برمجيات

Numerical solution → ~~approt~~ aproximate solution
accurcy اقل from analytical solution.

* decimal {0, 1, 2, 3, -- , 9}

$\boxed{\text{decimal}}$ x = 2 $\xrightarrow{\text{confer}}$ Binary
$\boxed{\text{decimal}}$ y = 3 → ،،
z = 5 → ،،

① conversion. → ② Back conferger.
② arthmetics → Three steps

~~single presion → singl ward → n bit~~
one bit لكل خانة

نوع ارقام number
Integer — Real

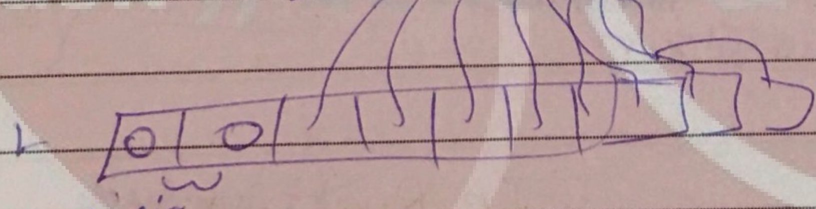
KLMNOB → كل ارقام → Integer
الارقام تبدا من هنا الى Real

كل → Real number 2.25
Area → ،، A = 2.25
L → Integer number L = 2

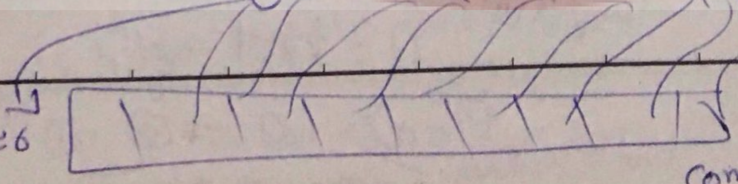
I = 2.0 عشري Real → Integer I / Real L → لازم اعرفه
I, J, K → Integer number

Integer لازم نعرفها بالبداية
comp اذا نعرفه في

let L = 245 3 bit Integer (decimal)
L = 11110101 binary


L — 0 0 (فراغ) — 10 bit

M = 24567
M = 0101111111110111


تضيع هنا — over flow — الرقم المدخل اكبر من ذاكرة computer

Fromp 2 option of Integer number — بيصير ويخلي / overflow

variable تعين

1) ل Doble Presion → doble word

capacity [illegible]

old 32^{bit} →

(excel) defult 64 → bit

excel جماعة

64 bit → variable

$L = 245$

$L = -245$

sign. 32 bit

(sign) 31 bit avalay for number

$DL = 245$

dopel presion. 32 32

sign. 31 32

4 presion

1	1	1	1
2^3	2^2	2^1	2^0

→ Integ numbr

$$= 1\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3$$
$$= 1+2+4+8$$
max number $= 15$

16 ← اذا رقم اكثر

overflow

31

max number

single

double

In Real number. Realword

float point

0.1101×2^{x4}

$A = 1101$

float $A = \mp 0 \,.\, 1\, 0\, 1\, 1 \times 2^{4}$

default

mantissa

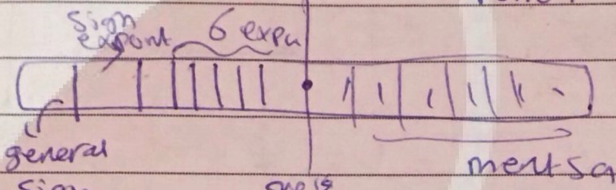
$0.001101 \rightarrow \boxed{0.1101} \times 2^{-2} \rightarrow$ exponent

$A = \pm \boxed{0} . \boxed{1} \boxed{0} \boxed{1} \boxed{0} \times 2^{\boxed{\pm | \; | \; | \;}}$

sign

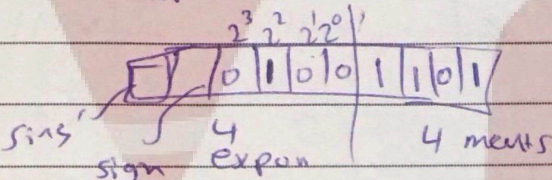
singl pre $32 - 2 = 30$

8 exponent 12 mantisa



→ Reu number.

inored bit
expont , ment.



$0.1101 \times 2^{-2} \rightarrow 2^2 = 4$

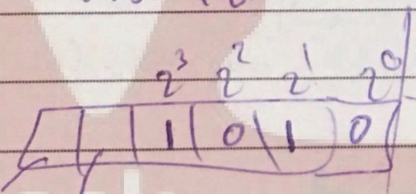
0.111101×2^{-2}

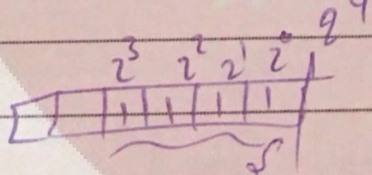
0.000001×2^{-2}

mental تقليل قيمة → 0.1×2^{-7}

over and under flow.

اذا ما يستوعب exponent





اذا العدد 6 ما يستوعب اكبر

overflow or under flow.

sign < over flow / under flow. } → exponent فقط

① Conversion from Integer decimal to binary.

no fraction

base 2

tria 3

quart 4

$N \rightarrow$ any integer number

base = $\frac{N}{2}$

$N = 2Q_0 + \boxed{b_0}$ remainder $< {0 \atop 1}$ $= 0$ (b_0)

Base 2 < 0, 1

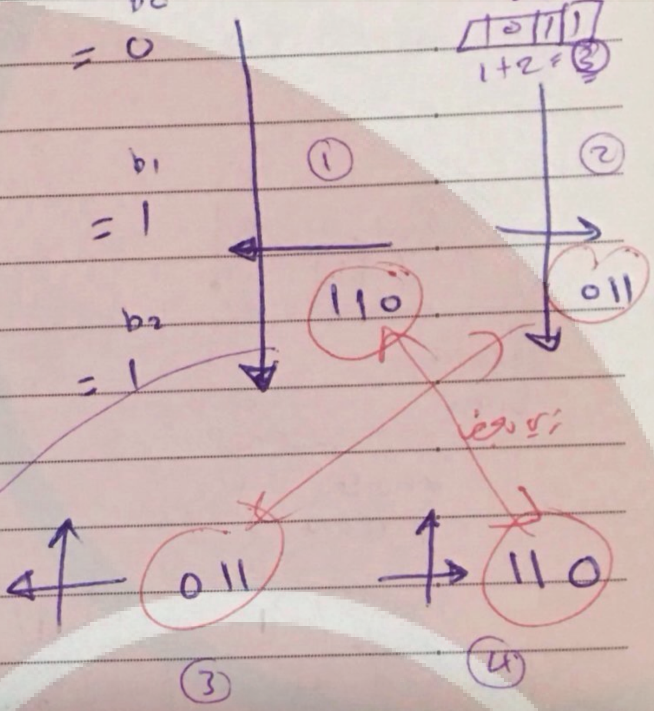
Base 3 < 0, 1, 2

$Q_0 = 2Q_1 + b_1 < {0 \atop 1}$ $= 1$ (b_1)

$Q_1 = 2Q_2 + b_2 < {0 \atop 1}$ $= 1$ (b_2)

nep = Q

$\vdots$



when Q_n = zero then stop.

Then binary num = $b_4 b_3 b_2 b_1 b_0$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$ | | | 1 | 1 | 0 |

$0+2+4 = 6$

$\underline{3} = 2 \times 1 + 1$

$1 = 2 \times 0 + 1$

$\underline{1}$ stop

$3 = \boxed{1\,|\,1}$ ($2^1\ 2^0$) binary

$1+2=\underline{3}$

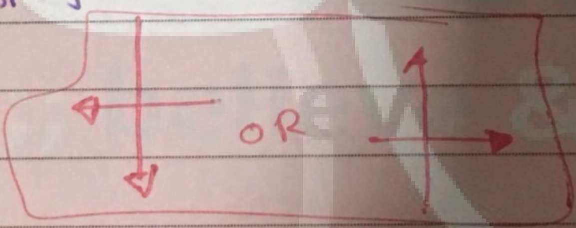
$\underline{6} = 2 \times 3 + 0$

$3 = 2 \times 1 + 1$

$1 = 2 \times 0 + 1$

$2^2\ 2^1\ 2^0$ | 1 | 1 | 0 |

$0+2+4 = 6$



$N = 17$

$17 = 2 \times 8 + 1$ b_0

$8 = 2 \times 4 + 0$ b_1

$4 = 2 \times 2 + 0$ b_2

$2 = 2 \times 1 + 0$ b_3

$1 = 2 \times 0 + 1$ b_4

$b_4 b_3 b_2 b_1 b_0$

$10001 = N_2$ (base 2)

$16+0+0+0+1$

$= \underline{17}$

Scanned by CamScanner

17

$17 = 5 \times 3 + 2 \; b_0$

$3 = 5 \times 0 + 3 \; b_1$

zero

$5^1 \quad 5^0$

$\boxed{3 \mid 2} = 17_5$

$2 \times 1 + 3 \times 5$

$2 + 15 = 17$

0, 1, 2, 3, 4 → Base

$2 \rightarrow$ base − 1

$5 \rightarrow 4$

$7 \rightarrow 6$

input x

$\ominus \rightarrow 1$

$\oplus \rightarrow 0 \quad x = \boxed{1 \mid 0 \mid 0 \mid 1 \mid 0 \mid 0 \mid 0 \mid 1}$

$2^4 \; 2^3 \; 2^2 \; 2^1 \; 2^0$

17

31

real < Integ / fraction

2.5

2

6.5

② conversion from Real decimal to binary.

Real = Integer . fraction R

$\frac{N}{2}$

0, 1, {0 … 9}

$R * 2 = \boxed{\;} . \boxed{\;}$ → $F \times 2 = \boxed{\partial} . \boxed{F}$

∂ Integ part, F

$F \times 2 = \boxed{\partial} . \boxed{F}$

N R

$\boxed{2}.\boxed{25}$

$N = 2 \times 1 + 0$

$1 = 2 \times 0 + 1$

$2 = 10$

$2^{-1} \; 2^{-2} \; 2^{-3}$

$\frac{1}{2} \; \frac{1}{4} \; \frac{1}{8}$

$0.\partial_0 \partial_1 \partial_2 \partial_3 = 0.010 = 0.25 \rightarrow 2.25$

$0.\partial_2 \partial_1 \partial_0 = 0.010$

$R = 0.25$

$2 * R = 2 \times 0.25 = \underset{\partial_0}{0}.\underset{F_1}{5}$

$2 \times F_1 = 2 \times 0.5 = \underset{\partial_1}{1}.\underset{F_2}{0}$

$2 \times F_2 = 2 \times 0 = \underset{\partial_2}{0}.\underset{F_3}{0}$

$F \rightarrow \partial$

$\partial_0 \; 1$

$\partial_1 \; 0$

$\partial_2 \; 1$

$\partial_3 \; 1$

$\partial_4 \; 1$

$2^{-1} \; 2^{-2} \; 2^{-3} \; 2^{-4} \; 2^{-5}$

$0.10111 \quad 0.718$

or

$= .11101 = 0.906$

$2^{-1} \; 2^{-2} \; 2^{-3} \; 2^{-4} \; 2^{-5}$

integer fraction

mantsa

2.25 = 10.01 binary number ⇒ $0.1001 \times 2^{+2}$ → exponent

Float number 0.1 □□□□ * 2 → base

3,4,5

$\pm n$

-n

+n

±	±	0	0	1	0	0	1	0	0	1

6

$1\times0.5+\frac{1}{16}\times1 = 0.5625 \times 2^2 = 2.25$

* Conversion of Real decimal to teriary ③

Real

x = 18.639

18 = 3x6 +0

6 = 3x2 +0

2 = 3x0 +2

$= 18_3 = 200$

$3^2\ 3^1\ 3^0$

9 3 1

0+0+2x9=18

0.639 → 3 x 0.639 = 1.917

3x 0.917 → 2.751

3x0.751 = 2.253

0.253 x3 = 0.759

3x0.759 = 2.277

$0.\partial_0\partial_1\partial_2\partial_3\partial_4\partial_5$

$0.639_3 = 0.12202$

$18.639_3 = 200.12202$

mentsa exponent

6 bit

$0.20012202 \times 3^{+3}$

±	±	1	0	0	0	0. 2	0	0	1	2	2	0	2

No exact value limited over mentsa

approxim

tranktion → $0.2001 \times 3^3 = 18.333$

±	±	1	0	0	0	2	0	0	2

aproximation → (rounding) → $0.2002 \times 3^3 = 18.6667$

رقم ليه منتهي ما ينفعش exact بنحطه error mensa.
مش بنحسب error دي اساسيه بتفرق.

n → + over flow اكبر من صفر
n → - under flow -1×16^{0} اقل من صفر

رقم صغير اكبر من صفر
الرقم الي صغيره صفر (بتكون خطأ)

C ← A x B Limited Capacity.

كل bit ← رقم واحد.

جمع، طرح، ضرب، قسمة

x = 0 1 1 0 1 1 1 +
y = 0 0 1 0 0 1 1
x + y = z = 1 0 0 1 0 1 0

1001010

x = 0 1 1 0 1
y = 0 0 1 1 1
z = 0 1 0 0 1
x+y+z = 1 1 1 0 1

= 11101

(الفكرة) Idia نفس Base لو غيرنا

x = 1 1 0 1 1 1 -
y = 0 1 0 0 1 1
x - y = 1 0 0 1 0 0
0 1 1 1 0 0

x = 1 0 0 1 1 1 -
y = 0 1 1 0 1 1
x - y = 0 1 1 1 0 0

x - y = 0 11100 you take from next bit.

homwork → (*, ÷) → قبل
ضرب

number of bit X cary
y بتحمل
x > y
num bit x x - y

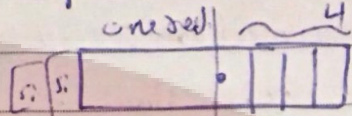
x ± 1 0 0 1 1 1
y = 1 1 1 0 1 1

(y - x) → نطرح ونضيف الاشارة سالبة.

نبدأ من أقصى اليمين
$0 + 1 = 1$
$1 + 1 = 10$ (0 وباقي 1)
$1 + 1 + 1 = 11$ (1 وباقي 1)

Error

① Adding or subtracting one larg and one small.



رقم صغير
4 bit

$+\; x = 0.01 \rightarrow 0.1000 \times 10^{-1}$

$\;\; y = 5000 \rightarrow 0.5000 \times 10^{+4}$

رقم كبير

$(10^0 \ldots 10^9)$ one digel

$10^{10} \rightarrow$ two diget

0.1000	$\times 10^{-1+5}$
0.5000	$\times 10^4$

اول ما نجمع او نطرح لازم تكون expo نفسها

$+\; 0.000001 \;\; \times 10^{-1} \times 10^{5} \rightarrow \times 10^4$

$\;\; 0.500000 \;\; \times 10^4 \rightarrow \times 10^4$ نفسها

$\;\; 0.500001$

$\rightarrow 0.5000|01 \times 10^4 \rightarrow 0.5000 \times 10^{+4} \rightarrow 5000 \rightarrow$ error 100%

OR Rounding

this is not accpectible

رقم صغير + رقم كبير = رقم كبير $\rightarrow$ error 100%.

② Round off in subtracting two almost (المتساوية) number.

$-\; x = 0.1235 \;|\; \times 10^0$

$\;\; y = 0.1234 \;|\; \times 10^0$

$\;\; 0.0001 \;|\; \times 10^0$

0.1000×10^{-3}

	0	1	1	1	
	1	0	0	0	1
		1	1	1	
		1	0	1	0
		1	1	1	
			1	1	1
			1	1	1
			0	0	0

③ over flow ~~and~~ under flow.

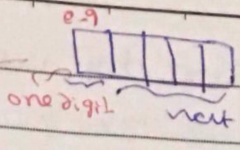
$x = 0.1000 \times 10^{6}$

$y = 0.2000 \times 10^{7}$

0.0200×10^{13}

↓

$0.2000 \times 10^{+12}$ → two digit → over flow.



ضرب والقسمة power

مش مع كده بيعوض

$X = 0.1000 \times 10^{+7}$ very larg

$y = 0.1000 \times 10^{-5}$ very small ÷

1.0000×10^{12}

$= 0.1000 \times 10^{+11}$ two digit (over flow if word is limiting)

4 bit one digit

$A = 0.1000 \times 10^{-6}$

$B = 0.2000 \times 10^{-5}$

$C = 0.3000 \times 10^{7}$

① $x = A \times B \times C$ ÷ x − +

$A = 0.1000 \times 10^{-6}$

$B = 0.2000 \times 10^{-5}$

$= 0.0200 \; 10^{-11}$

0.2000×10^{-12} → two digit → under flow

Approximation $0.2 \times 10^{-12} \approx 0$

$A \times B = 0 \times C = 0$

$0 \times 0.300 \times 10^{7}$

% → 100 % error.

$= A \times C \times B$

$A = 0.1000 \times 10^{-6}$

$C = 0.3000 \times 10^{7}$ x

0.0300×10^{1}

0.300×10^{0}

$(A \times C) \times B$

0.300×10^{0}

$B = 0.2000 \times 10^{-5}$

0.0600×10^{-5}

$0.6000 \times 10^{-6} \neq 0$ فرق جامد

① True error = True valu − Approximet

② % relative error = True error / true value × 100%

$$\frac{\text{True} - \text{Aproxim}}{\text{True valu}} \rightarrow \text{actual}$$

1st iteration
2nd iteration
3rd iteration

$$\left.\begin{matrix}\text{1st iteration}\\ \text{2nd iteration}\\ \text{3rd iteration}\end{matrix}\right\} \frac{\text{New} - \text{old}}{\text{New}} \Big\} \text{sessive Relative error}$$

Solving single non-linear equation.

$f(x) = 0$ $x^3 - 2x = \ln x \rightarrow \boxed{x^3 - 2x - \ln x = 0}$

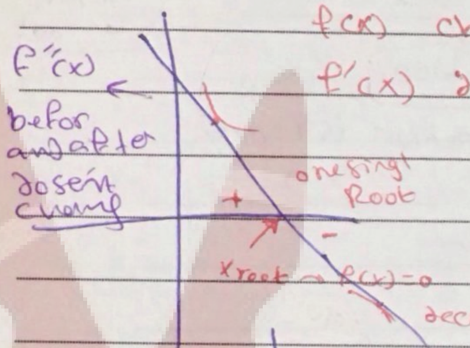
$f_1(x) = x^3 + 3x$
$f_2(x) = \sin x$ } when interset twogther.

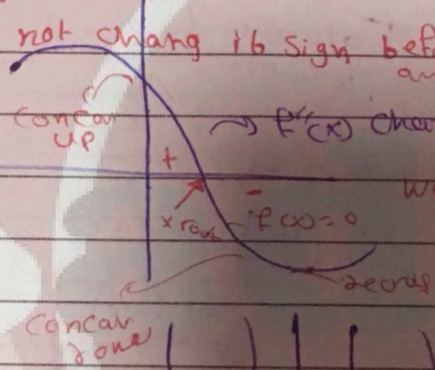
equal zero OR aproxim zero.

$x^3 + 3x = \sin x \rightarrow x^3 + 3x - \sin(x) = 0.$

f(x) changes its sign befor and after

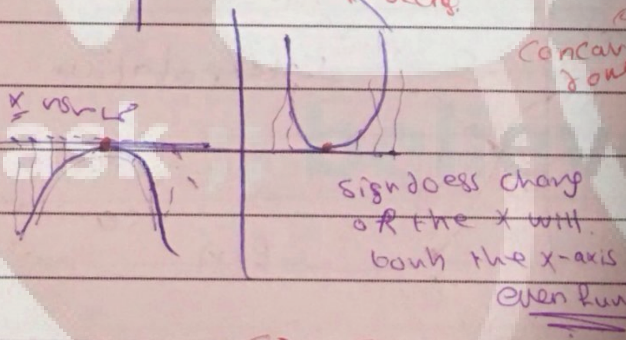
f'(x) does not chang its sign befor Root and after → x-axis

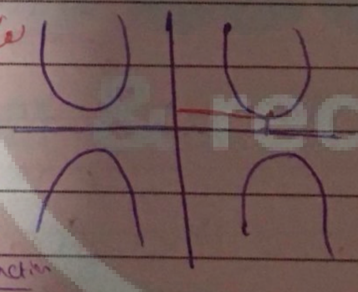




→ f''(x) chang it's sign,

we have odd number of Root 3, 5, 7





→ Imaginary Root (Complex Root)

Real , imag

f'' → concave (+) → (−)

f' → ⊕ ⊕
⊖ ⊖

$(x-2)(x-2) = 0$
$r_1 = 2$ $r_2 = 2$
Repeted.
even number of Root

x-axis
y-axis

even f → Root

f → Root } → odd

حفظ الطريقة اسهل

$$x_2 = x_0 - f(x_0)\left[\frac{x_0 - x_1}{f(x_0) - f(x_1)}\right]$$

OR

$$x_2 = x_1 - f(x_1)\left\{\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right\}$$

$f(x) = 3x + \sin x - e^x$

$x = \{0, 1\}$ tolem $= 1\times10^{-5}$

$x_0 = 0 \longrightarrow f(x_0) = f(0) = -1$ -ve

$x_1 = 1$ $f(1) = 3 + \sin 1 - e^1 = 1.123189$ +ve.

$x_2^1 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5 \rightarrow f(0.5) = {+}0.330704$

$|0.330704| > 1\times10^{-5}$

$\therefore x_2^1$ is not asolution.

- +ve ~~+ve~~

0 0.5 1

x_0 x_1

$x_0 = 0$ $x_1 = 0.5$

$x_0 = 0.25$ $x_1 = 0.5$

$x_2^2 = \frac{0 + 0.5}{2} = 0.25 \Longleftrightarrow f(0.25) = -0.28662$

$|0.28662| > 1\times10^{-5}$

x_2^2 is not asolution.

~~-ve~~ -ve +ve

0 0.25 0.5

x_0 x_1

$x_2^3 = \frac{0.25 + 0.5}{2}$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	check
0		1				copepase

posibr 1

negaiv 0

$\longrightarrow$ if $f(x_2) = f(x_0)$; x_2 ; x_0

if $(x_0 = x_0)$; x_2 ; x_1

نفس الـ sign

$f(x_0) = f(x_2)$; x_2 ; x_0

$f(x_1) = f(x_2)$; x_2 ; x_1

sign sign

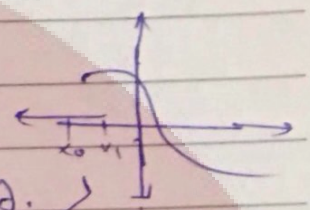
if abs $f(x_2) \geq 0.0001$

another iv elope

6 iteration

$$x_2' = 0 - (-1) \cdot \left\{ \frac{0-1}{-1-1.123181} \right\} = 0.47099$$

$$f(x_2') = f(0.47099) = |0.26516| > 1\times10^{-5}$$



+ + → مفيش solu

(+ −) → في solution.

Open method

① secant method شبه false position method.

بس بيختلف بشرط

مش لازم ين Root

Go to next step

* if $|f(x_0)| > |f(x_1)|$ then keep $x_0 = x_0$, $x_1 = x_1$ نفضل زي ما هو

if $|f(x_0)| < |f(x_1)|$ then swap the values $x_0 = x_1$, $x_1 = x_0$

نعكس

Go to next step

علشان يحصل convergen

نعملها اول خطوة بس (before the first iteration.)

* Next step calculate the expected root x_2

$$x_2^{①} = x_0 - f(x_0) \left\{ \frac{x_0 - x_1}{f(x_0) - f(x_1)} \right\}$$

OR

$$x_2^{①} = x_1 - f(x_1) \left\{ \frac{x_1 - x_0}{f(x_1) - f(x_0)} \right\}$$

Find $f(x_2)$

if $|f(x_2^{①})| >$ Tole, then another iteration is required.

if $|f(x_2^{①})| <$ Tol, the x_2 is the solution.

for the next iteration eleminate x_0 and remain $x_0^{new} = x_1^{old}$, $x_1^{new} = x_2^{①}$

Calcul $x_2^{②}$

$|f(x_0)| > |f(x_1)| > |f(x_2^{①})| > |f(x_2^{②})|$

$\cancel{x_0}$ $\cancel{x_1}$ x_2 $x_2^{②}$

$\cancel{x_0}$ x_1 x_1

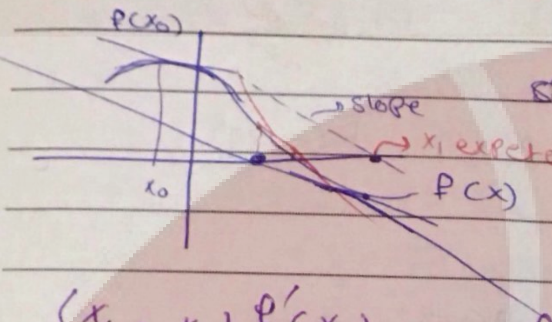
x_0

نلف دايماً الـ... نحو ناحية الشمال

x_0 (f) x_1 (f) x_2 (f)

$f(x) = 0$

② Newton's method → one initial guess is requiered. x_0



Share the slope to optain convergence.

$$\text{slope} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f'(x_0)$$

$(x_1 - x_0)\, f'(x_0) = -f(x_0)$

$$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)} \rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 3$
$f(3) = 5$
angle = 57
tan = f'
tan 57 = f'
1.54
$x = 3 - \frac{f(3)}{f'}$

$|f(x_1^{①})| > Tol$, another iteration

$|f(x_1^{①})| \le Tol$ ~~another iteration~~. stop. → eliminat x_0 old and swap it by $x_1^{①}$

$x_0^{new} = x_1^{①}$

now calculate $x_1^{②}$

$f(x) = 3x + \sin x - e^x$; $x_0 = 0$; $x_1 = 1$; $Tol = 1 \times 10^{-5}$

secant method.

$f(x_0) = f(0) = -1$
$f(x_1) = f(1) = 1.123189156$ } → since $|1.123181| > |-1|$ swap the names.

$\boxed{x_0 = 1 \;;\; x_1 = 0}$

$$x_2^{1} = x_0 - f(x_0)\left\{\frac{x_0 - x_1}{f(x_0) - f(x_1)}\right\}$$

$$x_2^{1} = 1 - 1.123\left[\frac{1-0}{1.1232 + 1}\right] = 0.4709895947$$

$|f(x_2)| > |f(x_0)| \; 0.265$

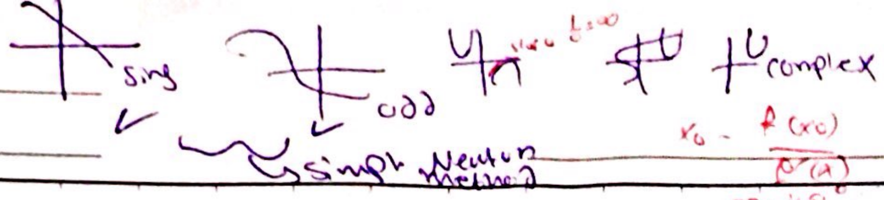
$x_0 = 1$ $x_1 = 0$ $x_2 = 0.475$

$f(x_2^{1}) = f(0.47) = |0.265158161| > 1 \times 10^{-5}$

for the 2nd iterat. $x_0^{new} = x_1^{old} = 0$

$x_1^{new} = x_2^{1} = 0.4709$

$$x_2^{②} = 0 - (-1)\left\{\frac{0 - 0.470989}{-1 - 0.26515}\right\}$$



Newtons :- $x_0 = 0$ (لازم نعرف نشتق)

$f'(x) = 3 + \cos(x) - e^x$

$f(x_0) = f(0) = -1$

$f'(x_0) = f'(0) = 3$

$x_1^{(1)} = 0 - \frac{(-1)}{3} = 0.333$

$f(x_1') = f(0.333)$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	check
0	$3x+\sin x-e^x$	1 → if (3 = 2), E2, A2)				if abs f(x2) > 1 × 10^{-5} > "anoth" , "stop"
if(abs(f(x0)) > abs(f(x1)), x0, x1						
1	1.12	0	-1	0.479		

if (x0 = x0, x1, x0

نغير

$x_0 = x_1$ $x_1 = x_2$ $x_2 = 0.364 \rightarrow f(x_2) = 5.5E{-}06$ نقف

Newtons

x_0	$f(x_0)$	$f'(x_0)$	x_1	$f(x_1)$	check
0	-1	$= 3 + \cos x_0 - e^{x_0}$	$x_0 - \frac{f(x_0)}{f'(x)}$		

$x_0 = x_1$

if $x_0 = 1$ نتغير

simple Newt نتغير كتير وطبيعي

بعطي ملاحظة Root
مش ضروري انو Root
بعطي نفسو حسب Range
عند نقاط الانعطاف نتغير
(max, min)

Modified Newton's :- Mainly used for even number of roots.

$$x_1^{(1)} = x_0 - \left[\frac{f(x_0)\, f'(x_0)}{[f'(x_0)]^2 - f(x_0)\, f''(x_0)}\right]$$

$f(x) = (x-2)(x-2)$

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$

$f''(x) = 2$

simple Newton $x_0 = 0$

modified $x_0 = 6$

|| iteration vs very simple eq

x_0	$f(x_0)$	$f'(x_0)$	x_1	$f(x_1)$	check
0	4	-4	$x_0 - \frac{f(x)}{f'(x)} = 1$	1	if(abs(f(x1))>1x10^6) another ith, stop

$= x_1$

مش اول ما ittor stop اكو

x_0	$f(x_0)$	$f'(x_0)$	$f''(x_0)$	x_1
0	4	-4	2	$x_0 - \frac{f(x)\, f'(x_0)}{f'(x)^2\, f(x_0)\, f''(x_0)} = 2$

even لا موضوع بعبر عن العادلة

بسبب (1.1) ع

* (5) Muller's method (most effient). اكثر كفاءة

3 initial gusses.

x_0, x_1, x_2 (midpoint)

$x_2 = \frac{x_0 + x_1}{2}$ اكبر ← اصغر

Step (1): name the gesses — $x_2 < x_0 < x_1$ (ترتيب حسب value)

2, 4, -1 — $-1 < 2 < 4$

For all etteriation. بترتيب العرفي. — $f(x_2)\ f(x_0)\ f(x_1)$ No trend of f

Step (2) $c_1 = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$ (slope اول)

slope ← 3 point $c_2 = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{f(x_2) - f(x_0)}{x_2 - x_0}$

$d_1 = \frac{c_2 - c_1}{x_0 - x_2}$ $\quad \delta = c_2 + d_1 (x_0 - x_1)$

now the expected $x^{(1)}_{exp} = x_0 - \left\{ \frac{2 f(x_0)}{\delta \pm \text{sign of } \delta \sqrt{\delta^2 - 4 d_1 f(x_0)}} \right.$

value of sign — الجذر

$|f(x^{(1)}_{exp})| >$ tolerance then another iteration → go to step (1)

$\leq$ the x_{exp} is the root → stop

~~$x_2 = x_{exp}$~~ $x_0 = x_2$ $x_0 = x_{exp}$

(1) $\boxed{x_2 < x_0 < x_1}$ x_{exp} $x_{exp} < x_2$ X x_2 x_0 x_1 $x_1 = x_0$.

(2) $\boxed{x_2 < x_0 < x_1}$ $x_2 = x_2$ $x_1 = x_0$ X x_2 x_{exp} x_0 x_1 $x_{exp} < x_0$

كل مرة نلغي البعد مرة كذا x_{exp} $x_1 = x_{exp}$

(3) X x_2 x_0 x_{exp} x_1 $exp < x_1$ الابعد $x_2 < x_0 < x_1$ $x_1 = x_1$ $x_1 = x_0$

(4) X الابعد x_2 x_0 x_1 x_{exp} $x_2 < x_0 < x_1$ $x_2 = x_0$

2- case eleminal x_1
2- case elemin x_2 } [x_{exp} and x_0] دايما موجودين

← رتب , الغي , سمي

* in programe we need Instcl if statment 3 if stament

x_2 | $f(x_2)$ | x_0 | $f(x_0)$ | x_1 | $f(x_1)$ | c_1 | c_2 | y_1 | S
0 | | 0.5 | ? | 1 | ? | ? | ? | ?

→ if($x_{exp} < x_2$, x_{exp}, if($x_{exp} < x_0$, x_2, if($x_{exp} < x_1$, x_0, x_0)))

if($x_{exp} < x_2$, x_2, if($x_{exp} < x_0$, x_{exp}, if $x_{exp} < x_1$, x_{exp}, x_1)))

→ if ($x_{exp} < x_2$

x_{exp} | $f(x_{exp})$ | chek

if abs($f(exp)$) > 1×10^{-5}, "anti", stop'

x_0: if ($x_{exp} < x_2$, x_2, if $x_{exp} < x_0$, x_{exp}, if ($x_{exp} < x_1$, x_{exp}, x_1)

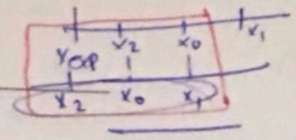
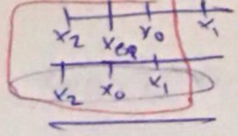
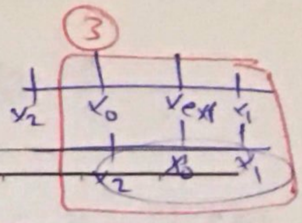
$f(x) = x^2 - 4x + 4$

$x_0 = 0$ $x_1 = 5$

5 2.5 0

x_2 = if ($x_{exp} < x_2$, x_{exp}, if ($x_{exp} < x_0$, x_2, if $x_{exp} < x_1$, x_0, x_0)))

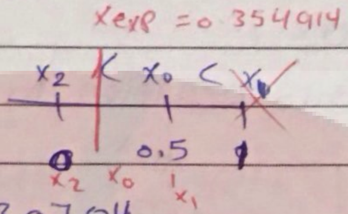
x_1: if ($x_{exp} < x_2$, x_0, if ($x_{exp} < x_0$, x_0, if ($x_{exp} < x_1$, x_1, x_{exp})

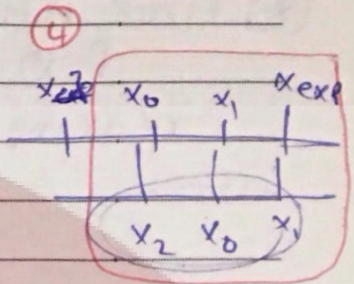
①  ②  ③ 

Ex: $f(x) = 3x + \sin(x) - e^x$

$x_0 = 0 \qquad x_1 = 1$

$x_2 = \frac{0+1}{2} = 0.5$



④ 

$f(x_0) = f(0.5) = 0.330704$

$f(x_1) = f(1) = 1.123189156$

$f(x_2) = f(0) = -1$

$c_1 = \frac{1.123189156 - (-1)}{1-0} = 2.213189$

$c_2 = \frac{0.330704 - 1.123189156}{0.5 - 1} = 1.58497$

$d_1 = \frac{c_2 - c_1}{x_0 - x_2} = \frac{1.58497 - 2.213189}{0.5 - 0} = -1.07644$

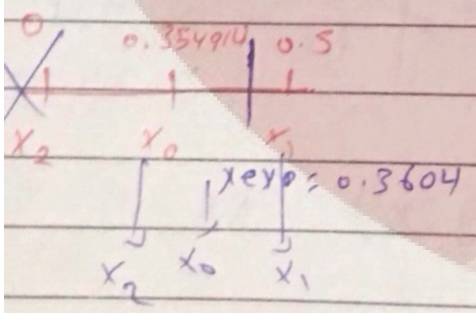
$\delta = c_2 + d_1(x_0 - x_1) = 1.58497 + (-1.07644)(0.5-1)$

$= \boxed{+2.123189} = c_1$ ~~in~~ alwyes in first iteration only.

$x_{exp} = 0.5 - \left[\frac{2 \times 0.330704}{(+2.123189) + (+)\sqrt{(2.123189)^2 - 4 \times (-1.07644)(0.330704)}} \right]$

$= \cancel{0.354} = 0.354914$ $\; x_{exp}^{①}$

$f(0.354914) = |-0.01381| > 1 \times 10^{-5}$ we need anothe itteratio



$c_1 = 2.66144$

$c_2 = 2.37453$

$d_1 = -0.80831$

$\delta = 2.49418$

$x_{ep} = 0.36046$

$f(x_{ep}) = 0.00011$

Scanned by CamScanner

Single non linear equation.

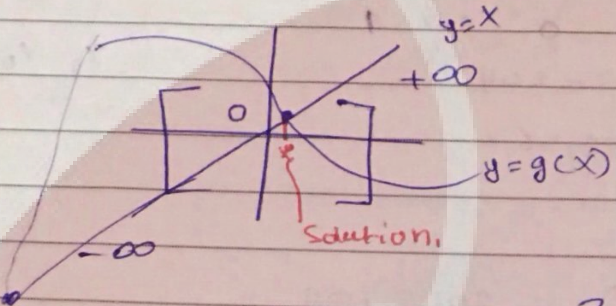
[*] Fixed Point iteration one initial guess only. (Newton, fixed)

$f(x) = 0$

rearang this equation to have $x = g(x)$ intersection of point is the soli

$y = x$ → $f_1(x)$

$y = g(x)$ → $f_2(x)$



$g(2) = 2$

$f(x) = x^2 + 2x - 3$

$2x = \frac{x^2 - 3}{2}$

$x = \cancel{\frac{x^2-3}{2}}$ $\frac{3 - x^2}{2}$ → 2 Solution

$g_1(x)$

$x = \sqrt{3 - 2x}$ $g_2(x)$

one solution.

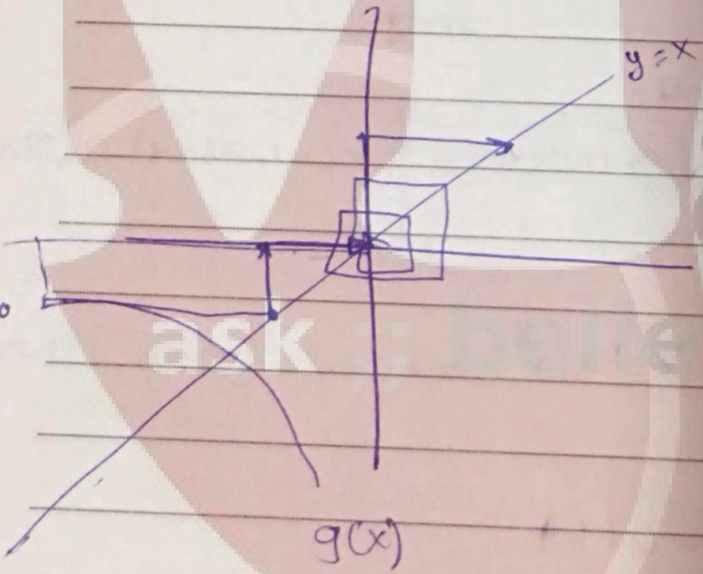
$3 - 2x = 0$

$3 = 2x$

$x^2 + 2x = 3$

$x(x+2) = 3$ → $x = \frac{3}{x} - 2$ $g_4(x)$

$x = \frac{3}{x+2}$ $g_3(x)$



$x = g_1(x) = \frac{3 - x^2}{2}$

$x_1^1 = g_1(x_0)$ نشتغل vs Relative error.

if $g(x_0) = x_0$ then x_0 is a solution

$R.\varepsilon = \left| \frac{x_1^{(1)} - x_0}{x_1^{(1)}} \right|$ < stop, > another iter

$x_0 = x_1$

$x_1^2 = g(x_1^1)$

$R.\varepsilon = \frac{x_1^2 - x_1^1}{x_1^2}$

$x_0 = 0.5$

مش شرط

$x_1^1 = g_1(x) = \frac{3-x^2}{2} = \frac{3-(0.5)^2}{2} = 1.375$

مش شرط

$x_1^2 = g(1.375) = \frac{3-1.375^2}{2} = 0.554688$

$x_1^3 = g(0.554688) = \frac{3-(0.554688)^2}{2}$

OR	x_0	$g_1(x)$	~~Chex~~ R.E	Chek
	0.5	+	$\frac{g_1 x - x_0}{g_1(x_0)}$	abs|R.E| >
	$= g_1(x)$	−		
		+		
		−		

اي g ناخذ g' ونعوض x_0 اذا اقل

$|f(x_0)| < 1$

$g_1(x) = \frac{3-x^2}{2} \to g_1'\left(\frac{-2x}{2}\right) = -x$

at $x_0 = 0.5 \to g_1'(0.5) = -0.5 < 1$ اذا صار

يعني ~~Solution~~ نعدل فيها

$g = \frac{3}{x+2}$

$g'(x) = \frac{-3}{(x+2)^2} \to g'(0.5) = \frac{-3}{(0.5+2)^2} = \left|\frac{3}{(0.5+2)^2}\right| = 0.48 < 1$

وهو من g نعدل فيها .

$g(x) = \sqrt{3-2x}$

$g'(x) = \frac{-2}{2\sqrt{3-2x}} = \frac{-1}{\sqrt{3-2x}}$

$g'(0.5) = \frac{-1}{\sqrt{3-2(\frac{1}{2})}} = \frac{-1}{\sqrt{2}} = 0.7 < 1$ مش بعيده

تعمل solution

up to 3 equation: ① graphical ② elimination

Small system

Large system.

up to ∞

Solution directly.

③ Cramer's Rule

up to 3 eq.

System of ~~lienier~~ Linear Equation. any number of equation.

Direct

① Gauss elimination

② Thomas (spesial matrix)

③ Gauss - Jordan

④ LU Factorization.

Iterative.

① Jacobi (2 init. gess)

② Gauss - seidel

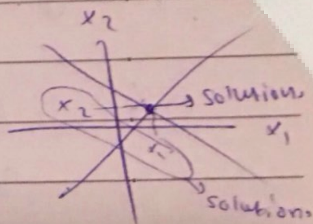
③

$2x_1 + 3x_2 = 5$

$3x_1 - 5x_2 = 11$

① Graphical (for 2 ...) solution of

② elimination



varing axis.

$x_1 = \frac{5 - 3x_2}{2}$

$\Rightarrow 3\left(\frac{5-3x_2}{2}\right) - 5x_2 = 11 \rightarrow$ one unknown.

coeffients

constant

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$

no. of equat. = no. of variables.

variables

$x_1 = \frac{\begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$

coeffient

Det[A]

$x_2 = \frac{\begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix}}{Det[A]}$

c_1, c_2, c_3

$x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}}{Det[A]}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = + a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$$

$$- a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$$

$$+ a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & - & - & - \\ - & - & - & a_{44} \end{bmatrix}$$ we can't determ. matrix 4x4

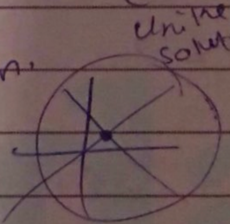
1) only have squer system of matrix

2) ~~not~~ all equation must be Indpent.

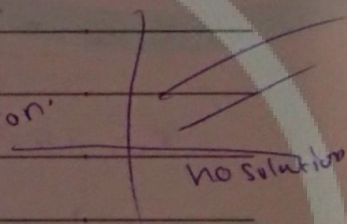
$2x_1 + 3x_2 = 5$
$2x_1 + 3x_2 = 5$ → عند رسم فوق بعض

Infinit # of solution.

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \boxed{= 0}$$

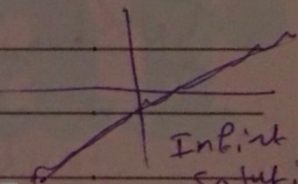


(well) Unike solution.



no solution

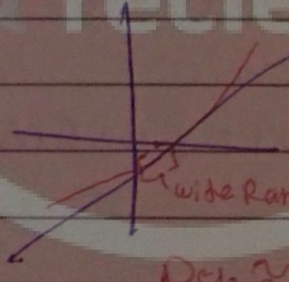
$2x_1 + 3x_2 = 5$
$4x_1 + 6x_2 = 10$ — Parllel no solutiv



Infinit solutio

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = \boxed{Det = 0}$$

3) inp = det تكون System معرفة



wide Rang of solution.

$\boxed{Det = Slope \approx 0}$

Det ≈ 0

4) must be well condition System

ill condition System very Sentsbi of Coffient.

Varition in Coffient

equ = # varidble.

square يعني تكون 6

$2x_1 + 3x_2 - 5x_3 = 7$

$x_1 - x_2 + 4x_3 = 5.2$

$-x_1 + 2.5x_2 + 3.5x_3 = -12$

$[A]\{x\} = \{c\}$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array}$$

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$\text{Det}\{A\} = 2(-3.5 - 4(2.5)) - 3(3.5+4) + (-5)(2.5-1)$

$= -57$

$$x_1 = \frac{\begin{bmatrix} 7 & 3 & -5 \\ 5.2 & -1 & 4 \\ -12 & 2.5 & 3.5 \end{bmatrix}}{-57} \qquad x_2 = \frac{\begin{bmatrix} 2 & 7 & -5 \\ 1 & 5.2 & 4 \\ -1 & -12 & 3.5 \end{bmatrix}}{-57} \qquad x_3 = \frac{\begin{bmatrix} 2 & 3 & 7 \\ 1 & -1 & 5.2 \\ -1 & 2.5 & -12 \end{bmatrix}}{-57}$$

$$x_1 = \frac{-298.1}{-57} \qquad x_2 = \frac{113.9}{-57} \qquad x_3 = \frac{28.9}{-57}$$

$x_1 = 5.2298 \qquad x_2 = -1.99825 \qquad x_3 = -.50072$

$\boxed{\text{ل8}}$ كل coeffic نجيبه

					constant
2	3	-5	x_1	7	
1	-1	4	x_2	5.2	
-1	2.5	3.5	x_3	-12	

حوّل خليه واحده، فيه الجواب نختار فيش الغز نحجم size

= MDETERM (selection كل coefficient) → بعض الجواب مباشره → all

منوع copy بعد Transe عشان اذا نغير دي الكل يتغير

F4 → dolor sign.

$ $ $ $

Polymath.

Lineraqut

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$ → بعض ① → size ② → شرح ③ →

main upper

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \neq 0$$

lower Triangle.

Gauss - Elimation.

$$\underset{3\times3}{\begin{bmatrix} a_{11} & \cdots \end{bmatrix}} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} \rightarrow [A]\{x\} = \{c\}$$

$$\{x\} = \frac{\{c\}}{[A]}$$

Step ① forward elimanition.

elemn of lower Triangle.

✓ Rows بعض

chang ← colm بين X

Variabl and constant

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c'_2 \\ c''_3 \end{Bmatrix}$$

Reduction of Varia

$a'_{22} x_2 + a'_{23} x_3 = c'_2$

$a''_{33} x_3 = c''_3 \rightarrow x_3 = \dfrac{c''_3}{a''_{33}}$

$x_2 = \dfrac{c'_2 - a'_{23} x_3}{a'_{22}}$

$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = c_1$

Step ② Backward substitution.

x_3

x_2

x_1

forward بعد upper نضيف substitution

elemintion on nonsquar matrix

بشتغل عكس

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array}\right] =$$

نفتش تحت main كلها

نزيد Raw > Count

نعد main ... 1

$$\frac{a_{11}}{a_{11}} \quad \frac{a_{12}}{a_{11}} \quad \frac{a_{13}}{a_{11}} \quad \frac{c_1}{a_{11}}$$

Pivot elements $= -\frac{a_{21}}{a_{11}}$

$$\left[1 \quad \frac{a_{12}}{a_{11}} \quad \frac{a_{13}}{a_{11}} \quad \frac{c_1}{a_{11}}\right] \times -a_{21} \quad \times -a_{31}$$

$$\left[-a_{21} \quad -\frac{a_{21}a_{12}}{a_{11}} \quad -\frac{a_{21}a_{13}}{a_{11}} \quad -\frac{a_{21}c_1}{a_{11}}\right]$$ Pivot equation for Row 2

$$\begin{bmatrix} a_{11} & a_{21} & a_{13} & c_1 \\ 0 & \underbrace{a_{22} - \frac{a_{21}a_{12}}{a_{11}}}_{a_{22}'} & \underbrace{a_{23} - \frac{a_{21}a_{13}}{a_{11}}}_{a_{23}'} & \underbrace{c_2 - \frac{a_{21}c_1}{a_{11}}}_{c_2'} \\ 0 & a_{32}' & a_{32}' & c_3' \end{bmatrix}$$

Pivot elem $-\frac{a_{31}}{a_{11}}$

$$\left[-a_{31} \quad -\frac{a_{31}a_{12}}{a_{11}} \quad \frac{a_{13} \times a_{31}}{a_{11}} \quad -\frac{a_{31}c_1}{a_{11}}\right]$$ pivot eq for R_3

نكمل العمل

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a_{22}' & a_{23}' & c_2' \\ 0 & a_{32}' & a_{33}' & c_3' \end{bmatrix}$$ matrix الناتجة

Row 2

column 2

الخطوة الثانية

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a_{22}' & a_{23}' & c_2' \\ 0 & 0 & a_{33}'' & c_3'' \end{bmatrix}$$

$$\left\{a_{33}' - \frac{a_{23}' \, a_{33}'}{a_{22}'}\right\}$$

$$\left[\frac{0}{a_{22}'} \quad \frac{a_{22}'}{a_{22}'} \quad \frac{a_{23}'}{a_{22}'} \quad \frac{c_2'}{a_{22}'}\right]$$

Pivot element $-\frac{a_{32}'}{a_{22}'}$

$$\left[0 \quad 1 \quad \frac{a_{23}'}{a_{22}'} \quad \frac{c_2'}{a_{22}'}\right] \times -a_{23}'$$

$$\left[0 \quad -a_{23}' \quad -\frac{a_{23}' \, a_{23}'}{a_{22}'} \quad -\frac{a_{23}' \, c_2'}{a_{22}'}\right]$$ Pivot equation.

$$\begin{array}{ccc} \frac{2}{2} & \frac{3}{2} & -\frac{5}{2} \\ \left[\begin{matrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{matrix}\right] \end{array} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix} \quad \frac{7}{2}$$

(1 1.5 -2.5 3.5)

$\times -1$ [-1 -1.5 2.5 3.5] Pivot

$\times +1$ [1 1.5 -2.5 3.5)

$$\left[\begin{matrix} 2 & 3 & -5 & 7 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 4 & 1 & -8.5 \end{matrix}\right]$$

First sub matrix

$$\left(\frac{0}{-2.5} \quad \frac{-2.5}{-2.5} \quad \frac{6.5}{-2.5} \quad \frac{1.7}{-2.5} \right)$$

$-4\times$ (0 1 -2.6 -0.68)

(0 -4 10.4 2.72)

$$\left[\begin{matrix} 2 & 3 & -5 & 7 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 0 & 11.4 & -5.78 \end{matrix}\right]$$

second submatrix

Fourow Elmin

$0x_1 + 0x_2 + 11.4x_3 = -5.78$

$$\boxed{x_3 = \frac{-5.78}{11.4} = -0.507}$$

$0x_1 - 2.5x_2 + 6.5x_3 = 1.7$

$-2.5x_2 + 6.5(-0.507) = 1.7$

$$\boxed{x_2 = -1.998}$$

$2x_1 + 3x_2 - 5x_3 = 7$

$2x_1 + 3(-1.998) - 5(-0.507) = 7$

coffient

"الأول

2 3 -5 7

1 -1 4 5.2

-1 2.5 3.5 -12

=(2 / 2(F4)) * -1 (F4) → سبب

sub = (2 / 2 * F4) * 1

= 2 →

= 1 + 1

→ $-\frac{2.5}{-2.5} f_4$ (-4) F4

= 2 3 -5 7

1 -1 4 5.2

= pivot +

x_1 =

x_2 =

x_3 = c_{31}

بنشيت
pivot element

invers → فقط square matrices

inverse →

minverse(select)

ctrl + shift + enter → ظلل الكل

x_1 =	mmult(invers , const)
x_2 =	
x_3 =	

ctr + shift + enter

~~① Gauss Elimination~~ $\left[\triangle\right]\begin{Bmatrix}x_1\\x_2\end{Bmatrix}=\begin{Bmatrix}c_1\\c_2\\c_3\end{Bmatrix}$

② Gauss-Jordan

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{Bmatrix}c_1\\c_2\\c_3\end{Bmatrix}$$

no need for any Backword elimination

الوصول على solution علشان اعمل inverces.

$[A]\{x\}=\{c\}$

① Direct solution

$\{x\}=\{c\}[A]^{-1}$

② find inverse علشان الضرب ب constant

normalization

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{Bmatrix}c_1'''\\c_2'''\\c_3'''\end{Bmatrix} \rightarrow 1\times x_1 = c_1'''$$

→ fourward and Backword

column ال column اكنس

$$\left[\begin{array}{ccc|c} 2 & 3 & -5 & 7 \\ 1 & -1 & 4 & -5.2 \\ -1 & 2.5 & 3.5 & -12 \end{array}\right]$$

$[A]\rightarrow[I]$

R_1

$$\left[\begin{array}{cccc} \cancel{\frac{2}{2}}\ 1 & \cancel{\frac{3}{2}}\ 1.5 & \cancel{\frac{-5}{2}}\ -2.5 & \cancel{\frac{7}{2}}\ 3.5 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 4 & 1 & -8.5 \end{array}\right]$$

نستخدم the Row الذي coffin ①

$R_1(x-1)$

$(-1\quad -1.5\quad 2.5\quad 3.5)$

$R_1\times 1$

$[1\quad 1.5\quad -2.5\quad 3.5]$

first sub matrix

+

R_2

$$\left[\begin{array}{cccc} 1 & 0 & 1.4 & 4.52 \\ 0 & \frac{-2.5}{-2.5} & \frac{6.5}{-2.5}\ -2.6 & \frac{1.7}{-2.5}\ -0.68 \\ 0 & 0 & 11.4 & -5.78 \end{array}\right]$$

$R_2(-1.5)=[0\quad -1.5\quad 3.9\quad 1.02]$

$R_2(-4)=[0\quad -4\quad 10.4\quad 2.76]$

Second submatrix

$$\left[\begin{array}{cccc} 1 & 0 & 1.4 & 4.52 \\ 0 & 1 & -2.6 & -0.68 \\ 0 & 0 & 11.4 & -5.78 \end{array}\right]$$

$\downarrow$

[I] — Solution

$$\begin{array}{l} \\ \\ R_3 \rightarrow \end{array}\left[\begin{array}{cccc} 1 & 0 & 0 & 5.2298 \\ 0 & 1 & 0 & -1.9982 \\ \frac{0}{11.4} & \frac{0}{11.4} & \frac{11.4}{11.4}=1 & \frac{-5.78}{11.4} = -0.507 \end{array}\right]$$

third sub matrix

$R_3(-1.4) = [\,0 \quad 0 \quad -1.4 \quad 0.7098\,]$ +

$R_3(2.6) \quad [\,0 \quad 0 \quad 2.6 \quad -1.3182\,]$ +

$$\left[\begin{array}{l} x_1 = 5.2298 \\ x_2 = -1.9982 \\ x_3 = -0.507 \end{array}\right] \text{Solution.}$$

only not solution

To find Inverse / in Gauss Jordan.

constant فقط بدونه [A]

→ How to find Inverse of [A]

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{\qquad}_{[I]}$ — لا يتغير

$\underbrace{\qquad}_{[A]^{-1}}$ — بيتغير ٣ مرات

Form $[I]_{3\times3}$ matrix من $[A]_{3\times3}$ نفس size

No inverse of (2×2)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & a_{13}^{-1} \\ 0 & 1 & 0 & a_{21}^{-1} & a_{22}^{-1} & a_{23}^{-1} \\ 0 & 0 & 1 & a_{31}^{-1} & a_{32}^{-1} & a_{33}^{-1} \end{array}\right]$$

$[A]_{2\times3} \quad [B]_{2\times3}$ ضفا Inner تنفع

$[A]_{3\times3}[B]_{3\times3} \neq [B]_{3\times3}[A]_{3\times3}$

Check 1 $\underset{3\times3}{[A]}\,\underset{3\times3}{[A]^{-1}} = I_{3\times3}$

$[A]^{-1}[A] = I_{3\times3}$ → matrix فقط طريقة ضرب مع inverse شكل تبادلي نفسه

Check 2 $\left[[A]^{-1}\right]^{-1} = [A]$

$\underset{3\times1}{\{\;\}}\ \underset{3\times3}{[A]^{-1}}\ X$

$\underset{3\times3}{[A^{-1}]}\,\{C\}_{3\times1} = \{\;\}_{3\times1}$ ✓ solution.

[LAB]

2	3	-5	7
1	-1	4	5.2
-1	2.5	3.5	-12

$= \square / \$\square$ →

$\$\square$ محذوف $\times$ Raw1 $=$ Raw2

sub 1

$R_2 \left[\quad \square \quad\right]$

$\square / \$\square$ →

$- \$\square \times$ Raw 2 →

لحد Raw1

نحسب بنفس الشكل

x_1 5.22981

x_2 -1.99825

x_3 -0.50702

To find inverse.

3x3 نظلل

نعرف نينكس

= minverse (   ) ctrl + shift + enter نضغط

3x3

drag. نضغط جدول ونسحب

2	3	-5	1	0	0
1	-1	4	0	1	0
-1	2.5	3.5	0	0	1

drg نسحب

= mmult (inver, orgin matrix)

format cell → number → 2 digit)

Copy invers على

original matrix → يعطي Inverse.

③ Thomas method Tri . Bandel. :

[Tridiagonal, banded] matrix No elimination

$$\begin{bmatrix} \overset{d_1}{a_{11}} & \overset{a_1}{a_{12}} & 0 & 0\\ \overset{b_2}{a_{21}} & \overset{d_2}{a_{22}} & \overset{a_2}{a_{23}} & 0 \\ 0 & \overset{b_3}{a_{32}} & \overset{d_3}{a_{33}} & \overset{a_3}{a_{34}} \\ 0 & 0 & \overset{b_4}{a_{43}} & \overset{d_4}{a_{44}}\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\\x_4\end{Bmatrix} = \begin{Bmatrix}c_1 \to r_1\\c_2 \to r_2\\c_3 \to r_3\\c_4 \to r_4\end{Bmatrix}$$

upper

lower

main $\neq 0$

one element at least

slope

$(a_1, a_2, a_3) \rightarrow$ matrix $(4,4)$

(b_2, b_3, b_4) Row

① for the 1st Row $A_1 = \dfrac{a_1}{d_1}$ $R_1 = \dfrac{r_1}{d_1}$

Start from Row② and above

For Row$_i$ $\Rightarrow A_i = \dfrac{a_i}{d_i - b_i A_{i-1}}$ and $R_i = \dfrac{r_i - R_{i-1} b_i}{d_i - A_{i-1} \cdot b_i}$

the last Row no A_4

Row$_2$ $\rightarrow A_2 = \dfrac{a_2}{d_2 - b_2 A_1}$, $R_2 = \dfrac{r_2 - b_2 R_1}{d_2 - b_2 A_1}$

For the last row ④ $\rightarrow R_4 = \dfrac{r_4 - b_4 R_3}{d_4 - b_4 A_3}$

step② :- to evaluate the Unknowns

$X_4 = R_4$

for all other X's

$X_i = R_i - A_i X_{i+1}$

$X_3 = R_3 - A_3 X_4$

$X_2 = R_2 - A_2 X_3$

$x_1 = R_1 - A_1 (X_2)$

$$\begin{bmatrix} -1200 & 100 & 0 & 0 \\ 1100 & -1500 & 100 & 0 \\ 0 & 1100 & -1240 & 100 \\ 0 & 0 & 1100 & -1250 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$A_1 = \frac{100}{-1200} = -0.08\bar{3} \qquad R_1 = \frac{-1000}{-1200} = 0.83\bar{3}$

$A_2 = \frac{100}{-1500 - 1100(-0.08\bar{3})} = -0.071005917$

$R_2 = \frac{\cancel{1000} 0 - 1100(0.83\bar{3})}{-1500 - 1100(-0.08\bar{3})} = 0.670887574$

$A_3 = \frac{100}{-1240 - 1100(-0.071005917)} = -0.086066409$

$R_3 = \frac{0 - 1100(0.670887574)}{1240 - 1100(-0.08\bar{3})} = 0.616215115$

$A_4 = 0 \qquad R_4 = \frac{0 - 1100(0.616215115)}{-1250 - 1100(-0.086066\ldots)}$

$= 0.58670 5457$

$x_4 = R_4 = 0.586705457$

$x_3 = 0.616215115 - (-0.86066409)(0.586705457)$

$= 0.666710747$

$x_2 = 0.698227982$

$x_1 = 0.891518999.$

lower upper

(4) L.U factorization.

$[A]\{x\} = \{c\}$

$[L]$ $[U]$

matrix 1, matrix 2

step (1) By Guass elemination

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$$

$\frac{a_{21}}{a_{11}}$, $\frac{a_{31}}{a_{11}}$, $\frac{a_{32}'}{a_{22}'}$

elemination

Pivot element with sign.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & a_{32}' & a_{33}' \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{bmatrix}$$

$[L][U] = [A]$

$[U][L] \neq [A]$

Rout 1 to find asolution directly

step (2) Solve the following system

$[L]\{D\} = \{C\} \rightarrow [I]$

Inderteminatic variables. consta

forward supstitio

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} \Rightarrow$$

(1) $D_1 = c_1$

$L_{21} D_1 + (1) D_2 = c_2$

$L_{31} D_1 + L_{32} D_2 + D_3 = c_3$

Pesozo variable

one colow, two colow, the colum

D_1, D_2, D_3

Step (3) Solve the following system By Backward substitutio to obtain x_1, x_2

$[U]\{X\} = [D]$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

?

$x_3 = \frac{c_3}{a_{33}''}$

one colu, bow colu (thhree col)

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} * \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -5 \\ 0 & -2.5 & 6.5 \\ 0 & 4 & 1 \end{bmatrix}$$

[L]

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & \frac{-4}{2.5} & 1 \end{bmatrix}$$

sup matrix الاولى.

$\downarrow$

$$\begin{bmatrix} 2 & 3 & -5 \\ 0 & -2.5 & 6.5 \\ 0 & 0 & 11.4 \end{bmatrix}$$

U6

$[U] \longrightarrow$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \quad [L]$$

ثم دون تشتيت

orig

sup

$\text{mmult}(U \times L) \times \longrightarrow \text{mmult}(L \times U) = A$ ✓

$\neq A$

نفك

$$[L] * \begin{matrix} D_1 \\ D_2 \\ D_3 \end{matrix} = \begin{pmatrix} 7 \\ 5.2 \\ -12 \end{pmatrix}$$

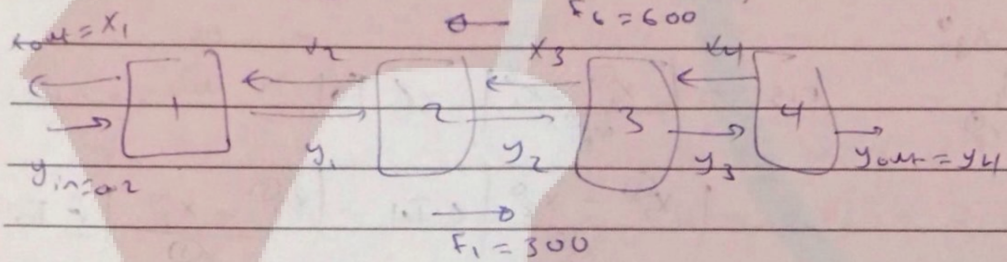
mmult(minverse(L) * C)

(قبل u) & x_1, x_2, x_3 = بعد D (min U & D)

Thomas

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

d_1, d_2, d_3, a_1, a_2, b_2, b_3 — main



$600x_2 + 0.2 \times 300 = 300y_1 + 600x_1$ (Thomas)

$2 = \frac{x_i}{y_i}$

$$\begin{bmatrix} & \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix}$$

*

Iterative Methods. Not anlytical. يعني run اكتر من مرة

اذا ماكان موجود Initial بنحط "الصفر" سكشن

1) Jacobi — (n) initial guesses (x_1^0, x_2^0, x_3^0)

2) Gauss seidel — initial gessus (n-1)

$a_{11}x_1 + a_{21}x_2 + a_{31}x_3 = c_1$

$a_{12}x_1 + a_{22}x_2 + a_{32}x_3 = c_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$

$x_1 = \frac{c_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$

$x_2 = \frac{c_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$

$x_3 = \frac{c_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$