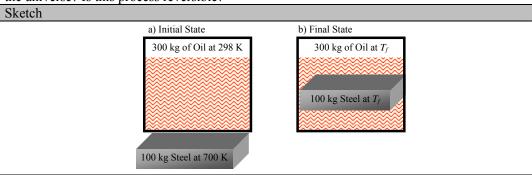
|                          | <b>(A)</b> | <b>(B)</b> | <b>(C)</b> | <b>(D)</b> |
|--------------------------|------------|------------|------------|------------|
| 00                       |            | 0          | (C)<br>O   | 0          |
| 02                       | 0          | •          | 0          | 0          |
| 0 8<br>0 4<br>0 5<br>0 6 | 0          | 0          | 0          |            |
| 04                       | 0          | •          | 0          | 0          |
| 06                       | 0          | 0          |            | 0          |
| 06                       | 0          | 0          | 0          |            |
| 00                       | 0          | 0          |            | 0          |
| 08                       | •          | 0          | 0          | 0          |
| 09                       | 0          | •          | 0          | 0          |
| 00                       | 0          | •          | 0          | 0          |
| 00                       | 0          | 0          | 0          |            |
| 00                       | 0          | 0          | •          | 0          |
| 08                       | 0          | 0          | 0          |            |
| 04                       | 0          | 0          |            | 0          |
| 06                       | 0          | 0          | 0          |            |

#### Question 2 (25 points)

A carbon-steel engine casting  $[C_P = 0.5 \text{ kJ/kg.}^{\circ}\text{C}]$  weighing 100 kg and having a temperature of 700 K is heat-treated to control hardness by quenching in 300 kg of oil  $[C_P = 2.5 \text{ kJ/kg.}^{\circ}\text{C}]$  initially at 298 K. If there are no heat losses from the system, what is the change in entropy of: the casting; the oil; and the universe? Is this process reversible?



## Assumptions

Closed and adiabatic system.

Both steel and oil are incompressible.

Constant heat capacity for the steel and oil.

The final temperature of oil and steel is the same.

Denote steel with the subscript w and oil with the subscript o.

## Summary & Comments

**M.B.** (closed system)  $\Rightarrow M_f = M_i$ 

**Energy B.** (closed system):  $\Rightarrow U_f = U_i$ 

$$m_{w}C_{P,w}(T_{f}-T_{i,w}) = m_{o}C_{P,o}(T_{i,o}-T_{f})$$

$$T_{f} = \frac{m_{w}C_{P,w}T_{i,w} + m_{o}C_{P,o}T_{i,o}}{m_{w}C_{P,w} + m_{o}C_{P,o}} = \frac{(100)(.5)(700) + (300)(2.5)(298)}{(100)(.5) + (300)(2.5)} = 323.125 \text{ K}.$$

**Entropy B.** (closed system):  $\Rightarrow S_f - S_i = S_{\text{gen}}$ 

The entropy change of an incompressible material is given by:

$$\Delta S = m \int_{T}^{T_{f}} \frac{C_{p}}{T} dT = mC_{p} \ln \left( \frac{T_{f}}{T_{i}} \right)$$

Consequently, the entropy change for the oil and steel can be calculated as

$$\Delta S_w = m_w C_{P,w} \ln \left( \frac{T_f}{T_{i,w}} \right) = (100)(0.5) \ln \left( \frac{323.125}{700} \right) = -38.7 \text{ kJ/K}.$$

$$\Delta S_o = m_o C_{P,o} \ln \left( \frac{T_f}{T_{i,o}} \right) = (300)(2.5) \ln \left( \frac{323.125}{298} \right) = 60.65 \text{ kJ/K}.$$

The change in the entropy of the universe is equal to the change in the entropy of the oil and steel or the generation of entropy i.e.,

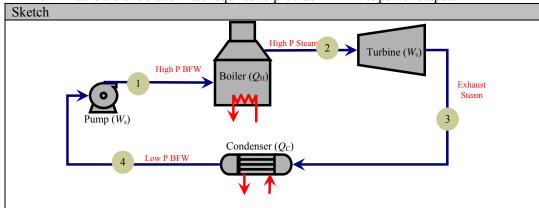
$$\Delta S_{\text{Universe}} = \Delta S_o + \Delta S_w = 60.65 - 38.7 = 21.9 \text{ kJ/K}.$$

Clearly, the generation term is positive indicating that the process is irreversible.

## Question 3 (37.5 points)

A power plant uses the Rankine cycle. The maximum desired temperature in the boiler is 500°C. The turbine operating pressures are 1.6 MPa and 25 kPa.

- 1. What is the efficiency of this plant?
- 2. What would be the efficiency for the plant if the efficiency of the turbine is 80% of the efficiency of the isentropic turbine?
- 3. What is the Carnot's efficiency operating at the temperatures of the boiler and the ambient?
- 4. What is the circulation rate required to provide 1 MW net power output?



# Assumptions

Rankine cycle applies.

Steam tables used for properties.

Turbine and pump are isentropic; condenser is isothermal—isobaric; evaporator is isobaric.

No pressure drop or heat transfer in the pipes.

## Summary & Comments

## **Stream Summary**

| Stream                    | 1              | 2                | 3                | 4              |
|---------------------------|----------------|------------------|------------------|----------------|
| State                     | Subcooled L.   | Superheated V.   | V+L, x=0.9584    | Saturated L.   |
| (quality x)               | (Subcooled L.) | (Superheated V.) | (Superheated V.) | (Saturated L.) |
| $T(\mathbf{K})$           | 338            | 773              | 338              | 338            |
| P (MPa)                   | 1.6            | 1.6              | 0.025            | 0.025          |
| <u>H</u> (kJ/kg)          | 273.8          | 3472.6           | 2519.7 (2710.3)  | 272.0          |
| $\underline{S}$ (kJ/kg.K) | ≈0.8933        | 7.5409           | 7.5409           | 0.8933         |

**Equipment Summary** 

|           | Pump   |        | Boiler |        | Turbine |        | Condenser |         |
|-----------|--------|--------|--------|--------|---------|--------|-----------|---------|
|           | Case 1 | Case 2 | Case 1 | Case 2 | Case 1  | Case 2 | Case 1    | Case 2  |
| W (kJ/kg) | 1.8    | 1.8    | 0      |        | -952.9  | -762.3 | 0         |         |
| O(kJ/kg)  | (      | 0      | 3198.8 |        | 0       |        | -2247.7   | -2438.3 |

**Process Summary** 

| $\eta_1$ $\eta_2$ |      | $\eta_{Carnot}$ | Circulation rate (kg/s) |  |
|-------------------|------|-----------------|-------------------------|--|
| 29.7              | 23.8 | 61.4            | 1.31 (4723 kg/h)        |  |

1. Assume the turbine is isentropic and start solving at the turbine inlet (boiler outlet) since this is the point with specified *T* and *P*.

# a. Turbine

$$\underline{S}_2(T_2 = 773 \text{ K}, P_2 = 1.6 \text{ MPa}) = \underline{S}_3(T_2 = ? \text{ K}, P_2 = 0.025 \text{ MPa}) = 7.5409 \text{ kJ/kg.K}$$

From the steam tables at the outlet pressure; the saturation entropy is higher than the outlet entropy. Consequently, the outlet stream is a mixture of vapor and liquid. Saturation temperature is 338 K and the quality is given by

$$x = \frac{\underline{S}_2 - \underline{S}_{3,L}}{\underline{S}_{3,V} - \underline{S}_{3,L}} = \frac{7.5409 - 0.8933}{7.8928 - 0.8933} = 0.9584$$

The enthalpy of this stream is given by:

$$\underline{H}_3 = x\underline{H}_{3,V} + (1-x)\underline{H}_{3,L} = 2519.7 \text{ kJ/kg}.$$

Energy balance on the turbine

$$W_T = \underline{H}_3(338 \text{ K}; 0.025 \text{ MPa}) - \underline{H}_2(773 \text{ K}; 1.6 \text{ MPa}) = 2519.7 - 3472.6 = -952.9 \text{ kJ/kg}.$$

b. Condenser energy balance

$$Q_C = \underline{H}_{4,L}(338 \text{ K}; 0.025 \text{ MPa}) - \underline{H}_3(338 \text{ K}; 0.025 \text{ MPa}) = 272.0 - 2519.7 = -2247.7 \text{ kJ/kg}.$$

c. **Pump** energy balance

$$W_P = P_1 V_1(338 \text{ K}; 1.6 \text{ MPa}) - P_1 V_{4,L}(338 \text{ K}; 0.025 \text{ MPa})$$
  
=  $(0.001159)(1600) - (0.00102)(25) = 1.83 \text{ kJ/kg}.$ 

$$\underline{H}_1(338 \text{ K}; 1.6 \text{ MPa}) = \underline{H}_{4,L}(338 \text{ K}; 0.025 \text{ MPa}) + W_P = 273.8 \text{ kJ/kg}.$$

d. Boiler energy balance

$$Q_B = \underline{H}_2(773 \text{ K;1.6 MPa}) - \underline{H}_1(338 \text{ K;1.6 MPa}) = 3198.8 \text{ kJ/kg}.$$

e. Cycle efficiency

$$\eta_1 = \frac{|W_T + W_P|}{Q_P} = \frac{|-952.9 + 1.8|}{3198.8} \times 100\% = 29.7\%.$$

- 2. Turbine is 80% efficient.
  - a. Calculate the actual work and from it the outlet enthalpy from the turbine  $W_{T,\text{actual}} = 0.8W_{T,\text{actual}} = (0.8)(-952.9) = -762.3 \text{ kJ/kg}.$

$$\underline{H}_3$$
(? K;0.025 MPa) =  $\underline{H}_2$ (773 K;1.6 MPa) +  $W_{T \text{ actual}}$  = 2710.3 kJ/kg.

Notice that you need a double interpolation to match the outlet enthalpy and pressure to find that the outlet stream is superheated.

- b. Recalculate the condenser duty from the new enthalpy as -2438.3 kJ/kg.
- c. Cycle efficiency

$$\eta_2 = \frac{|W_T + W_P|}{Q_P} = \frac{|-762.3 + 1.8|}{3198.8} \times 100\% = 23.8\%.$$

3. **Carnot engine** efficiency

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{298}{773} \times 100\% = 61.4\%.$$

4. Circulation rate.

The power generation in the previous calculations is based on a kilogram of steam. The circulation rate required is nothing but the ratio between the power output to that of one kilogram i.e.,

$$m = \frac{\text{Power output required}}{\text{Power output/kg}} = \frac{1000}{762.3} = 1.31 \text{ kg/s or } 4723 \text{ kg/h}.$$

#### Comment

The pump consumes less than 1% of the work produced in the turbine. Consequently, the cycle efficiency is determined by the turbine efficiency. Also, notice that the efficiency of the cycle is less than 50% of the maximum possible efficiency i.e., that of a Carnot engine. The circulation rate of about 5 ton steam per hour is on the small to medium size range of power plants. However, notice that the steam used in this plant is of medium classification rather than high pressure steam.