Chapter 9

GAS POWER CYCLES

(Otto, Diesel, Brayton)

Thermodynamics: An Engineering Approach
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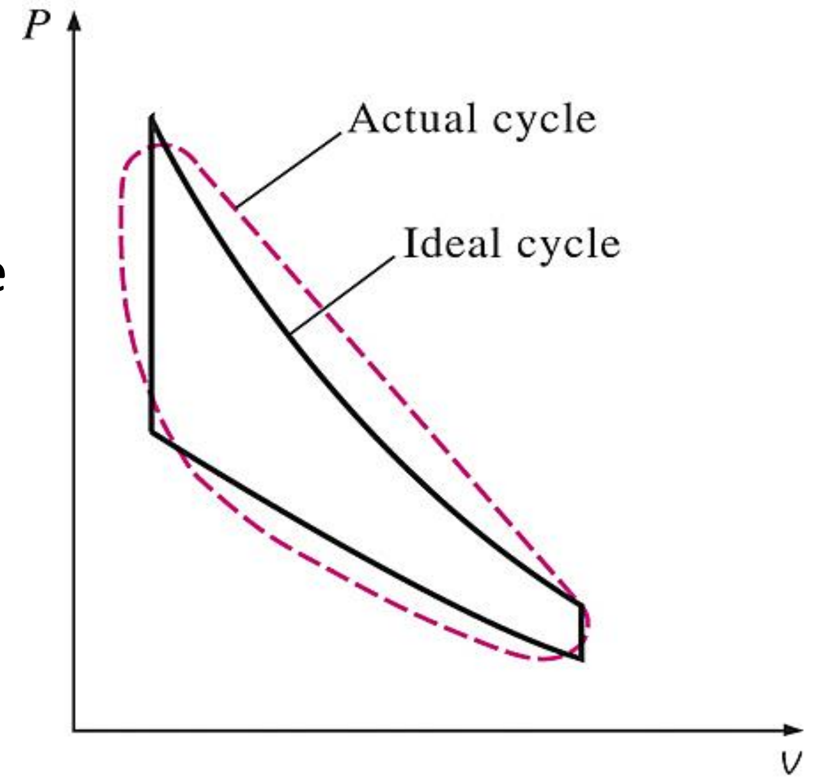
BASIC CONSIDERATIONS IN THE ANALYSISOF POWER CYCLES

Thermal efficiency of heat engines

$$\eta_{th} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$

- Most power-producing devices operate on cycles.
- **Ideal cycle:** A cycle that resembles the actual cycle closely but is made up of *internally reversible* processes.
- **Carnot cycle** have the highest thermal efficiency of all heat engines operating between the same temperature levels.
- Unlike ideal cycles, **Carnot cycles** are *totally reversible*, and unsuitable as a realistic model.



The analysis of many complex processes can be reduced to a manageable level by utilizing some idealizations.

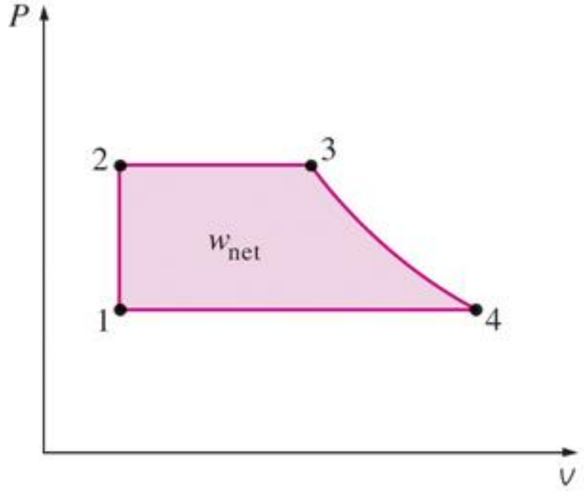
- On a ***T-s* diagram**, the ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the **thermal efficiency** of the cycle.

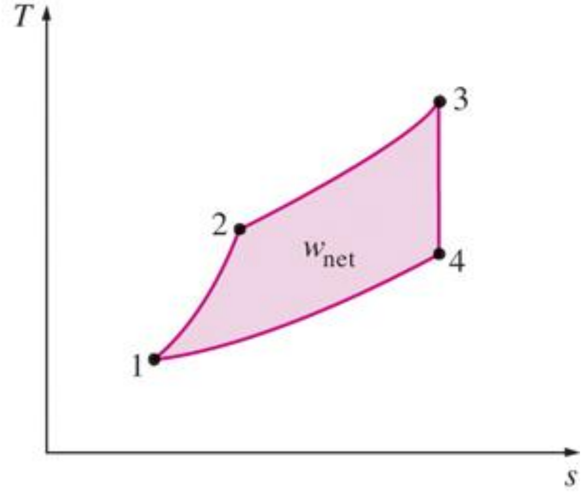
- **Any modification that increases the ratio of these two areas will also increase the thermal efficiency of the cycle.**

Idealizations / simplifications in the analysis of power cycles:

1. The cycle does not involve any *friction.* Therefore, the working fluid does not experience any *pressure drop* as it flows in pipes or devices such as heat exchangers.
2. All expansion and compression processes take place in a *quasi-equilibrium* manner.
3. The pipes connecting the various components of a system are *well insulated,* and *heat transfer* through them is *negligible.*

On both *P-v* and *T-s* diagrams, the area enclosed by the process curve represents the net work of the cycle.

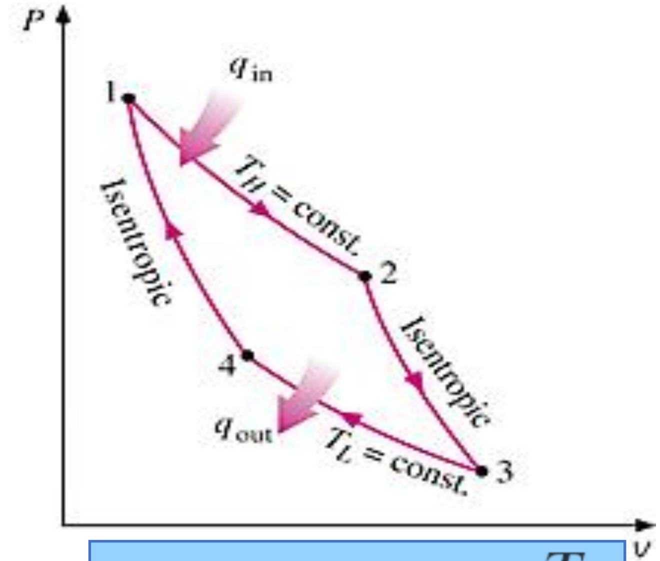




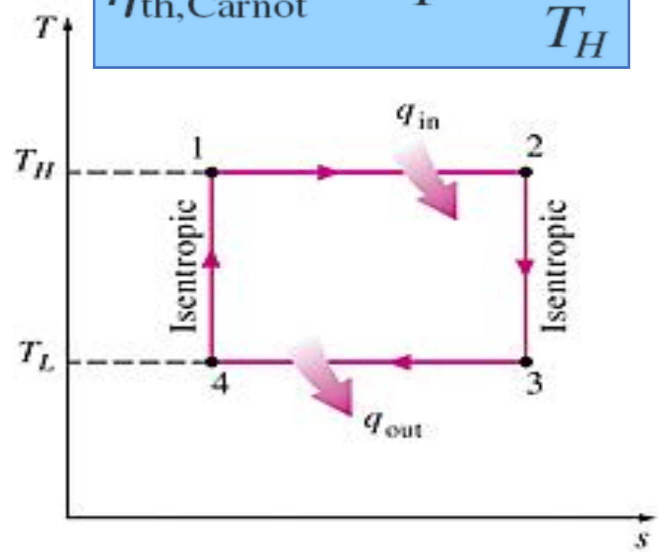
CARNOT CYCLE AND ITS VALUE IN ENG.

- **Carnot cycle is composed of four totally reversible processes** (*isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression*).

- ***For both ideal and actual cycles:*** *Thermal efficiency increases with:*
  1. *Increase in the temperature at which heat is supplied to the system, or*
  2. *Decrease in the temperature at which heat is rejected from the system.*

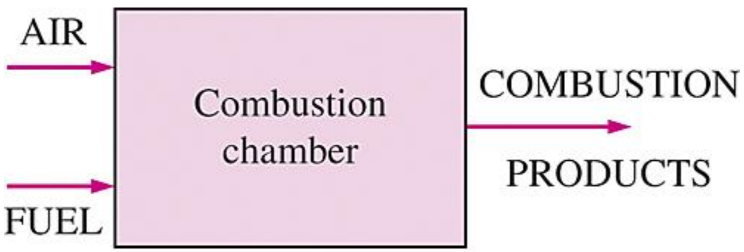


$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

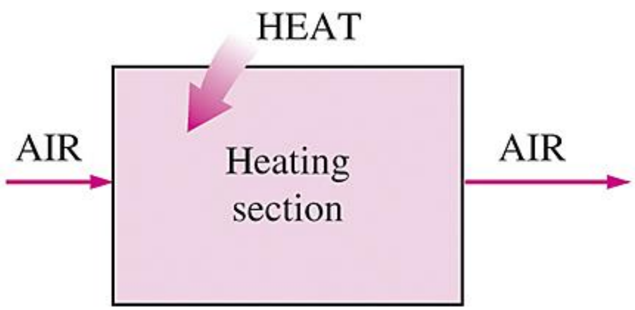


***P-v* and *T-s* diagrams of a Carnot cycle.**

AIR-STANDARD ASSUMPTIONS



(*a*) Actual



(*b*) Ideal

The combustion process is replaced by a heat-addition process in ideal cycles.

Air-standard assumptions:

1. The working fluid is air**,** which *continuously circulates* in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are *internally reversible*.
3. The combustion process is replaced by a ***heat-addition*** process from an external source.
4. The exhaust process is replaced by a ***heat-rejection*** process that restores the working fluid to its initial state.

Cold-air-standard assumptions: When the working fluid is considered to be air with constant specific heats at ***room temperature (25°C).***

Air-standard cycle: A cycle for which the air-standard assumptions are applicable.

AN OVERVIEW OF RECIPROCATING ENGINES:

1) Spark-ignition (SI) engines
2) Compression-ignition (CI) engines

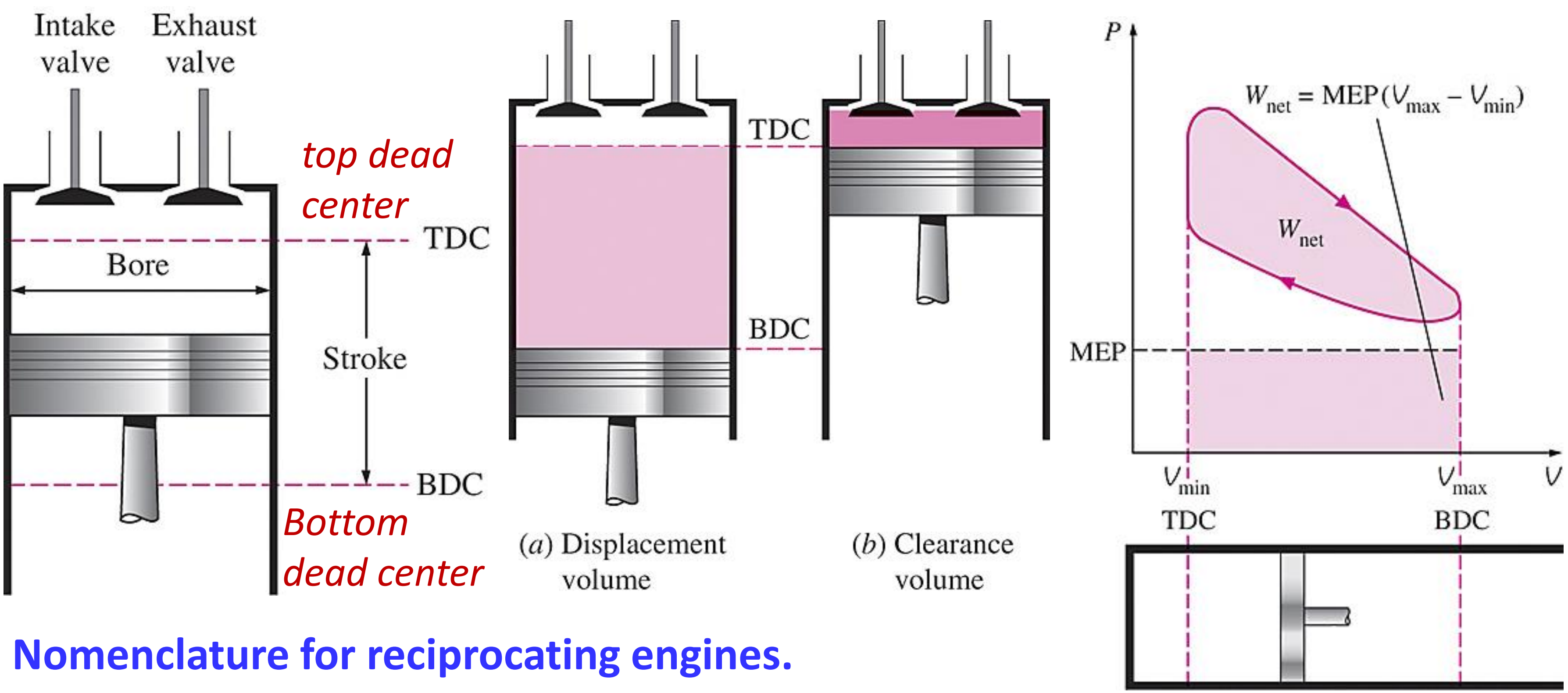
Compression ratio

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$

Mean effective pressure

$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \quad \text{(kPa)}$$

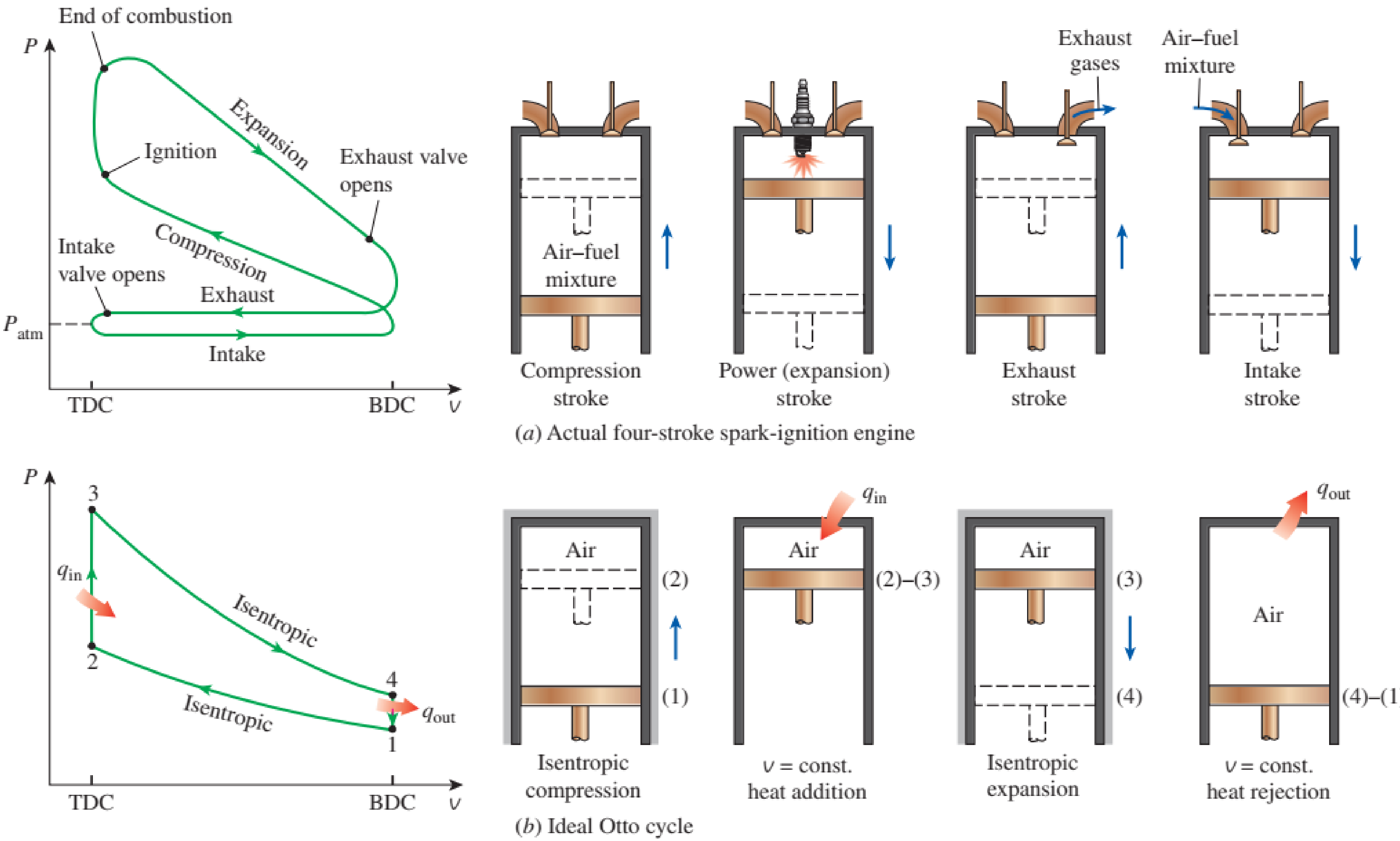
$$W_{net} = \text{MEP} \times \text{Piston area} \times \text{Stroke} = \text{MEP} \times \text{Displacement volume}$$



Nomenclature for reciprocating engines.

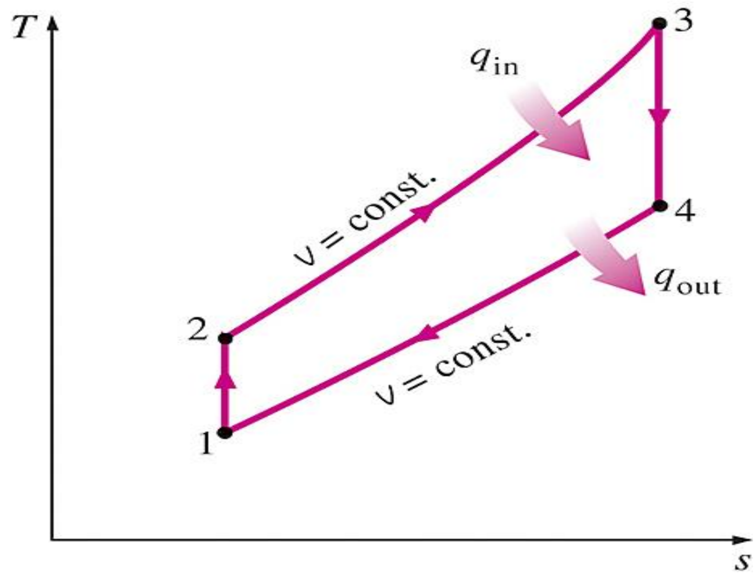
OTTO CYCLE: IDEAL CYCLE FOR SPARK-IGNITION (SI) ENGINES

1-2	Isentropic compression
2-3	Constant-volume heat addition
3-4	Isentropic expansion
4-1	Constant-volume heat rejection



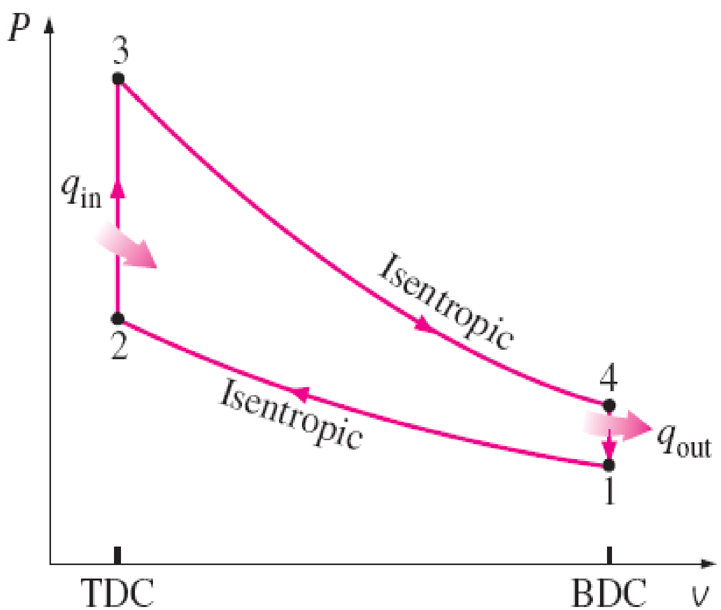
Actual and ideal cycles in spark-ignition (SI) engines and their *P-v* diagrams.

IDEAL OTTO CYCLE



T-s diagram of the ideal Otto cycle

1-2 Isentropic compression
2-3 Constant-volume heat addition
3-4 Isentropic expansion
4-1 Constant-volume heat rejection



P-v diagram of the ideal Otto cycle

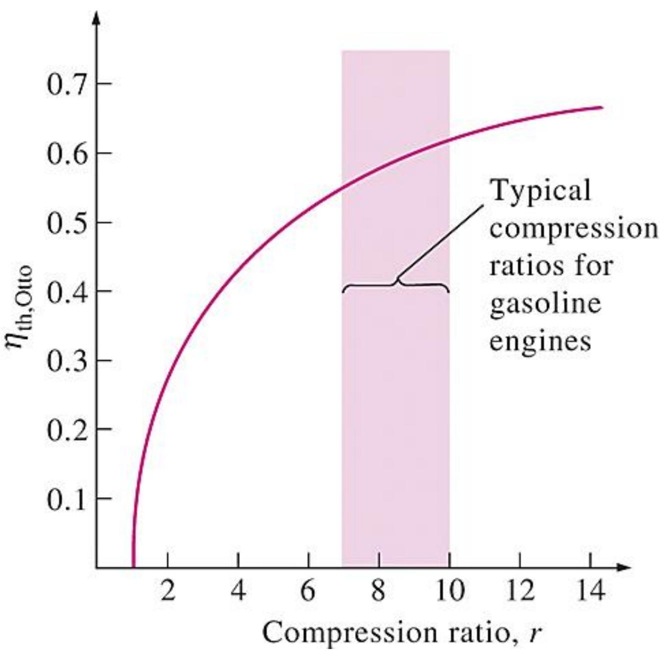
OTTO CYCLE

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$



Thermal efficiency of the ideal Otto cycle as a function of compression ratio (*k* = 1.4).

Use adiabatic relationships:

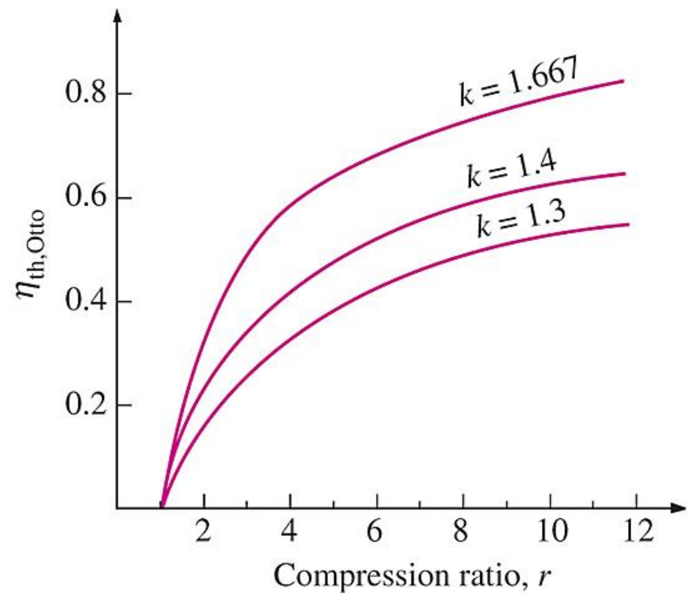
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

Considering:

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$

$$r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$$



The thermal efficiency of the Otto cycle increases with the specific heat ratio *k* of the working fluid.

EXAMPLE 9–2 **The Ideal Otto Cycle**

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (*a*) the maximum temperature and pressure that occur during the cycle, (*b*) the net work output, (*c*) the thermal efficiency, and (*d*) the mean effective pressure for the cycle.

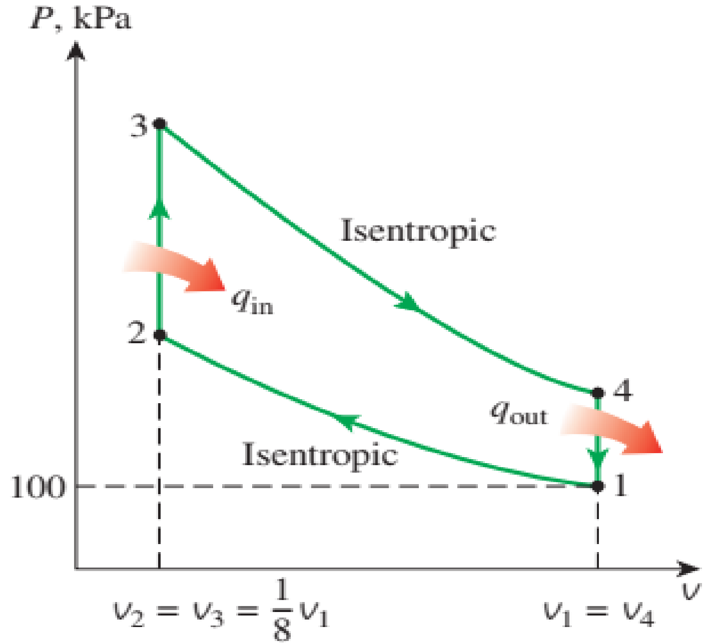


FIGURE 9–19

P-*v* diagram for the Otto cycle discussed in Example 9–2.

(*a*) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). But first we need to determine the temperature and pressure of air at the end of the isentropic compression process (state 2), using data from Table A–17:

$$T_1 = 290 \text{ K} \quad \rightarrow \quad u_1 = 206.91 \text{ kJ/kg}$$
$$v_{r1} = 676.1$$

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \quad \rightarrow \quad T_2 = 652.4 \text{ K}$$
$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$$
$$= (100 \text{ kPa})\left(\frac{652.4 \text{ K}}{290 \text{ K}}\right)(8) = 1799.7 \text{ kPa}$$

Process 2-3 (constant-volume heat addition):

$$q_{\text{in}} = u_3 - u_2$$
$$800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$$
$$u_3 = 1275.11 \text{ kJ/kg} \quad \rightarrow \quad T_3 = \mathbf{1575.1 \text{ K}}$$
$$v_{r3} = 6.108$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right)$$

$$= (1.7997 \text{ MPa})\left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right)(1) = \mathbf{4.345 \text{ MPa}}$$

(*b*) The net work output for the cycle is determined either by finding the boundary ($P\,dV$) work involved in each process by integration and adding them or by finding the net heat transfer that is equivalent to the net work done during the cycle. We take the latter approach. However, first we need to find the internal energy of the air at state 4:

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = r v_{r3} = (8)(6.108) = 48.864 \quad \rightarrow \quad T_4 = 795.6 \text{ K}$$

$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1$$

$$q_{out} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

Thus,

$$w_{net} = q_{net} = q_{in} - q_{out} = 800 - 381.83 = \mathbf{418.17 \text{ kJ/kg}}$$

(*c*) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = \mathbf{0.523 \text{ or } 52.3\%}$$

Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be (Eq. 9–8)

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \text{ or } 56.5\%$$

which is considerably different from the value obtained above. Therefore, care should be exercised in utilizing the cold-air-standard assumptions.

(*d*) The mean effective pressure is determined from its definition, Eq. 9–4:

$$\text{MEP} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1 - v_1/r} = \frac{w_{net}}{v_1(1 - 1/r)}$$

where

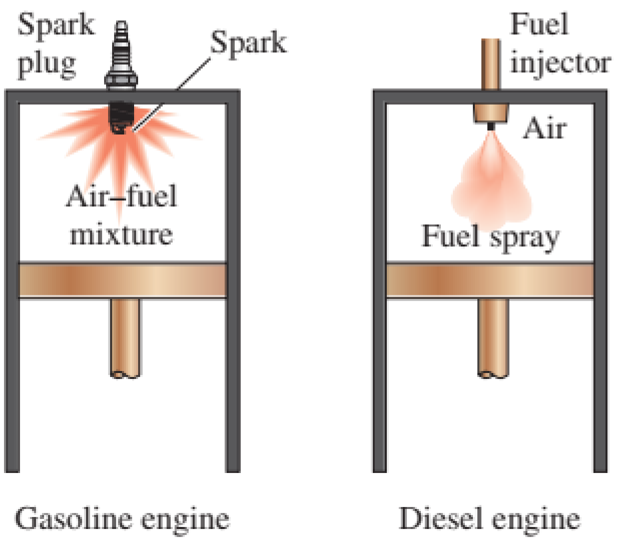
$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290 \text{ K})}{100 \text{ kPa}} = 0.8323 \text{ m}^3/\text{kg}$$

Thus,

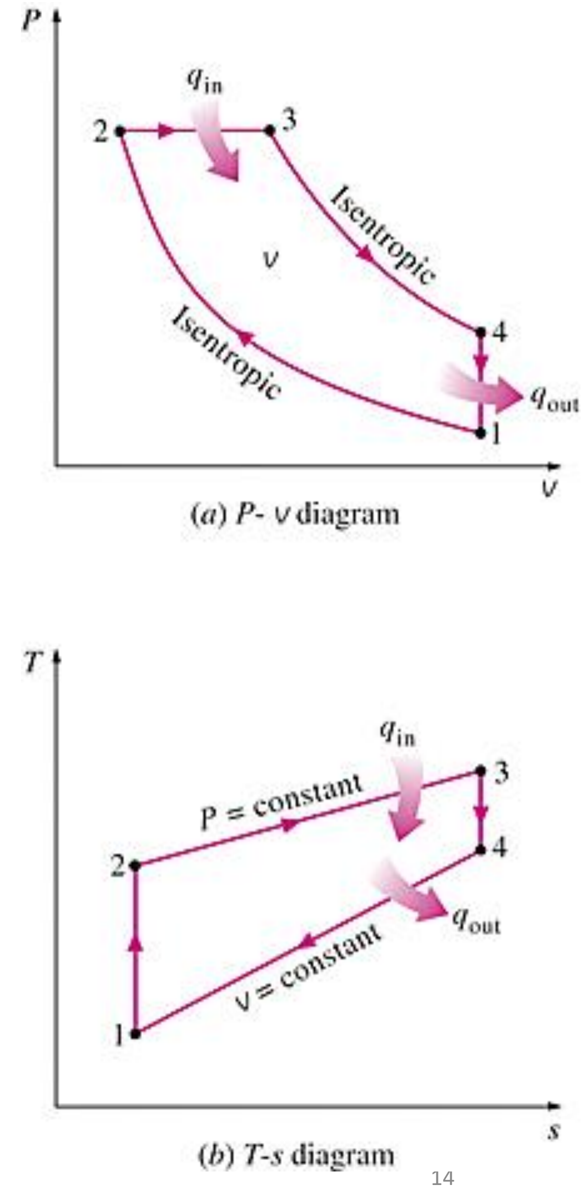
$$\text{MEP} = \frac{418.17 \text{ kJ/kg}}{(0.8323 \text{ m}^3/\text{kg})(1 - \frac{1}{8})}\left(\frac{1 \text{ kPa}\cdot\text{m}^3}{1 \text{ kJ}}\right) = \mathbf{574 \text{ kPa}}$$

DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION (CI) ENGINES

- In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition (engine knock).
- Diesel engines can be designed to operate at much higher compression ratios than SI engines, typically between 12 and 24.

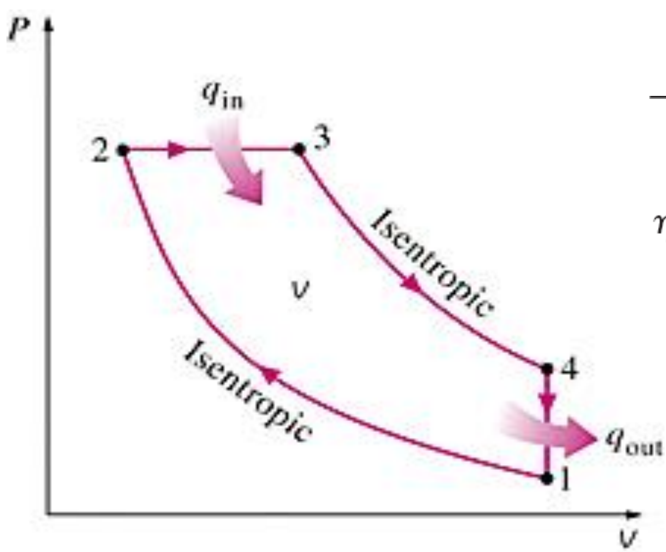


- **1-2** isentropic compression
- **2-3** constant-volume heat addition
- **3-4** isentropic expansion
- **4-1** constant-volume heat rejection.



In diesel engines, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.

DIESEL CYCLE



(a) P- v diagram

$$q_{in} - w_{b,out} = u_3 - u_2 \rightarrow q_{in} = P_2(v_3 - v_2) + (u_3 - u_2)$$
$$= h_3 - h_2 = c_p(T_3 - T_2)$$
$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Diesel} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

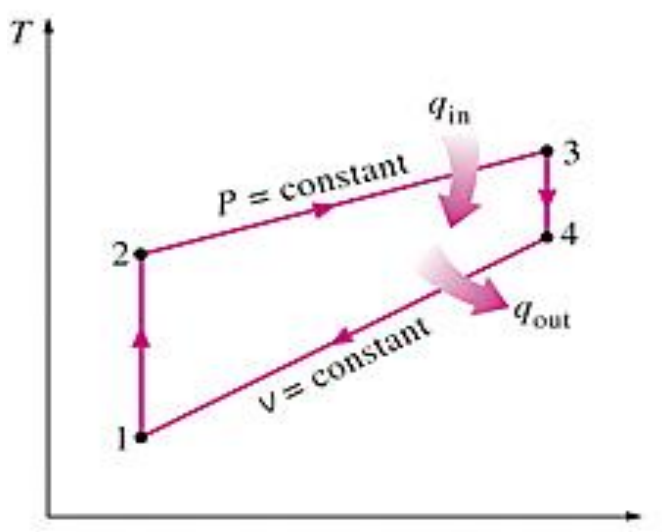
$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

Cutoff ratio

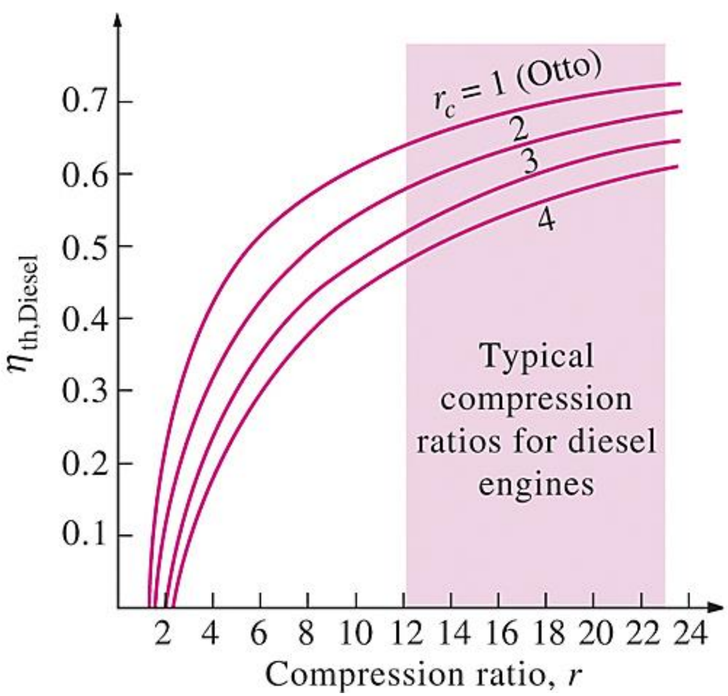
$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

$$\eta_{th,Otto} > \eta_{th,Diesel}$$

for the same compression ratio

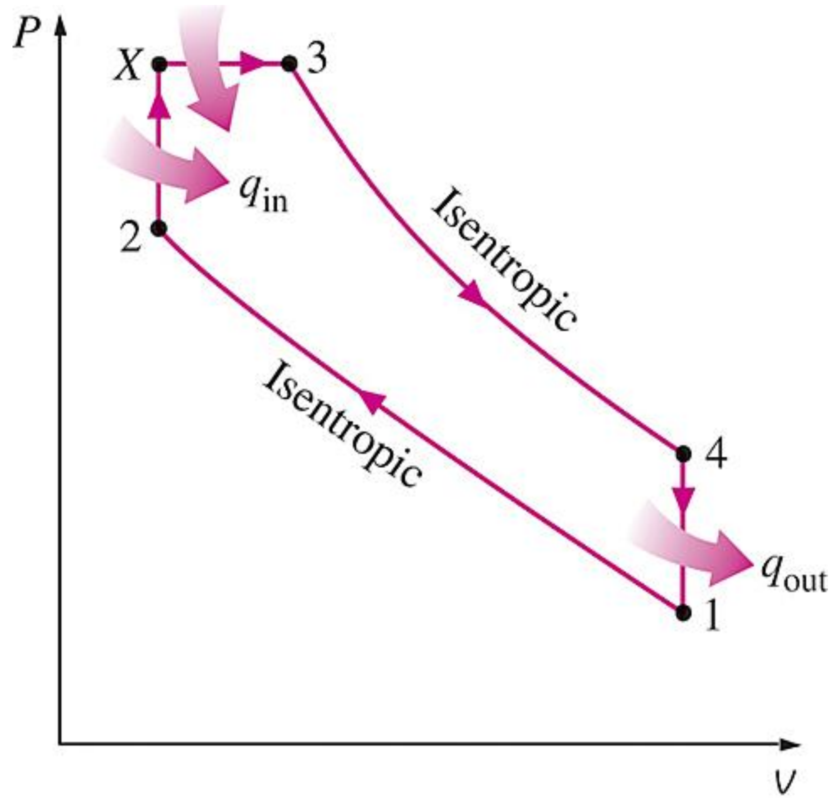


(b) T-s diagram



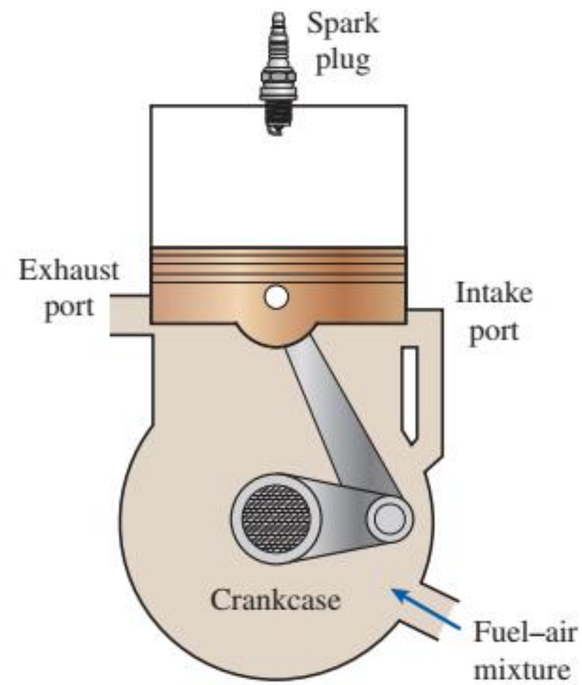
Thermal efficiency of the ideal Diesel cycle as a function of compression and cutoff ratios (*k*=1.4).

Dual cycle: **A more realistic ideal cycle model for modern, high-speed compression ignition engine.**



P-v diagram of an ideal dual cycle

- **The two-stroke engines are generally less efficient than their four-stroke counterparts, but they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios.**



Schematic of a two-stroke reciprocating engine.

BRAYTON CYCLE:
THE IDEAL CYCLE FOR GAS-TURBINE ENGINES

- **The combustion process is replaced by a *constant-pressure heat-addition* process from an external source, and the exhaust process is replaced by a *constant-pressure heat-rejection* process to the ambient air.**

  1-2 Isentropic compression (in a compressor)
  2-3 Constant-pressure heat addition
  3-4 Isentropic expansion (in a turbine)
  4-1 Constant-pressure heat rejection

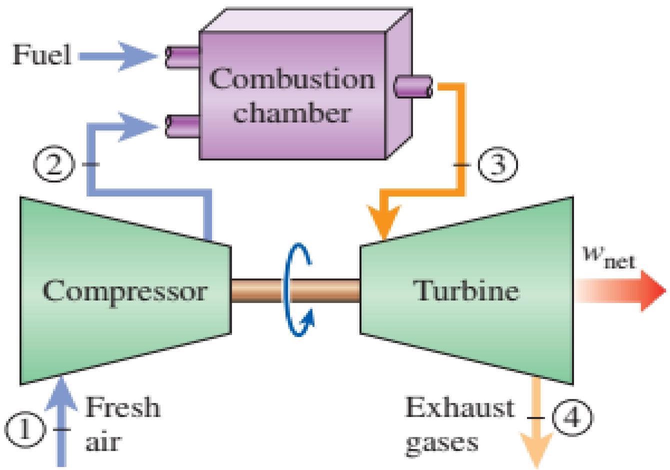


FIGURE 9–29
An open-cycle gas-turbine engine.

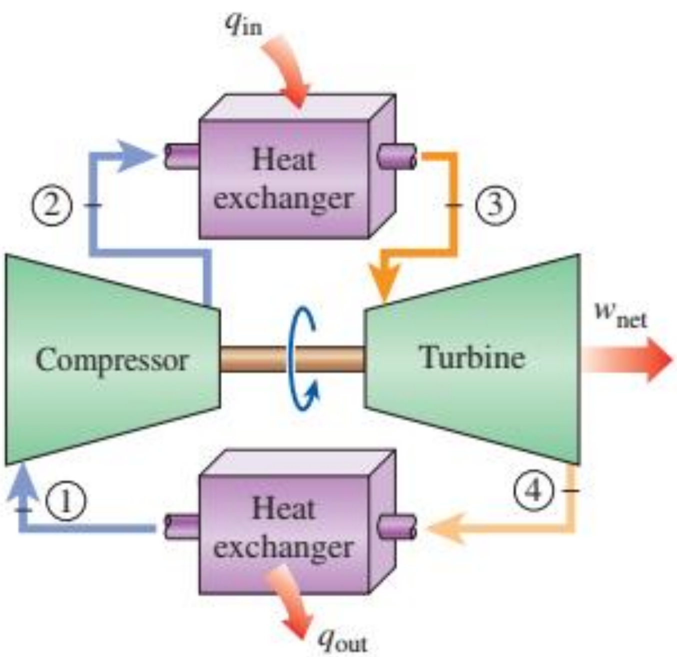
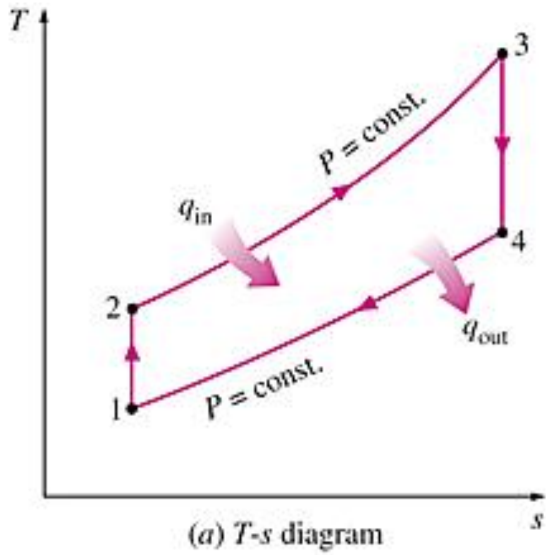


FIGURE 9–30
A closed-cycle gas-turbine engine.

An open-cycle gas-turbine engine.

A closed-cycle gas-turbine engine.



(a) T-s diagram

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

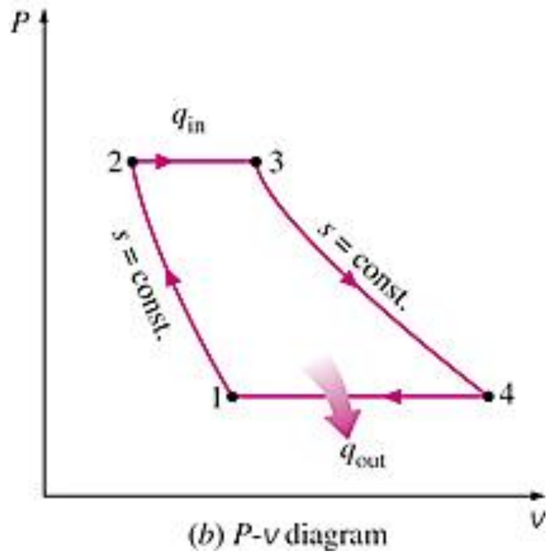
$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

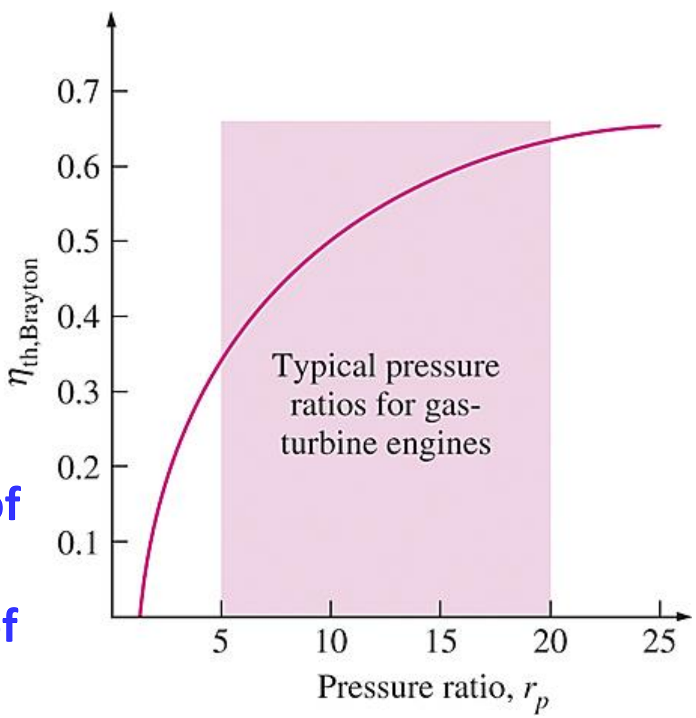
$$r_p = \frac{P_2}{P_1}$$

Pressure ratio



(b) P-v diagram

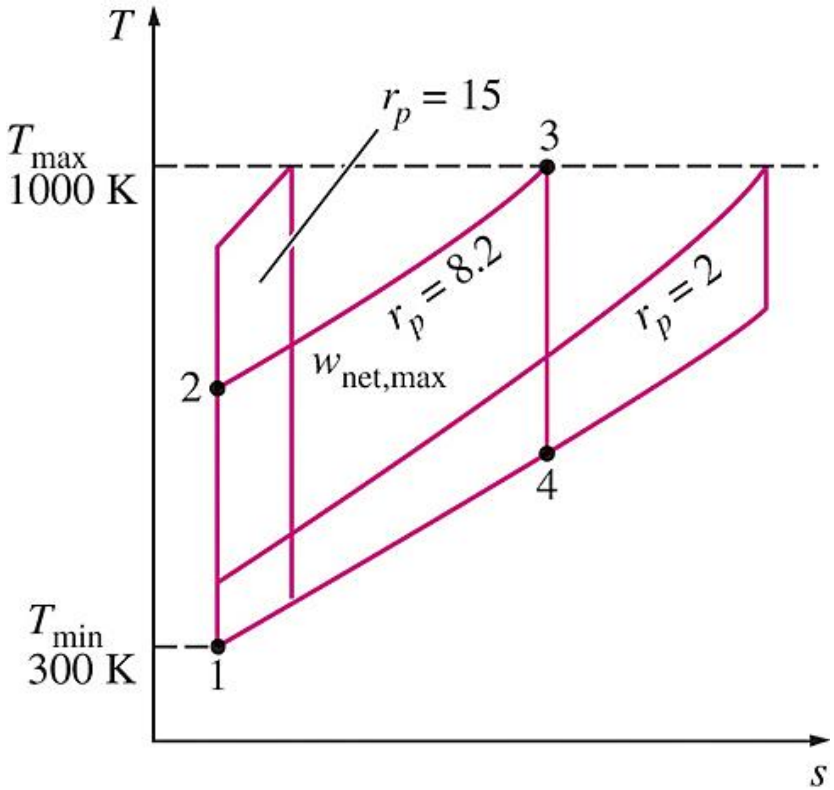
$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$



***T-s* and *P-v* diagrams for the ideal Brayton cycle.**

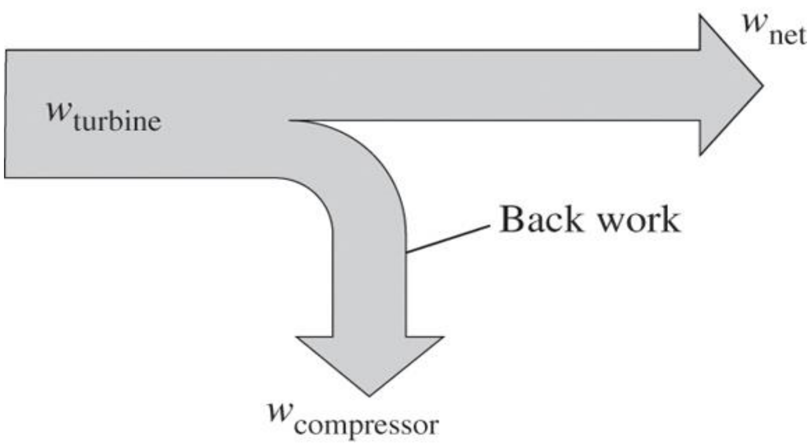
Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.

- **The two major application areas of gas-turbine engines are *aircraft propulsion* and *electric power generation.***



For fixed values of T_{min} and T_{max}, the net work of the Brayton cycle first increases with the pressure ratio, then reaches a maximum at $r_p = (T_{max}/T_{min})^{k/[2(k-1)]}$, and finally decreases.

- The highest temperature in the cycle is limited by the maximum temperature that the turbine blades can withstand. This also limits the pressure ratios that can be used in the cycle.
- The air in gas turbines supplies the necessary oxidant for the combustion of the fuel, and it serves as a coolant to keep the temperature of various components within safe limits. An air–fuel ratio of 50 or above is not uncommon.



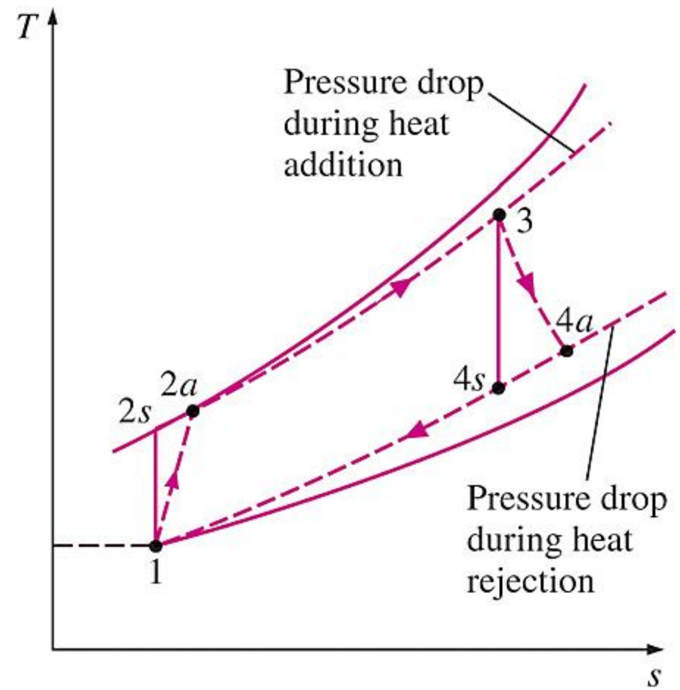
The fraction of the turbine work used to drive the compressor is called the *back work ratio*.

Development of Gas Turbines

1. Increasing the turbine inlet (or firing) temperatures
2. Increasing the efficiencies of turbomachinery (turbines, compressors)
3. Adding modifications to the basic cycle (intercooling, regeneration, and reheating).

Deviation of Actual Gas-Turbine Cycles from Idealized Ones

Reasons: Irreversibilities in turbine and compressors, pressure drops, heat losses



The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

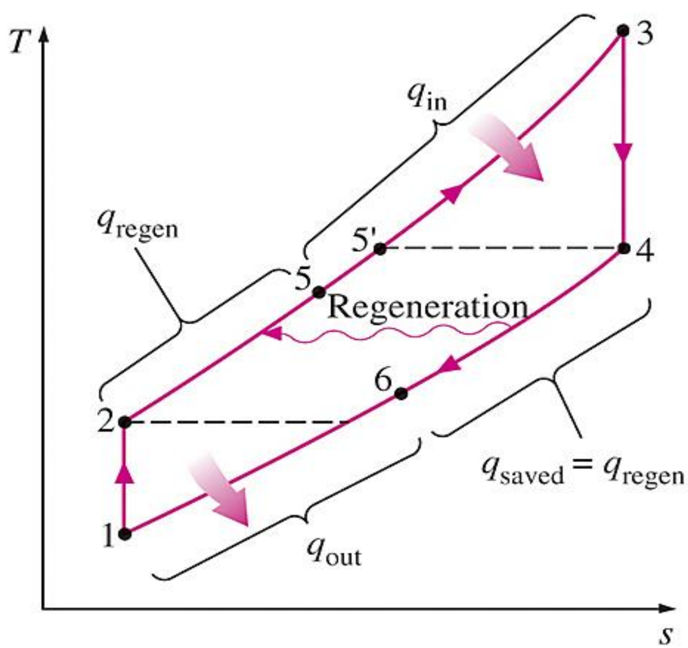
Isentropic efficiencies of the compressor and turbine:

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

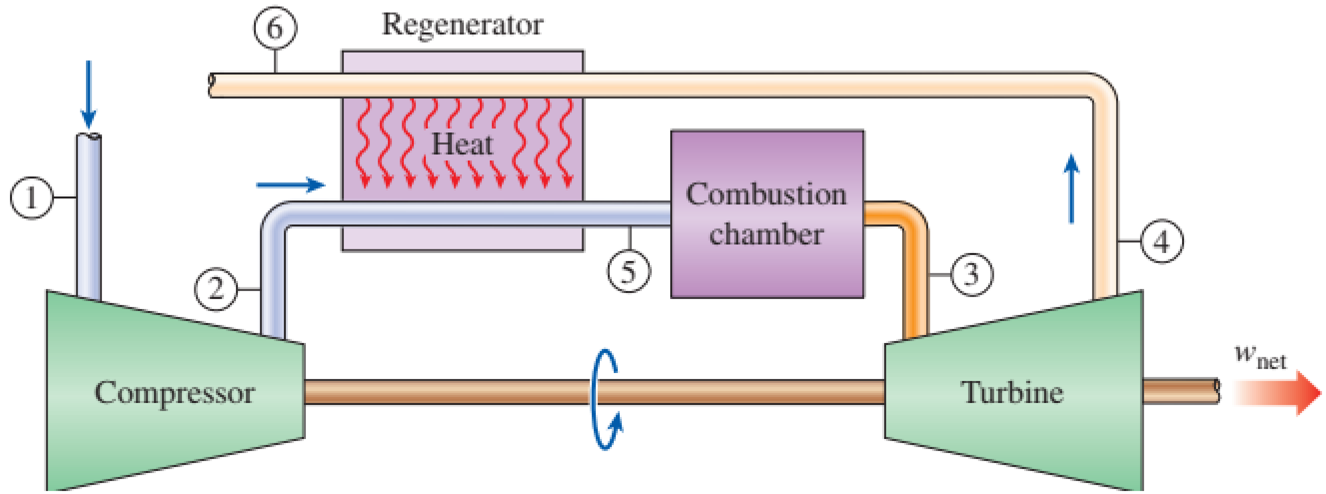
THE BRAYTON CYCLE WITH REGENERATION-1

- In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.
- Therefore, the high-pressure air leaving the compressor can be heated by the hot exhaust gases in a counter-flow heat exchanger (**a *regenerator***).
- The *thermal efficiency of the Brayton cycle increases* as a result of regeneration since less fuel is used for the same work output.

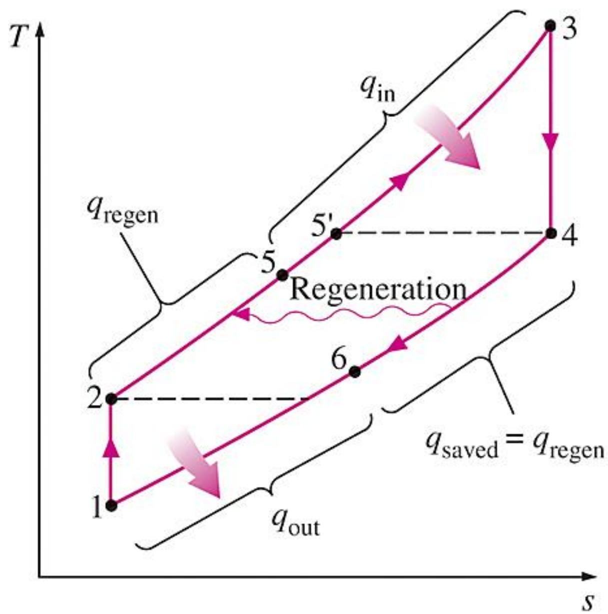


***T-s* diagram of a Brayton cycle with regeneration.**

THE BRAYTON CYCLE WITH REGENERATION-2



A gas-turbine engine with regenerator.



T-s diagram of a Brayton cycle with regeneration.

$$q_{regen,act} = h_5 - h_2$$

$$q_{regen,max} = h_{5'} - h_2 = h_4 - h_2$$

Effectiveness of regenerator:

$$\epsilon = \frac{q_{regen,act}}{q_{regen,max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2}$$

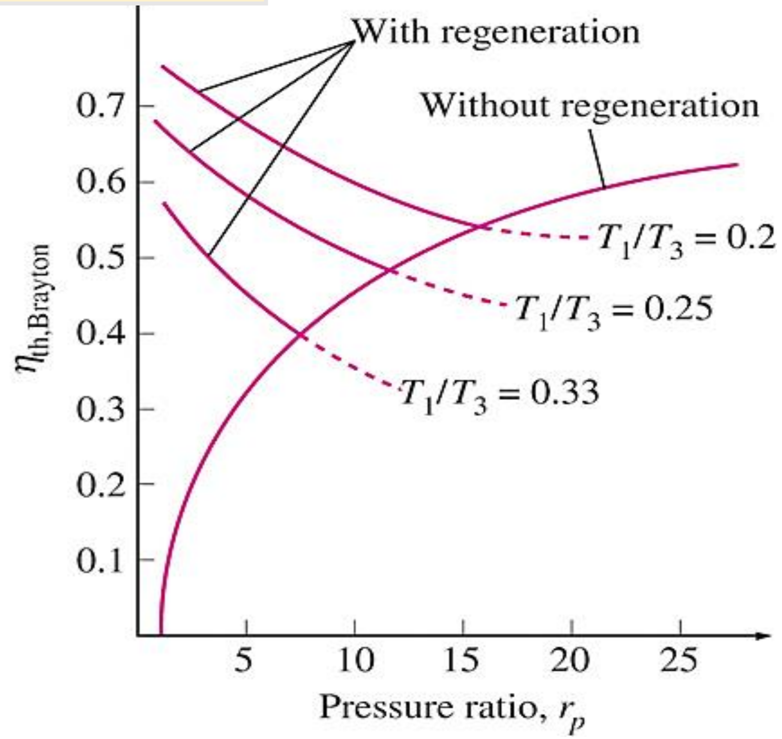
Effectiveness under cold-air standard assumptions

$$\eta_{\text{th,regen}} = 1 - \left(\frac{T_1}{T_3}\right)(r_p)^{(k-1)/k}$$

Under cold-air standard assumptions

Can regeneration be used at high pressure ratios?

- The thermal efficiency depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio.
- Regeneration is most effective at lower pressure ratios and low minimum-to-maximum temperature ratios.



Thermal efficiency of the ideal Brayton cycle with and without regeneration.

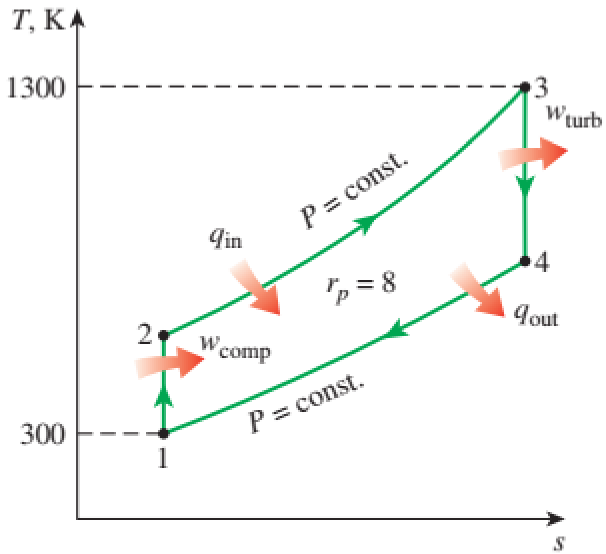


FIGURE 9–35
T-s diagram for the Brayton cycle discussed in Example 9–5.

EXAMPLE 9–5 The Simple Ideal Brayton Cycle

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (*a*) the gas temperature at the exits of the compressor and the turbine, (*b*) the back work ratio, and (*c*) the thermal efficiency.

SOLUTION A power plant operating on the ideal Brayton cycle is considered. The compressor and turbine exit temperatures, back work ratio, and the thermal efficiency are to be determined.
Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** The variation of specific heats with temperature is to be considered.
Analysis The *T-s* diagram of the ideal Brayton cycle described is shown in Fig. 9–35. We note that the components involved in the Brayton cycle are steady-flow devices.

(*a*) The air temperatures at the compressor and turbine exits are determined from isentropic relations:

Process 1–2 (isentropic compression of an ideal gas):

$$T_1 = 300\text{ K} \rightarrow h_1 = 300.19\text{ kJ/kg}$$

$$P_{r1} = 1.386$$

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = \mathbf{540\ K} \quad \text{(at compressor exit)}$$

$$h_2 = 544.35\text{ kJ/kg}$$

Process 3–4 (isentropic expansion of an ideal gas):

$$T_3 = 1300\text{ K} \rightarrow h_3 = 1395.97\text{ kJ/kg}$$

$$P_{r3} = 330.9$$

$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36 \rightarrow T_4 = \mathbf{770\ K} \quad \text{(at turbine exit)}$$

$$h_4 = 789.37\text{ kJ/kg}$$

(*b*) To find the back work ratio, we need to find the work input to the compressor and the work output of the turbine:

$$w_{\text{comp,in}} = h_2 - h_1 = 544.35 - 300.19 = 244.16\text{ kJ/kg}$$

$$w_{\text{turb,out}} = h_3 - h_4 = 1395.97 - 789.37 = 606.60\text{ kJ/kg}$$

Thus,

$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{244.16\ \text{kJ/kg}}{606.60\ \text{kJ/kg}} = \mathbf{0.403}$$

That is, 40.3 percent of the turbine work output is used just to drive the compressor.

(*c*) The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

$$q_{in} = h_3 - h_2 = 1395.97 - 544.35 = 851.62\ \text{kJ/kg}$$

$$w_{net} = w_{out} - w_{in} = 606.60 - 244.16 = 362.4\ \text{kJ/kg}$$

Thus,

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{362.4\ \text{kJ/kg}}{851.62\ \text{kJ/kg}} = \mathbf{0.426} \text{ or } \mathbf{42.6\%}$$

The thermal efficiency could also be determined from

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

where

$$q_{out} = h_4 - h_1 = 789.37 - 300.19 = 489.2\ \text{kJ/kg}$$

EXAMPLE 9–6 **An Actual Gas-Turbine Cycle**

Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (*a*) the back work ratio, (*b*) the thermal efficiency, and (*c*) the turbine exit temperature of the gas-turbine cycle discussed in Example 9–5.

SOLUTION The Brayton cycle discussed in Example 9–5 is reconsidered. For specified turbine and compressor efficiencies, the back work ratio, the thermal efficiency, and the turbine exit temperature are to be determined.

Analysis (*a*) The *T-s* diagram of the cycle is shown in Fig. 9–37. The actual compressor work and turbine work are determined by using the definitions of compressor and turbine efficiencies, Eqs. 9–19 and 9–20:

Compressor: $$w_{\text{comp,in}} = \frac{w_s}{\eta_C} = \frac{244.16 \text{ kJ/kg}}{0.80} = 305.20 \text{ kJ/kg}$$

Turbine: $$w_{\text{turb,out}} = \eta_T w_s = (0.85)(606.60 \text{ kJ/kg}) = 515.61 \text{ kJ/kg}$$

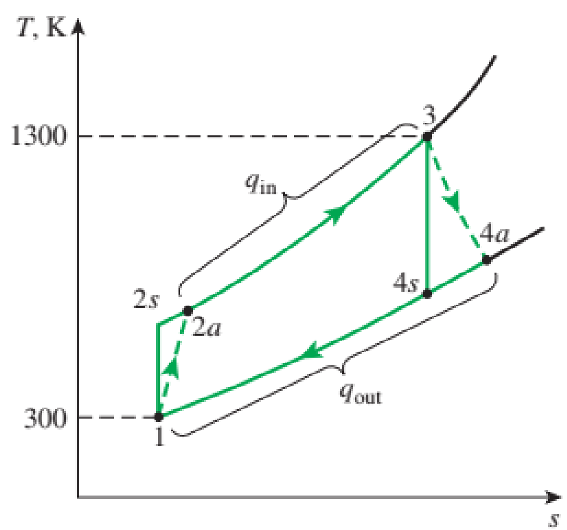


FIGURE 9–37
T-s diagram of the gas-turbine cycle discussed in Example 9–6.

Thus,

$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{305.20 \text{ kJ/kg}}{515.61 \text{ kJ/kg}} = \mathbf{0.592}$$

That is, the compressor is now consuming 59.2 percent of the work produced by the turbine (up from 40.3 percent). This increase is due to the irreversibilities that occur within the compressor and the turbine.

(*b*) In this case, air leaves the compressor at a higher temperature and enthalpy, which are determined to be

$$\begin{aligned} w_{comp,in} = h_{2a} - h_1 \rightarrow h_{2a} &= h_1 + w_{comp,in} \\ &= 300.19 + 305.20 \\ &= 605.39 \text{ kJ/kg} \quad (\text{and } T_{2a} = 598 \text{ K}) \end{aligned}$$

Thus,

$$q_{in} = h_3 - h_{2a} = 1395.97 - 605.39 = 790.58 \text{ kJ/kg}$$

$$w_{net} = w_{out} - w_{in} = 515.61 - 305.20 = 210.41 \text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{210.41 \text{ kJ/kg}}{790.58 \text{ kJ/kg}} = \mathbf{0.266} \text{ or } \mathbf{26.6\%}$$

That is, the irreversibilities occurring within the turbine and compressor caused the thermal efficiency of the gas turbine cycle to drop from 42.6 to 26.6 percent. This example shows how sensitive the performance of a gas-turbine power plant is to the efficiencies of the compressor and the turbine. In fact, gas-turbine efficiencies did not reach competitive values until significant improvements were made in the design of gas turbines and compressors.

(*c*) The air temperature at the turbine exit is determined from an energy balance on the turbine:

$$w_{\text{turb,out}} = h_3 - h_{4a} \rightarrow h_{4a} = h_3 - w_{\text{turb,out}}$$
$$= 1395.97 - 515.61$$
$$= 880.36 \text{ kJ/kg}$$

Then, from Table A–17,

$$T_{4a} = \mathbf{853\ K}$$

Discussion The temperature at turbine exit is considerably higher than that at the compressor exit (T_{2a} = 598 K), which suggests the use of regeneration to reduce fuel cost.

EXAMPLE 9–1 **Derivation of the Efficiency of the Carnot Cycle**

Show that the thermal efficiency of a Carnot cycle operating between the temperature limits of T_H and T_L is solely a function of these two temperatures and is given by Eq. 9–2.

SOLUTION It is to be shown that the efficiency of a Carnot cycle depends on the source and sink temperatures alone.

Analysis The *T-s* diagram of a Carnot cycle is redrawn in Fig. 9–7. All four processes that comprise the Carnot cycle are reversible, and thus the area under each process curve represents the heat transfer for that process. Heat is transferred to the system during process 1-2 and rejected during process 3-4. Therefore, the amount of heat input and heat output for the cycle can be expressed as

$$q_{in} = T_H(s_2 - s_1) \quad \text{and} \quad q_{out} = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$

since processes 2-3 and 4-1 are isentropic, and thus $s_2 = s_3$ and $s_4 = s_1$. Substituting these into Eq. 9–1, we see that the thermal efficiency of a Carnot cycle is

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L(s_2 - s_1)}{T_H(s_2 - s_1)} = 1 - \frac{T_L}{T_H}$$

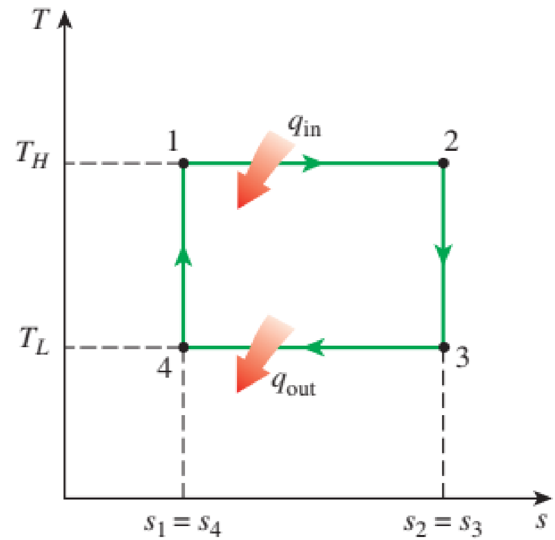


FIGURE 9–7
T-s diagram for Example 9–1.

Discussion Notice that the thermal efficiency of a Carnot cycle is independent of the type of the working fluid used (an ideal gas, steam, etc.) or whether the cycle is executed in a closed or steady-flow system.