Student Name:

	(A)	(B)	(C)	(D)
⓿❶	●	○	○	○
⓿❷	○	○	●	○
⓿❸	○	○	○	●
⓿❹	○	●	○	○
⓿❺	○	○	●	○
⓿❻	●	○	○	○
⓿❼	○	●	○	○
⓿❽	○	●	○	○
⓿❾	○	●	○	○
❶⓿	○	●	○	○
❶❶	○	○	●	○
❶❷	○	●	○	○
❶❸	○	○	○	●
❶❹	○	●	○	○
❶❺	○	○	○	●

Question 2 (20 points)

An intended plan to phase out the LPG cylinders used for heating in Jordan is to provide Amman with a natural gas network. The natural gas is to be imported from Egypt via the extension of Arabian pipeline in Aqaba. Natural gas is to be delivered from Amman central distribution system at 300 K and ≈1 bar. Base your calculations on the data below.

Population for Amman: 2,000,000 capita.
Electric consumption per capita per year: 2000 kW.h/yr/capita.
Municipal consumption out of the total consumption: 20%.
Assume the temperature of combustion products of methane to be 800 K.
The efficiency of utilizing the generated heat is estimated to be 85%.

The decision makers are in need of a good estimate for the flowrate of the imported natural gas per year. What is your recommended value for the imports per year?

Calculate the heat of combustion (per mole) of methane at the flue gases temperature then calculate the total consumption of Amman then divide this total consumption with the heat of combustion to get the required estimate.

The following reaction is the combustion reaction for methane with water vapor generation.

$$\mathrm{CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(g)} \quad ;\Delta H_C^\circ = -802.32 \text{ kJ/mol.}$$

There is inert nitrogen involved in this reaction as well as the heat of the reaction is raising the We need to account for the inert nitrogen and the temperature effects by including the heat capacities as a function of temperature.

	CH_4	O_2	CO_2	H_2O	N_2		
ν	-1	-2	1	2	7.52381		
CpA	19.875	25.46	22.243	32.218	28.883		
CpB	5.02E-02	1.52E-02	5.98E-02	1.92E-03	-1.57E-03		
CpC	1.27E-05	-7.15E-06	-3.50E-05	1.06E-05	8.08E-06		
CpD	-1.10E-08	1.31E-09	7.46E-09	-3.59E-09	-2.87E-09		
νCpA	-19.875	-50.92	22.243	64.436	217.3102	ΣνCpA	303.9892
νCpB	-0.05021	-0.03038	0.05977	0.00384	-0.01181	ΣνCpB	0.051798
νCpC	-1.3E-05	1.43E-05	-3.5E-05	2.11E-05	6.08E-05	ΣνCpC	4.69E-05
νCpD	1.1E-08	-2.6E-09	7.46E-09	-7.2E-09	-2.2E-08	ΣνCpD	-2.1E-08

The heat of reaction at any temperature is given by the following formula where the constants are calculated using the summation values

$$\Delta H_{rxn} = \Delta H^\circ_{rxn} + \int_{T_1}^{T_1} \left(\sum \nu_i C^*_{P,i} \right) dT$$

$$= \Delta H^\circ_{rxn} + A(T_2 - T_1) + \frac{B}{2}(T_2^2 - T_1^2) + \frac{C}{3}(T_2^3 - T_1^3) + \frac{D}{4}(T_2^4 - T_1^4)$$

$$= -802.32 + \frac{303.99}{1000}(800 - 298.15) + \frac{0.0518}{2\times 1000}(800^2 - 298.15^2)$$

$$+ \frac{4.69\times 10^{-5}}{3\times 1000}(800^3 - 298.15^3) - \frac{2.1\times 10^{-8}}{4\times 1000}(800^4 - 298.15^4)$$

$$= -802.32 + 152.56 + 14.27 + 7.59 - 2.14 = -630.04 \text{ kJ/mol.}$$

We can correct this heat of reaction for inefficiencies such as incomplete combustion of methane by multiplying by the given efficiency 85%

$$\Delta H = \eta \Delta H_{rxn} = (0.85)(-630.04) = -535.54 \text{ kJ/mol.}$$

We need to calculate the total energy consumption in Amman to determine the amount of natural gas to be imported.

$$Q = \text{Population} \times \text{Electric consumption} \times \text{Municipal Fraction}$$

$$= (2\times10^6 \text{ Capita})(2000 \frac{\text{kW.h}}{\text{Capita. year}})(0.2) = 8\times10^8 \text{ kW.h/year.}$$

Convert the kilowatt hours to Joules by using the conversion $\text{kW.h} = \frac{\text{kJ.h}}{\text{s}} \frac{3600\text{s}}{\text{h}}$ to get the total energy requirements for municipal use as 2.88×10^{12} kJ.

The mass of methane can be determined as the ratio between the total requirements to that of energy requirements per one mole then using the molecular weight to convert to mass.

$$\text{Moles CH}_4 = \frac{2.88\times10^{12}}{535.54} = 5.38\times10^9 \text{ mol/year.}$$

$$\text{Mass CH}_4 = 5.38\times10^9 \times 0.016 = 8.6\times10^7 \text{ kg/year}$$

$$= 86000 \text{ tons/year.}$$

Notice that the numbers are just for illustration of areal life situation and they do not reflect the actual requirements for the Jordanian market. Jordan consumed about 276000 ton of LPG in the year 2001. (http://www.environment.gov.jo/Tuea17.html).

Question 3 (30 points)

A natural gas stream available at 1 bar and 180 K is to be used to produce liquefied methane. The compressed stream will be leaving the cooler at 180 K. The flash drum is adiabatic and operates at 1 bar, and each compressor stage can be assumed to operate reversibly and adiabatically. The compression ratio is the same in each compression stage and is equal to 5. Between each stage the gas is to be isobarically cooled to 180 K. (**Use the provided chart for methane**)

1. How many stages of compression are required for an outlet pressure from the cooler of 125 bars?
2. Calculate the amount of work required for each kilogram of methane that passes through the compressor in the simple liquefaction process.
3. Calculate the fractions of vapor and liquid leaving the flash drum in the simple liquefaction process and the amount of compressor work required for each kilogram of LNG produced.
4. What is the maximum permissible operating temperature out from the cooler?
5. Assuming that the recycled methane leaving the heat exchanger in the Linde process is at 1 bar and 180 K, calculate the amount of compressor work required per kilogram of LNG produced.

Assumptions
1. Isentropic compression. 2. Equal compression ratio within compression stages. 3. Isobaric cooling and heat exchange. 4. Isenthalpic throttling valve. 5. Adiabatic flash drum. 6. No heat transfer or pressure drop through the pipes of the process. 7. Methane properties are described by the provided chart.

1. Since the outlet pressure is 125 bar and the compression ratio is the same and equal 5 then the number of stages is

$$(r)^n = (5)^n = 125 = 5^3 \Rightarrow n = 3$$

2. The amount of work is calculated by stepping in the chart isentropically for the compression stages and isobarically for the cooling stages.

$$
\begin{aligned}
W &= W_1 + W_2 + W_3 \\
&= (900-730) + (880-720) + (800-650) \\
&= 480 \text{ kJ/kg feed natural gas.}
\end{aligned}
$$

3. The liquid fraction is about 0.5 using the isenthalpic expansion for the valve and flash drum section i.e., 50% of the feed is liquefied. Therefore the amount of work is 1040 kJ/kg LNG produced.
4. The maximum permissible temperature out of the cooler is the maximum temperature that will let the expansion process produce a stream in the two phase region i.e., saturated vapor at 1 bar and the same enthalpy coming out of the cooler. Using the chart again yields that the temperature is about 235 K.
5. The Linde process calculations are done around two sections (using the notation in the flowsheet in the lecture)

$$
\begin{aligned}
H_3 &= x_L H_6 - (1-x_L) H_{5'} \\
320 &= x_L (90) + (1-x_L)(720) \Rightarrow x_L = 0.635.
\end{aligned}
$$

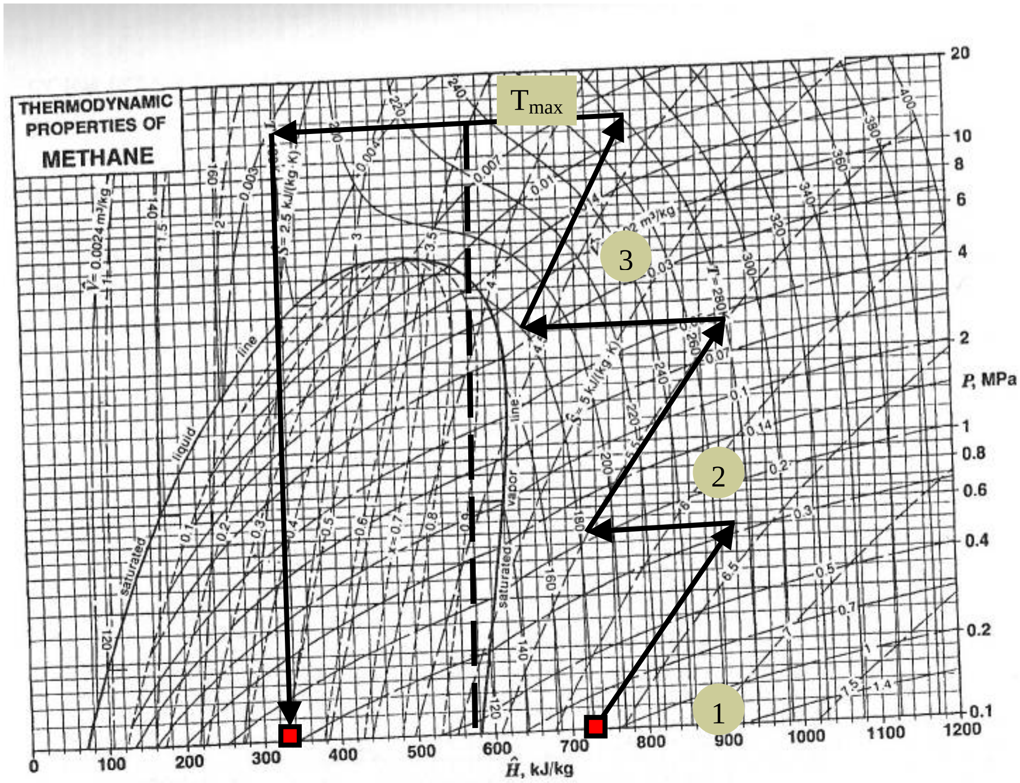
The work per kilogram feed is the same as that in the simple process. However, the feed in the Linde process is the same as the product. Consequently, the work per kilogram LNG is given as

$$
W_{\text{Linde}} = \frac{480}{0.635} = 756 \text{ kJ/kg LNG.}
$$

Notice that the Linde process increased the liquid fraction by about 27% while reducing the compressor work duty by about 25%.

Process summary

1. Number of compression stages	3
2. *W* (kJ/kg CH_4)	480
3.a Liquid fraction (Simple)	0.5
3.b *W* (kJ/kg LNG)	960
4. Max operating *T* (K)	235
5.a Liquid fraction (Linde)	0.635
5.b *W* (kJ/kg LNG)	756


THERMODYNAMIC PROPERTIES OF METHANE
T max
1
2
3
P, MPa
Ĥ, kJ/kg
saturated liquid line
saturated vapor line

Question 4 (20 points)

The "Quick Fill" bicycle tire filling system consists of a small cylinder filled with nitrogen to a pressure of 140 bar. **Use Departure Charts to solve this problem**.

You may assume the contents are initially at the ambient temperature of 298.15 K.
Cylinder dimensions: 2-cm-diameter and 6.5-cm-long.
You may use the ideal gas heat capacity of nitrogen up to the linear term i.e.; $C_p = A + BT$.

1. Estimate the explosive equivalent of the gas contained in the cylinder as grams of TNT.
2. What is the mass of nitrogen in the cylinder?

1. Obtain the critical constants and coefficients of the ideal gas heat capacity

T_c(K)	126.2
P_c (MPa)	3.394
C_p =	28.883-0.00157T + 0.00000808T^2 - 2.871×10^{-9}T^3

2. Find the reduced temperature and pressure at the initial state. Also, get the reduced pressure at the final state.

$$P_{r,1} = \frac{P_1}{P_c} = \frac{14}{3.394} = 2.36; P_{r,2} = \frac{P_2}{P_c} = \frac{0.1}{3.394} = 0.029$$

$$T_{r,1} = \frac{T_1}{T_c} = \frac{298.15}{126.2} = 4.12.$$

3. You can solve for the mass of nitrogen first since it is easier than finding the TNT equivalent. Use the reduced compressibility chart at the initial reduced conditions to find Z then find the molar volume and from the volume of cylinder and the molecular weight find the mass of nitrogen.

$$Z(T_r = 2.36; P_r = 4.12) = 0.95$$

$$\underline{V} = \frac{ZRT}{P} = \frac{(0.95)(8.314)(298.15)}{1.4\times10^7} = 1.68\times10^{-4} \text{ m}^3\text{/mol}$$

$$V(\text{cylinder}) = \pi r^2 l = \pi(0.01)^2(0.065) = 2.042\times10^{-5} \text{ m}^3$$

$$\text{Moles} = \frac{V}{\underline{V}} = \frac{2.042\times10^{-5}}{1.68\times10^{-4}} = 0.121 \text{ mol}$$

$$\text{Mass} = (0.121)(0.028) = 0.0034 \text{ g N}_2.$$

4. To solve for the TNT equivalent we start from the isentropic condition which yields the following equation in departure quantities

$$\underline{S}(T_2, P_2) - \underline{S}(T_1, P_1) = 0 = \int_{T_1}^{T_2} \frac{C_P^*}{T} dT - R\ln\frac{P_2}{P_1} + \left[\underline{S} - \underline{S}^{IG}\right]_{T_{r,2},P_{r,2}} - \left[\underline{S} - \underline{S}^{IG}\right]_{T_{r,1},P_{r,1}}$$

The only unknown in this equation is the final temperature which must be solved for by trial and error. An initial guess can be used from the ideal gas law with a constant heat capacity i.e., taking only the constant term. The equation for an ideal gas undergoing isentropic expansion is given as

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{R/C_P^*} = 298.15\left(\frac{1}{140}\right)^{8.314/28.883} = 71.9 \text{ K.}$$

Of course, a better initial guess would be solving the nonlinear equation resulting from the cubic form of the heat capacity.

The best simplification to solve this problem is to note that the final reduced pressure is so low that we can assume the final state to be in the ideal gas state. Therefore, the departure term involved is set to zero.

$$0=\int_{T_1}^{T_2}\frac{C_P^*}{T}dT-R\ln\frac{P_2}{P_1}-\left[\underline{S}-\underline{S}^{IG}\right]_{T_{r,1},P_{r,1}}$$

$$=\int_{T_1}^{T_2}\frac{A+BT}{T}dT-R\ln\frac{P_2}{P_1}-\left[\underline{S}-\underline{S}^{IG}\right]_{T_{r,1},P_{r,1}}$$

$$=A\ln\frac{T_2}{T_1}+B(T_2-T_1)-R\ln\frac{P_2}{P_1}-\left[\underline{S}-\underline{S}^{IG}\right]_{T_{r,1},P_{r,1}}$$

$$=A\ln\frac{T_2}{T_1}+B(T_2-T_1)+41.1+2.1=A\ln\frac{T_2}{T_1}+B(T_2-T_1)+43.2$$

Solve this equation by trial and error to find the final temperature = 66 K or T_r = 0.52. There is a contradiction now between the assumption of ideal gas state at the final condition and the value we get which puts the final condition in the liquid state. Consequently, grading for this problem will be used for the ideal gas law solution with variable heat capacity which yields a final temperature of 72.9 K or T_r = 0.58.

The work is the internal energy change for the ideal gas i.e.,

$$W=N\int_{T_1}^{T_2}C_V^*dT=N\int_{T_1}^{T_2}\frac{(A-R)+BT}{T}dT=N(A-R)(T_2-T_1)+NB(T_2^2-T_1^2)$$

$$=0.121\left[(28.883-8.314)(72.9-298.15)-0.00157(72.9^2-298.15^2)\right]=-547\text{ J.}$$

One gram of TNT is equivalent to 4600 J which means that the explosion of nitrogen in this cylinder is equivalent to 0.119 grams of TNT.

Results Summary

Final temperature (K)	72.9
Work (J)	547
TNT equivalent (g)	0.119
Mass (kg)	0.0034

Question 5 (15 points)

A LNG stream (may be assumed methane) at 20 bar and 140 K flows at a rate of 100 kg/s through a pipe of cross sectional area of 0.5 m^2. What is the volumetric flowrate and velocity for this stream? **Base your calculations on the Peng-Robinson EOS**.

Carry out three iterations starting with a reasonable initial guess then use Z = 0.065 for subsequent calculations.

Apply the Peng-Robinson EOS at the given conditions. I used the Newton-Raphson method to obtain the final answer of Z = 0.065796243.

$$b=0.07779\frac{RT_c}{P_c}=0.07779\frac{(8.314)(190.6)}{(4.6\times10^6)}=2.679\times10^{-5}$$

$$\kappa=0.37464+1.5422\omega-0.26992\omega^2=0.3869$$

$$\alpha(T)=\left[1+\kappa\left(1-\sqrt{\frac{T}{T_c}}\right)\right]^2=\left[1+0.3869\left(1-\sqrt{\frac{140}{190.6}}\right)\right]^2=1.114$$

$$a(T)=0.45724\frac{(RT_c)^2}{P_c}\alpha(T)=0.45724\frac{(8.314\times190.6)^2}{(4.6\times10^6)}(1.114)=0.278$$