Linear

Non-Linear
$F(x,y,y',y'')$

- $F(x,y',y'')$
  let $z=\frac{dy}{dx}$
- $F(y,y',y'')$
  let $y''=\frac{dz}{dy}\cdot z$
  $z=\frac{dy}{dx}$

Non-homogeneous

$y_g = y_h + y_N$

y_h: homogeneous, y_N: non-homogeneous

- Undetermined coefficient
  - By assumption
  - Just for polynomial / expon. / trigeometric functions.
  - check that they not exist in y_h, if exist multiply
    - constant coefficient → x
    - variable coefficient → $\ln x$
  - * فقط ضرب الحد المتكرر
- Variation of Parameters

Imp. You must normalize to get $r(x)$

$y_N = y_1 \int \frac{-y_2}{w} r(x)\,dx + y_2 \int \frac{y_1}{w} r(x)\,dx$

*Apply conditions on y_h

Homogeneous

- Constant coefficient
  1. Normalize the equation
     $y'' + Ay' + By = 0$
  2. let $y = e^{\lambda x} \rightarrow e^{\lambda x}(\lambda^2 + A\lambda + B) = 0$
     - $y_g = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$
       (λ is Two distinct value)
     - $y_g = e^{\lambda x}(C_1 + C_2 x)$
       (λ is repeated) $[y_2 = u\,y_1]$
       $[u = C_1 x + C_2]$
     - $y_g = e^{\frac{-a}{2}\lambda}(C_1 e^{wi x} + C_2 e^{-wi x})$ $\left[w = \sqrt{\frac{4b-a^2}{4}}\right]$
       $y_g = e^{\frac{-a}{2}\lambda}[A\cos(wx) + B\sin(wx)]$ (complex root)
- Variable coefficient
  - ~~Normalize the equation~~
  - we will Just Deal with Euler equation $x^2y'' + axy' + by = 0$
  - let $y = x^{\mu} \rightarrow x^{\mu}(\mu^2 + \overset{a-1}{(\ldots)}\mu + b) = 0$
    - $y_g = C_1 x^{\mu_1} + C_2 x^{\mu_2}$
      (μ is Two real distinct values)
    - $y_g = x^{\frac{1-a}{2}}(E + F\ln x)$
      (μ is repeated)
    - $y_g = x^{\mu}[K\cos w\ln x + L\sin w\ln x]$ (complex root).

Higher order differential equ.

أسئلة فقط على 3rd order

Non-homogeneous

*عندما يكون هناك مجموعة مضروبة لا يوجد دائماً يكون هناك ثابت لكل اقتران.

- Indetermined
  As previous
  The Details of this method:
  1. Poly.
     نفرض كثير حدود قوته على الاقل نفس قوة $r(x)$ لكن مع معاملات مجهولة.
  2. Exp. e^{*x}
     $y_N = Ae^{*x}$
  3. Tri.
     نفرض على شكل ثنائي اذا كانت الاقترانات لنفس الزاوية.
- Variation of Parameters
  - Normalize the equation
  - $y_N = y_1\int \frac{w_1}{w} R(x)\,dx + y_2\int\frac{w_2}{w} R(x)\,dx + y_3\int\frac{w_3}{w} R(x)\,dx + \ldots$

$w_1 / w_2 / w_3 / \ldots \rightarrow$ constant or function But Not Zero

- How to get $w \rightarrow$ square matrix of y and it's derivatives.
- How to get $w_1 / w_2 / w_3 \ldots$
  حسب رقم w نفرغ العمود من المصفوفة ب $\begin{matrix}0\\0\\\vdots\\1\end{matrix}$

Homogeneous

- Variable coef.
  $e^{\lambda x}(\lambda^3 + a\lambda^2 + b\lambda + c) = 0$
- constant coef.
  $x^{\mu}(\mu^3 + \mu^2(a-3) + \mu(2-a+b) + c) = 0$

First order differential equation

$y' = F(x,y)$

- Seperable D.E
- Reduction to seperable D.E
  - $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ or $F\left(\frac{x}{y}\right) \rightarrow v = \frac{y}{x}$
  - $\frac{dy}{dx} = [F(x+y)]^{\frac{1}{n}} \rightarrow v^n = x+y$
  - $\frac{dy}{dx} = [F(x+y)]^n \rightarrow v = x+y$
  - $\frac{dy}{dx} = \sin(F(x,y)) \rightarrow v = F(x,y)$
  - $\frac{dy}{dx} = f(ax+by+c) \rightarrow v = ax+by+c$
  - $(a_1x+b_1y+c_1)dx + (a_2x+b_2y+c_2)dy = 0$
    - if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ $\quad z = a_1x+b_1y$ or a_2x+b_2y
    - if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\quad$ Let $x = X+h$, $dx = dX$ $\quad y = Y+K$, $dy = dY$
      - To obtain h & K
        عوض بحل x و y في المعادلتين
- Exact D.E $\quad Mdx + Ndy = 0$
  (when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$)
  * How to solve ??
    1. $u = \int M dx + K(y)$
    2. $\frac{\partial u}{\partial y} = \frac{\partial [\int M dx]}{\partial y} + \frac{\partial K}{\partial y} = N(x,y)$
    3. $u = \int M dx + K(y) = 0$ $\quad$ or vise versa
- Integrating Factor
  (for non exact D.E)
  * How do we find I.F?
    1. $R_1 = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $\quad R_1$ is function of x <u>only</u>
    2. $I.F = e^{\int R_1 dx}$ $\quad$ or vice versa (R_2 اذا ما طلعت R_1 متغير بنعمل)
- Linear D.E $\quad \frac{dy}{dx} + P(x)y = r(x) \Rightarrow y = e^{-\int P(x)dx}\left[C_1 + \int e^{\int P(x)dx} \cdot r(x)\,dx\right]$
  - valid for homogeneous & nonhomogeneous
- Applications :-
  - Mixing
  - Heating/cooling
    $+ \Delta \quad - \Delta$
    $\frac{dT}{dt} = \pm K(T - T_o)$
  - cylinders
    $q = KA\frac{dT}{dr}$

Second order differential equation

- Linear combination of solutions → Just for homogeneous & linear D.E
- Solutions for general solution must be "Basis of solution" (Independent), You can check dependency by
  - Ratio ($\neq$ constant ✓ independent)
  - Determinate for a square matrix ($\neq$ zero ✓ independent)