Past Papers

MATHEMATICAL

Madar

University of Jordan

Faculty of Engineering and Technology
Chemical Engineering Department
0905231 Mathematical Methods for Chemical Engineering

40/40

Name:

Q1) Complete the following table:

Equation	Ordinary or Partial	Order	Independent variables	Dependent variables	Linear or non-linear	Degree
$t\frac{d^2m}{dt^2}-4m\sin(2m)=t\frac{dm}{dt}$	ordinary	second	t	m	non-Linear	first
$4\frac{d^3u}{dx^3}-x^3\frac{du}{dx}+\cos(u)=e^{2x}$	ordinary	third	x	u	non-Linear	first ~~thr~~
$\frac{d^2y}{dx^2}+\frac{dy}{dx}=3x\sin(x)+3y$	ordinary	second	x	y	Linear	first
$\frac{\partial y}{\partial t}+x\left(\frac{\partial y}{\partial x}\right)^3=\frac{x+t}{x-t}$	partial	~~third~~ first	x, t	y	non-Linear	third
$v\frac{\partial^2 v}{\partial t^2}=c^2\frac{\partial^2 v}{\partial x^2}$	partial	second	x, t	v	non-Linear	first
$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial t}+C\frac{\partial^2 u}{\partial t^2}=H\ln(u)$	partial	second	x, t	u	non-Linear	first
$3t^4\frac{d^3u}{dt^3}-t^3\frac{du}{dt}+e^tu=\sqrt[3]{2t}$	ordinary	third	t	u	Linear	first
$\left(\frac{d^2y}{dx^2}\right)^4+3\left(\frac{dy}{dx}\right)^2+4y=\frac{12}{x}$	ordinary	second	x	y	non-Linear	fourth
$3x^2\frac{d^2y}{dx^2}+2\tan(x)\frac{dy}{dx}+e^xy=3y\cos(x)$	ordinary	second	x	y	Linear	first
$x^2\frac{d^2v}{dx^2}+2x\frac{dv}{dx}+ve^v=x$	ordinary	second	x	V	non-Linear	first
$\sin(y)\frac{\partial^2 u}{\partial y^2}+3x\frac{\partial^2 u}{\partial x^2}+2=u$	partial	second	x, y	u	Linear	first
$y\cdot\frac{d^2y}{dx^2}+\frac{x}{y}=x^2\log(x)\cdot y$	ordinary	second	x	Y	non Linear	first

Q2) For the following differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-10y=0$, which one of the given functions is a solution to it?

a) $y(x)=e^{-2x}$ [✓ b) $y(x)=e^{-5x}$] c) Both are solutions

$\frac{dy}{dx}=-2e^{-2x}$

$\frac{d^2y}{dx^2}=-2*-2e^{-2x}$

$\frac{d^2y}{dx^2}=4e^{-2x}$

$4e^{-2x}+\underset{-6}{\cancel{3(-2)}}e^{-2x}-10(e^{-2x})=0$

$\frac{dy}{dx}=-5e^{-5x}$

$\frac{d^2y}{dx^2}=-5*-5e^{-5x}$

$\frac{d^2y}{dx^2}=25e^{-5x}$

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Name: ______________________

Q1) Complete the following table:

Equation	Ordinary or Partial	Order	Independent variables	Dependent variables	Linear or non-linear	Degree
$\frac{dy}{dx} = x^2 + 5y$	ordi ✓	1 ✓	x ✓	y ✓	linear ✓	1 ✓
$\frac{d^3y}{dx^3} + x^2 y = xe^x$	ordin ✓	3 ✓	x ✓	y ✓	linear ✓	1 ✓
$\frac{dr}{d\phi} = \sqrt{r\phi}$	ordin ✓	1 ✓	Φ ✓	r ✓	non ✓	1 ✓
$\frac{\partial^2 V}{\partial x^2} = \sqrt[3]{\frac{\partial V}{\partial y}}$	Partial ✓	2 ✓	x, y ✓	V ✓	non ✓	3 ✓
$\frac{d^3S}{dt^3} + \left(\frac{d^2S}{dt^2}\right)^3 = S - 3t$	ordin ✓	3 ✓	t ✓	S ✓	non ✓	1 ✓
$\frac{\partial U}{\partial t} = 4\frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial y}$	Partial ✓	2 ✓	y t, x ✓	U ✓	linear ✓	1 ✓
$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$	ordin ✓	2 ✓	x ✓	Y ✓	non linear	2/3 3/2 ✗

$\left(\frac{dy}{dx}\right)^{2}$ deg

$\frac{d^2y}{dx^2}$ order

Q2) For the following differential equation $y\frac{dy}{dx} = x$, which one of the given functions is a solution to it?

1) $y = x^2$ 2) $\boxed{y^2 = 1 + x^2}$ 3) $y^2 = x$

$y y' = x \Rightarrow y' = x/y$

① $y = x^2$ ✗ $y' = 2x$

② $y^2 = 1 + x^2$
$2y y' = 2x$
✓ $y' = \frac{2x}{2y}$

③ $2y y' = x$
✗ $y' = \frac{x}{2y}$

Q3) Which of these functions is an explicit function and which is an implicit function?

Function	explicit	implicit
$V = e^{3x}\sin 2y$	/ ✓	
$y^2 - 3x + 3y = 5$		/ ✓
$y = \sqrt{9 - x^2}$	/	

$y^2 + 3y = 3x + 5$

$y = a -$

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Name:

Find the particular solution for the following differential equation

$$10y''+5y'+0.625y=0$$
$$y(0)=2 \quad y'(0)=-4.5$$

$10y''+5y'+0.625y=0 \quad \div 10$

$y''+0.5y'+0.0625y=0$ ✓ homogenous .. S.O.D.G

let $y=\lambda e^{x}$

$e^{x\lambda}(\lambda^2+\underset{a}{0.5}\lambda+\underset{b}{0.0625})$

$\lambda=\frac{-a\mp\sqrt{a^2-4b}}{2}$

$\sqrt{a^2-4b}=\sqrt{0.25-0.25}=0$

so, there is two equal root

$0.25-0.25=0$

$\lambda=\frac{-0.5\mp\sqrt{(0.5)^2-4\times0.0625}}{2}$

$\lambda=\frac{-0.5\mp\sqrt{=}}{2}$

$\lambda_1=\lambda_2=-\frac{0.5}{2}=-0.25$ ✓

$\frac{625}{4}$

2500

0.20

$2500\times\frac{1}{10000}$

$\frac{5}{10}\times\frac{1}{2}$ 2.5

$\therefore y_1=c_1e^{-0.25x}$

$y_2=c_2\cdot x\cdot e^{-0.25x}$ ✓

$\therefore y_g=c_1\cdot e^{-0.25x}+c_2\cdot x\cdot e^{-0.25x}$ ✓

$y_g'=-0.25.c_1.e^{-0.25x}+c_2x\cdot-0.25\cdot e^{-0.25}+e^{-0.25x}\times c_2$

and when $(y(0))=2$, $y'(0)=-4.5$

$2=c_1\cdot1+c_2\times0\times1$

$\boxed{2=c_1}$ ✓

$-4.5=-0.25\times2\times1+0+c_2\times1$

$\boxed{c_2=-4}$ ✗ $c_2=-4$

$\therefore y_p=2e^{-0.25x}+-4.5$

$y_p=2e^{-0.25x}+-4.5\cdot x\cdot e^{-0.25x}$

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Name:

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Find the general solution for the following differential equation

$$\underbrace{\left(-\frac{y}{x^2}+2\cos(2x)\right)}_{M}dx+\underbrace{\left(\frac{1}{x}-2\sin(2y)\right)}_{N}dy=0$$

$\frac{\partial M}{\partial y} = \frac{-1}{x^2}$ ✓ $\frac{\partial N}{\partial x} = \frac{-1}{x^2}$ ✓

so they are equal ...

1. $u = \int N_{(x,y)}\,dy + L(x) \Rightarrow u \int \frac{1}{x} - 2\sin(2y)\,dy + L(x)$ ✓

$= \cos(2y) + L(x) + \frac{y}{x}$

$= \cos(2y) + L(x)$

$\frac{du}{dx} = \frac{\partial u}{\partial x}\left(\int N_{(x,y)}\right) + \frac{du}{dx} L(x) = M(x,y)$

2. $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}\left[\cos 2y\right] + L'(x) = \frac{-y}{x^2} + 2\cos(2x)$

$= \quad \therefore + L'(x) = \frac{-y}{x^2} + 2\cos(2x)$

$L = \int \frac{-y}{x^2} + 2\cos(2x)\,dx + C$

$L(x) = \frac{-y}{2x}\ln(x^2) + \frac{2}{2}\sin(2x) + C$ ✗

3. $u = \frac{1}{x} - 2\sin(2y) + \frac{-y}{2x}\ln(x^2) + \sin(2x) + C$

and this is the general solution ✗

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Find a power series solution about $x_0 = 0$ for the following differential equation:

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$$

①

Solution:

$$xy'' + 2y' - xy = 0$$

$2a_0 +$ [illegible]

$$y'' + \frac{2}{x}y' - y = 0 \qquad 0 \text{ is singular pt.}$$

$$\lim_{x\to 0}\frac{(x-0)\cdot 2}{x} = 2 \qquad \lim_{x\to 0}\frac{(x-0)^2\cdot x}{x} = 0$$

both limit are exist ∴ regular singular pt

$$y = \sum_{j=0}^{\infty} a_j x^{j+r}, \quad y' = \sum a_j (j+r) x^{j+r-1}, \quad y'' = \sum a_j (j+r)(j+r-1) x^{j+r-2}$$

$$x\sum_{j=0}^{\infty} a_j (j+r)(j+r-1) x^{j+r-2} + 2\sum_{j=0} a_j (j+r) x^{j+r-1} - x\sum_{j=0}^{\infty} a_j x^{j+r}$$

$$\sum_{j=0}^{\infty} a_j (j+r)(j+r-1) x^{j+r-1} + 2\sum_{j=0} a_j (j+r) x^{j+r-1} - \sum a_j x^{j+r+1}$$

$$\sum_{j=0}^{\infty} a_j (j+r)(j+r-1) x^{j+r-1} + 2\sum_{j=0}^{\infty} a_j (j+r) x^{j+r-1} - \sum_{j=2} a_{j-2} x^{j+r-1}$$

or

$$\frac{2.5}{5}$$

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Name:

Q1) Complete the following table:

Equation	Ordinary or Partial	Order	Independent variables	Dependent variables	Linear or non-linear	Degree
$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=3x$	ordinary	second	X	y	non-linear	first
$\frac{\partial y}{\partial t}+y\frac{\partial y}{\partial x}=\frac{x+t}{x-t}$	partial	first	t, x	y	non-linear	first
$3x^4\left(\frac{d^3u}{dx^3}\right)^3-x^3\frac{du}{dx}+e^xu=0$	ordinary	third	X	u	non-linear	third
$4x\frac{d^4y}{dx^4}-x^3\frac{dy}{dx}+\cos(x)=5e^{2x}$	ordinary	fourth	X	y	linear	first
$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$	partial	second	t, x	y	linear	first
$A\frac{\partial^2 u}{\partial x^2}+B\left(\frac{\partial^2 u}{\partial x\partial y}\right)^2+C\frac{\partial^2 u}{\partial y^2}=H$	partial	second	X, y	u	non-linear	second
$(t)^{\frac{1}{2}}\frac{d^3x}{dt^3}^{2}=t\sqrt[3]{\frac{dx}{dt}}$ x^2	ordinary	third	t	X	non-linear	second
$\sin(x)\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2\log(x)-y$	ordinary	second	X	y	linear	first
$\left(\frac{d^2y}{dx^2}\right)^2+3\left(\frac{dy}{dx}\right)^3+4\cos(x)=0$	ordinary	second	X	y	non-linear	second
$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}+2u=xy$	partial	second	y, x	u	linear	first
$3x^2\frac{d^2y}{dx^2}+2\ln(x)\frac{dy}{dx}+e^xy=3x\cos(y)$	ordinary	second	X	y	non-linear	first
$\frac{d^2v}{dt^2}+2\frac{dv}{dt}+e^t=t$	ordinary	second	t	v	linear	first

Q2) For the following differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-12y=0$, which one of the given functions is a solution to it?

a) $y(x)=e^{3x}$ b) $y(x)=e^{-4x}$ [✓ c) Both are solutions]

a) $y' = 3e^{3X}$ $y'' = 9e^{3X} \rightarrow 9e^{3X} + 3e^{3X} - 12e^{3X} = 0$ ✓ ∴ its solutions

b) $y' = -4e^{-4X}$ $y'' = 16e^{-4X} \rightarrow 16e^{-4X} + -4e^{-4X} - 12e^{-4X} = 0$ ✓ ∴ its solutions

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Name:

Find the particular solution for the following differential equation

$$xy' = y + 4x^5 \cos^2\left(\frac{y}{x}\right), \quad y(2) = 0$$

2/10

$$y' = \frac{y}{x} + 4x^4 \cos^2\left(\frac{y}{x}\right) \quad , \text{let } \frac{y}{x} = v$$

$$\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} = v + 4x^4 \cos^2 v$$

$$dv = dx\,(v + 4x^4 \cos^2 v)$$

$$dv = v\,dx + 4x^4 \cos^2 v\,dx$$

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Name:

Find the particular solution for the following differential equation

$ax + by + c = 0$

$\sqrt{b^2 - 4ac}$

$y + ay' + by$

$$y'' + 4y' + 5y = 0$$
$$y(0) = 2 \quad y'(0) = -5$$

$\therefore y'' + \overset{a}{4} y' + 5y = 0$, $\sqrt{a^2 - 4ac}$ $= \sqrt{16 - 16} = 0$ $\therefore$ we have 2 similer solution

-25

$\lambda = \frac{-4 \mp \sqrt{4^2 - \boxed{16}}}{2}$ $\cancel{0}16$

$16 - 25 = -$

$\lambda = \frac{-4}{2} = -2$

$y_1 = e^{-2x}$, $y_2 = x e^{-2x}$

$y_g = c_1 e^{-2x} + c_2 x e^{-2x}$ $\rightarrow y_0 = c_1 e^0 + 0 = 2 \rightarrow \boxed{c_1 = 2}$

~~$y_g = c_1 + c_2 x (e^{-2x})$~~ $\rightarrow$ ~~$y(0) = c_1 + c_2(0)$~~

$y'_g = -2 c_1 e^{-2x} - 2 x c_2 e^{-2x}$

$y'(0) = -2 c_1 e^0 - 0 = -5$ $\longrightarrow c_2 = 3$

$\therefore$ particular solution

$y_p = 2 e^{-2x} + 3 x e^{-2x}$

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Name: ______________________________

Q1) Complete the following table:

Equation	Ordinary or Partial	Order	Independent variables	Dependent variables	Linear or non-linear	Degree
$\frac{d^2x}{dy^2}+y\sin x=0$	ordinary	2nd	y	X	non-linear	first
$x^2dy+y^2dx=0$	ordinary	first	X	y	non-linear	first
$\left(\frac{dr}{ds}\right)^3=\sqrt{\frac{d^2r}{ds^2}+1}$	ordinary	2nd	s	r	non-linear	first
$\frac{\partial U}{\partial t}=k\left(\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}\right)$	partial	2nd	t, x, y	U	linear	first
$y\frac{d^2x}{dy^2}+\frac{dx}{dy}+yx^2=0$	ordinary	2nd	y	X	non-linear	first
$\left(\frac{\partial z}{\partial t}\right)=\frac{\partial^2 z}{\partial x^2}$	partial	2nd	t, x	z	linear	first

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Q2) For the following differential equation. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=0$, state if the following equation is a solution or not $y=4e^{-x}\sin(3x)$?

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$y' = -12e^{-x}\cos 3x + -4e^{-x}\sin 3x = -12e^{-x}\cos 3x - 4e^{-x}\sin 3x$

$y'' = \left(-36e^{-x}\sin 3x + 12e^{-x}\cos 3x\right) + \left(12e^{-x}\cos 3x + 4e^{-x}\sin 3x\right) =$

$-32e^{-x}\sin 3x + 24e^{-x}\cos 3x$

$\rightarrow -32e^{-x}\sin 3x + 24e^{-x}\cos 3x - 24e^{-x}\cos 3x - 8e^{-x}\sin 3x + 40e^{-x}\sin 3x$

$0 = 0$

∴ it's solution. ✓

Q3) Which of these functions is an explicit function and which is an implicit function?

12/12

Function	explicit	implicit
$x-1=5e^{-2y}+2y^2+2y$	x dep ✓	
$\sqrt[3]{x}-y^2=c$	x dep ✓	
$\cos(y)+\sin(x)+x=5$	y dep ✓	

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$\frac{7}{10}$

Name.

Use the method of undetermined coefficients to find the general solution for the following system of first order differential equations.

$$y_1' = y_1 + y_2 \boxed{+5\cos(x)}$$
$$y_2' = 3y_1 - y_2 \boxed{-5\sin(x)}$$

$y_h \rightarrow y_1' = y_1 + y_2$

$y_2' = 3y_1 - y_2$

$\rightarrow \det \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{vmatrix} = 0 \rightarrow (1-\lambda)(-1-1) - 3 = 0$

$-1-\lambda+\lambda+\lambda^2-3=0$

$\rightarrow \lambda^2 - 4 = 0 \rightarrow \lambda_1 = 2$

$\lambda_2 = -2$ ✓

For $\lambda_1 = 2$

$\begin{bmatrix} 1-2 & 1 \\ 3 & -1-2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow -A + B = 0 \rightarrow$ if $A = 1 \therefore B = 1$

$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = c_1 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} e^{2x}$ ✓

For $\lambda_2 = -2$

$\begin{bmatrix} 1--2 & 1 \\ 3 & -1--2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow 3A + B = 0$ if $A = -1 \therefore B = 3$

$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = c_2 \begin{Bmatrix} -1 \\ 3 \end{Bmatrix} e^{-2x}$ ✓

$y_h = c_1 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} e^{2x} - c_2 \begin{Bmatrix} -1 \\ 3 \end{Bmatrix} e^{2x}$ ✓

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$\frac{7}{10}$

Name: ______________________________

Use the method of undetermined coefficients to find the general solution for the following system of first order differential equations:

$$y_1' = y_1 + y_2 \boxed{+5\cos(x)}$$
$$y_2' = 3y_1 - y_2 \boxed{-5\sin(x)}$$

$v(x)$

$y_h \rightarrow y_1' = y_1 + y_2$
$y_2' = 3y_1 - y_2$

$\rightarrow \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{vmatrix} = 0 \rightarrow 1-\lambda(-1-\lambda) - 3 = 0$

$-1 - \lambda + \lambda + \lambda^2 - 3 = 0$

$\rightarrow \lambda^2 - 4 = 0 \rightarrow \lambda_1 = 2$
$\lambda_2 = -2$ ✓

for $\underline{\lambda_1 = 2}$

$\begin{bmatrix} 1-2 & 1 \\ 3 & -1-2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow -A + B = 0 \rightarrow \text{if } A = 1 \therefore B = 1$

$\therefore \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = c_1 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} e^{2x}$ ✓

for $\underline{\lambda_2 = -2}$

$\begin{bmatrix} 1--2 & 1 \\ 3 & -1--2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow 3A + B = 0 \quad \text{if } A = -1 \therefore B = 3$

$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = c_2 \begin{Bmatrix} -1 \\ 3 \end{Bmatrix} e^{-2x}$ ✓

$\therefore y_h = c_1 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} e^{2x} + c_2 \begin{Bmatrix} -1 \\ 3 \end{Bmatrix} e^{-2x}$ ✓

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Name: ____________________

Find the general solution for the following system of first order differential equations:

$$y_1' = y_1 + y_2$$
$$y_2' = y_1 - 3y_2$$

$\begin{bmatrix}1 & 1\\ 1 & -3\end{bmatrix}\begin{Bmatrix}y_1\\ y_2\end{Bmatrix} = \begin{Bmatrix}y_1'\\ y_2'\end{Bmatrix}$

$\begin{bmatrix}1-\lambda & 1\\ 1 & -3-\lambda\end{bmatrix}\begin{Bmatrix}y_1\\ y_2\end{Bmatrix} = \begin{Bmatrix}y_1'\\ y_2'\end{Bmatrix}$

Det = $(1-\lambda)(-3-\lambda) - 1 = 0$

$-3 - \lambda + 3\lambda + \lambda^2 - 1 = 0$

$\lambda^2 + 2\lambda - 4 = 0$

$\lambda = \frac{-2 \mp \sqrt{4+16}}{2} = \frac{-2 \mp \sqrt{20}}{2} = -1 \mp \sqrt{\frac{20}{4}} = -1 \mp \sqrt{5}$

[illegible] $\approx 2.1 \approx 2$

$= -1 \mp 2.1$

$\approx -1 \mp 2 = 1, -3$

$\boxed{\lambda_1 = -3 \quad \lambda_2 = 1}$

wrong eign values

$\lambda_1 = -3$

$\begin{bmatrix}1-(-3) & 1\\ 1 & -3-(-3)\end{bmatrix}\begin{Bmatrix}A\\ B\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \Rightarrow \begin{bmatrix}4 & 1\\ 1 & 0\end{bmatrix}\begin{Bmatrix}A\\ B\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$

$4A + B = 0$

$4A = -B$ when $A = 1$, $B = -4$

$y = e^{-3x}\begin{Bmatrix}1\\ -4\end{Bmatrix}$

$$y_h = c_1 \begin{Bmatrix} A_1 \\ B_1 \end{Bmatrix} e^{\lambda_1 x} + c_2 \begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} e^{\lambda_2 x} \Rightarrow y_h = c_1 \begin{Bmatrix} -3 \\ 1 \end{Bmatrix} e^{x} + c_2 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} e \quad \boxed{\lambda_1 \ \lambda_2 = -1}$$

$$Y = \begin{bmatrix} A_1 y_1 & A_2 y_2 \\ B_1 y_1 & B_2 y_2 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} -3e^{x} & -e^{-x} \\ e^{x} & e^{-x} \end{bmatrix} \qquad \text{Det}[Y] = -3 - (-1) = -2$$

$$Y^{-1} = \frac{\begin{bmatrix} e^{-x} & e^{-x} \\ -e^{x} & -3e^{x} \end{bmatrix}}{-2}$$

$$g(x) \cdot Y^{-1} \Rightarrow g(x) = \begin{Bmatrix} -2e^{-x} \\ 0 \end{Bmatrix} \Rightarrow Y^{-1} \cdot g(x) = \frac{-1}{2} \begin{bmatrix} e^{-x} & e^{-x} \\ -e^{x} & -3e^{x} \end{bmatrix} \begin{Bmatrix} -2e^{-x} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2e^{-2x} + 0 \\ 2 + 0 \end{Bmatrix} = \frac{-1}{2} \begin{Bmatrix} -2e^{-2x} \\ 2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} +e^{-2x} \\ -1 \end{Bmatrix}.$$

$$u(x) = \int_0^x Y^{-1}(y) \cdot g(x) = \int_0^x \begin{Bmatrix} +e^{-2x} \\ -1 \end{Bmatrix} dy + C$$

$$\begin{Bmatrix} \frac{e^{-2x}}{\mp 2} \\ -x \end{Bmatrix}_0^x + C \Rightarrow u(x) = \begin{Bmatrix} -\frac{1}{2} e^{-2x} \; \boxed{-\frac{1}{2}} \\ +x \end{Bmatrix}$$

$$y_p = Y(x) \cdot u(x) = \begin{bmatrix} -3e^{x} & -e^{-x} \\ e^{x} & e^{-x} \end{bmatrix} \begin{Bmatrix} \frac{1}{2} e^{-2x} \; \boxed{-\frac{1}{2}} \\ \mp x \end{Bmatrix} = \begin{Bmatrix} \frac{3}{2} e^{-x} + x e^{-x} - \frac{3}{2} e^{x} \\ -\frac{1}{2} e^{-x} - x e^{-x} + \frac{1}{2} e^{x} \end{Bmatrix}.$$

$$y_1 = -3c_1 e^{x} - c_2 e^{-x} + \frac{3}{2} e^{-x} + x e^{-x} \mp \frac{3}{2} e^{x} \Rightarrow \underbrace{(-c_2 + \tfrac{3}{2})}_{C_2'} e^{-x} + 3C_1' e^{x} + x e^{-x}$$

$$y_2 = c_1 e^{x} + c_2 e^{-x} - \frac{1}{2} e^{-x} - x e^{-x} + \frac{1}{2} e^{x} \qquad y_1 = C_2' e^{-x} + 3c_1' e^{x} + x e^{-x}$$

$$\Rightarrow \boxed{C_2' = (1 + C_2'')}\,!$$

$$y_2 = c_1 e^{x} + \underbrace{(c_2 - \tfrac{1}{2})}_{C_2'} e^{-x} - x e^{-x} \qquad \boxed{y_2 = -C_1' e^{x} - C_2'' e^{-x} - x e^{-x}} \qquad (1 + C_2'') e^{-x} + 3C_1' e^{x} + x e^{-x}$$

$$\boxed{y_1 = C_2'' e^{-x} + e^{-x} + 3c_1 e^{x} + x e}$$

Madar
ask :: believe and recieve

أسئلة سنوات

Mathmatical

Madar
ask :: believe and recieve

امتحانات سابقة

MID EXAMS

University of Jordan

Faculty of Engineering and Technology
Chemical Engineering Department
0905231 Mathematical Methods for Chemical Engineering
First Semester 2009/2010 **Midterm EXAMINATION** 90 minutes

Student Name	
Student Number	
Section	1

Question Number	Full Mark	Mark
1	6	5.0
2	7	7.0
3	9	8.0
4	8	5.0
Total	30	25

عهد والتزام

انا الطالب رقمي الجامعي

اقسم بالله العظيم انني سالتزم الصدق والامانة اثناء اداني لامتحان

الطرق الرياضية في الهندسة الكيميائية

وانني لن اغش او احاول الغش، ولن اقدم المساعدة لاي شخص او اتلقاها من اي شخص، طيلة فترة الامتحان. وان اجابتي على كامل الاسئلة ستكون نتاج جهدي الشخصي وحدي، وانني اتعهد بتحمل كافة المسؤوليات والعقوبات القانونية، المنصوص عليها في انظمة وتعليمات الجامعة، في حال عدم التزامي بذلك.

التاريخ: 2009-11-24 توقيع الطالب

Question 1 [6 *Marks*]:

Solve the following first order differential equation:

$$\left[2\cos(y)+4x^2\right]dx=\left[x\sin(y)\right]dy$$

Solution:

$$\underbrace{(2\cos(y)+4x^2)}_{M}dx+\underbrace{(-x\sin(y))}_{N}dy=0$$

$$\frac{\partial M}{\partial y}=-2\sin y \qquad \frac{\partial N}{\partial x}=-\sin y \qquad \frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

$$R=\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$$

$$=\frac{1}{-x\sin y}\left(-2\sin y+\sin y\right)=\frac{1}{-x\sin y}(-\sin y)=\frac{1}{x}=R(x)$$

$$\therefore IF=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$\rightarrow \acute{M}=(2x\cos y+4x^3)dx \qquad \acute{N}=-x^2\sin y$$

$$\frac{\partial \acute{M}}{\partial y}=-2x\cos y \qquad \frac{\partial \acute{N}}{\partial x}=-2x\sin y$$

$$\rightarrow ① \; u=\int(2x\cos y+4x^3)\,dx+k_y$$

$$u=x^2\cos y+x^4+k_y$$

5

$$② \; \frac{du}{dy}=-\cancel{x^2\sin y}+\frac{dk_y}{dy}=-x^2\sin y$$

$$\rightarrow \frac{dk_y}{dy}=1 \rightarrow \int dk_y=\int dy$$

1

$$\boxed{k=y+c}$$

1

$$③ \; \boxed{u=x^2\cos y+x^4+y+c}$$

1

Question 2 [7 Marks]:

Newton's law of cooling states that the temperature of an object changes at a rate proportional to its surroundings. If the temperature of a cake is 150°C when it leaves the oven and is 90°C 10 minutes later, when will it be practically equal to the room temperature of 15°C, say, when will it be 16°C?

Newton's law of cooling $\frac{dT}{dt} = k(T - T_{amb})$

Condition $T = 90$ $t = 10$

$T = 150$ $t = 0$

Solution:

$\rightarrow \frac{dT}{dt} = k(T - 15)$

$\frac{dT}{T-15} = \int k\,dt$

$\boxed{\ln(T-15) = kt + C}$

$\rightarrow$ at $t=0$ $T=150 \longrightarrow \ln(150-15) = k(0) + C$

$\rightarrow \boxed{C = 4.91}$

$\rightarrow$ at $t=10$ $T=90 \rightarrow \ln(90-15) = k(10) + 4.91$

$k = -0.0588$

$\therefore \ln(T-15) = -0.0588\,t + 4.91$

when $T = 16°C$ $t = ??$

$\ln(16-15) = -0.0588\,t + 4.91$

$\rightarrow \boxed{t = 83.45 \text{ min}}$

7

Question 3 [8 Marks]:

A- Find the general solution for the following ordinary differential equation:

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=0$$

$y=e^{-3x}\left[A\cos(16)x+B\sin(16)x\right]$	*3 Marks*

(2.0

B- Find the particular solution for the following ordinary differential equation:

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$$
$$y(0)=0,\ y'(0)=2$$

$y=2xe^{3x}$	*3 Marks*

(3

C- Find an ordinary differential equation for which the given functions are solutions

$$y_1=e^{-0.25x},\ y_2=e^{-4x}$$

$y''+4.25y'+y=0$	*3 Marks*

(3

Ⓐ $y''+6y'+13y=0$ homo with constant.

$\therefore y=e^{\lambda x}$

$(\lambda^2+6\lambda+13)e^{\lambda x}=0 \rightarrow \lambda^2+6\lambda+13=0 \rightarrow \lambda=\frac{-6\pm\sqrt{6^2-4(13)}}{2}$

$=-3\pm\sqrt{\frac{4(13)-6^2}{4}}\sqrt{-1}$

$\boxed{\omega=16}$

$$\boxed{\therefore y=e^{-3x}\left[A\cos 16x+B\sin 16x\right]}$$

Ⓑ $y''-6y'+9y=0$ hom. cons.

$y=e^{\lambda x}$

$\lambda^2-6\lambda+9=0 \quad \lambda=\frac{6\pm\sqrt{6^2-4(9)}}{2}$

$\boxed{\lambda_1=3}=\lambda_2$

$y=c_1e^{3x}+c_2xe^{3x}$

$(c_1+c_2x)e^{3x}$

$y(0)=0$

$\boxed{0=c_1}$

$y'=3c_1e^{3x}+c_2e^{3x}+3c_2xe^{3x}$

$2=3c_1+c_2$

Question 4 [8 Marks]:

Using the method of variation of parameters to find the solution for the following differential equation:

$$x^3\frac{d^3y}{dx^3}-3x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}-6y=24x^5$$

$$y(1)=1,\ y'(1)=3,\ y''(1)=14$$

Solution:

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 24x^5$$

$$x^3y''' - 3x^2y'' + 6x - 6 = 0$$

$$y = x^m \quad y' = mx^{m-1} \quad y'' = m(m-1)x^{m-2} = (m^2-m)x^{m-2} \quad y''' = m(m-1)(m-2)x^{m-3} = (m^3-3m^2+2m)x^{m-3}$$

$$x^3(m^3-3m^2+2m)x^{m-3} - 3x^2(m^2-m)x^{m-2} + 6x\,mx^{m-1} - 6x^m = 0$$

$$(m^3-3m^2+2m)x^m - 3(m^2-m)x^m + 6mx^m - 6x^m = 0$$

$$x^m(m^3-6m^2+11m-6) = 0$$

5

$$m_1 = 1,\ m_2 = 3,\ m_3 = 2$$

$$y_{homo} = C_1\underset{y_1}{x} + C_2\underset{y_2}{x^3} + C_3\underset{y_3}{x^2}$$

$$W = \begin{vmatrix} x & x^3 & x^2 \\ 1 & 3x^2 & 2x \\ 0 & 6x & 2 \end{vmatrix} \rightarrow \text{Det}|W| = x\begin{vmatrix} 3x^2 & 2x \\ 6x & 2 \end{vmatrix} - x^3\begin{vmatrix} 0 & 2x \\ 0 & 2 \end{vmatrix} + x^2\begin{vmatrix} 1 & 3x^2 \\ 0 & 6x \end{vmatrix}$$

$$\text{Det}|W| = x(6x^2-12x^2) - x^3(2) + x^2(6x)$$

$$= -6x^3 - 2x^3 + 6x^3 = \boxed{-2x^3}$$

Good Luck

$$\therefore y_{whole} = -2.914x + 2.6x^3 + 1.2x^2 + 0.14285x^5$$

University of Jordan

Faculty of Engineering and Technology
Chemical Engineering Department
0905231 Mathematical Methods for Chemical Engineering

Midterm EXAMINATION

Second Semester 2008/2009

80 minutes

Student Name	
Student Number	
Section	ح ث خ (١٠:٠٠ - ١١:٠٠)

Question Number	Full Mark	Mark
1	6	5.0
2	7	3.0
3	9	9.0
4	8	5.5
Total	30	22.5

عهد والتزام

انا الطالب رقمي الجامعي

اقسم بالله العظيم انني سالتزم الصدق والامانة اثناء اداني لامتحان

الطرق الرياضية في الهندسة الكيميائية

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التاريخ: ٢٠٠٩-٤-٠٩

توقيع الطالب

Question 1 [6 Marks]:

Solve the following nonlinear first order differential equation:

$$\underbrace{\left(x^4+y^2\right)}_{M}dx+\underbrace{(-xy)}_{N}dy=0$$

$$y(2)=1$$

Solution:

$$\frac{\partial M}{\partial y}=2y \qquad \frac{\partial N}{\partial x}=-y$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x} \quad \text{Not exact}$$

$$R_1=\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{1}{-xy}\left(2y--y\right)=\frac{1}{-xy}(3y)=-\frac{3}{x}$$

∴ it's function of x only..

$$I.F=e^{\int R_1\,dx}=e^{\int\frac{-3}{x}dx}=e^{-3\int\frac{1}{x}dx}=e^{-3\ln x}=e^{\ln x^{-3}}=\frac{1}{x^3}$$

we multiply by $\frac{1}{x^3}$

$$(x^4+y^2)*\frac{1}{x^3}dx+(-xy)*\frac{1}{x^3}dy=0$$

$$\underbrace{(x+y^2x^{-3})}_{M'}dx+\underbrace{\left(\frac{-y}{x^2}\right)}_{N'}dy=0$$

5

$$\frac{\partial M}{\partial y}=x^{-3}\cdot 2y \qquad \frac{\partial N}{\partial x}=\frac{y*2x}{x^4}=2y.x^{-3}$$

so $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ ∴ it's exact.

① $U=\int M_{(x,y)}\,dx+K(y)$

$=\int(x+y^2x^{-3})\,dx+K(y)$

$=\frac{1}{2}x^2+-\frac{y^2}{2}x^{-2}+K(y)$

② $\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\int M'(x,y)\,dx+\frac{\partial}{\partial y}K(y)=N'(x,y)$ $\quad \frac{\partial}{\partial y}\left(\frac{1}{2}x^2-\frac{y^2}{2}\right.$

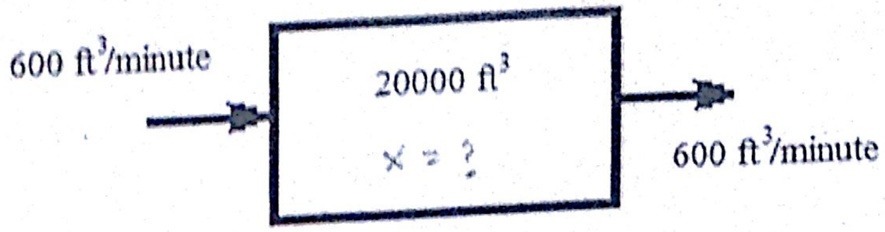
$\frac{\partial}{\partial y}K(y)=N'(x,y)-\frac{\partial}{\partial y}\int M'(x,y)\,dx$

$=-\frac{y}{x^2}-\left(-\frac{2y}{2}x^{-2}\right)$

$\frac{\partial}{\partial y}K(y)=$ ∴

Question 2 [7 Marks]:

In a room containing 20000 ft^3 of air, 600 ft^3 of fresh air flows in per minute, and the mixture (made practically uniform by circulating fans) is exhausted at a rate of 600 ft^3 per minute. What is the amount of fresh air $y(t)$ at any time if $y(0) = 0$? After what time will 90% of the air be fresh?



Solution:

$$\frac{dy}{dt} = 600 - \frac{600\,y}{2000}$$

$$\frac{dy}{dt} + (600 - 0.3\,y)$$

How many dependent variables do you have?

$$\frac{dy}{600-0.3x} - dt$$

$$\int \frac{dy}{600-0.3x} - \int dt = 0$$

$$\frac{1}{-0.3}\int \frac{-0.3}{600-0.3x}\,dy - \int dt = 0$$

$$\frac{1}{-0.3}\ln(-0.3y+600) - t = c$$

3.0

at $y(0)=0$

$$\frac{1}{-0.3}\ln(0+600) - 0 = c$$

$$\frac{1}{-0.3}\ln(600) = c \Rightarrow c = -21.32$$

when $y = \frac{90}{100}$

$$\frac{1}{-0.3}\ln(-0.3 \times \frac{90}{100} + 600) - t = -21.32$$

X

Question 3 [9 Marks]:

a) Find an ODE $y'' + ay' + by = 0$ for the following basis $e^{0.5x}, e^{-3.5x}$

Answer = ⇨ $y'' + 3y - 1.75 = 0$

4.5

b) Find the general solution for the following O.D.E.

$$2x^2y'' + 4xy' + 5y = 0$$

Answer = ⇨ $x^{-\frac{1}{2}}\left(A\cos(\tfrac{1}{2}\ln x) + B\sin(\tfrac{1}{2}\ln x)\right)$

4.5

c) Find the general solution for the following O.D.E.

$$y''' + 4y'' + 12y' + 8y = 0$$

Answer = ⇨ $e^{[illegible]} + e^{-2x}\left[A\cos 2x + B\sin 2x\right]$

Question 4 [8 Marks]:

Using the method of variation of parameters find the particular solution for the following differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin(x)$$

$$y(0) = 3,\ y'(0) = 1$$

Solution:

$y'' - 2y' + y = e^x \sin(x)$ ⇒ Non-homo. constants..

① solving homogeneous part.

$y_h = ?$

$y'' - 2y' + y = 0$

assume $y = e^{\lambda x} \rightarrow y' = \lambda e^{\lambda x} \rightarrow y'' = \lambda^2 e^{\lambda x}$

$y'' - 2y' + y = 0 \rightarrow e^{\lambda x}(\lambda^2 - 2\lambda + 1) = 0$

$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = \frac{-a \mp \sqrt{a^2 - 4b}}{2}$

$= \frac{2 \mp \sqrt{4 - 4}}{2} = 1$

$\lambda_1 = \lambda_2 = 1$

$\therefore y_1 = e^x, \quad y_2 = x e^x$

$\therefore y_h = c_1 e^x + c_2 x e^x$

② find $y_p = ?$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix} \rightarrow$ determinate $= e^x(xe^x + e^x) - (e^x \cdot xe^x)$

$= xe^{2x} + e^{2x} - xe^{2x} = e^{2x}$

$\therefore W = e^{2x}$

$y_p = -y_1 \int \frac{y_2 \cdot r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx$

$= -e^x \int \frac{\cancel{x e^x} \cdot (e^x \sin x)}{\cancel{e^{2x}}} + x e^x \int \frac{e^x (e^x \sin(x))}{\cancel{e^{2x}}}$

$= -e^x \int x \sin x \, dx + x e^x \int \sin(x)$

$= -e^x \int x \sin(x) \, dx + x e^x (-\cos(x))$

$= -e^x \left[\int x \sin(x) dx + x \cos(x)\right)$

5.5

③ $y_g = y_h + y_p = c_1 e^x + c_2 x e^x + -e^x \left[\int x \sin(x) \, dx + x \cos(x)\right]$ → $x \sin x$ ✗

يتبع

University of Jordan

Faculty of Engineering and Technology
Chemical Engineering Department
Mathematical Methods for Chemical Engineering
Midterm Exam

$\frac{26}{30}$

Instructor: Dr. Y. Mubarak
Student Name:
13/11/2007
Student No.:

Question 1 [4 Marks]:

Find the trajectpries of the following curves:

$$y = ce^{\left(\frac{x^2}{2}\right)}$$

Solution:

$y = ce^{x^2/2}$

$\frac{y}{e^{x^2/2}} = c$ ✓

$\frac{dy}{dx}$

$e^{x^2/2} \cdot y' + \frac{\overset{-2x}{\boxed{x^2}}}{2} e^{(-x^2/2)} y = 0$

$e^{x^2/2} \cdot y' = -\frac{x^2}{2} e^{x^2/2} y$

$y' = \frac{-\frac{x^2}{2} e^{x^2/2} y}{e^{x^2/2}}$

3.0

$y' = -\frac{x^2}{2} y$

$\frac{dy}{dx} = \frac{-1}{2} x^2 y$

$\frac{dy}{dx} = \frac{-1}{\cancel{(-)}0.5 x^2 y}$

$dx = 0.5x^2 y \, dy$

$\frac{dx}{0.5x^2} = \int y \, dy$

$\frac{-2}{x} = 0.5y^2 \Rightarrow -4\cancel{x} = y^2 \Rightarrow y = \left(-\frac{4}{x}\right)^{1/2} + c$

where is the constant of integration

Question 4 [7 Marks]:

Using the method of variation of parameters to find the particular solution for the following differential equation:

$$x^2 y'' - 3xy' + 4y = x^2 . \ln x$$

$$y(1) = 2 \text{ and } y'(1) = 5 .$$

[Hint: $\int \frac{\ln^n x dx}{x} = \frac{\ln^{n+1} x}{n+1}$]

Solution:

$y_h = x^2 y'' - 3xy' + 4y = 0$

let $y = x^m$

$m^2 + (-3-1)m + 4 = 0$

$m^2 - 4m + 4 = 0$

$\frac{+4 \pm \sqrt{16-16}}{2} = \underline{\underline{+2 \pm 0}}$

$m_1 = m_2 = +2$

$y_h = c_1 x^{+2} + c_2 \ln x \, x^{+2} = \boxed{(c_1 + c_2 \ln x) x^{-2}}$

$$w = \begin{vmatrix} x^{+2} & x^2 . \ln x \\ +2x & x^2 . \frac{1}{x} + \ln x (2x) \end{vmatrix}$$

$= x^2 (x + 2x \ln x) - 2x^3 \ln x$

$x^3 + 2x^3 \ln x - 2x^3 \ln x$

$w = x^3$

$y_p = -y_1 \int \frac{y_2 . r(x)}{w} dx + y_2 \int \frac{y_1 . r(x)}{w} dx$

$-x^2 \int \frac{x^2 \ln x . x^2 \ln x}{x^3} dx + x^2 \ln x \int \frac{x^2 . x^2 \ln x}{x^3} dx$

$-x^2 \int \frac{x^4 \ln^2 x}{x^3} dx + x^2 \ln x \int \frac{x^4 \ln x}{x^3} dx$

Question 5 [8 Marks]:

Using the method of undetermined coefficients to find the general solution for the following differential equation:

$$\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}-4\frac{dy}{dx}+8y=e^{-3x}+8x^2$$

$y'''-2y''-4y'+8y=0$

Solution:

let $y=e^{\lambda x}$

$y_h=$

$\lambda^3-2\lambda^2-4\lambda+8=0$

$\lambda_1=2$; $(\lambda-2)=0$

$$\begin{array}{r|l} & \lambda^2-4 \\ \lambda-2 & \lambda^3-2\lambda^2-4\lambda+8 \\ & \cancel{\lambda^3}\mp 2\lambda^2 \\ & -4\lambda+8 \\ & \pm 4\lambda\mp 8 \\ & 0 \end{array}$$

$\lambda^2-4=0$

$\lambda^2=4$

$\boxed{\lambda_2=2}$, $\boxed{\lambda_3=-2}$

$\boxed{y_h=C_1e^{2x}+C_2xe^{2x}+C_3e^{-2x}}$

$$\cancel{\begin{array}{r|l} & \lambda^2+\lambda+4 \\ \lambda+2 & \lambda^3+4\lambda^2+4\lambda+8 \\ & \lambda^3+2\lambda^2 \\ & 2\lambda^2+4\lambda+8 \\ & \lambda^2+4\lambda \\ & 4\lambda+8 \\ & \lambda^2+\lambda \\ & 8\lambda+8 \end{array}}$$

$y_p=K_1e^{-3x}+K_2x^2+K_3x+K_4$

$y_p'=-3K_1e^{-3x}+2K_2x+K_3$

$y_p''=9K_1e^{-3x}+2K_2$

$y_p'''=-27K_1e^{-3x}$

substitute $-27K_1e^{-3x}-18K_1e^{-3x}-4K_2+12K_1e^{-3x}-8K_2x-4K_3+8K_1e^{-3x}$
$+8K_2x^2+8K_3x+8K_4=e^{-3x}+8x^2$

Good Luck

$-25K_1e^{-3x}=e^{-3x}$ $\boxed{K_1=\frac{-1}{25}}$

$8K_2x^2=8x^2$ $\boxed{K_2=1}$

$-8K_2x+8K_3x=0$

$8K_3x=8$ $\boxed{K_3=1}$

$-4K_2-4K_3+8K_4=0$

$8K_4-4K_2=4$

$K_4=1$

$$y_p = -\frac{1}{25} e^{-3x} + x^2 + x + 1$$

$$y_{Net} = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{-3x} - \frac{1}{25} e^{-3x} + x^2 + x + 1$$

8

19.5

University of Jordan

Faculty of Engineering and Technology
Chemical Engineering Department
Mathematical Methods for Chemical Engineering
Midterm Exam

Instructor: Dr. Y. Mubar
09/4/2007
Student Name:
Student No.:

Question 1 [3.5 Marks]:

Find a second order homogeneous linear differential equation for which the given functions are solutions

$$y_1 = \sqrt{x} \text{ and } y_2 = \sqrt{x^3}$$

Using the Wronskian, show that these solutions are linearly independent.

$y_1 = x^{1/2}$ $\quad$ $y_2 = x^{1/3}$ $\quad \Rightarrow \quad m_1 = \frac{1}{2}$ ✓ , $m_2 = \frac{1}{3}$

$y_1 = x^{m_1}$ $\quad$ $y_2 = x^{m_2}$

α

$$(m - \tfrac{1}{2})(m - \tfrac{1}{3}) = m^2 - \tfrac{1}{3}m - \tfrac{1}{2}m + \tfrac{1}{6}$$

$$m^2 \underbrace{- \tfrac{5}{6}}_{(1-a)} m + \underbrace{\tfrac{1}{6}}_{b}$$

$$1 - a = -\frac{5}{6} \Rightarrow \frac{6}{6} + \frac{5}{6} = a \Rightarrow \boxed{a = \frac{11}{6}}$$

$$b = \frac{1}{6}.$$

$$\boxed{x^2y'' + \frac{11x}{6}y - \frac{1}{6}y = 0}$$ ✓

α $\quad$ α

$\frac{1}{2}x^{-\frac{1}{2}} \Leftarrow x^{\frac{1}{2}}$

$x^{\frac{3}{2}} \Rightarrow \frac{3}{2}x^{\frac{1}{2}}$

$\frac{1}{2\sqrt{x}}$

$\frac{3\sqrt{x}}{2}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \sqrt{x} & \sqrt{x3} \\ \frac{1}{2\sqrt{x}} & \frac{3\sqrt{x}}{2} \end{vmatrix} = \frac{3x}{2} - \frac{1}{2}x = \boxed{x}$$ ✓

$x \neq$ const

so they are independent.

3

Question 2 [5 Marks]:

Two chemicals *A* and *B* react to form another chemical *C*. It is found that the rate at which *C* is formed varies as the product of the instantaneous amounts of chemical *A* and *B* present. The formation requires 3 *lb* of *A* for each pound of *B*. If *5 lb* of *A* and 15 *lb* of *B* are present initially, and if 2 *lb* of *C* are formed in 10 *minutes*, find the amount of chemical *C* at *any time*? Find amount of *C* after *long time*?

㊀

$3A + B = C$

$5 lb_m \quad 15 lb_m$

$$\frac{dx}{dt} = (\overset{A}{5 - \frac{3}{4}x})(4)(\overset{B}{15 - \frac{1}{4}x})$$

$$\frac{dx}{dt} = 5 - \frac{3}{4}x + 15 - \frac{1}{4}x$$

$$\frac{dx}{dt} = 20 - \frac{4}{4}x \Rightarrow \frac{dx}{dt} = 20 - x$$

$$dx = (20 - x)\,dt$$

$$\frac{dx}{20 - x} = dt \Rightarrow -\ln(20 - x) = t + C$$

$$\ln\left(\frac{1}{20 - x}\right) = t + C \qquad t = 10\,min \quad x = 2\,lb$$

$$\ln\left(\frac{1}{20 - 2}\right) = 10 + C$$

$$-2.89 = 10 + C \Rightarrow \boxed{C = -12.89}$$

$$\ln\left(\frac{1}{20 - x}\right) = t - 12.89$$

$$\text{at } t = \infty \Rightarrow \ln\left(\frac{1}{20 - x}\right) = \infty$$

$$\frac{1}{20 - x} = 0$$

2

Question 3 [10.5 Marks]:

Find the general solution for 3 of the following differential equations:

1. $\frac{dy}{dx} = \frac{-(y^3 + 2ye^x)}{(e^x + 3y^2)}$
2. $\frac{dy}{dx} = \sqrt{2x + 3y}$ ✓
3. $\frac{d^2y}{dx^2} + \left(1 + \frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2 = 0$
4. $(x-2)^2 \frac{d^2y}{dx^2} + 5(x-2)\frac{dy}{dx} + 3y = 0$

$\boxed{2}$ $\frac{dy}{dx} = \sqrt{2x+3y}$

$2x + 3y = v^2$

$2 + 3\frac{dy}{dx} = 2v\frac{dv}{dx}$

$\frac{2v}{3}\frac{dv}{dx} - \frac{2}{3} = \frac{dy}{dx}$

$\frac{2v}{3}\frac{dv}{dx} - \frac{2}{3} = v \Longrightarrow 2v\frac{dv}{dx} - 2 = 3v$

$\frac{dv}{dx} = \frac{3v+2}{2v}$

$$\begin{array}{r|l} & \frac{2}{3} \\ 3v+2 & 2v \\ & -2v + \frac{4}{3} \\ \hline & \frac{-4}{3} \end{array}$$

$\frac{2v}{3v+2}dv = dx$

$\int \frac{2}{3} + \frac{\frac{-4}{3}}{3v+2} dv = \int dx$

$\frac{2v}{3} - \frac{4}{3 \times 3}\ln(3v+2) = x + C$

$\frac{2v}{3} - \frac{4}{9}\ln(3v+2) = x + C$

$\frac{2\sqrt{2x+3y}}{3} - \frac{4}{9}\ln(3\sqrt{2x+3y}+2) = x + C$

3.5

10.5

$\boxed{4}$ $(x-2)^2 \frac{d^2y}{dx^2} + 5(x-2)\frac{dy}{dx} + 3y = 0.$

$u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$u^2 y'' + \underset{a}{5} u y' + \underset{b}{3} y = 0.$

$y = u^m$

$m^2 + (a-1)m + b = 0.$

$m^2 + 4m + 3 = 0.$

$(m+1)(m+3)$

$m_1 = -1$ $m_2 = -3.$

$y_1 = u^{1}$ $y_2 = u^3$

$y_1 = (x-2)$ $y_2 = (x-2)^3$

$y_G = c_1(x-2)^{-1} + c_2(x-2)^{-3}$

3.0

$\boxed{1}$ $$\frac{dy}{dx} = \frac{-(y^3+2ye^x)}{(e^x+3y^2)}$$

$$-(y^3+2ye^x)dx = (e^x+3y^2)dy.$$

$$\underbrace{(y^3+2ye^x)}_{M}dx + \underbrace{(e^x+3y^2)}_{N}dy = 0.$$

$$\frac{\partial M}{\partial y} = 3y^2+2e^x \neq \frac{\partial N}{\partial x} = e^x$$

$$R_1 = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \Rightarrow \frac{1}{e^x+3y^2}(3y^2+2e^x-e^x) = \frac{3y^2+e^x}{e^x+3y^2} = 1$$

$$\boxed{R_1 = 1}$$

$$I.F = e^{\int R_1 dx} \Rightarrow IF = \boxed{e^x}$$

3.5

$$\underbrace{(e^x y^3 + 2ye^{2x})}_{M'}dx + \underbrace{(e^{2x}+3y^2e^x)}_{N'}dy = 0.$$

$$\frac{\partial M}{\partial y} = 3e^xy^2 + 2e^{2x} = \frac{\partial N}{\partial x} = 2e^{2x} + 3y^2e^x$$

$$u = \int M\,dx + k(y).$$

$$u = \int e^xy^3 + 2ye^{2x}\,dx + k(y).$$

$$u = y^3e^x + ye^{2x} + ky$$

$$\frac{du}{dy} = \cancel{3y^2e^x} + \cancel{e^{2x}} + \frac{dk}{dy} = \cancel{e^{2x}} + \cancel{3y^2e^x}$$

$$\frac{dk}{dy} = 0$$

$$k(y) = C$$

$$u = y^3e^x + ye^{2x} + C$$

Question 4 [5 Marks]:

Find the general solution for **1** of the following differential equations:

1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = xe^x \ln x + y$, (Hint; $\int x^m \ln x dx = \frac{x^{m+1}}{m+1}\left[\ln(x) - \frac{1}{m+1}\right]$).
2. $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} = e^{2x}\sin(x) + 6y$

$y_p =$ [illegible]

$$y''' - 6y'' + 11y' - 6y = e^{2x}\sin(x)$$

① $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$$\begin{array}{r|l} & \lambda^2 - 5\lambda + 6 \\ \hline \lambda - 1 & \lambda^3 - 6\lambda^2 + 11\lambda - 6 \\ & -\lambda^3 + \lambda^2 \\ \hline & -5\lambda^2 + 11\lambda - 6 \\ & +5\lambda^2 \mp 5\lambda \\ \hline & 6\lambda - 6 \\ & \mp 6\lambda + 6 \\ \hline & 0 \end{array}$$

$\lambda_1 = 1$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 3)(\lambda - 2) = 0$

$\boxed{\lambda_3 = 3}$ $\boxed{\lambda_2 = 2}$

$y_1 = e^x$, $y_2 = e^{2x}$, $y_3 = e^{3x}$

$$W = \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix} \Rightarrow e^x(\underbrace{2e^{2x} * 9e^{3x} - 4e^{2x} * 3e^{3x}}_{6e^{6x}}) - e^{2x}(\underbrace{e^x * 9e^{3x} - e^x * 3e^{3x}}_{6e^{6x}})$$

$$+ e^{3x}(\underbrace{e^x * 4e^{2x} - e^x * 2e^{2x}}_{2e^{6x}})$$

$W = 2e^{6x}$

$$W_1 = \begin{vmatrix} 0 & e^{2x} & e^{3x} \\ 0 & 2e^{2x} & 3e^{3x} \\ 1 & 4e^{2x} & 9e^{3x} \end{vmatrix} \Rightarrow -e^{2x}(0 - 3e^{3x}) + e^{3x}(0 - 2e^{2x})$$

$$3e^{5x} - 2e^{5x} = \boxed{e^{5x}}$$

$$W_2 = \begin{vmatrix} e^x & 0 & e^{3x} \\ e^x & 0 & 3e^{3x} \\ e^x & 1 & 9e^{3x} \end{vmatrix} = e^x(0 - 3e^{3x}) + e^{3x}(e^x - 0)$$

$$-3e^{4x} + e^{4x} = \boxed{-2e^{4x}}$$

$$W_3 \begin{vmatrix} e^x & e^{2x} & 0 \\ e^x & 2e^{2x} & 0 \\ e^x & 4e^{2x} & 1 \end{vmatrix}$$

$e^x(2e^{2x} - 0) - e^{2x}(e^x -$

$= e^x(2e^{2x}) - e^{2x}(e^x)$

$2e^{3x} - e^{3x} = \boxed{e^{3x}}$

3.5

Question 5 [6 Marks]:

Find the particular solution for the following differential equation:

$$x^3\frac{d^3y}{dx^3}+x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\frac{1}{x^2}$$

$$y(1)=2.5\quad y'(1)=6.5\quad y''(1)=5$$

① $x^3y'''+x^2y''-2xy'+2y=0$

$x^m=y$ $\quad a=1 \quad b=-2$

~~$a=-2 \quad b=-2$~~

$m^3+(1-a)^2m^2+(a-2+b)m+2=0.$

$m^3-3m+2=0$

$m_1=1 \quad (m-1)$

$$\begin{array}{r|l} & m^2+m-2 \\ \hline m-1 & m^3-3m+2 \\ & m^3-m^2 \\ \hline & m^2-3m+2 \\ & -m^2+m \\ \hline & -2m+2 \\ & -2m+2 \\ \hline & 0 \end{array}$$

$m^2+m-2=0.$

$(m+2)(m-1)$

$m_3=-2 \quad , \quad m_2=1$

$m_1=m_2=1$

$m_3=-2$

$y_1=x^{-2}$

$y_2=x\ln(x)$

$y_3=x$

$$w=\begin{array}{c} + \quad\quad - \quad\quad + \\ \left|\begin{matrix} x^{-2} & x\ln(x) & x \\ -2x^{-3} & 1+\ln(x) & 1 \\ 6x^{-4} & \frac{1}{x} & 0 \end{matrix}\right| \end{array}$$

$$w=x^{-2}\left(0-\frac{1}{x}\right)-x\ln(x)\xcancel{\left[0-6x^{-4}\right]}+x\left(\frac{-2}{x^4}-\frac{6}{x^4}-\frac{\ln}{x^2}\right.$$

$$w=\frac{-1}{x^3}+\frac{6\ln(x)}{x^3}+-\frac{8-\ln(x)}{x^3}=\boxed{\frac{-9+5\ln(x)}{x^3}}$$

Good Luck

1.0

$$w_1 = \begin{array}{|ccc|} & - & + \\ 0 & x\ln(x) & x \\ 0 & 1+\ln(x) & 1 \\ 1 & \frac{1}{x} & 0 \end{array} = +1$$

Question 4 [7 Marks]:

Using the method of variation of parameters to find the particular solution for the following differential equation:

$$x^2y''-3xy'+4y=x^2.\ln x$$
$$y(1)=2 \text{ and } y'(1)=5 .$$

[Hint: $\int\frac{\ln^n x dx}{x}=\frac{\ln^{n+1}x}{n+1}$]

$y_h = x^2y'' - 3xy' + 4y = 0$

Solution:

let $y = x^m$

$m^2 + (-3-1)m + 4 = 0$

$m^2 - 4m + 4 = 0$

$\frac{+4 \mp \sqrt{16-16}}{2} = +2 \pm 0$

$m_1 = m_2 = +2$

$y_h = c_1x^{+2} + c_2 \ln x \, x^{+2} = \boxed{(c_1 + c_2 \ln x)x^{-2}}$

$w = \begin{vmatrix} x^{+2} & x^{+2}.\ln x \\ +2x & x^{+2}.\frac{1}{x} + \ln x(2x) \end{vmatrix}$

$= x^2(x + 2x\ln x) - 2x^3 \ln x$

$x^3 + 2x^3\ln x - 2x^3 \ln x$

$w = x^3$

$y_p = -y_1\int\frac{y_2 . r(x)}{w}dx + y_2\int\frac{y_1 . r(x)}{w}dx$

$-x^2\int\frac{x^2\ln x . x^2\ln x}{x^3}dx + x^2\ln x\int\frac{x^2 . x^2\ln x}{x^3}dx$

$-x^2\int\frac{x^4\ln^2 x}{x^3}dx + x^2\ln x\int\frac{x^4\ln x}{x^3}dx$

Question 2 [5 Marks]:

Newton's law of cooling states that the temperature of an object changes at a rate proportional to its surroundings. If the temperature of a cake is 300°F when it leaves the oven and is 200°F 10 minutes later, when will it be practically equal to the room temperature of 60°F, say, when will it be 61°F?

Newton's law of cooling $\frac{dT}{dt} = k(T - T_{amb})$

[illegible]

Solution:

$$\frac{dT}{dt} = k(T - T_{amb})$$

$$dT = k(T - T_{amb})\,dt$$

$$\int \frac{dT}{T-60} = \int k\,dt \quad ✓$$

$$\ln(T-60) = kt + \boxed{C}$$

2.5

take T = 300

$$\ln 240 = kt \quad --① \rightarrow 240 = e^{kt}$$

Tak. T = 200

$$\ln 140 = kt \quad --② \rightarrow 140 = e^{kt}$$

$$5.4806 = kt \rightarrow \frac{5.4806}{k} = t$$

$$4.9416 = kt$$

$$\rightarrow \frac{4.9416}{t} = k$$

$$\boxed{t = 23.9 \text{ min}}$$