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The University of Jordan
Faculty of Engineering & Technology
Chemical Engineering Department

(0905231) Mathematical Methods in Chemical Engineering — Summer Semester – 2015/2016

First Exam

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Solve the following ODEs. Show the details of your work.

1. [8 points]
$y'' + 9y = -\sin 3x$

8

$y = y_h + y_p$

$y'' + 9y = 0$

$\lambda^2 + 9 = 0$

$a = 0 \quad b = 9$

$\omega^2 = 9 - \frac{0^2}{4}$

$\omega^2 = 9$

$\omega = 3$

$y_h = A\cos 3x + B\sin 3x$

$y = A\cos 3x + B\sin 3x + \frac{x\cos 3x}{6} + \frac{\sin 6x \cos 3x}{36} + \frac{\sin 3x \cos 6x}{36}$

2. [8 points]

8 $(2xe^x - y^2)dx + 2y\,dy = 0, \quad y(0) = \sqrt{2}$

$M \qquad N$

$$\frac{\partial M}{\partial y} = -2y = \frac{\partial P}{\partial y}$$

$$\frac{\partial N}{\partial x} = 0 = \frac{\partial Q}{\partial x}$$

$$R(x) = \frac{1}{2y}(-2y - 0)$$

$$= -1$$

$$F(x) = e^{\int -1\,dx}$$

$$= e^{-x}$$

$$(2x - e^{-x}y^2)dx + 2ye^{-x}dy = 0$$

$$\frac{\partial M}{\partial y} = -2ye^{-x} \qquad \frac{\partial N}{\partial x} = -2ye^{-x}$$

$$u = \int N\,dy + L(x)$$

$$u = \int 2e^{-x}y\,dy + L(x)$$

$$u = e^{-x}y^2 + L(x)$$

$$\frac{du}{dx} = -e^{-x}y^2 + \frac{dL}{dx}$$

$$2x - e^{-x}y^2 = -e^{-x}y^2 + \frac{dL}{dx}$$

$$\int dL = \int 2x\,dx$$

$$L(x) = x^2$$

$$c = e^{-x}y^2 + x^2$$

$$c = e^0 2 + 0$$

$$c = 2$$

$$\boxed{e^{-x}y^2 + x^2 = 2}$$

2- [8 points]

$y' + \frac{1}{2}y = y^3, \; y(0) = \frac{1}{3}$

(5)

$u = y^{1-3}$

$u = y^{-2}$

$u' + 2 \times \frac{1}{2} u = -2$

$u' - u = -2$

$u' = u - 2$

$\frac{du}{dx} = u - 2$

$\int \frac{du}{u-2} = \int dx$

$\ln u - 2 = x + C$

$u - 2 = e^x$

$y^{-2} - 2 = e^x + c$

$7 = e^0 + c$

$c = 7$

$y^{-2} - 2 = e^x + 7$

$y_p = kx\cos 3x + Mx\sin 3x$

$y_p' = Kx.-3\sin 3x + \cos 3x.k + 3Mx\cos 3x + \sin 3x.M$

$y_p'' = -3kx.3\cos 3x + \sin 3x.-3k - 3k\sin 3x - 9Mx\sin 3x + \cos 3x.3M + 3M\cos 3x$

$-9kx\cos 3x - 3k\sin 3x - 3k\sin 3x - 9Mx\sin 3x + 3M\cos 3x + 3M\cos 3x + 9kx\cos 3x + 9Mx\sin 3x = -\sin 3x$

$-3k - 3k = -1$

$k = \frac{1}{6}$

$3M + 3M = 0$

$M = 0$

$y = A\cos 3x + B\sin 3x + \frac{1}{6}x\cos 3x$

4. [6 points] 6/5

A copper ball whose temperature is 100 °C is placed in water that is maintained at a temperature of 30 °C. At the end of 3 minutes, the temperature of the ball is reduced to 70 °C. How long does it take to reduce the ball temperature to 31 °C?

$$\frac{dT}{dt} = k(T - T_n)$$

$$\int \frac{dT}{T-30} = \int k\,dt$$

$t = 0 \text{ min}, T = 100\,C^\circ$

$t = 3 \text{ min}, T = 70\,C^\circ$

$$\ln T - 30 = kt + c$$

$$\ln 70 = 0 + c$$

$$\boxed{c = 4.2}$$

$$\ln T - 30 = kt + 4.2$$

$$\ln 40 = 3k + 4.2$$

$$3.7 = 3k + 4.2$$

$$3k = -0.5$$

$$\boxed{k = -0.17}$$

too much rounding

$$\ln T - 30 = -0.17t + 4.2$$

$$\ln 1 = -0.17t + 4.2$$

$$-0.17t = -4.2$$

$$\boxed{t = 25 \text{ min}}$$

OK