NUMERICAL

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$x^2 + 3x + 6 = 0$ analatical Technic.

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$x^5 - x - 1 = 0$

$f(x) = x^5 - x - 1 = 0$

$x^5 - 1 = x$

$x^5 = x + 1$

$x = (x+1)^{1/5}$

x_i	x_{cal}	$\lvert D \rvert$
0	-1	1
-1	-2	1
-2	-33	31
-33	Large No.	

Divergance

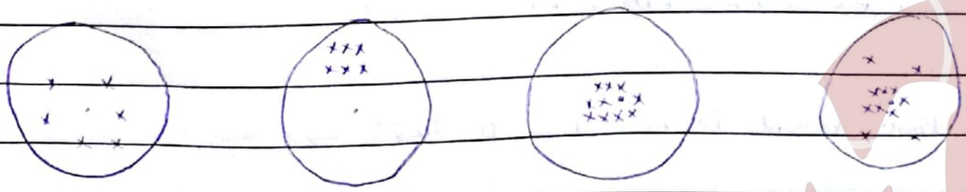
x_i	x_{cal}	$\lvert D \rvert$
0	1	1
1	1.148	0.148
1.148	1.165	0.017
1.165	1.167	0.002
1.167	1.167...	~ 0

Convergance.

* لحتى احصل على قيمة X الصحيحة

1- initial guss

2- طريقة كتابة المعادلة،



not precision   not accurate

Precision

accurate   Precision

not Precision   accurate.

Conversion between binary and Decimal.

Decimal:

$N = a_1 10^{k-1} + a_2 10^{k-2} + a_3 10^{k-3} + \dots$ $\therefore k$ عدد منازل الرقم

$165 = a_1 10^2 + a_2 10^1 + a_3 10^0$

$= 1 \times 10^2 + 6 \times 10 + 5$

$= 100 + 60 + 5 = 165$

$165 \rightarrow 10^2 \; 10^1 \; 10^0$

5×1

6×10

1×100

Binary No.

$N = b_j 2^j + b_{j-1} 2^{j-1} + b_{j-2} 2^{j-2} + \dots$

$N = Q_0 + b_0$

$Q_0 = Q_1 + b_1$

$Q_1 = Q_2 + b_2$

$\vdots$

example:

15

$15 = 2 \times 7 + 1$

$7 = 2 \times 3 + 1$

$3 = 2 \times 1 + 1$

$1 = 2 \times 0 + 1$ → binary system $= 1111$

Convert (10111) to decimal.

$10111 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 =$ $10111 \rightarrow 2^4 \; 2^3 \; 2^2 \; 2^1 \; 2^0$

$= 1 + 0 + 4 + 8 + 16 = 29$ ✗

Convert from Digit to binary $0.4375 \rightarrow$ binary.

$R = d_1 2^{-1} + d_2 2^{-2} + d_3 2^{-3} + \dots$

$0.4375 \rightarrow 2R = 0.875 \qquad d_1 = 0 \quad \times 2 \quad 0$

$2F_1 = 1.75 \qquad d_2 = 1 \times 2^{-2} + 1/4$

$2F_2 = 1.5 \qquad d_3 = 1 \times 2^{-3} + 1/8$

$2F_3 = 1.0 \qquad d_4 = 1 \times 2^{-4} + 1/16$

$0.4375 = \overrightarrow{0111}$

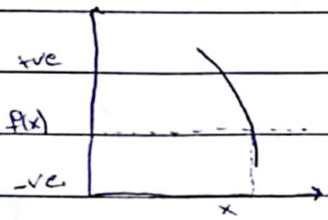
Solution of nonlinear equation

1. iterative method
2. Bisection method
3. Newton - Raphson method
4. Newton - 2nd order method
5. Secant method
6. Muller method

1. Fixed point (iterative method)

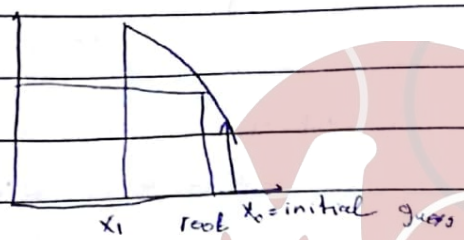
$f(x) = 0 \rightarrow x_i = ??$

initial guess



$f(x) = e^{-x} - x \qquad , x_0 = 0$

x	f(x)
0	1
1	0.367
0.367	0.692
0.692	0.5
:	
0.57	0.57



example 2.

$P(x) = x^3 + x^2 - 3x - 3$

1) $g(x) = \dfrac{x^3 + x^2 - 3}{3} = x$ Position initial guess

$x_0 = -0.1$

$x = -0.1$ -0.99 -0.999 1

$g(x) = -0.99$ -0.999 -0.9999 1

let $x_0 = 2$

$x = 2$ 3 11 483

$g(x) = 3$ 11 483 37637291 $\rightarrow$ divergance.

1. choice of initial guess will determine if the method converge or diverge.

2) $x^3 = 3 + 3x - x^2$

$g(x) = x = (3 + 3x - x^2)^{1/3}$

$x = 2$ 1.7 1.73

$g(x) = 1.7$ 1.73 1.73 $\rightarrow$ Convergence.

3) $x^3 + x^2 - 3x - 3 = 0$

$x(x^2 + x - 3) - 3 = 0$

$x(x^2 + x - 3) = 3$

$g(x) = \boxed{x = \dfrac{3}{x^2 + x - 3}}$

$x = 2$ 1 -3

$g(x) = 1$ -3 1 $\rightarrow$ no Conversion (osillation)

2. writing the function in proper way will determine if convergance or divergance occurse.

3. ~~We~~ ~~have~~ chose the form for higher order independant variabls

— a lot of differentiation

2) Newton – Raphson – method

+ you can get more than one root in the same time.

$$f(x) = f(x_0) + f'(x_0)\frac{\Delta x}{1} + f''(x_0)\frac{(\Delta x)^2}{2!} + \cdots \quad \text{taylor series.}$$

$$f(x) \approx f(x_0) + f'(x_0)\frac{\Delta x}{1}$$

$$\Delta x = \frac{f(x_0)}{f'(x_0)}$$

$$X_{new} - X_{old} = \frac{f(x_0)}{f'(x_0)}$$

$$X_{new} = X_{old} + \frac{f(x_0)}{f'(x_0)}$$

Example:

$$f(x) = x^3 + x^2 - 3x - 3$$

$$f'(x) = 3x^2 + 2x - 3$$

$$X_{new} = X_{old} + \frac{x^3 + x^2 - 3x - 3}{3x^2 + 2x - 3}$$

[illegible]

X_{old}	X_{new}
0.1	-1.08
-1.08	-0.989
-0.989	-0.999
-0.999	1
1	1

X_{old}	X_{new}
2	1.769
1.769	1.732
1.732	1.732.

في غلط
الأرقام يتعوض

X_{old}	X_{new}
-2	-1.8
-1.8	-1.738
-1.738	-1.738

Algorthm

1. Fixed point method.

set x

Assume x_0

① $x = g(x)$ $\qquad f(x) = x + g(x) = 0$

if $(|x - g(x)|) < \varepsilon$ (go to zero) $\rightarrow$ go print results.

$\therefore \varepsilon$: small value $0.1 - 10^{-6}$

$x_0 = x$

go to ①

set x

Assume x_0

for $i = 1$ to 100

$x = g(x)$

if $(|x - g(x)|) < \varepsilon$ $\rightarrow$ print results.

else

$x_0 = x$

Next i

Newton Raphson method $\Delta x = \frac{-f(x)}{f'(x)}$

set x (real, integer, ...)

set x_0 as initial guess

for $i=1$ to 100

$f =$

$df =$

$x_{new} = x_{old} - \frac{f}{df}$

if $(|x_{new} - x_{old}| < \varepsilon)$ $\rightarrow$ print x_{new}

else

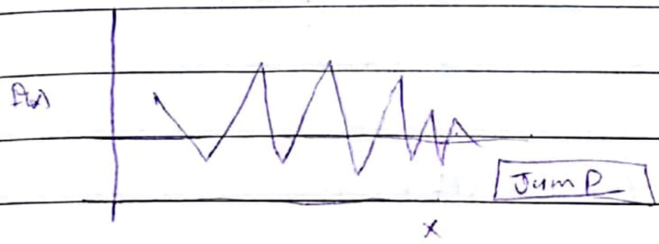
$x_{old} = x_{new}$

Next i

3. Newton second order method

$$f(x) = f(x_0) + f'(x_0)\frac{\Delta x}{1!} + f''(x_0)\frac{(\Delta x)^2}{2!} + \cdots$$

stiffness



roots

$$0 = f(x_0) + \Delta x\left(f'(x_0) + f''(x_0)\frac{\Delta x}{2}\right)$$

$$0 = f(x_0) + \Delta x\left(f'(x_0) + \frac{f''(x_0)}{2}\left(\frac{-f(x)}{f'(x)}\right)\right)$$

$$-f(x_0) = \Delta x\left[f'(x_0) - \frac{f''(x_0)}{f'(x_0)} \cdot \frac{f(x)}{2}\right]$$

$$\Delta x = \frac{-f(x_0)}{\left[f'(x) - \frac{f(x)\cdot f''(x_0)}{2f'(x)}\right]}$$

$$x_{new} = x_{old} - \frac{f(x)}{\left[f'(x) - \frac{f(x)\, f''(x)}{2f'(x)}\right]}$$

Ex. $f(x) = 2x^3 - 3x^2 + x - 6$

$f'(x) = 6x^2 - 6x + 1$

$f''(x) = 12x - 6$

$$x_{new} = x_{old} - \frac{2x^3 - 3x^2 + x - 6}{\left[(6x^2 - 6x + 1) - \frac{(2x^3 - 3x^2 + x - 6)\cdot(12x - 6)}{2(6x^2 - 6x + 1)}\right]}$$

x_i	x_{new}	$f(x)$
0	$-\frac{6}{17} = (-0.352)$	-6
-0.352	$-0.9.$	

Algorethm

1 set X
2 Assume X old.
3 Calculate F.
4 dF.
5 d2F.
6 $X_{new} = X_{old} - F/(dF - F * d2F / 2 dF)$
7 if $(|X_{new} - X_{old}|) < \varepsilon$ → print X_{new}.
8 Go to #3

4. Secant Method

$$f(x) = \lim_{\Delta X \to 0} \frac{\Delta F}{\Delta X}$$

$$= \lim_{\Delta X \to 0} \frac{F(X_i + 1) - F(X_i)}{}$$

From newton Raphson method.

$$X_i = X(i-1) - \frac{F(X_{i-1})}{F'(X_i - 1)}$$

→ $$X_i = X_{i-1} - F(X_{i-1}) \left[\frac{X_{i-1} - X_{i-2}}{F(X_{i-1}) - F(X_{i-2})} \right]$$

$$X_2 = X_1 - F(X_1) \left[\frac{X_1 - X_0}{F(X_1) - F(X_0)} \right]$$

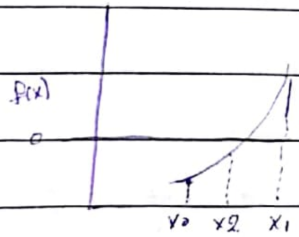
- this method advantage → first guess two Value.

$$X_1, IG < X_3\ IG < X_2, IG$$

if $(X_3 \cdot X_2)$ negative we will negligeble X_1

Algorithm

1) initially Define x_0, x_1
2) calculate $f(x_0), f(x_1)$
3) calculate x_2
4) set $x_0 = x_1$, $x_1 = x_2$



كل قيمة جديدة تبلش العمل ويكون ال range اصغر
واقرب لل real root.

Example:

$f(x) = \cos x - x$

$x_0 = 1/2$, $x_1 = \frac{\pi}{4}$ → (-/+) بختارهم بحيث يعطوني جواب

لازم احول لـ radian من degree

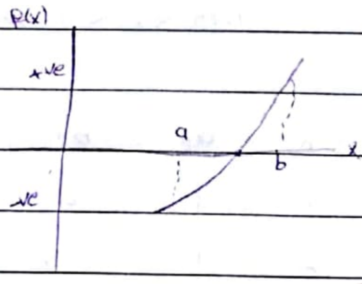
n	x_n	$f(x_n)$
0	$\frac{1}{2}$	$\cos\left(\frac{0.5 \times 180}{\pi}\right) - 0.5 = 0.37$
1	$\frac{\pi}{4}$	-0.077
2	$x_2 = x_1 - f(x_1)\left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] = 0.736$	$f(x_2) = 0.00448$
3	$x_3 = x_2 - f(x_2)\left[\frac{x_2 - x_1}{f(x_2) - f(x_1)}\right] = 0.739$	$f(x_3) = 4.4 \times 10^{-3}$

الخطوة التالية

تكرار

Target to have $f(x_n) = 0$

5) Bisection method



$a \le x \le b$

$x_n = \frac{a+b}{2}$, $f(x_n) = ??$

if $(f(a) * f(x_n)) > 0 \rightarrow a = x_n$

if $(f(b) * f(x_n)) > 0 \rightarrow b = x_n$

Example 8

$$f(x) = x^3 + 4x^2 - 10$$

$$1 < x < 2$$

$f(1) = -5$

$f(2) = 14$

وحدة موجب
وحدة سالب

$$x_n = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$$f(1.5) = (1.5)^3 + 4(1.5)^2 - 10$$
$$= 2.375$$

بضربها بالموجب $f(1.5) \times f(2) > 0 \rightarrow b = x_n = 1.5$

$$x_n = \frac{a+b}{2} = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$$

$f(1.25) = -1.79$

$f(a) \times f(x_n) = f(1) \times f(1.25) > 0$

$a = x_n = 1.25$

$1.25 < x < 1.5$

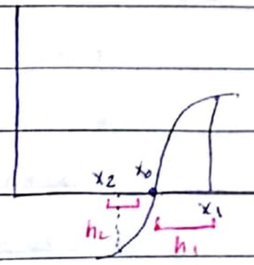
$x_n = \frac{1.25+1.5}{2} = 1.375$, $f(1.375) = 0.162$

$f(0.162) \times f(1.5) > 0 \rightarrow b = 1.375$

$1.25 < x < 1.375$

n	a_n	b_n	x_n	$f(x_n)$
1	1	2	1.5	2.375
2	1	1.5	1.25	-1.79
3	1.25	1.5	1.375	0.162
4	1.25	1.375	1.3125	-0.84
5	1.3125	1.375	1.34375	-0.35
⋮				
13	1.36409	1.3652		-0.001

6.) Muller Method:



$$h_1 = x_1 - x_0$$

$$h_2 = x_0 - x_2$$

-/+ [illegible] ← x_1/x_2

$$av^2 + bv + c = 0$$

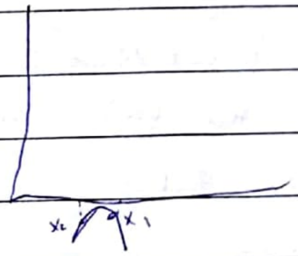
$\rightarrow a h_1^2 + b h_1 + c = 0 = f(h_1)$

$\rightarrow a h_2^2 + b h_2 + c = 0 = f(h_2)$

x_0 عند تعويض $\rightarrow \quad + c = 0 = f(0)$

$h = 0$ $(x_0 - x_0)$

لما تكون القيمة الصح.



$$a = \frac{\gamma f_1 - f_0(1+\gamma) + f_2}{\gamma h_1^2 (1+\gamma)}$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$

$$c = f(0) = f(x_0)$$

$$\gamma = \frac{h_2}{h_1}$$

$$\Longrightarrow \Delta X = \frac{-2c}{b \mp \sqrt{b^2 - 4ac}}$$

$$X = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$\Downarrow$

min value:

The sign of denomenator should be chosen to have $b \mp \sqrt{b^2 - 4ac}$ as max as possible.

sign of b is -ve $\Rightarrow$ sign -ve

sign of b is +ve $\Rightarrow$ sign = +ve.

Example:

$$f(x) = \sin x - \frac{x}{2}$$

Choose $h = 0.2$, $x_0 = 2$

$x_1 = x_0 + h = 2.2$

$x_2 = x_0 - h = 1.8$

$$f_0 = f(x_0) = \sin(\underset{\text{radian}}{2}) - \frac{2}{2} = -0.0907$$

$$f_1 = f(x_1) = \sin(2.2) - \frac{2.2}{2} = -0.291$$

$$f_2 = f(x_2) = \sin(1.8) - \frac{1.8}{2} = 0.0738$$

مرة موجبة
ومرة سالبة

$$\gamma = 1 \quad \frac{h_2}{h_1} = \frac{0.2}{0.2} = 1$$

$$a = \frac{1(-0.291) - (-0.0907)(1+1) + 0.0738}{1(0.2)^2(1+1)} = -0.453$$

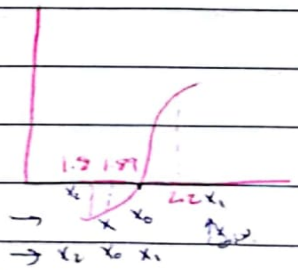
$$b = \frac{-0.291 - (-0.0907) - (-0.453)(0.2)^2}{0.2} = -0.913$$

$$c = f(x_0) = f_0 = -0.0907.$$

$$x = 2 - \frac{2(-0.0907)}{(-0.913) \pm \sqrt{(-0.913)^2 - 4(-0.453)(-0.0907)}}$$

$= 1.89526$

هذا يبقى x_0 الجديدة.



Trial 2

$x_0 = 1.89526 \rightarrow f_0 = \sin(1.89526) - \frac{1.89526}{2}$

$= 1.918 \times 10^{-4}$

يعني 1.8 مثلاً لازم تغير وحدة تحولها
موجب ومرة سالب + بشيل الأبعد x

$h_1 = x_1 - x_0$

$= 2 - 1.89526 = 0.104$

$x_2 = 1.8 \rightarrow f_2 = 0.07385$

$x_1 = 2 \rightarrow f_1 = -0.0907$

$h_2 = x_0 - x_2$

$h_2 = 0.09526$

$$\gamma = \frac{h_2}{h_1} = \frac{0.09526}{0.0104} = 0.0909.$$

$$c = f(x_0) = f(1.89526) = 1.918 \times 10^{-4}$$

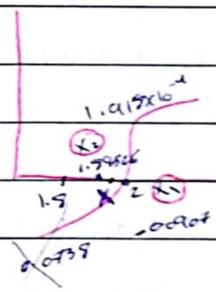
$$a = \frac{(0.0909)(-0.0907) - (1.918 \times 10^{-4})(1 + 0.0909) + 0.07384}{0.0909(0.0104)^2(1 + 0.0909)}$$

$$= -0.4728$$

$$b = \frac{(-0.0907) - (1.918 \times 10^{-4}) - (-0.4728)(0.0104)^2}{(0.0104)} = -0.81820$$

$$x = 1.89526 - \frac{2(1.918 \times 10^{-4})}{-0.81820 - \sqrt{(-0.81820) - 4(0.4729)(1.918 \times 10^{-4})}}$$

$$= 1.8954.$$



تصحيح الأستاذ ←

12/12.

Solution of system of linear equation

$$x_1 + 3x_2 = 5$$
$$2x_1 + x_2 = 3$$

1. Gaussion Elimination method
2. Gauss - Jordan method
3. Jacobi iterative method
4. Gauss - Saidel method

1. Gaussion Elemination method.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$Ax = b$$
$$A^{-1}Ax = A^{-1}b$$
$$\boxed{x = A^{-1}b}$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 & 0 & - & 0 \\ 0 & 1 & 0 & - & 0 \\ \vdots \\ 0 & 0 & 0 & - & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}^{-1} \times \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Example :-

$$2x_1 - x_2 + 3x_3 = 2 \quad \cdots \; E_1$$
$$x_1 + 2x_2 - x_3 = 3 \quad \cdots \; E_2$$
$$3x_1 - 2x_2 + x_3 = -1 \quad \cdots \; E_3$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$E_{1_{old}} - 2E_{2_{old}} = E_{2_{new}}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & -5 & 5 & -4 \\ 3 & -2 & 1 & -1 \end{array}\right]$$

$$E_{1_{old}} - \frac{2}{3} E_{3_{old}} = E_{3_{new}}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & -5 & 5 & -4 \\ 0 & 1/3 & \frac{7}{3} & \frac{8}{3} \end{array}\right]$$

$$E_{2_{old}} + 15 E_{3_{old}} = E_{3_{new}}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & -5 & 5 & -4 \\ 0 & 0 & 40 & 36 \end{array}\right]$$

$$40x_3 = 36 \Rightarrow x_3 = \frac{36}{40}$$

$$\alpha_1 - 5x_2 + 5\left(\frac{36}{40}\right) = -4$$

$$x_2 = \frac{17}{10}$$

$$2x_1 - x_2 + 3x_2 = 2$$

$$x_1 = 1/2.$$

Crammer's Rule

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_i = \frac{D_i}{D}$$

$$D = \left[\begin{array}{ccc|c|c} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right]$$

$$D = a_{11}\,a_{22}\,a_{33} + a_{12}\,a_{23}\,a_{31} + a_{13}\,a_{21}\,a_{32}$$
$$- a_{12}\,a_{21}\,a_{33} - a_{11}\,a_{23}\,a_{32} - a_{13}\,a_{22}\,a_{31}$$

$$D_1 = \left[\begin{array}{ccc|cc} b_1 & a_{12} & a_{13} & b_1 & a_{12} \\ b_2 & a_{22} & a_{23} & b_2 & a_{22} \\ b_3 & a_{32} & a_{33} & b_3 & a_{32} \end{array}\right]$$

Code.

```
A = [a11  a12  a13 ; a21 .... ]
b = [b1 ; b2 ; b3]
X  = inv(A*)*b
```

Gauss - Jordan method :

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$AX = b$$

$$A^{-1}AX = A^{-1}b$$

$$IX = A^{-1}b$$

$$\boxed{X = A^{-1}b}$$

Contenue last example :

$$\left[\begin{array}{ccc|c} \cancel{1}\,2 & -1 & 3 & 2 \\ 0 & -5 & 5 & -4 \\ 0 & 0 & 40 & 36 \end{array}\right]$$

÷ diagonal by a_{ii}

$$\begin{array}{c} \div 2 \\ \div -5 \\ \div 40 \end{array} \left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & 1 \\ 0 & 1 & -1 & \frac{4}{5} \\ 0 & 0 & 1 & \frac{36}{40} \end{array}\right]$$

$R_{2\,old} + R_{3\,old} \rightarrow R_{2\,new}$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & 1 \\ 0 & 1 & 0 & \frac{68}{40} \\ 0 & 0 & 1 & \frac{36}{40} \end{array}\right]$$

$R_{2\,old} * \frac{1}{2} + R_{1\,old} = R_{1\,new}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & \frac{14}{40} \\ 0 & 1 & 0 & \frac{68}{40} \\ 0 & 0 & 1 & \frac{36}{40} \end{array}\right] \rightarrow 34/40$$

$R_3 \times \frac{-3}{2} + R_1 = R_{1\,new}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{80} \\ 0 & 1 & 0 & \frac{68}{40} \\ 0 & 0 & 1 & \frac{36}{40} \end{array}\right]$$

بنحكم و نطلع الاصفار ← 1
و بنعزل [illegible]

$x_1 = \frac{20}{40}$

$x_1 = \frac{4}{80}$

$x_2 = \frac{68}{40}$

$x_3 = \frac{36}{40}$

calculation? في علوا

Algorithm :

Set $K = 1 , N-1$

Set $i = K+1 , N$

Set $j = N+1 , K , -1$

$$a_{ij} = a_{ij} - \frac{a_{iK}}{a_{KK}} \, a_{Kj}$$

Next j

Next i

Next K

Jacobi iterative method

$\prod_{i=1}^{N} a_{ii}$ should be max

Arranging the equation to have the elements on the diagonal as max as possible.

Example :

$$2x_1 + 7x_2 - 8x_3 = 10$$

$$5x_1 - 2x_2 + 6x_3 = 6$$

$$3x_1 + 5x_2 - 7x_3 = 3$$

$a_{11} a_{22} a_{33} = 28$
$\left[\begin{array}{ccc|c} 2 & 7 & -8 & 10 \\ 5 & -2 & +6 & 6 \\ 3 & 5 & -7 & 3 \end{array}\right]$

(16)
$\left[\begin{array}{ccc|c} 2 & 7 & -8 & 10 \\ 3 & 5 & -7 & 3 \\ 5 & -2 & 6 & 6 \end{array}\right]$

(245)
$\left[\begin{array}{ccc|c} 5 & -2 & 6 & 6 \\ 2 & 7 & -8 & 10 \\ 3 & 5 & -7 & 3 \end{array}\right]$

(48)
$\left[\begin{array}{ccc|c} 3 & 5 & -7 & 3 \\ 5 & -2 & 6 & 6 \\ 2 & 7 & -8 & 10 \end{array}\right]$

x_1: $5x_1 - 2x_2 + 6x_3 = 6$

x_2: $2x_1 + 7x_2 - 8x_3 = 10$

x_3: $3x_1 + 5x_2 - 7x_3 = 3$

$$5x_1 = 6 + 2x_2 - 6x_3$$

$$7x_2 = 10 + 8x_3 - 2x_1$$

$$7x_3 = -3 + 3x_1 + 5x_2$$

$$x_1 = \frac{6 + 2x_2 - 6x_3}{5}$$

$$x_2 = \frac{10 + 8x_3 - 2x_1}{7}$$

$$x_3 = \frac{-3 + 3x_1 + 5x_2}{7}$$

initial guess

	#1	#2	#3	#4	....	#40
x_1	0	$\frac{6}{5}$	2.28	6.11		-0.305
x_2	0	$\frac{10}{7}$	0.5415	2.04		4.76
x_3	0	$\frac{-3}{7}$	1.66	0.98		2.84

بكل trial بعوض في القيم يلي قبلها.

Converge after 40 iterative.

Gauss-sidel method

$x_1 = (6 + 2x_2 - 6x_3)/5$

$x_2 = (10 - 2x_1 + 8x_3)/7$

$x_3 = (3 - 3x_1 - 5x_2)/-7$

Jacobi iterative method

	#1	#2	#3	.....	#40
x_1	0	6/5	2.28		-0.306
x_2	0	10/7	0.595		4.767
x_3	0	-3/7	1.106		2.844

Gauss-Sidel

				.....	#22
x_1	0	6/5	0.6		0.306
x_2	0	$[10 - 2(\frac{6}{5}) + 8(0)]/7 = 1.08$	2.28		4.76
x_3	0	$[3 - 3(\frac{6}{5}) - 5(1.08)]/-7 = 0.86$	1.42		2.34

...

Algorithm

i) Jacobi method

1. Assume initial guess for x_i
2. set $k=1$

For $i = 1 \dots$ n-equation

$$x_i^k = \frac{-\sum_{j=1, j\neq i} a_{ij} x_j^{k-1} + b_i}{a_{ii}}$$

Next i

3. if $|x_i^k - x_i^{k+1}| < \varepsilon$ Goto step 4

else set $k = k+1$, Go to # 2

4. print Results

2) Gauss - sidel method

$$x_i^k = \frac{\sum_{j=1, j\neq i} a_{ij} x_j^k - \sum_{j=j+1} a_{ij} x_j^{k-1} + b_i}{a_{ii}}$$

Lower - upper (LU) Decomposition method

$$Ax = b$$

$A = [n \times n]$, $b = [n \times 1]$, $x = [n \times 1]$

$$x = A^{-1} b$$

$$A = \begin{bmatrix} \checkmark & \emptyset \\ \checkmark & \checkmark \end{bmatrix} \times \begin{bmatrix} \checkmark & \checkmark \\ \emptyset & \checkmark \end{bmatrix} = \begin{bmatrix} \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark \end{bmatrix}$$

$Ax = b$

$LUx = b$

$y = Ux$

$Ly = b$

Example :-

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & U_{11} & a_{12} \\ 0 & 1 & a_{21} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$L \quad \times \quad U \quad = \quad A$

to get L , $\ell_{ij} = \frac{a_{ij}}{a_{jj}}$ $\qquad a_{ij} = a_{ij} - L_{ij} a_{ij}$

$\ell_{21} = \frac{64}{25} = 2.56$ $\qquad a_{21} = a_{21} - \ell_{21} a_{11}$

$= 64 - 2.56(25) = 0$

$a_{22} = a_{22} - \ell_{21} a_{12}$

$= 8 - 2.56(5) = -4.75$

$a_{23} = a_{23} - l_{21}\, a_{13}$

$= 1 - 2.56(1) = -1.56$

$l_{31} = \frac{144}{25} = 5.76$

$a_{31} = a_{31} - l_{31}\, a_{11}$

$= 144 - (5.76)(25) = 0$

$a_{32} = 12 - (5.76)(5) = -16.8$

$a_{33} = 1 - (5.76)(1) = -4.76$

$l_{32} = \frac{a_{32}}{a_{22}} = \frac{-16.8}{-4.76} = 3.53$

$a_{32} = a_{31} - l_{32}\, a_{22}$

$= -16.8 - (3.53)(-4.75)$

$= 0$

$a_{31} = 0$

$a_{33} = (-4.76) - (3.53)(-1.55)$

$= 0.7$

$$U = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.75 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.53 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

24/2

تكملة من آخر الصفحة الماضية

Example 8:

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$Ax = b$

$LUx = b$

$Ly = b$

$Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.53 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Solve by Forward substitution

$y_1 = 1$

$2.56(1) + (1)\, y_2 = 2 \longrightarrow y_2 = -0.56$

$5.76\,(y_1) + 3.53\,(y_2) + y_3 = 3$

$y_3 = -0.78$

$Ux = y$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.75 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.56 \\ -0.78 \end{bmatrix}$$

Backword Substitution

$0.7\, x_3 = -0.78 \longrightarrow \boxed{x_3 = -1.11}$

$-4.75\,(x_2) - 1.56\,(x_3) = -0.56 \longrightarrow \boxed{x_2 = 0.48}$

$25x_1 + 5x_2 + x_3 = 1 \longrightarrow \boxed{x_1 = -0.0116}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.0116 \\ 0.48 \\ -1.11 \end{bmatrix}$$

Algorithm

$Ax = b \;,\; LUx = b \;,\; L\,y = b$

Set $U = A \;,\; L = I$

$k = 1 : m-1$

$j = k+1 : m$

$L_{jk} = U_{jk} / U_{kk}$

$U_{j,k:m} = U_{j,k:m} - L_{jk}\, U_{k,k:m}$

Next j

Next k

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Numerical ~~Solution~~ Solution of system of non-Linear Equation

1. iterative method
2. Newton – Raphson method.

1. iterative method.

* high order in side

$$x_1 + x_2^3 = 1 \longrightarrow x_2^3 = 1 - x_1 \rightarrow x_2 = (1-x_1)^{1/3}$$

$$3x_1^2 - x_2 = 5 \longrightarrow 3x_1^2 = 5 + x_2 \longrightarrow x_1 = \left(\frac{5+x_2}{3}\right)^{1/2}$$

x_1	x_2
0	$(1-0)^{1/3} = 1$
$\left(\frac{5+1}{3}\right)^{1/2} = 1.4$	$(1-1.4)^{1/3} = -0.74$
1.19 ~~[illegible]~~	-0.57
1.2	-0.59
1.2	-0.59

انتباه! odd or even

$$\boxed{x_1 = 1.2 \quad x_2 = -0.59}$$

2) Newton - Raphson method

$f(x, y) = 0$

$g(x, y) = 0$

$$f(x,y) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)\frac{\Delta x}{1!} + \frac{\partial^2 f}{\partial x^2}(x_0, y_0)\frac{(\Delta x)^2}{2!} + \cdots$$

$$+ \frac{\partial f}{\partial x}(x_0, y_0)\frac{\Delta y}{1!} + \frac{\partial^2 f}{\partial x^2}(x_0, y_0)\frac{(\Delta y)^2}{2!} + \cdots$$

$$+ \frac{\partial f}{\partial z} \cdots$$

اقل تأثير

$$f = f_0 + f_x \Delta x + \cancel{f_{xx}\frac{(\Delta x)^2}{2!}} + \cdots$$

$$+ f_y \Delta y + \cancel{f_{yy}\frac{(\Delta y)^2}{2!}} + \cdots$$

$$f = f_0 + f_x \Delta x + f_y \Delta y = 0$$

$$g = g_0 + g_x \Delta x + g_y \Delta y = 0$$

$$f_x \Delta x + f_y \Delta y = -f_0$$

$$g_x \Delta x + g_y \Delta y = -g_0$$

$$\underbrace{\begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -f_0 \\ -g_0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}^{-1} \begin{bmatrix} -f_0 \\ -g_0 \end{bmatrix}$$

$X_{new} = X_{old} + \Delta X$

$y_{new} = y_{old} + \Delta y$

$$\begin{bmatrix} f_{1x_1} & f_{1x_2} & f_{1x_3} & \cdots & f_{1x_n} \\ f_{2x_1} & f_{2x_2} & f_{2x_3} & & f_{2x_n} \\ \vdots & & & & \\ f_{nx_1} & f_{nx_2} & f_{nx_3} & \cdots & f_{nx_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \vdots \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} -f_1 \\ -f_2 \\ -f_3 \\ \vdots \\ -f_n \end{bmatrix}$$

$\boxed{26/2}$

Example: Newton Raphson method.

$$A + B \rightleftharpoons C + 3D \quad \xi_1 \quad K_{eq_1} = 3.76 \text{ mol/L}$$

$$C + B \rightleftharpoons E + D \quad \xi_2 \quad K_{eq_2} = 1.2 \text{ mol/L}$$

$$C_A^\circ = 50 \text{ Kmol/L}$$

$$C_B^\circ = 125 \text{ Kmol/L}$$

$$\Rightarrow \quad C_A = C_{A0} - \xi_1$$

$$C_B = C_{B0} - \xi_1 - \xi_2$$

$$C_C = \cancel{C_{C0}} + \xi_1 - \xi_2$$

$$C_D = \cancel{C_{D}^\circ} + 3\xi_1 + \xi_2$$

$$C_E = \cancel{C_{E0}} + \xi_2$$

$$K_{eq_1} = \frac{C_C\, C_D^3}{C_A\, C_B} = 3.76 = \frac{(\xi_1 - \xi_2)(3\xi_1 + \xi_2)^3}{(50 - \xi_1)(125 - \xi_1 - \xi_2)}$$

$$K_{eq_2} = \frac{C_E\, C_D}{C_C\, C_B} = 1.2 = \frac{(\xi_2)(3\xi_1 + \xi_2)}{(\xi_1 - \xi_2)(125 - \xi_1 - \xi_2)}$$

$$f_1 = \frac{(\xi_1 - \xi_2)(3\xi_1 + \xi_2)^3}{(50 - \xi_1)(125 - \xi_1 - \xi_2)} - 3.76 = 0$$

$$f_2 = \frac{\xi_2 (3\xi_1 + \xi_2)}{(\xi_1 - \xi_2)(125 - \xi_1 - \xi_2)} - 1.2 = 0$$

$$\bar{X}_{new} = \bar{X}_{old} - \begin{bmatrix} \frac{\partial P_1}{\partial \Sigma_1} & \frac{\partial P_1}{\partial \Sigma_2} \\ \frac{\partial P_2}{\partial \Sigma_1} & \frac{\partial P_2}{\partial \Sigma_2} \end{bmatrix}^{-1} * \begin{bmatrix} -P_1 \\ -P_2 \end{bmatrix}$$

$$\begin{bmatrix} \Sigma_1 \\ \Sigma_2 \end{bmatrix}_{new} = \begin{bmatrix} \Sigma_1 \\ \Sigma_2 \end{bmatrix}_{old} - \cdots * \cdots$$

$$\frac{\partial P_1}{\partial \Sigma_1} = -\frac{9(\Sigma_1 - \Sigma_2)(3\Sigma_1 + \Sigma_2)^2}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)} + \frac{(\Sigma_1 - \Sigma_2)(3\Sigma_1 + \Sigma_2)^3}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)^2}$$

$$-\frac{(3\Sigma_1 + \Sigma_2)^3}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)} + \frac{(\Sigma_1 - \Sigma_2)(3\Sigma_1 + \Sigma_2)^3}{(-50+\Sigma_1)^2(-125+\Sigma_1+\Sigma_2)}$$

$$\frac{\partial P_1}{\partial \Sigma_2} = \frac{3(\Sigma_1 - \Sigma_2)(3\Sigma_1 + \Sigma_2)^2}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)} + \frac{(\Sigma_1 - \Sigma_2)(3\Sigma_1 + \Sigma_2)^3}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)^2}$$

$$+\frac{3(\Sigma_1 + \Sigma_2)^3}{(-50+\Sigma_1)(-125+\Sigma_1+\Sigma_2)}$$

$$\frac{\partial P_2}{\partial \Sigma_1} = \frac{3\Sigma_2}{(\Sigma_1 - \Sigma_2)(-125+\Sigma_1+\Sigma_2)} - \frac{\Sigma_2(3\Sigma_1 + \Sigma_2)}{(\Sigma_1 - \Sigma_2)(-125+\Sigma_1+\Sigma_2)^2}$$

$$-\frac{\Sigma_2(3\Sigma_1 + \Sigma_2)}{(\Sigma_1 - \Sigma_2)^2(-125+\Sigma_1+\Sigma_2)}$$

$$\frac{\partial P_2}{\partial \Sigma_2} = \frac{\Sigma_2}{(\Sigma_1 - \Sigma_2)(-125+\Sigma_1+\Sigma_2)} - \frac{\Sigma_2(3\Sigma_1 + \Sigma_2)}{(\Sigma_1 - \Sigma_2)(-125+\Sigma_1+\Sigma_2)^2}$$

$$+\frac{3\Sigma_1 + \Sigma_2}{(\Sigma_1 - \Sigma_2)(-125+\Sigma_1+\Sigma_2)} + \frac{\Sigma_2(3\Sigma_1 + \Sigma_2)}{(\Sigma_1 - \Sigma_2)^2(-125+\Sigma_1+\Sigma_2)}$$

Assume $\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 44.42 \\ 15.33 \end{bmatrix}$

$$\begin{bmatrix} \frac{\partial P_1}{\partial \tau_1} & \frac{\partial P_1}{\partial \tau_2} \\ \frac{\partial P_2}{\partial \tau_1} & \frac{\partial P_2}{\partial \tau_2} \end{bmatrix} = \begin{bmatrix} -7587 & -298 \\ -0.0013 & -0.146 \end{bmatrix}$$

$$\text{Inv (Jacobian)} = \begin{bmatrix} -1.3 \times 10^{-5} & 0.026 \\ 1.23 \times 10^{-7} & -68 \end{bmatrix}$$

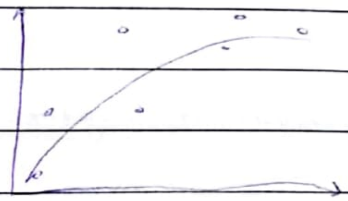
$$F = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} -262115 \\ -0.00007 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 44.42 \\ 15.33 \end{bmatrix}$$

Curve fitting and interpolation

x	y_{ex}	y_{cal}
0	0	0
1	2	2
2	4	4
3	6	6

$y = 2x$



irregular

1. Least square method

$$y = a + bx$$

$$error = \Sigma (y_{exp} - y_{cal})^2 \rightarrow min$$

$$\frac{\partial error}{\partial a} = 0 = 2 \Sigma (y_{exp} - y_{cal}) \frac{dy_{cal}}{da}$$

$$= 2 \Sigma (y_{exp} - a - bx)(-1)$$

$$= -2 \sum_{i=1}^{n} (y_{exp} - a - bx)$$

$$\frac{\partial error}{\partial b} = 0 = -2 \sum_{i=1}^{n} (y_{exp} - a - bx) \frac{dy}{db}^{-x}$$

$$\Sigma (y_{exp} - a - bx) = 0$$

$$\Sigma (y_{exp} - a - bx) x = 0$$

$$\Sigma y_{exp} = \sum_{i=1}^{n} a + \Sigma bx$$

$$\Sigma y_{exp} x = \Sigma a x + \Sigma b x^2$$

$$\Sigma y_{exp} = na + b \Sigma x$$

$$\Sigma y_{exp} = a \Sigma x + b \Sigma x^2$$

Variables a, b

$$\begin{bmatrix} n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma y_{exp} \\ x\Sigma y_{exp} \end{bmatrix}$$

For system of 2 equation with 2 unknowns

$$a = \frac{\Sigma y_{exp,i} - b\,\Sigma x_i}{n}$$

$$b = \frac{n\,\Sigma y_{exp,i} - \Sigma y_{exp,i}\,\Sigma x_i}{n\,\Sigma x_i^2 - (\Sigma x_i)^2}$$

Example 8:

[illegible]

i	x_i	y_i	$x_i y_i$	x_i^2	$(y_{exp} - y_{cal})^2$	y_{cal}
1	1	4	4	1	0.49	3.3
2	2	6	12	4	0.16	5.6
3	3	5	15	9	8.41	7.9
4	4	12	48	16	3.24	10.2
	10	27	79	30	12.30	

$\boxed{y, x}$ [illegible] is not factor to determinate accurate.

$$b = \frac{n\Sigma y_i x_i - \Sigma x_i\,\Sigma y_i}{n\,\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{4(79) - (10)(27)}{4(30) - (10)^2} = 2.3$$

$$a = \frac{\Sigma y_i - b\,\Sigma x_i}{n} = \frac{27 - 2.3(10)}{4} = 1$$

$$y_{cal} = a + bx$$

$$\boxed{y = 1 + 2.3x}$$

$$R^2 = \frac{-\Sigma(y_{cal\,i} - y_{exp})^2 + \Sigma(y_{i\,cal} - y_{i\,avg})^2}{\Sigma(y_{i\,cal} - y_{i\,avg})^2}$$

$$0 \le R^2 \le 1$$

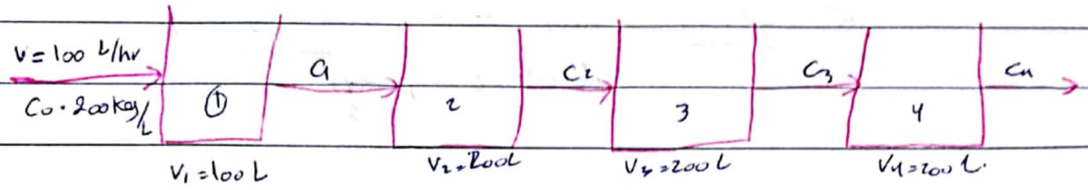
$$R^2 \gg 0.9 \checkmark$$

$$y_{avg} = \frac{\Sigma y_i}{n} = \frac{27}{4} = 6.75$$

$(y_i - y_{avg})^2$
7.56
0.56
3.06
27.56
38.74

$$R^2 = \frac{-12.3 + 38.74}{38.74} = 0.68$$ Por fit For this model

• ولكن 0.99 كان كانت ممتازة



$r = KC_A = 2C_A$

$\Rightarrow \quad F_0 - F_1 + r_1 V = \frac{dc}{dt} \nearrow 0$

$F = VC$

$\frac{mol}{time} = \frac{\cancel{L}}{time} \times \frac{mol}{\cancel{L}}$

$VC_0 - VC_1 - 2C_1V_1 = 0$ in - out - rate of reaction = 0

$VC_1 - VC_2 - 2C_2V_2 = 0$

$VC_2 - VC_3 - 2C_3V_3 = 0$

$VC_3 - VC_4 - 2C_4V_4 = 0$

$100 \times 200 - 100\, C_1 - 2 \times 100\, C_1 = 0$ $\quad 20{,}000 - 100\, C_1 - 200\, C_1 = 0$

$100 \times C_1 - 100\, C_2 - 2 \times 200\, C_2 = 0$ $\quad 100\, C_1 - 100\, C_2 - 400\, C_2 = 0$

$100 \times C_2 - 100\, C_3 - 2 \times 200\, C_3 = 0 \Rightarrow$ $\quad 100\, C_2 - 100\, C_3 - 400\, C_3 = 0$

$100 \times C_3 - 100\, C_4 - 2 \times 200\, C_4 = 0$ $\quad 100\, C_3 - 100\, C_4 - 400\, C_4 = 0$

$20{,}000 - 300\, C_1 = 0$

$100\, C_1 - 500\, C_2 = 0$

$100\, C_2 - 500\, C_3 = 0$

$100\, C_3 - 500\, C_4 = 0$

$$\begin{bmatrix} 300 & 0 & 0 & 0 \\ 100 & -500 & 0 & 0 \\ 0 & 100 & -500 & 0 \\ 0 & 0 & 100 & 500 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 20{,}000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \frac{20{,}000}{300} = 66.7$$

$$C_2 = \frac{100(66.7)}{500} = 13.3$$

$$C_3 = \frac{100\ (13.3)}{500} = 2.67$$

$$C_4 = \frac{100\ (2.67)}{500} = 0.5$$

Code → Fittness 8

```
A = [x1 y1 ; x2 y2 ; .....]

sum x = 0
sum y = 0
sum x^2 = 0
sum xy = 0
[n, m] = size (A)
For i=1:n
sumx = sumx + x(i,1)
sumy = sumy + y(i,2)
sumx^2 = sumx^2 + x(i,1)^2
sumxy = sumxy + x(i,1) x y(i,2)
end
Jac = [n  sumx ; sumx  sumx^2]
y = [sumy ; sumxy]
ab = inv(Jac) * y
a = ab(1)
b = ab(2)
```

12/3/2019

Multiple Linear Regression (Fitting)

$$y = a_1 + a_2 x_1 + a_3 x_2 + a_4 x_3 + \cdots$$

x_1	x_2	y	x_1^2	x_2^2	$x_1 x_2$
1	0.1	2			
2	3	8			
3	6	10			
4	8	15			

$$Err = \Sigma (y_{cal} - y_{iexp})^2$$

$$Err = \Sigma (a_1 + a_2 x_1 + a_3 x_2 - y_{exp})^2$$

$$\frac{\partial Err}{\partial a_1} = 0 = \Sigma a_1 + \Sigma a_2 x_1 + \Sigma a_3 x_2 - \Sigma y_{exp}$$

$$\frac{\partial Err}{\partial a_2} = 0 = \Sigma a_1 x_1 + \Sigma a_2 x_1^2 + \Sigma a_3 x_1 x_2 - \Sigma x_1 y_{exp}$$

$$\frac{\partial Err}{\partial a_3} = 0 = \Sigma a_1 x_2 + \Sigma a_2 x_1 x_2 + \Sigma a_3 x_2^2 - \Sigma x_2 y_{exp}$$

$$\Rightarrow \Sigma a_1 + a_2 \Sigma x_1 + a_3 \Sigma x_2 = \Sigma y_{exp}$$

$$a_1 \Sigma x_1 + a_2 \Sigma x_1^2 + a_3 \Sigma x_1 x_2 = \Sigma x_1 y_{exp}$$

$$a_1 \Sigma x_2 + a_2 \Sigma x_1 x_2 + a_3 \Sigma x_2^2 = \Sigma x_2 y_{exp}$$

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \Sigma y_{exp} \\ \Sigma x_1 y_{exp} \\ \Sigma x_2 y_{exp} \end{bmatrix}$$

كمان تطبيق

$$y = a_1 + a_2 x_1 + a_3 x_2 + a_4 x_3$$

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 & \Sigma x_3 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 & \Sigma x_1 x_3 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 & \Sigma x_2 x_3 \\ \Sigma x_3 & \Sigma x_1 x_3 & \Sigma x_2 x_3 & \Sigma x_3^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \Sigma y_{exp} \\ \Sigma x_1 y_{exp} \\ \Sigma x_2 y_{exp} \\ \Sigma x_3 y_{exp} \end{bmatrix}$$

Example 8:

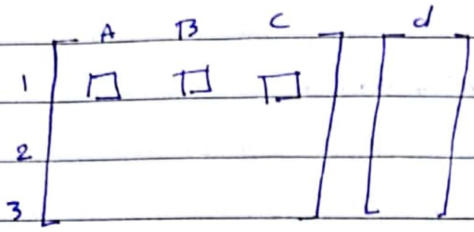
#	x_1	x_2	y	x_1^2	x_2^2	$x_1 x_2$	$x_1 y$	$x_2 y$
1	0	0	3	0	0	0	0	0
2	1	3	1	1	9	3	1	3
3	2	2	7	4	4	4	14	14
4	3	5	5	9	25	15	15	25
5	4	7	5	16	49	28	20	35
6	5	1	21	25	1	5	105	21
	$\Sigma x_1 = 15$	$= 18$	$= 42$	$= 55$	$= 88$	55	$= 155$	$= 98$

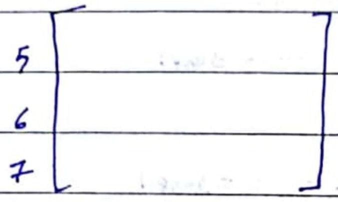
$$\begin{bmatrix} 6 & 15 & 18 \\ 15 & 55 & 55 \\ 18 & 55 & 88 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 42 \\ 155 \\ 98 \end{bmatrix}$$

بنحلها بأي طريقة من الطرق يلي مرت

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 6 & 15 & 18 \\ 15 & 55 & 55 \\ 18 & 55 & 88 \end{bmatrix}^{-1} \begin{bmatrix} 42 \\ 155 \\ 98 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

Excel





F1 = INVERSE (بنظلل)

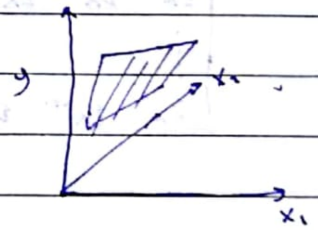
CTR + Shift + Enter

حل اليوم بتعطيني.

F = mmult (A5 : A7) * (d1 : d3)

Fitting of nonLinear Equation

x	y	$\ln x$ (K)	$\ln y$ (z)



$$y = a x^b$$

$$\ln y = \ln a + b \ln x$$

$$z = \alpha + bK$$

$$b = b$$

$$\alpha = \ln a \rightarrow a = \exp(\alpha)$$

17/3/2019

polynomial Regression

$$y = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + \dots$$

$$err = \sum (y - y_{exp})^2$$
$$= \sum (a_1 + a_2 x + a_3 x^2 + \dots - y_{exp})^2$$

eq.1 $\frac{\partial err}{\partial a_1} = 0 = 2 \sum (a_1 + a_2 x \dots - y_{exp})$

eq.2 $\frac{\partial err}{\partial a_2} = 0 = 2x \sum (a_1 + a_2 x \dots - y_{exp})$

eq.3 $\frac{\partial err}{\partial a_3} = 0 = 2x^2 \sum (a_1 + a_2 x \dots - y_{exp})$

$\vdots$

eq.n $\frac{\partial err}{\partial a_n} = 0 = 2x^{n-1} \sum (a_1 + a_2 x + \dots - y_{exp})$

$$\begin{bmatrix} n & \sum x & \sum x^2 & \sum x^3 & \dots & \sum x^n \\ \sum x & \sum x^2 & \sum x^3 & \sum x^4 & \dots & \sum x^{n+1} \\ \sum x^2 & \sum x^3 & \sum x^4 & \sum x^5 & \dots & \sum x^{n+2} \\ \vdots & & & & & \\ \sum x^n & \sum x^{n+1} & \sum x^{n+2} & \sum x^{n+3} & \dots & \sum x^{2n} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \\ \vdots \\ \sum x^n y \end{bmatrix}$$

Special case

$$y = a_1 + a_2 x + a_3 x^2$$

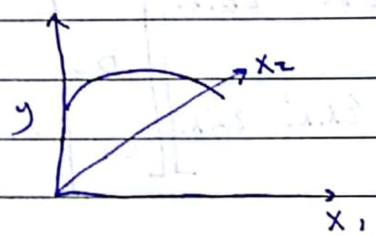
$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \end{bmatrix}$$

Multi Variable , Polynomial Regression

$$y = a_0 + a_1 x_1 + a_2 x_1^2 + b_1 x_2 + b_2 x_2^2 + c_1 x_1 x_2$$

$$Err = \Sigma (y - y_{exp})^2 = \Sigma (a_0 + a_1 x_1 + a_2 x_1^2 + b_1 x_2 + b_2 x_2^2 + c_1 x_1 x_2 - y_{exp})^2$$

x_1	x_2	y
⋮	⋮	⋮



$\Rightarrow$ $\dfrac{\partial err}{\partial a_0} = 0 = 2\,\Sigma (y - y_{exp})$

$\dfrac{\partial err}{\partial a_1} = 0 = 2 x_1 \Sigma (y - y_{exp})$

$\dfrac{\partial err}{\partial a_2} = 0 = 2 x_1^2 \Sigma (y - y_{exp})$

$\dfrac{\partial err}{b_1} = 0 = 2 x_2 \Sigma (y - y_{exp})$

$\dfrac{\partial er}{b_2} = 0 = 2 x_2^2 \Sigma (y - y_{exp})$

$\dfrac{\partial err}{\partial c_1} = 0 = 2 x_1 x_2 \Sigma (y - y_{exp})$.

$$
\begin{bmatrix}
n & \Sigma x_1 & \Sigma x_1^2 & \Sigma x_2 & \Sigma x_2^2 & \Sigma x_1x_2 \\
\Sigma x_1 & \Sigma x_1^2 & \Sigma x_1^3 & \Sigma x_1x_2 & \Sigma x_1x_2^2 & \Sigma x_1^2x_2 \\
\Sigma x_1^2 & \Sigma x_1^3 & \Sigma x_1^4 & \Sigma x_1^2x_2 & \Sigma x_1^2x_2^2 & \Sigma x_1^3x_2 \\
\Sigma x_2 & \Sigma x_1x_2 & \Sigma x_1^2x_2 & \Sigma x_2^2 & \Sigma x_2^3 & \Sigma x_1x_2^2 \\
\Sigma x_2^2 & \Sigma x_1x_2^2 & \Sigma x_1^2x_2^2 & \Sigma x_2^3 & \Sigma x_2^4 & \Sigma x_1x_2^3 \\
\Sigma x_1x_2 & \Sigma x_1^2x_2 & \Sigma x_1^3x_2 & \Sigma x_1x_2^2 & \Sigma x_1x_2^3 & \Sigma x_1^2x_2^2
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_1 \\ b_2 \\ c_1 \end{bmatrix}
=
\begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_1^2 y \\ \Sigma x_2 y \\ \Sigma x_2^2 y \\ \Sigma x_1x_2 y \end{bmatrix}
$$

جاي بيجي سؤال بالامتحان

Example

x_1	x_2	y	x_1^2	x_2^2	x_1x_2	x_1^3	$x_1x_2^2$	$x_1^2x_2$	x_2^3	x_1^4	$x_1^2x_2^2$
0	0	0									
6	1	8									
2	0	20									
1	1	20									
2	2	70									
5	6	488									
Σ = 10	10	606	34	42	35	142	189	159	226	658	917

$x_1^3x_2$	x_2^4	$x_1x_2^3$	x_1y	x_2y	x_1^2y	x_2^4y	x_1x_2y
767	1314	1097	2640	3096	12580	17876	14970

$$\begin{bmatrix} 6 & 10 & 10 & 34 & 42 & 35 \\ 10 & 34 & 35 & 142 & 189 & 159 \\ 10 & 35 & 42 & 159 & 226 & 189 \\ 34 & 142 & 159 & 658 & 917 & 767 \\ 42 & 189 & 226 & 917 & 1314 & 1097 \\ 35 & 159 & 189 & 767 & 1097 & 917 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} 606 \\ 2640 \\ 3096 \\ 12580 \\ 17876 \\ 14940 \end{bmatrix}$$

$C_1 = 0$

$C_2 = 2$

$C_3 = 3$

$C_4 = 4$

$C_5 = 5$

$C_6 = 6$

19/3/2019

Interpolation

- one dimentional
- two dimentinal

→ Lagrange interpolation

→ Newton's "

→ Spline "

→ Lagrange interpolation

x	y
1	2
3	8
5	12
7	18

$4 \rightarrow$ ← ?

$$\frac{y - y_0}{x - x_0} = \frac{y_2 - y_0}{x_2 - x_0}$$

not acurate

$$f(x) = a_1 + a_2 x + a_3 x^2 + \cdots$$

$$f(x) = \sum_{j=0}^{n} L_j(x)\, f(x_j)$$

$$L_j(x) = \prod_{\substack{k=0 \\ k \neq j}}^{n} \frac{x - x_k}{x_j - x_k}$$

Example (case 1)

$$f(x) = a + bx$$

$$f_1(x) = \sum_{j=0}^{1} L_j(x)\, f(x_j)$$

$$= L_0(x) f(x_0) + L_1(x) f(x_1)$$

$$L_0(x) = \cancel{\frac{x - x_0}{x_0 - x_0}} + \frac{x - x_1}{x_0 - x_1}$$

$\frac{x}{0}$ باطل

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} + \cancel{\frac{x - x_1}{x_1 - x_1}}$$

$$f_1(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right) f(x_1)$$

Example (2)

	x	y	
$x_0 \rightarrow$	2	1.5	$\leftarrow f(x_0)$
$3 \rightarrow$			$\leftarrow ?$
$x_1 \rightarrow$	5	4	$\leftarrow f(x_1)$
	8	6.5	
	11	9	

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$= \frac{x - 5}{2 - 5}(1.5) + \frac{x - 2}{5 - 2}(4)$$

$$f_1(x) = \frac{2.5}{3} x - \frac{1}{6}$$

$$f_1(3) = \frac{2.5}{3}(3) - \frac{1}{6} = 2.33$$

Quadratic interpolation

$n=2$

$$P_2(x) = \sum_{j=0}^{2} L_j(x)\, f(x_j)$$

$$= L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$$

$$L_0(x) = \prod_{\substack{k=0 \\ k\neq j}}^{2} \frac{x - x_k}{x_j - x_k} = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2}$$

$$L_1(x) = \prod_{\substack{k=0 \\ k\neq j}}^{2} \frac{x - x_k}{x_j - x_k} = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

$$L_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1}$$

Example

	x	y	
	1	0	
$x_0 \rightarrow$	2	1	$\leftarrow f(x_0)$
$x_1 \rightarrow$	3	2	$\leftarrow f(x_1)$
$x_2 \rightarrow$	4	3	$\leftarrow f(x_2)$
	5	5	
	6	7	

$f(2.5) = ??$

2 value before - 1 value after

or 1 value after, 2 value before.

$$F(x) = \frac{x-3}{2-3} \times \frac{x-4}{2-4} F(x_0) + \frac{x-2}{3-2} \times \frac{x-4}{3-4} F(x_1)$$

$$+ \frac{x-2}{4-2} \times \frac{x-3}{4-3} F(x_2)$$

$$F(2.5) = 0.375 + 2(0.75) + 3(-0.125)$$

$$= 1.5$$

24/3/2019

$$y = f(x) = \sum L_i(x)\, y_i$$

$$L_i(x) = \frac{(x-x_0)(x-x_1)(x-x_2)\cdots}{(x_i-x_0)(x_i-x_1)(x_i-x_2)\cdots}$$

where:- $(x_i - x_k) \neq 0$

$k = 0, 1, 2, 3, \ldots$

Algorithem

مش فاهم .

```
for i=1 , n-data
 For j=1 , n-date
  if (i≠j) then
  L(j:) = L(i:) * (x - x(j)) / (x(i) - x(j))
   ond
  end
end
 y=0
for i=1 , n data
 y = y + y(i) * L(i:)
end.
```

Newton's interpolation

$$f(x) = a_0 + a_1(x - x_0)$$

$n = 1$ 8

$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0} \cdot (x - x_0)$$

$n = 2$ 8

$$f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$+ \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}(x - x_0)(x - x_1)$$

For higher orders

$$f(x) = a_0 + \sum_{i=0}^{n} \prod_{j=0}^{i} (x - x_j) \quad a_{i+1}$$

Example :

$i =$	0	1	2	3	4
$x_i =$	0	1	2	3	4
$y_i =$	1	2.25	3.75	4.25	5.81

Find $f(2.4)$ using third order N.R-method.

$$F_3(x) = \sum_{k=0}^{3} a_k \prod_{i=0}^{k-1} (x - x_i)$$

$$= \overset{a_0}{f(x_0)} + \overset{a_1}{f(x_0, x_1)}(x - x_0)$$

$$+ \overset{a_2}{f(x_0, x_1, x_2)}(x - x_0)(x - x_1)$$

$$+ \overset{a_3}{f(x_0, x_1, x_2, x_3)}(x - x_0)(x - x_1)(x - x_2)$$

$a_0 = f(x_0) = 1$

$a_1 = f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{2.25 - 1}{1 - 0} = 1.25$

$a_2 = f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$b_1 \; f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{3.75 - 2.25}{2 - 1} = 1.5$

$a_2 = \frac{1.5 - 1.25}{2 - 0} = 0.125$

$a_3 = f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$

$\Rightarrow b_3 \; f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$

$\Rightarrow b_2 \; f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \frac{4.25 - 3.75}{3 - 2} = 0.5$

$f(x_1, x_2, x_3) = \frac{0.5 - 1.5}{3 - 1} = -0.5$

$a_3 = \frac{-0.5 - 0.125}{3 - 0} = -0.208333$

$\Rightarrow F_3(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$

$= 1 + 1.25(x - 0) + 0.125(x - 0)(x - 1) + -0.20833(x - 0)(x - 1)(x - 2)$

$f(2.4) = 4.22$

i	x	y
1	x_1	y_1
2	x_2	y_2
3	x_3	y_3
4	x_4	y_4
5	x_5	y_5
6	x_6	y_6

d_1

$\frac{y_2 - y_1}{x_2 - x_1} = f_{11}$

$\frac{y_3 - y_2}{x_3 - x_2} = f_{21}$

$\frac{y_4 - y_3}{x_4 - x_3} = f_{31}$

$\frac{y_5 - y_4}{x_5 - x_4} = f_{41}$

$\frac{y_6 - y_5}{x_6 - x_5} = f_{51}$

d_2

$\frac{f_{21} - f_{11}}{x_3 - x_1} = f_{12}$

$\frac{f_{31} - f_{21}}{x_4 - x_2} = f_{22}$

$\frac{f_{41} - f_{31}}{x_5 - x_3} = f_{32}$

$\frac{f_{51} - f_{41}}{x_6 - x_4} = f_{42}$

d_3

$\frac{f_{22} - f_{12}}{x_4 - x_1} = f_{13}$

$= f_{14}$

$\frac{f_{32} - f_{22}}{x_5 - x_2} -$

$\frac{f_{42} - f_{32}}{x_6 - x_3} = f_{15}$

d_4

$\frac{f_{14} - f_{13}}{x_5 - x_1} = f_{14}$

$\frac{f_{15} - f_{14}}{x_6 - x_2} = f_{24}$

d_5

$\frac{f_{24} - f_{14}}{x_6 - x_1} = f_{15}$

$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$

$F(x) \quad \sum_{k=0}^{n} a_k \prod_{j=0}^{k-1} (x - x_{j-1})$

Example :

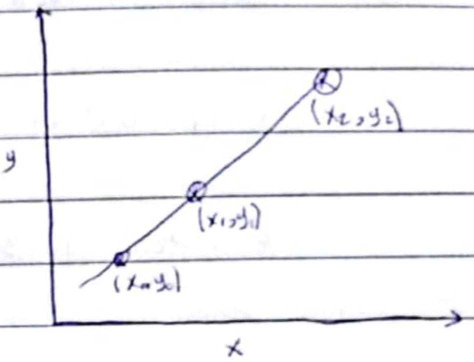
x	y	d_1	d_2	d_3	d_4
0	1				
		1.25			
1	2.25		0.125		
		1.5		-0.208	
2	3.75		-0.5		0.137
		0.5		0.34	
3	4.25		~~0.53~~ 0.53		
		~~0.44~~ 1.56			
4	5.81				

2nd Midterm exam

26/3/2019

Spline interpolation

$$\frac{y-y_0}{x-x_0} = \frac{y_2-y_1}{x_2-x_1} = \frac{y_1-y_0}{x_1-x_0}$$



$$y = \left(\frac{y_1-y_0}{x_1-x_0}\right)(x-x_0) + y_0$$

$$= \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1) + y_1$$

example

x	y
0	0
10	227
15	362
→ 20	517
30	901

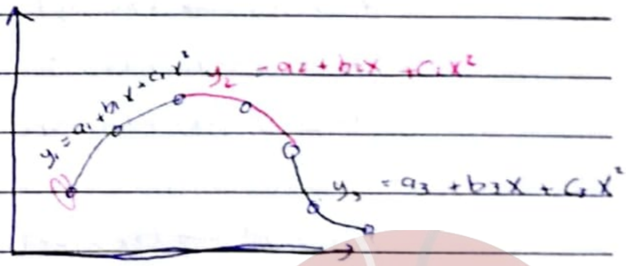
$y(16) = ?$

$$\frac{y-y_0}{x-x_0} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-362}{x-15} = \frac{517-362}{20-15}$$

@ $x = 16 \rightarrow y = 393$

يعني 3 معادلات بـ 9 متغيرات .



عدد المعادلات $= \dfrac{(\text{عدد النقاط} - 1)}{2}$

$f(x) = ax^2 + bx + c$

$f_1(x) = a_1 x^2 + b_1 x + c_1$

$f_2(x) = a_1 x^2 + b_1 x + c_1$

$\vdots$

$f_n(x) = a_n x^2 + b_n x + c_n$

equation $3N-3$, we need $3N-3$ parameter.

$(x_0, y_0) \rightarrow (x_1, y_1)$

$a_1 x_0^2 + b_1 x_0 + c_1 = y_0$

$a_1 x_1^2 + b_1 x_1 + c_1 = y_1$

$\frac{d}{dx}(a_1 x_1^2 + b_1 x_1 + c_1) = \frac{d}{dx}(a_2 x_1^2 + b_2 x_1 + c_2)$

$(x_1, y_1) \rightarrow (x_2, y_2)$.

$a_2 x_1^2 + b_2 x_1 + c_2 = y_1$

$2a_1 x_1 + b_1 = 2a_2 x_1 + b_2$

$a_2 x_2^2 + b_2 x_2 + c_2 = y_2$

$(x_2, y_2) \rightarrow (x_3 - y_3)$

$2a_2 x_2 + b_2 = 2a_3 x_2 + b_3$

$a_3 x_2^2 + b_3 x_2 + c_3 = y_2$

$a_3 x_3^2 + b_3 x_3 + c_3 = y_3$

$(x_3, y_3) \rightarrow (x_4, y_4)$

$2a_3 x_3 + b_3 = 2a_4 x_3 + b_4$

$a_4 x_3^2 + b_4 x_3 + c_4 = y_3$

$a_4 x_4^2 + b_4 x_4 + c_4 = y_4$

$\vdots$

$(x_6, y_6) \rightarrow (x_7, y_7)$

$a_7 x_6^2 + b_7 x_6 + c_7 = y_6$

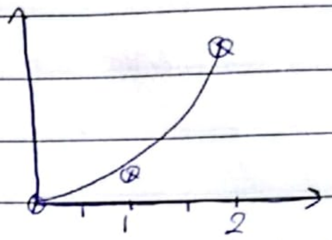
Last equation

$a_7 x_7^2 + b_7 x_7 + c_7 = y_7$

$a_1 = 0$

or $a_6 = 0$

	x	y	
x_0	0	0	y_0
x_1	1	1	y_1
x_2	2	4	y_2



n data = 3

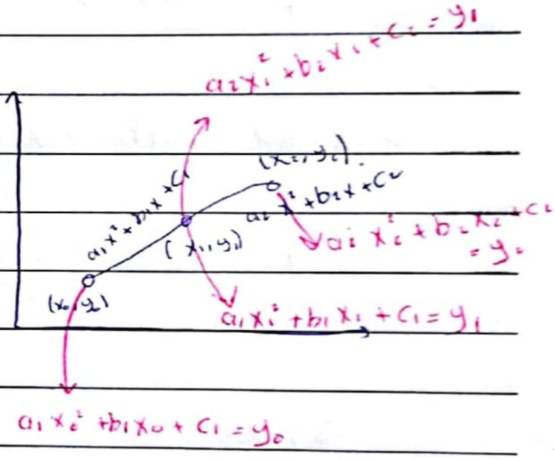
n - equation = 3(3) - 3 = 6

Eq 1 $\rightarrow$: $a_1 x_0^2 + b_1 x_0 + c_1 = y_0$

$\rightarrow a_1 x_1^2 + b_1 x_1 + c_1 = y_1$

$\rightarrow a_2 x_1^2 + b_2 x_1 + c_2 = y_1$

$\rightarrow a_2 x_2^2 + b_2 x_2 + c_2 = y_2$



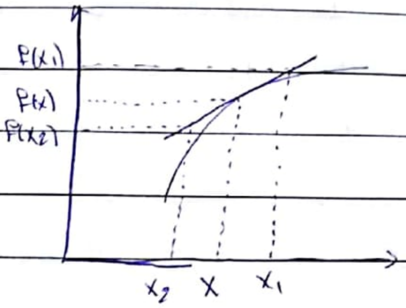
~~$a_1 x_2^2 + b_2 x_2 + c_2 = y_2$~~

$\frac{d}{dx}(a_1 x^2 + b_1 x + c_1)\big|_{x=1} = \frac{d}{dx}(a_2 x^2 + b_2 x + c_2)\big|_{x_1}$

$\rightarrow 2a_1 x_1 + b_1 = 2a_2 x_1 + b_2.$

$\rightarrow a_1 = 0$

Numerical Differentiation



$$f'(x) = \frac{d f(x)}{dx} = \frac{\Delta f(x)}{\Delta X}$$

For word Differentiation:

$$f'(x) = \frac{\Delta f(x)}{\Delta X}$$

$$f'_F(x) = \frac{f_{i+1} - f_i}{x_{i+1} - x_i} = \frac{f_{i+1} - f_i}{\Delta X}$$

F

Back word Differatiation:

B

$$f'_B(x) = \frac{f_i - f_{i-1}}{x_i - x_{i-1}} = \frac{f_i - f_{i-1}}{\Delta X}$$

Contral:

$$f'_c = \frac{f_{i+1} - f_{i-1}}{x_{i+1} - x_{i-1}} = \frac{f_{i+1} - f_{i-1}}{2\Delta X}$$

Example:

$f(x) = e^x$

$\Delta X = 0.01$

$x = 1$

$f'(x) = e^x$

$f'(1) = e^1 = 2.71$

$i \to x_i$
$1 \to \Delta x$

Forword

$$f'(x) = \frac{f(i+1) - f(i)}{x_{i+1} - x_i} = \frac{f(x_i + \Delta x) - f(x_i)}{(x_i + \Delta x) - (x_i)} = \frac{f(x_i + \Delta x) - f(x)}{\Delta x}$$

$$= \frac{e^{(1+0.01)} - e^{(1)}}{1+0.01-1} = \frac{e^{1.01} - e^{1}}{0.01} = 2.7319$$

$$err = \frac{2.7319 - 2.71}{2.71} = 0.8\%$$

Backword

$$f'(x) = \frac{f(x_i) - f(x_i - \Delta x)}{\Delta x} = \frac{e^{1} - e^{1-0.01}}{0.01} = \frac{e^{1} - e^{0.99}}{0.01} = 2.704$$

$$err = \frac{2.704 - 2.71}{2.71} = 0.4\%$$

Central

$$f'(x) = \frac{f(x_i + \Delta x) - f(x_i - \Delta x)}{2\Delta x}$$

$$= \frac{e^{1.01} - e^{0.99}}{2(0.01)} = 2.7183$$

$err = 0.001\ \%$

3-point Derivative.

$$f'(x) = \frac{-f(i+2) + 4f(i+1) - 3f(i)}{2\Delta x}$$

$$= \frac{-f(x_i - 2\Delta x) + 4(x_i + \Delta x) - 3f(x_i)}{2\Delta x}$$

4-point Derivative.

$$f'(x) = \frac{-f(x_i - 2\Delta x) + 8f(x_i + \Delta x) - 8f(x_i - \Delta x) + f(x_i - 2\Delta x)}{12\Delta x}$$

Example

$$f(x) = \frac{-e^{(1+2(0.01))} + 4e^{1+0.01} - 3e^{1}}{2(0.01)}$$

$$= \frac{-e^{1.02} + 4e^{1.01} - 3e^{1}}{0.02} = \frac{-2.77 + 10.98 - 8.15}{0.02} = 2.71$$

2nd Derivation

$$f''_F(x) = \frac{f_{i+2} - 2f_{i+1} + f_i}{(Dx)^2}$$ Forword.

$$f_{B,C}(x) = \frac{f_{i+1} - 2f_i + f_{i-1}}{(Dx)^2}$$ Backword + contral.

Example

$f(x) = e^x$, $x = 1$, $Dx = 0.01$

$$f''_F(1) = \frac{f(1+2(0.01)) - 2f(1+0.01) + f(1)}{(0.01)^2}$$

$$= \frac{e^{1.02} - 2e^{1.01} + e^1}{(0.01)^2} = 2.7456$$

$$f''_{B,C}(x) = \frac{e^{1.01} - 2e^1 + e^{0.99}}{(0.01)^2} =$$

3rd Derivation

$$f'''_F(x) = \frac{f(x_i + 3Dx) - 3f(x_i + 2Dx) + 3f(x_i + Dx) - f(x_i)}{(Dx)^3}$$

$$f'''_C(x) = \frac{f(x_i + 2Dx) - 2f(x_i + Dx) + 2f(x_i - Dx) - f(x_i - 2Dx)}{2(Dx)^3}$$

Example

$f(x) = x^3$, $x = 1$, $\Delta x = 0.01$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f'''(x) = 6$

$$f'''_F(x) = \frac{(1.03)^3 - 3(1.02)^3 + 3(1.01)^3 - (1)^3}{(0.01)^3} = 6$$

$$f'''_C(x) = \frac{(1.02)^3 - 2(1.01)^3 + 2(0.99)^3 - (0.98)^3}{2(0.01)^3} = 6.$$

4th Derivative

$$f^{IV}_F(x) = \frac{f(x_i + 4\Delta x) - 4f(x_i + 3\Delta x) + 6f(x_i + 2\Delta x) - 4f(x + \Delta x) + f(x_i)}{(\Delta x)^4}$$

$$f^{IV}_C(x) = \frac{f(x_i + 2\Delta x) - 4f(x + \Delta x) + 6f(x_i) - 4f(x_i - \Delta x) + f(x_i - 2\Delta x)}{(\Delta x)^4}$$

Homework

$f(x) = x^6 \sin x \quad e^x$

$x = 1$

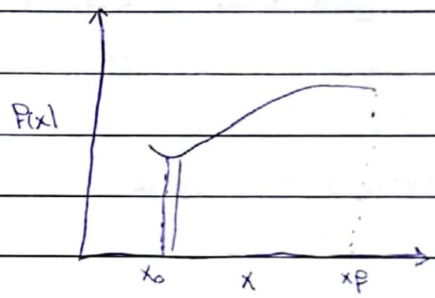
$\Delta x = 0.01$

14/4/2019

Numerical integration.

1- Trapizodal rule
2. Simpsons Rule.
3- Gaussian
4-

1. Trapizoidal rule



$$I = \Delta x \left[\frac{f(x_0) + f(x_1)}{2} + \frac{f(x_1) + f(x_2)}{2} + \frac{f(x_2) + f(x_3)}{2} + \cdots + \frac{f(x_{n-1}) + f(x_n)}{2} \right]$$

$$= \frac{\Delta x}{2} \left[f(x_2) + \left(f(x_1) + f(x_1) \right) + \left(f(x_2) + f(x_2) \right) + \cdots + f(x_n) \right]$$

$$I = \frac{\Delta x}{2} \left[f(x_0) + 2 f(x_1) + 2 f(x_2) + \cdots + f(x_n) \right]$$

$$I_T = \frac{\Delta x}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

Example مهم نتبه له.

$f(x) = x^2 \quad , \quad 0 < x < 1$

$n = 10 \quad , \quad \Delta x = \frac{x_p - x_0}{n} = \frac{1-0}{10} = 0.1$

$$I = \frac{\Delta x}{2} \left[f(0) + f(1) + 2 \sum_{i=1}^{n} f(x_i) \right] \qquad f(x_i) = x_0 + n_i \Delta x_i$$

$$= \frac{0.1}{2} \left[(0)^2 + (1)^2 + 2\left[(0.1)^2 + (0.2)^2 + \cdots + (0.9)^2 \right] \right]$$

2 - Simpson's 1/3 Rule

n - data should be even.

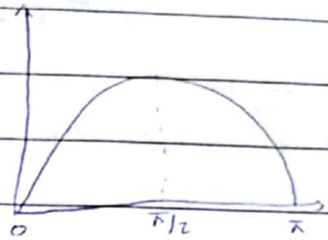
$$I_{1/3} = \frac{\Delta x}{3} \Big[f(x_0) + f(x_n) + 4 f(x_0 + \Delta x)$$
$$+ 2 f(x_0 + 2\Delta x)$$
$$+ 4 f(x_0 + 3\Delta x)$$
$$+ 2 f(x_0 + 4\Delta x)$$
$$+ \cdots \Big)$$

$\Delta x = \frac{b-a}{n} \quad , \quad a < x < b$

Example

$$\int_0^{\pi} \sin(x)\,dx = -\cos x \Big|_0^{\pi} = 2.$$

apply Simpson's 1/3 rule $\quad \Delta X = \frac{\pi - 0}{2}$



$$I_{1/3} = \frac{\pi/2}{3}\left[f(0) + f(\pi) + 4f\left(0+\frac{\pi}{2}\right)\right]$$

$$= 2.09$$

3- Simpson's 3/8 rule

$\boxed{n \text{ - should be divided by } 3}$

$$I_{3/8} = \frac{3}{8}\Delta X\,[f(x_0) + f(x_n) + 3f(x_0 + \Delta X)$$
$$+ 3f(x_0 + 2\Delta X)$$
$$+ 2f(x_0 + 3\Delta X)$$
$$+ 3f(x_0 + 4\Delta X)$$
$$+ 3f(x_0 + 5\Delta X)$$
$$+ \ldots\ldots]$$

Example :

$$\int_0^{\pi} \sin x\,dx$$

اذا قسمها $\quad n = 3$

$$\Delta X = \frac{\pi - 0}{3} = \frac{\pi}{3}$$

$$I_{3/8} = \frac{3}{8} \Delta x \left[\qquad \qquad \right]$$

$$= \frac{3}{8} \cdot \frac{\pi}{3} \left[f(0) + f(\pi) + 3f\left(\frac{\pi}{3}\right) + 3f\left(\frac{2\pi}{3}\right) \right]$$

$$= 2.039$$

→ choose $n = 4$ (wrong) لا تقسم على 3

$I_{3/8} = 1.423$

→ choose $n = 6$

$I_{3/8} = 2.0009$

Algorithm

Simpson's 1/3 rule

$$I = \frac{\Delta x}{3} \left[f(x_0) + 4 \sum_{i=1,3,5}^{n-1} f(x_i) + 2 \sum_{i=2,4,6}^{n-2} f(x_i) + f(x_n) \right]$$

Simpson's 3/8 rule

$$I = \frac{3}{8} \Delta x \left[f_0 + 3 \sum_{\substack{i=1 \\ i \neq 3,6,9}}^{n-1} f(x_i) + 2 \sum_{i=3,6,9}^{n-3} f(x_i) + f(x_n) \right]$$

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شرح الفا د

21/4/2019

Romberg integration.

$h = (b-a)\, 2^{-k}$

$K=1 \rightarrow h_1 = (b-a)\, 2^{-1} = \frac{(b-a)}{2}$ يستخدم h بدل DX

$K=2 \rightarrow h_2 = (b-a)\, 2^{-2} = \frac{b-a}{4} = \frac{h_1}{2}$

$K=3 \rightarrow h_3 = (b-a)\, 2^{-3} = \frac{b-a}{8} = \frac{h_2}{2} = \frac{h_1}{4}$

Trapizoidal Rule

$T_{11} = \frac{b-a}{2}\left[f(a) + f(b)\right]$

$n=2 \quad T_{21} = \frac{b-a}{4}\left[f(a) + 2f(h_1) + f(b)\right]$

$T_{11} = I_{exact} + k_1 h^2 + err$

$T_{21} = I_{exact} + k_1\left(\frac{h}{2}\right)^2 + err$

$T_{21} - T_{11} = k_1\left(\frac{1}{4} - 1\right) h^2$

$k_1 = \frac{T_2 - T_{11}}{h^2}\left(\frac{-4}{3}\right)$

$$T_{21} = I + k_1 \frac{h^2}{4}$$

$$= I + \left(\frac{T_{21} - T_{11}}{\cancel{h^2}}\right)\left(\frac{-\cancel{4}}{3}\right) \frac{\cancel{h^2}}{\cancel{4}}$$

$$T_{22} = T_{21} + \frac{T_{21} - T_{11}}{3} + \cancel{err}^{\to 0}$$

$$T_{j1} = \frac{1}{2} T_{j-1,1} + h \sum_{j=1}^{2^{(j-1)}} f(a + (i-1)h)$$

$\boxed{\text{Example}}$

$$\int_0^1 e^{-x^2} dx$$

$n = 1$

$$h = \frac{b-a}{n} = \frac{1-0}{1} = 1$$

$$T_{11} = \frac{1-0}{2}\left[f(0) + f(1)\right]$$

$$= \frac{1}{2}\left[1 + e^{-1}\right] = 0.6839$$

$n = 2$

$$h = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}, \quad x_1 = \frac{1}{2} \quad 0 - \frac{1}{2} - 1$$

$$T_{21} = \frac{h}{2}\left[f(a) + 2f(x_1) + f(b)\right]$$

$$= \frac{1/2}{2}\left[1 + 2f(1/2) + 0.367\right]$$

$$= 0.7313$$

$$T_{22} = T_{21} + \frac{T_{21} - T_{11}}{3}$$

$$= 0.7313 + \frac{0.7313 - 0.6839}{3} = 0.747.$$

$n = 4$

$h = \frac{b-a}{n} = \frac{1-0}{4} = 1/4$ $\qquad 0 - 0.25 - 0.5 - 0.75 - 1$

$x_1 = 1/4 \qquad x_2 = \frac{1}{2} \qquad x_3 = \frac{3}{4}$

$$T_{31} = \frac{h}{2}\left[f(a) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(b)\right]$$

$= 0.7429$

Algorithim

Define a, b, n

$h_1 = DX$

① Set $i = 1,\ j = 1$

$$T_{11} = \frac{h_1}{2}\left[f(a) + f(b)\right]$$

② Set $i = 1 + i$

if $i > n \rightarrow$ stop

Set $h_i = \frac{h_{i-1}}{2}$

$$T_{i1} = \frac{1}{2}\left[T_{i-1,1} + h_{i-1}\sum_{K=1}^{2^{(i-2)}} f\left(a + \left(K - \frac{1}{2}\right)h_{i-1}\right)\right]$$

Set $j = j + 1$

$\Rightarrow$

if $j > i$ then set $j=1$ Goto step 2

else

$$T_{i,j} = \frac{2^{2(j-1)}(T_{i,j-1}) - T_{i-1,j-1}}{2^{2(j-1)}-1}$$

Go to 5

Tabulated Integrals.

x	$f(x)$
⋮	⋮

Data: regular; irregular

irregular: err in DX small; err = large.

Regular x : 1 2 3 4 5 DX = fixed

$f(x)$: $f(1)$ $f(2)$ $f(3)$ $f(4)$ $f(5)$

irregular x : 1 3 8 20 100

$f(x)$: $f(1)$ ....

Regular Data

$$I_T = \frac{DX}{2}\left[f(a) + \Sigma\, 2\, f(x_i) + f(b)\right]$$

$$I_{S\,1/3} = \frac{DX}{3}\left[f(a) + 2f(x_i) + 4f(x_i) + \cdots + f(b)\right]$$

x:	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$:	2.1	2.54	3.6	3.75	4.62	6.6	7.8	9.3

$$I_T = \frac{0.2}{2}\left[2.1 + 2(2.54 + \cdots + 7.8) + 9.3\right] = 6.9$$

irregular Data

(err : small)

$$DX = \frac{\Sigma(x_{i+1} - x_i)}{n}$$

Example 8·

→ indicator
err small

		0·15	0·2	0·07	0·133	
X :	0	0·15	0·35	0·42	0·55	0·68
F(x) :	2·1	2·5	3·6	3·9	4·6	6·6

$$DX = \frac{0.15 + 0.2 + 0.07 + 0.13 + 0.13}{5} = 0.136$$

$$I_T = \frac{0.136}{2}\left[2.1 + 2(2.5 + 3.6 + 4.9 + 4.6) + 6.6\right]$$

$$= 2.577$$

Irregular : err → Large·

DX	1	2	5	3	
X :	1	2	4	9	12
F(x) :	1	3	5	8	17

$$I_{T1} = \frac{DX}{n}\left[f(a) + f(b)\right]$$

$$= \frac{1}{1}\left[1 + 3\right] = 4$$

$$I_{T2} = \frac{2}{1}\left[3 + 5\right] = 16$$

$$I_{T3} = \frac{4}{1}\left[5 + 8\right] = 65$$

$$I_{T4} = \frac{3}{1}(8 + 17) = 75$$

$$I_{TT} = 160$$

$$\frac{1}{e^x + \frac{1}{e^x}}$$

$$\frac{e^x}{e^{2x}+1}$$

another method

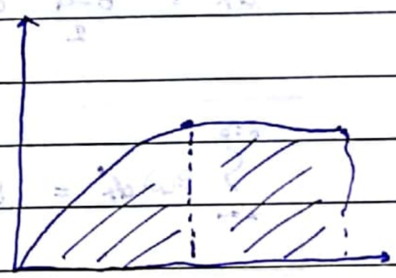
DX		1	1	1	5	32	50
X :	0	1	2	3	8	40	90
f(x) :	3	5	7	16	20	22	80

reeg

مجمع

Integration of infinity intervals.

$$\int_0^\infty \frac{dx}{e^x + e^{-x}}$$



$x = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$f(x) = e^1 + \frac{1}{e^1} \quad e^2 + \frac{1}{e^2} \quad \frac{e^3+1}{e^3} \longrightarrow$

الباقي

$$\int_0^\infty \frac{dx}{e^x+e^{-x}} = \int_0^{10} \frac{dx}{e^x+e^{-x}} + \int_{10}^\infty \frac{dx}{e^x}$$

فوق 10 بتصغر كتير

integrate Numerically

$$\int_0^5 \frac{dx}{e^x+e^{-x}} \approx 1.55$$

$$\int_5^\infty \frac{dx}{dx} \approx 0.0067$$

$$\int_0^\infty = 1.55 + 0.0067 = 1.56$$

Gaussian Quadrature integration

$$\int_{x=a}^{x=b} f(x)\,dx = \int_{t_1}^{t_2} f(t)\,dt = \sum_{i=1}^{n} c_i f(t_i)$$

$$x = \frac{(b-a)t + b+a}{2}$$

$$dx = \frac{b-a}{2}\,dt$$

$$\boxed{\int_{x=a}^{x=b} f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left[\frac{(b-a)t + b+a}{2}\right] dt}$$

Example 8

$$\int_{0}^{\pi/2} \sin(x)\,dx \qquad x = \frac{(\pi/2 - 0)t + \frac{\pi}{2} + 0}{2}$$

$n=2$

$$dx = \frac{\pi/2 - 0}{2}\,dt = \frac{\pi}{4}\,dt$$

$$I_{2term} = \frac{\pi}{4}\left[c_1 \sin\left(\frac{\frac{\pi}{2}t_1 + \frac{\pi}{2}}{2}\right) + c_2 \sin\left(\frac{\frac{\pi}{2}t_2 + \frac{\pi}{2}}{2}\right)\right]$$

$$I = \frac{\pi}{4}\left[1 \sin\left(\frac{\frac{\pi}{2}(0.5773) + \frac{\pi}{2}}{2}\right) + 1 \sin\left(\frac{\frac{\pi}{2}(-0.5773) + \frac{\pi}{2}}{2}\right)\right.$$

$$= 0.998$$

# of terms	c_1, c_2	t_1, t_2, t_3	
2-term	$c_1 = c_2 = 1$	$t_1 = 0.5773$	$t_2 = -0.5773$
3-term	$c_1 = 0.555$	$t_1 = -0.7745$	
	$c_2 = 0.888$	$t_2 = 0$	
	$c_3 = 0.555$	$t_3 = 0.7745$	

I 3 term

$$I = \frac{b-a}{2}\left[c_1 f(t_1) + c_2 f(t_2) + c_3 f(t_3)\right]$$

28/4

Solution of ODE's

$$y' = f(x, y)$$

ex $\frac{dy}{dx} = x + y$, $y(x_0) = y_0$

$$y' = f(x, y)$$

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = f(x, y)$$

$$\Delta y = \Delta x \; f(x, y)$$

Euler method

$$\Delta y = \Delta x * f(x, y)$$

$$y_{i+1} - y_i = \Delta x * f(x, y)$$

$$\boxed{y_{i+1} = y_i + \Delta x * f(x, y)}$$

Examples:

Solv $\dot{y} = x + 2y$

@ $x = 0$, $y = 0$

$$y_{i+1} = y_i + \Delta x \; f(x, y)$$

Assume $\Delta x = 0.25$

كل ما كانت اصغر بتكون احسن

	x	y
x_0	0	0
x_1	0.25	0
x_2	0.5	0.0625
x_3	0.75	0.21875
x_4	1.0	0.5156

$$y_1 = y_0 + \Delta x \; f(x_0, y_0)$$
$$= 0 + 0.25\,(x_0 + 2y_0)$$
$$= 0$$

$$y_2 = y_1 + \Delta x \; f(x_1, y_1)$$
$$= 0 + 0.25\,(0.25 + 2(0))$$
$$= 0.0625$$

$$y_3 = y_2 + \Delta x \; f(x_2, y_2)$$
$$= 0.0625 + 0.25\,(x_2 + 2y_2)$$
$$= 0.0625 + 0.25\,(0.5 + 2(0.0625))$$
$$= 0.21875$$

Runge - Kutta - method

$y_{i+1} = y_i + a_1 k_1 + a_2 k_2 + a_3 k_3 + \cdots$

4th order R.K method

$y_{i+1} = y_i + a_1 k_1 + a_2 k_2 + a_3 k_3 + a_4 k_4$

$$\boxed{y_{i+1} = y_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]}$$

مهم

$k_1 = DX * f(x_i , y_i)$

$k_2 = DX * f(x_i + \frac{DX}{2} , y_i + \frac{k_1}{2})$

$k_3 = DX * f(x_i + \frac{DX}{2} , y_i + \frac{k_2}{2})$

$k_4 = DX * f(x_i + DX , y_i + k_3)$

Example:

$y' = x + 2y$

$DX = 0.25 , y(0) = 0$

$y_{i+1} = y_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

y_0	y_1	y_2	y_3	y_4
0	0.25	0.5	0.75	1
x_0	x_1	x_2	x_3	x_4

NOTES

To obtain y_1

$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$k_1 = DX * f(x_i , y_i) = 0.25\,[x_0 + 2y_0] = 0.25(0) = 0$

$k_2 = DX * f(x_0 + \frac{DX}{2} , y_0 + \frac{k_1}{2})$

$= 0.25 * [(x_0 + \frac{DX}{2}) + 2(y_0 + \frac{k_1}{2})]$

$= 0.25 * [(0 + \frac{0.25}{2}) + 2(0 + \frac{0}{2})]$

$= 0.03125$

$$K_3 = DX * F\left(X_0 + \frac{DX}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= 0.25 * \left(\left(x_0 + \frac{DX}{2}\right) + 2\left(y_0 + \frac{K_2}{2}\right)\right)$$

$$= 0.25 * \left(\left(0 + \frac{0.25}{2}\right) + 2\left(0 + \frac{0.03125}{2}\right)\right)$$

$$= 0.039$$

$$K_4 = DX * F(X_0 + DX, y_0 + K_3)$$

$$= 0.25 * [(0 + 0.25) + 2(0 + 0.039)]$$

$$= 0.082$$

$$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= 0 + \frac{1}{6}[0 + 2(0.03125) + 2(0.039) + 0.082]$$

$$\boxed{y_1 = 0.037}$$

لما بدي y_2 ← بعوض جوا x_1

y_3 ← x_2 وهكذا.

مش داخل

For Future

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{T_{i+1,j} - T_{i,j}}{\Delta t} = \alpha \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta x)^2}$$