary

For a closed system:

$$PV^n = \text{constant}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$$

Expansion/Compression (Moving Boundary) Work (Ideal Gas OR liquid):

$$\int_1^2 P \cdot dV = \frac{P_2V_2 - P_1V_1}{1-n}, (n \neq 1)$$

$$\int_1^2 P \cdot dV = P_1V_1 \ln\left(\frac{V_2}{V_1}\right), (n = 1)$$

Ideal Gases ONLY:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} = \left(\frac{V_1}{V_2}\right)^{(n-1)}$$

$$\int_1^2 P \cdot dV = \frac{mR(T_2 - T_1)}{1-n}, (n \neq 1)$$

$$\int_1^2 P \cdot dV = mRT \ln\left(\frac{V_2}{V_1}\right)$$

Energy balance for a constant-pressure expansion or compression process

- General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. Q is *to* the system and W is *from* the system.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \Delta \cancel{KE}^{\,0} + \Delta \cancel{PE}^{\,0}$$

$$Q - W_{other} - W_b = U_2 - U_1$$

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{other} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{other} = H_2 - H_1$$

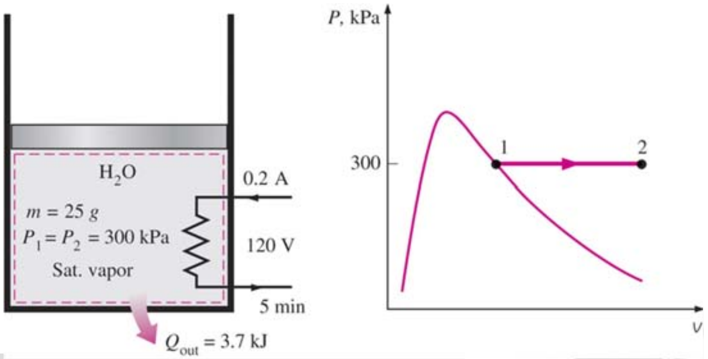
For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e,in} - Q_{out} - W_b = \Delta U$$

$$W_{e,in} - Q_{out} = \Delta H = m(h_2 - h_1)$$



Constant-Volume and Constant-Pressure Processes

The energy balance for a homogeneous closed system of n moles :

$$d(nU) = dQ + dW$$

The work of a mechanically reversible, closed-system process is

$$dW = -P\,d(nV)$$

$$\Rightarrow \quad d(nU) = dQ - P\,d(nV) \qquad dQ = d(nU) + P\,d(nV)$$

the general first-law equation for a mechanically reversible, closed-system process.

If the process occurs at constant total volume, the work is zero.

Since for closed systems, n is also constant

$$d(nU) = dQ - \overset{\text{zero}}{\cancel{P\,d(nV)}}$$

$$dQ = d(nU) \qquad \text{(const V)} \qquad\qquad Q = n\,\Delta U \qquad \text{(const V)}$$

Constant-Volume and Constant-Pressure Processes

For a constant-pressure change of state:

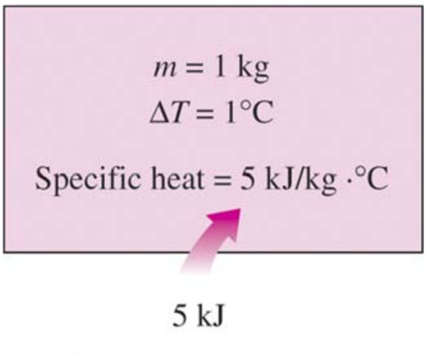
$$dQ = d(nU) + d(nPV) = d[n(U + PV)]$$

Since $\boxed{H \equiv U + PV}$ $\qquad dH = dU + d(PV)$

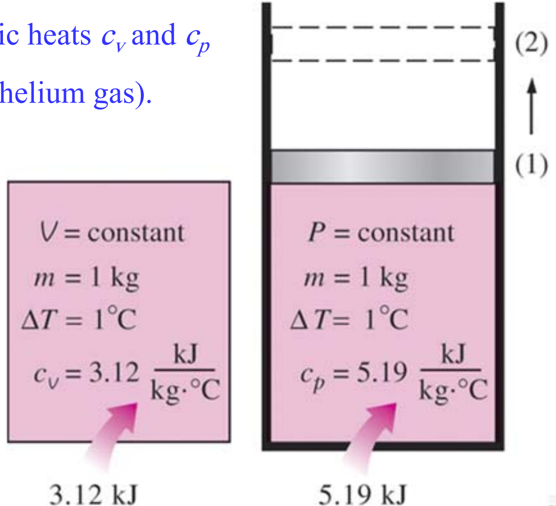
$$dQ = d(nH) \qquad (\text{const } P) \qquad\qquad Q = n\,\Delta H \qquad (\text{const } P)$$

Specific Heat

- **Specific heat at constant volume, c_v**: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.
- **Specific heat at constant pressure, c_p**: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



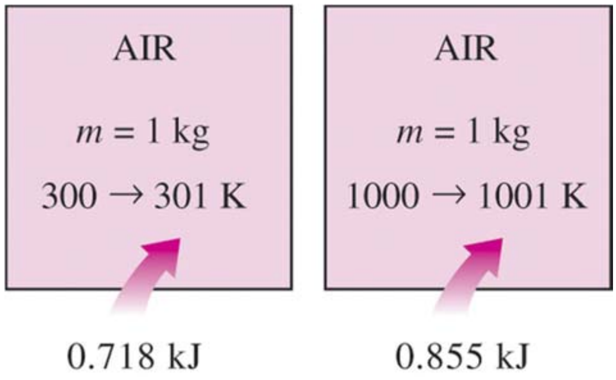
Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).

Specific Heat

- The equations in the figure are valid for *any* substance undergoing *any* process.
- c_v and c_p are properties.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- A common unit for specific heats is kJ/kg · °C or kJ/kg · K. **Are these units identical?**



The specific heat of a substance changes with temperature.

True or False?

c_p is always greater than c_v.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

= the change in internal energy with temperature at constant volume

Formal definitions of c_v and c_p.

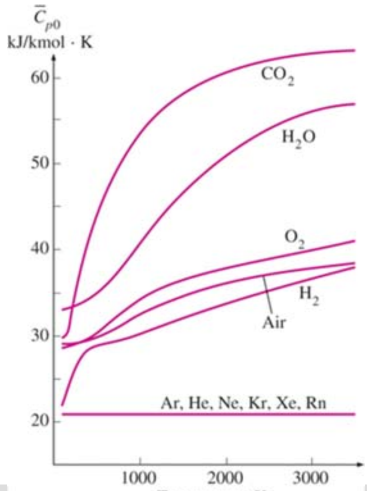
$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

---

U, H and Specific Heat of Ideal Gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.



- Ideal-gas constant-pressure specific heats for some gases (see Table A–2*c* for c_p equations).

- *u* and *h* data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

AIR

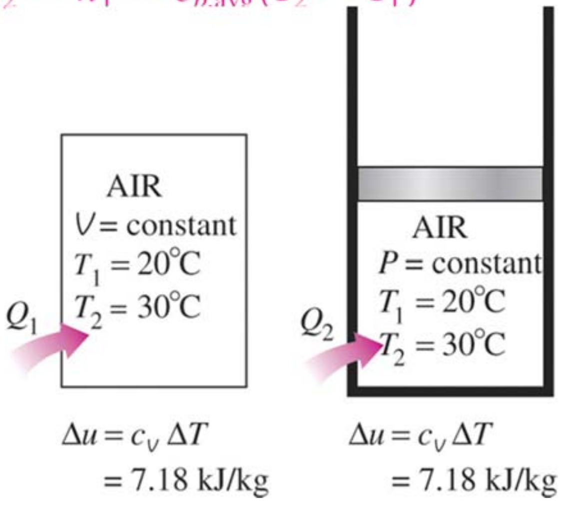
T, K	u, kJ/kg	h, kJ/kg
0	0	0
·	·	·
·	·	·
300	214.07	300.19
310	221.25	310.24
·	·	·
·	·	·

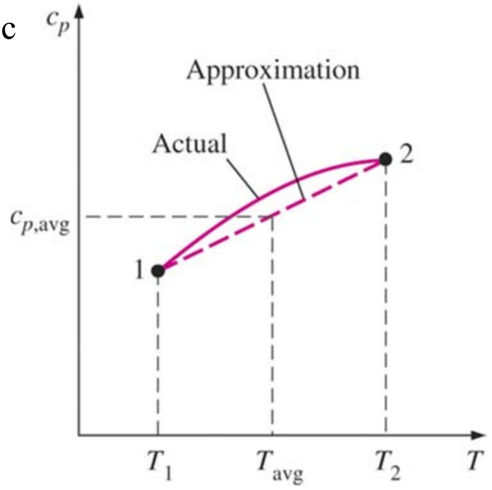
- In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

U, H and Specific Heat of Ideal Gas

- Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_2 - u_1 = c_{v,\mathrm{avg}}(T_2 - T_1)$$
$$h_2 - h_1 = c_{p,\mathrm{avg}}(T_2 - T_1) \quad \text{(kJ/kg)}$$

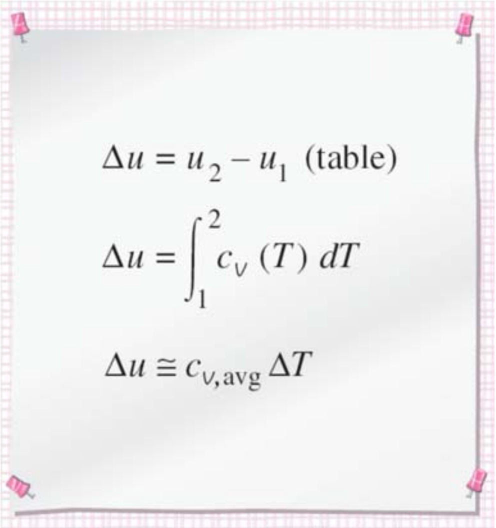




- For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.
- The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not.

Three Ways of Calculating Δ*u* and Δ*h*

1. By using the tabulated *u* and *h* data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.



$$\Delta u = u_2 - u_1 \ \text{(table)}$$
$$\Delta u = \int_1^2 c_v(T)\, dT$$
$$\Delta u \cong c_{v,\mathrm{avg}} \Delta T$$

Three ways of calculating Δ*u*.

Specific Heat Relations of Ideal Gases

$$h = u + RT,$$
$$dh = du + R\,dT$$
$$dh = c_p dT \text{ and } du = c_v dT$$

$\rightarrow$ The relationship between c_p, c_v and R

$$c_p = c_v + R \qquad (\text{kJ/kg}\cdot\text{K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\text{kJ/kmol}\cdot\text{K})$$

$$k = \frac{c_p}{c_v} \quad \text{Specific heat ratio} = 1 + \frac{R}{C_V}$$

AIR at 300 K

$$\left.\begin{array}{l} c_v = 0.718 \text{ kJ/kg}\cdot\text{K} \\ R = 0.287 \text{ kJ/kg}\cdot\text{K} \end{array}\right\} c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$$

or

$$\left.\begin{array}{l} \bar{c}_v = 20.80 \text{ kJ/kmol}\cdot\text{K} \\ R_u = 8.314 \text{ kJ/kmol}\cdot\text{K} \end{array}\right\} \bar{c}_p = 29.114 \text{ kJ/kmol}\cdot\text{K}$$

- The c_p of an ideal gas can be determined from a knowledge of c_v and R.

- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

Equations for Process Calculations: Ideal Gases

- For **an ideal gas** in any **mechanically reversible closed-system process**

$$dQ + dW = C_V\,dT$$

Since $dW = -P\,dV$

$$dQ = C_V\,dT + P\,dV$$

$$dQ = C_V\,dT + RT\frac{dV}{V}$$

$$dW = -RT\frac{dV}{V}$$

Alternatively, let $V = RT/P$:

$$dQ = C_V\,dT + P\left(\frac{R}{P}dT - \frac{RT}{P^2}dP\right)$$

Equations for Process Calculations: Ideal Gases

But
$$C_P = \frac{dH}{dT} = \frac{dU}{dT} + R = C_V + R$$

$$dQ = C_P\,dT - RT\frac{dP}{P}$$

$$dW = -R\,dT + RT\frac{dP}{P}$$

let $T = PV/R$:

$$dQ = C_V\left(\frac{V}{R}dP + \frac{P}{R}dV\right) + P\,dV$$

$$dQ = \frac{C_V}{R}V\,dP + \frac{C_P}{R}P\,dV$$

Isothermal Process

$$\Delta U = \Delta H = 0$$

$$Q = \mathrm{RT}\ln\frac{V_2}{V_1} = -RT\ln\frac{P_2}{P_1}$$

$$W = -RT\ln\frac{V_2}{V_1} = \mathrm{RT}\ln\frac{P_2}{P_1}$$

$Q = -W,$

$$Q = -W = \mathrm{RT}\ln\frac{V_2}{V_1} = -RT\ln\frac{P_2}{P_1} \qquad (\text{const } T)$$

Isobaric Process

$$\Delta U = \int C_V \, dT \qquad \text{and} \qquad \Delta H = \int C_P \, dT$$

$$Q = \int C_P \, dT \qquad \text{and} \qquad W = -R(T_2 - T_1)$$

$$Q = AH$$

$$Q = \Delta H = \int C_P \, dT \qquad (\text{const } P)$$

Isochoric (Constant- V) Process

$$\Delta U = \int C_V \, dT \qquad \text{and} \qquad \Delta H = \int C_P \, dT$$

$$Q = \int C_V \, dT \qquad \text{and} \qquad W = O$$

$$Q = \Delta U = \int C_V \, dT \qquad (\text{const } V)$$

Adiabatic Process: Constant Heat Capacities

- An adiabatic process is one for which there is no heat transfer between the system and its surroundings

$$dQ = 0.$$

$$\frac{dT}{T} = -\frac{R}{C_V}\frac{dV}{V}$$

Integration with C_V constant then gives:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_V}$$

$$\frac{T_2}{T_1} - \left(\frac{P_2}{P_1}\right)^{R/C_P} \quad \text{and} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{C_P/C_V}$$

Adiabatic Process: Constant Heat Capacities

$$TV^{\gamma-1} = \text{constant}$$

$$TP^{(1-\gamma)/\gamma} = \text{constant}$$

$$PV^{\gamma} = \text{constant}$$

$$\gamma \equiv \frac{C_P}{C_V}$$

- Apply to an ideal gas with constant heat capacities undergoing a mechanically reversible adiabatic process

The work of an adiabatic process

$$dW = dU = C_V\,dT$$

$$W = \Delta U = C_V\,\Delta T$$

Adiabatic Process: Constant Heat Capacities

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

Whence, $$C_V = \frac{R}{\gamma - 1}$$

Therefore $$W = C_V\,\Delta T = \frac{R\,\Delta T}{\gamma - 1}$$

Since $RT_1 = P_1V_1$ and $RT_2 = P_2V_2$, this expression may be written:

$$W = \frac{RT_2 - RT_1}{\gamma - 1} = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

V_2 is usually not known, and is eliminated

$$W = \frac{P_1V_1}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] = \frac{RT_1}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

Polytropic Process: Constant Heat Capacities

$$PV^{\delta} = \text{constant}$$

$$TV^{\delta-1} = \text{constant}$$

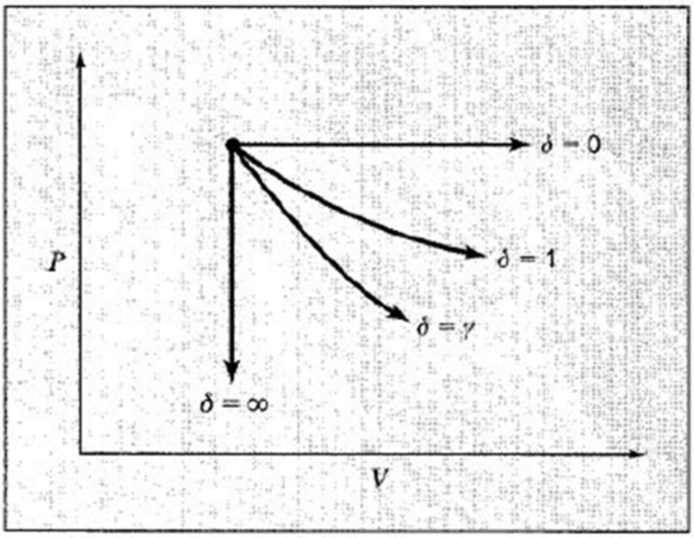
$$TP^{(1-\delta)/\delta} = \text{constant}$$

$$W = \frac{RT_1}{\delta - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\delta-1)/\delta} - 1\right]$$

$$Q = \frac{(\delta - \gamma)RT_1}{(\delta - 1)(\gamma - 1)}\left[\left(\frac{P_2}{P_1}\right)^{(\delta-1)/\delta} - 1\right]$$

Polytropic Process

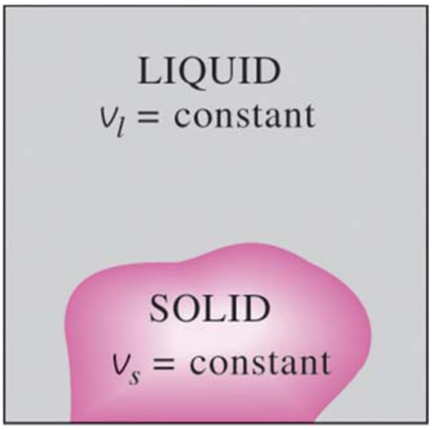
- Isobaric process: δ = 0.
- Isothermal process:, δ = 1.
- Adiabatic process: δ = *γ*.
- Isochoric process: dV/dP = V/Pδ; for constant V, δ = ±∞



Paths of polytropic processes characterized by specific values of δ

U, H and Specific Heat of Solids and Liquids

- **Incompressible substance**: A substance whose specific volume (or density) is constant. Solids and liquids are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.

The c_v and c_p values of incompressible substances are identical and are denoted by *c*.

Internal Energy Changes

$$du = c_v\, dT = c(T)\, dT \qquad \Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad \text{(kJ/kg)}$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \qquad \text{(kJ/kg)}$$

Enthalpy Changes

$$h = u + Pv \qquad dh = du + v\,dP + \cancelto{0}{P\,dv} = du + v\,dP$$

$$\Delta h = \Delta u + v\,\Delta P \cong c_{\text{avg}}\,\Delta T + v\,\Delta P \qquad \text{(kJ/kg)}$$

For *solids*, the term $v\,\Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v\,\Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T})$$

The enthalpy of a compressed liquid

- A more accurate relation than $h_{@P,T} \cong h_{f@T}$

Approximations for liquids and solids

Using Saturated Liquid Data
(‘Compressed Liquid Rule’)

$$v \approx v_f$$
$$h \approx h_f$$
$$u \approx u_f$$
$$s \approx s_f$$

- Using ‘Incompressible Substance Model’

$$c_p = c_v = c$$
$$u_2 - u_1 = c(T_2 - T_1)$$
$$h_2 - h_1 = c(T_2 - T_1) + \underline{v(P_2 - P_1)}$$
$$h_2 - h_1 \approx c(T_2 - T_1)$$

- Generalized equations for the estimation of molar volumes of *saturated* liquids (Rackett equ.)

$$V^{\text{sat}} = V_c Z_c^{(1-T_r)^{0.2857}}$$