Chapter 4

ENERGY ANALYSIS OF CLOSED SYSTEMS (Control Mass)

Thermodynamics: An Engineering Approach
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ENERGY BALANCE FOR CLOSED SYSTEMS

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,}\\ \text{kinetic, potential, etc., energies}}} \quad (\text{kW})$$

Energy balance in the rate form

- ☐ **The total quantities are related to the quantities per unit time is:**

$$Q = \dot{Q}\,\Delta t, \quad W = \dot{W}\,\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t \qquad (\text{kJ})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \qquad (\text{kJ/kg})$$

Energy balance per unit mass basis

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$$

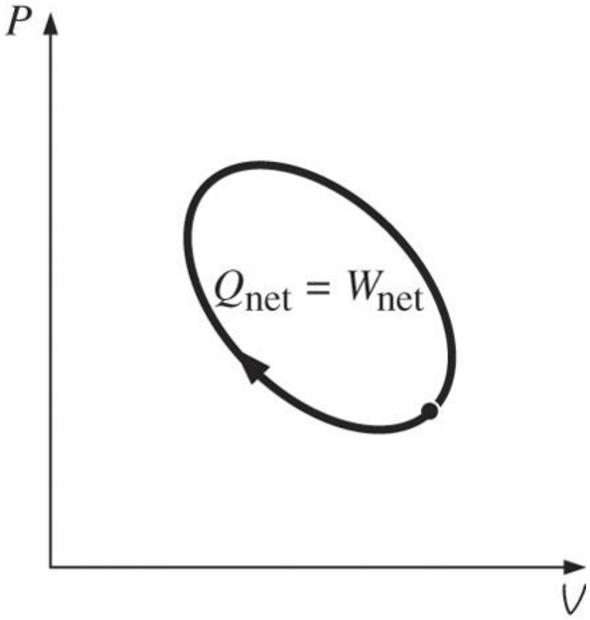
Energy balance for a cycle

$$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$$

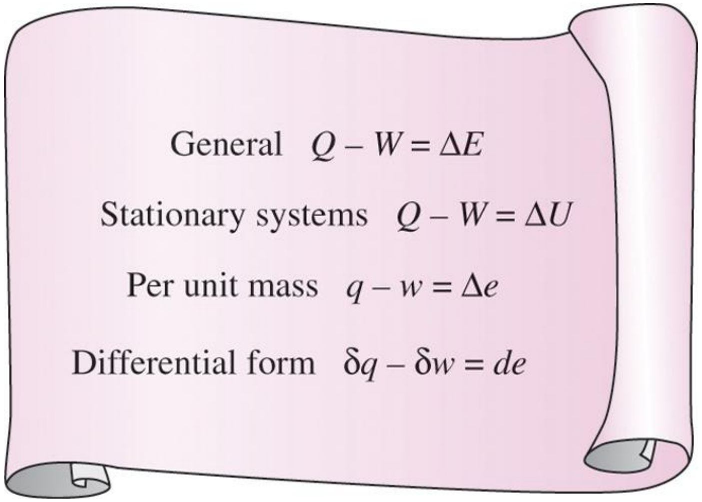
$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E$$

$$W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$$

- ☐ *Energy balance when sign convention is used (i.e., heat input and work output are **positive**; heat output and work input are **negative**).*



For a cycle $\Delta E = 0$, thus $Q = W$.



General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

MOVING BOUNDARY WORK

Moving boundary work ($P\,dV$ work):

The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

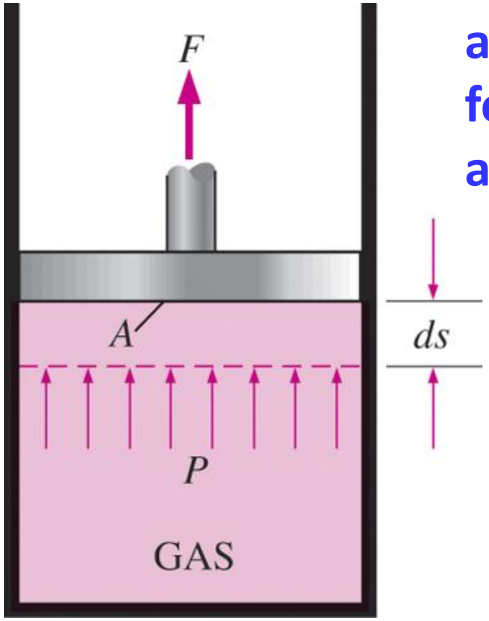
$$W_b = \int_1^2 P\,dV \qquad (\text{kJ})$$

Quasi-equilibrium process:

A process during which the system remains nearly in equilibrium at all times.

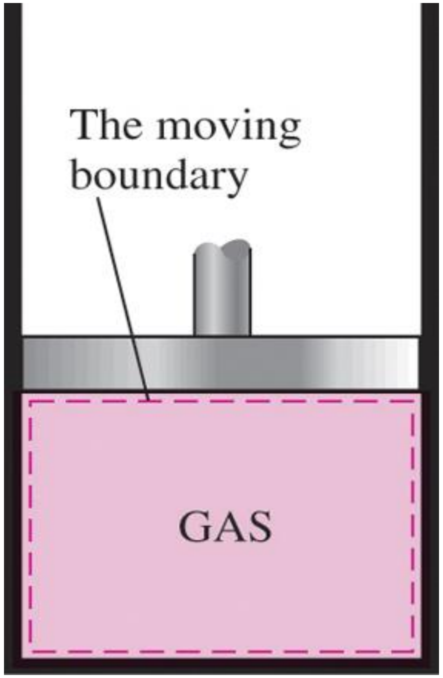
W_b is positive → for expansion

W_b is negative → for compression

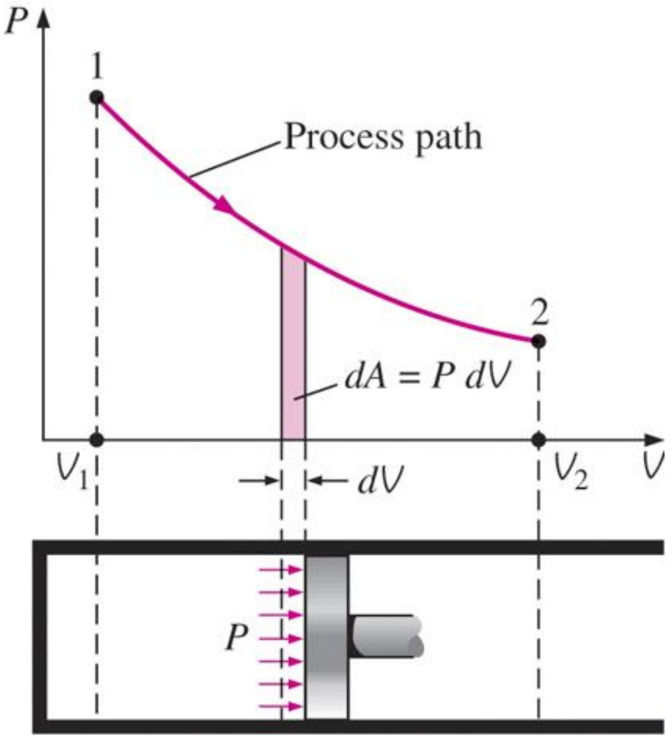


A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds.

The work associated with a *moving boundary* is called *boundary work.*

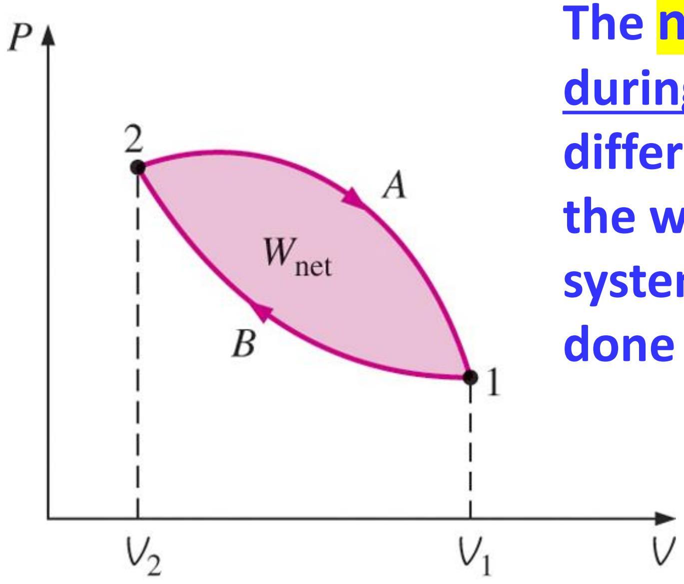


MOVING BOUNDARY WORK

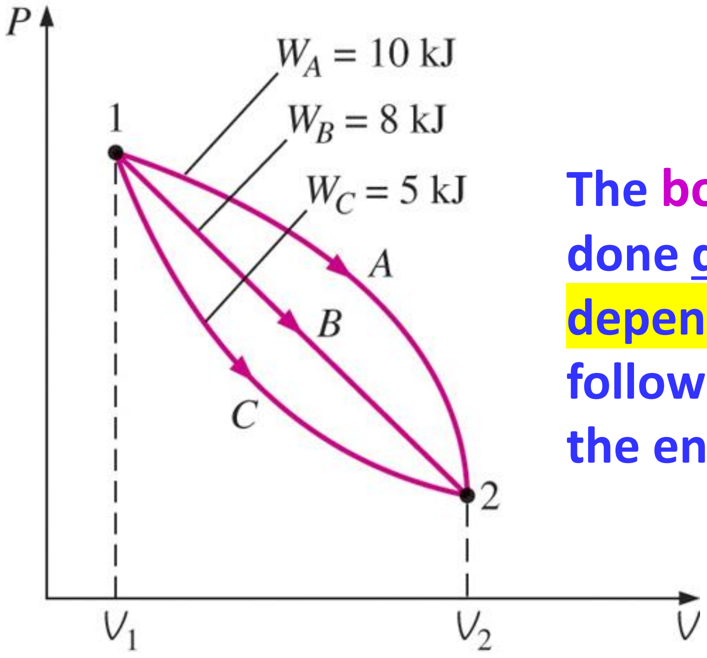


The area under the process curve on a *P*-*V* diagram represents the boundary work.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV$$



The net work done during a cycle is the difference between the work done by the system and the work done on the system.



The boundary work done during a process depends on the path followed as well as the end states.

Polytropic, Isothermal, and Isobaric processes

- The **polytropic process** can describe gas expansion or compression which includes heat transfer.

$$\boxed{P = CV^{-n}}$$

C*, *n (polytropic exponent) constants

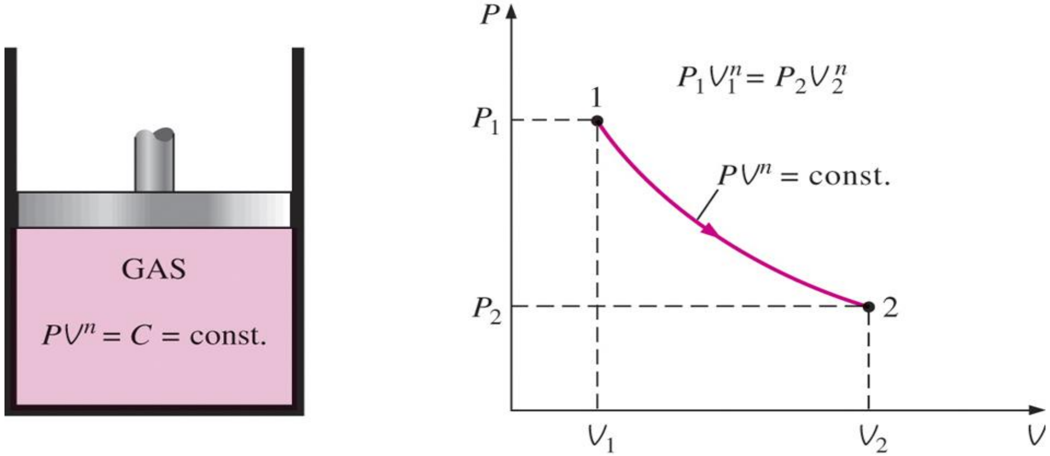
Polytropic process work for any gas:

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

Ideal gas EOS: $PV = mRT$ **OR** $Pv = RT$

Polytropic process work for ideal gas:

$$W_b = \frac{mR(T_2 - T_1)}{1-n}$$



Schematic and *P-V* diagram for a polytropic process.

Polytropic, Isothermal, and Isobaric processes

When *n* = 1 (isothermal process)

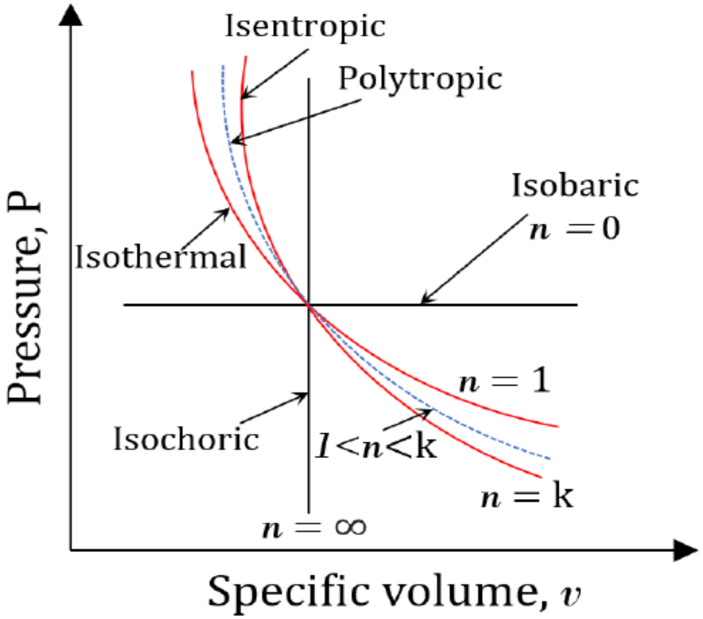
$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$

When *n* = 0 (isobaric) process

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$

When *n* = ∞ (isometric) process

What is the boundary work for a constant-volume process?



n = 1.3 polytropic

n = k = Cp/Cv = 1.4 adiabatic (isentropic)

Energy balance for a constant-pressure expansion or compression process

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

General analysis for a closed system undergoing a quasi-equilibrium, *constant-pressure process:*

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE}$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

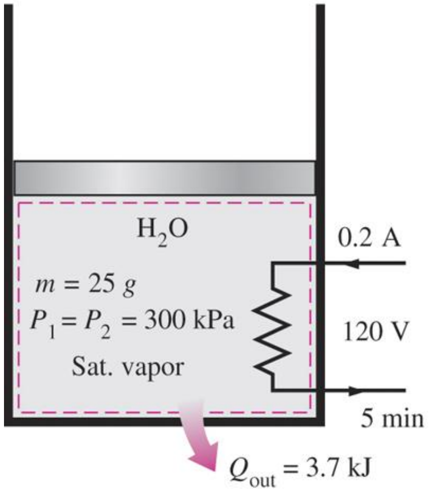
$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

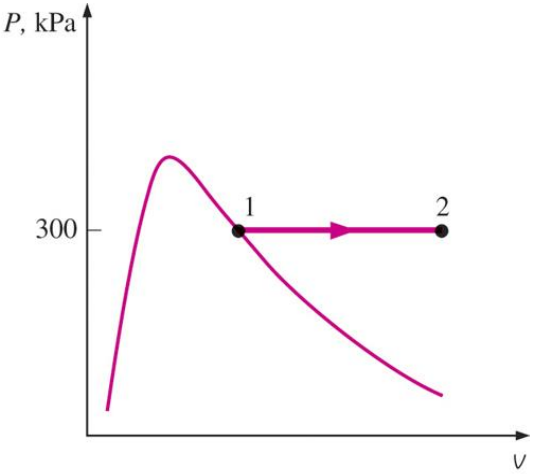
$$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

Example:





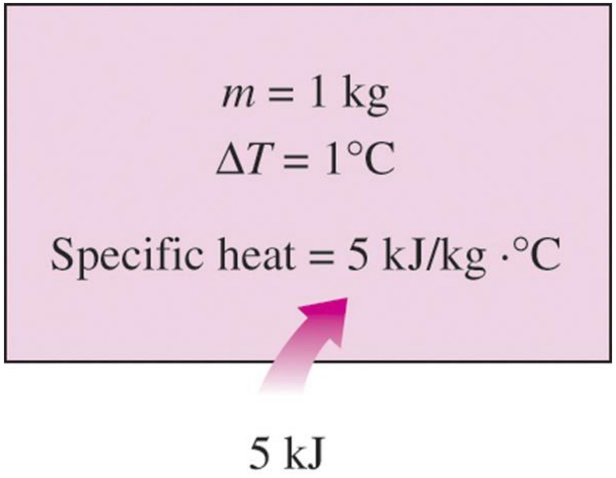
$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$

SPECIFIC HEATS

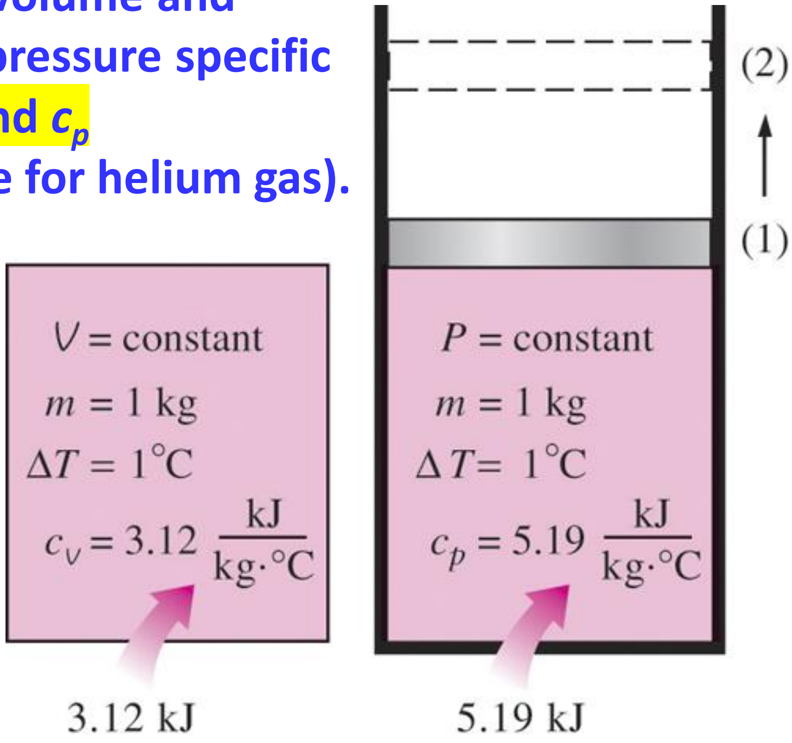
Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



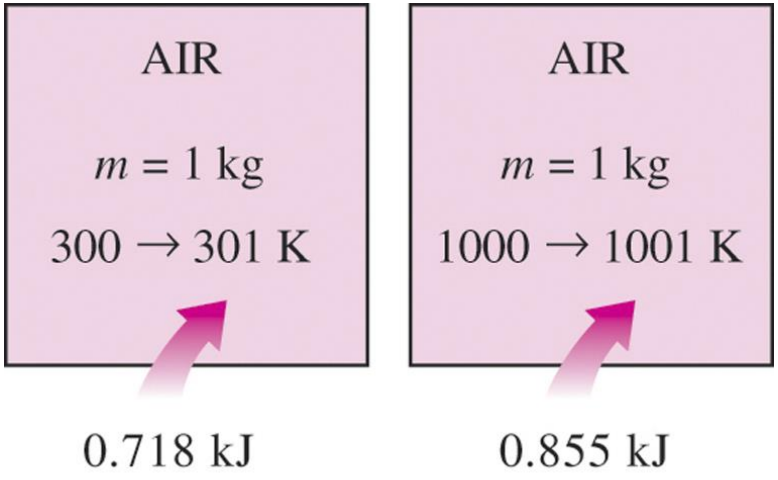
Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).



SPECIFIC HEATS

- The equations in the figure are valid for *any* substance undergoing *any* process.
- c_v and c_p are properties.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- A common unit for specific heats is kJ/kg·°C or kJ/kg·K. ***Are these units identical?***



The specific heat of a substance changes with temperature.

Q: True or False?

c_p is always greater than c_v

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

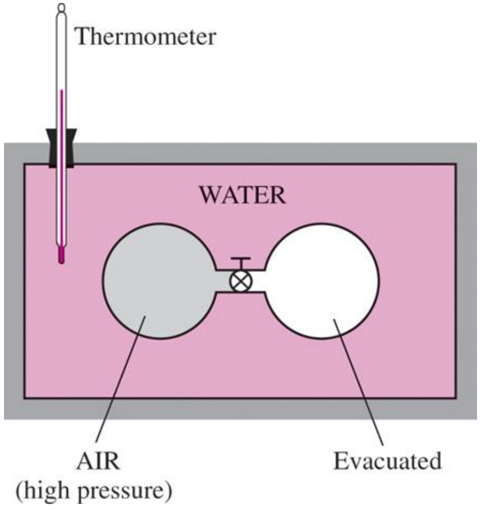
= the change in internal energy with temperature at constant volume

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

Formal definitions of c_v and c_p.

INTERNAL ENERGY and ENTHALPY of *IDEAL GASES*



Joule showed using this experimental apparatus that $u=u(T)$

$$\left.\begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned}\right\} \quad h = u + RT$$

$$u = u(T) \quad h = h(T)$$

$$du = c_v(T)\,dT \quad dh = c_p(T)\,dT$$

A cylinder filled with gas at high pressure connected via a stopcock to a second cylinder with gas at a low pressure. The two cylinders were immersed in a water bath, and the stopcock was opened so that gas from the high-pressure cylinder flowed into the evacuated cylinder. No heat was supplied to or lost from the system, nor did the gas do any work, so the *internal energy* was constant during the expansion. Joule found no temperature fall as a result of the expansion.

$$\boxed{\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}}$$

For ideal gases, u, h, c_v, and c_p vary with temperature only.

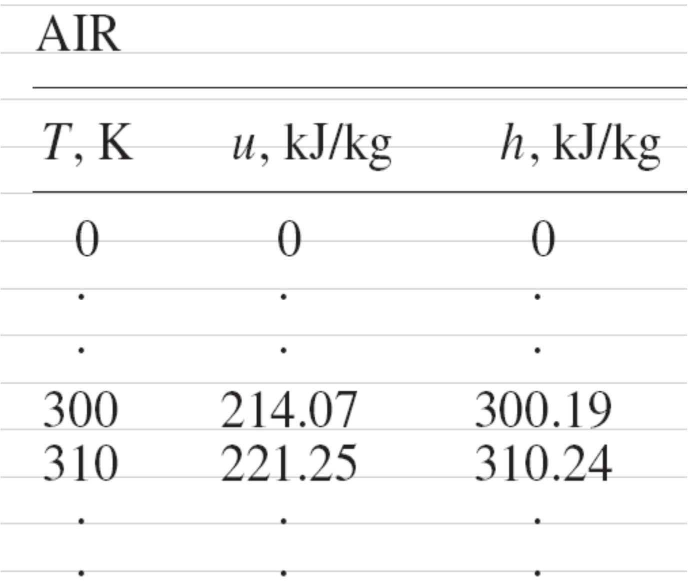
Internal energy and enthalpy change of an ideal gas:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\,dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\,dT$$

SPECIFIC HEATS

- At low pressures, all real gases approach ideal-gas behavior, and therefore their ***specific heats depend on temperature only.***
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats,* and are often denoted c_{p0} and c_{v0}.
- ***u* and *h*** data for a number of gases have been **tabulated.**
- These **tables** (See Textbook) are obtained by choosing an *arbitrary reference point* and performing the integrations by treating state 1 as the reference state.



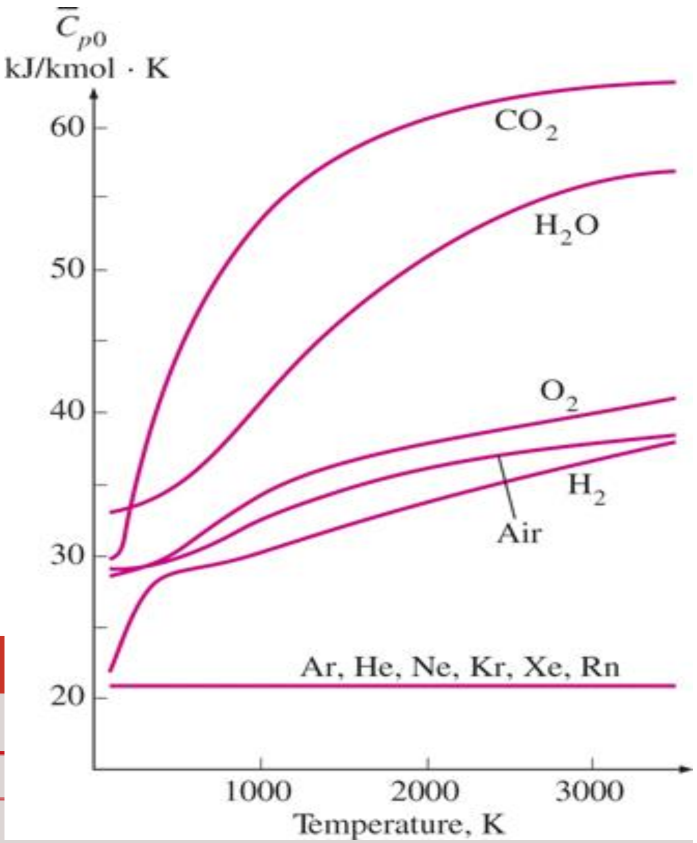
AIR

T, K	u, kJ/kg	h, kJ/kg
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

SPECIFIC HEATS

Ideal-gas constant-pressure specific heats for some gases
(see Table A–2*c* for c_p equations, NEXT).



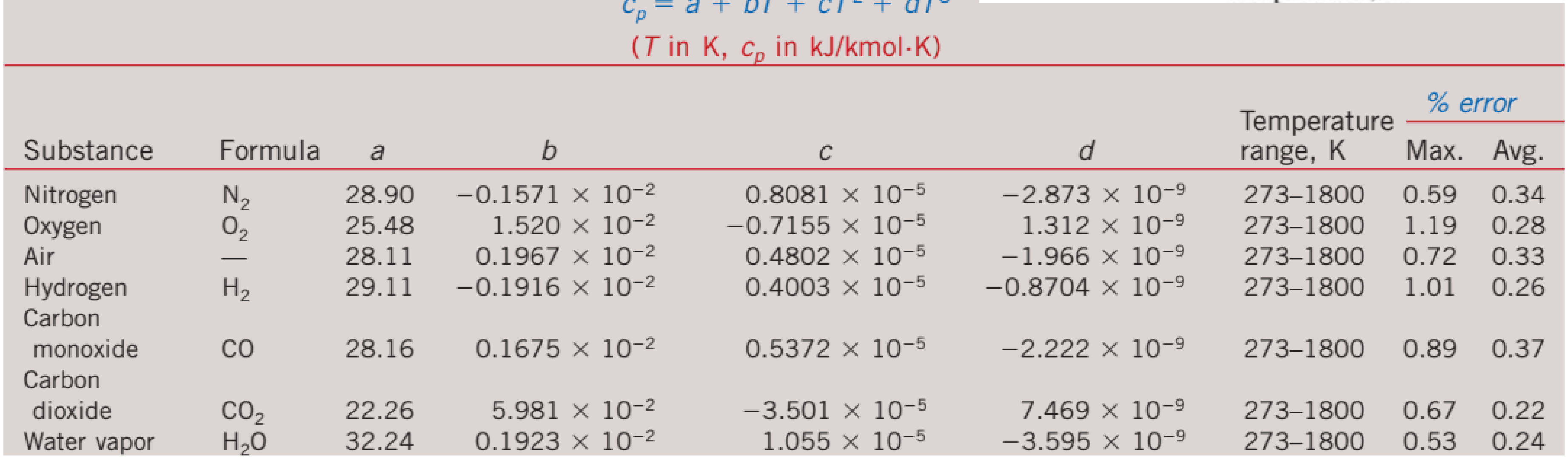


TABLE A–2

Ideal–gas specific heats of various common gases (*Concluded*)

(*c*) As a function of temperature

$$\bar{c}_p = a + bT + cT^2 + dT^3$$

(T in K, c_p in kJ/kmol·K)

Substance	Formula	*a*	*b*	*c*	*d*	Temperature range, K	*% error* Max.	*% error* Avg.
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	273–1800	0.59	0.34
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	273–1800	1.19	0.28
Air	—	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	273–1800	0.72	0.33
Hydrogen	H_2	29.11	-0.1916×10^{-2}	0.4003×10^{-5}	-0.8704×10^{-9}	273–1800	1.01	0.26
Carbon monoxide	CO	28.16	0.1675×10^{-2}	0.5372×10^{-5}	-2.222×10^{-9}	273–1800	0.89	0.37
Carbon dioxide	CO_2	22.26	5.981×10^{-2}	-3.501×10^{-5}	7.469×10^{-9}	273–1800	0.67	0.22
Water vapor	H_2O	32.24	0.1923×10^{-2}	1.055×10^{-5}	-3.595×10^{-9}	273–1800	0.53	0.24

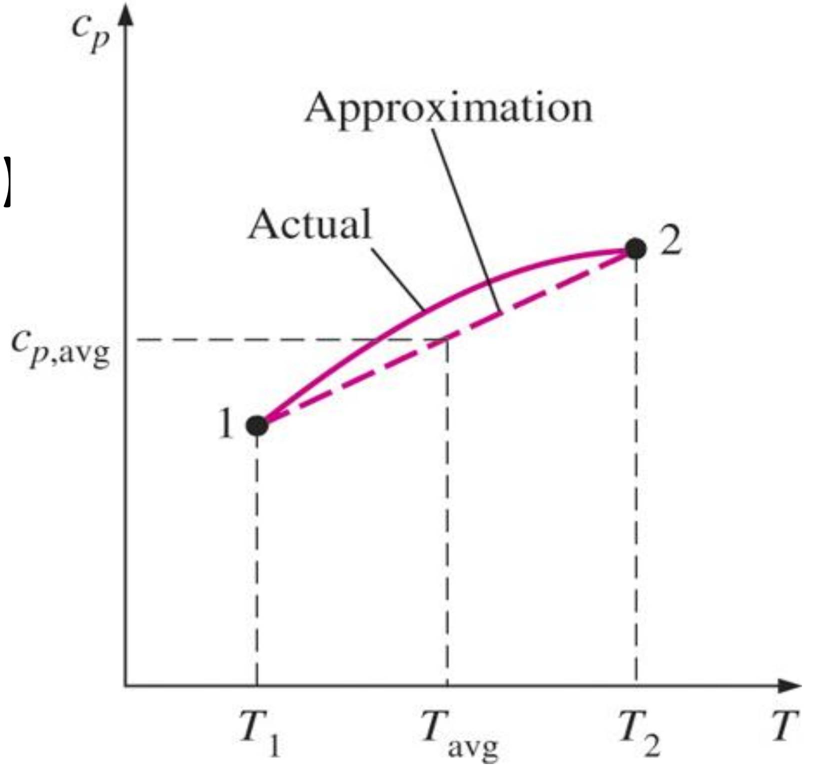
Internal energy and enthalpy changes when specific heat are taken constant at an average value:

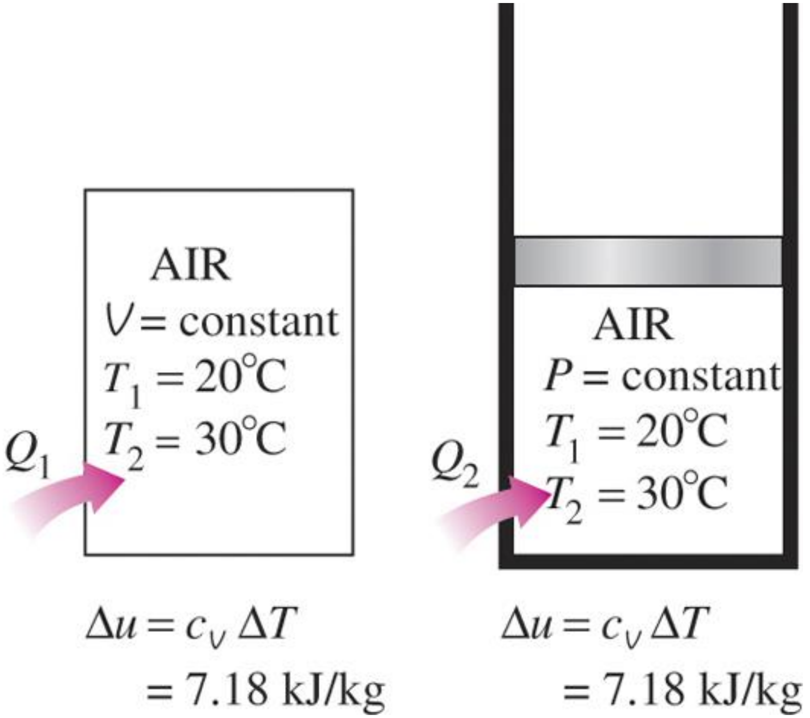
For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

$$u_2 - u_1 = c_{v,\mathrm{avg}}(T_2 - T_1)$$

(kJ/kg)

$$h_2 - h_1 = c_{p,\mathrm{avg}}(T_2 - T_1)$$



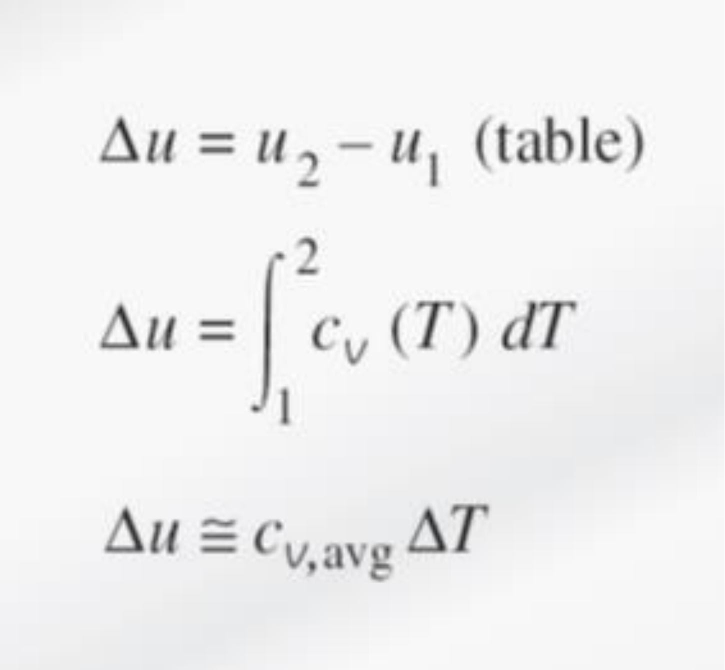


The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not, i.e., also valid for constant-pressure process.

Three ways of calculating Δ*u and* Δ*h*

1. By using the tabulated ***u* and *h*** data. This is the easiest and ***most accurate*** way when **tables** are readily available.
2. By using the **c_v or c_p** relations **(Table A-2c,** see above**)** as a function of temperature and performing the integrations. This is very inconvenient for hand calculations, but desirable for computerized calculations. The results obtained are ***very accurate***.
3. By using **average specific heats**. This is very simple and certainly very convenient *when property tables are not available.* The results obtained are ***reasonably accurate*** if the temperature interval is not very large.

Three ways of calculating Δu:



$$\Delta u = u_2 - u_1 \quad \text{(table)}$$

$$\Delta u = \int_1^2 c_v\,(T)\,dT$$

$$\Delta u \cong c_{v,\mathrm{avg}}\,\Delta T$$

Specific Heat Relations of Ideal Gases

$$h = u + RT,$$
$$dh = du + R\,dT$$

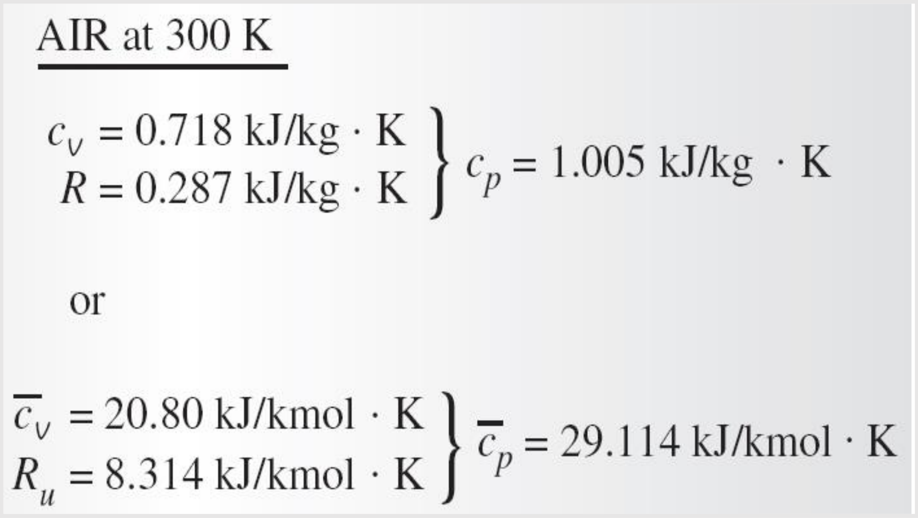
$dh = c_p dT$ and $du = c_v dT$

The relationship between c_p, c_v and R

$$\xrightarrow{\hspace{2em}} c_p = c_v + R \qquad (\mathrm{kJ/kg \cdot K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\mathrm{kJ/kmol \cdot K})$$



AIR at 300 K

$$\left.\begin{array}{l} c_v = 0.718\ \mathrm{kJ/kg \cdot K} \\ R = 0.287\ \mathrm{kJ/kg \cdot K} \end{array}\right\} c_p = 1.005\ \mathrm{kJ/kg \cdot K}$$

or

$$\left.\begin{array}{l} \bar{c}_v = 20.80\ \mathrm{kJ/kmol \cdot K} \\ R_u = 8.314\ \mathrm{kJ/kmol \cdot K} \end{array}\right\} \bar{c}_p = 29.114\ \mathrm{kJ/kmol \cdot K}$$

The c_p of an ideal gas can be determined from a knowledge of c_v and R.

Specific heat ratio:

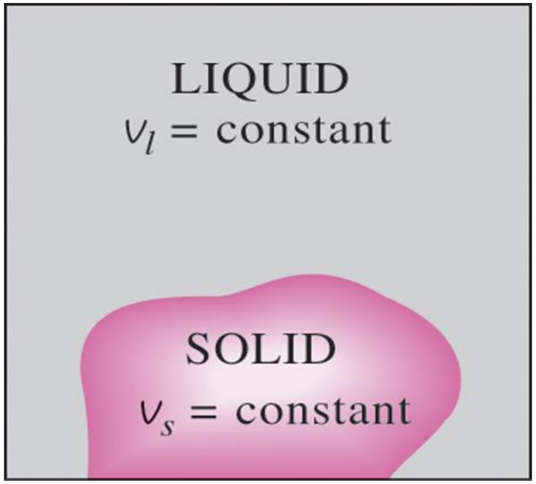
$$k = \frac{c_p}{c_v}$$

- ☐ The specific ratio varies with temperature, but this variation is very mild.
- ☐ For monatomic gases (helium, argon, etc.), its value is essentially constant at **1.667.**
- ☐ Many diatomic gases, including air, have a specific heat ratio of about **1.4** at room temperature.

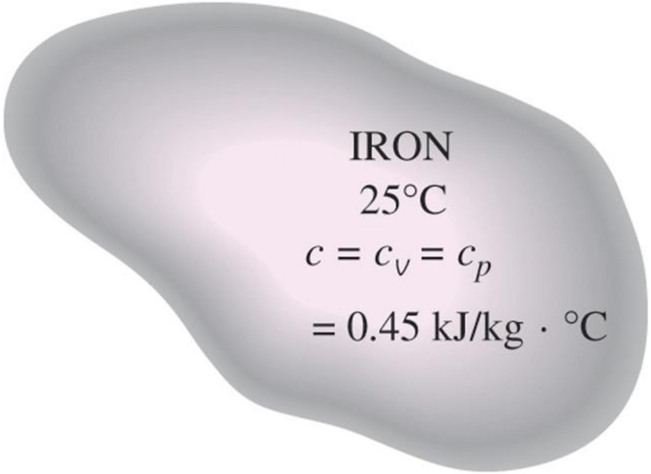
INTERNAL ENERGY, ENTHALPY, and SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Incompressible substance:

- A substance whose specific volume (or density) is constant.
- **Solids and liquids** are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by c.

Internal Energy Changes *(liquids & solids)*

$$du = c_v\, dT = c(T)\, dT \qquad \Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad (\mathrm{kJ/kg})$$

$$\Delta u \cong c_{\mathrm{avg}}(T_2 - T_1) \qquad (\mathrm{kJ/kg})$$

Enthalpy Changes *(liquids & solids)*

$$dh = du + v\, dP + P\, d\overset{\nearrow 0}{v} = du + v\, dP \qquad \boxed{h = u + Pv}$$

$$\Delta h = \Delta u + v\, \Delta P \cong c_{\mathrm{avg}}\, \Delta T + v\, \Delta P \qquad (\mathrm{kJ/kg})$$

For *solids*, the term $v\,\Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\mathrm{avg}}\Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\mathrm{avg}}\Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v\,\Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\mathrm{sat}@T})$$

Table 3-38

A more accurate relation than: $h_{@P,T} \cong h_{f@T}$

The enthalpy of a compressed liquid

EXAMPLE 4–1 Boundary Work for a Constant-Volume Process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

SOLUTION Air in a rigid tank is cooled, and both the pressure and temperature drop. The boundary work done is to be determined.

Analysis A sketch of the system and the *P*-*V* diagram of the process are shown in Fig. 4–6. The boundary work can be determined from Eq. 4–2 to be

$$W_b = \int_1^2 P\, \cancelto{0}{dV} = \mathbf{0}$$

Discussion This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the *P*-*V* diagram of the process (the area under the process curve is zero).

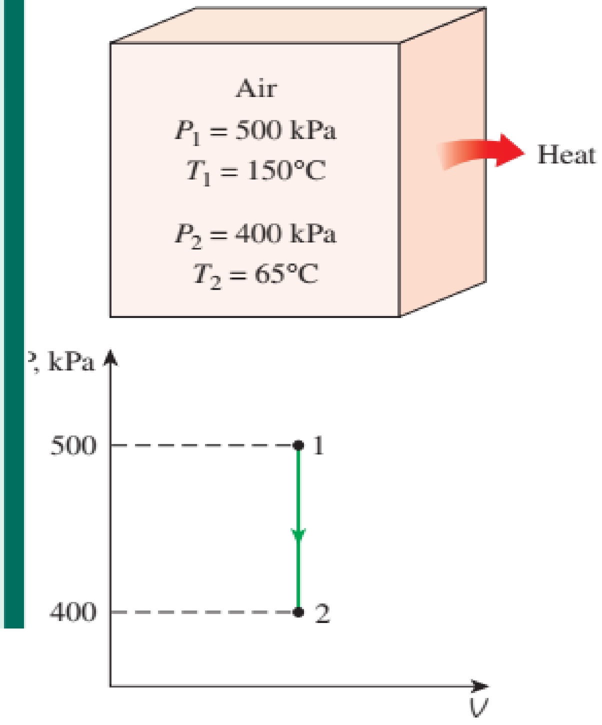
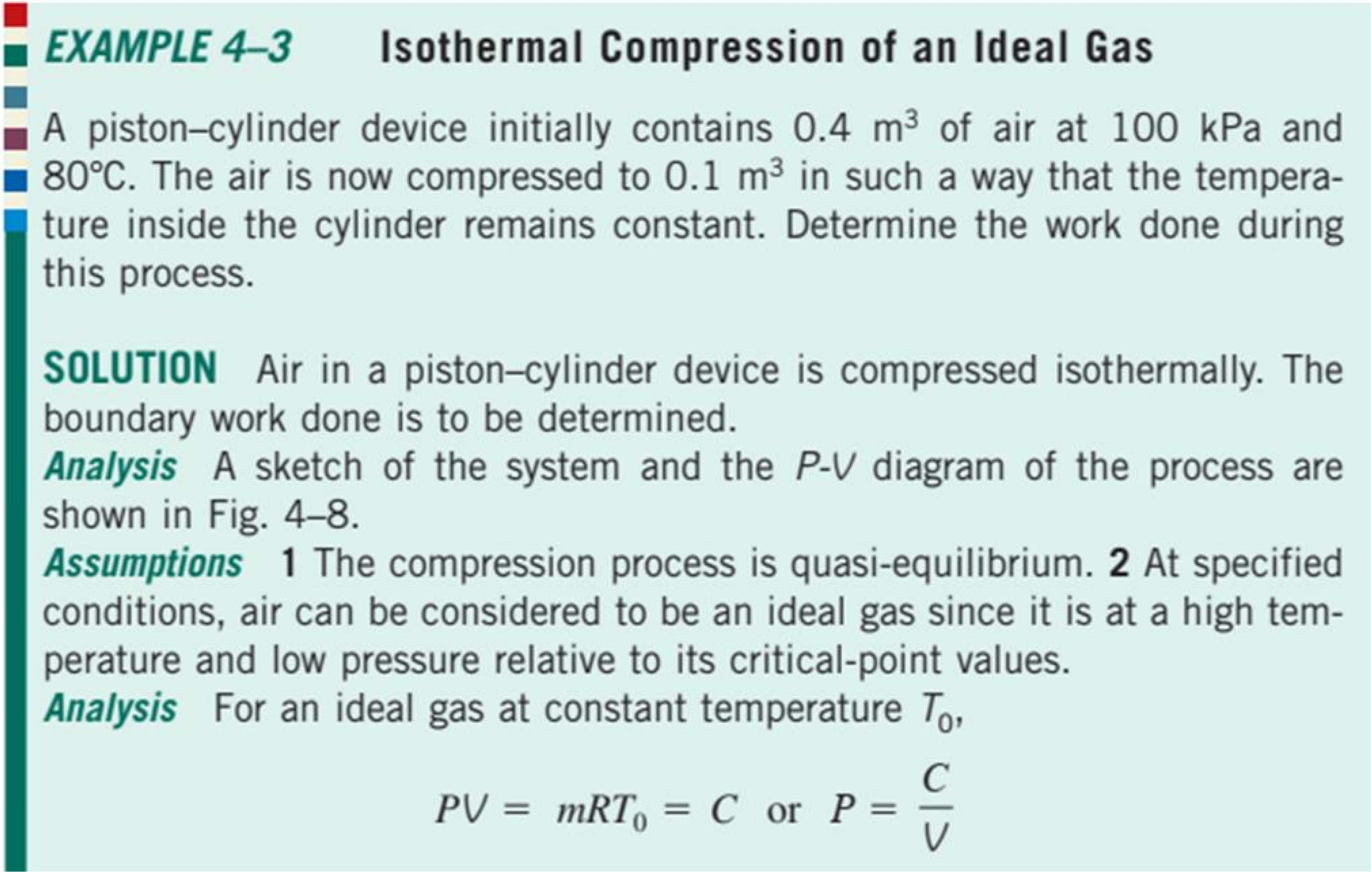


FIGURE 4–6
Schematic and *P*-*V* diagram for Example 4–1.



EXAMPLE 4–3 **Isothermal Compression of an Ideal Gas**

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

SOLUTION Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.

Analysis A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–8.

Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

Analysis For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

$$Pv = RT$$

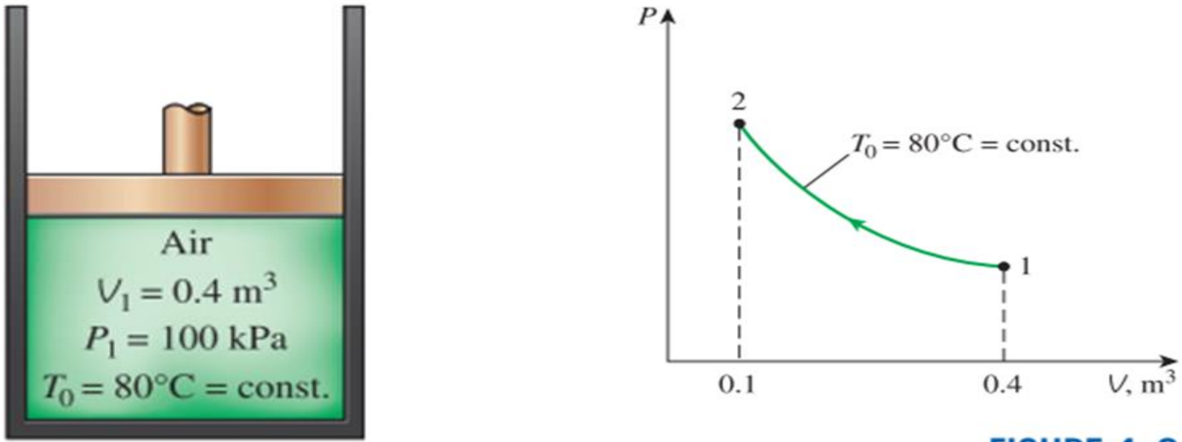


FIGURE 4–8
Schematic and P-V diagram for Example 4–3.

where C is a constant. Substituting this into Eq. 4–2, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1} \qquad \textbf{(4–7)}$$

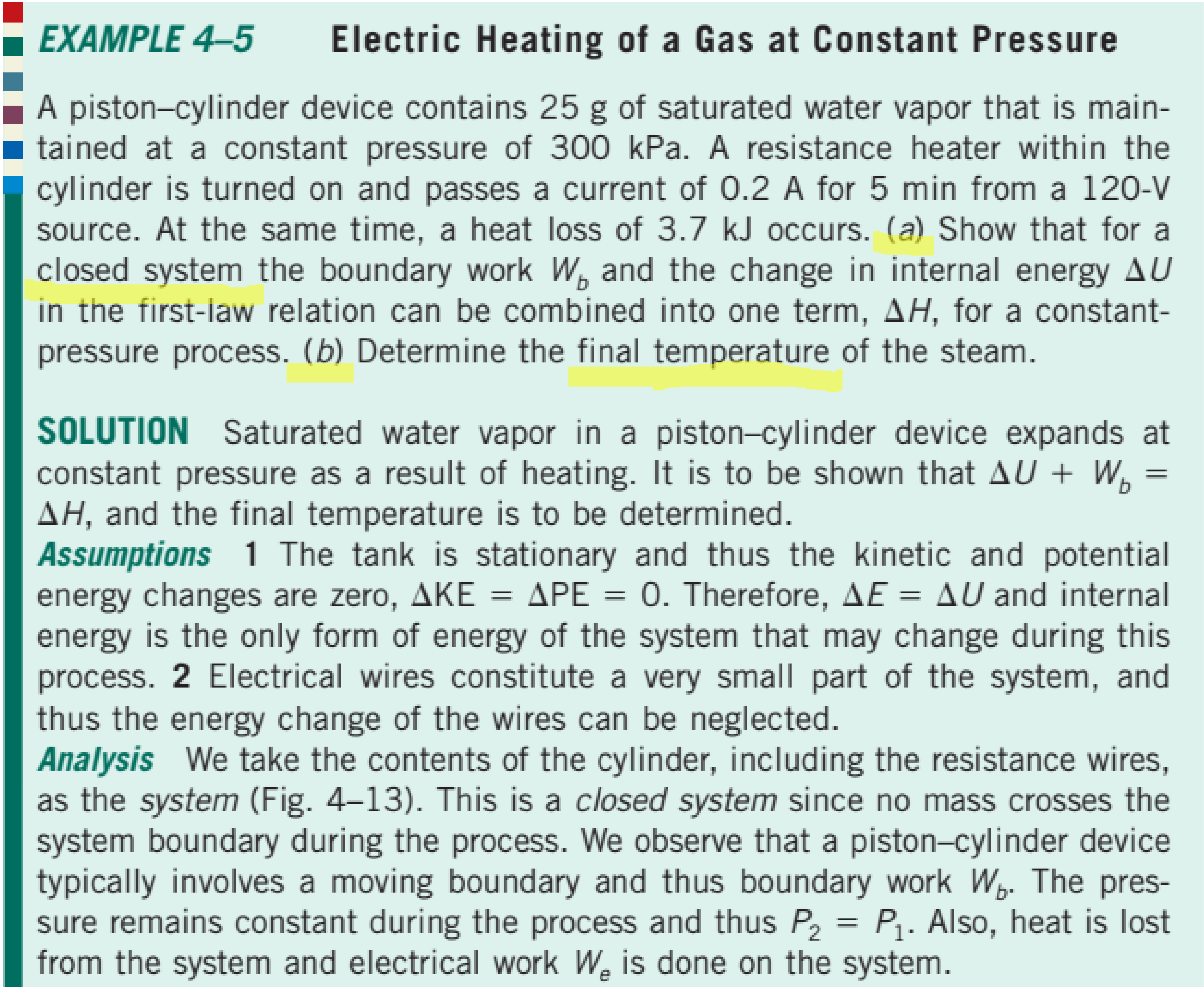
In Eq. 4–7, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.

Substituting the numerical values into Eq. 4–7 yields

$$W_b = (100\ \mathrm{kPa})(0.4\ \mathrm{m^3})\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\ \mathrm{kJ}}{1\ \mathrm{kPa{\cdot}m^3}}\right)$$

$$= \mathbf{-55.5\ kJ}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.



EXAMPLE 4–5 Electric Heating of a Gas at Constant Pressure

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (*a*) Show that for a closed system the boundary work W_b and the change in internal energy ΔU in the first-law relation can be combined into one term, ΔH, for a constant-pressure process. (*b*) Determine the final temperature of the steam.

SOLUTION Saturated water vapor in a piston–cylinder device expands at constant pressure as a result of heating. It is to be shown that $\Delta U + W_b = \Delta H$, and the final temperature is to be determined.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$. Therefore, $\Delta E = \Delta U$ and internal energy is the only form of energy of the system that may change during this process. **2** Electrical wires constitute a very small part of the system, and thus the energy change of the wires can be neglected.

Analysis We take the contents of the cylinder, including the resistance wires, as the *system* (Fig. 4–13). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work W_b. The pressure remains constant during the process and thus $P_2 = P_1$. Also, heat is lost from the system and electrical work W_e is done on the system.

(*a*) This part of the solution involves a general analysis for a closed system undergoing a quasi-equilibrium constant-pressure process, and thus we consider a general closed system. We take the direction of heat transfer Q to be to the system and the work W to be done by the system. We also express the work as the sum of boundary and other forms of work (such as electrical and shaft). Then, the energy balance can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \cancelto{0}{\Delta \mathrm{KE}} + \cancelto{0}{\Delta \mathrm{PE}}$$

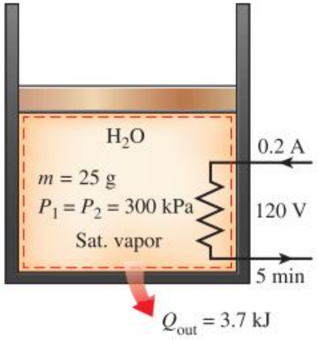
$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

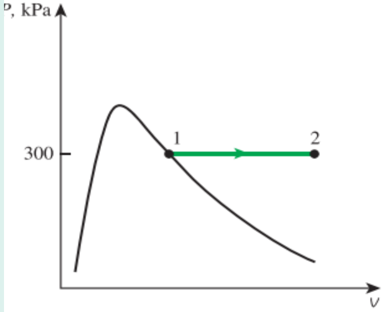
For a constant-pressure process, the boundary work is given as $W_b = P_0(V_2 - V_1)$. Substituting this into the preceding relation gives

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

However,

$$P_0 = P_2 = P_1 \rightarrow Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

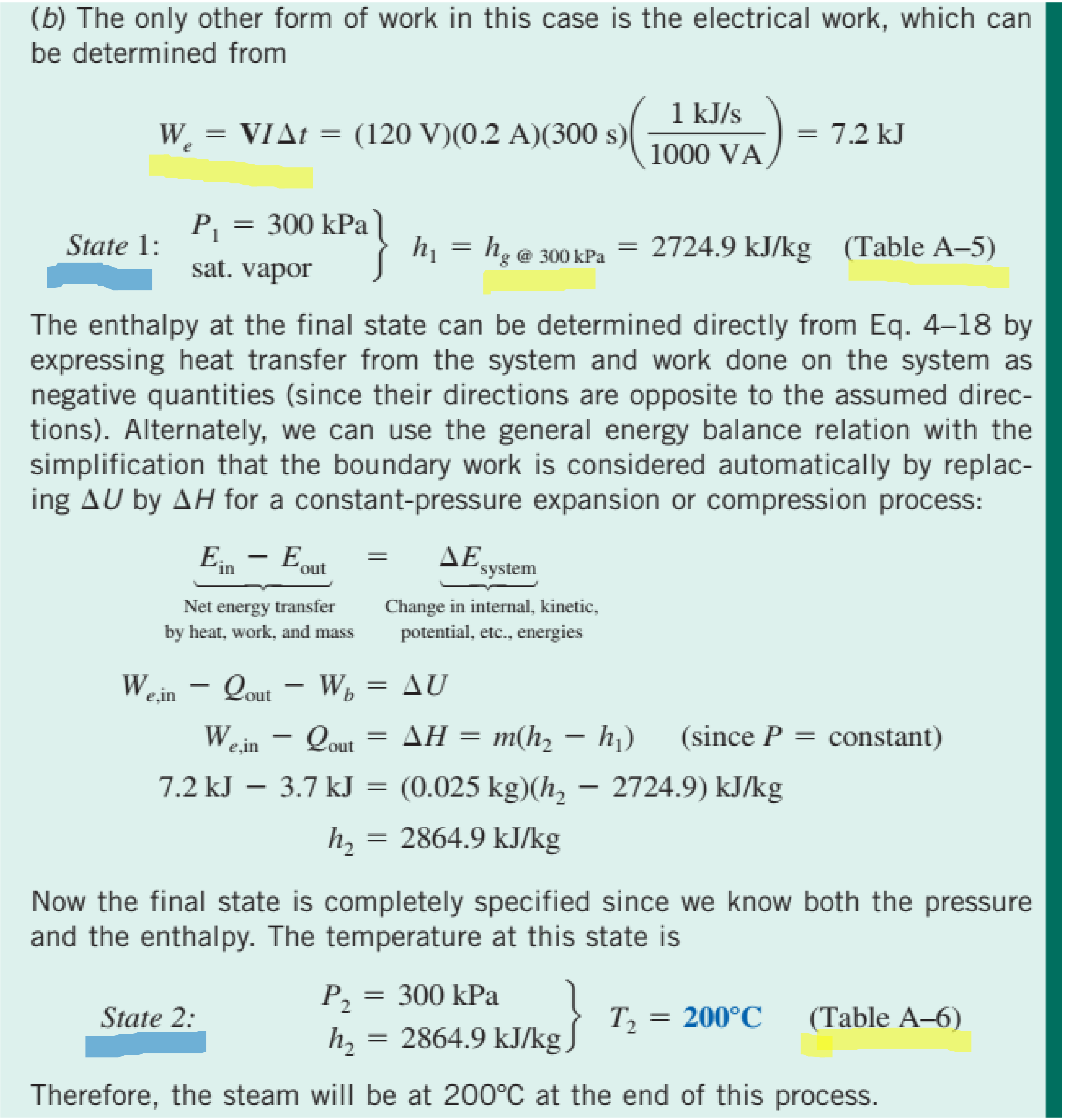




Also $H = U + PV$, and thus

$$Q - W_{\text{other}} = H_2 - H_1 \quad \text{(kJ)} \qquad \textbf{(4–18)}$$

which is the desired relation (Fig. 4–14). This equation is very convenient to use in the analysis of closed systems undergoing a ***constant-pressure quasi-equilibrium process*** since the boundary work is automatically taken care of by the enthalpy terms, and one no longer needs to determine it separately.



(*b*) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = \mathbf{V}I\,\Delta t = (120\ \text{V})(0.2\ \text{A})(300\ \text{s})\left(\frac{1\ \text{kJ/s}}{1000\ \text{VA}}\right) = 7.2\ \text{kJ}$$

$$\textit{State 1:}\quad \left.\begin{array}{l} P_1 = 300\ \text{kPa} \\ \text{sat. vapor} \end{array}\right\}\ h_1 = h_{g\ @\ 300\ \text{kPa}} = 2724.9\ \text{kJ/kg}\qquad (\text{Table A–5})$$

The enthalpy at the final state can be determined directly from Eq. 4–18 by expressing heat transfer from the system and work done on the system as negative quantities (since their directions are opposite to the assumed directions). Alternately, we can use the general energy balance relation with the simplification that the boundary work is considered automatically by replacing ΔU by ΔH for a constant-pressure expansion or compression process:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$\begin{aligned} W_{e,\text{in}} - Q_{\text{out}} - W_b &= \Delta U \\ W_{e,\text{in}} - Q_{\text{out}} &= \Delta H = m(h_2 - h_1) \qquad (\text{since } P = \text{constant}) \\ 7.2\ \text{kJ} - 3.7\ \text{kJ} &= (0.025\ \text{kg})(h_2 - 2724.9)\ \text{kJ/kg} \\ h_2 &= 2864.9\ \text{kJ/kg} \end{aligned}$$

Now the final state is completely specified since we know both the pressure and the enthalpy. The temperature at this state is

$$\textit{State 2:}\quad \left.\begin{array}{l} P_2 = 300\ \text{kPa} \\ h_2 = 2864.9\ \text{kJ/kg} \end{array}\right\}\ T_2 = \mathbf{200°C}\qquad (\text{Table A–6})$$

Therefore, the steam will be at 200°C at the end of this process.

EXAMPLE 4–7 Evaluation of the Δu of an Ideal Gas

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using (*a*) data from the air table (Table A–17), (*b*) the functional form of the specific heat (Table A–2*c*), and (*c*) the average specific heat value (Table A–2*b*).

SOLUTION The internal energy change of air is to be determined in three different ways.

Assumptions At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

Analysis The internal energy change Δu of ideal gases depends on the initial and final temperatures only, and not on the type of process. Thus, the following solution is valid for any kind of process.

(*a*) One way of determining the change in internal energy of air is to read the u values at T_1 and T_2 from Table A–17 and take the difference:

$$u_1 = u_{@\,300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_2 = u_{@\,600\text{ K}} = 434.78\text{ kJ/kg}$$

Thus,

$$\Delta u = u_2 - u_1 = (434.78 - 214.07)\text{ kJ/kg} = \mathbf{220.71\text{ kJ/kg}}$$

(*b*) The $\bar{c}_p(T)$ of air is given in Table A–2*c* in the form of a third-degree polynomial expressed as

$$\bar{c}_p(T) = a + bT + cT^2 + dT^3$$

where $a = 28.11$, $b = 0.1967 \times 10^{-2}$, $c = 0.4802 \times 10^{-5}$, and $d = -1.966 \times 10^{-9}$. From Eq. 4–30,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

From Eq. 4–25,

$$\Delta \bar{u} = \int_1^2 \bar{c}_v(T)\,dT = \int_{T_1}^{T_2} [(a - R_u) + bT + cT^2 + dT^3]\,dT$$

Performing the integration and substituting the values, we obtain

$$\Delta \bar{u} = 6447 \text{ kJ/kmol}$$

The change in the internal energy on a unit-mass basis is determined by dividing this value by the molar mass of air (Table A–1):

$$\Delta u = \frac{\Delta \bar{u}}{M} = \frac{6447 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}} = \mathbf{222.5 \text{ kJ/kg}}$$

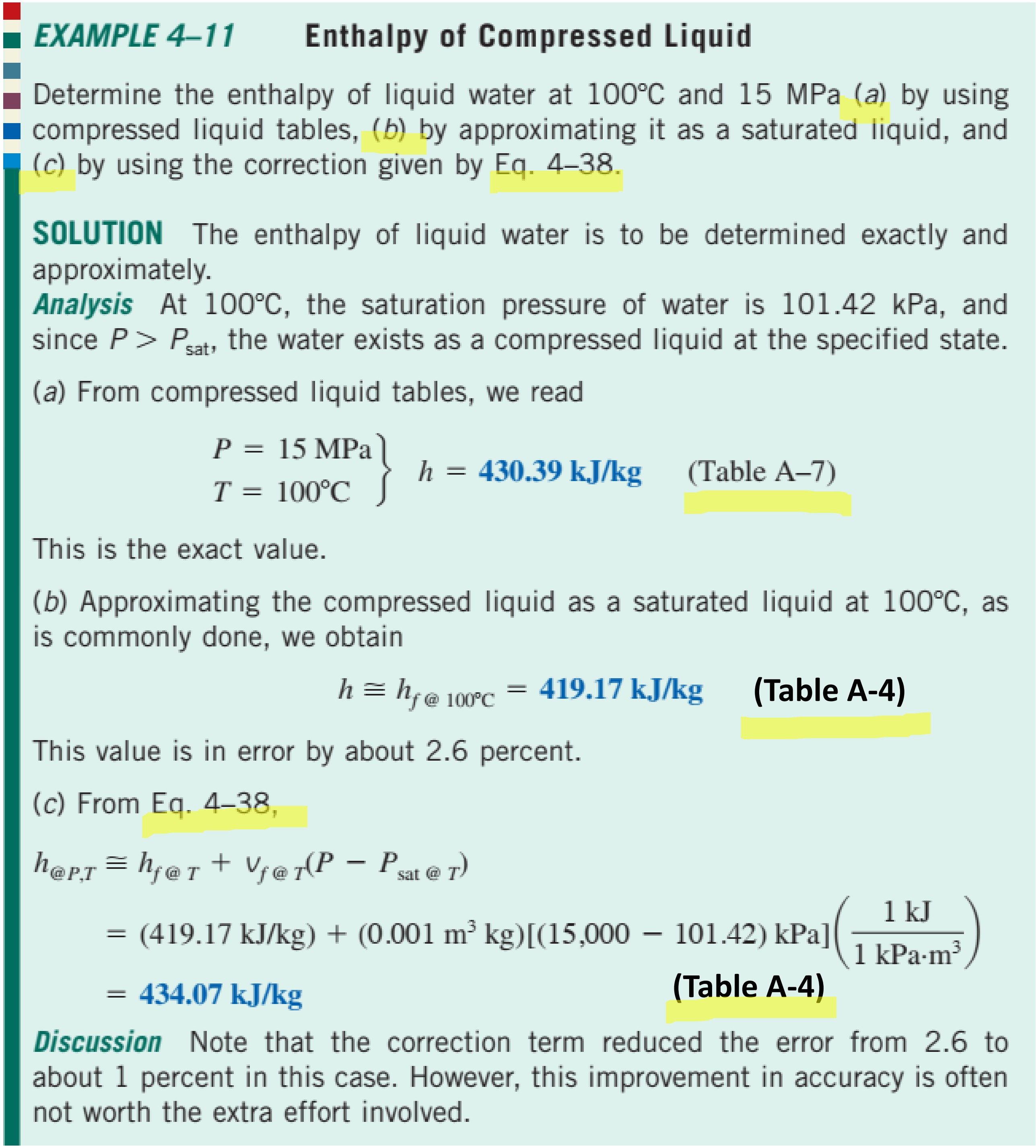
which differs from the tabulated value by 0.8 percent.

(*c*) The average value of the constant-volume specific heat $c_{v,\text{avg}}$ is determined from Table A–2*b* at the average temperature of $(T_1 + T_2)/2 = 450$ K to be

$$c_{v,\text{avg}} = c_{v\,@\,450\text{ K}} = 0.733 \text{ kJ/kg·K}$$

Thus,

$$\Delta u = c_{v,\text{avg}}(T_2 - T_1) = (0.733 \text{ kJ/kg·K})[(600 - 300)\text{K}]$$
$$= 220 \text{ kJ/kg}$$



EXAMPLE 4–11 **Enthalpy of Compressed Liquid**

Determine the enthalpy of liquid water at 100°C and 15 MPa (*a*) by using compressed liquid tables, (*b*) by approximating it as a saturated liquid, and (*c*) by using the correction given by Eq. 4–38.

SOLUTION The enthalpy of liquid water is to be determined exactly and approximately.

Analysis At 100°C, the saturation pressure of water is 101.42 kPa, and since $P > P_{\text{sat}}$, the water exists as a compressed liquid at the specified state.

(*a*) From compressed liquid tables, we read

$$\left.\begin{aligned} P &= 15\text{ MPa} \\ T &= 100°\text{C} \end{aligned}\right\} \quad h = \mathbf{430.39\ kJ/kg} \qquad \text{(Table A–7)}$$

This is the exact value.

(*b*) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain

$$h \cong h_{f\,@\,100°\text{C}} = \mathbf{419.17\ kJ/kg} \qquad \textbf{(Table A-4)}$$

This value is in error by about 2.6 percent.

(*c*) From Eq. 4–38,

$$\begin{aligned} h_{@P,T} &\cong h_{f\,@\,T} + v_{f\,@\,T}(P - P_{\text{sat}\,@\,T}) \\ &= (419.17\text{ kJ/kg}) + (0.001\text{ m}^3\text{ kg})[(15{,}000 - 101.42)\text{ kPa}]\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) \\ &= \mathbf{434.07\ kJ/kg} \qquad \textbf{(Table A-4)} \end{aligned}$$

Discussion Note that the correction term reduced the error from 2.6 to about 1 percent in this case. However, this improvement in accuracy is often not worth the extra effort involved.