# Numerical Methods

1st
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DR :

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## * Conversion from decimal to binary:

$(56)_{10}$ نقسمة على 2

$56/2 = 28 + 0$

$28/2 = 14 + 0$

$14/2 = 7 + 0$

$7/2 = 3 + 1$

$3/2 = 1 + 1$

$1/2 = 0 + 1$

$(56)_{10} = (111000)_2$

من تحت لفوق

العشري نضرب 2

$(0.75)_{10}$

$0.75 * 2 = 1.50$ (1)

$0.5 * 2 = 1.00$ (1)

$(0.11)_2$ من فوق لتحت

## from binary to decimal:

$(10111)_2$

الخانات الي فيها واحد نجمع

$1 + 2 + 4 + 16 = 23$

العشري

$(0.0111)_2$

الخانات الي فيها واحد نجمع

$2^{-2} + 2^{-3} + 2^{-4} =$

$(0.4375)_{10}$

الانتباه للترتيب

## Types of errors:

- Non numerical:
  1. Modiling
  2. Human
  3. Uncertainty in information data
- Numerical:
  1. Round off errors
  2. Truncation errors
  3. Propagation
  4. Mathematical approximation errors

Relative error

$e = X_1 - X_0$

لو الرقم حقيقي

الي قبله

## Solution of Non-linear Eq:

### 1- Fixed point (iterative) method:

initial guess? كيف نختار

$f(a) < 0$ and $f(b) > 0$

أو العكس

$x_0 = \frac{a+b}{2}$

Ex: $2x^3 - 2x - 5 = 0$

$f(1) = -5$ / $f(2) = 7$

$x_0 = \frac{1+2}{2} = 1.5$

$x = \left(\frac{2x+5}{2}\right)^{\frac{1}{3}}$ نشتق الأكبر وضع الثانية

$x_1 = \left(\frac{2*1.5+5}{2}\right)^{\frac{1}{3}} = 1.5874$

$x_2 = \left(\frac{2*1.5874}{2}\right) = 1.5984$

إلى متى؟

### 2- Newton - Raphson method:

$$X_1 = X_0 - \frac{f(x_0)}{f'(x_0)}$$

Ex: $f(x) = x - 2 + \ln x$

$x_0 = 1.5$ $f'(x) = 1 + \frac{1}{x}$

$f'(x_0) = 1.5 - \frac{1}{1.5} = \frac{5}{3}$

$f(x_0) = -0.0945$

$x_1 = 1.5 - \frac{-0.0945}{\frac{5}{3}} = 1.5567$

$x_2 = 1.5567 - \frac{-0.0007}{1.6423} = 1.5571\ldots$

إذا الإيرور الجديدة أقل من القديمة

إلى متى؟

### 3- Bisection method:

بدي قيمتين حيث

$f(a) * f(b) < 0$

$x_n = \frac{a+b}{2}$

If $f(a) * f(x_n) > 0$

$a = x_n$

If $f(b) * f(x_n) > 0$

$b = x_n$

Ex: $f(x) = x^3 + 4x^2 - 10$

$f(1) = -5$ (a)

$f(2) = 14$ (b) $\rightarrow x_n = \frac{1+2}{2} = 1.5$

$f(1.5) = 2.375$

$f(b) * f(x_n) > 0 \rightarrow b = x_n$

$x_n = \frac{1.5+1}{2} = 1.25$

$f(1.25) = -1.79$

$f(a) * f(x_n) > 0 \rightarrow a = x_n$

وهكذا

السابقة $x_n = \frac{1.5 + 1.25}{2}$

### 4- Newton 2nd order method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0) - f''(x_0)\frac{f(x)}{2f'(x)}}$$

Newton Raphson بنفس طريقة

* هاي بتعطينا الحل أسرع من 1st

### 5- The Secant method:

$$X_{i+1} = X_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

true value لو أعطاني

طبعت الإيرور مقارنة فقط

$$E = \frac{X - X_t}{X_t} * 100\%$$

Ex: $f(x) = e^{-x} - X \rightarrow$ نفرض قيمتين

$x_{i-1} = 0$ $x_i = 1$

* دائماً في البداية بنحط المطلوب في القانون

$f(1) = -0.6212$

$f(0) = 1$ $\rightarrow x_1 = 1 - \frac{-0.63212(0-1)}{1-(-0.63212)} = 0.61270$

Trial 2

$x_i = 0.61270$

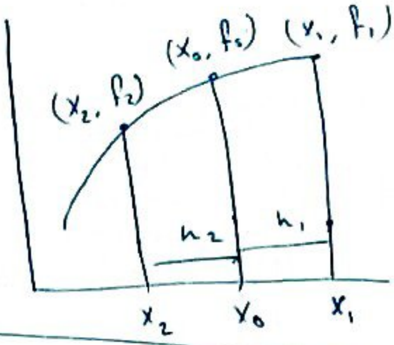
$x_{i-1} = 1$

....

## Solution using Muller method:

$h_2 = x_0 - x_2$

$h_1 = x_1 - x_0$




**Example:** Find the root near the value of $x_0 = 2$

choose $h_1 = h_2 = 0.2$

$$f(x) = \sin(x) - \frac{x}{2}$$

solution

$\boxed{x_1 = x_0 + h_1} = 2 + 0.2 = 2.2$

$\boxed{x_2 = x_0 - h_1} = 2 - 0.2 = 1.8$

$f_0 = f(x_0) = f(2) = -0.0907$

$f_1 = f(x_1) = f(2.2) = -0.291$

$f_2 = f(x_2) = f(1.8) = 0.0738$

$\boxed{\gamma = \frac{h_2}{h_1}} = 1$

$$\boxed{a = \frac{\gamma f_1 - (1+\gamma) f_0 + f_2}{\gamma h_1^2 (1+\gamma)}} = -0.453$$

$$\boxed{b = \frac{f_1 - f_0 - a h_1^2}{h_1}} = -0.913$$

$\boxed{c = f_0} = f(2) = -0.0907$

$$\boxed{x = x_0 - \frac{2 f_0}{b \pm \sqrt{b^2 - 4ac}}}$$

نفس إشارة $b$

$x = 1.8952$ Trial 1

Now New trial with $x_0 = 1.8952$

بس ما معي قيم h !

* كان عندي قبل $x_0, x_1, x_2$

وكانت $x_0$ بينهم

فمن الـ 3 قيم اللي قبل يختار اللي قبل واللي بعد $x_0$ الجديدة (1.8952) ويستبعد الثالثة.

Trial 2:

$x_0 = 1.8952$

$x_1 = 2$

$x_2 = 1.8$

ومنهم

$h_1 = x_1 - x_0 = 0.0104$

$h_2 = x_0 - x_2 = 0.0952$

$\gamma = \frac{h_2}{h_1}$

....

ewton - Raphson method for Complex root :

Ex: $f(x) = x^2 + 1$

→ intial guess

$z_0 = 1 + i$ where $x_0 = 1$, $y_0 = 1$

$$\boxed{z_{k+1} = z_k - \frac{F(z_k)}{F'(z_k)}}$$

↓

$$z_{k+1} = z_k - \frac{z_k^2 + 1}{2 z_k}$$

$$z_{k+1} = z_k - \frac{1}{2}\left(\frac{z_k^2 + 1}{z_k}\right)$$

$$= z_k - \frac{1}{2}\left(z_k + \frac{1}{z_k}\right) = \frac{1}{2}\left(z_k - \frac{1}{z_k}\right)$$

$$\boxed{z_1 = z_0 - \frac{z_0^2 + 1}{2 z_0}} = \frac{1}{2}\left(z_0 - \frac{1}{z_0}\right)$$

$z_0 = 1 + i$

نعوض

$$z_1 = \frac{1}{2}\left((1+i) - \frac{1}{(1+i)}\right)$$

$$z_1 = \frac{1}{2}\left((1+i) - \frac{1}{(1+i)} * \frac{(1-i)}{(1-i)}\right)$$

where $(1+i) \times (1-i) = 2$ → $1 - \cancel{i} + \cancel{i} - i^2$

$i^2 = \sqrt{-1} * \sqrt{-1} = -1$

$1 - -1 = 2$

$$z_1 = \frac{1}{2}\left(1 + i - \frac{(1-i)}{2}\right)$$

توحيد مقامات ←

$$z_1 = \frac{2 * (1+i)}{2 * 2} - \frac{(1-i)}{4}$$

$$z_1 = \frac{2 + 2i}{4} \quad \frac{-1 + i}{4}$$

$$z_1 = \frac{2}{4} + \frac{2i}{4} - \frac{1}{4} + \frac{i}{4}$$

→ real number together and the imaginary ones in the same manner:

$$z_1 = \frac{1}{4} + \frac{3}{4} i = \boxed{0.25 + 0.75i}$$

Trial 2

$$z_2 = \frac{1}{2}\left(0.25 + 0.75i - \frac{1}{0.25 + 0.75i}\right)$$

بذات الطريقة

$$z_n = -0.075 + 0.975i$$

Notes:

→ $z^2 + 1 = 0$ → $z = \pm\sqrt{-1}$ → $0 + i$ (real)

→ $x_{i+1} + y_{i+1} = \left[x_i - \frac{f(x_i)}{f'(x_i)}\right] + \left[y_i - \frac{f(y_i)}{f'(y_i)}\right] i$

real ↵ complex ↵

## Gaussion Elimination method:

$$x_1 + x_2 - x_3 = -2 \rightarrow E_1$$
$$2x_1 - x_2 + x_3 = 5 \rightarrow E_2$$
$$-x + 2x_2 + 2x_3 = 1 \rightarrow E_3$$

$$\overset{a}{\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ -1 & 2 & 2 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \overset{b}{\begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}}$$

بدنا نصفر

$E_2 - 2E_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 3 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ 1 \end{bmatrix}$$

$a_{21} = 2 - 2(1) = 0$

$a_{22} = -1 - 2(1) = -3$

$a_{23} = 1 - 2(-1) = 3$

$b_{22} = 5 - 2(-2) = 9$

$E_1 + E_3$

$a_{31} = 1 - 1 = 0$

$a_{32} = 1 + 2 = 3$

$a_{33} = -1 + 2 = 1$

$b_{31} = -2 + 1 = -1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ -1 \end{bmatrix}$$

* لازم نصفر باستخدام ثاني معادلة عشان نصفر $a_{31} = 0$

$E_2 + E_3$

ثابت الثانية مع الثالثة

$a_{31} = 0$

$a_{32} = 0$

$a_{33} = 1 + 3 = 4$

$b_{31} = 9 - 1 = 8$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ 8 \end{bmatrix} \longrightarrow$$

$4x_3 = 8 \rightarrow \boxed{x_3 = 2}$

$-3x_2 + 3x_3 = 9$

$-3x_2 + 6 = 9 \rightarrow \boxed{x_2 = -1}$

$x_1 + x_2 - x_3 = 2$

$x_1 - 1 - 2 = -2 \rightarrow \boxed{x_1 = 1}$

## Gauss-Jordan method:

بنعمل نفس الخطوات الي عملناها ب Gaussion

بالعودة للمثال السابق

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

بدنا نصفر

Ex:
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

وصلناها حل بنفس الطريقة السابقة

$E_3 - 3E_2$ (المعادلة الثانية والثالثة ثابتة)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -6 & 0 & -18 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$E_3 + 3E_1$ (المعادلة الأولى مع الثالثة ثابتة)

$$\left[\begin{array}{ccc|c} 3 & 3 & 0 & 15 \\ 0 & -6 & 0 & -18 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$E_2 + 2E_1$ (المعادلة الأولى مع الثانية ثابتة)

$$\left[\begin{array}{ccc|c} 6 & 0 & 0 & 12 \\ 0 & -6 & 0 & -18 \\ 0 & 0 & -3 & -6 \end{array}\right] \begin{array}{l} \rightarrow \div 6 \\ \rightarrow \div -6 \\ \rightarrow \div -3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x_1 = 2 \quad x_3 = 2$

$x_2 = 3$

# Jacobi iterative method:

$|a_{11} * a_{22} * a_{33} * \dots| = max$

Ex:

$$2x_1 + 7x_2 - 8x_3 = 10$$
$$5x_1 - 2x_2 + 6x_3 = 6 \qquad 2 * 2 * 7 = 28$$
$$3x_1 + 5x_2 - 7x_3 = 3$$

بدها اعادة ترتيب

$$5x_1 - 2x_2 + 6x_3 = 6$$
$$2x_1 + 7x_2 - 8x_3 = 10 \qquad 5 * 7 * 7 = 245 \text{ max}$$
$$3x_1 + 5x_2 - 7x_3 = 3$$

$$x_1 = \frac{6 + 2x_2 - 6x_3}{5}$$

$$x_2 = \frac{10 - 2x_1 + 8x_3}{7}$$

$$x_3 = \frac{3 - 3x_1 - 5x_2}{-7}$$

* بفرض من عندي $x_1$ و $x_2$ و $x_3$ لأول تريل

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

Trial (1)

$$x_1 = \frac{6 + 2(0) - 6(0)}{5} = \frac{6}{5}$$

$$x_2 = \frac{10 - 2(0) + 8(0)}{7} = \frac{10}{7}$$

$$x_3 = \frac{3 - 3(0) - 5(0)}{-7} = \frac{-3}{7}$$

Trial (2) $x_1 = \frac{6}{5}$, $x_2 = \frac{10}{7}$, $x_3 = \frac{-3}{7}$

$$x_1 = \frac{6 + 2(\frac{10}{7}) - 6(\frac{-3}{7})}{5} = 2.28$$

$$x_2 = \frac{10 - 2(\frac{6}{5}) + 8(\frac{-3}{7})}{7} = 0.59$$

$$x_3 = \frac{3 - 3(\frac{6}{5}) - 5(\frac{10}{7})}{-7} = 1.106$$

Trial (3) $x_1 = 2.28$, $x_2 = 0.59$
$x_3 = 1.106$

وبنرجع الطريقة

## Gauss - Siedle method:

نصل فيها إلى الحل أسرع من جاكوبي

* نفس جاكوبي لكن الفرق بطريقة التعويض.

على نفس المثال السابق

$$x_1 = \frac{6 + 2x_2 - 6x_3}{5}$$

$$x_2 = \frac{10 - 2x_1 + 8x_3}{7}$$

$$x_3 = \frac{3 - 3x_1 - 5x_2}{-7}$$

Trial (1) $x_1 = 0$, $x_2 = 0$, $x_3 = 0$

$$x_1 = \frac{6 + 2(0) - 6(0)}{5} = 6/5$$

بستخدمها بـ $x_2$ و $x_3$

$$x_2 = \frac{10 - 2\left(\frac{6}{5}\right) - 8(0)}{7} = 1.08$$

بستخدمها بـ $x_3$

$$x_3 = \frac{3 - 3\left(\frac{6}{5}\right) - 5(1.08)}{-7} = 0.85$$

Trial (2) $x_1 = \frac{6}{5}$, $x_2 = 1.08$, $x_3 = 0.85$

$$x_1 = \frac{6 + 2(1.08) - 6(0.85)}{5} = 0.60$$

$$x_2 = \frac{10 - 2(0.60) + 8(0.85)}{7} = 2.22$$

$$x_3 = \frac{3 - 3(0.60) - 5(2.22)}{-7} = 1.41$$

Trial (3) $x_1 = 0.60$, $x_2 = 2.22$, $x_3 = 1.41$

وبنكمل الطريقة

# Eigen values and Eigen vectors :

$A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} \longrightarrow \det \begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix}$

3×3

$(7-\lambda)(-1-\lambda) - 9 = 0$

$-7 - 7\lambda + \lambda + \lambda^2 - 9 = 0$

$\lambda^2 - 6\lambda - 16 = 0$

$(\lambda - 8)(\lambda + 2) = 0 \longrightarrow \left.\begin{matrix} \lambda_1 = 8 \\ \lambda_2 = -2 \end{matrix}\right]$ Eigen Values

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when $\lambda = 8$ → عوضنا $\lambda$

$\begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

نعمل معادلتين

$-x_1 + 3x_2 = 0 \longrightarrow -x = -3x_2$

$3x_1 - 9x_2 = 0 \longrightarrow 3x_1 = -9x_2$

$x_1 = +3x_2$

نفس بعض

let $x_2 = 1 \longrightarrow x_1 = 3$

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when $\lambda = -2$

$\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$9x_1 + 3x_2 = 0 \longrightarrow \frac{9x_1}{-3} = \frac{-3x_2}{-3}$

$3x_1 + x_2 = 0 \qquad x_2 = -3x_1$

let $x_1 = 1 \longrightarrow x_2 = -3$

---

$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \gamma \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Egin vectors

check:

$\lambda = 8$ when $x_1 = 3$, $x_2 = 1$

$8 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$

[illegible] $x_1$

$\lambda = -2$ when $x_1 = 1$, $x_2 = -3$

$-2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

Inverse based on adj factor:

$$A=\begin{array}{c}\begin{array}{ccc}+ & - & +\end{array}\\ \begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}\end{array} \qquad A^{-1}=\frac{1}{Det}[adj]$$

$$Det\ A = 2\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} - 1\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} + 1\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}$$

$$= 2(1*2-1(-1)) - 1(1*2-1*1) + 1(1*-1-1*1)$$

$$= 2(3) - 1 + -2$$

$$= 6 - 1 - 2 = 3$$

$$\begin{bmatrix} +\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & -\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & +\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}\\ -\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & +\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & -\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}\\ +\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & -\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix} & +\begin{bmatrix}2 & 1 & 1\\ 1 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}\end{bmatrix}$$

$$[adj]=\begin{bmatrix} +\begin{bmatrix}1 & 1\\ -1 & 2\end{bmatrix} & -\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix} & +\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\\ -\begin{bmatrix}1 & 1\\ -1 & 2\end{bmatrix} & +\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} & -\begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix}\\ +\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} & -\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix} & +\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\end{bmatrix}$$

$$a_{11} = 1*2-(1*-1) = 3$$

وهيك للكل

* الانتباه للإشارة اللي برا

$$[adj]=\begin{bmatrix}3 & -1 & -2\\ -3 & 3 & 3\\ 0 & -1 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{Det}[adj]=\frac{1}{3}\begin{bmatrix}3 & -1 & -2\\ -3 & 3 & 3\\ 0 & -1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1/3 & -2/3\\ -1 & 1 & 1\\ 0 & -1/3 & 1/3\end{bmatrix}$$

2*2

$$Ex: \begin{array}{c}\begin{array}{cc}+ & -\end{array}\\ \begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\end{array}$$

$$Det\ A = 2*5 - 3*4 = -2$$

$$\begin{bmatrix} +\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix} & -\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\\ -\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix} & +\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\end{bmatrix}$$

$$[adj]=\begin{bmatrix}5 & -4\\ -3 & 2\end{bmatrix}$$ → بتقدر الحكي قلبنا ال diagonal مع الانتباه للإشارة

$$A^{-1}=\frac{1}{Det}[adj]=\begin{bmatrix}-5/2 & 2\\ 3/2 & -1\end{bmatrix}$$

## Cramer's rule (3*3)

$x + 2y - z = 2$
$2x - 3z = 1$
$y - 4z = -3$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

نقل أول عمودين

$$|A| = \left[\begin{array}{ccc} 1 & 2 & -1 \\ 2 & 0 & -3 \\ 0 & 1 & -4 \end{array}\right] \begin{array}{cc} 1 & 2 \\ 2 & 0 \\ 0 & 1 \end{array}$$

ممكن نسميها D أيضا

$\text{Det } A = (1*0*-4) + (2*-3*0) + (-1*2*1)$
$\quad - ((2*2*-4) + (1*-3*1) + (-1*0*0))$
$\quad = 17$

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نقل أول عمودين

$$[A_x] = \left[\begin{array}{ccc} 2 & 2 & -1 \\ 1 & 0 & -3 \\ 3 & 1 & -4 \end{array}\right] \begin{array}{cc} 2 & 2 \\ 1 & 0 \\ 3 & 1 \end{array}$$

شلنا عمود الـ X وحطينا بداله b

$\text{Det } A_x = (2*0*-4) + (\overset{-18}{2*-3*3}) + (-1*1*1)$
$\quad - ((\overset{-8}{2*1*-4}) + (\overset{-6}{2*-3*1}) + (-1*0*3))$
$\quad = -19 + 14 = -5$

$$x = \frac{\text{Det}[A_x]}{\text{Det}[A]} = \boxed{\frac{-5}{17}}$$

---

نقل أول عمودين

$$[A_y] = \left[\begin{array}{ccc} 1 & 2 & -1 \\ 2 & 1 & -3 \\ 0 & 3 & -4 \end{array}\right] \begin{array}{cc} 1 & 2 \\ 2 & 1 \\ 0 & 3 \end{array}$$

شلنا عمود الـ y وحطينا b

$\text{Det } A_y = (\overset{-4}{1*1*-4}) + (2*-3*0) + (\overset{-6}{-1*2*3})$
$\quad - ((\overset{-16}{2*2*-4}) + (\overset{-9}{1*-3*3}) + (-1*1*0))$
$\quad = -10 + 25 = 15$

$$y = \frac{\text{Det}[A_y]}{\text{Det}[A]} = \boxed{\frac{15}{17}}$$

---

نقل أول عمودين

$$[A_z] = \left[\begin{array}{ccc} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{array}\right] \begin{array}{cc} 1 & 2 \\ 2 & 0 \\ 0 & 1 \end{array}$$

شلنا عمود Z وحطينا b

$\text{Det } A_z = (\overset{0}{1*0*3}) + (\overset{0}{2*1*0}) + (\overset{4}{2*2*1})$
$\quad - ((\overset{12}{2*2*3}) + (1*1*1) + (2*0*0))$
$\quad = 4 - 13 = -9$

$$z = \frac{\text{Det}[A_z]}{\text{Det}[A]} = \boxed{\frac{-9}{17}}$$

check :

$x + 2y - z = 2$

$\frac{-5}{17} + 2\left(\frac{15}{17}\right) + \frac{9}{17} \overset{?}{=} 2$

$= \frac{34}{17} = 2$ ✓

---

## Cramer's rule (2*2)

$3x - 2y = 4$
$2x + y = -3$

$$[D] = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$\text{Det}[D] = 3*1 - (-2*2) = 7$

$$[D_x] = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \rightarrow \text{Det}[D_x] = 4*1 - (-2*-3) = -2$$

شلنا عمود x وحطينا b

$$x = \frac{\text{Det}[D_x]}{\text{Det}[D]} = \boxed{\frac{-2}{7}}$$

$$[D_y] = \begin{bmatrix} 3 & 4 \\ 2 & -3 \end{bmatrix}$$

شلنا عمود y وحطينا b

$\text{Det}[D_y] = 3*-3 - 4*2 = -17$

$$y = \frac{\text{Det}[D_y]}{\text{Det}[D]} = \boxed{\frac{-17}{7}}$$

* ممكن نعمل check سواء لـ 2*2 أو 3*3 مأسا نرجع نعوض القيم اللي طلعت معنا في المعادلات ونشوف لو صح

# LU Decomposition

Ex: $2x_1 + x_2 + 3x_3 = 1$

$6x_1 + 2x_2 + 7x_3 = 0$

$4x_1 + 8x_2 + 2x_3 = 2$

$$\overset{A}{\begin{bmatrix} 2 & 1 & 3 \\ 6 & 2 & 7 \\ 4 & 8 & 2 \end{bmatrix}} \overset{x}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{b}{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}}$$

$$\overset{A}{\begin{bmatrix} 2 & 1 & 3 \\ 6 & 2 & 7 \\ 4 & 8 & 2 \end{bmatrix}} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 6 & 2 & 7 \\ 4 & 8 & 2 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{23} \end{bmatrix}$$

$\boxed{U_{11} = 2}$, $\boxed{U_{12} = 1}$, $\boxed{U_{13} = 3}$

$\rightarrow L_{21} U_{11} = 6 \rightarrow \boxed{L_{21} = 3}$

$\rightarrow L_{21} U_{12} + U_{22} = 2 \rightarrow \boxed{U_{22} = -1}$

$\rightarrow L_{21} U_{13} + U_{23} = 7 \rightarrow \boxed{U_{23} = -2}$

$\rightarrow L_{31} U_{11} = 4 \rightarrow \boxed{L_{31} = 2}$

$\rightarrow L_{31} U_{12} + L_{32} U_{22} = 8 \rightarrow \boxed{L_{32} = -6}$

$2 - L_{32} = 8$

$\rightarrow L_{31} U_{13} + L_{32} U_{23} + U_{33} = 2$

$2 * 3 + -6 * -2 + U_{33} = 2$

$6 + 12 + U_{33} = 2 \rightarrow \boxed{U_{33} = -16}$

$$\underset{A}{\begin{bmatrix} 2 & 1 & 3 \\ 6 & 2 & 7 \\ 4 & 8 & 2 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -6 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -16 \end{bmatrix}}$$

$Ax = b \rightarrow A = LU \rightarrow LUx = b$

$Ux = y \rightarrow Ly = b$

$Ly = b$

$$\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -6 & 1 \end{bmatrix}} \underset{y}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}}$$

$\hookrightarrow y_1(1) + y_2(0) + y_3(0) = 1$

$y_1(3) + y_2(1) + y_3(0) = 0$

$y_1(2) + y_2(-6) + y_3(1) = 2$

$\hookrightarrow y_1 = 1, \; y_2 = -3, \; y_3 = -18$

$UX = y$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -18 \end{bmatrix}$$

$2x_1 + x_2 + 3x_3 = 1$

$-x_2 - 2x_3 = -3$

$-16x_3 = -18$

$x_3 = 1.125$

$x_2 = 0.75$

$x_1 = 1.5625$

# Soln of System of non linear Eqns :

## (11) Alternative method :

Ex :

$$3x_1^5 + 2x_2^2 = 5$$

$$x_1 + 2x_2^2 = 3$$

$$\xrightarrow{(1)} x_1 = \left[\frac{5 - 2x_2^2}{3}\right]^{\frac{1}{5}}$$

$$\xrightarrow{(2)} x_2 = \left[\frac{3 - x_1}{2}\right]^{\frac{1}{2}}$$

| * | #1 | #2 | #3 |
|---|---|---|---|
| $x_1$ | 0 (initial) | 0.922 | 0.995 |
| $x_2$ | 1.224 | 1.02 | 1.001 |

عوض بمعادلة 2

* بنختار تكون $x_2$ أكبر قوة بمعادلة و $x_1$ أكبر قوة بالمعادلة الثانية ، لو مثلاً بالمعادلتين $x_1$ الأكبر ، بنختار الأكبر لـ $x_1$ بمعادلة والأخرى لـ $x_2$ حتى لو كانت الأصغر.

* لو $x_1$ الأكبر بالمعادلتين والقوتين متساويتين؟ بنحتكم للمعاملات الأكبر لـ $x_1$ .

* لازم ننتبه باختيار القيمة الأولية حتى ما يطلع complex root

## ② Newton Raphson's method:

Ex: $3x_1^5 + 2x_2^2 = 5$

$x_1 + 2x_2^2 = 3$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{New} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{old} - \underbrace{\begin{bmatrix} f'_{11} & f'_{12} \\ f'_{21} & f'_{22} \end{bmatrix}^{-1}_{old}}_{\text{Jacobian matrix}} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_{old}$$

①
$$\rightarrow \begin{aligned} f_1 &= 3x_1^5 + 2x_2^2 - 5 = 0 \\ f_2 &= x_1 + 2x_2^2 - 3 = 0 \end{aligned}$$

بنساوي بالصفر وبنقسم المعادلتين

②
$$\rightarrow \begin{aligned} f'_{11} &= \frac{df_1}{dx_1} = 15x_1^4 \\ f'_{12} &= \frac{df_1}{dx_2} = 4x_2 \\ f'_{21} &= \frac{df_2}{dx_1} = 1 \\ f'_{22} &= \frac{df_2}{dx_2} = 4x_2 \end{aligned}$$

نشتق المعادلتين مرة بالنسبة لـ $x_1$ ومرة لـ $x_2$

* Assume $x_1 = 0.1$ / $x_2 = 0.5$ as intial guesse

③
$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{New} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} - \begin{bmatrix} 1.5\times10^{-3} & 2 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -4.49 \\ -2.4 \end{bmatrix}$$

نعوض في المعادلة قيم $x_1$ و $x_2$

④
$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{New} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} - \begin{bmatrix} -1 & 1.0 \\ 0.5 & -0.00074 \end{bmatrix} \begin{bmatrix} -4.49 \\ -2.4 \end{bmatrix}$$

نجد معكوس المصفوفة عن طريق الآلة الحاسبة

⑤
$$-1 * -4.49 + 1 * -2.4 = 2.09$$
$$(0.5 * -4.49) + (-0.00074 * -2.4) = -2.243$$

نضرب المصفوفتين

$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{New} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} - \begin{bmatrix} 2.09 \\ -2.243 \end{bmatrix}$$

⑥
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{New} = \begin{bmatrix} -2 \\ 2.75 \end{bmatrix}$$

طرح المصفوفتين

$$\rightarrow \begin{aligned} x_1 &= -2 \\ x_2 &= 2.75 \end{aligned}$$

* لو عملنا أكثر من مرة

$$\text{error} = \sum_{i=1}^{n} (x_{iN} - x_{io})^2$$

* لو عنا مصفوفة 3*3 كله بنفس المبدأ،

Jacopian matrix

$$\begin{bmatrix} f'_1x_1 & f'_1x_2 & f'_1x_3 \\ f'_2x_1 & f'_2x_2 & f'_2x_3 \\ f'_3x_1 & f'_3x_2 & f'_3x_3 \end{bmatrix}^{-1}$$

مشتقة المعادلات بالنسبة لـ $x_1$ — بالنسبة لـ $x_2$ — بالنسبة لـ $x_3$

## Curve fitting:

### 1) Linear regression:

$$y = a + bx$$

Ex:

①

①, ② ... الخطوة.

| $x$ | $y$ | $xy$ | $x^2$ |
|---|---|---|---|
| 0 | 0.9 | 0 | 0 |
| 1 | 3.1 | 3.1 | 1 |
| 2 | 5 | 10 | 4 |
| 3 | 6.5 | 19.5 | 9 |
| 4 | 9.5 | 38 | 16 |
| $\Sigma = 10$ | 25 | 70.6 | 30 |

③ $$a = \frac{\Sigma y - b\,\Sigma x}{n} = \frac{(25) - (2.06)(10)}{5} = 0.88$$

$$\boxed{y = 0.88 + 2.06x}$$

② $$b = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{n\,\Sigma x^2 - (\Sigma x)^2} = \frac{(5)(70.6) - (10)(25)}{(5)(30) - (10)^2} = 2.06$$

(أولاً)

### Coefficient of determination:

$$R^2 = \frac{\Sigma (y_i - y_{avg})^2 - \Sigma (y_i - y_{model})^2}{\Sigma (y_i - y_{avg})^2}$$

④ $$y_{avg} = \frac{\Sigma y_i}{n} = \frac{25}{5} = 5$$

⑤ $y_{model}$

$0.88 + 2.06(0) = 0.88$

$0.88 + 2.06(1) = 2.94$

$0.88 + 2.06(2) = 5$

$0.88 + 2.06(3) = 7.06$

$0.88 + 2.06(4) = 9.12$

⑥ →

| $x_i$ | $y_i$ | $(y_i - y_{avg})^2$ | $y_{model}$ | $(y_i - y_{model})^2$ |
|---|---|---|---|---|
| 0 | 0.9 | 16.81 | 0.88 | $4 \times 10^{-4}$ |
| 1 | 3.1 | 3.61 | 2.94 | 0.0256 |
| 2 | 5 | 0 | 5 | 0 |
| 3 | 6.5 | 2.25 | 7.06 | 0.3136 |
| 4 | 9.5 | 20.25 | 9.12 | 0.1444 |
| $\Sigma$ 10 | 25 | 42.92 | 25 | 0.484 |

⑦ $$R^2 = \frac{42.92 - 0.484}{\cancel{0.484}\ 42.92} = 0.9887$$

$$\boxed{R = 0.9943}$$

← مقياس للفعالية كلّما كانت أقرب لـ 1 يعني إعتباره linear مناسب
(ويفضلها فيه [illegible])

## ② curve fitting for non linear:

Ex:

$$\ln y = \ln a + b \ln x$$

$$Y = A + b X$$

القيم اللي معنا
زابط لساي
ارجعة بكون
الخطأ كبير لو
افترضناها linear
منفرضلها معادلة
ونعملها linearization

| $x$ | $y$ | $Y = \ln y$ | $X = \ln x$ | $YX$ | $X^2$ |
|---|---|---|---|---|---|
| 1 | 3 | 1.1 | 0 | 0 | 0 |
| 2 | 12 | 2.48 | 0.69 | 1.71 | 0.48 |
| 3 | 27 | 3.3 | 1.1 | 3.63 | 1.21 |
| 4 | 48 | 3.87 | 1.39 | 5.3793 | 1.9321 |
| 4 | | 10.75 | 3.18 | 10.72 | 3.62 |

$\Sigma$

$$b = \frac{n \Sigma XY - \Sigma X \Sigma Y}{n \Sigma X^2 - (\Sigma x_i)^2} = \frac{4(10.72) - (3.18)(10.75)}{4(3.62) - (3.18)^2} = 1.99$$

$$a = \frac{\Sigma Y - b \Sigma X}{n} = \frac{10.75 - 1.99(3.18)}{4} = 1.11$$

$$\ln y = \ln 1.11 + 1.99 \ln x$$

$$\boxed{\ln y = 0.104 + 1.99 \ln x}$$

## ③ Fitting using polynomial :

$$y = a + bx + cx^2$$

$$\begin{bmatrix} n & \Sigma x & \Sigma x^2 \\ \Sigma x & \Sigma x^2 & \Sigma x^3 \\ \Sigma x^2 & \Sigma x^3 & \Sigma x^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma xy \\ \Sigma x^2 y \end{bmatrix}$$

Ex :

| | $x$ | $y$ | $x^2$ | $x^3$ | $x^4$ | $xy$ | $x^2y$ |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 3 | 1 | 1 | 1 | 3 | 3 |
| | 2 | 7 | 4 | 8 | 16 | 14 | 28 |
| | 3 | 13 | 9 | 27 | 81 | 39 | 117 |
| $\Sigma$ | 6 | 24 | 14 | 36 | 98 | 56 | 148 |

$$\begin{bmatrix} 4 & 6 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 24 \\ 56 \\ 148 \end{bmatrix}$$

من آلة الحاسبة

$a = 1$
$b = 1$
$c = 1$

$$\boxed{y = 1 + x + x^2}$$

## (4) Multiple linear Regression:

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 \cdots$$

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 & \Sigma x_3 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 & \Sigma x_1 x_3 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 & \Sigma x_2 x_3 \\ \Sigma x_3 & \Sigma x_1 x_3 & \Sigma x_2 x_3 & \Sigma x_3^2 \\ \vdots & \vdots & \vdots & \vdots \\ \Sigma x_n & \Sigma x_1 x_n & \Sigma x_2 x_n & \Sigma x_3 x_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \Sigma y_1 \\ \Sigma x_1 y \\ \Sigma x_2 y \\ \Sigma x_3 y \\ \vdots \\ \Sigma x_n y \end{bmatrix}$$

Ex:

| # | $x_1$ | $x_2$ | $y$ | $x_1^2$ | $x_1 x_2$ | $x_2^2$ | $x_1 y$ | $x_2 y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 3 | 1 | 1 | 3 | 9 | 1 | 3 |
| 3 | 2 | 2 | 7 | 4 | 4 | 4 | 14 | 14 |
| 4 | 3 | 5 | 5 | 9 | 15 | 25 | 15 | 25 |
| 5 | 4 | 7 | 5 | 16 | 28 | 49 | 20 | 35 |
| 6 | 5 | 1 | 21 | 25 | 5 | 1 | 105 | 21 |
| | 15 | 18 | 42 | 65 | 55 | 88 | 155 | 98 |

$$\begin{bmatrix} 6 & 15 & 18 \\ 15 & 65 & 55 \\ 18 & 55 & 88 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 155 \\ 98 \end{bmatrix}$$

$a_0 = 5.63$

$a_1 = 2.37$

$a_2 = -1.52$

$$\boxed{y = 5.63 + 2.37\, x_1 - 1.52\, x_2}$$

## 5 Multi variable polynomial fitting :

$$y = a_0 + a_1 x_1 + a_2 x_1^2 + a_3 x_2 + a_4 x_2^2$$

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_1^2 & \Sigma x_2 & \Sigma x_2^2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1^3 & \Sigma x_1 x_2 & \Sigma x_1 x_2^2 \\ \Sigma x_1^2 & \Sigma x_1^3 & \Sigma x_1^4 & \Sigma x_1^2 x_2 & \Sigma x_1^2 x_2^2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_1^2 x_2 & \Sigma x_1^2 & \Sigma x_2^3 \\ \Sigma x_2^2 & \Sigma x_1 x_2^2 & \Sigma x_1^2 x_2^2 & \Sigma x_2^3 & \Sigma x_2^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_1^2 y \\ \Sigma x_2 y \\ \Sigma x_2^2 y \end{bmatrix}$$

### Ex :

| | $x_1$ | $x_2$ | $y$ | $x_1^2$ | $x_2^2$ | $x_1^3$ | $x_1^4$ | $x_2^3$ | $x_2^4$ |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 5 | 41 | 0 | 25 | 0 | 0 | 125 | 625 |
| | 1 | 4 | 32 | 1 | 16 | 1 | 1 | 64 | 256 |
| | 2 | 3 | 27 | 4 | 9 | 8 | 16 | 27 | 81 |
| | 3 | 2 | 26 | 9 | 4 | 27 | 81 | 8 | 16 |
| | 4 | 1 | 29 | 16 | 1 | 64 | 256 | 1 | 1 |
| | 5 | 0 | 36 | 25 | 0 | 125 | 625 | 0 | 0 |
| $\Sigma$ | 15 | 15 | 191 | 55 | 55 | 225 | 979 | 225 | 979 |

| $x_1 x_2$ | $x_1 x_2^2$ | $x_1^2 x_2$ | $x_1^2 x_2^2$ | $x_2 y$ | $x_1 y$ | $x_1^2 y$ | $x_2^2 y$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 205 | 0 | 0 | 1025 |
| 4 | 16 | 4 | 16 | 128 | 32 | 32 | 512 |
| 6 | 18 | 12 | 36 | 81 | 54 | 108 | 243 |
| 6 | 12 | 18 | 36 | 52 | 78 | 234 | 104 |
| 4 | 4 | 16 | 16 | 29 | 116 | 416 | 29 |
| 0 | 0 | 0 | 0 | 0 | 180 | 900 | 39 |
| 20 | 50 | 50 | 104 | 445 | 460 | 1690 | 1949 |

$$\begin{bmatrix} 6 & 15 & 55 & 15 & 55 \\ 15 & 55 & 225 & 20 & 50 \\ 55 & 225 & 979 & 50 & 104 \\ 15 & 20 & 50 & 55 & 225 \\ 55 & 50 & 104 & 225 & 979 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 191 \\ 460 \\ 1690 \\ 495 \\ 1949 \end{bmatrix}$$

- ~~linear interpolation~~

Langrage

$$\boxed{f(x)} = L_0(x)\, f(x_0) + L_1(x)\, f(x_1) \qquad \text{interpolation(1)}$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \quad \Bigg| \quad L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

Ex:

| | $x$ | $f(x)$ |
|---|---|---|
| $x_0 \to$ | 2 | 1.5 |
| $x \to$ | | |
| $x_1 \to$ | 5 | 4 |
| | 2 | 6.5 |
| | 11 | 9 |

find $f(3)$ $\rightarrow$ $f(3) = \left(\frac{3-5}{2-5}\right)(1.5) + \left(\frac{3-2}{5-2}\right)(4) = \underline{\underline{2.33}}$

---

Quadratic interpolation

Langrage case 2

$$\boxed{f(x)} = L_0(x) * f(x_0) + L_1(x)\, f(x_1) + L_2(x)\, f(x_2)$$

$$L_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$L_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$L_2(x) = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

Ex

| | $x$ | $f(x)$ |
|---|---|---|
| | 1 | 0 |
| $x_0 \to$ | 2 | 1 |
| $x \to$ | | |
| $x_1 \to$ | 3 | 2 |
| $x_2 \to$ | 4 | 3 |
| | 5 | 5 |
| | 6 | 7 |

لازم أختار ٣ نقاط القيمة بينهم ٣ منهم ولازم بالترتيب بس مش شرط ورا بعض

find $f(2.5)$: $\left(\frac{2.5-3}{2-3}\right)\left(\frac{2.5-4}{2-4}\right)(1) + \left(\frac{2.5-2}{3-2}\right)\left(\frac{2.5-4}{3-4}\right)(2) + \left(\frac{2.5-2}{4-2}\right)\left(\frac{2.5-3}{4-3}\right)(3)$

$$\boxed{= 1.5}$$

---

special case: $n = 3$

$$L_3(x) = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

$$\boxed{f(x)} = L_0(x)\, f(x_0) + L_1(x)\, f(x_1) + L_2(x)\, f(x_2) + L_3(x)\, f(x_3)$$

$$L_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

Ex:

| | $x$ | $f(x)$ |
|---|---|---|
| | 1 | 0 |
| $x_0 \to$ | 2 | 1 |
| $x \to$, $x_1 \to$ | 3 | 2 |
| $x_2 \to$ | 4 | 3 |
| $x_3 \to$ | 5 | 5 |
| | 6 | 7 |

find $f(2.5)$

# Newton interpolation



a) Linear interpolation $n = 1$

$$f(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0)$$

b) 2nd order $n = 2$

$$f(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0) + \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}(x - x_0)(x - x_1)$$

Ex

| | x | f(x) |
|---|---|---|
| $x_0$ | 2 | 1.5 |
| $x_1$ | 5 | 4 |
| | 8 | 6.5 |
| $x_2$ | 11 | 9 |

find $f(3)$ using Newton 2nd order method.

$$f(3) = 1.5 + \left(\frac{4 - 1.5}{5 - 2}\right)(3 - 2) + \frac{\dfrac{9 - 4}{11 - 5} - \dfrac{4 - 1.5}{5 - 2}}{11 - 2}(3 - 2)(3 - 5)$$

$$= \boxed{2.33}$$

عادي 8 ول و

# Newton's divided table

متتالي

| $x$ | $f(x)$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ |
|---|---|---|---|---|---|---|
| $x_1$ | $f_1$ | | | | | |
| | | $d_{11} = \frac{f_2 - f_1}{x_2 - x_1}$ | | | | |
| $x_2$ | $f_2$ | | $d_{21} = \frac{d_{12} - d_{11}}{x_3 - x_1}$ | | | |
| | | $d_{12} = \frac{f_3 - f_2}{x_3 - x_2}$ | | $d_{31} = \frac{d_{22} - d_{21}}{x_4 - x_1}$ | | |
| $x_3$ | $f_3$ | | $d_{22} = \frac{d_{13} - d_{12}}{x_4 - x_2}$ | | $d_{41} = \frac{d_{32} - d_{31}}{x_5 - x_1}$ | |
| | | $d_{13} = \frac{f_4 - f_3}{x_4 - x_3}$ | | $d_{32} = \frac{d_{23} - d_{22}}{x_5 - x_2}$ | | $d_{51} = \frac{d_{42} - d_{41}}{x_6 - x_1}$ |
| $x_4$ | $f_4$ | | $d_{23} = \frac{d_{14} - d_{13}}{x_5 - x_3}$ | | $d_{42} = \frac{d_{33} - d_{32}}{x_6 - x_2}$ | |
| | | $d_{14} = \frac{f_5 - f_4}{x_5 - x_4}$ | | $d_{33} = \frac{d_{24} - d_{23}}{x_6 - x_3}$ | | |
| $x_5$ | $f_5$ | | $d_{24} = \frac{d_{15} - d_{14}}{x_6 - x_4}$ | | | |
| | | $d_{15} = \frac{f_6 - f_5}{x_6 - x_5}$ | | | | |
| $x_6$ | $f_6$ | | | | | |

$$f(x) = f(x_1) + d_{11}(x - x_1) + d_{21}(x - x_1)(x - x_2) + d_{31}(x - x_1)(x - x_2)(x - x_3) + d_{41}(x - x_1)(x - x_2)(x - x_3)(x - x_4) \ldots$$

Ex :

| | x | f(x) | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|---|---|---|
| $x_1$ | 0 | 1 | | | | |
| | | | $d_{11} = \frac{2.25-1}{1-0} = 1.25$ | | | |
| $x_2$ | 1 | 2.25 | | $d_{21} = \frac{1.5-1.25}{2-0} = 0.125$ | | |
| | | | $d_{12} = \frac{3.75-2.25}{2-1} = 1.5$ | | $d_{31} = \frac{-0.5-0.125}{3-0} = -0.208$ | |
| $x_3$ | 2 | 3.75 | | $d_{22} = \frac{0.5-1.5}{3-1} = -.05$ | | $d_{41} = \frac{0.343+0.208}{4-0} = 0.1378$ |
| | | | $d_{13} = \frac{4.25-3.75}{3-2} = 0.5$ | | $d_{32} = \frac{0.53+0.5}{4-1} = 0.343$ | |
| $x_4$ | 3 | 4.25 | | $d_{23} = \frac{1.56-0.5}{4-2} = 0.53$ | | |
| | | | $d_{14} = \frac{5.81-4.25}{4-3} = 1.56$ | | | |
| $x_5$ | 4 | 5.81 | | | | |

find (2.4)

$\cancel{f(x)}$

$$f(4.2) = 1 + 1.25(2.4-0) + 0.125(2.4-0)(2.4-1) + 0.1378(2.4-0)(2.4-1)$$
$$-0.208(2.4-0)(2.4-1)(2.4-2)$$
$$(2.4-2)(2.4-3) = \boxed{4.029}$$

Spline (linear) $y = y_0 + \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)$ interpolation 4

Ex :

find f(16)

| t | v |
|---|---|
| 0 | 0 |
| 10 | 227 |
| 15 | 362 |
| → 20 | 517 |
| 30 | 901 |

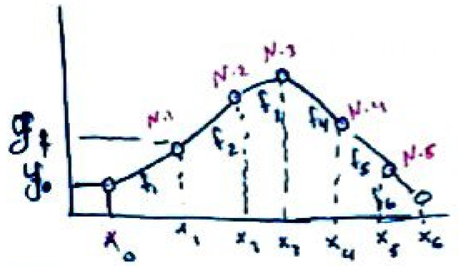
$$f(16) = f(15) + \frac{f(20) - f(15)}{(20 - 15)}(16 - 15)$$

$$= 362 + \frac{517 - 362}{20 - 15}(1)$$

$$= 393$$

Spline method: (interpolation)

Second order.




على 3 اطراف

ما بحل عيهم.

نعتمد على عدد

النقاط

عندي 6 ← 18 مجهول

$f_1(x) = a_1 x^2 + b_1 x + c_1$

6 معادلات

بـ 18 مجهول

$(a, b, c)$

$f_2(x) = a_2 x^2 + b_2 x + c_2$

$f_3(x) = a_3 x^2 + b_3 x + c_3$

$f_4(x) = a_4 x^2 + b_4 x + c_4$

$f_5(x) = a_5 x^2 + b_5 x + c_5$

$f_6(x) = a_6 x^2 + b_6 x + c_6$

من الرسمة معي f و x لكل

اقتران.

بتبني معادلات تا يتجاوب أصل

وصير الحل

Node 1 $\frac{d}{dx} f_1 = \frac{d}{dx} f_2 \rightarrow 2a_1 x + b_1 = 2a_2 x + b_2$

5 معادلات

Node 1: $2a_1 x - 2a_2 x + b_1 - b_2 = 0$

Node 2: $2a_2 x - 2a_3 x + b_2 - b_3 = 0$

Node 3: $2a_3 x - 2a_4 x + b_3 - b_4 = 0$

Node 4: $2a_4 x - 2a_5 x + b_4 - b_5 = 0$

Node 5: $2a_5 x - 2a_6 x + b_5 - b_6 = 0$

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N.1 بتعرب $f_1$ و $f_2$

N.1: $a_1 x_1^2 + b_1 x_1 + c_1 = y_1$

$a_2 x_1^2 + b_2 x_1 + c_2 = y_1$

$N_2$: $a_2 x_2^2 + b_2 x_2 + c_2 = y_2$

$a_3 x_2^2 + b_3 x_2 + c_3 = y_2$

$N_3$: $a_3 x_3^2 + b_3 x_3 + c_3 = y_3$

$a_4 x_3^2 + b_4 x_3 + c_4 = y_3$

N.4: $a_4 x_4^2 + b_4 x_4 + c_4 = y_4$

$a_5 x_4^2 + b_5 x_4 + c_5 = y_4$

N.5: $a_5 x_5^2 + b_5 x_5 + c_5 = y_5$

$a_6 x_5^2 + b_6 x_5 + c_6 = y_6$

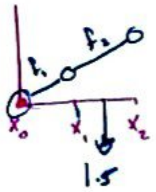
$N_0$: $a_1 x_0^2 + b_1 x_0 + c_1 = y_0$

$N_6$: $a_6 x_6^2 + b_6 x_6 + c_6 = y_6$

$a_1 = 0 \mid a_6 = 0$

Ex :

| | $x$ | $y$ |
|---|---|---|
| $x_0$ | 0 | 0 |
| $x_1$ | 1 | 1 |
| $x_2$ | 2 | 4 |




$n.eq = 3N - 3 = 3(3) - 3 = 6$

$f_1(x) = a_1 x_1^2 + b_1 x_1 + c_1$

$f_2(x) = a_2 x_2^2 + b_2 x_2 + c_2$

$y_0 = a_1 x_0^2 + b_1 x_0 + c_1$

$$\begin{cases} y_1 = a_1 x_1^2 + b_1 x_1 + c_1 \\ y_1 = a_2 x_1^2 + b_2 x_1 + c_2 \end{cases}$$

$y_2 = a_2 x_2^2 + b_2 x_2 + c_2$

$\frac{df_1}{dx} = \frac{df_2}{dx} \rightarrow 2a_1 x_1 + b_1 = 2a_2 x_1 + b_2$

$a_1 = 0$

$\downarrow$

$0(a_1) + (0)b_1 + c_1 = 0$

$1a_1 + (1)b_1 + c_1 = 1$

$1a_2 + (1)b_2 + c_2 = 1$

$4a_2 + 2b_2 + c_2 = 4$

$2a_1 + b_1 - 2a_2 - b_2 = 0$

$a_1 = 0$

بالحالة هاي حذف وتعويض بنحل.

$a_1 = 0$

$c_1 = 0$

$b_1 = 1$

عوض. $\rightarrow$

٣ معادلات بـ٣ مجاهيل

$$\begin{cases} a_2 + b_2 + c_2 = 1 \\ 4a_2 + 2b_2 + c_2 = 4 \\ 1 - 2a_2 - b_2 = 0 \end{cases}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 4 & 2 & 1 \\ 2 & 1 & 0 & -2 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

بنحلها جاوسيه إليمنيشن بطلع

$a_1 = 0$

$b_1 = 1$

$c_1 = 0$

$a_2 = 2$

$b_2 = -3$

$c_2 = 2$

find $y(1.5)$

كل الحالة الموقرآن

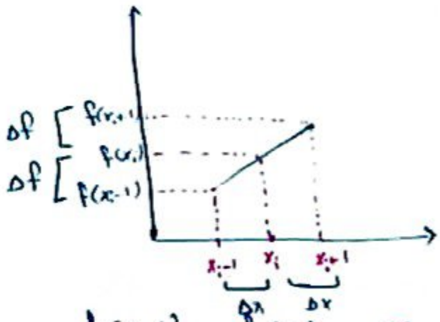
$y(1.5) = a_2 x^2 + b_2 x + c_2$

$= 2(1.5)^2 + (-3)(1.5) + 2$

$= 2$

# Numerical Differentiation

$$f'(x) = \frac{\Delta f}{\Delta x}$$




$$f'_F(x) = \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1}) - (x_i)} = \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$$ Forword diff

$$f'_B(x) = \frac{f(x_i) - f(x_{i-1})}{(x_i) - (x_i - 1)} = \frac{f(x_i) - f(x_{i-1})}{\Delta x}$$ Backword diff

$$f'_c(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{(x_i + 1) - (x_i - 1)} = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}$$ Centeral diff → أدق

Ex: $f(x) = e^x \rightarrow f'(x) = e^x \rightarrow f'(x) = 2.718$ → حكي بالسؤال انه قيمة $x=1$.

$$f'_F(x) = \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x} = \frac{e^{1+0.1} - e^1}{0.1} = 2.86$$

$$f'_B(x) = \frac{f(x_i) - f(x_i - \Delta x)}{\Delta x} = \frac{e^1 - e^{1-0.1}}{0.1} = 2.59$$

$$f'_c(x) = \frac{f(x_i + \Delta x) - f(x_i - \Delta x)}{2\Delta x} = \frac{e^{1+0.1} - e^{1-0.1}}{2(0.1)} = 2.72$$

$\Delta x = 0.1$

$$err\ f'_F = \frac{2.86 - 2.718}{2.718} = 5\%$$

$\boxed{C > B > F}$ → من حيث الدقة.

$$err\ f'_B = \frac{2.718 - 2.59}{2.718} = 4.7\%$$

$$err\ f'_c = \frac{|2.718 - 2.721|}{2.718} = 0.07\%$$

$$f'_F = \frac{e^{1.01} - e^1}{0.01} = 2.73,\ err = 0.44\%$$

$$f'_B = \frac{e^1 - e^{0.99}}{0.01} = 2.704,\ err = 0.4\%$$

$$f'_c = \frac{e^{1.01} - e^{0.99}}{2(0.01)} = 2.718,\ err = 0\%$$

- كلما قلت $\Delta x$ زادت الدقة.

$\Delta x = 0.01$

3-point diff

$$f'(x) = \frac{-f(x_i+2\Delta x) + 4 f(x_i+\Delta x) - 3 f(x_i)}{2\Delta x}$$

4-point diff

$$f'(x) = \frac{-f(x_i+2\Delta x) + 8 f(x_i+\Delta x) - 8 f(x_i-\Delta x) - f(x_i-2\Delta x)}{12\Delta x}$$

[2nd Derv]

$$f''_F = \frac{f(x_i+2\Delta x) - 2 f(x_i+\Delta x) + f(x_i)}{(\Delta x)^2}$$

$$f''_{B,C} = \frac{f(x_i+1) - 2 f(x_i) + f(x_i-\Delta x)}{(\Delta x)^2}$$

Ex: $f(x) = e^x$, $f' = e^x$, $f'' = e^x$

$$f''_F(1) = \frac{e^{1+0.02} - 2e^{1.01} + e^{1}}{(0.01)^2} = 2.745$$

$$f''_{B,C} = \frac{e^{1.01} - 2e^{1} + e^{0.99}}{(0.01)^2} = 2.7183$$

$\Delta x = 0.01$

Third derv:

$$f'''_F(x) = \frac{f(x_i+3\Delta x) - 3f(x_i+2\Delta x) + 3f(x_i+\Delta x) - f(x_i)}{(\Delta x)^3}$$

$$f'''_{B,C}(x) = \frac{f(x_i+2\Delta x) - 2f(x_i+\Delta x) + 2f(x_i-\Delta x) - f(x_i-2\Delta x)}{2(\Delta x)^3}$$

Fourth derv:

$$f^{IV}_F = \frac{f(x_i+4\Delta x) - 4f(x_i+3\Delta x) + 6f(x_i+2\Delta x) - 4f(x_i+\Delta x) + f(x_i)}{(\Delta x)^4}$$

$$f^{IV}_{B,C} = \frac{f(x_i+2\Delta x) - 4f(x_i+\Delta x) + 6f(x_i) - 4f(x_i-\Delta x) + f(x_i-2\Delta x)}{(\Delta x)^4}$$

## Numerical Integration

### Tropizoidal rule

$$I_T = \frac{\Delta x}{2}\left[f(a) + 2f(x_i+\Delta x) + 2f(x_i + 2\Delta x) + \dots + f(b)\right]$$

$$I_T = \frac{\Delta x}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a + i\Delta x)\right]$$

Ex : $\int_0^{\pi} \sin x \, dx \longrightarrow = -\cos x \Big|_0^{\pi} = 2 \rightarrow$ الطريقة العادية *

① $n = 2$, $a = 0$, $b = \pi$, $\boxed{\Delta x = \frac{b-a}{n}} = \frac{\pi}{2}$ ($n$ → no. of intervals) بدنا نقارن جوابها بالطريقة .

$$I_T = \frac{\pi/2}{2}\left[\sin(0) + \sin(\pi) + 2\left[\sin\left(0+\frac{\pi}{2}\right) + \cancel{\sin\left(0+2\frac{\pi}{2}\right)} \dots\right]\right]$$

$= \sin(\pi)$ وهي نفس قيمة $f(b)$ أول ما تظهر معنا بنوقف عند الحد الي قبلها .

$$= \frac{\pi/2}{2}\left[0 + 0 + 2*1\right] = \frac{\pi}{4}*2 = \frac{\pi}{2} = \frac{3.14}{2} = \boxed{1.57}$$

② $n = 5 \rightarrow \Delta x = \frac{\pi}{5}$

$$I_T = \frac{\pi/5}{2}\left[\sin(0) + \sin(\pi) + 2\left[\sin\left(0+\frac{\pi}{5}\right) + \sin\left(0+\frac{2\pi}{5}\right) + \sin\left(0+\frac{3\pi}{5}\right) + \sin\left(0+\frac{4\pi}{5}\right) + \cancel{\sin\left(0+\frac{5\pi}{5}\right)}\right]\right]$$

No $= f(b)$

$$= \frac{\pi}{10}\left[0 + 0 + 2\left[0.59 + 0.95 + 0.95 + 0.59\right]\right]$$

$$= \boxed{1.93}$$

* كلما زادت قيمة $n$ زادت الدقة .

### Simpson's 1/3 rule → أدق من Tropizoidel

$$I_{\frac{1}{3}} = \frac{\Delta x}{3}\left[f(x_0) + f(x_n) + 4f(x_0+\Delta x) + 2f(x_0+2\Delta x) + 4f(x_0+3\Delta x) + 2f(x_0+4\Delta x)\dots\right]$$

* لما نوصل لـ $f(x_n)$ بنوقف عند الحد الي قبلها

* Rule 4 - 2 - 4 - 2 ...

4 with odd

2 with even

$\Delta x = \frac{x_n - x_0}{n}$ → $n$ should be even

## Simpson's 3/8 rule

$$I_{\frac{3}{8}} = \frac{3\Delta x}{8}\Big[f(x_0) + f(x_n) + 3f(x_0+\Delta x) + 3f(x_0+2\Delta x) + 2f(x_0+3\Delta x) + 3f(x_0+4\Delta x) \cdots\Big]$$

لحد ما نوصل
إلى ما قبل $f(x_n)$

rule 3-3-2

$$\Delta x = \frac{x_n - x_0}{n}$$

$n \rightarrow$ لازم تنقسم على $\underline{3}$

**Ex:** $\int_0^{\pi} \sin x \, dx$

→ Simpson's 1/3 rule, $\underbrace{n=2}_{\text{even}}$

$$\Delta x = \frac{\pi - 0}{2} = \frac{\pi}{2}$$

$$I_{\frac{1}{3}} = \frac{\pi/2}{3}\Big[\sin(0) + \sin(\pi) + 4(\sin(0+\tfrac{\pi}{2})) + \cancel{2(\sin(0+\tfrac{2\pi}{2})}\Big]$$

NO

$$= \frac{\pi}{6}\big[0 + 0 + 4(1)\big] = \frac{4 * 3.14}{6} = \boxed{2.09}$$

لو جربنا نحط
$n = 3$
بطلع $\underline{1.81}$
بعيد عن القيمة الحقيقية
عشان هيك لازم $n$ زوجي.

→ Simpson's 3/8 rule

$n = 3$ (تقسم على 3) $\qquad \Delta x = \frac{\pi - 0}{3} = \frac{\pi}{3}$

$$I_{\frac{3}{8}} = \frac{3}{8}\left(\frac{\pi}{3}\right)\Big[\sin(0) + \sin(\pi) + 3\sin(0+\tfrac{\pi}{3}) + 3\sin(0+\tfrac{2\pi}{3}) + \cancel{2\sin(0+\tfrac{3\pi}{3})}\Big]$$

NO

$$= \frac{3\pi}{24}\big[0 + 0 + 3(0.866) + 3(0.866)\big] = \boxed{2.039}$$

## Integration for function without formula for it :

Ex :

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 0.6 | 0.36 |
| 1.2 | 1.44 |
| 1.8 | 3.24 |
| 2.4 | 5.76 |
| 3 | 9 |

$$I_T = \frac{0.6}{2}\left[f(0) + f(3) + 2\left[f(0.6) + f(1.2) + f(1.8) + f(2.4)\right]\right]$$

$$= \frac{0.6}{2}\left[0 + 9 + 2\left[0.36 + 1.44 + 3.24 + 5.76\right]\right]$$

$$= 9.18$$

$\underline{n = 5}$ → فقط بقدر استخدم Tropizoidal ; الفترات بين قيم x منتظمة → لو كانت $n = 6$ بقدر استخدم :

$$\Delta x = \frac{3-0}{5} = \boxed{0.6}$$

1. Tropizoidal
2. simpson's 1/3 rule
3. simpson's 3/8 rule.

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* لو الفترات بين قيم x غير منتظمة ؟

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 0.5 | 0.36 |
| 1.2 | 1.44 |
| 2 | 3.24 |
| 2.4 | 5.76 |
| 3 | 9 |

$\Delta x_1 = 0.5 - 0 = 0.5$

$\Delta x_2 = 0.7$

$\Delta x_3 = 0.8$

$\Delta x_4 = 0.4$

$\Delta x_5 = 0.6$

$\Delta x_{avg} = 0.6$

$$I = \frac{0.6}{2}\left[f(0) + f(3) + 2\left[f(0.5) + f(1.2) + f(2) + f(2.4)\right]\right]$$

$$= 9.18$$

* لو الفترات بين القيم غير منتظمة وكانت في البداية الفترة بينها منظمة بجانبه كبيرة؟

عند الفترة المنظمة $I_T$ ونعمل avg $\Delta x$

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 0.6 | 1 |
| 1 | 3 |
| 1.4 | 7 |
| 3 | 18 |
| 8 | 25 |

(1) 0 – 1.4, (2) 1.4 – 3 ($\Delta x = 1.6$), (3) 3 – 8 ($\Delta x = 5$)

$\Delta x_1 = 0.6$ / $\Delta x_2 = 0.4$ / $\Delta x_3 = 0.4$

$\Delta x_{avg} = 0.46$

$$I_{T_1} = \frac{0.46}{2}\left[0 + 7 + 2(1) + 2(3)\right] = \underline{3.45}$$

بجانبه اللي بعاد عن بعض كل نقطتين لحال.

$$I_{T_2} = \frac{1.6}{2}(7 + 18) = \underline{20}$$

$$I_{T_2} = \frac{5}{2}(18 + 25) = \underline{107.5}$$

بنجمع

$$I_T = 3.45 + 20 + 107.5 = 130.95$$

## Numerical Soln of ODE

Euler method :

$y_{i+1} = y_i + \Delta x \, F(x_i, y_i)$

Ex: Solve $\frac{dy}{dx} + y = x+1$ from $x=0$ to $x = 0.8$ , $y(0) = 1$.

$\frac{dy}{dx} = x + 1 - y = f(x,y) \rightarrow$ $\frac{dy}{dx}$ نعطي موضوع القانون

$y_{i+1} = y_i + \Delta x \left[ x_i + 1 - y_i \right]$

$\rightarrow \underline{x_0 = 0}$ , $\underline{y_0 = 1}$

Assume $\Delta x = 0.2$

$y_1 = y_0 + 0.2 [0 + 1 - 1] = 1 \leftarrow y_1 = y_0 + \Delta x \, f(x_0, y_0)$

$y_1 = y_0 + \Delta x [x_0 + 1 - y_0]$

$x_1 = x_0 + \Delta x = 0 + 0.2 = 0.2$

$\rightarrow \underline{x_1 = 0.2}$ , $\underline{y_1 = 1}$

$y_2 = 1 + 0.2 [0.2 + 1 - 1] = 1.04 \leftarrow y_2 = y_1 + \Delta x \, f(x_1, y_1)$

$y_2 = y_1 + \Delta x [x_1 + 1 - y_1]$

$x_2 = x_0 + 2\Delta x = 0.4$

$\rightarrow \underline{x_2 = 0.4}$ , $\underline{y_2 = 1.04}$

$y_3 = 1.04 + 0.2 [0.4 + 1 - 1.04] = 1.112 \leftarrow y_3 = y_2 + \Delta x \, f(x_2, y_2)$

$y_3 = y_2 + \Delta x [x_2 + 1 - y_2]$

$x_3 = x_0 + 3\Delta x = 0.6$

$\rightarrow \underline{x_3 = 0.6}$ , $\underline{y_3 = 1.112}$

$y_4 = 1.112 + 0.2 [0.6 + 1 - 1.112] = 1.209 \leftarrow y_4 = y_3 + \Delta x \, f(x_3, y_3)$

$y_4 = y_3 + \Delta x [x_3 + 1 - y_3]$

$x_4 = x_0 + 4\Delta x = \underline{0.8}$ stop → من السؤال

$\rightarrow \underline{x_4 = 0.8}$ , $\underline{y_4 = 1.209}$

| x | y |
|---|---|
| 0 | 1 |
| 0.2 | 1 |
| 0.4 | 1.04 |
| 0.6 | 1.112 |
| 0.8 | 1.209 |

curve fitting ونعمل

حتى نطلع المعادلة .

The end :)

13/1/2025.