CH: 2 : Second order ODEs

→ $y'' + p(x)y' + q(x)y = r(x) \rightarrow$ Linear "S.O" ODE

→ $y'' + p(x)y' + q(x)y = 0 \rightarrow$ Homogenous

→ The solution of $y'' + p(x)y' + q(x)y = 0$; is given by

$$y = c_1 y_1 + c_2 y_2$$

→ Basic of the function is $\{y_1, y_2\}$

$\boxed{*}$ Reduce to First order

$\boxed{I}$ y missing

* لما ما يكون عندي (y) بالمعادلة
* بتحل من خلال الفرض $\leftarrow \boxed{\begin{array}{l} z = y' \\ z' = y'' \end{array}}$

EX2 $x^2 y'' + (y')^2 = 0$

$\boxed{\begin{array}{l} z = y' \\ z' = y'' \end{array}}$ $x z' + z^2 = 0 \rightarrow$ by seperable

$x \dfrac{dz}{dx} = -z^2 \rightarrow \displaystyle\int \frac{dz}{z^2} = \int \frac{-dx}{x^2} \rightarrow \frac{-1}{z} = \frac{c_1 x + 1}{x}$

برجع الـ z

$-\dfrac{1}{dy/dx} = \dfrac{c_1 x + 1}{x} \xrightarrow{\text{تقليب}} \dfrac{-dy}{dx} = \dfrac{x}{c_1 x + 1}$

$$\int -dy = \int \frac{x}{c_1 x + 1}\, dx$$

$$\begin{array}{r|l} & 1/c_1 \\ \hline c_1 x + 1 & x \\ & -x + \frac{1}{c_1} \\ \hline & \frac{-1}{c_1} \end{array}$$

$$-y = \int \frac{1}{c_1} + \frac{\left(\frac{-1}{c_1}\right)}{c_1 x + 1}\, dx$$

$$\boxed{y = \frac{-1}{c_1} x + \frac{1}{c_1^2} \ln|c_1 x + 1| + c_2}$$

Ex.2 $x\,y'' = y' + (y')^3$

$$\begin{cases} z = y' \\ z' = y'' \end{cases}$$

$x z' = z + z^3$

$z' = \dfrac{z + z^3}{x} \longrightarrow \underbrace{\int \dfrac{dz}{z(1+z^2)}}_{\text{partial fraction}} = \int \dfrac{dx}{x}$

$$\frac{A}{z} + \frac{Bz + C}{z^2+1} = 1$$

$A(z^2+1) + (Bz+C)(z) = 1$

$z = 0 \rightsquigarrow A = 1$

$z = 1 \rightsquigarrow 2 + B + C = 1$

$z = -1 \rightsquigarrow 2 + B - C = 1$

$$\begin{cases} B = -1 \\ C = 0 \end{cases}$$

$$\int \left[\frac{1}{z} - \frac{z}{z^2+1}\right] dz = \ln|x| + c_1$$

$$\left(\ln|z| - \frac{1}{2}\ln|z^2+1| = \ln x + c_1\right) \times 2$$

$2\ln|z| \to \ln z^2$

$2\ln|x| = \ln|x|^2$

$$\left(\ln\left|\frac{z^2}{z^2+1}\right| = \ln x^2 + c_1^*\right)$$

e

$$\frac{z^2}{z^2+1} = x^2 \cdot c_1 \cdot e = x^2 c_2$$

$$z^2 = c_2 x^2 z^2 + c_2 x^2$$

⋮

2) X-missing

- طالما (X) ما تكون موجودة بالسؤال
- بنحل من خلال الفرضية

$$\boxed{Z = y'}$$

$$y'' = y'\frac{dZ}{dy} = Z\frac{dZ}{dy}$$

$$y'' = ZZ'$$

Ex: $yy'' - (y')^2 = y^2y'$

$$Z = y'$$

$$y'' = Z\frac{dZ}{dy}$$

$$y'' = ZZ'$$

$$y\left(Z\frac{dZ}{dy}\right) - (Z)^2 = y^2Z$$

$$yZZ' - Z^2 = y^2Z$$

$$\div (yZ)$$

become $\longrightarrow Z' - \frac{Z}{y} = y$ (Linear)

$$u(y) = e^{\int \frac{-1}{y}dy} = \frac{1}{y}$$

$$Z = \frac{1}{u(y)}\int u(y)\,Q(y)\,dy = y\int \frac{1}{y}\cdot y\,dy = y^2 + c_1y$$

$$Z = y^2 + c_1y$$

$$y' = y^2 + c_1y \longrightarrow \frac{dy}{dx} = y^2 + c_1y$$ → separable

$$\int \frac{dy}{y(y+c_1)} = \int dx$$

⋮

$\boxed{*}$ one solution Given:

$$y'' + P(x)y' + q(x)y = r(x)$$

$\boxed{*}$ y_1 : is solution for corresponding homogenous ordinary DIFF eq^n

$\boxed{*}$ $y = y_1 u$

$$\Longrightarrow \boxed{u'' + \left(\frac{2y_1'}{y_1} + P(x)\right)u' = \frac{r(x)}{y_1}}$$

Let $Z = u'$

$$Z' = u''$$

<u>EX</u>: solve: $x^2 y'' - 2y = x^2$, $y_1 = \frac{1}{x}$

$\rightarrow y = y_1 u = \frac{u}{x}$, $\boxed{y_1 = x^{-1}}$

$\boxed{y_1' = \frac{-1}{x^2}}$ $\quad P(x) = 0$; $\boxed{r(x) = 1}$

$$\Longrightarrow u'' - \frac{2u'}{x} = x$$

$\boxed{\begin{aligned} Z &= u' \\ Z' &= u'' \end{aligned}}$ $\quad Z' - \frac{2Z}{x} = x \longrightarrow$ Linear

$$u(x) = e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$$

$$Z = x^2 \int \frac{1}{x^2} \cdot x\,dx \longrightarrow Z = x^2 \ln x + x^2 C_1$$

$$\cancel{u' = x^2 \int \frac{1}{x^2} x\,d}$$

$$\frac{du}{dx} = \int u' = \int x^2 \ln x + x^2 C_1$$

ونكمل حل الأمر

※ Given one solution [special Case]:

when $y'' + p(x)y' + q(x)y = 0$, and given y_1

then $$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2}\,dx$$

General solution is $y = c_1 y_1 + c_2 y_2$

Ex: solve: $x(y'') - (x+2)y' + 2y = 0$, $y_1 = e^x$

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2}\,dx = e^x \int \frac{e^{-\int \frac{1+2}{x}dx}}{e^{2x}}\,dx = e\int \frac{x^2 e^x}{e^{2x}}\,dx$$

(بالتجزئة)

$$\rightarrow \boxed{y'' - \left(1+\frac{2}{x}\right)y' + \frac{2}{x}y = 0} \longrightarrow \text{General form}$$

$$y_2 = e^x \int x^2 e^{-x}\,dx \rightarrow e^x\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]$$

$$y_2 = -(x^2+2x+2)$$

$$y = c_1 y_1 + c_2 y_2 = c_1 e^x - c_2\,(x^2+2x+2)$$

Ex: IF $xy'' + 2y' + xy = 0$ and $y_1 = \frac{\sin x}{x}$ Find y_2

G.F $\rightarrow$ $y'' + \frac{2y'}{x} + y = 0$

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2}\,dx = \frac{\sin x}{x} \int \frac{e^{-\int \frac{2}{x}dx}}{\frac{\sin^2 x}{x^2}}\,dx = \frac{\sin x}{x} \int \frac{x^{-2}}{\frac{\sin^2 x}{x^2}}\,dx$$

$$= \frac{\sin x}{x} \int \frac{1}{\sin^2 x}\,dx = \frac{\sin x}{x}\left[-\cot x\right] = \frac{\sin x}{x} \cdot \frac{-\cos x}{\sin x} = -\frac{\cos x}{x}$$

$$y_2 = -\frac{\cos x}{x}$$

* Second order Linear homogenous ODE with Constant Cofficient :

G.F $\rightarrow$ $y'' + ay' + by = 0$, [a, b $\rightarrow$ are Constants]

G. solution $\rightarrow$ $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

Basic. solution $\rightarrow$ $\{ e^{\lambda_1 x}, e^{\lambda_2 x} \}$

$\rightarrow$ 3 - Cases for solving:
- $\rightarrow$ distict roots $\rightarrow$ $\lambda_1 \neq \lambda_2$
- $\rightarrow$ repeated roots $\rightarrow$ $\lambda_1 = \lambda_2$
- $\rightarrow$ Complex roots $\rightarrow$ $\lambda = \alpha \pm \beta i$

* عشان نطلع قيم (λ) بنفرض معادلة هي مفتاح لكل 3 -

$\rightarrow$ characteristic equation

$\cancel{a r^2 + b y' + c y = 0}$

$$\begin{array}{ccccc} a r^2 & + & b r & + & c = 0 \\ \downarrow & & \downarrow & & \downarrow \\ y'' & & y' & & y \end{array}$$

or $\boxed{a\lambda^2 + b\lambda + c = 0}$

اختلاف رموز

Ex : $y'' - 5y' + 6y = 0$

I] write characteristic equation $\rightarrow$ $r^2 - 5r + 6 = 0$

$\boxed{\lambda^2 - 5\lambda + 6 = 0}$

$\cancel{(r-3)(r-2) = 0}$

$(\lambda - 3)(\lambda - 2) = 0$

$\lambda_1 = 3 , \lambda_2 = 2$

$\lambda_1 \neq \lambda_2$ Case 1

$$\begin{aligned} y &= C_1 y_1 + C_2 y_2 \\ &= C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} \\ &= C_1 e^{3x} + C_2 e^{2x} \end{aligned}$$

يجب أن يكون y_1 و y_2 Independent لأنه إذا كانا dependent راح نعتبرها حل واحد وهذا لا يجوز لأنه (second order) يجب أن يكون لدينا حلين مختلفين فإذا كانا عندنا y_1 و y_2 (dependent) راح نخليهم (indep)

← كيف نعرف إذا كانوا الحلول dependent أو لا ؟

IF $\frac{y_1}{y_2}$ or $\frac{y_2}{y_1} = k \rightarrow$ dependent , $k =$ constant

IF $\frac{y_1}{y_2}$ or $\frac{y_2}{y_1} \neq k \rightarrow$ indep , $k =$ constant

EX :- $y_1 = x^2$, $y_2 = x^3$ dep or indep ?

$$\frac{y_1}{y_2} = \frac{x^2}{x^3} = \frac{1}{x} \rightarrow \frac{1}{x} \neq k \rightarrow \text{indep}$$

EX : $y_1 = 150e^{3x}$ & $y_2 = 3e^{3x}$ dep or indep?

$$\frac{y_1}{y_2} = \frac{150\cancel{e^{3x}}}{3\cancel{e^{3x}}} = \frac{150}{3} \rightarrow 50 = k \quad \text{dep}$$

EX (Case 2) : $2y'' + 8y' + 8y = 0 \rightarrow$

$y'' + 4y' + 4y = 0$ (G.F)

I ch. eq $\rightarrow \lambda^2 + 4\lambda + 4 = 0$

$(\lambda + 2)(\lambda + 2) = 0$

$\lambda_1 = \lambda_2 = 0$

dep ← هل هذا

$y = c_1 e^{2x} + c_2 e^{2x} \rightarrow$ هنا لا يجوز لأنه الحلين (dep) فبنضرب y_2 بـ (x) حتى يصيروا (indep)

G.S $\left\{ y = c_1 e^{2x} + c_2 x e^{2x} \right\}$ indep

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EX (Case 3) : solve : $y'' - 6y' + 10y = 0$ Complex = $\boxed{\sqrt{-1} = i}$

1] ch.eq → $\lambda^2 - 6\lambda + 10 = 0$

Use $\lambda = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 \pm \sqrt{-4}}{2} = \dfrac{6 \pm \sqrt{-1}\sqrt{4}}{2} = \underbrace{3}_{\mu} \pm 1i$

$1 = \beta$

→ $y_1 = e^{\mu x} \cos(\beta x)$ $y_2 = e^{\mu x} \sin(\beta x)$

G.S → $y = c_1 e^{\mu x} \cos(\beta x) + c_2 e^{\mu x} \sin(\beta x)$

→ $y = c_1 e^{3x} \cos(x) + c_2 e^{3x} \sin(x)$

$= e^{3x} [c_1 \cos(x) + c_2 \sin(x)]$

EX : Find Basic solution of $y'' - 10y' + 25y = 0$

$\lambda^2 - 10\lambda + 25 = 0$

$(\lambda - 5)(\lambda - 5) = 0$

$\lambda_1 = \lambda_2 = 5$ → repeated

$y_1 = e^{5x}$ & $y_2 = x e^{5x}$

Basic solution = $[e^{5x}, x e^{5x}]$

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EX : Solve : $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = 2$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = c_1 \cos 2x + c_2 \sin 2x$

$y' = -2c_1 \sin 2x + 2c_2 \cos 2x$

* $y'(0) = 2$

$2 = 2c_2$ → $\boxed{c_2 = 1}$

* $y(0) = 0$

$\boxed{0 = c_1}$

G.S : $\boxed{y = \sin 2x}$

هذا النوع من الأسئلة يعطينا الجواب ويطلب شكل السؤال .

EX: $y = (c_1 e^{5x} + c_2 e^{x})$

From solution $\rightarrow \lambda_1 = 5 \neq \lambda_2 = 1$

$(\lambda - 5)$ $\qquad$ $(\lambda - 1)$

Characteristic eq $= (\lambda - 5)(\lambda - 1) = 0$

$\lambda^2 - \lambda - 5\lambda + 6 = 0$

$\lambda^2 - 6\lambda + 6 = 0$

$$\boxed{y'' - 6y' + 6y = 0}$$

EX: $y = e^{2x}(c_1 + c_2 x) \rightarrow$ repeated Form

From solution $\rightarrow \lambda_1 = \lambda_2 = 2$

$(\lambda - 2)(\lambda - 2) = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$$\boxed{y'' - 4y' + 4y = 0}$$

EX: $y = e^{3x}(c_1 \cos 2x + c_2 \sin 2x) \rightarrow$ Complex Form $\boxed{\alpha \pm \beta i}$

From solution $\rightarrow \lambda = 3 \pm 2i$

$(\lambda - (3 + 2i))(\lambda - (3 - 2i)) = 0$

$(\lambda - 3 - 2i)(\lambda - 3 + 2i) = 0$ $\qquad$ $i = \sqrt{-1}$ $\qquad$ $\boxed{i^2 = -1}$

~~$\lambda^2 - 3\lambda + 2\lambda i - 3\lambda + 9 - 6i - 2\lambda i + 6i$~~

$\lambda^2 - 3\lambda + \cancel{2\lambda i} - 3\lambda + 9 - \cancel{6i} - \cancel{2\lambda i} + \cancel{6i} - 4(i)^2 = 0$

$\lambda^2 - 6\lambda + 9 + 4 = 0 \longrightarrow \lambda^2 - 6\lambda + 13 = 0$

$$\boxed{y'' - 6y' + 13y = 0}$$

* Euler - Caughy ODE [Non- Constant Coff] :

→ General Form For second order : $\boxed{x^2 y'' + a x y' + b y = 0}$

→ G.s :- $y_1 = x^{\lambda_1}$, $y_2 = x^{\lambda_2}$

$$\boxed{y = c_1 x^{\lambda_1} + c_2 x^{\lambda_2}}$$

→ Characteristic eq For Euler :

→ $\boxed{m(m-1) + am + b = 0}$

1. $m(m-1) x^m + a m x^m + b m = 0 \rightarrow$ G.s بنستخدمها لما يعطيني المعادلة ويطلب G.s

2. $m^2 + (a-1) m + b = 0 \rightarrow$ بنستخدمها لما يعطيني G.s ويطلب المعادلة

بنحط λ بدل m كلها رموز

3- Cases

$\lambda_1 \neq \lambda_2$

$y_1 = x^{\lambda_1}$, $y_2 = x^{\lambda_2}$

G.s : $y = c_1 x^{\lambda_1} + c_2 x^{\lambda_2}$

$\lambda_1 = \lambda_2$

$y_1 = x^{\lambda}$; $y_2 = x^{\lambda} \ln x$

G.s : $y = c_1 x^{\lambda} + c_2 \ln x \, x^{\lambda}$

$\lambda = \mu \pm \beta i$

$y_1 = x^{\mu} \cos(\beta \ln x)$

$y_2 = x^{\mu} \sin(\beta \ln x)$

$y = x^{\mu} [c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x)]$

EX : $x^2 y'' - x y' - 3 y = 0 \rightarrow$ اعطاني المعادلة بدون حل

→ $m(m-1) + am + b = 0$

نعوض ↳ $\lambda(\lambda - 1) + a\lambda + b = 0$

$\lambda^2 - \lambda + -1\lambda - 3 = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\lambda_1 = +3$, $\lambda_2 = -1$

$$\boxed{G.s : y = c_1 x^{3} + c_2 x^{-1}}$$

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Ex: $x^2y'' - xy' + y = 0$

$\lambda(\lambda-1) + a\lambda + b = 0$

$\lambda(\lambda-1) - \lambda + 1 = 0$

$\lambda^2 - \lambda - \lambda + 1 = 0 \longrightarrow \lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)(\lambda-1) = 0$

$\lambda_1 = \lambda_2 = 0 \longrightarrow$ repeated

$$\boxed{G.S: \; y = c_1 x + c_2 \ln x \, x}$$

Ex: $x^2y'' + 7xy' + 10y = 0$

$\lambda(\lambda-1) + 7\lambda + 10 = 0$

$\lambda^2 - \lambda + 7\lambda + 10 = 0$

$\lambda^2 + 6\lambda + 10 = 0$

$$\lambda = \frac{-6 \pm \sqrt{-4}}{2} = -3 \pm i$$

$$\left.\boxed{y = x^{-3}\left[c_1 \cos(\ln x) + c_2 \sin(\ln x)\right]}\right\} G.S$$

Ex: Find S.O ODE whose solution is: $y = c_1 x^2 + c_2 x^5$

$(\lambda-2)(\lambda-5) = 0$

$\lambda^2 - 5\lambda - 2\lambda + 10 = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$\lambda^2 + (a-1)\lambda + 10 = 0 \longrightarrow$ راح نختصر طاي لأنه اعطاني الحل وطلب المعادلة

$a - 1 = -7$

$\boxed{a = -6}$

$\longrightarrow \lambda^2 - 6\lambda + 10 = 0$

$$\boxed{x^2y'' - 6xy' + 10y = 0}$$

Ex : Find S.O ODE whose solutions

$$y = x\left[c_1 \cos(2\ln x) + c_2 \sin(2\ln x)\right]$$

$\lambda = 1 \pm 2i$

$(\lambda - (1+2i))(\lambda - (1-2i)) = 0$

$\lambda^2 - \lambda + 2\lambda i - \lambda + 1 - 2i - 2\lambda i + 2i - 4i^2 = 0$

$\lambda^2 - 2\lambda + 5 = 0$

$\lambda^2 + (a-1)\lambda + b = 0$

$a - 1 = -2$

$\boxed{a = -1}$

$\lambda^2 - \lambda + 5 = 0$

$\boxed{x^2 y'' - x y' + 5y = 0}$

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* NON - Homogenous ODE

G.F :- $y'' + p(x)y' + q(x) = r(x)$

G.S : $y = y_h + y_p$, ($y_h \rightarrow$ The solution when ean equation is Homogenous $r(x) = 0$)

y_p → Called particular solution
→ solution for NON-Homo

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NON-Homo. solved by
→ undetermined coeff. method
→ Variation of parameters method

Undetermined Coff :

* في هذه الطريقة شكل y_p موجود بالجدول اللي معطيناه إياه مباشرة لو إذا لقينا الشكل بالجدول بنحل undetermined و إذا لا بنروح ع Variation .

* يجب أن لا يكون هناك أي تشابه بين شكل y_h و y_p ، إذا لقينا تشابه راح أغير شكل y_p .

$r(x)$	y_p
$k e^{\lambda x}$	$A e^{\lambda x}$
$k x^n$	$A x^n + B x^{n-1} + C x^{n-2} \dots$ Constant
$k \cos \omega x$ $k \sin \omega x$	$k_1 \cos \omega x + k_2 \sin \omega x$
$k e^{ax} \cos \omega x$ $k e^{ax} \sin \omega x$	$e^{ax}(k_1 \cos \omega x + k_2 \sin \omega x)$

$\boxed{*}$ لتأثيرات حل y_p :-

١- إذا كانت $(kx^n + Ae^{\lambda x} + \dots) = r(x)$ فإن $y_p = y_{p_1} + y_{p_2} + \dots$

EX : $r(x) = x^2 + 3\cos 3x$

$$y(p) = y_{p_1} + y_{p_2} = \underbrace{Ax^2 + Bx + C}_{y_{p_1}} + \underbrace{k_1 \cos 3x + k_2 \sin 3x}_{y_p}$$

٢- إذا كان هناك تشابه بين y_h و y_p بنضرب y_p ب $(x, x^2, x^3, \dots)$

إذا كانت المسألة (Cons coff) و $(\ln x, \ln x^2, \dots)$ إذا كانت المسألة Euler

EX : $y_h = C_1 e^{3x} + C_2 e^{-x}$

$y_p = A e^{7x} + B x e^{-x}$

← ضربناها ب X مشان التشابه

EX: Find the form of y_p for the ODE :-

$$y'' - 3y' + 2y = 6e^{-x} + e^{x} + e^{2x}$$

$y_h \rightarrow y'' - 3y' + 2y = 0$

$$\lambda^2 - 3\lambda + 2 = 0 \rightarrow \lambda_1 = 2 , \lambda_2 = 1$$

$$\boxed{y_h = c_1 e^{2x} + c_2 e^{x}}$$

$y_p = Ae^{-x} + Be^{x} + Ce^{2x} \rightarrow$ from table

بنلاحظ في تشابه بين و
y_h عند e^x و e^{2x} وبما انه
نوع المعادلة Constant Cofficient
لبضربهم
ب x

$$\boxed{y_p = Ae^{-x} + Bxe^{x} + Cxe^{2x}}$$

EX: Find the form of y_p for the ODE :-

$$x^2 y'' - x y' + y = e^{x}$$

$y_h = c_1 x + c_2 x \ln|x|$

$\boxed{y_p = Ae^{x}}$ $\rightarrow$ بضل زي ما هي
لأنه ما في تشابه
بين y_p , y_h

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EX: If $y_h = c_1 x^2 + c_2 x^2 \ln(x)$ and $r(x) = e^{7x} + 3x^2$, Find y_p ?

$$\boxed{y_p = c_1^{*}(\ln(x))^2 x^2 + \cancel{c_2 x^2 \ln} \; Ae^{7x}}$$

ضربنا الحد الأول
ب $(\ln|x|)^2$ مشان ما يصير
في تشابه بحدود y_p

Ex: solve: $x^2y'' - xy' + y = e^x$

$$\boxed{y_h = C_1 x + C_2 x \ln x}$$

$$\boxed{y_p = Ae^x} \text{ from table}$$

$y'_p = Ae^x$

$y''_p = Ae^x$

بنعوضهم بالمعادلة لإيجاد قيمة (A)

$$\Longrightarrow x^2(Ae^x) - x(Ae^x) + Ae^x = e^x$$

$$Ae^x = e^x \longrightarrow \boxed{A = 1}$$

G.S: $y = y_h + y_p$

$$= \boxed{C_1 x + C_2 x \ln(x) + e^x}$$

Ex: $y'' + 9y = 6\sin 3x$ solve:

$$\boxed{y_h = C_1 \cos 3x + C_2 \sin 3x}$$

$$\boxed{y_p = x(A\cos 3x + B\sin 3x)} \to \text{from table}$$

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$y'_p = (A\cos 3x + B\sin 3x) + x(-3A\sin 3x + 3B\cos 3x)$

$y''_p = (-3A\sin 3x + 3B\cos 3x) + (-3A\sin 3x + 3B\cos 3x) + x(-9A\cos 3x - 9B\sin 3x)$

$\to y'' + 9y = -6(A\sin 3x - B\cos 3x) - 9x(A\cos 3x + B\sin 3x) + 9x(A\cos 3x + B\sin 3x)$

$y_p = -x\cos 3x$

$-6A = 6$, $\boxed{A = -1}$

$6B = 0$, $\boxed{B = 0}$

$$\nearrow \boxed{y = C_1\cos 3x + C_2 \sin 3x - x\cos 3x}$$

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Ex: Find S.O ODE whose solution is:

$$\cancel{y_h = C_1 \cos 2x + C_2 \sin 2x}$$

$$y = C_1 \cos 2x + C_2 \sin 2x - \sin 3x$$

$\boxed{*}$ $y_h = C_1 \cos 2x + C_2 \sin 2x$

$\boxed{*}$ $y_p = -\sin 3x$

$\rightarrow$ From $y_h \rightarrow \lambda = \pm 2i$

$(\lambda - 2i)(\lambda + 2i) = 0$

$\lambda^2 + 4 = 0$

$y'' + 4y = r(x)$

$\rightarrow$ $y_p = -\sin 3x$

$y'_p = -3\cos 3x$

$y''_p = 9 \sin 3x$

نعوض بالمعادلة

$y''_p + 4y_p = r(x)$

$9\sin 3x - 4\sin 3x = r(x) \longrightarrow \boxed{r(x) = 5 \sin 3x}$

$\Rightarrow y'' + 4y = 6 \sin 3x$

Variation of parameters method

$\rightarrow y = y_h + y_p \quad , \quad y_h = c_1 y_1 + c_2 y_2$

$$\boxed{y_p = y_1 \int \frac{W_1}{W} + y_2 \int \frac{W_2}{W}}$$

Wronskain:

$\Rightarrow y'' + p(x) y' + q(x) y = 0$

$y = c_1 y_1 + c_2 y_2$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

IF $y_1 y_2' - y_2 y_1' = 0 \rightarrow$ Linearly dependent

" " $\neq 0 \rightarrow$ " independent

Ex: $W(x, 2x) = \begin{vmatrix} x & 2x \\ 1 & 2 \end{vmatrix} = 2x - 2x = 0$ L.D

Ex: $W(x, x^2) = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2$ L.InD

$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

↳ الموجودة بالقانون

$W_1 = \begin{vmatrix} 0 & y_2 \\ r(x) & y_2' \end{vmatrix}$ و $W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & r(x) \end{vmatrix}$

W_1 ↳ الموجودة بالقانون

W_2 ↳ الموجودة بالقانون

Ex: $y'' - 3y' + 2y = \underbrace{\cos(e^{-x})}$

↳ not exist in the table, then we will use Variation

$y_h \Rightarrow$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$\lambda_1 = 2, \lambda_2 = 1$

$\boxed{y_h = C_1 e^{x} + C_2 e^{2x}}$

① $W(e^x, e^{2x}) = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = e^{3x}$

② $W_1 = \begin{vmatrix} 0 & e^{2x} \\ \cos(e^{-x}) & 2e^{2x} \end{vmatrix} = -e^{2x}\cos(e^{-x})$

③ $W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \cos(e^{-x}) \end{vmatrix} = e^{x}\cos(e^{-x})$

* $\int \frac{W_1}{W} = \int \frac{-e^{2x}\cos(e^{-x})}{e^{3x}} = \int \underbrace{e^{-x}\cos(e^{-x})\,dx}_{\text{substitution}}$

* $\int \frac{W_2}{W} = \int \frac{e^{x}\cos(e^{-x})}{e^{3x}} = \int \underbrace{e^{-2x}\cos(e^{-x})\,dx}_{\text{by part}}$ → $z = e^{-x}$ بفرض نجد ما أجزاء تكامل

$y_p = e^{x}\sin(e^{-x}) + e^{2x}\left(\cancel{-}e^{-x}\cos(e^{-x}) - \cos(e^{-x})\right) = -e^{2x}\cos e^{-x}$

$\boxed{y = C_1 e^{x} + C_2 e^{2x} - e^{2x}\cos(e^{-x})}$