1 ST 2023/2024

Numerical Methods

DR:

Yousef Mubarak

Subject: Introduction

Equation

analytically — numerically

Computing & computers :-

Numbers
- Decimal (0, 1, 2, ... 9)
- Binary (0, 1)

computer $\xrightarrow{\text{from decimal to binary}}$ Binary $\xrightarrow{\text{operation (Binary)}}$ screen

أنا بدخل على الحاسوب dicemal والحاسوب بحولها

لـ وتظهر بالشاشة decimal

* Number
  - integer
  - Real

① Integer ⟶ 4 Bits لما أدخل رقم صحيح الحاسوب يحجز مكان للرقم كإضافة

+ ⟶ 1   - ⟶ 0

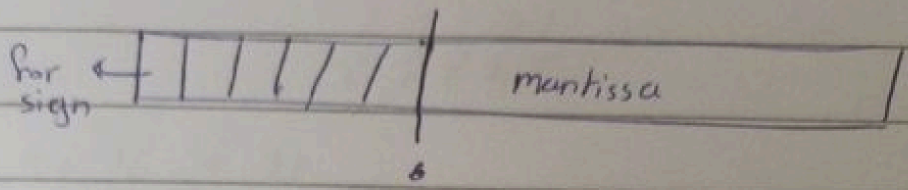
* اتجاه الأرقام من اليمين لليسار

* اذا دخلنا 1000 ح يطلع عندي Error بصير over flow

* Single precision ⟶ one word (32 bits)

* douple precision ⟶ Two word (64 bits)

② Real Number $x = Fq * 2^{\pm n}$



$x = 12.5 \longrightarrow 0.125 * 10^2$

* shift left (+ve) * shift right (-ve)

* كلما زادت عدد ال Bits زادت ال accuracy

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ 2^{-5}$

$0.1011 * 2^0$ | + | | | | | | 1 | 1 | 0 | 1 | 1 |

* over flow ⟶ stop
* under flow ⟶ stop
  ⟶ very small ⟶ round to be zero

* Conversion from Decimal Integer to Binary

$N = 2Q_0 + b_0$

$Q_0 = 2Q_1 + b_1$ $N = \leftarrow \cdots\cdots b_3 b_2 b_1 b_0$

$Q_1 = 2Q_2 + b_2$

$Q_2 = 0$ We stop only Q are equal zero

* ازا كان الرقم زوجي ال b بتساوي zero
* ازا كان الرقم فردي ال b تساوي 1

Example :- Convert $N=23$ to a binary base on 2 ?

$23 = 2(11) + 1$

$11 = 2(5) + 1$ $N = 10111$

$5 = 2(2) + 1$

$2 = 2(1) + 0$

$1 = 2(0) + 1$

Subject :

Example②:- Convert (10101) to decimal number?

$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 1 \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$1 + 4 + 16 = 21$

Example③:- Convert (10) to a binary number base on 3?

$10 = 3(3) + 1$

$3 = 3(1) + 0$ $\qquad N = 101$

$1 = 3(0) + 1$

* Conversion from Decimal Fraction to Binary :-

* أول اشي الرقم الصحيح (integer) زي الطريقة الي فوق

* الجزء الثاني بعد الفاصلة (R)

$2R = \underset{}{\overset{d_1}{\boxed{0/1}}} , \overset{\text{Fraction}}{\boxed{F_1}}$

$2F_1 = \overset{d_2}{\boxed{0/1}} , \boxed{F_2}$ $\qquad R = 0, d_1 d_2 d_3 \dots$

$2F_2 = \overset{d_3}{\boxed{0.1}} , \boxed{F_3}$

Example①:- Convert (3,5) from decimal to Binary Base on 2?

$3 = 2(1) + 1$

$1 = 2(0) + 1$ $\qquad N = 11$

$2(0{,}5) = 1{,}0$ $\qquad R = 0{,}10$

$2(0) = 0{,}0$

$3{,}5 = 11{,}10$

$= 0{,}111 * 2^{+2}$

Subject : / /

Example ② :- Convert (16,75) decimal to Binary Base on 3?

$16 = 3(5) + 1$

$5 = 3(1) + 2 \qquad N = 121$

$1 = 3(0) + 1$

$3(,75) = 2,25$

$3(0,25) = 0,75$

$R = 0,\overline{20}$

$16,75 = 121,20\overline{20}$

$= ,121 20\overline{20} * 3^3$

	3^3	3^2	3^1	3^0							
	1	0	0	0	1	2	1	2	0	2	0

or 0 1 1 0,

*Operations

- Addition

```
 1 1 1
  0 1 1
  1 0 1  +
 ------
 1 0 0 0
```

```
 1 1 1 1 1
    1 0 1 1 1
    1 1 0 0 1  +
 ------------
  1 1 0 0 0 0
```

- suptraction :-

```
   0 X' 11
  1 X 0 0 1
  1 0 1 1 1
 ----------
  0 0 0 1 0   = -10
```

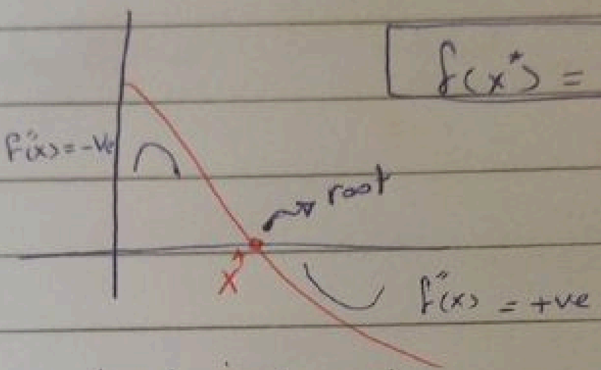
$-,10 * 2^{+2}$

Subject :

Chapter 1 single none linear Equation

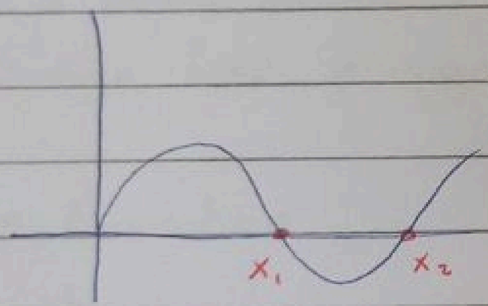
solutions = roots

x	f(x)
⋮	⋮



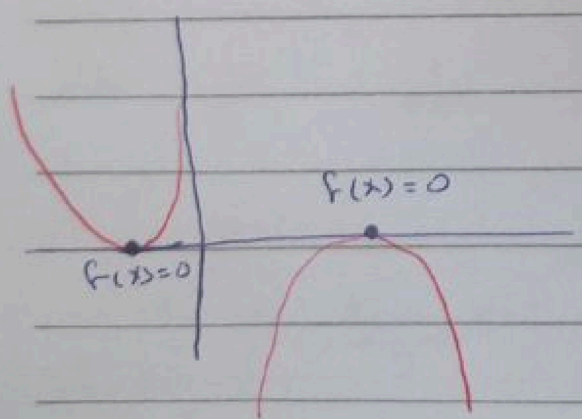
$$\boxed{f(x^*) = 0}$$

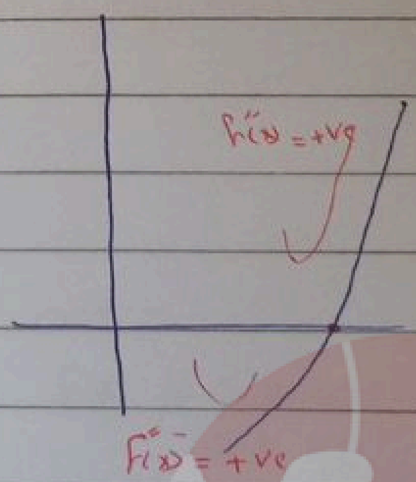
is not single root



(more than one root)

$f(x_1) = f(x_2) = 0$





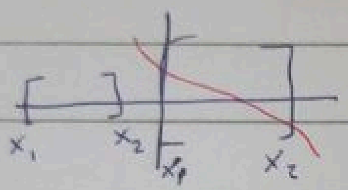
مش شرط الاقتران يمر من خلال الـ x axis عشان يكونلها root

* لما ما تتغير اشارة المشتقة الثانية يعني الوحيد single root

Subject : / /

Method

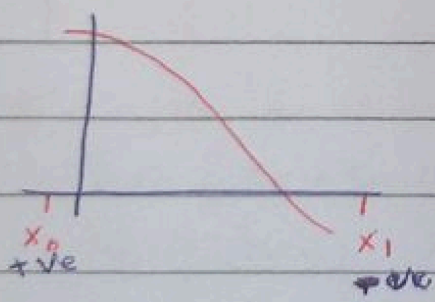
- closed (Brackted)
  - Bisection
  - false Position
- open
  - Muller's — 3 = =
  - Fixed point — 1 = =
  - Newton's — 1 = =
  - secant — 2 initial guses



* Bisection

1) 2 initial guses
2) sign $f(x_0) \neq$ sign $f(x_1)$

* Bisection



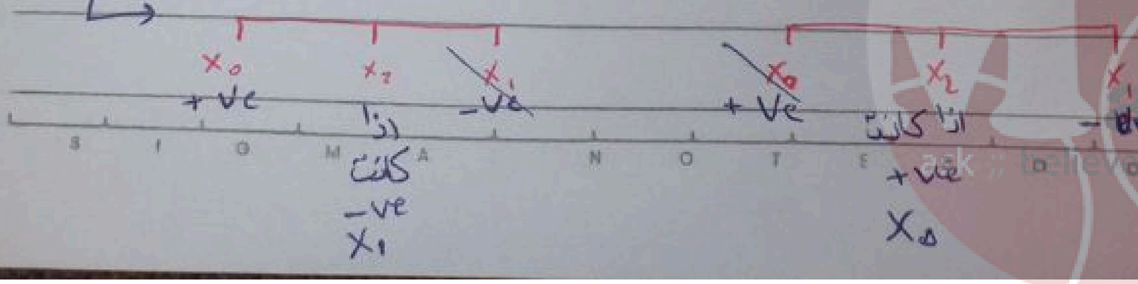
1) 2 initial gues
2) sign $f(x_0) \neq$ sign $f(x_1)$

- The expected value of the root

$$\boxed{x_2 = \frac{x_0 + x_1}{2}}$$

if $f(x_2) = 0{,}00001$ it is the root $|f(x_2)| \leq TR$ or relative error

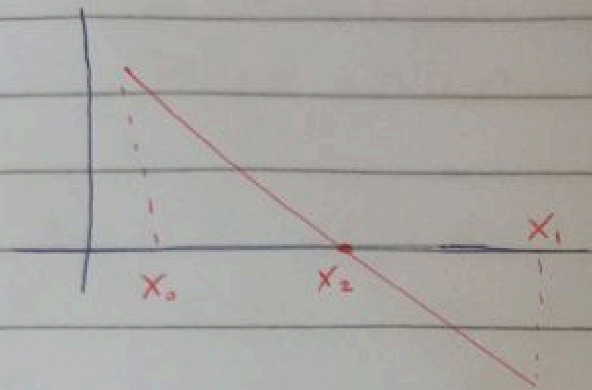
if No →



Subject :

* False position Method :- (أكثر دقة من Bisection)

* sign $f(x_0) \neq$ sign $f(x_1)$



معادلة خط مستقيم X_2

$$y - y_1 = m(x - x_1)$$

$$y - 0 = m(x - x_2)$$

$$y = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}(x - x_2)$$

$$f(x_1) = \frac{f(x_0) - f(x_1)}{x_1 - x_0}(x_1 - x_2)$$

$$\boxed{x_2 = x_0 - f(x_0)\left|\frac{x_0 - x_1}{f(x_0) - f(x_1)}\right|}$$

$$\boxed{x_2 = x_1 - f(x_1)\left|\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right|}$$

Example on Bisection method :-

$f(x) = 3x + \sin(x) - e^{x}$ $\quad x_0 = 0 \quad x_1 = 1$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
0	-1	1	1,123189	0,5	1,37284
0	-1	0,5	1,37289	0,25	0,2186
0	-1	0,25	0,21860		

Subject :

مثال خارجي

Example 2 :- Find the root of the function by Bisection Method using $x_0 = 5$, $x_1 = 10$

$$f(x) = -0.4x^2 + 2,2x + 4,7 \qquad \varepsilon_s = 5\%$$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	$\varepsilon_a\%$
5	5,7	10	-13,3	7,5	-1,3	-
5	5,7	7,5	-1,3	6,25	2,825	20%
6,25	2,825	7,5	-1,3	6,875	0,91875	9%
6,875	0,91875	7,5	-1,3	7,1875	-0,1516	4,34% < 5%

$$\boxed{x_r = 7,1875}$$

The same example By false position Method :-

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	$\varepsilon_a\%$
5	5,7	10	-13,3	6,5	2,1	—
6,5	2,1	10	-13,3	6,97727	0,57707	6,8%
6,97727	0,57707	10	-13,3	7,10296	0,145695	1,76%

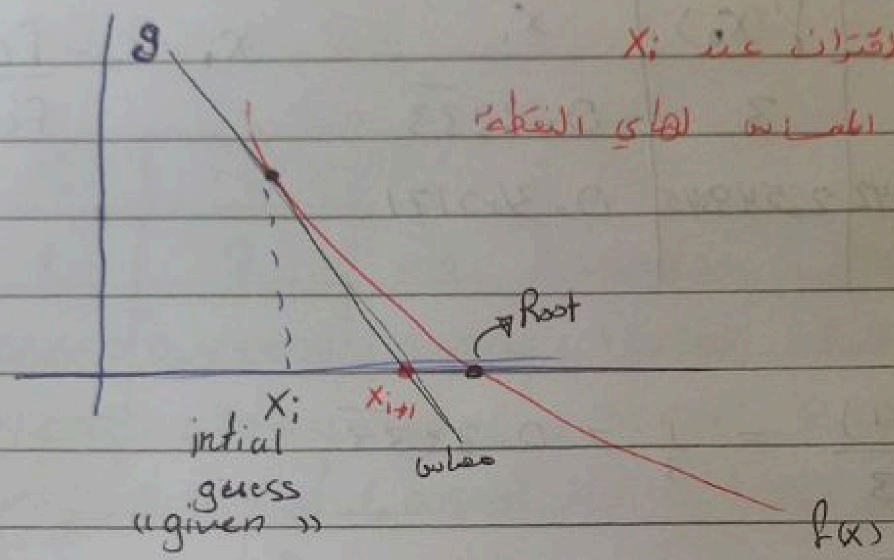
$$x_2 = x_1 - f(x_1) \left| \frac{x_1 - x_0}{f(x_1) - f(x_0)} \right|$$

$$\boxed{x_r = 7,10296}$$

Subject :

* open Method

① Newton - Raphson method :-



* مشتقة الاقتران عند x_i تساوي ميل المماس لهاي النقطة

$$f'(x_i) = \frac{\overset{0}{\cancel{g(x_{i+1})}} - \overbrace{g(x_i)}^{= f(x_i)}}{x_{i+1} - x_i}$$

$$f'(x_i) = \frac{-f(x_i)}{x_{i+1} - x_i}$$ (بدنا نجعل x_{i+1} موضوعا للقانون)

$$-f(x_i) * \frac{1}{f'(x_i)} = \frac{x_{i+1} - x_i}{-f(x_i)} \xrightarrow{* -f(x_i)} x_{i+1} - x_i = \frac{-f(x_i)}{f'(x_i)}$$

$$\boxed{x_{i+1} = x_i - \left(\frac{f(x_i)}{f'(x_i)}\right)}$$

Example 2:- $f(x) = 3x + \sin x - e^x$ $\quad x_0 = 0$

$f'(x) = 3 + \cos x - e^x$

x_0	$f(x_0)$	$f'(x_0)$	x_1
0	-1	3	$0.33\overline{33}$
$0.3\overline{33}$	-0.06842	2.549345	0.360171

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1^{①} = 0 - \frac{(-1)}{3} = \frac{1}{3} = 0.33\overline{33}$

$x_2^{②} = 0.33\overline{33} - \frac{(-0.06842)}{2.549345} = 0.360171$

$f(x_1^{②}) = -0.00063$

Subject : / /

Example خارجي

$f(x) = x^2 - 5x + 6$

find the Root of the function using Newton Raphson Method with $x_0 = 1$ $\varepsilon_s = 1\%$ $f'(x) = 2x - 5$

x_0	$f(x_i)$	$f'(x_i)$	x_{i+1}	$\varepsilon_a\%$
1	2	-3	1,66667	40%
1,66667	0,44453	-1,66666	1,933388	13,79%
1,933388	0,070863	-1,133224	1,9959202	3,13%
1,9959202	0,004096	-1,008159	1,999983	0,203%

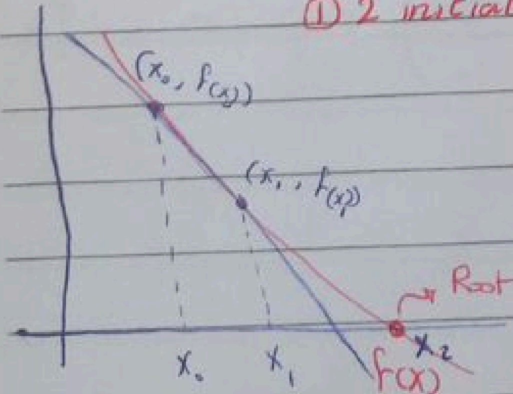
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= 1 - -\left(\frac{2}{+3}\right) = 1,666$$

$$\boxed{x_{i+1} = x_r = 1,999983}$$

* Secant Methode

① 2 initial ~~Method~~ guesses



$$\boxed{x_2 = x_0 - f(x_0)\left(\frac{x_0 - x_1}{f(x_0) - f(x_1)}\right)}$$

Subject : ______ / /

Example :- $f(x) = 3x + \sin(x) - e^{x}$

$x_0 = 0 \qquad x_1 = 1$

$f(x_0) = f(0) = -1$

$f(x_1) = f(1) = 1,123189$

$|f(x_0)| > |f(x_1)| \rightarrow$ اذا ما تحقق هذا الشرط يتم

عكس x_0 مع x_1

$\therefore x_0 = 1 \ \& \ x_1 = 0$

$$x_2 = x_0 - f(x_0)\left(\frac{x_0 - x_1}{f(x_0) - f(x_1)}\right)$$

$$x_2 = 1 - 1,123189\left(\frac{1 - 0}{1,123189 - (-1)}\right) = 0,470989$$

$f(x_2) = f(0,470989) = |0,2651588| > 1 * 10^{-4}$

$x_0 = 0 \quad , \ x_1 = 0,4709895$

$$x_2^{(2)} = 0 - (-1)\left(\frac{0 - 0,4709895}{-1 - 0,2651588}\right) = 0,37227$$

Subject :

Example مثال $f(x) = e^{-x} - x$

find the root of the function using secant Method with $x_0 = 0$ $x_1 = 1$ $\varepsilon_s = 1\%$.

$f(0) = 1$ $f(1) = -0.63212$ $|f(0)| > |f(1)|$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	$\varepsilon\%$
0	1	1	-0.63212	0.6127	-0.070814	63.21%
1	-0.63212	0.6127	-0.070814	0.5638	0.00524258	8.67%
0.6127	-0.070814	0.5638	0.00524258	0.5671706		0.594% < 1%

$$\boxed{x_2 = 0.5671706}$$

* fixed point iteration

all other method $f(x) = 0$

- fixed point $x = g(x)$

$y_1 = x$ $y_2 = g(x)$

Example :- $f(x) = x^2 - 2x - 3$ at $x_0 = 4$

$2x = x^2 - 3$

$$\boxed{x = \frac{x^2 - 3}{2}} \rightarrow g(x)$$

$$\sqrt{x^2} = \sqrt{2x+3} \quad \boxed{x = \sqrt{2x+3}} \rightarrow g(x)$$

$$x(x-2) - 3 \quad \boxed{x = \frac{3}{x-2}} \rightarrow g(x)$$

Subject :

- $f(x) = x$, $x_1 = \sqrt{2x+3}$ $\qquad x_0 = 4$

$x_1 = \sqrt{11} = 3,31662$

$x_1^{①} = \sqrt{2(3,31662)+3} =$

$x_1^{②} = 3,10375$

$x_1^{③} = 3,034385 \quad \dots \quad x_1^{④} = 3,01144$

$x_1^{⑦} = 3,000141$

$\vdots$

$x_1^{8} = 3,000047$

- $f(x) = x \qquad x = \dfrac{x^2-3}{2} \qquad x_0 = 4$

$x_1^{①} = \dfrac{16-3}{2} = 6,5 \qquad x_1^{②} = \dfrac{(6,5)^2-3}{2} = 19,625$

$x_1^{③} = \dfrac{(19,625)^2-3}{2} = 191,07$

* الفارق بالقيم جدا كبير

so will diverge

- $f(x) = x \qquad x = \dfrac{3}{x-2} \qquad x_0 = 4$

$x_1^{①} = \dfrac{3}{4-2} = 1,5 \qquad x_1^{②} = \dfrac{3}{1,5-2} = -6$

$x_1^{③} = \dfrac{3}{-6-2} = -0,375 \qquad x_1^{④} = \dfrac{3}{-0,375-2} = -1,26315$

$x_1^{⑤} = -0,91935 \quad x_1^{⑥} = -1,027624 \quad x_1^{⑧} = -0,9998871$

$x_1^{12} = -1,0000376$

Subject :

Hint if $|g'(x_0)| < 1$

$\hookrightarrow$ احتمالية أكبر يطلع convergence

if $|g'(x_0)| > 1$ $\rightarrow$ احتمالية أكبر يطلع divergence

$g(x) = \frac{3}{x-2} \Rightarrow g'(x_0) = \frac{-3}{(x-2)^2}$ $\quad g'(4) = \frac{-3}{(4-2)^2}$

$= -0.75$

< 1

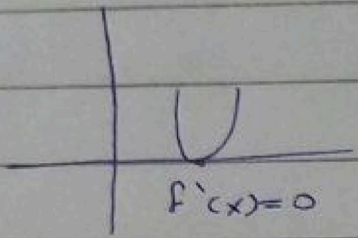
$g(x) = \frac{x^2-3}{2} \rightsquigarrow g'(x) = \frac{2x}{2}$

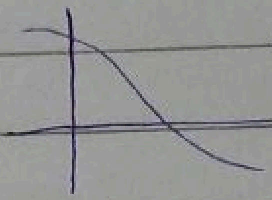
$g(4) = 4 > 1$

Subject :

Modified Newton's Raphson Method

Remember : simple $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$





* sign of $F'_{befor} = F'(x)_{after}$

* sign $f''(x)_{befor} \neq f''(x)_{after}$

* sign of $f'(x)_{befor} \neq f'(x)_{after}$

* sign of $f(x)_{befor} = f(x)_{after}$

* sign of $f''(x)_{befor} = f'(x)_{after}$

Modification newton's = $\cancel{x} = x_0 - \frac{F(x_0) \cdot f'(x_0)}{(f'(x_0))^2 - f(x_0) \cdot f''(x_0)}$

Example $f(x) = x^3 - 5x^2 + 7x - 3 \;,\; x_0 = 0$

$f'(x) = 3x^2 - 10x + 7$

Simple Newton's

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

x_0	$f(x_0)$	$f'(x_0)$	x_1
0	-3	7	0,42857
0,42857	-0,83965	3,265316	0,6857135
0,6857135	-0,22859	1,553469	
0,832865	-0,0[illegible]	0,75234	
0,91333	-0,01567	0,369216	
0,955783	-0,004	0,182732	
0,977655			

يتبع →

Modifide N.M

$$f''(x) = 6x - 10 \qquad x_0 = 0$$

$$f(0) = -3 \qquad f'(0) = 7 \qquad f''(0) = -10$$

$$x_1^{(1)} = x_0 - \left\{ \frac{f(x_0)\, f'(x_0)}{(f'(x_0))^2 - f(x_0) f''(x_0)} \right\}$$

$$= 0 - \left\{ \frac{-3(7)}{7^2 - (-3)(-10)} \right\} = 1.105263158$$

$$x_1^{(2)} = 1.105263158 - \left\{ \frac{(-0.3878116)(-0.0209943)}{(0.3878116)^2 - (0.0209943)(-3.36842)} \right\}$$

$$x_1^{(2)} = 1.003081660 \qquad x_1^{(3)} = 1.000002382$$

$$f(x_1^{(3)}) = |1.3 \times 10^{-11}| < 1 \times 10^{-5}$$

* Type of Error :-

1) True Error = True Answer − Approximate Answer

2) True % relative error = $\dfrac{\text{True answer} - \text{Approximate ans}}{\text{True ans}} \times 100$

3) Approximate % relative error

$= \dfrac{\text{present approximation} - \text{previous aproximation}}{\text{present approximation}} \times 100$

Subject :

(4) Prespecified % tolerance

$$\varepsilon = 0.5 * 10^{2-n} \% \qquad n = \text{significant figure}$$

* Muller's Method

1) 3 initial guesses are required x_0, x_1, x_2
if only 2 guesses are given then $x_2 = \frac{x_0 + x_1}{2}$

2) arrange the values as follow (يجب الالتزام)

$$x_2 < x_0 < x_1$$

$f(x_2) \quad f(x_0) \quad f(x_1)$

3) calculate the following constants :-

$$c_1 = \frac{f(x_1) - f(x_2)}{x_1 - x_2} \qquad c_2 = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

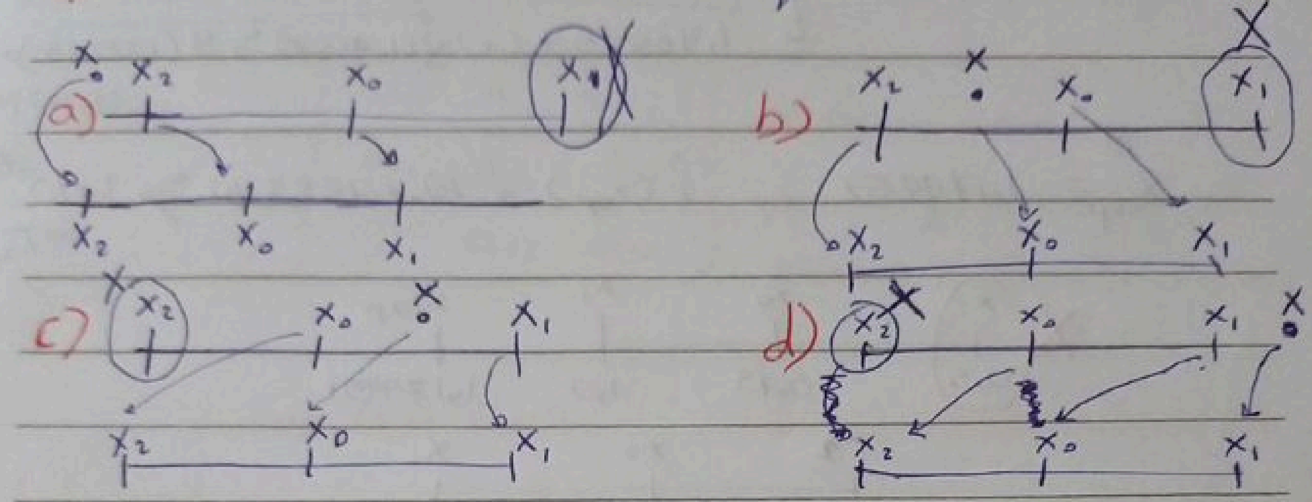
$$d_1 = \frac{c_2 - c_1}{x_0 - x_2} \qquad s = c_2 + d_1 (x_0 - x_1)$$

4) calculate the expected root

$$x = x_0 - \left\{ \frac{2 f(x_0)}{s + (\text{sign of } s) * \sqrt{s^2 - 4 f(x_0) - d_1}} \right\}$$

5) if $|f(x)| \le Tol$, then x is the root
if $|f(x)| > Tol$, then another iteration is required

b) if onother iteration is required then



Example $f(x) = x^6 - 2$ $\quad x_0 = 0{,}5 \quad x_1 = 1{,}0$

$$x_2 = \frac{0{,}5 + 1}{2} = 0{,}75$$

	x_2	x_0	x_1
$x =$	0,5	0,75	1,0
$f(x)$	−1,984375	−1,82202484	−1

$$c_1 = \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{(-1) - (-1{,}984375)}{1 - 0{,}5} = 1{,}96875$$

$$c_2 = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{-1{,}82202484 - (-1)}{0{,}75 - (+1)} = 3{,}28808592$$

$$d_1 = \frac{c_2 - c_1}{x_0 - x_2} = \frac{3{,}28808592 - 1{,}96875}{0{,}75 - 0{,}5} = 5{,}27734368$$

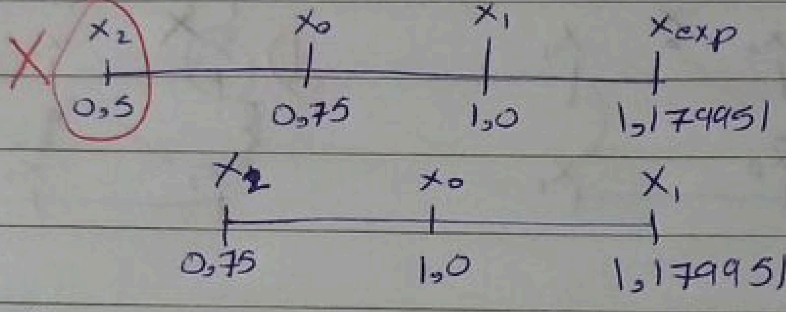
$$s = c_2 + d_1(x_0 - x_1) = 3{,}28808592 + 5{,}27734368(0{,}75 - 1)$$
$$= 1{,}96875$$

Subject : / /

$$x_{exp} = 0,75 - \left[\frac{2(-1,82202484)}{1,96875 + (+)\sqrt{(1,96875)^2 + 4(1,82202484) * (5,27734368)}} \right.$$

①

$x_{exp} = 1,179951$, $f(x_{exp}) = 10,69888161 > 1\times10^{-6}$ ↳ Tol



$f(x)$ ~~0,6~~	$-1,82202484$	-1	$0,6988816$

$C_1 = 5,86324$ $C_2 = 9,440801$

$d_1 = 14,310244$ $S = 6,8656583$

②

$$x_{exp} = 1,0 - \left[\frac{2(-1)}{6,8656583 + (+)\sqrt{(6,8656583)^2 - 4(-1)(14,310244)}} \right.$$

$x^2_{exp} = 1,11708$

$$\%R.E = \left(\left| \frac{1,11708 - 1,179951}{1,11708} \right| \right) \times 100$$

$= 5,62\%$

$x^3_{exp} = 1,12237$

$x^4_{root} = 1,122462$

Subject: ______ / /

Chapter 2 System of linear & non linear Eqs.

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \boxed{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

* Matrix : consists of a rectangular array of element represented by a single symbols

* مجموعة من الأعمدة و الأسطر at least $2*2$

* اذا كان عندي $2*1$ / $1*2$ / 3×1 (عدد الصفوف او الاعمدة 1) بسمي ~~[illegible]~~ vector (not matrix)

$$A = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \rightarrow \text{column vector}$$

$$B = \{1 \; 2 \; 3\} \quad \text{row vector}$$

$$[A] = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 8 & -10 & 11 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 5 \\ 7 \\ 12 \end{Bmatrix}$$

↳ square matrix (عدد المتغيرات = عدد المعادلات)

* Addition & subtraction

شرط * size of the first matrix = size the second matrix

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 9 & 8 \\ 6 & 5 \end{bmatrix}$$

Subject :

* multipication of matrix

* Number of column in first matrix must equal number of row in second matrix

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 4 & 5 & 7 \end{bmatrix} \quad (2\times3) \qquad B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \quad (3\times2)$$

$a_{11} = 2\cdot2 + 3\cdot4 + 6\times6 = 52$

$a_{12} = 2\cdot3 + 3\cdot5 + 6\cdot7 = 63$

$a_{21} = 4\cdot2 + 5\cdot4 + 7\cdot6 = 70$

(2×2) المصفوفة الناتجة

$$A * B = \begin{bmatrix} 52 & 63 \\ 70 & 86 \end{bmatrix}$$

* notes :

* $A(B+C) = AB + AC$
* $(AB)C = A(BC)$
* $A \xrightarrow{\text{Transpose}} [A]^T$ (تبديل الاعمدة بأسطر و العكس)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \longrightarrow \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

* $(AB)^T \longrightarrow A^T B^T$

$$[A] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \qquad [A]^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

$$\underbrace{B = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \qquad B^T = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}}_{\text{symtrical}}$$

$$[A] = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \rightarrow \text{diagonal}$$

$$[A] = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$ bonded matrix (Tri-diagonal)

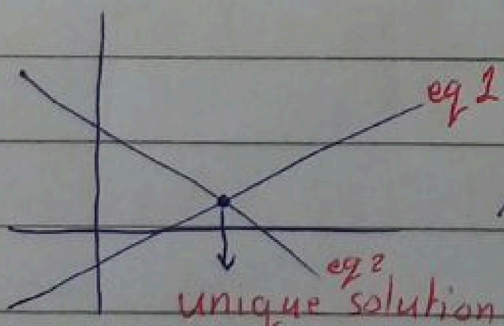
$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$ upper-triangle Matrix

$$[A] = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$ lower-triangle Matrix

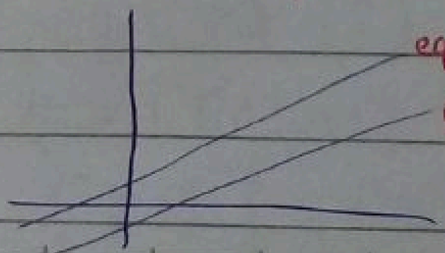
* singular matrix

$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \quad 16 - 16 = 0$$

*في هاي الحالة بكون عندي المعادلتين dependent

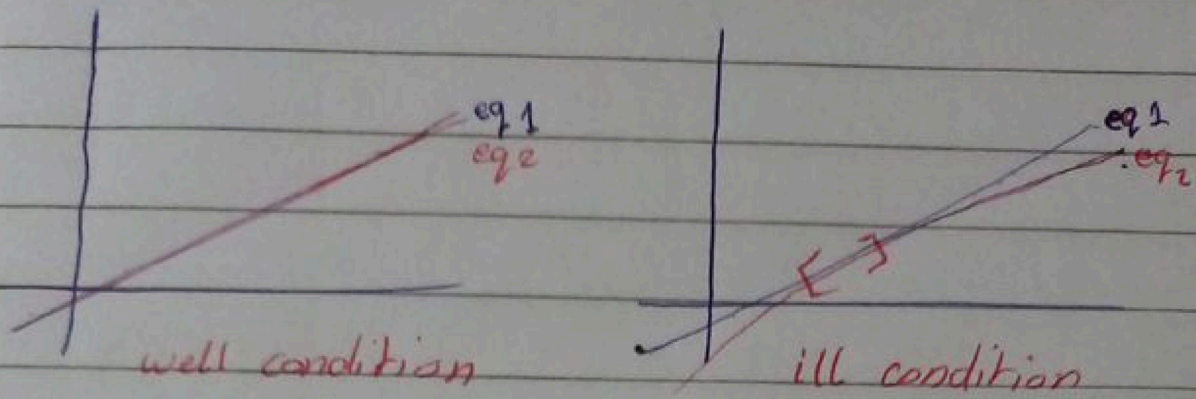


$\rightsquigarrow$ det $A \neq$ zero



(There is no solution)

det $A = 0$



well condition ill condition

[-∞,∞] solution لل

* مشتغلين احنا على square matrix
(# equation = # variable)

*Inverse of A matrix

$$[A] \longrightarrow [A]^{-1}$$

$$[A]\{x\} = \{c\}$$

$$\{x\} = \frac{\{c\}}{[A]}$$

$$\{x\} = \{c\} \cdot [A]^{-1}$$

① $[A].[A]^{-1} = [I]$ طريقتين
$[A]^{-1}[A] = [I]$ للتأكد اذا الinverse
② $\left[[A]^{-1}\right]^{-1} = [A]$ لي طلعتو صح

Some notes :

① $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ إذا أي column/row كانو يساوي صفر فـ Det = zero

② $\begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ هون عندي تكرار للصف لذلك Det = zero

③ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot 2$ Det = 20

$\rightarrow$ Det = (20)(2)

اذا بضرب عناصر صف (وعمود برقم
الـ Det رح ينضرب بنفس الرقم

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

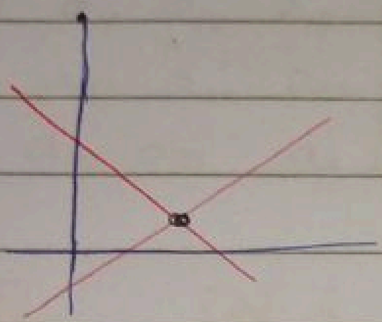
$$\left|\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array}\right|$$

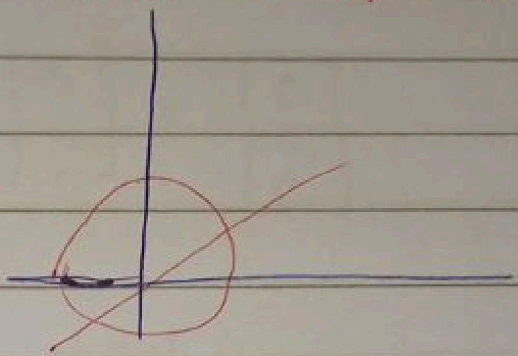
$\leftarrow$ بستخدم هاد التمثيل
والهدف منه (إذا عدلت
على طرف بتعدل الطرف الثاني
فلازم لا أنسى التعديل)

Subject : / /

* Solving small system

① Graphical solution (up to 3 equation)





② Elimination of unknown

* بقدر احل عليها مع مجاهيل بس صعب فبستخدمها

لـ up to 2equ / 3equ

Example : $2x_1 + 3x_2 = 5 \longrightarrow \boxed{x_2 = \frac{5 - 2x_1}{3}}$

$x_1 - 4x_2 = 10$

$$x_1 - 4\left(\frac{5 - 2x_1}{3}\right) = 10$$

$$\frac{3*}{3*}\frac{x_1}{1} - \frac{20}{3} + \frac{8}{3}x_1 = 10$$

$$\frac{11}{3}x_1 = \frac{30}{3} + \frac{20}{3}$$

$$\frac{3}{11} * \frac{11}{3}x_1 = \frac{50}{3} * \frac{3}{11}$$

$$\boxed{x_1 = 4.545454}$$

Subject : ____ / /

⑬ Cramer's Rule (up to 3eq)

$$[A] \cdot \{x\} = \{C\}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix}$$

① Det A ايجاد

② $\text{Det} \begin{bmatrix} C_1 & a_{12} & a_{13} \\ C_2 & a_{22} & a_{23} \\ C_3 & a_{32} & a_{33} \end{bmatrix}$ $\text{Det} \begin{bmatrix} a_{11} & C_1 & a_{13} \\ a_{21} & C_2 & a_{23} \\ a_{31} & C_3 & a_{33} \end{bmatrix}$

$\text{Det} \begin{bmatrix} a_{11} & a_{12} & C_1 \\ a_{21} & a_{22} & C_2 \\ a_{31} & a_{32} & C_3 \end{bmatrix}$

③ $x_1 = \dfrac{\text{Det} \begin{bmatrix} C_1 & & \\ C_2 & & \\ C_3 & & \end{bmatrix}}{\text{Det } A}$ $x_2 = \dfrac{\text{Det} \begin{bmatrix} & C_1 & \\ & C_2 & \\ & C_3 & \end{bmatrix}}{\text{Det } A}$

$x_3 = \dfrac{\text{Det} \begin{bmatrix} & & C_1 \\ & & C_2 \\ & & C_3 \end{bmatrix}}{\text{Det } A}$

Subject :

Example :
$$7x_1 - 3x_2 + 3x_3 = -49$$
$$x_1 - 2x_2 - 5x_3 = 5$$
$$3x_1 - 6x_2 + 10x_3 = -84$$

[A]
$$\begin{bmatrix} 7 & -3 & 3 \\ 1 & -2 & -5 \\ 3 & -6 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -49 \\ 5 \\ -84 \end{Bmatrix}$$

$$D_A = \begin{bmatrix} 7 & -3 & 3 \\ 1 & -2 & -5 \\ 3 & -6 & 10 \end{bmatrix} = 7\begin{vmatrix} -2 & -5 \\ -6 & 10 \end{vmatrix} + 3\begin{vmatrix} 1 & -5 \\ 3 & 10 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix}$$
$$7(-20-30) + 3(10+15) + 3(-6+6)$$
$$= -275$$

$$D_{x_1} = \begin{bmatrix} -49 & -3 & 3 \\ 5 & -2 & -5 \\ -84 & -6 & 10 \end{bmatrix} = -49(-20-30) + 3(50-420) + 3(-30-168)$$
$$= 746$$

$$D_{x_2} \begin{bmatrix} 7 & -49 & 3 \\ 1 & 5 & -5 \\ 3 & -84 & 10 \end{bmatrix} = 7(50-420) + 49(10+15) + 3(-84-15)$$
$$= -1662$$

$$D_{x_3} \begin{bmatrix} 7 & -3 & -49 \\ 1 & -2 & 5 \\ 3 & -6 & -84 \end{bmatrix} =$$
$$7(168+30) + 3(-84-15) - 49(-6+6)$$
$$= 1089$$

$$x_1 = \frac{D_{x_1}}{D_A} = \frac{746}{-275} = -2{,}7127$$

$$x_2 = \frac{D_{x_2}}{D_A} = \frac{-1662}{-275} = 6{,}0436$$

$$x_3 = \frac{D_{x_3}}{D_A} = \frac{1089}{-275} = -3{,}96$$

Subject :

Example مثال

$$2x + 3y - 5z = 1$$
$$x + y - z = 2$$
$$2y + z = 8$$

$$\underset{[A]}{\begin{bmatrix} 2 & 3 & -5 \\ 1 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 8 \end{Bmatrix}$$

$$x = \frac{D_x}{D_A} = \frac{-7}{-7} = 1$$

$$y = \frac{D_y}{D_A} = \frac{-21}{-7} = 3$$

$$z = \frac{D_z}{D_A} = \frac{-14}{-7} = 2$$

$$D_A = 2\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 5\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 2\{1+2\} - 3\{1-0\} - 5\{2-0\}$$

$$D_A = -7$$

$$D_x = \begin{bmatrix} 1 & 3 & -5 \\ 2 & 1 & -1 \\ 8 & 2 & 1 \end{bmatrix} = 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & -1 \\ 8 & 1 \end{vmatrix} - 5\begin{vmatrix} 2 & 1 \\ 8 & 2 \end{vmatrix}$$

$$= 1\{1+2\} - 3\{2+8\} - 5\{4-8\}$$

$$= -7$$

$$D_y = \begin{bmatrix} 2 & 1 & -5 \\ 1 & 2 & -1 \\ 0 & 8 & 1 \end{bmatrix} = 2\begin{vmatrix} 2 & -1 \\ 8 & 1 \end{vmatrix} - 1\begin{Bmatrix} 1 & -1 \\ 0 & 1 \end{Bmatrix} - 5\begin{Bmatrix} 1 & 2 \\ 0 & 8 \end{Bmatrix}$$

$$2\{2+8\} - 1(1-0) - 5(8-0)$$

$$= -21$$

$$D_z = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 8 \end{bmatrix} = 2\begin{vmatrix} 1 & 2 \\ 2 & 8 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 0 & 8 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$2(8-4) - 3(8-0) + 1(2-0) = -14$$

* Solving large system (2 upto ∞)

① direct Method

A- Gauss Elimination

B- Gauss - Jordan ⤳ [inverse]

C- LU - factorization ⤳ $[A] . \{x\} = \{C\}$

Lower upper

D- Thomas Methods ⤳ just for spicial matrix [Banded Matrix]

$$\begin{bmatrix} a & a & 0 \\ a & a & a \\ 0 & a & a \end{bmatrix}$$

mian diagonal وما يكون في صفر في ال

② interative method

We need gusses $x_0 , x_1 , x_2 \ldots\ldots$

A- Jacobi

B- Gauss - Seidel

Gauess Elimination

↳ two mian step

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}$$

$$\left| \begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array} \right|$$

① الغاء هدول العناصر ونجعلهم zero

① forword Elimination

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & a'_{32} & a'_{33} & c'_3 \end{bmatrix} \longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & a_{32} & a''_{33} & c''_3 \end{bmatrix}$$

* يتم تغيير العناصر في العمود الاول باستخدام الصف الاول
* يتم تغيير العنصر في العمود الثاني باستخدام الصف الثاني

$$x_3 = \frac{c''_3}{a''_{33}} \qquad x_2 = \frac{c'_2 \; x_3 \; a'_{23}}{a'_{22}}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$

ومنطلع x_1

② Backward substitution

لو عندنا upper بجيبر forward subst

بستخدمها للحذف

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ \boxed{a_{21}} & a_{22} & a_{23} & c_2 \\ \boxed{a_{31}} & a_{32} & a_{33} & c_3 \end{array}\right] \quad -a_{21}\left\{ \frac{a_{11}}{a_{11}} \quad \frac{a_{12}}{a_{11}} \quad \frac{a_{13}}{a_{11}} \quad \frac{c_1}{a_{11}} \right\} + R_2$$

$$\left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & \underbrace{a_{22} - \frac{a_{21}a_{12}}{a_{11}}}_{a'_{22}} & \underbrace{a_{23} - \frac{a_{21}a_{13}}{a_{11}}}_{a'_{23}} & \underbrace{c_2 - a_{21}\frac{c_1}{a_{11}}}_{c'_2} \\ a_{31} & a_{32} & a_{33} & c_3 \end{array}\right] \quad \left(-a_{31}\left\{ \frac{a_{11}}{a_{11}} \quad \frac{a_{12}}{a_{11}} \quad \frac{a_{13}}{a_{11}} \quad \frac{c_1}{a_{11}} \right\} + R_3\right)$$

$$\left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & \underbrace{a_{32} - a_{31}\frac{a_{12}}{a_{11}}}_{a'_{32}} & \underbrace{a_{33} - a_{31}\frac{a_{13}}{a_{11}}}_{a'_{33}} & \underbrace{c_3 - a_{31}\frac{c_1}{a_{11}}}_{c'_3} \end{array}\right] \quad \left(-a'_{32}\left\{ \frac{0}{a'_{22}} \quad \frac{a'_{22}}{a'_{22}} \quad \frac{a'_{23}}{a'_{22}} \quad \frac{c'_2}{a'_{22}} \right\}\right) + R_3$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & 0 & \underbrace{a'_{33} - a'_{32}\frac{a'_{23}}{a'_{22}}}_{a''_{33}} & \underbrace{c'_3 - a'_{32}\frac{c'_2}{a'_{22}}}_{c''_3} \end{array}\right]$$

$$\left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & 0 & a''_{33} & c''_3 \end{array}\right]$$

Subject :

Example

$$3x_1 - 2x_2 + x_3 = -10$$
$$2x + 6x_2 - 4x_3 = 44$$
$$-x_1 - 2x_2 + 5x_3 = -26$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 6 & -4 \\ -1 & -2 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 44 \\ -26 \end{Bmatrix}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 2 & 6 & -4 & 44 \\ -1 & -2 & 5 & -26 \end{array}\right] \quad -2\left\{\frac{3}{3} \quad \frac{-2}{3} \quad \frac{1}{3} \quad \frac{-10}{3}\right\} \; +$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 0 & 7{,}333 & -4{,}\overline{66} & 50{,}\overline{66} \\ -1 & -2 & 5 & -26 \end{array}\right] \quad \times 1\left\{\frac{3}{3} \quad \frac{-2}{3} \quad \frac{1}{3} \quad \frac{-10}{3}\right\} \; +$$

$$\begin{bmatrix} 3 & -2 & 1 & -10 \\ 0 & 7{,}333 & -4{,}\overline{66} & 50{,}\overline{66} \\ 0 & -2{,}\overline{66} & 5{,}\overline{33} & -29{,}\overline{33} \end{bmatrix} \quad 2{,}\overline{66}\left\{\frac{0}{7{,}33} \quad \frac{7{,}33}{7{,}33} \quad \frac{-4{,}\overline{66}}{7{,}33} \quad \frac{50{,}\overline{66}}{7{,}33}\right\} \; +$$

$$\begin{bmatrix} 3 & -2 & 1 & -10 \\ 0 & 7{,}333 & -4{,}\overline{66} & 50{,}\overline{56} \\ 0 & 0 & 3{,}6384 & -10{,}945 \end{bmatrix}$$

$$x_3 = \frac{-10{,}945}{3{,}6384} = -3$$

$$x_2 = \frac{50{,}66 + 4{,}66(-3{,}008)}{7{,}33}$$

$$\boxed{x_2 = 5}$$

$$\boxed{x_1 = 1} \qquad x_1 = -10 - (-3) + 2(5)$$

Subject:

Gauss Jordan Method

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix}$$

Forward & Backward

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} C_1''' \\ C_2''' \\ C_3''' \end{Bmatrix}$$

بحذف هدول باستخدام Gauss elimination

$x_1 = C_1'''$

$x_2 = C_2'''$ ~ الحل بطلع مباشرة

$x_3 = C_3'''$

Example

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 2 & 6 & -4 & 44 \\ -1 & -2 & 5 & -26 \end{array}\right]$$ using Gauss Jordan

$+$

$-2 * \left\{ \frac{3}{3} \quad \frac{-2}{3} \quad \frac{1}{3} \quad \frac{-10}{3} \right\}$

$1 * \left\{ \frac{3}{3} \quad \frac{-2}{3} \quad \frac{1}{3} \quad \frac{-10}{3} \right\}$

$$\begin{bmatrix} 3 & -2 & 1 & -10 \\ 0 & 7,\overline{33} & -4,\overline{66} & 50,\overline{66} \\ 0 & -2,\overline{66} & 5,\overline{33} & -29,\overline{33} \end{bmatrix}$$ $+$

$2,\overline{66} * \left\{ \frac{0}{7,33} \quad \frac{7,\overline{33}}{7,33} \quad \frac{-4,\overline{66}}{7,33} \quad \frac{50,\overline{66}}{7,33} \right\}$

$$\begin{bmatrix} 3 & -2 & 1 & -10 \\ 0 & 7,\overline{33} & -4,\overline{66} & 50,65 \\ 0 & 0 & 3,638 & -10,945 \end{bmatrix}$$ ~ للتبع →

Subject : ___

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 0 & 7.\overline{33} & -4.\overline{66} & 50.\overline{66} \\ 0 & 0 & 3.638 & -10.945 \end{array}\right]$$

$\frac{R_1}{a_{11}} \left\{ \frac{3}{3} \quad \frac{-2}{3} \quad \frac{1}{3} \quad \frac{-10}{3} \right\} - a_{12} R_2$

① $\frac{R_2}{a_{22}} \left\{ \frac{0}{7.\overline{33}} \quad \frac{7.33}{7.33} \quad \frac{-4.\overline{66}}{7.\overline{33}} \quad \frac{50.66}{7.33} \right\} - a_{23} R_3$

$\frac{R_3}{a_{33}} \left\{ \frac{0}{3.638} \quad \frac{0}{3.638} \quad \frac{3.638}{3.638} \quad \frac{-10.945}{3.638} \right\}$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -0.\overline{66} & 0.\overline{33} & -3.\overline{33} \\ 0 & 1 & -0.639 & 6.911 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

~~$R_2 - a_{23} R_3$~~ ① $R_2 - a_{23} R_3$

~~0.639~~ ② $R_1 - a_{12} R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.33 & -0.03 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

③ $R_1 - a_{13} R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

Subject : ______ / /

* Find A^{-1} using Gauss-Jordan

$$[A]\{X\} = \{C\} \rightsquigarrow \{X\} = \frac{\{C\}}{[A]}$$

$$\{X\} = [A]^{-1}\{C\}$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & C_1 \\ a_{21} & a_{22} & a_{23} & C_2 \\ a_{31} & a_{32} & a_{33} & C_3 \end{array}\right]$$

بشيلهم وبحط identity matrix تكون
نفس الـ size للـ matrix
الي بدي اطلعلها inverse

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & a_{13}^{-1} \\ 0 & 1 & 0 & a_{21}^{-1} & a_{22}^{-1} & a_{23}^{-1} \\ 0 & 0 & 1 & a_{31}^{-1} & a_{32}^{-1} & a_{33}^{-1} \end{array}\right]$$

Example

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 6 & -4 \\ -1 & -2 & 5 \end{bmatrix} \qquad C = \begin{Bmatrix} -10 \\ 44 \\ -26 \end{Bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 3 & -2 & 1 & 1 & 0 & 0 \\ 2 & 6 & -4 & 0 & 1 & 0 \\ -1 & -2 & 5 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & -0,\overline{666} & 0,\overline{333} & 0,\overline{33} & 0 & 0 \\ 0 & 7,333 & -4,666 & -0,\overline{66} & 1 & 0 \\ 0 & -2,666 & 5,333 & 0,\overline{33} & 0 & \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & -0,09091 & 0,272727 & 0,09090 & 0 \\ 0 & 1 & -0,6363 & -0,09091 & 0,13636 & 0 \\ 0 & 0 & 3,6363 & 0,09091 & 0,36363 & 0 \end{bmatrix}$$

يتبع

Subject

$$\langle I \rangle \qquad\qquad [A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.275 & 0.1 & 0.025 \\ 0 & 1 & 0 & -0.075 & 0.2 & 0.025 \\ 0 & 0 & 1 & 0.025 & 0.1 & 0.025 \end{array}\right]$$

$$\begin{bmatrix} 0.275 & 0.1 & 0.025 \\ -0.075 & 0.2 & 0.175 \\ -0.025 & 0.1 & 0.275 \end{bmatrix} \begin{Bmatrix} -10 \\ 44 \\ -26 \end{Bmatrix}$$

$$x_1 = 0.275 * -10 + 0.1\,(44) + 0.025\,(-26) = 1$$

$$x_2 = (-0.075 * -10) + 0.2\,(44) + 0.175\,(-26) = 5$$

$$x_3 = (0.025 * -10) + (0.1 * 44) + 0.175\,(-26) = -3$$

Subject : / /

Thomas Method

$\hookrightarrow$ Banded matrix بتحل بس

(يجب الالتزام بالتسمية)

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}\begin{bmatrix} \overset{d_1}{a_{11}} & \overset{a_1}{a_{12}} & 0 & 0 \\ \overset{b_2}{a_{21}} & \overset{d_2}{a_{22}} & \overset{a_2}{a_{23}} & 0 \\ 0 & \overset{b_3}{a_{32}} & \overset{d_3}{a_{33}} & \overset{a_3}{a_{34}} \\ 0 & 0 & \overset{b_4}{a_{43}} & \overset{d_4}{a_{44}} \end{bmatrix}$$

$$r_1 = d_1 X_1 + a_1 X_2$$
$$r_2 = b_2 X_1 + d_2 X_2 + a_2 X_3$$
$$r_3 = b_3 X_2 + d_3 X_3 + a_3 X_4$$
$$r_4 = b_4 X_3 + d_4 X_4$$

No zero's

After know all unknown

$$X_n = R_n$$
$$X_i = R_i - A_i X_{(i+1)}$$
$$X_3 = R_3 - A_3 X_4$$

* for Row No. 1

$$A_1 = \frac{a_1}{d_1} \quad , \quad R_1 = \frac{r_1}{d_1}$$

* Row 2 ...... n-1

$$A_i = \frac{a_i}{d_i - b_i A_{(i-1)}} \qquad R_i = \frac{r_i - b_i R_{(i-1)}}{d_i - b_i A_{(i-1)}}$$

2^{nd} row $\quad A_2 = \dfrac{a_2}{d_2 - b_2 A_2} \qquad R_2 = \dfrac{r_2 - b_2 R_1}{d_2 - b_2 A_1}$

for last row (n)

$$A_n = 0 \qquad R_n = \frac{r_n - b_n R_{(n-1)}}{d_n - b_n A_{(n-1)}}$$

$$A_4 = 0 \qquad R_4 = \frac{r_4 - b_4 R_{(3)}}{d_4 - b_4 A_3}$$

Subject : ______ / /

Example خارجي

$$\begin{bmatrix} 2,04 & -1 & 0 & 0 \\ -1 & 2,04 & -1 & 0 \\ 0 & -1 & 2,04 & -1 \\ 0 & 0 & -1 & 2,04 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{Bmatrix} 40,8 \\ 0,8 \\ 0,8 \\ 200,8 \end{Bmatrix}$$

* This is tridiagonal matrix so I can solve by Thomas method

for first Row

$$A_1 = \frac{a_1}{d_1} = \frac{-1}{2,04} = -,49 \quad R_1 = \frac{r_1}{d_1} = \frac{40,8}{2,04} = 20$$

for R_2

$$A_2 = \frac{a_2}{d_2 - b_2 A_1} = \frac{-1}{2,04-(-1)(-,49)} = -0,645$$

$$R_2 = \frac{r_2 - b_2 R_1}{d_2 - b_2 A_1} = \frac{0,8-(-1)(20)}{2,04-(-1)(-0,49)} = 13,419$$

for R_3

$$A_3 = \frac{a_3}{d_3 - b_3 A_2} = \frac{-1}{2,04-(-1)(-0,645)} = -0,7168$$

$$R_3 = \frac{r_3 - b_3 R_2}{d_3 - b_3 A_2} = \frac{0,8-(-1)(13,419)}{2,04-(-1)(-0,7168)} = 10,745$$

for R_4 $\quad A_4 = 0$

$$R_4 = \frac{r_4 - b_4 R_3}{d_4 - b_4 A_3} = \cancel{98,43}\ 159,87$$

Subject :

$T_4 = 159.87$

$T_3 = R_3 - A_3 T_4 = 10.745 - (-0.7168)(159.87) = 124.59$

$T_2 = R_2 - A_2 T_3 = 13.419 - (-0.645)(124.59) = 93.77$

$T_1 = R_1 - A_1 T_2 = 20 - -0.49(93.77) = 95.96$

LU factorization Method

↳ (lower - upper)

Step 1 Decomposition of the orginal matrix

$[A] \longrightarrow [L].[U]$ بحيث لما أضرب $L * U$ يعطيني $[A]$

$$\begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \qquad \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix}$$

$L_{11} = 1$, $L_{22} = 1$, $L_{33} = 1$, $L_{44} = 1$

$+\frac{a_{21}}{a_{11}}$ $\quad \frac{a'_{32}}{a'_{22}}$ $\quad \frac{a''_{43}}{a''_{33}}$

$+\frac{a_{31}}{a_{11}}$ $\quad \frac{a'_{42}}{a'_{22}}$

$\frac{a_{41}}{a_{11}}$

↓

Gauss Elimination عن طريق $[A]$

To cheak : $[L] * [U] = [A]$

* ناتج ضرب $[L].[U]$ رح يعطينا الـ original matrix

Subject : ________ / /

Example :-

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 6 & -4 \\ -1 & -2 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 44 \\ -26 \end{Bmatrix}$$

* we use Gauss eléminition (step ①)

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 7{,}33 & -4{,}66 \\ 0 & -2{,}666 & 5{,}333 \end{bmatrix}$$

$L_{21} = \frac{2}{3} = 0{,}667$ $L_{31} = \frac{-1}{3} = -0{,}333$

$L_{32} = \frac{2{,}666}{7{,}33} = -0{,}3635$

$$[U] = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 7{,}33 & -4{,}665 \\ 0 & 0 & 3{,}6363 \end{bmatrix} \qquad [L] = \begin{bmatrix} 1 & 0 & 0 \\ 0{,}667 & 1 & 0 \\ -0{,}333 & -0{,}3635 & 1 \end{bmatrix}$$

step 2 to find solution

$[L]\{D\} = \{C\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0{,}667 & 1 & 0 \\ 0{,}333 & -0{,}3635 & 1 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 44 \\ -26 \end{Bmatrix}$$

$D_1 = -10$

$D_2 = 44 - 0{,}667(-10)$

$= 50{,}67$

$D_3 = -26 - 0{,}33(-10) + 0{,}3635(50{,}67)$

$D_3 = -10{,}9041$

Subject :

$[U]\{X\} = \{D\}$

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 7.33 & -4.66 \\ 0 & 0 & 3.6363 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 50.667 \\ -10.9091 \end{Bmatrix}$$

$$X_3 = \frac{-10.9091}{3.6363} = -3$$

$$X_2 = \frac{50.667 + \cancel{4.66}(-3)}{7.33} = 5$$

$$X_1 = \frac{-10 + 2(\cancel{5}) - (1)(-3)}{3} = 1$$

step ③ to find inverse of [A]

① $[L]\{D^{①}\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$ → $\begin{Bmatrix} D_{11} \\ D_{21} \\ D_{31} \end{Bmatrix}$

② $[L]\{D^{②}\} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$ → $\begin{Bmatrix} D_{12} \\ D_{22} \\ D_{32} \end{Bmatrix}$

③ $[L]\{D^{③}\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$ → $\begin{Bmatrix} D_{13} \\ D_{23} \\ D_{33} \end{Bmatrix}$

الناتج → $\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$

④ $[U]\{X^{-1}\} = \{D^{①}\}$ → $\begin{Bmatrix} X_{11}^{-1} \\ X_{21}^{-1} \\ X_{31}^{-1} \end{Bmatrix}$

⑤ $[U]\{X^{-1}\} = \{D^{②}\}$ → $\begin{Bmatrix} X_{12}^{-1} \\ X_{22}^{-1} \\ X_{32}^{-1} \end{Bmatrix}$

⑥ $[U]\{X^{-1}\} = \{D^{③}\}$ → $\begin{Bmatrix} X_{13}^{-1} \\ X_{23}^{-1} \\ X_{33}^{-1} \end{Bmatrix}$

الناتج $\begin{bmatrix} X_{11}^{-1} & X_{12}^{-1} & X_{13}^{-1} \\ X_{21}^{-1} & X_{22}^{-1} & X_{23}^{-1} \\ X_{31}^{-1} & X_{32}^{-1} & X_{33}^{-1} \end{bmatrix}$

Subject :

Iterative Method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = C_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = C_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = C_3$$

$$x_1 = \frac{C_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

x_1^0, x_2^0, x_3^0 (initial gausses)

$$x_2 = \frac{C_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$$

بروح بطلع
$x_1^{(1)}, x_2^{(1)}\ x_3^{(1)}$

$$x_3 = \frac{C_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

$$\left|\frac{x_1^{(1)} - x_1^0}{x_1^{(1)}}\right| \leq Tol \quad \therefore \text{goto cheak } x_2$$

$$\left|\frac{x_2^{(1)} - x_2^0}{x_2^{(1)}}\right| \leq Tol \quad \therefore \text{goto cheak } x_3$$

$$\left|\frac{x_3^{(1)} - x_3^0}{x_3^{(1)}}\right| \leq Tol \quad \therefore x_1^{(1)}, x_2^{(1)}, x_3^{(1)} \text{ is solution}$$

* اذا كان كلهم اصغر من Tol هيك بقدر اعتبرهم حل
أما اذا ما كانوا حتى لو وحدة منهم بروح بعمل
2 iteration

Subject :

Jacobi $\quad x_1^{0}, x_2^{0}, x_3^{0}$ (initial gausses all them I need)

$$x_1^{1} = \frac{c_1 - a_{12}x_2^{0} - a_{13}x_3^{0}}{a_{11}} \quad \text{(old)}$$

$$x_2^{1} = \frac{c_2 - a_{21}x_1^{0} - a_{23}x_3^{0}}{a_{22}} \quad \text{(old)}$$

$$x_3^{1} = \frac{c_3 - a_{31}x_1^{0} - a_{32}x_2^{0}}{a_{33}} \quad \text{(old)}$$

Gauss Seidel → (اسرع)

بس بحتاج هدول → x_2^{0}, x_3^{0}

$$x_1^{1} = \frac{c_1 - a_{12}x_2^{0} - a_{13}x_3^{0}}{a_{11}} \quad \text{(old)}$$

$$x_2^{1} = \frac{c_2 - a_{21}x_1^{1} - a_{23}x_3^{0}}{a_{22}} \quad \text{(mix (old, new))}$$

$$x_3^{1} = \frac{c_3 - a_{31}x_1^{1} - a_{32}x_2^{1}}{a_{33}} \quad \text{(new)}$$

* لازم اهتم بهاي الطرق بالترتيب عشان يطلع معي solution صح

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = C_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = C_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = C_3$$

$$\begin{bmatrix} |a_{11}| > |a_{12}| + |a_{13}| \\ a_{21} \quad |a_{22}| > |a_{23}| + |a_{21}| \\ |a_{31}| + |a_{32}| < |a_{33}| \end{bmatrix}$$

Example :-
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$
$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$(x_1^0, x_2^0, x_3^0 = 0) \quad 0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

* الحل دون الانتباه للترتيب :-

$$x_1^{1} = \frac{71.4 + 0.2x_2 - 10x_3}{0.3}$$

$$x_2^{1} = \frac{7.85 - 3x_1 + 0.2x_3}{-0.1}$$

$$x_3^{1} = \frac{-19.3 - 0.1x_1 - 7x_2}{-0.3}$$

Jacobi
$$x_1^{1} = \frac{71.4 + 0 + 0}{0.3} = 238$$
$$x_2^{1} = \frac{7.85}{-0.1} = -78.5$$
$$x_3^{1} = \frac{-19.3}{-0.3} = 64.33$$

يتبع

$X_1^{2)}$ 2nd iteration

$$X_1^{2)} = \frac{71,4 + 0,2(-78,5) - 10(64,33)}{0,3} = -1958,66$$

$$X_2^{2)} = \frac{7,85 - 3(238) + 0,2(64,33)}{-0,1} = 6932,84$$

$$X_3^{2)} = \frac{-19,3 - 0,1(238) - 7(-78,5)}{-0,3} = -1685,66$$

Gauss seidel

$$X_1^{1)} = 238$$

$$X_2^{1)} = \frac{7,85 - 3(238) + 0,2(0)}{-0,1} = 7061,5$$

$$X_3^{1)} = \frac{-19,3 - 0,1(238) - 7(7061,5)}{-0,3} = 164900,833$$

* أما الحل بعد الترتيب

$$3X_1 - 0,1X_2 - 0,2X_3 = 7,85$$

$$0,1X_1 + 7X_2 - 0,3X_3 = -19,3 \qquad (X_1^0, X_2^0, X_3^0 = 0)$$

$$0,3X_1 - 0,2X_2 + 10X_3 = 71,4$$

Jacobi

$$X_1 = \frac{7,85 + 0,1X_2 + 0,2X_3}{3} \qquad X_1^{1)} = \frac{7,85}{3} = 2,6166$$

$$X_2^{1)} = \frac{-19,3}{7} = -2,7575$$

$$X_2 = \frac{-19,3 - 0,1X_1 + 0,3X_3}{7} \qquad X_3^{1)} = \frac{71,4}{10} = 7,14$$

2nd iteration

$$X_3 = \frac{71,4 - 0,3X_1 + 0,2X_2}{10} \qquad X_1^{2)} = \frac{7,85 + 0,1(-2,75) + 0,2(7,14)}{3}$$

$$\boxed{X_1^{2)} = 3,0007}$$

$$\boxed{X_3^{2)} = 7,006} \qquad X_2^{2)} = \frac{-19,3 - 0,1(2,616) + 0,3(7,14)}{7}$$

$$\boxed{X_2^{2)} = -2,4}$$

Subject

Seidel $x_2^0 = 0$ $x_1^0 = 0$

$$x_1^{1)} = \frac{7{,}85}{3} = 2{,}6166$$

$$x_2^{1)} = \frac{-19{,}3 - 0{,}1(2{,}6166) + 0{,}3(0)}{7} = -2{,}7945$$

$$x_3^{1)} = \frac{71{,}4 - 0{,}3(2{,}6166) + 0{,}2(-2{,}7575)}{10} = 7{,}005003$$

2nd iteration

$$x_1^{2)} = \frac{7{,}85 + 0{,}1(-2{,}7945) + 0{,}2(7{,}005003)}{3} = 2{,}9905$$

$$x_2^{2)} = \frac{-19{,}3 - 0{,}1(2{,}9905) + 0{,}3(7{,}005003)}{7} = -2{,}4995$$

$$x_3^{2)} = \frac{71{,}4 - 0{,}3(2{,}9905) + 0{,}2(-2{,}4996)}{10} = 7{,}0002$$

System of nonlinear Equation

$$\left.\begin{aligned} f_1(x, y, z) &= 0 \\ f_2(x, y, z) &= 0 \\ f_3(x, y, z) &= 0 \end{aligned}\right\} \rightarrow$$

if the system is nonlinear

1. Newton's Method
2. fixed point

1) Newton's Method (are required initial gausses x_0, y_0, z_0)

step 1 find the Jacobi matrix

$$J(x, y, z) \begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} & \frac{df_1}{dz} \\ \frac{df_2}{dx} & \frac{df_2}{dy} & \frac{df_2}{dz} \\ \frac{df_3}{dx} & \frac{df_3}{dy} & \frac{df_3}{dz} \end{bmatrix}$$

step 2

$$J(x, y, z) \cdot \begin{Bmatrix} \Delta x \\ \Delta z \\ \Delta y \end{Bmatrix} = - \begin{Bmatrix} f_1(x_0, y_0, z_0) \\ f_2(x_0, y_0, z_0) \\ f_3(x_0, y_0, z_0) \end{Bmatrix}$$

corrections
(Gauss Elimination)

Step 3 find expected values

$$x^{1)} = x_0 + \Delta x \qquad \left| \frac{x^{1)} - x_0}{x^{1)}} \right| < Tol$$

$$y^{1)} = y_0 + \Delta y \qquad \left| \frac{y^{1)} - y_0}{y^{1)}} \right| < Tol$$

$$z^{1)} = z_0 + \Delta z \qquad \left| \frac{z^{1)} - z_0}{z^{1)}} \right| < Tol$$

* إذا وجدة
ما طلعت أصغر
من Tol
…

after the first iteration x^0 y^0 z^0

$$\left\{ \left|\frac{df_1}{dx}\right| + \left|\frac{df_1}{dy}\right| + \left|\frac{df_1}{dz}\right| \right\} < 1$$

تعويض x^0 y^0 z^0

$$\left\{ \left|\frac{df_2}{dx}\right| + \left|\frac{df_2}{dy}\right| + \left|\frac{df_2}{dz}\right| \right\} < 1$$

check I will find solution or not

تعويض x^1 y^1 z^1

$$\left\{ \left|\frac{df_3}{dx}\right| + \left|\frac{df_3}{dy}\right| + \left|\frac{df_3}{dz}\right| \right\} < 1$$

Example

$$f(x,y) = y^2 e^{2x} - \sin(x) \qquad x_0 = 0.8$$

$$f(x,y) = x^2 e^{3y} - \cos(y) \qquad y_0 = 0.3$$

$$J(x,y) = \begin{bmatrix} 2y^2 e^{2x} - \cos(x) & 2y e^{2x} \\ 2x e^{3y} \cancel{- \sin(x)} & 3x^2 e^{3y} + \sin(y) \end{bmatrix}$$

initial gausses نعوض في J *

$$J(x,y) = \begin{bmatrix} 0.194839 & 2.971819 \\ 3.93536 & \cancel{0.97788} \; 5.017452 \end{bmatrix}$$

$$D = -\begin{Bmatrix} f_1(x_0, y_0) \\ f_2(x_0, y_0) \end{Bmatrix} = -\begin{Bmatrix} -0.271588 \\ 0.618809 \end{Bmatrix}$$

following,

Subject :

$$\begin{bmatrix} 0.194839 & 2.971849 \\ 3.93536 & 5.017452 \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \begin{Bmatrix} 0.271503 \\ -0.61881 \end{Bmatrix}$$

$$\Delta y = \frac{-0.61881 - 3.93536(\Delta x)}{5.017452}$$

$$\Delta x = \frac{0.271503 - 2.971849\,(-0.12333 - 0.78433\Delta x)}{0.194839}$$

$$\Delta y = 0.110972 \qquad \Delta x = -0.298689$$

$$x^1 = 0.8 + -0.298689 = 0.501311$$

$$y^1 = 0.3 + 0.110972 = 0.410972$$

② fixed point

$$f_1(x, y, z) = 0 \qquad g_1(x, y, z) = x$$

$$f_2(x, y, z) = 0 \qquad g_2(x, y, z) = y$$

$$f_3(x, y, z) = 0 \qquad g_3(x, y, z) = z$$

seidel

$$x^{1)} = g_1(x_0, y_0, z_0)$$

$$y^{1)} = g_2(x^1, y_0, z_0)$$

$$z^{1)} = g_3(x^1, y^1, z_0)$$

Jacobi

$$x^{1)} = g_1(x_0, y_0, z_0)$$

$$y^{1)} = g_2(x_0, y_0, z_0)$$

$$z^{1)} = g_3(x_0, y_0, z_0)$$

$$\left|\frac{x^{1)} - x_0}{x_0}\right| < Tol$$

$$\left|\frac{y^{1)} - y_0}{y^{1)}}\right| < Tol$$

$$\left|\frac{z^{1)} - z_0}{z^{1)}}\right| < Tol$$

ازا وصلة
طلعت أكبر
من Tol
بعمل 2nd iteration

$$x^{2)} = g_1(x^{1)} y^{1)} z^{1)})$$

$$y^{2)} = g_2(x^{2)} y^{1)} z^{1)})$$

$$z^{2)} = g_3(x^{2)} y^{2)} z^{1)})$$

after the first ~~eq~~ iteration

$$\left|\frac{dg_1}{dx}\right| + \left|\frac{dg_1}{dy}\right| + \left|\frac{dg_1}{dz}\right| < 1$$

$$\left|\frac{dg_2}{dx}\right| + \left|\frac{dg_2}{dy}\right| + \left|\frac{dg_2}{dz}\right| < 1$$

$$\left|\frac{dg_3}{dx}\right| + \left|\frac{dg_3}{dy}\right| + \left|\frac{dg_3}{dz}\right| < 1$$

ازا هاد الشرط
ما تطبق
يا بغامر و بكمل حل
يا بغير initial gausses
يا بغير ال g

Subject :

Example $f_1(x, y) = x^3 + 10x - y - 5 = 0$

$f_2(x, y) = x + y^3 - 10y + 1 = 0$

$x_0 = 0.6 \qquad y_0 = 0.6$

$$x = \frac{y + 5 - x^3}{10} \qquad y = \frac{x + y^3 + 1}{10}$$

or or

$$x = (y + 5 - 10x)^{\frac{1}{3}} \qquad y = (10y - x - 1)^{\frac{1}{3}}$$

$$x^{1)} = \frac{0.6 - 5 - 0.6^3}{10} = 0.5384$$

$$y^{1)} = \frac{0.5384 - 0.6^3 + 1}{10} = 0.17544$$

$$x^{2)} = \frac{0.17544 + 5 - (0.5384)^3}{10} = 0.50194$$

$$y^{2)} = \frac{0.50194 + (0.17544)^3 + 1}{10} = 0.15073$$

Subject :

Chapter 4 Interpolation & Curve fitting

- Curve fitting

1) linear or Polynomial ⇝ using least square Method
2) Simple non-linear ↓
3) General non linear (Minimization of summation of square Error)

$$Error = (y_i - y_{x_i})$$

where y_i is the given value

y_{x_i} the calculated value

$$- S = \sum_{i=0}^{n} (\sigma)^2 = \sum_{i=0}^{n} (y_i - y_{x_i})^2$$

- Minimization the value of S $\left\{ \frac{ds}{d\,paramiters} \right\}$

1) linear Model

$$y(x_i) = A + Bx_i \qquad \sigma = y_i - A - Bx_i$$

x_i	$y(x_i)$
x_0	$y(x_0)$
x_1	$y(x_1)$
⋮	⋮

$$S = \sum_{i=0}^{1} (y_i - A - Bx_i)^2$$

$$\frac{ds}{dA} = 2\sum_{i=0}^{1} (y_i - A - Bx_i)(-1) = zero$$

$$\frac{ds}{dB} = 2\sum_{i=0}^{1} (y_i - A - Bx_i)(-x_i) = zero$$

$$\cancel{-2}\left(\sum_{i=0}^{1} y_i - \sum_{i=0}^{1} A - \sum_{i=0}^{1} Bx_i\right) = 0$$

$$\cancel{-2}\left(\sum y_i x_i - \sum Ax_i - \sum Bx_i^2\right) = 0$$

S I G M A

Subject:

$$y = ae^{bx}$$

$$S = \sum_{i=0}^{n} (y - ae^{bx})^2$$

$$\frac{ds}{da} = 0 = 2\sum_{i=0}^{n} (y_i - ae^{bx})(-e^{bx_i}) \quad X$$

لاتطلع
لأنه مابصير يطلع عندي في الاشتقاق ال parameter

* اذا طلع معي parameter . لازم اغير ال Model

$$\frac{ds}{db} = 2\sum_{i=0}^{n} (y_i - ae^{bx_i})(-ax_i e^{bx_i}) \quad X$$

$$y(x_i) = A + bx_i$$

$$S = \sum_{i=0}^{n} (y_i - A - bx_i)^2$$

$$\frac{ds}{dA} = 2\sum_{i=0}^{n} (y_i - A - bx_i)(-1) = 0$$

$$\frac{ds}{dA} = 0 = \sum y_i - \sum a - \sum bx_i$$

$$\frac{ds}{db} = 0 = \sum x_i y_i - \sum ax_i - \sum bx_i^2$$

number of

$$a\sum_{i=0}^{n}(1) + \sum x_i = \sum y_i \rightsquigarrow an + b\sum x_i = \sum y_i$$

$$a\sum_{i=0}^{n} x_i + b\sum_{i=0}^{n} x_i^2 = \sum x_i y_i$$

معطى

x_i	y_i	x_i^2	$x_i y_i$
x_0	$y(x_0)$	x_0^2	$x_0\, y(x_0)$
x_1	$y(x_1)$	x_1^2	$x_1\, y(x_1)$
x_2	$y(x_2)$	x_2^2	$x_2\, y(x_2)$
x_3	$y(x_3)$	x_3^2	$x_3\, y(x_1)$
$\sum x_i$	$\sum y_i$	$\sum x_i^2$	$\sum x_i y_i$

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{Bmatrix} \sum y_i \\ \sum x_i y_i \end{Bmatrix}$$

* نحصل على ايجاد a,b

* أفضل fitting يكون باستخدام a,b

$$y = a + bx$$

Example

$n=5$

x	2	3	4	6	7	$\sum$ 22
y	4	7	5	8	6	30
x_i^2	4	9	16	36	49	114
$x_i y_i$	8	21	20	48	42	139

$$\begin{bmatrix} 5 & 22 \\ 22 & 114 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} 30 \\ 139 \end{Bmatrix}$$

$$-22 * \left\{ \frac{5}{5} \quad \frac{22}{5} \quad \frac{30}{5} \right\}$$

$$\left[\begin{array}{cc|c} 5 & 22 & 30 \\ 0 & 17{,}2 & 7 \end{array}\right]$$

$$b = \frac{7}{17{,}2} = 0{,}406976$$

$$a = \frac{30 - 22(0{,}406976)}{5}$$

$$a = 4{,}2093056$$

$$\boxed{y = 4{,}2093056 + 0{,}40697x}$$

Subject :

Polynomial of 2^{nd} order

$$y(x_i) = A + Bx_i + Cx_i$$

$$0 = y_i - A - Bx_i - Cx_i^2$$

$$S = \sum (y_i - A - Bx_i - Cx_i^2)^2$$

$$\frac{ds}{dA} = 2\sum_{i=0}^{n} (y_i - A - Bx_i - Cx_i^2)(-x_i)$$

$$a\sum_{i=0}^{n}(1) + b\sum_{i=0}^{n} x_i + C\sum_{i=0}^{n} x_i^2 = \sum_{i=0}^{n} y_i$$

$$a\sum_{i=0}^{n} x_i + b\sum_{i=0}^{n} x_i^2 + c\sum_{i=0}^{n} x_i^3 = \sum_{i=0}^{n}$$

$$a\sum_{i=0}^{n} x_i^2 + b\sum_{i=0}^{n} x_i^3 + c\sum_{i=0}^{n} x_i^4 = \sum_{i=0}^{n} x_i^2 y_i$$

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
$\sum x_i$	$\sum y_i$	$\sum x_i^2$	$\sum x_i^3$	$\sum x_i^4$	$\sum x_i y_i$	$\sum x_i^2 y_i$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{Bmatrix}$$

* Polynomial for 4^{th} order

$$y = A + Bx_i + Cx_i^2 + Dx_i^3 + Ex_i^4$$

Just for polynomial

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 & \sum x_i^7 \\ \sum x_i^4 & \sum x_i^5 & \sum x_i^6 & \sum x_i^7 & \sum x_i^8 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \\ E \end{Bmatrix} = \begin{Bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \sum x_i^3 y_i \\ \sum x_i^4 y_i \end{Bmatrix}$$

* Simple non linear (Linear بقدر احولهم)

x_i	x_0	x_1	x_3
y_i	$f(x_0)$	$f(x_1)$	$f(x_3)$

$$y = a e^{bx}$$

$$\ln y = \ln a + bx_i$$

$$Y = A + bx_i$$

$$\sigma = Y_i - A - bx_i$$

$$S = \sum_{i=0}^{n} (Y_i - A - bx_i)^2$$

$$\frac{ds}{dA} = 2\sum (Y_i - A - bx_i)(-1) \rightsquigarrow A\sum(1) + b\sum x_i = \sum Y_i$$

$$\frac{ds}{dB} = 2\sum (Y_i - A - bx_i)(-x_i) \rightsquigarrow A\sum x_i - b\sum x_i^2 = \sum x_i Y_i$$

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{Bmatrix} A \\ b \end{Bmatrix} = \begin{Bmatrix} \sum Y_i \\ \sum x_i Y_i \end{Bmatrix}$$

Subject :

x_i	y_i	$Ln y = Y$	x_i^2	$x_i Y_i$
⋮	⋮	⋮	⋮	⋮
$\sum x_i$		$\sum Y_i$	$\sum x_i^2$	$\sum x_i Y_i$

Example ٢

$$y = a e^{bx}$$

$$Ln y = Ln a + b x_i \rightsquigarrow Y_i = A + b x_i$$

x_i	y_i	$Ln y = Y_i$	x_i^2	$x_i Y_i$
2	4	1,3862	4	2,7724
3	7	1,9459	9	5,8377
4	5	1,6094	16	6,4376
6	8	2,0794	36	12,4764
7	6	0,77815	49	5,44705
22		7,79905	114	32,9711

$$\begin{bmatrix} 5 & 22 \\ 22 & 114 \end{bmatrix} \begin{Bmatrix} A \\ b \end{Bmatrix} = \begin{Bmatrix} \sum Y_i \\ \sum x_i Y_i \end{Bmatrix}$$

$$\begin{bmatrix} 5 & 22 \\ 22 & 114 \end{bmatrix} \begin{Bmatrix} A \\ b \end{Bmatrix} = \begin{Bmatrix} 7,79905 \\ 32,9711 \end{Bmatrix} \quad -22\times\left\{ \frac{5}{5} \quad \frac{22}{5} \quad \frac{7,79905}{5} \right.$$

$$\left[\begin{array}{cc|c} 5 & 22 & 7,79905 \\ 0 & 17,2 & -1,34472 \end{array}\right]$$

$$\boxed{b = \cancel{-1,34472} = -0,07818}$$

$$A = 1,9038 \rightsquigarrow Ln a = 1,9038$$

$$\boxed{a = 6,7113}$$

$$\boxed{y = 6,7113 e^{-0,07818x}}$$

Subject :

other Model

$$y = e^{ax+b}$$

$$Lny = ax + b$$

$$Y_i = ax_i + b$$

$$y = \frac{1}{a + bx_i} \qquad Y_i = \frac{1}{y}$$

$$\frac{1}{y} = a + bx_i$$

$$Y_i = a + bx_i$$

S I G M A N O T E

Subject :

other Model

$$y = e^{ax+b}$$
$$\ln y = ax + b$$
$$Y_i = ax_i + b$$

$$y = \frac{1}{a + bx_i}$$
$$\frac{1}{y} = a + bx_i$$
$$Y_i = a + bx_i$$
$$Y_i = \frac{1}{y}$$

$$F = c_1 + c_2 x + c_3 y$$
$$\sigma = F_i - c_1 - c_2 x_i - c_3 y$$
$$S = \sum (F_i - c_1 - c_2 x_i - c_3 y)^2$$

$$\frac{ds}{dc_1} = 2 \sum (F_i - c_1 - c_2 x_i - c_3 y)(-1)$$

$$\frac{ds}{dc_2} = 2 \sum (F_i - c_1 - c_2 x_i - c_3 y)(-x_i)$$

$$\frac{ds}{dc_3} = 2 \sum (F_i - c_1 - c_2 x_i - c_3 y_i)(-y_i)$$

$$\begin{bmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum y_i^2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{bmatrix} \sum F_i \\ \sum F_i X_i \\ \sum F_i x_i^2 \end{bmatrix}$$

$$\cancel{F_i = c_1 + c_2 x_i + c_3 y_i + c_4}$$

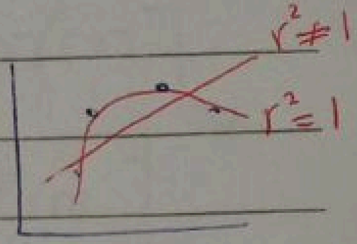
Subject :

$$\Sigma_i = c_1 + c_2 x_i + c_3 y_i + c_4 x_i^2 + c_5 y_i^2 + c_6 x_i y_i$$

$$\begin{bmatrix} n & \Sigma x & \Sigma y & \Sigma x_i^2 & \Sigma y_i^2 & \Sigma xy \\ \Sigma x & \Sigma x^2 & \Sigma xy & \Sigma x^3 & \Sigma xy^2 & \Sigma x^2 y \\ \Sigma y & \Sigma x_i y_i & \Sigma y^2 & \Sigma x^2 y & \Sigma y^3 & \Sigma xy^2 \\ & & & & & \\ & & & & & \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} = \begin{pmatrix} \Sigma z \\ \Sigma xz \\ \Sigma yz \\ \\ \\ \end{pmatrix}$$

Subject :

coeficient of Determination (r^2)



$$0 \leq r^2 \leq 1$$

$$\boxed{r^2 = \frac{S_t - S_r}{S_t}}$$

$$S_t = \sum (y_i - \bar{y})^2$$

$$S_r = \sum (y_i - y_{calculation})$$

3) Non linear Regression

$$y = a e^{bx}$$

(we need intial gausses a_0, b_0)

x_i	y_i
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$

step1 → find the Z matrix

$$[Z] = \begin{array}{cc} \frac{dy}{da_0} & \frac{dy}{db_0} \\ \end{array}$$
$$\begin{bmatrix} e^{b_0 x_0} & a x_0 e^{b x_0} \\ e^{b x_1} & a x_1 e^{b x_1} \\ e^{b_0 x_2} & a x_2 e^{b x_2} \\ e^{b_0 x_3} & a x_3 e^{b x_3} \end{bmatrix}_{n \times 2}$$

عدد ال parameter = عدد ال column

n = number of point

step2 Find {D} where

$$\{D\} = \begin{Bmatrix} f(x_0) - a_0 e^{b x_0} \\ f(x_1) - a_0 e^{b_0 x_1} \\ f(x_2) - a_0 e^{b_0 x_2} \\ f(x_3) - a_0 e^{b_0 x_3} \end{Bmatrix}$$

Subject : / /

step 3 Find $[Z]^T_{2\times4}[Z]_{4\times2} \longrightarrow \left[[Z]^T[Z]\right]_{2\times2}$

$$[Z]^T_{2\times4}\{D\}_{4\times1} \longrightarrow \left[[Z]^T\{D\}\right]_{2\times1}$$

step 4 solve the following system

$$\left[[Z]^T[Z]\right]_{2\times2}\begin{Bmatrix}\Delta a\\ \Delta b\end{Bmatrix} = \left\{[Z]\{D\}\right\}_{2\times1}$$

step 5 correct the initial gusses

$a_1 = a_0 + \Delta a$

$b_1 = b_0 + \Delta b$

stop when only $\left|\dfrac{a_1 - a_0}{a_1}\right| * 100\% < Tol$

$$\left|\frac{b_1 - b_0}{b_1}\right| * 100\% < Tol$$

Subject : / /

Interpolation

1) Lagrange
2) Newton's interpolation

* الطريقتين لازم يطلع نفس الاجابات

* Lagrange

$$f_n(x) = \sum_{i=0}^{n} L_i(x) \cdot f(x_i)$$

↳ order of interpolation

x_i	$f(x_i)$
⋮	
x_0	$f(x_0)$
x_1	$f(x_1)$
⋮	

* For first order $n=1$

$$f_1(x) = \sum_{i=0}^{1} L_i(x)\, f(x_i)$$
$$= L_0(x)\, f(x_0) + L_1(x)\, f(x_1)$$

* $L_i(x)$ is the Lagrange term

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}} \frac{x - x_j}{x_i - x_j}$$

multiplecation

$\cancel{f_1(x) = \sum_{i=0}^{1}}$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \qquad L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$\boxed{x_0 \leq x \leq x_1}$$

Subject: ______ / /

at $X = X_0$

$$f_1(x) = \frac{X_0 - X_1}{X_0 - X_1} f(x_0) + \frac{X_0 - X_0}{X_1 - X_0} f(x_1)$$

$$\underbrace{\frac{X_0 - X_1}{X_0 - X_1}}_{=1} \qquad \underbrace{\frac{X_0 - X_0}{X_1 - X_0}}_{=0}$$

$\# \; f_1(x_0) = f(x_0)$ * اذا الاقتران يمر من هذه النقطة

$$f(x_1) = \cancel{\left(\frac{X_1 - X_1}{X_0 - X_1}\right) f(x_0)} + \frac{X_1 - X_0}{X_1 - X_0} f(x_1)$$

$$\left(\frac{X_1 - X_1}{X_0 - X_1}\right) \downarrow = 0 \qquad \frac{X_1 - X_0}{X_1 - X_0} \downarrow = 1$$

$\# \; f(x_1) = f(x_1)$ * اذا نقطة مِن الاقتران

* $n = 2$ 2^{nd} order

$$f_2(x) = \sum_{i=0}^{2} L_i(x) f(x_i)$$

$$= L_0(x) f(x_0) + L_1^3(x) L(x_1) + L_2(x) f(x_1)$$

$$L_0(x) = \left(\frac{X - X_1}{X_0 - X_1}\right)\left(\frac{X - X_2}{X_0 - X_2}\right)$$

$$L_1(x) = \left(\frac{X - X_0}{X_1 - X_0}\right)\left(\frac{X - X_2}{X_1 - X_2}\right)$$

$$L_2(x) = \left(\frac{X - X_0}{X_2 - X_0}\right)\left(\frac{X - X_1}{X_2 - X_1}\right)$$

$$f_2(x) = \left(\frac{X - X_1}{X_0 - X_1}\right)\left(\frac{X - X_2}{X_0 - X_2}\right) f(x_0) + \left(\frac{X - X_0}{X_1 - X_0}\right)\left(\frac{X - X_2}{X_1 - X_2}\right) f(x_1)$$

$$+ \left(\frac{X - X_0}{\ldots}\right)\left(\frac{X - X_1}{\ldots}\right) f(x_2)$$

$$f_4(x) = \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)\left(\frac{x-x_3}{x_0-x_3}\right)\left(\frac{x-x_4}{x_0-x_4}\right) f(x_0)$$

$$+ \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right)\left(\frac{x-x_4}{x_1-x_4}\right) f(x_1)$$

$$+ \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right)\left(\frac{x-x_4}{x_2-x_4}\right) f(x_2)$$

$$+ \left(\frac{x-x_0}{x_3-x_0}\right)\left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right)\left(\frac{x-x_4}{x_3-x_4}\right) f(x_3)$$

$$+ \left(\frac{x-x_0}{x_4-x_0}\right)\left(\frac{x-x_1}{x_4-x_1}\right)\left(\frac{x-x_2}{x_4-x_2}\right)\left(\frac{x-x_3}{x_4-x_3}\right) f(x_4)$$

x_i	$f(x_i)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$
x_4	$f(x_4)$

* عدد ال x الي بحتاجها هي n+1

* Newton's Method

$$f_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \dots\dots\dots$$

$$f_1(x) = a_0 + a_1(x-x_0)$$

$$f_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$

at $x = x_0$

$$f(x) = a_0 + a_1(x-x_0)$$

$$f(x_0) = a_0 + a_1(x_0-x_0)$$

$$\boxed{a_0 = f(x_0)}$$

$$f(x_1) = f(x_0) + a_1(x_1-x_0)$$

$$\boxed{a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

$$f_1(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0) \quad \boxed{x_0 \le x \le x_1}$$

Subject :

$$f_2(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0) + a_2 (x - x_0)(x - x_1)$$

$$\boxed{a_2 = \frac{\left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right) - \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)}{x_2 - x_0}}$$

$$f_2(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0) + \quad (x - x_0)(x - x_1)$$

$$\boxed{a_3 = \frac{\left(\frac{\left(\frac{f(x_3) - f(x_2)}{x_3 - x_2}\right) - \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right)}{x_3 - x_1}\right) - \left(\frac{\left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right) - \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)}{x_2 - x_0}\right)}{x_3 - x_0}}$$

3) Devided different Table

* هاد الجدول بساعدنا في طريقة Newton's

x_i	$f(x_i)$	1st DD	2nd DD
x_0	$f(x_0)$		
		$\frac{f(x_1) - f(x_0)}{x_1 - x_0}$	
x_1	$f(x_1)$		$\frac{\left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right) - \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)}{x_2 - x_0}$
		$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$	
x_2	$f(x_2)$		$\frac{\left(\frac{f(x_3) - f(x_2)}{x_3 - x_2}\right) - \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right)}{x_3 - x_1}$
		$\frac{f(x_3) - f(x_2)}{x_3 - x_2}$	
x_3	$f(x_3)$		

Subject :

Example

	x_i	$f(x_i)$
x_0	1	0
x_1	4	1,3852744
x_2	6	1,7917595

Lagrange

1^{st} order

$$f_1(x) = L_0(x)\, f(x_0) + L_1(x) . f(x_1)$$

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$= \frac{x-4}{1-4}(0) + \frac{x-1}{4-1}(1,3852744)$$

$$f(2) = \frac{2-4}{1-4}(0) + \frac{2-1}{4-1}(1,3852744) = 0,462098133$$

$$f_2(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$$

$$= \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) f(x_1)$$

$$+ \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) f(x_2)$$

$$= \left(\frac{x-1}{1-4}\right)\left(\frac{x-6}{1-6}\right)(0) + \left(\frac{x-1}{4-1}\right)\left(\frac{x-6}{4-6}\right)(1,3852744)$$

$$+ \left(\frac{x-1}{6-1}\right)\left(\frac{x-4}{6-4}\right)(1,791759)$$

$$f(2) = 0,56580437$$

Subject :

Newton's

x_i	$f(x_i)$
1	0
4	1,3862744
6	1,7917595

$$f_1(x) = a_0 + a_1(x - x_0)$$

$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$f(2) = 0 + \frac{1,3862744 - 0}{4 - 1}(2 - 1)$$

$$= 0,462098133$$

$$f_2(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \left(\frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_2}}{x_2 - x_0}\right)$$

$(x - x_0)$

$(x - x_1)$

$$f_2(2) = 0 + \frac{1,3862744 - 0}{4 - 1}(2 - 1) + \frac{\left(\frac{1,7917595 - 1,3862744}{6 - 4}\right) - \left(\frac{1,3862744 - 0}{4 - 1}\right)}{6 - 1}$$

$$* (2 - 1)(2 - 4)$$

$$= 0,5658[illegible]$$

Subject : / /

Differential & Integration

Differentiation

equally — unequally

ΔX متساوية

1) Equally spaced data points

A) forward direction

Taylor series

$$f(x_{i+1}) = f(x_i) + f'(x_i)\,h + \frac{f''(x_i)}{2} h^2 \ldots\ldots$$

متساويان

x_i	$f(x_i)$
0,1	
0,2	
0,3	
0,4	
0,5	

Δx_1, Δx_2, Δx_3, Δx_4

$$\boxed{f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)}$$

x	$f(x)$
1	
1,4	
1,8	
2,2	

$\Delta x = 0,4$

$$f'(1{,}4) = \frac{f(1{,}8) - f(1{,}4)}{0{,}4}$$

x_{i+1} — x_i — x_{i+1} 0,4 — x_{i+2} 0,4

$$\boxed{f'(x_i) = \frac{-f(x_{i+2}) - 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2)}$$

$$f'(1{,}4) = \frac{-f(2{,}2) - 4f(1{,}8) - 3f(1{,}4)}{2(0{,}4)}$$

- for word

x_{i-2} x_{i-1} x_i x_{i+1} x_{i+2}

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2)$$

* if the error of $O(h)$ then halving the interval step size will reduce the error by 50%

- Back word

$$f'(x) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h} + O(h^2)$$

- central

x_{i-1} x_i x_{i+1}

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h} + O(h^4)$$

* if the error of $O(h^2)$, then halving the step size the error will reduce by 75%

Subject : ______ / /

x_i	$f(x_i)$
Just → forword $\boxed{1}$	
2	
3	
4	
just ← Backword $\boxed{5}$	

* Un equally Spaced Data Points

if $f'(x)$ is required ~~so~~ at least a 2^{nd} order function must be obtained {langrage, Newton's }

$$f_2(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) f(x_1)$$

$$+ \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) f(x_2)$$

$$\boxed{f_2'(x) = \frac{2x - x_{i+1} - x_i}{(x_{i-1} - x_i)} y(x_{i-1}) + \frac{2x - x_{i+1} - x_{i-1}}{(x_i - x_{i-1})(x_i - x_{i+1})} y(x_i) + \frac{2x - x_i - x_{i-1}}{(x_{i+1} - x_{i-1})(x_{i+1} - x_{i-1})} y(x_{i+1})}$$

Subject : / /

Example: Forword & Backword $\{o(h), o(h^2)\}$

central $(o(h^2), o(h^4))$

for $f'(\frac{\pi}{4})$ of the function $y = \sin(x)$

using $\Delta x = \frac{\pi}{12}$

$y' = \cos(x)$ $y'(\frac{\pi}{4}) = 0,7071$ True answer

x	f(x)
$\frac{\pi}{3}$	0,866025
$\frac{\pi}{4}$	0,70710678
$\frac{\pi}{6}$	0,5

for word

$$f'(x) = \frac{f(x_{i+1}) - f(x_i)}{h} + o(h)$$

$$= \frac{0,866025 - 0,70710678}{\frac{\pi}{12}} + o(h)$$

$$= 0,607022$$

$\frac{5\pi}{12}$	0,965925
$\frac{\pi}{3}$	0,866025
$\frac{\pi}{4}$	0,70710678
$\frac{\pi}{6}$	0,5
$\frac{\pi}{18}$	0,258819

$$f'(x) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + o(h^2)$$

$$= \frac{-0,965925 + 4(0,866025) - 3(0,70710678)}{2(\frac{\pi}{12})}$$

$$= 0,7195872$$

Backword

$$f'(x) = \frac{f(x_i) - f(x_{i-1})}{h} + oh = \frac{0,70710678 - 0,5}{\frac{\pi}{12}}$$

$$= 0,791089$$

$$f'(x) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i+2})}{2h} = \frac{3(0,70710678) - 4(0,5) + 0,258819}{\frac{\pi}{6}}$$

$$= 0,726012$$

Subject :

Central $f'(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$

$$= \frac{0,866025 - 0,5}{\frac{\pi}{6}} = 0,699056 + O(h^2)$$

$$f'(x) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

$$= \frac{-0,965925 + 8(0,866025) - 8(0,5) + 0,258819}{\pi}$$

$$= 0,7069 + O(h^4)$$

* Richardson Extrapolation

$$f'(x_0)\Big|_{h_1=4} \frac{f(x_{i+4}) - f(x_0)}{4\,\Delta x} \qquad h_1 = 4$$

↳ Less Accurate

x_i	$f(x_i)$
x_0	
$\Delta x = 1$	
x_{i+1}	
$\Delta x = 1$	
x_{i+2}	
$\Delta x = 1$	
x_{i+3}	
$\Delta x = 1$	
x_{i+4}	

$$h_2 = \frac{1}{2} h_1 = \frac{1}{2}(4) = 2$$

$$f'(x_0)\Big|_{h_2=2} = \frac{f(x_{i+2}) - f(x_0)}{2} + O(h) \rightsquigarrow \text{أدق}$$

$$\boxed{D_{\substack{best\\ h_1/h_2}} = D\Big|_{h_2} + \frac{1}{\left(\frac{h_1}{h_2}\right)^2 - 1}\left[D\Big|_{h_2} - D\Big|_{h_1}\right] + O(h^2)}$$

h	$O(h)$	$O(h^2)$	$O(h^4)$	$O(h^5)$
$h_1 = 4$	$D\|_{h_1}$	$D\|_{h_1, h_2} = \frac{4}{3}D(h_2) - \frac{1}{3}D(h_1)$	$\frac{16}{15}D[h_2, h_1] - \frac{1}{15}D[h_1, h_2]$	
$h_2 = 2$	$D\|_{h_2}$	$\frac{4}{3}D(h_3) - \frac{1}{3}D(h_2)$		
$h_3 = \frac{1}{4}h_1$	$D\|_{h_3}$			

$$D = D(h_2) + \frac{1}{\left(\frac{h_1}{0.5h_1}\right)^2 - 1}\left[D(h_2) - D(h_1)\right]$$

Example $y = \ln x$ at $x = 4$ find $y'(4)$ using a step size of 2 and $h_2 = 1 = \frac{1}{2}h_1$, employ central differences of $O(h^2)$

	x_i	$f(x_i)$
x_{i-1}	2	0,693
x_i	4	1,3863
x_{i+1}	6	1,79176

$$D|_{h_1} = \frac{1,79176 - 0,693}{2*2} = 0,2747$$

add

3	1,099
5	1,6094

$$D|_{h_2} = \frac{1,6094 - 1,099}{2(1)} = 0,255 + O(h^3)$$

$$D|_{h_3} = \frac{1,5041 - 1,257}{2(0,5)} = 0,2513 + O(h^2)$$

add

3,5	1,257
4,5	1,5041

Subject :

$o(h^{3})$

$0,255 + \frac{1}{3}(0,255 - 0,2747) = 0,24815$

$0,2513 + \frac{1}{3}(0,2513 - 0,255) = 0,25007$

$o(h^{4})$

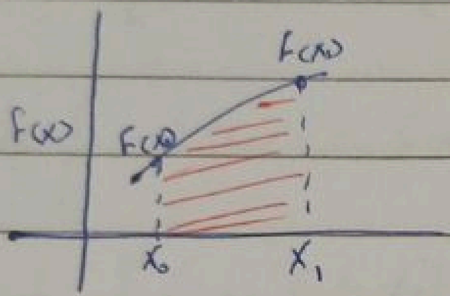
$0,25\cancel{7}007 + \frac{1}{15}(0,25007 - 0,24815) = 0,251135$

Subject : / /

* Numerical Integration

equation → Table

Equally | Un-Equally



x_i	$f(x_i)$
x_0	$f(x_0)$
x_1	$f(x_1)$

* Trapezoidal Rule

$$f_1(x) = a_0 + a_1 (x - x_0)$$

$$f_1(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0} \right) (x - x_0)$$

$$\int_{x_0}^{x_1} f_1(x) = \int_{x_0}^{x_1} \left[f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0} (x - x_0) \right] dx$$

$$f(x_0)(x_1 - x_0) + \left[\frac{f(x_1) - f(x_0)}{x_1 - x_0} \right] \left[\frac{1}{2} (x_1^2 - x_0^2) - x_0(x_1 - x_0) \right]$$

$$f_1(x) = (x_1 - x_0) \left[\frac{f(x_1) - f(x_0)}{2} \right] - \frac{1}{12} f''(\xi) h^3 \quad O(h^3)$$

$$= \frac{h}{2} \left[f(x_1) + f(x_0) \right]$$

Subject :

Total $A = A_{R} + R_{T} = f(x_0)(x_1 - x_0) + \frac{1}{2}[f(x_1) - f(x_0)](x_1 - x_0)$

$= f(x_0)x_1 - f(x_0)x_0 + \frac{1}{2} f(x_1)x_1 - \frac{1}{2} f(x_1)x_0 - \frac{1}{2} f(x_0)x_1 -$
$\frac{1}{2} f(x_0)x_0$

$= -\frac{1}{2} f(x_0)x_0 + \frac{1}{2} f(x_0)x_1 + \frac{1}{2} f(x_1)x_1 - \frac{1}{2} f(x_1)x_0$

$= \frac{1}{2}\left[-f(x_0)x_0 + f(x_0)x_1 + -f(x_1)x_1 - f(x_1)x_0\right]$

$= \frac{1}{2}\left[f(x_0)[x_1 - x_0] + f(x_1)[x_1 - x_0]\right]$

$= \frac{1}{2}(x_1 - x_0)[f(x_1) + f(x_0)]$ ← Trapezoidal Rule

$$\int_{x_0}^{x_4} f_1(x)\,dx = \int_{x_1}^{x_2} f_1(x)\,dx + \int_{x_2}^{x_3} f_1(x)\,dx$$

	x_i	$f(x_i)$
h_1	x_0	$f(x_0)$
$\neq$	x_1	$f(x_1)$
h_2	x_2	$f(x_2)$
$\neq$ h_3	x_3	$f(x_3)$

$\frac{h_1}{2}[f(x_1) + f(x_0)] + \frac{h_1}{2}[f(x_2) + f(x_1)] + \frac{h_1}{2}[f(x_3) + f(x_2)]$

$$\boxed{= \frac{h_1}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)\right]}$$

$= \frac{h_1}{2}\left\{f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right\} + nO(h^3)$ $\quad \boxed{h = \frac{a-b}{n}}$

Subject :

Example حل واجبي Find $\int_0^{10} f(x)\,dx \rightarrow f(x) = x^2$

$n=5$ used the Tarpezoidal Rule

x	0	2	4	6	8	10
f(x)	0	4	16	36	64	100

$$T = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5)\right]$$

$$\Delta x = \frac{10-0}{5} = 2$$

$$\frac{2}{2}\left[0 + 2(4) + 2(16) + 2(36) + 2(64) + 100\right]$$

$$= 340$$

* $\frac{1}{3}$ Sampson Rule

x_i	$f(x_i)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$

$$f_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$

$$\int_{x_0}^{x_2} f_2(x)\,dx = \int_{x_0}^{x_2} f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_0)$$

$$+ \frac{\left(\frac{f(x_2)-f(x_1)}{x_2-x_1}\right) - \left(\frac{f(x_1)-f(x_0)}{x_1-x_0}\right)}{x_2-x_0}(x-x_0)(x-x_1)$$

$$\boxed{= \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] + O(h^4)}$$

Subject : ______ / /

$$I = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)\right] + O(h^4)$$

$\frac{1}{3}$ simpson's Rule $\qquad h = \frac{b-a}{n}$

* if there are 4 equallyspaced points use 3/8 Rule
* if there are 6 equally spaced points use $\frac{1}{3}$ for the first two & 3/8 for the last three

Example مثال :-

the data table shows the instantaneous velocity of a car every 5 minute use simpson's $\frac{1}{3}$ Rule to determine the displacment of the car in the first 30 minute what is the avarge speute of the car for the entine 30 minute

T	0	5	10	15	20	25	30
V	25	28	32	30	29	26	23

$n = \frac{30}{5} = 6$

$$\int_0^{30} V(T)\cdot dT = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right]$$

$h = \frac{30-0}{6}$

$h = 5$

$$= \frac{5}{3}\left[25 + 4(28) + 2(32) + 4(30) + 2(29) + 4(26) + 23\right]$$

$= 843.333$

Subject : / /

* Simpson's $\frac{3}{8}$ Rule

$$\boxed{\int_{x_0}^{x_3} f_3(x)\,dx = \frac{3}{8}h\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]}$$

x_i	$f(x)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$

$$\text{Boole's} = \frac{b-a}{a_0}\left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)\right] + O(h)$$

→ Highe order Newton cotes

$$\boxed{I = \frac{3h}{8}\left[f(x_0) + 3\sum_{\substack{i=1 \\ i\neq 3,6,9.}} f(x_i) + 2\sum_{i=3,6,9}^{n-3} f(x_i) + f(x_n)\right] + O(h^2)}$$

Example خارجي find $\int_0^6 f(x)\,dx \rightarrow \frac{1}{1+x^2}$ used simpson's $\frac{3}{8}$ Rule

$n = 6$

$h = \frac{6-0}{6} = 1$

0	1	2	3	4	5	6
1	0,5	0,2	0,1	0,0588	0,0384	0,027

$$I = \frac{3*6}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) + f(x_6)\right]$$

$$= \frac{3}{8}\left[1 + 3(0,5) + 3(0,2) + 2(0,1) + 3(0,0588) + 3(0,0384) + 0,027\right]$$

$$= 1,357$$

Subject : ______ / /

* Unequally Spaced Data points

- Gaussian Quadrature (Gauss legendre) + Runburg

*(integrate if we don't have common step size)

Q	weighting test	function
2	$C_0 = 1$	$x_0 = -0.5773502$
	$C_1 = 1$	$x_1 = 0.5773502$

$$I = \int_a^b f(x)\, dx \quad \text{2point} \quad \begin{matrix} c_0 & c_1 \\ x_0 & x_1 \end{matrix}$$

step 2 $\quad I = \int_a^b f(x)\, dx = c_0 f(x_0) + c_1 f(x_1)$

$$I = c_0 f(-0.577350289) + c_1 f(0.577350289)$$

Step 3 $\quad x = \dfrac{(b-a)\, T + b + a}{2}$ (value of x in terme of T to find f

step 4 $\quad dx = \dfrac{b-a}{2}\, dT$

Subject :   / /

<u>step 5</u>

$$T = \int_a^b f(x)\,dx = \int_{-1}^{1} f(t)\,dt$$

$$\int_{-1}^{1} f\left(\frac{(b-a)t + b + a}{2}\right)\left(\frac{b-a}{2}\right) dt$$

Example خارجي

find the value of the integral $\int^{1.5} e^{-x^2} dx$ using 2point & 3point Gaussian quadrature method

$$x = \frac{(b-a)t + b + a}{2} = \frac{(1.5-1)t + 1.5 + 1}{2}$$

$$= \frac{0.5t + 2.5}{2} = \frac{t+5}{4}$$

$$dx = \frac{d-a}{2}dt = \frac{1.5-1}{2} = 0.25\,dt$$

$$\int_1^{1.5} e^{-x^2} dx = 0.25\int_{-1}^{1} e^{-\left(\frac{t+5}{4}\right)^2} dt$$

at 2point

$$= 0.25\,(c_0 f(x_0) + c_1 f(x_1))$$

1

من الجدول

at 3point

$$= 0.25 * (c_0 f(x_0) + c_1 f(x_1) + c_2 f(x_2))$$

1

من الجدول

Subject : / /

* Ordinary Differential Equation

initial value — $y(0)$ $y'(0)$ $y''(0)$ constant

Boundary Value — $y(0)$, $y'(1)$ $y''(0)$ not constant

initial value:

- 1^{st} O.O.D.E
  - Euler's
    - * Heun's method
    - * polygon method
    - Modification of Eulers
  - Runge Kutta
- Higher O.O.D.E (2^{nd}, 3^{rd})

* على الأقل لازم يكون المعادلة second order عشان اقدر اصنف initial or Boundary

* ولازم ال condition المعطى تكون أقل من order تبع المعادلة عشان اقدر احلها مثلا (first order need $y(\ )=$

* it is very important to know if condition is initial or Bandary

initial value

- 1^{st} order
- Higher order — there is no solution for Higher directly

يجب تحويلها

* 2^{nd} order → 2 first order Eq

* 3^{rd} order → 3 first order Eq

Subject :   / /

1^{st} order differential Equation

1) Euler method
2) Polygon method
3) Heun's method
4) Rung-Kutta method

* Euler method

$$y(x_{i+1}) = y(x_i) + f'(x_i, y(x_i))h$$

where $f(x_i, y(x_i))$ is the differential Equation evaluated at x_i, y_i, $h \rightarrow$ is the step size

Example $\quad f'(x) = -2x^3 + 12x^2 - 20x + 8{,}5$

$y(0) = 1$ from $x = 0$ to $x = 4$ using stepsize $= 1$?

$y(x_{i+1}) = y(x_i) + f'(x_i, y(x_i))h$

$y(1) = y(0) + f'(0, 1)(1)$

$= 1 + 8{,}5(1) = 9{,}5$

$y(2) = y(1) + f'(1, 9{,}5)(1) = 9{,}5 + (-1{,}5)(1)$

$y(2) = 8$

$y(3) = y(2) + f'(2, 8)(1)$

$= 8 + 0{,}5(1) \quad y(3) = 8{,}5$

$y(4) = y(3) + f'(3, 8{,}5)(1)$

$= 8{,}5 + 2{,}5(1) \quad \boxed{y(4) = 11}$

Subject : ___ / /

Ordinary Differential Equation

1) first O.O.D.E

A- Euler method

$$y(x_{i+1}) = y(x_i) + f'(x_i, y(x_i))\, h$$

B- Heun's Method

$$y(x_{i+1}) = y(x_i) + \left[\frac{f'(x_i, y(x_i)) + f'(x_{i+1}, y(x_{i+1}))}{2} \right]$$

$y(x_{i+1})$ I need calculatede using Euler method

$$y^P(x_{i+1}) = y(x_i) + f'(x_i, y(x_i))\, h$$

C- The improved polygon method

$$y(x_{i+1}) = y(x_i) + y'\left(x_{i+\frac{1}{2}}, \boxed{y(x_{i+\frac{1}{2}})}\right) h$$

I need calculated using Eluer

$$y(x_{i+\frac{1}{2}}) = y(x_i) + f'(x_i, y(x_i)) \times \frac{h}{2}$$

Example $\frac{dy}{dx} = 4e^{0,8x} - 0,5y \quad y(0) = 2$

from $x=0$ to $x=4$ at $h=1$?

using Eluer

$y(1) = y(0) + f'(0, 2)(1)$

$= 2 + (4e^{0,8(0)} - 0,5(2))(1)$

$y(1) = 2 + 3(1) = 5$

$y(2) = y(1) + f'(1, 5)(h)$

$= 5 + (4e^{0,8(1)} - 0,5(5))(1)$

$= 5 + 6,402163714(1) = 11,402163714$

$y(3) = y(2) + f'(2, 11,402163714)(1)$

$= 11,402163714 + (4e^{0,8(2)} - 0,5(11,402163714))(1)$

$= 11,402163714 + 14,11104784 = 25,51321155$

$y(4) = y(3) + f'(3, 25,51321155)$

$= 25,51321155 + (4e^{0,8(3)} - 0,5(25,51321155))(1)$

$= 25,51321155 + 31,33609975 = 56,8493113$

Using Heun's

$$y(x_{i+1}) = y(x_i) + \left\{ \frac{f'(x_i, y(x_i)) + f'(x_{i+1}, y(x_{i+1}))}{2} \right.$$

$$y(1) = y(0) + \left\{ \frac{f'(0,2) + f'(1, y(1))}{2} \right.$$

$$y^{p}(1) = y(0) + f'(0,2)(1)$$

$$= 2 + 3(1) = 5$$

$$y(1) = y(0) + \left\{ \frac{f'(0,2) + f'(1,5)}{2} \right\}(1)$$

$$2 + \left\{ \frac{3 + 6{,}02163714}{2} \right\}(1)$$

$$\boxed{y(1) = 6{,}7010818}$$

$$y(2) = y(1) + \left\{ \frac{f'(1, 6{,}7010818) + f'(2, y(2))}{2} \right\}(1)$$

$$y(2) = y(1) + f'(1, 6{,}7010818)(1)$$

$$= 6{,}7010818 + 5{,}551622814(1) = 12{,}25270461$$

$$y(2) = 6{,}7010818 + \left\{ \frac{5{,}551622814 + f'(2, 12{,}25270461)}{2} \right\} h$$

$$= 6{,}7010818 + \left\{ \frac{5{,}551622814 + 13{,}68577739}{2} \right\}(1)$$

$$y(2) = 16{,}3197819$$

Subject :

$$y(3) = y(2) + \left\{ \frac{f'(2, 16{,}3197819) + f'(3, y(3))}{2} \right\} h$$

$$y(3) = y(2) + f'(2, 16{,}3197819)(1)$$

$$= 16{,}3197819 + \left(4e^{0{,}8\times 2} - 0{,}5(16{,}3197819)\right)(1)$$

$$y(3) = 27{,}97202065$$

$$y(3) = y(2) + \left\{ \frac{11{,}65223875 + f'(3, 27{,}202065)}{2} \right\}(1)$$

$$= 16{,}3197819 + \left\{ \frac{11{,}65223875 + 30{,}1066952}{2} \right\}(1)$$

$$y(3) = 37{,}19924887$$

$$y(4) = y(3) + \left\{ \frac{f'(3, 37{,}19924887) + f'(4, y(4))}{2} \right\} h$$

$$y(4) = y(3) + f'(3, 37{,}19924887)$$

$$= 37{,}19924887 + 25{,}49308109 = 62{,}69232996$$

$$y(4) = y(3) + \left\{ \frac{f'(3, 37{,}19924887) + f'(4, 62{,}69232996)}{2} \right\}$$

$$37{,}19924887 + \left(\frac{25{,}49308109 + 66{,}78395581}{2} \right)$$

$$\boxed{y(4) = 83{,}33776732}$$

Subject :

using polygon

$$y(x_{i+1}) = y(x_i) + f'(x_{i+\frac{1}{2}}, y(x_{i+\frac{1}{2}}))\, h$$

$$y(1) = y(0) + f'(0{,}5, y(0{,}5))(1)$$

$$y(0{,}5) = y(0) + f'(0, 2)\frac{1}{2}$$
$$= 2 + 3 \times \frac{1}{2} = 3{,}5$$
$$y(1) = 2 + f'(0{,}5, 3{,}5)(1)$$
$$= 2 + 4{,}217298791 = 6{,}217298791$$
$$y(2) = y(1) + f'(1{,}5, y(1{,}5))(1)$$
$$y(1{,}5) = y(1) + f'(1, 6{,}217298791)\left(\frac{1}{2}\right)$$
$$= 6{,}217298791 + \frac{5{,}793514318}{2}$$

$$y(1{,}5) = 9{,}11405595$$

$$y(2) = y(1) + f'(1{,}5, y(1{,}5))(1)$$
$$= 6{,}217298791 + 9{,}11405595$$
$$y(2) = 15{,}33135474$$
$$y(3) = y(2) + f'(2{,}5, y(2{,}5))(1)$$
$$y(2{,}5) = y(2) + f'(2, 15{,}33135474)\frac{1}{2}$$
$$= 15{,}33135474 + \frac{12{,}14645233}{2}$$
$$y(2{,}5) = 21{,}4045809$$
$$y(3) = 15{,}33135474 + f'(2{,}5, 21{,}4045809)(1)$$
$$y(3) = 15{,}33135474 + 18{,}85393394(1) = 34{,}18528868$$

Subject :

$$y(4) = y(3) + f'(3{,}5,\ y(3{,}5))(1)$$

$$y(3{,}5) = y(3) + f'(3,\ 34{,}18528868)\left(\frac{1}{2}\right)$$

$$y(3{,}5) = 34{,}18528868 + \frac{27{,}00006118}{2} = 47{,}68531927$$

$$y(4) = y(3) + f'(3{,}5,\ 47{,}68531927)(1)$$

$$= 34{,}18528868 + 47{,}68531927$$

$$y(4) = 81{,}87060795$$