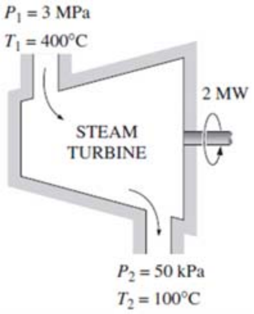
University Of Jordan

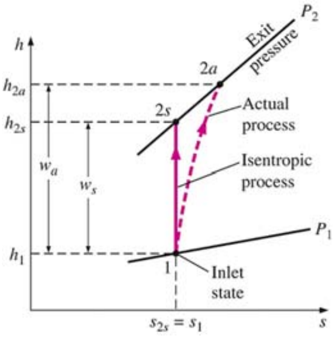
Faculty of Engineering and Technology
Department of Chemical Engineering



Thermodynamics I

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Entropy Analysis for Open Systems



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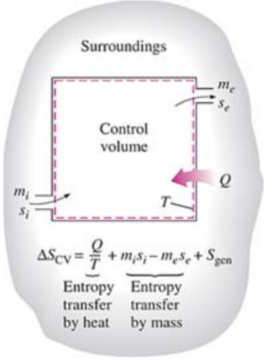
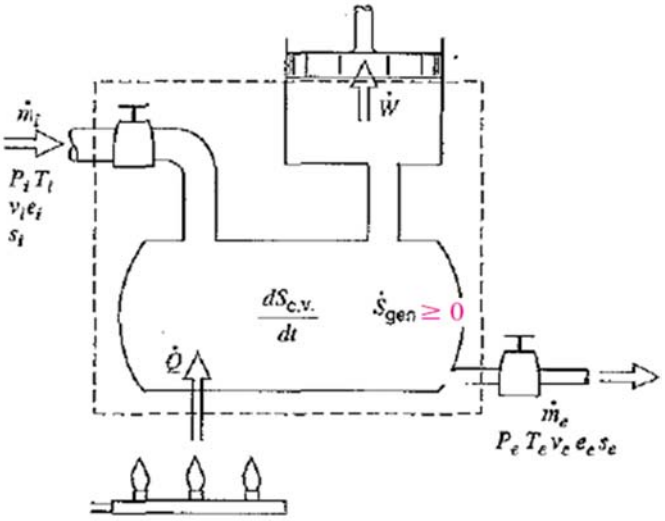


Content

- Entropy balance for Open Systems
- Reversible steady-flow work (Ideal work)
- Reversible steady-Processes
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices

Entropy Balance for Open Systems

- *Entropy* is *not conserved.*
- The second law states that the *total* entropy change associated with any process must be positive, with a limiting value of zero for a reversible process.
- This requirement is taken into account by writing the entropy balance for both the system and its surroundings, considered together, and by including an *entropy-generation* term to account for the irreversibilities of the process.



$$\left\{\begin{array}{c}\text{Net rate of}\\ \text{change in}\\ \text{entropy of}\\ \text{flowing streams}\end{array}\right\} + \left\{\begin{array}{c}\text{Time rate of}\\ \text{change of}\\ \text{entropy}\\ \text{in control}\\ \text{volume}\end{array}\right\} + \left\{\begin{array}{c}\text{Time rate of}\\ \text{change of}\\ \text{entropy in}\\ \text{surroundings}\end{array}\right\} = \left\{\begin{array}{c}\text{Total rate}\\ \text{of entropy}\\ \text{generation}\end{array}\right\}$$

Entropy Balance for Open Systems

$$\Delta(S\dot{m})_{fs} + \frac{d(mS)_{cv}}{dt} + \frac{dS^t_{surr}}{dt} = \dot{S}_G \geq 0$$

The net rate of gain in entropy of the flowing streams, i.e., the difference between the total entropy transported out by exit streams and the total entropy transported in by entrance streams

The time rate of change of the total entropy of the fluid contained within the control volume.

Entropy changes in the surroundings, the result of heat transfer between system and surroundings.

The rate of entropy generation

Let rate of heat transfer $\dot{Q}_j$ with respect to a particular part of the control surface be associated with $T_{\sigma,j}$ where subscript σ, j denotes a temperature in the surroundings.

The rate of entropy change in the surroundings as a result of this transfer is $-\dot{Q}_j/T_{\sigma,j}$

- The minus sign converts Q_j, defined with respect to the system, to a heat rate with respect to the surroundings

Entropy Balance for Open Systems

$$\frac{dS^t_{\text{surr}}}{dt} = -\sum_j \frac{\dot{Q}_j}{T_{\sigma,j}}$$

➡

$$\Delta(S\dot{m})_{\text{fs}} + \frac{d(mS)_{\text{cv}}}{dt} - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = dS_{\text{CV}}/dt \qquad (\text{kW/K})$$

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}} \qquad (\text{kJ/K})$$

The rate of entropy change within the control volume during a process is equal to the sum of the rate of entropy transfer through the control volume boundary by heat transfer, the net rate of entropy transfer into the control volume by mass flow, and the rate of entropy generation within the boundaries of the control volume as a result of irreversibilities.

Entropy Balance for Open Systems

- For a steady-state flow process the mass and entropy of the fluid in the control volume are constant

$$\Delta(S\dot{m})_{\text{fs}} - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0$$

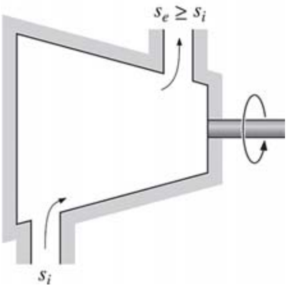
$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

- For *single-stream* (one inlet and one exit) steady-flow devices

Steady-flow, single-stream:

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

$$s_e = s_i + \sum \frac{q}{T} + s_{\text{gen}}$$



- For the case of an *adiabatic* single-stream device

Steady-flow, single-stream, adiabatic:

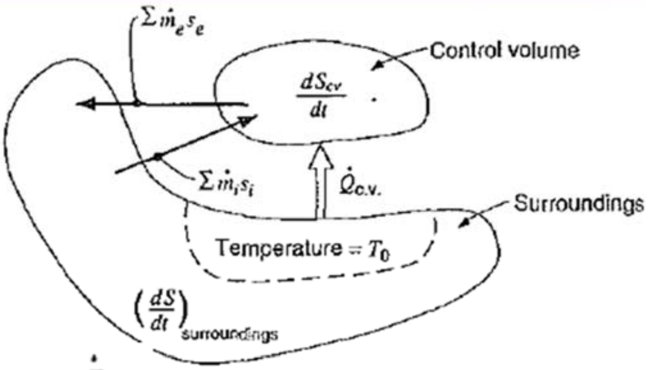
$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$

$$s_e = s_i + s_{\text{gen}} \geq s_i$$

The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

The Principle of the Increase of Entropy

- System that exchanges both mass and heat with its surroundings. At the point in the surroundings where the heat transfer occurs, the temperature is T_o.



the second law for this process

$$\frac{dS_{c.v.}}{dt} + \sum \dot m_e s_e - \sum \dot m_i s_i \ge \sum_{c.v.} \frac{\dot Q_{c.v.}}{T}$$

the rate of change of Entropy within the control volume

the net entropy flow out of the control volume resulting from the mass flow

$$\frac{dS_{surr}}{dt} = \sum \dot m_e s_e - \sum \dot m_i s_i - \frac{\dot Q_{c.v.}}{T_0}$$

➡

The Principle of the Increase of Entropy

$$\frac{dS_{net}}{dt} = \frac{dS_{c.v.}}{dt} + \frac{dS_{surr}}{dt} \ge \sum_{c.v.} \frac{\dot Q_{c.v.}}{T} - \frac{\dot Q_{c.v.}}{T_0}$$

➡ $$\frac{dS_{net}}{dt} = \frac{dS_{c.v.}}{dt} + \frac{dS_{surr}}{dt} = \sum \dot S_{gen} \ge 0$$

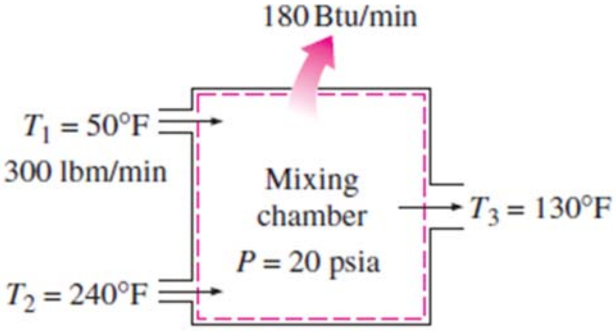
$$\Delta S_{net} = \Delta S_{c.v.} + \Delta S_{surr}$$

Where

$$\Delta S_{c.v.} = (m_2 s_2 - m_1 s_1)_{c.v.} \qquad \Delta S_{surr} = \frac{-Q_{c.v.}}{T_0} + \sum m_e s_e - \sum m_i s_i$$

Example

Water at 20 psia and 50F enters a mixing chamber at a rate of 300 lbm/min where it is mixed steadily with steam entering at 20 psia and 240F. The mixture leaves the chamber at 20 psia and 130F, and heat is lost to the surrounding air at 70F at a rate of 180 Btu/min. Neglecting the changes in kinetic and potential energies, determine the rate of entropy generation during this process.

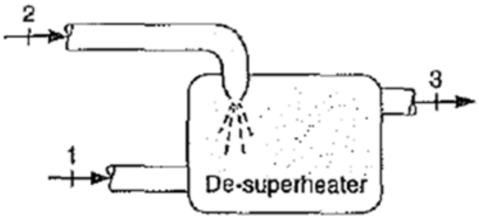


Example Cont.

Example Cont.

Example

A de-superheater works by injecting liquid water into a flow of superheated steam. With 2 kg/s at 300 kPa, 200^{o}C, steam flowing in, what mass flow rate of liquid water at 20°C should be added to generate saturated vapor at 300 kPa? What is the rate of entropy generation in the process.



Example Cont.

Example

Steam enters a steam turbine at a pressure of 1 MPa, a temperature of 300°C, and a velocity of 50 *m/s.* The steam leaves the turbine at a pressure of 150 kPa and a velocity of 200 m/s. Determine the work per kilogram of steam flowing through the turbine, assuming the process to be reversible and adiabatic.

Example Cont.

Transient Process

$$\frac{d}{dt}(ms)_{\text{c.v.}} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}$$

Integrating over the time interval *t*, $\int_0^t \frac{d}{dt}(ms)_{\text{c.v.}}\,dt = (m_2 s_2 - m_1 s_1)_{\text{c.v.}}$

And

$$\int_0^t \left(\sum \dot{m}_i s_i\right) dt = \sum m_i s_i, \qquad \int_0^t \left(\sum \dot{m}_e \dot{s}_e\right) dt = \sum m_e s_e, \qquad \int_0^t \dot{S}_{\text{gen}}\,dt = {}_1S_{2\text{gen}}$$

$$\Longrightarrow \quad (m_2 s_2 - m_1 s_1)_{\text{c.v.}} = \sum m_i s_i - \sum m_e s_e + \int_0^t \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T}\,dt + {}_1S_{2\text{gen}}$$

If the temperature is uniform throughout the control volume at any instant of time

$$\int_0^t \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T}\,dt = \int_0^t \frac{1}{T} \sum_{\text{c.v.}} \dot{Q}_{\text{c.v.}}\,dt = \int_0^t \frac{\dot{Q}_{\text{c.v.}}}{T}\,dt$$

the second law for the transient process

$$(m_2 s_2 - m_1 s_1)_{\text{c.v.}} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{\text{c.v.}}}{T}\,dt + {}_1S_{2\text{gen}}$$

Calculation of the Ideal Work

- In any steady-state flow process requiring work, there is an absolute minimum amount which must be expended to accomplish the desired change of state of the fluid flowing through the control volume.
- In a process producing work, there is an absolute maximum amount which may be accomplished as the result of a given change of state of the fluid flowing through the control volume.
- In either case, the limiting value obtains when the change of state associated with the process is accomplished *completely reversibly.*

For such a process, the entropy generation is zero

$$\Delta(S\dot{m})_{\text{fs}} - \frac{\dot{Q}}{T_\sigma} = 0$$

$$Q = T_\sigma\,\Delta(S\dot{m})_{\text{fs}}$$

Calculation of the Ideal Work

$$\Delta\left[\left(H + \tfrac{1}{2}u^2 + zg\right)\dot{m}\right]_{\text{fs}} = T_\sigma\,\Delta(S\dot{m})_{\text{fs}} + \dot{W}_s(\text{rev})$$

$\dot{W}_s(\text{rev})$ the work of a completely reversible process or *ideal work,* $\dot{W}_{\text{ideal}}$

$$\dot{W}_{\text{ideal}} = \Delta\left[\left(H + \tfrac{1}{2}u^2 + zg\right)\dot{m}\right]_{\text{fs}} - T_\sigma\,\Delta(S\dot{m})_{\text{fs}}$$

Neglecting the kinetic- and potential-energy terms

$$W_{\text{ideal}} = \Delta(H\dot{m})_{\text{fs}} - T_\sigma\,\Delta(S\dot{m})_{\text{fs}}$$

For the special case of a single stream flowing through the control volume,

$$W_{\text{ideal}} = \dot{m}(\Delta H - T_\sigma\,\Delta S)$$

Or per unit-mass

$$W_{\text{ideal}} = \Delta H - T_\sigma\,\Delta S$$

Calculation of the Ideal Work

- A completely reversible process is hypothetical, devised solely for determination of the ideal work associated with a given change of state

The only connection between the hypothetical reversible process and an actual process is that it brings about the same change of state as the actual process

When W_{ideal} (or W_{ideal}) is positive,

It is the *minimum work required* to bring about a given change in the properties of the flowing streams and it is smaller than the actual work

$$\eta_t(\text{work required}) = \frac{\dot{W}_{\text{ideal}}}{\dot{W}_s}$$

When $\dot{W}_{\text{ideal}}$ (or W_{ideal}) is negative,

The absolute value is the *maximum work obtainable* from a given change in the properties of the flowing streams, and it is larger than the actual work

$$\eta_t(\text{work produced}) = \frac{\dot{W}_s}{\dot{W}_{\text{ideal}}}$$

Lost Work

$$W_{\text{lost}} \equiv W_s - W_{\text{ideal}} \quad \textbf{Or} \quad \dot{W}_{\text{lost}} \equiv \dot{W}_s - \dot{W}_{\text{ideal}}$$

But $$\dot{W}_s = \Delta\left[\left(H + \tfrac{1}{2}u^2 + zg\right)\dot{m}\right]_{\text{fs}} - Q$$

And $$\dot{W}_{\text{ideal}} = \Delta\left[\left(H + \tfrac{1}{2}u^2 + zg\right)\dot{m}\right]_{\text{fs}} - T_\sigma\,\Delta(S\dot{m})_{\text{fs}}$$

$$\longrightarrow \quad \dot{W}_{\text{lost}} = T_\sigma\,\Delta(S\dot{m})_{\text{fs}} - \dot{Q}$$

For the case of a single surroundings temperature T_σ,

$$\dot{S}_G = \Delta(S\dot{m})_{\text{fs}} - \frac{\dot{Q}}{T_\sigma} \quad \longrightarrow \quad T_\sigma \dot{S}_G = T_\sigma\,\Delta(S\dot{m})_{\text{fs}} - Q$$

$$\longrightarrow \quad \dot{W}_{\text{lost}} = T_\sigma \dot{S}_G$$

Since $$\dot{S}_G \geq 0 \quad \longrightarrow \quad W_{\text{lost}} \geq 0$$

The Reversible Steady-State Process

- When a steady-state process involves a single flow of fluid into and out of the control volume,

The first law,

$$q + h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i = h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e + w$$

The second law

$$\dot{m}(s_e - s_i) = \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}$$

- If the process is reversible and adiabatic, the second-law equation reduces

$$s_e = s_i$$

from the property relation $T\,ds = dh - v\,dP$

With $q = 0,$

$$\longrightarrow \quad h_e - h_i = \int_i^e v\,dP$$

The Reversible Steady-State Process

$$\longrightarrow \quad w = (h_i - h_e) + \frac{\mathbf{V}_i^2 - \mathbf{V}_e^2}{2} + g(Z_i - Z_e)$$

$$= -\int_i^e v\,dP + \frac{\mathbf{V}_i^2 - \mathbf{V}_e^2}{2} + g(Z_i - Z_e)$$

- If The process is reversible and isothermal, the second law reduces to

$$\dot{m}(s_e - s_i) = \frac{1}{T}\sum_{\text{c.v.}} \dot{Q}_{\text{c.v.}} = \frac{\dot{Q}_{\text{c.v.}}}{T} \quad \longrightarrow \quad T(s_e - s_i) = \frac{\dot{Q}_{\text{c.v.}}}{\dot{m}} = q$$

$$\longrightarrow \quad T(s_e - s_i) = (h_e - h_i) - \int_i^e v\,dP$$

Again

$$w = -\int_i^e v\,dP + \frac{\mathbf{V}_i^2 - \mathbf{V}_e^2}{2} + g(Z_i - Z_e)$$

- The equation is valid for any reversible, steady-state process without the restriction that it be either adiabatic or isothermal

The Reversible Steady-State Process

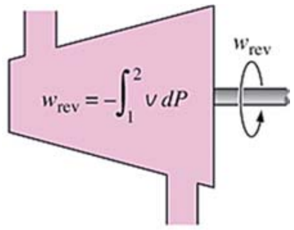
- For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (Bernoulli equation):

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

- The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

- When kinetic and potential energies are negligible

$$w_{rev} = -\int_1^2 v\,dP$$



(*a*) Steady-flow system

(*b*) Closed system

Reversible work relations

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{act} - \delta w_{act} = dh + dke + dpe$$ Actual

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$ Reversible

$$\delta q_{act} - \delta w_{act} = \delta q_{rev} - \delta w_{rev}$$

$$\delta w_{rev} - \delta w_{act} = \delta q_{rev} - \delta q_{act}$$

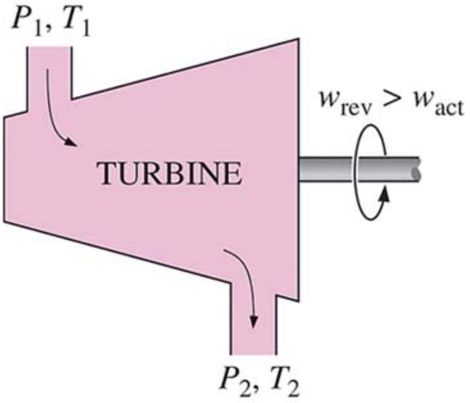
$$\delta q_{rev} = T\,ds \qquad ds \geq \frac{\delta q_{act}}{T}$$

$$\frac{\delta w_{rev} - \delta w_{act}}{T} = ds - \frac{\delta q_{act}}{T} \geq 0$$

$$\partial w_{rev} \geq \partial w_{act}$$

$$w_{rev} \geq w_{act}$$

Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.



A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

Minimizing the Compressor Work

$$w_{\text{rev,in}} = \int_1^2 v\,dP$$ When kinetic and potential energies are negligible

Isentropic (Pv^k = constant):

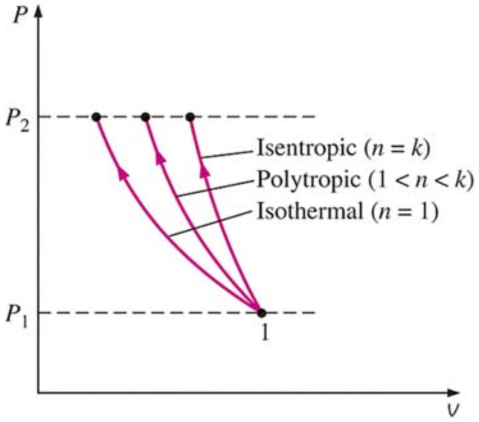
$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Polytropic (Pv^n = constant):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Isothermal (Pv = constant):

$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$

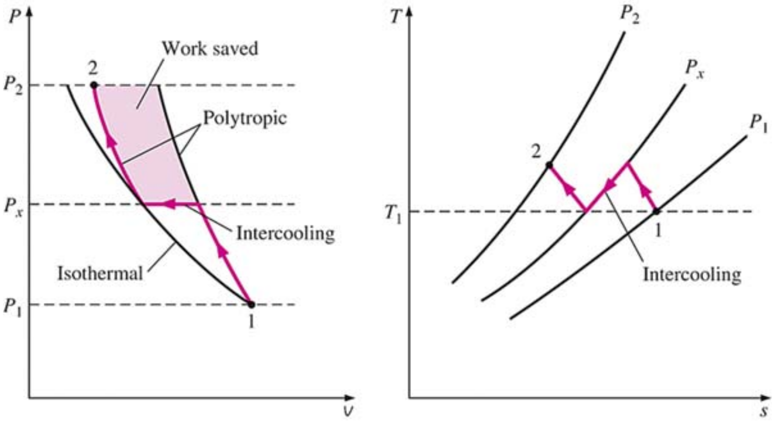


P-v diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

The adiabatic compression (Pv^k = constant) requires the maximum work and the isothermal compression (T = constant) requires the minimum. **Why?**

Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an *intercooler*.



P-v and *T-s* diagrams for a two-stage steady-flow compression process.

$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

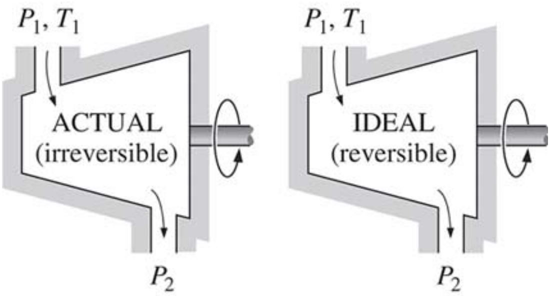
$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

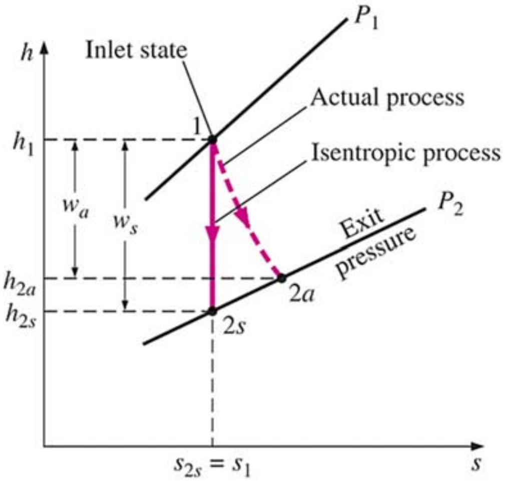
$$P_x = (P_1 P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

Isentropic Efficiencies of Steady-Flow Devices

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.





Isentropic Efficiency of Turbines

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine.

Isentropic Efficiencies of Compressors and Pumps

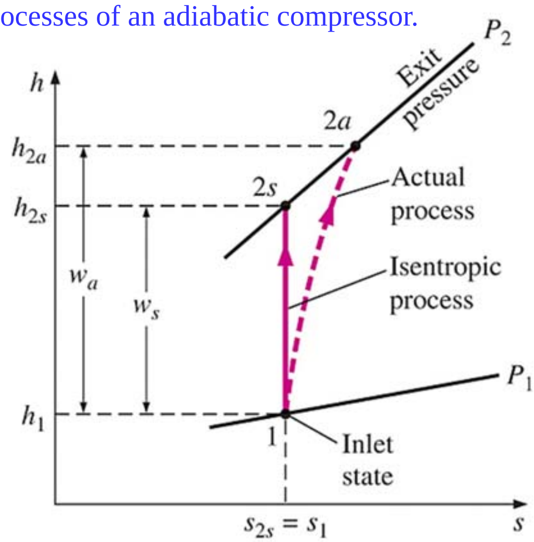
$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

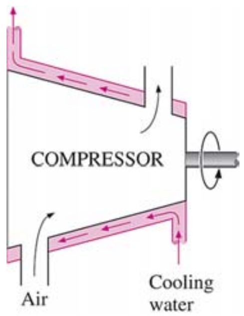
$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$ When kinetic and potential energies are negligible

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$ For a pump

$$\eta_C = \frac{w_t}{w_a}$$ Isothermal efficiency

The *h-s* diagram of the actual and isentropic processes of an adiabatic compressor.





Compressors are sometimes intentionally cooled to minimize the work input.

Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

Isentropic Efficiency of Nozzles

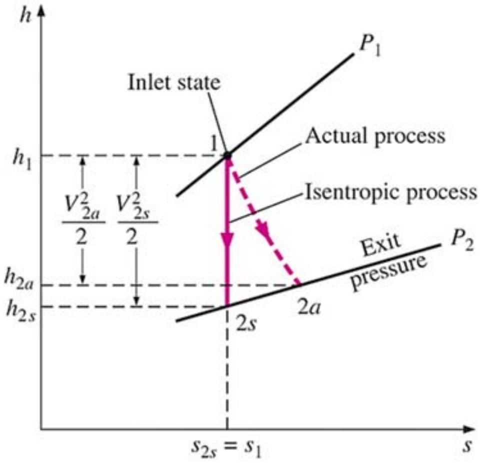
$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

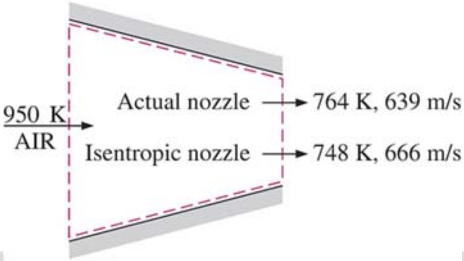
$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

Then,

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle.



A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.