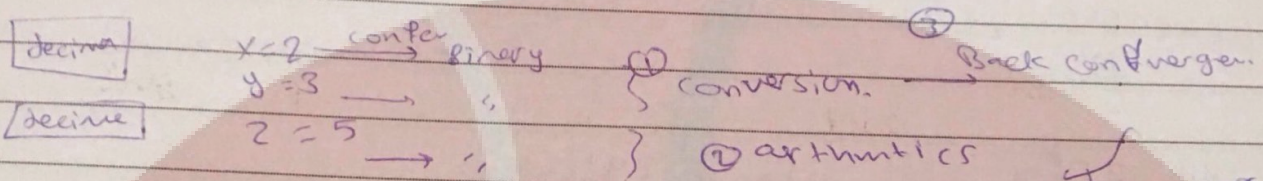


Numerical solution $\rightarrow$ approx approximate solution
 accuracy of from analytical solution.
 * decimal {0, 1, 2, 3, ... 19}



single precision $\rightarrow$ single word $\rightarrow$ 32 bits
 one bit $\rightarrow$ 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 131072, 262144, 524288, 1048576, 2097152, 4194304, 8388608, 16777216, 33554432, 67108864, 134217728, 268435456, 536870912, 1073741824, 2147483648, 4294967296, 8589934592, 17179869184, 34359738368, 68719476736, 137438953472, 274877906944, 549755813888, 1099511627776, 2199023255552, 4398046511104, 8796093022208, 17592186044416, 35184372088832, 70368744177664, 140737488355328, 281474976710656, 562949953421312, 1125899906842624, 2251799813685248, 4503599627370496, 9007199254740992, 18014398509481984, 36028797018963968, 72057594037927936, 144115188075855872, 288230376151711744, 576460752303423488, 1152921504606846976, 2305843009213693952, 4611686018427387904, 9223372036854775808, 18446744073709551616, 36893488147419103232, 73786976294838206464, 147573952589676412928, 295147905179352825856, 590295810358705651712, 1180591620717411303424, 2361183241434822606848, 4722366482869645213696, 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From 2 Option of Integer number

division
overflow

variable

1) Single Precision $\rightarrow$ double word

capacity ~ 10

old 32 bit

$L = 245$

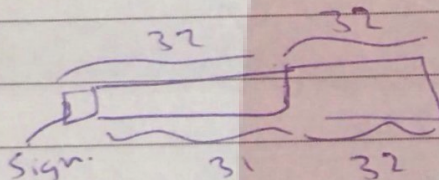
$L = -245$

Sign 32 bit

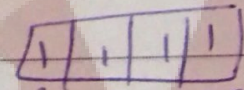
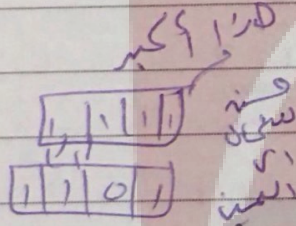
(ex) 32 Sign 31 bit available for number

$D1 = 245$

double precision



4 precision



$2^3 2^2 2^1 2^0 \rightarrow$ Integer number $= 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$

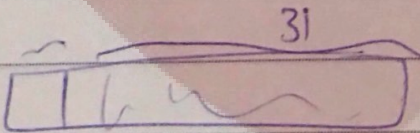
$= 1 + 2 + 4 + 8$

max number $= 15$

16

max number

overflow



max number
single &
double

In Real number

Contribute

Real word

Float point

0.1101×2^4
 $A = 2.44$

Float $A = 70 10 10 10 \times 10$

default

0.001101 $\rightarrow$ $\begin{bmatrix} 0 & 1101 \end{bmatrix} \times 2^{-2}$

mantissa

exponent

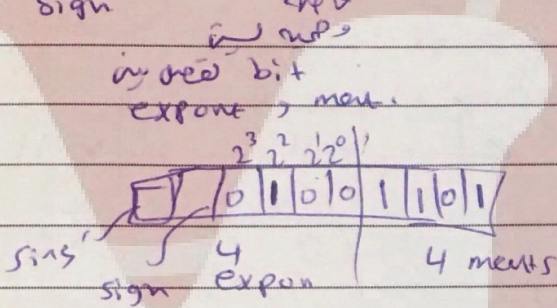
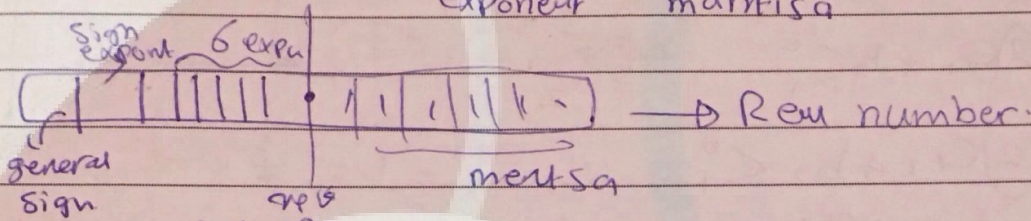
$n = \pm 0.11010 \times 2^{\pm 1}$

single pre 32-2=30

sign

8 exponent

12 mantissa

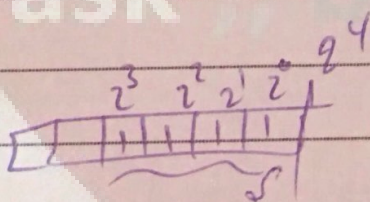
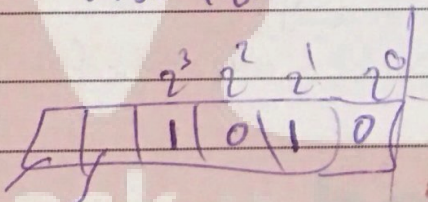


$0.1101 \times 2^{-2} \rightarrow 2^2 = 4$

0.111101×2^{-2}

overflow or underflow

exponent 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31



overflow or underflow

sign

exponent

① Conversion From Integer decimal to binary.

no fraction

base 2

bit 3
quart 4

N → any integer number

base = $\frac{N}{2}$

$$N = 2Q_0 + b_0$$

remainder (div)

Base 2

Base 3

$$Q_0 = 2Q_1 + b_1$$

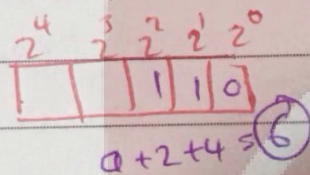
$$Q_1 = 2Q_2 + b_2$$

rep = Q is 0 is 0000

when $Q_n = \text{zero}$ then stop.

Then binary

num: $b_4 b_3 b_2 b_1 b_0$



$$3 = 2 \times 1 + 1$$

$$1 = 2 \times 0 + 1$$

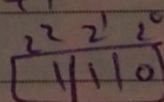
$$3 = \begin{matrix} 2^1 & 2^0 \\ 1 & 1 \end{matrix}$$

$$1+2=3$$

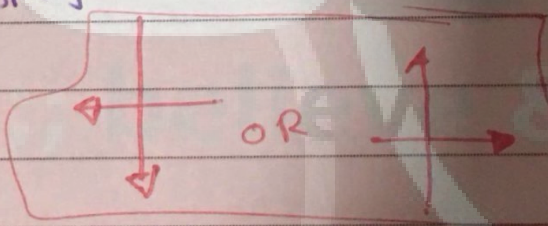
$$6 = 2 \times 3 + 0$$

$$3 = 2 \times 1 + 1$$

$$1 = 2 \times 0 + 1$$



$$0+2+4=6$$



$$N = 17$$

$$17 = 2 \times 8 + 1 \quad b_0$$

$$8 = 2 \times 4 + 0 \quad b_1$$

$$4 = 2 \times 2 + 0 \quad b_2$$

$$2 = 2 \times 1 + 0 \quad b_3$$

$$1 = 2 \times 0 + 1 \quad b_4$$

$$b_4 b_3 b_2 b_1 b_0$$

$$10001 = N_2$$

$$16+0+0+0+1$$

$$= 17$$

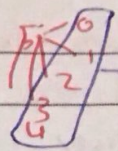
17

$$17 = 5 \times 3 + 2 \text{ } b_0$$

$$3 = 5 \times 0 + 3 \text{ } b_1$$

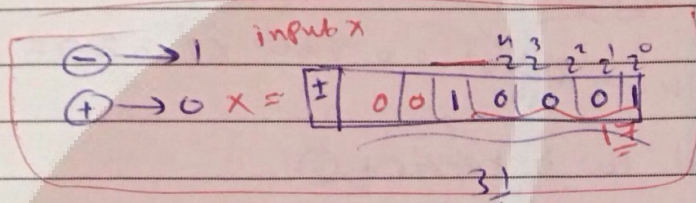
$$\begin{matrix} 5^1 & 5^0 \\ 3 & 2 \end{matrix} = 17_5$$

$$2 \times 1 + 3 \times 5 = 17$$



Base

2 → Base-1
5 → 4
7 → 6



Integer 2-3
Fraction 2
6.5

② conversion from Real decimal to binary.

$$\text{Real} = \text{Integer} + \text{Fraction} \quad R$$

$$\frac{N}{2}$$

$$R \times 2 = \boxed{1} . \boxed{F} \rightarrow F \times 2 = \boxed{0} . \boxed{F}$$

Integer part

$$\begin{matrix} N & R \\ 2 & 25 \end{matrix}$$

$$R = 0.25$$

$$N = 2 \times 1 + 0$$

$$1 = 2 \times 0 + 1$$

$$2 = 10$$

$$2 \times R = 2 \times 0.25 = 0.5$$

$$2 \times F_1 = 2 \times 0.5 = 1.0$$

$$0.20212223 = 0.010 = 0.25 + 2.25$$

$$0.222, 20 = 0.010$$

$$2 \times F_2 = 2 \times 0 = 0.0$$

$$F = 0.25$$

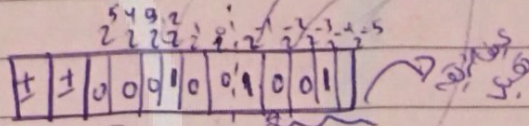
20
01
02
03
04

$$0.10111 = 0.718$$

$$= 0.906$$

$2.25 = 10.01$ *integer fraction*
 $\Rightarrow 0.1001 \times 2^{+2}$ *exponent*
 $\Rightarrow 0.100000 \times 2^{+2}$ *base 2*
 $\Rightarrow 0.100000 \times 2^{+2}$ *base 2*

$-n$
 $+n$



$1 \times 0.5 + \frac{1}{16} \times 1 = 0.5625 \times 2^2 = 2.25$

* Conversion of Real decimal to ternary ③

$x = 18.639$ *Real*

$18 = 3 \times 6 + 0$
 $6 = 3 \times 2 + 0$
 $2 = 3 \times 0 + 2$
 $\Rightarrow 18_3 = 200$
 $0 + 0 + 2 \times 9 = 18$

$0.639 \rightarrow 3 \times 0.639 = 1.917$

$3 \times 0.917 = 2.751$
 $d_1 \quad F_2$

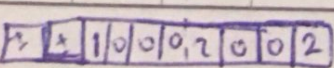
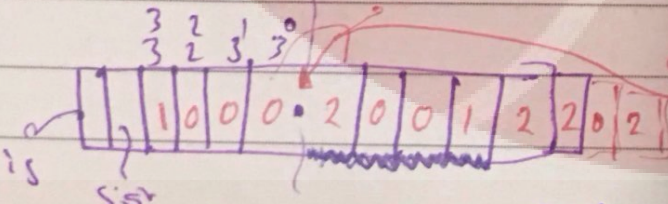
$3 \times 0.751 = 2.253$
 $d_2 \quad F_3$

$0.253 \times 3 = 0.759$
 $d_3 \quad F$

$3 \times 0.759 = 2.277$
 d_4

$0.639_3 = 0.12202$

$18.639_3 = 200.12202$



$18.639_3 \approx 18.333$
 $18.639_3 \approx 18.666$

error detection exact error rate is mensa.
 error detection no error mensa

$n \rightarrow +$ over flow

$n \rightarrow -$ under flow -1×10^8

you have under flow

$$C = A \times 8$$

Limited Capacity.

(if it is)

$$\begin{array}{r} X = 0110111 \\ Y = 0010011 \\ \hline X + Y = 1101000 \end{array}$$

$$1001010$$

$$\begin{array}{r} X = 0110111 \\ Y = 0010011 \\ \hline Z = 0100100 \end{array}$$

$$X + Y + Z = 11101$$

$$= 11101$$

you have error

$$\begin{array}{r} X = 0110111 \\ Y = 0010011 \\ \hline X - Y = 0100100 \end{array}$$

$$\begin{array}{r} X = 0110111 \\ Y = 0010011 \\ \hline X - Y = 0100100 \end{array}$$

$X - Y = 0111000$ you take from next bit.

ask & recieve

homework $\rightarrow (*, \div) \rightarrow$

number of bit X $\geq$ $\text{larg } Y$
 $X > Y$

$$\begin{array}{r} X = 0110111 \\ Y = 0010011 \\ \hline X - Y = 0100100 \end{array}$$

number of bit $X - Y$

$(Y - X) \rightarrow$

$$\begin{array}{r} 1 = 1 + 0 \\ (1 \text{ ~} 490) 10 = 1 + 1 \\ (1 \text{ ~} 101) 11 = 1 + 1 + 1 \end{array}$$

Error

① Adding or subtracting one large and one small.

one red | 4

5	5				
---	---	--	--	--	--

$10^{10} \rightarrow$ two riget

اولاً ما ينبغي لو
نظرة اعم
تكون exposure نفسها

$$\begin{array}{r}
 + \quad 0.000001 \times 10^{-1} \times 10^5 \rightarrow \times 10^4 \\
 \quad 0.500000 \times 10^4 \rightarrow \times 10^4 \\
 \hline
 0.500001
 \end{array}$$

→ 0.50001 $\times 10^4 \rightarrow 0.5000 \times 10^4 \rightarrow 5000 \rightarrow$ error 100%
 ✗
 OR
 Rounding
 this is not acceptable

$$P_{\text{K}_2} + P_{\text{K}_1} = P_{\text{K}_2} \rightarrow \text{error } 100\%$$

equal

② Round off in subtracting two almost number.

$$\begin{array}{r} x = 0.1235 \times 10^0 \\ y = 0.1234 \times 10^0 \\ 0.0001 \times 10^0 \end{array}$$

recieve

③ overflow and underflow.

$$x = 0.1000 \times 10^6$$

$$y = 0.2000 \times 10^7$$

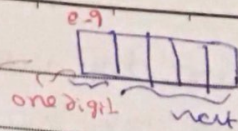
$$0.0200 \times 10^{13}$$

$$\downarrow$$

$$0.2000 \times 10^{+12}$$

two digit

→ overflow.



power of 2 is 4

very long

$$x = 0.1000 \times 10^{+7}$$

very long

$$y = 0.1000 \times 10^{-5}$$

very small

$$1.0000 \times 10^{12}$$

$$= 0.1000 \times 10^{+11}$$

two digit (overflow if word is limiting)

$$A = 0.1000 \times 10^{+6}$$

4 bit

one digit

$$B = 0.2000 \times 10^{-5}$$

$$C = 0.3000 \times 10^{+7}$$

$$① x = A \times B \times C$$

$$A = 0.1000 \times 10^{-6}$$

$$B = 0.2000 \times 10^{-5}$$

$$= 0.0200 \times 10^{-11}$$

$$0.2000 \times 10^{+12}$$

two digit

underflow is
approximation
 0.2×10^{-12}

$$A \times B = 0 \times C = 0$$

$$0 \times 0.3000 \times 10^{+7}$$

0% → 100% error.

$$① \quad ②$$

$$= A \times C \times B$$

$$A = 0.1000 \times 10^{-6}$$

$$C = 0.3000 \times 10^{+7}$$

$$= 0.3000 \times 10^{+1}$$

$$0.3000 \times 10^0$$

$$(A \times C) \times B$$

$$0.3000 \times 10^{+1}$$

$$= 0.2000 \times 10^{-5}$$

$$0.0600 \times 10^{-5}$$

$$0.6000 \times 10^{-6} \neq 0$$

① True error = True value - Approximate

② % relative error = $\frac{\text{True error}}{\text{True value}} \times 100\%$

1st iteration
2nd iteration
3rd iteration

$\frac{\text{True} - \text{Approx}}{\text{True value}} \rightarrow \text{actual}$

$\frac{\text{New} - \text{old}}{\text{New}} \rightarrow \text{successive relative error}$

Solving single non-linear equation.

$$f(x) = 0$$

$$x^3 - 2x = \ln x \rightarrow x^3 - 2x - \ln x = 0$$

$$f(x) = x^3 + 3x$$

$$g(x) = \sin x$$

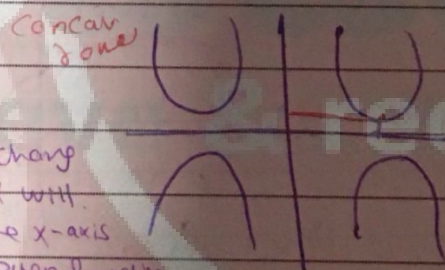
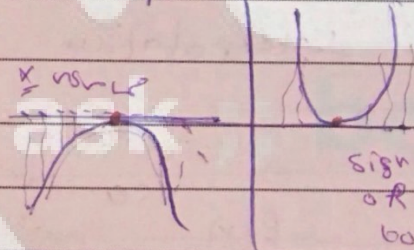
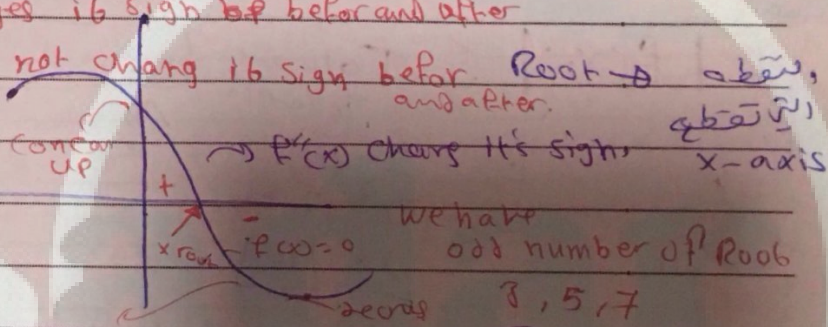
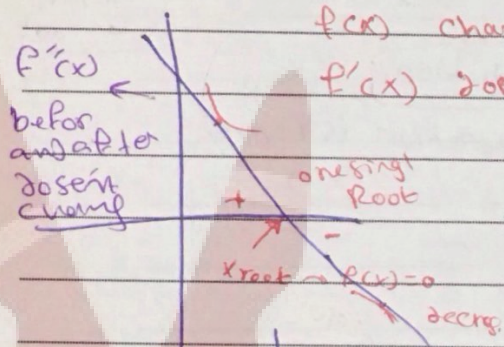
When intersect two graphs.

$$x^3 + 3x = \sin x \rightarrow x^3 + 3x - \sin x = 0$$

if $f(x)$ or $g(x)$ equal zero or approx zero.

$f(x)$ changes its sign before and after

$f'(x)$ does not change its sign before and after.



imaginary Root (Complex Root)

Real + imag

$f' \rightarrow \oplus \rightarrow \ominus$
Concave
 $f' \rightarrow \oplus \oplus$
 $\ominus \ominus$

$$(x-2)(x-2) = 0$$

$$x_1 = 2, x_2 = 2$$

Repeated

even number of Root

$2 + 3i$
 $2 - 3i$
when x -axis is y -axis
 $\sqrt{40}$

even $f \rightarrow$ $\oplus \rightarrow \oplus$ or $\ominus \rightarrow \ominus$
Roots are 2

$f \rightarrow$ $\oplus \rightarrow \ominus$ or $\ominus \rightarrow \oplus$
Roots are 3

→ 1st iteration

$$x_2 = x_0 - f(x_0) \left[\frac{x_0 - x_1}{f(x_0) - f(x_1)} \right]$$

OR

$$x_2 = x_1 - f(x_1) \left\{ \frac{x_1 - x_0}{f(x_1) - f(x_0)} \right\}$$

$$f(x) = 3x + \sin x - e^x$$

$$x \in [0, 1]$$

$$\text{tolerance} = 1 \times 10^{-5}$$

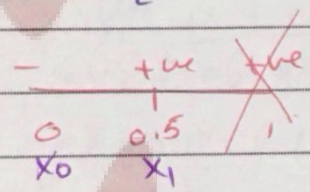
$$x_0 = 0 \rightarrow f(x_0) = f(0) = -1 \quad \text{---ve}$$

$$x_1 = 1 \quad f(1) = 3 + \sin 1 - e^1 = 1.123189 \quad \text{+ve}$$

$$x_2^1 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5 \rightarrow f(0.5) = 0.330704$$

$$|0.330704| > 1 \times 10^{-5}$$

∴ x_2^1 is not a solution.



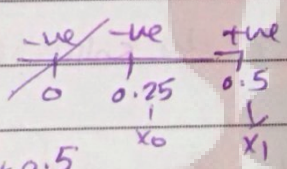
$$x_0 = 0 \quad x_1 = 0.5$$

$$x_0 = 0.25 \quad x_1 = 0.5$$

$$x_2^2 = \frac{0 + 0.5}{2} = 0.25 \Rightarrow f(0.25) = -0.28662$$

$$|0.28662| > 1 \times 10^{-5}$$

x_2^2 is not a solution.



$$x_2^3 = \frac{0.25 + 0.5}{2}$$

ask & receive

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	check
0		1				copy paste

positive $\ominus$
negative $\oplus$

→ if $f(x_2) = f(x_0)$ & $x_2 > x_0$
if $f(x_2) = f(x_1)$ & $x_2 > x_1$

if $f(x_0) \neq f(x_2)$ & $x_2 > x_0$
if $f(x_1) \neq f(x_2)$ & $x_2 > x_1$
sign

if $abs(f(x_2)) > 0.00001$
another in else

6 iteration

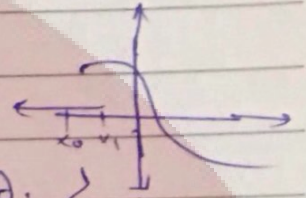
$$x_2' = 0 - (-1) \Delta \left(\frac{0 - 1}{-1 - 1.123181} \right) = 0.47699$$

$$f(x_2') = f(0.47699) = |6.26516| > 1 \times 10^{-5}$$

++ → 2nd solution

Open method

(+ -) → 2nd solution



① secant method

False position method.

Root

Go to next step

* if $|f(x_0)| > |f(x_1)|$ then keep $x_0 = x_0$, $x_1 = x_1$
 if $|f(x_0)| < |f(x_1)|$ then swap the values $x_0 = x_1$, $x_1 = x_0$

convergence
 قبل
 iteration
 Before the first iteration.

* Next step calculate the expected root x_2

$$x_2^{(1)} = x_0 - f(x_0) \left\{ \frac{x_0 - x_1}{f(x_0) - f(x_1)} \right\}$$

$$\text{OR } x_2^{(1)} = x_1 - f(x_1) \left\{ \frac{x_1 - x_0}{f(x_1) - f(x_0)} \right\}$$

Find $f(x_2)$

if $|f(x_2)| > \text{Tol}$, then another iteration is required.

if $|f(x_2)| \leq \text{Tol}$, the x_2 is the solution.

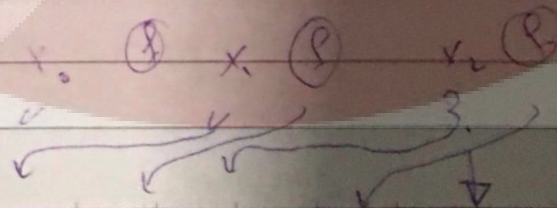
for the next iteration eliminate x_0 and remain $x_0^{\text{new}} = x_1^{\text{old}}$

$$|f(x_0)| > |f(x_1)| > |f(x_2)| > |f(x_2^{(1)})|$$

x_0	x_1	x_2	$x_2^{(1)}$
x_0	x_1	x_1	x_2

تلفظ
 في
 الشرح

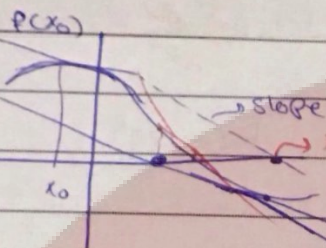
calculate $x_2^{(1)}$



Problem 2.1

$$f(x) = 0$$

② Newton's method → one initial guess is required. x_0



Share the slope to obtain convergence.

$$\text{slope} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f'(x_0)}{1}$$

$$(x_1 - x_0) f'(x_0) = -f(x_0)$$

$$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)} \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$|f(x_1)| > \text{Tol}$, another iteration

$|f(x_1)| \leq \text{Tol}$ another iteration. Stop. → eliminate x_0 old and swap it by x_1

$$x_0^{\text{new}} = x_1$$

now calculate x_1

$$f(x) = 3x + \sin x - e^x \quad ; \quad x_0 = 0 \quad ; \quad x_1 = 1 \quad ; \quad \text{Tol} = 1 \times 10^{-5}$$

secant method.

$$f(x_0) = f(0) = -1$$

$$f(x_1) = f(1) = 1.123189156$$

→ since $|1.123189156| > 1$ swap the names.

$$\boxed{x_0 = 1 \quad ; \quad x_1 = 0}$$

$$x_2' = x_0 - f(x_0) \left\{ \frac{x_0 - x_1}{f(x_0) - f(x_1)} \right\}$$

$$x_2' = 1 - 1.23 \left[\frac{1 - 0}{1.1232 + 1} \right] = 0.4709895947$$

$$f(x_2') = f(0.47) = |0.2651588161| > 1 \times 10^{-5}$$

for the 2nd iteration

$$x_0^{\text{new}} = x_1^{\text{old}} = 0$$

$$x_1^{\text{new}} = x_2' = 0.4709$$

$$x_2^{\text{new}} = 0 - (-1) \left\{ \frac{0 - 0.4709897}{-1 - 0.26515} \right\}$$

$\frac{1}{x}$ sing $\frac{1}{x^2}$ odd $\frac{1}{x^3}$ even $\frac{1}{x^4}$ complex
 $x_0 = \frac{f(x_0)}{f'(x_0)}$
 Simple Newton Method

Newton's: $x_0 = 0$ (if $f'(x) \neq 0$)

$$P'(x) = 3 + \cos(x) - e^x$$

$$P(x_0) = P(0) = -1$$

$$P'(x_0) = P'(0) = 3$$

$$x_1^{(0)} = 0 - \frac{(-1)}{3} = 0.333$$

$$P(x_1) = P(0.333)$$

x_0	$P(x_0)$	x_1	$P(x_1)$	x_2	$P(x_2)$	check
0	$3x + \sin x - e^x$	$1 \rightarrow i P(3=2), \sqrt{2}, A_1$				$ P(x_2) < 10^{-5}$
						$> 10^{-5}$
						"one" steps

$$x_0 = x_1$$

$$x_1 = x_2$$

$$x_2 = 0.364 \rightarrow P(x_2) = -5.5 \times 10^{-6}$$

Newton's

x_0	$P(x_0)$	$P'(x_0)$	x_1	$P(x_1)$	check
0	-1	$3 + \cos x_0 - e^{x_0}$	$\frac{x_0 - P(x_0)}{P'(x_0)}$		

$$x_0 = x_1$$

$$if x_0 = 1$$

Simple Newton

Roots are not real
Roots are complex
Range of real roots

Modified Newton's: Mainly used for even number of roots.

$$x_1^{(0)} = x_0 - \frac{P(x_0) P'(x_0)}{[P'(x_0)]^2 - P(x_0) P''(x_0)}$$

$$P(x) = (x-2)(x-2)$$

$$P(x) = x^2 - 4x + 4$$

$$P'(x) = 2x - 4$$

$$P''(x) = 2$$

Simple Newton

$$x_0 = 0$$

modified

$$x_0 = 0$$

11 iterations vs very simple eq

x_0	$f(x_0)$	$f'(x_0)$	x_1	$f(x_1)$	check
0	4	-4	$x_0 - \frac{f(x_0)}{f'(x_0)} = 1$	1	$\downarrow$ if $(\text{abs}(f(x_1)) > 1 \times 10^{-6})$ another iteration stop

$= x_1$

stop 50 iterations

x_0	$f(x_0)$	$f'(x_0)$	$f''(x_0)$	x_1
0	4	-4	2	$x_0 - \frac{f(x_0)}{f'(x_0) + \frac{f''(x_0)}{2} f(x_0)}$

even it avoids oscillating

7. (B) Muller's method (most efficient) requires 3 initial guesses.

mid point
 x_0, x_1, x_2
 $x_2 = \frac{x_0 + x_1}{2}$

Step ①: name the guesses
 2, 4, -1

For all iteration, $f(x_0), f(x_1), f(x_2)$

if $x_2 < x_0 < x_1$
 $-1 < 2 < 4$
 $f(x_2), f(x_0), f(x_1)$
 No trend of f

Step ②: $c_1 = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$ (slope 2 pt)

slope 3 point
 $c_2 = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{f(x_2) - f(x_0)}{x_2 - x_0}$

$d_1 = \frac{c_2 - c_1}{x_0 - x_2}$ $d_2 = c_2 + d_1(x_0 - x_1)$

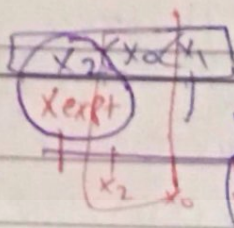
now the expected $x_{exp}^{(1)} = x_0 - \sqrt{2 f(x_0) / d_2}$

$|f(x_{exp}^{(1)})| > \epsilon$ continues then another iteration → go to step ①
 $\leq$ the x_{exp} is the root → stop

$$x_2 = x_{exp} \quad x_0 = x_2$$

$$x_0 = x_{exp}$$

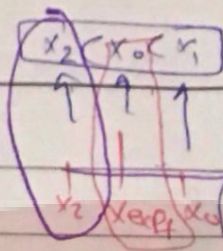
①



$$x_{exp} < x_2$$

$$x_1 = x_0$$

②



$$x_2 = x_2$$

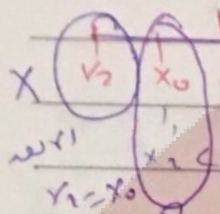
$$x_1 = x_0$$

نظير

and

is

③



$$x_{exp} < x_1$$

$$x_{exp} < x_1$$

$$x_1 = x_1$$

$$x_2 = x_0$$

$$x_2 = x_0$$

$$x_2 = x_0$$

$$x_2 = x_0$$

$$x_2 = x_0$$

$$x_2 = x_0$$

2 case eliminat x_1

2 case eliminat x_2

[x_{exp} and x_0]

www, (a), (b), (c)

* in programme we need instel if statment 3 if statment

u)

(x_2)

$f(x_2)$

(x_0)

$f(x_0)$

(x_1)

$f(x_1)$

c_1

c_2

d_1

s

0

?

0.5

?

1

?

?

?

?

?

?

if ($x_{exp} < x_2$; x_{exp} ; if ($x_{exp} < x_0$; x_2 ; if ($x_{exp} < x_1$; x_0 ; x_0)))

if ($x_{exp} < x_2$; x_2 ; if ($x_{exp} < x_0$; x_{exp} ; if ($x_{exp} < x_1$; x_{exp} ; x_1)))

if ($x_{exp} < x_2$

x_{exp}

$f(x_{exp})$

check

if $abs(f(x_{exp})) > 10^{-5}$; "anti" ; stop

(x_0)

if ($x_{exp} < x_2$; x_2 ; if ($x_{exp} < x_0$; x_{exp} ; if ($x_{exp} < x_1$; x_{exp} ; x_1)))

if ($x_{exp} < x_1$; x_{exp} ; x_1)

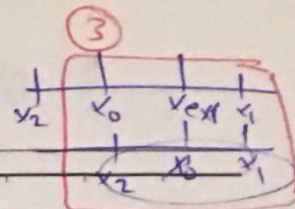
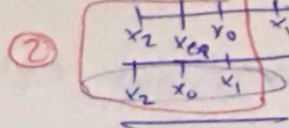
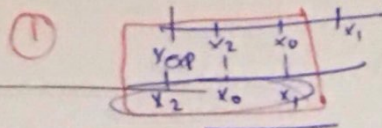
$$f(x) = x^2 - 4x + 4$$

$$x_0 = 0 \quad x_1 = 5$$

$$x_2 = x_0 = 0$$

$$x_2 = \text{if} (x_{exp} < x_2 ; x_{exp} ; \text{if} (x_{exp} < x_0 ; x_2 ; \text{if} (x_{exp} < x_1 ; x_0 ; x_0)))$$

$$x_1 : \text{if} (x_{exp} < x_2 ; x_0 ; \text{if} (x_{exp} < x_0 ; x_0 ; \text{if} (x_{exp} < x_1 ; x_1 ; x_{exp})))$$

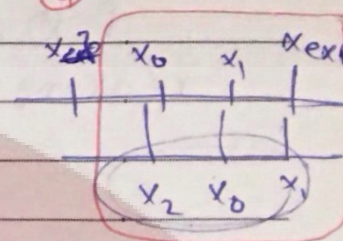
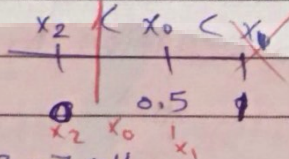


Ex: $f(x) = 3x + \sin(x) - e^x$

$x_0 = 0$ $x_1 = 1$

$x_2 = \frac{0+1}{2} = 0.5$

$x_{exp} = 0.354914$



$f(x_0) = f(0.5) = 0.330704$

$f(x_1) = f(1) = 1.123189156$

$f(x_2) = f(0) = -1$

$c_1 = \frac{1.123189156 - (-1)}{1 - 0} = 2.123189$

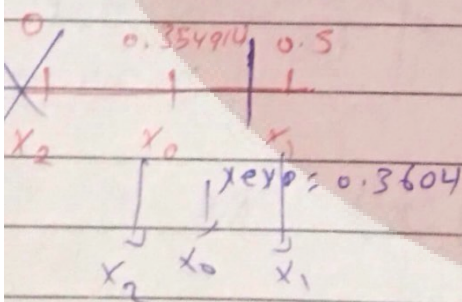
$c_2 = \frac{0.330704 - 1.123189156}{0.5 - 1} = 1.58497$

$d_1 = \frac{c_2 - c_1}{x_0 - x_1} = \frac{1.58497 - 2.123189}{0.5 - 0} = -1.07644$

$\delta = c_2 + d_1(x_0 - x_1) = 1.58497 + (-1.07644)(0.5 - 1)$
 $= \boxed{+2.123189} = c_1$ in always in first iteration only.

$x_{exp} = 0.5 - \frac{2 \times 0.330704}{(+2.123189) + (+)\sqrt{(2.123189)^2 - 4 \times (-1.07644)(0.330704)}}$
 $= 0.354914$

$f(0.354914) = |-0.01381| > 1 \times 10^{-5}$ we need another iteration



$c_1 = 2.66144$

$c_2 = 2.37453$

$d_1 = 0.80831$

$\delta = 2.4918$

$x_{exp} = 0.36046$

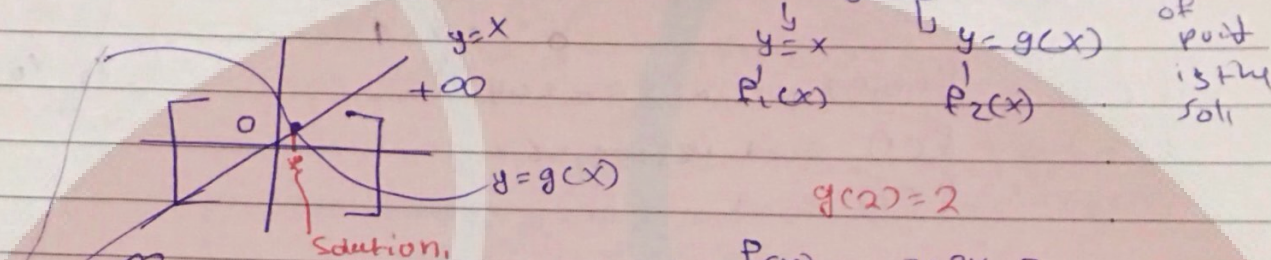
$f(x_{exp}) = 0.00011$

Single non linear equation.

* Fixed Point iteration one initial guess only. (Newton's)

$$f(x) = 0$$

Rearrang this equation to have $x = g(x)$ intersection of



$$g(x) = 2$$

$$f(x) = x^2 + 2x - 3$$

$$2x = \frac{x^2 - 3}{2}$$

$$x = \frac{x^2 - 3}{2} \rightarrow 2 \text{ Solution}$$

$$x = \sqrt{3 - 2x} \quad g_2(x)$$

one solution.

$$x^2 + 2x = 3$$

$$x(x+2) = 3$$

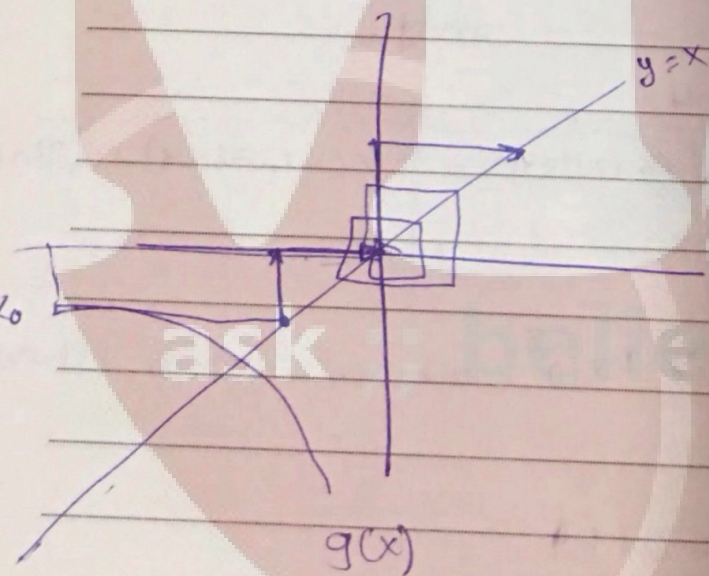
$$x = \frac{3}{x+2} \quad g_3(x)$$

$$x = \frac{3 - x^2}{x} \quad g_4(x)$$

$$3 - 2x = 0$$

$$3 = 2x$$

$$x = \frac{3}{2}$$



$$x = g_1(x) = \frac{3 - x^2}{2}$$

$$x_1' = g_1(x_0)$$

define
vs
Relative
error

if $g(x_0) = x_0$
then x_0
is a solution

$$R.E = \left| \frac{x_1^{(1)} - x_0}{x_1^{(1)}} \right| \leq \text{stop}$$

> another
it...

$$x_0 = x_1$$

$$x_1^2 = g(x_1')$$

$$R.E = \frac{x_1^2 - x_1}{x_1^2}$$

$$x_0 = 0.5$$

$$x_1^1 = g_1(x) = \frac{3-x^2}{2} = \frac{3-(0.5)^2}{2} = 1.375$$

$$x_1^2 = g(1.375) = \frac{3-1.375^2}{2} = 0.554688$$

$$x_1^3 = g(0.554688) = \frac{3-(0.554688)^2}{2}$$

x_0	$g_1(x)$	check P.S.	check
0.5	$+0.875$ -0.5 $= g_1(x)$	$\frac{g_1(x) - x_0}{g_1(x)}$	$abs(1.375) > 1$

في كل مرة g نأخذ g ونقوض x_0 إذا انزل

$$|f(x_0)| < 1$$

$$g_1(x) = \frac{3-x^2}{2} \rightarrow g'_1\left(\frac{-2x}{2}\right) = -x$$

$$\text{at } x_0 = 0.5 \rightarrow g'_1(0.5) = -0.5 < 1 \text{ valid}$$

هذا هو الحل

$$g = \frac{3}{x+2}$$

$$g'(x) = \frac{-3}{(x+2)^2} \rightarrow g'(0.5) = \frac{-3}{(0.5+2)^2} = \frac{-3}{(2.5)^2} = -0.48 < 1$$

في كل مرة g نأخذ g ونقوض

$$g(x) = \sqrt{3-2x}$$

$$g'(x) = \frac{-2}{2\sqrt{3-2x}} = \frac{-1}{\sqrt{3-2x}}$$

$$g'(0.5) = \frac{-1}{\sqrt{3-2(\frac{1}{2})}} = \frac{-1}{\sqrt{2}} = -0.7 < 1$$

هذا هو الحل

Solution directly.

Cramer's Rule

up to 3 eq.

Small System

Large System.

up to 10

System of Linear Equation. any number of equation.

Direct

Iterative.

① Gauss elimination

② Thomas (special matrix)

③ Gauss-Jordan

④ LU Factorization.

① Jacobi (Zinib goss)

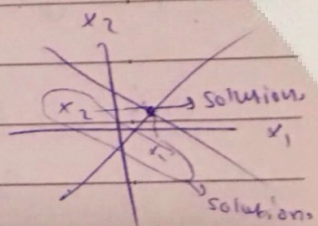
② Gauss-Seidel

③

$$2x_1 + 3x_2 = 5$$

$$3x_1 - 5x_2 = 11$$

① Graphical (Point & line)
② elimination



$$x_1 = \frac{5 - 3x_2}{2}$$

$$\Rightarrow 3(5 - 3x_2) - 5x_2 = 11 \rightarrow \text{one unknown}$$

Variable of axis.

coefficients

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= c_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= c_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= c_3 \end{aligned}$$

if $A \times \text{equation} = X \text{ variables}$

variables

$$x_1 = \frac{\begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}}{\det[A]}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix}}{\det[A]}$$

$$\det[A]$$

$$\text{Solution } x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\det[A]}$$

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}}{\det[A]}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, a_{11}a_{22} - a_{12}a_{21}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{pmatrix} + \\ - \\ + \end{pmatrix} a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$$

$$\begin{pmatrix} - \\ + \end{pmatrix} a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$$

$$\begin{pmatrix} + \\ - \end{pmatrix} a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & & & \\ & & & \\ & & & a_{44} \end{bmatrix} \quad \text{we can't determine matrix } 4 \times 4$$

① only have square system of matrix

② all equation must be independent.

$$2x_1 + 3x_2 = 5$$

$$2x_1 + 3x_2 = 5$$

no, is
no sol

Infinite no of solution.

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = 0$$

$$2x_1 + 3x_2 = 5$$

$$4x_1 + 6x_2 = 10$$

Parallel
no solution

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = \text{Det} = 0$$

③ $\det = 0$ is system of line

$$\text{Det} = \text{Slope} \neq 0$$

④ must be well condition system

$\# \text{ eqn} = \# \text{ variable}$

square is $n \times n$

(well)
Unique
solution.

No solution

Infinite
solution

Wide range of solution

$$\text{Det} \neq 0$$

ill condition

System very

Sensitive of

small

variation in input

$$2x_1 + 3x_2 - 5x_3 = 7$$

$$[A]\{x\} = \{c\}$$

$$x_1 - x_2 + 4x_3 = 5.2$$

$$-x_1 + 2.5x_2 + 3.5x_3 = -12$$

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$x_1 \quad x_2 \quad x_3$

$$\text{Det}[A] = 2(-3.5 - 4(2.5)) - 3(3.5 + 4) + (-5)(7.5 - 1)$$

$$= -57$$

$$x_1 = \frac{\begin{bmatrix} -7 & 3 & -5 \\ 5.2 & -1 & 4 \\ -12 & 2.5 & 3.5 \end{bmatrix}}{-57}$$

$$x_2 = \frac{\begin{bmatrix} 2 & 7 & -5 \\ -1 & 5.2 & 4 \\ -12 & -12 & 3.5 \end{bmatrix}}{-57}$$

$$x_3 = \frac{\begin{bmatrix} 2 & 3 & 7 \\ -1 & -1 & 5.2 \\ -12 & 2.5 & -12 \end{bmatrix}}{-57}$$

$$x_1 = \frac{-298.1}{-57}$$

$$x_2 = \frac{113.9}{-57}$$

$$x_3 = \frac{28.9}{-57}$$

$$x_1 = 5.2298$$

$$x_2 = -1.99825$$

$$x_3 = -5.072$$

08

constant

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$$\begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

constant

حساب واریان

الجاب
نحوه
حساب
ماتریس
مربعی

= DETERM (A selection) → all
constant

Trans and Copy for

4 → solution sign.

§§§

Polymath.

Linear

x_1, x_2, x_3, x_4, x_5

Row size
in

main upper

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$\neq 0$

lower Triangle.

Gauss - Elimination.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$\rightarrow [A][x] = \{C\}$$

$$\{x\} = \frac{[C]}{[A]}$$

Step ① Forward elimination.

Row a_{ij}

elem of lower triangle.

change $\rightarrow$ Column use x

Variable and constant

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2' \\ c_3'' \end{bmatrix}$$

Reduction of Variable

$$a_{22}'x_2 + a_{23}'x_3 = c_2'$$

$$a_{33}''x_3 = c_3'' \rightarrow x_3 = \frac{c_3''}{a_{33}''}$$

$$x_2 = \frac{c_2' - a_{23}'x_3}{a_{22}'}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$

Step ② Backward substitution.

Forward $\rightarrow$ upper triangular substitution

x_3
 x_2
 x_1

elimination on nonsquare matrix

Row $\rightarrow$ Column

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & | & c_1 \\ a_{21} & a_{22} & a_{23} & | & c_2 \\ a_{31} & a_{32} & a_{33} & | & c_3 \end{bmatrix}$$

Row $\rightarrow$ Column

Row $\rightarrow$ Column

main $\rightarrow$ upper

Pivot element
 $= -\frac{a_{21}}{a_{11}}$

$$\begin{bmatrix} \frac{a_{11}}{a_{11}} & \frac{a_{12}}{a_{11}} & \frac{a_{13}}{a_{11}} & \frac{c_1}{a_{11}} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{a_{12}}{a_{11}} & \frac{a_{13}}{a_{11}} & \frac{c_1}{a_{11}} \end{bmatrix} X' - a_{21} X - a_{31}$$

$$\begin{bmatrix} -a_{21} & -\frac{a_{21} a_{12}}{a_{11}} & -\frac{a_{21} a_{13}}{a_{11}} & -\frac{a_{21} c_1}{a_{11}} \end{bmatrix}$$

Pivot equation for Row 2

Pivot element
 $= -\frac{a_{31}}{a_{11}}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a_{22} - \frac{a_{21} a_{12}}{a_{11}} & a_{23} - \frac{a_{21} a_{13}}{a_{11}} & c_2 - \frac{a_{21} c_1}{a_{11}} \\ 0 & a_{32} & a_{33} & c_3 \end{bmatrix}$$

$$\begin{bmatrix} -a_{31} & -\frac{a_{31} a_{12}}{a_{11}} & -\frac{a_{31} a_{13}}{a_{11}} & -\frac{a_{31} c_1}{a_{11}} \end{bmatrix}$$

Pivot equation for Row 3

Pivot of Row 2

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a_{22}' & a_{23}' & c_2' \\ 0 & a_{32}' & a_{33}' & c_3' \end{bmatrix}$$

Column 2

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a_{22}' & a_{23}' & c_2' \\ 0 & 0 & a_{33}' - \frac{a_{23}' a_{13}}{a_{22}'} & c_3' - \frac{a_{23}' c_2'}{a_{22}'} \end{bmatrix}$$

$$\begin{bmatrix} \frac{0}{a_{22}'} & \frac{a_{22}'}{a_{22}'} & \frac{a_{23}'}{a_{22}'} & \frac{c_2'}{a_{22}'} \end{bmatrix}$$

Pivot element

$$\begin{bmatrix} 0 & 1 & \frac{a_{23}'}{a_{22}'} & \frac{c_2'}{a_{22}'} \end{bmatrix} X' - a_{23}'$$

$$\begin{bmatrix} 0 & -a_{23}' & -\frac{a_{23}' a_{23}'}{a_{22}'} & -\frac{a_{23}' c_2'}{a_{22}'} \end{bmatrix}$$

Pivot equation

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$x-1 \rightarrow \begin{bmatrix} 1 & 1.5 & -2.5 & 3.5 \end{bmatrix}$
 $x+1 \rightarrow \begin{bmatrix} -1 & -1.5 & 2.5 & 3.5 \end{bmatrix}$ Pivot

$$\begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 4 & 1 & -8.5 \end{bmatrix}$$

First sub matrix X

$$\begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 0 & 11.4 & -5.78 \end{bmatrix}$$

$\begin{pmatrix} 0 & -2.5 & 6.5 & 1.7 \\ -2.5 & -2.5 & -2.5 & -2.5 \end{pmatrix}$
 $-4 \times \begin{pmatrix} 0 & 1 & -2.6 & -0.68 \end{pmatrix}$
 $\begin{pmatrix} 0 & -4 & 10.4 & 2.72 \end{pmatrix}$
 Second sub matrix

$$0x_1 + 0x_2 + 11.4x_3 = -5.78$$

$$x_3 = \frac{-5.78}{11.4} = -0.507$$

$$0x_1 + 2.5x_2 + 6.5x_3 = 1.7$$

$$-2.5x_2 + 6.5(-0.507) = 1.7$$

$$x_2 = -1.998$$

$$2x_1 + 3x_2 - 5x_3 = 7$$

$$2x_1 + 3(-1.998) - 5(-0.507) = 7$$

coefficient

1 $\rightarrow$ 2 3 -5 7

1 -1 4 5.2

-1 2.5 3.5 -12

$$= (2 / 2(F_4)) * (-1)(F_4)$$

sub

$$= 2$$

$$= 2 + 1$$

$$= (2 / 2(F_4)) * 1$$

$$- \frac{2.5}{-2.5} (-4) F_4$$

= 2 3 -5 7
1 -1 4 5.2

= Pivot +

X1 =

X2 = 3

X3 = 3

پایه
pivot element

invers $\rightarrow$ kās
square matrixes.

inverse

inverse	select

ctrl + shift + enter

tricks etc

X1 = mmult(invers, const)

X2 =

X3 =

ctrl + shift + enter

ask

& recieve

① Gauss Elimination $\begin{bmatrix} \triangle \\ \triangle \\ \triangle \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$

② Gauss-Jordan.

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}$ No need for any Backward elimination

→ is a solution → is a complete solution

① Direct solution

② Find inverse matrix constant w.d. value

$$[A] \{x\} = \{c\}$$

$$\{x\} = \{c\} [A]^{-1}$$

Normalized $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} c_1'' \\ c_2'' \\ c_3'' \end{Bmatrix} \rightarrow 1 \times 1 \times 1 = c_1''$

→ Forward and Backward column at a time

$$\begin{bmatrix} 2 & 3 & -5 & 7 \\ 1 & -1 & 4 & -5.2 \\ -1 & 2.5 & 3.5 & -12 \end{bmatrix}$$

$[A] \rightarrow [I]$

$R_1 \rightarrow \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 4 & 1 & -8.5 \end{bmatrix}$

First submatrix

$R_2 \rightarrow \begin{bmatrix} 1 & 0 & 1.4 & 4.52 \\ 0 & -2.5 & 6.5 & 1.7 \\ 0 & 0 & 11.4 & -5.78 \end{bmatrix}$

↑ ↑

Second submatrix

$$R_2(-1.5) = \begin{bmatrix} 0 & -1.5 & 2.9 & 1.02 \end{bmatrix}$$

$$R_2(-4) = \begin{bmatrix} 0 & -4 & 10.4 & 2.76 \end{bmatrix}$$

$$\begin{array}{c}
 \left[\begin{array}{ccc|cc}
 1 & 0 & 1.4 & 4.52 & \\
 0 & 1 & -2.6 & -0.68 & \\
 0 & 0 & 11.4 & -5.78 &
 \end{array} \right] \begin{array}{l} + \\ + \\ + \end{array} \\
 \downarrow \\
 \left[\begin{array}{ccc|cc}
 1 & 0 & 0 & 5.2298 & \\
 0 & 1 & 0 & -1.9982 & \\
 0 & 0 & 1 & -0.507 &
 \end{array} \right] \begin{array}{l} \text{Solution} \\ \\ \text{Third submaxim} \end{array}
 \end{array}$$

[I]

R₃ → $\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 5.2298 & \\ 0 & 1 & 0 & -1.9982 & \\ 0 & 0 & 1 & -0.507 & \end{array} \right]$

$$R_3(-1.4) = \left[\begin{array}{ccc|cc} 0 & 0 & -1.4 & 0.7098 & \end{array} \right]$$

$$R_3(2.6) = \left[\begin{array}{ccc|cc} 0 & 0 & 2.6 & -1.3182 & \end{array} \right]$$

$$\left[\begin{array}{l}
 x_1 = 5.2298 \\
 x_2 = -1.9982 \\
 x_3 = -0.507
 \end{array} \right] \text{Solution.}$$

only not solution

To find Inverse in Gauss Jordan.

constant is $[A]$

→ How to find Inverse of $[A]$

$$\left[\begin{array}{ccc|ccc}
 a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\
 a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\
 a_{31} & a_{32} & a_{33} & 0 & 0 & 1
 \end{array} \right] \begin{array}{l} \text{Inverse } [I]_{3 \times 3} \\ \text{matrix} \\ [A]_{3 \times 3} \\ \text{is the} \\ \text{matrix} \\ \text{of } [A]^{-1} \end{array}$$

↓ [I]

$$\left[\begin{array}{ccc|ccc}
 1 & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & a_{13}^{-1} \\
 0 & 1 & 0 & a_{21}^{-1} & a_{22}^{-1} & a_{23}^{-1} \\
 0 & 0 & 1 & a_{31}^{-1} & a_{32}^{-1} & a_{33}^{-1}
 \end{array} \right]$$

$$[A]_{3 \times 3} [B]_{3 \times 3} \quad \text{فقط inner}$$

$$[A]_{3 \times 3} [B]_{3 \times 3} \neq [B]_{3 \times 3} [A]_{3 \times 3}$$

$$\text{check 1 } [A]_{3 \times 3} [A]^{-1}_{3 \times 3} = I_{3 \times 3}$$

$$[A]^{-1}_{3 \times 3} [A]_{3 \times 3} = I_{3 \times 3} \quad \left. \begin{array}{l} \text{فقط inner} \\ \text{matrix} \end{array} \right\} \text{inner}$$

$$\text{check 2 } [[A]^{-1}]^{-1} = [A]$$

$$\left\{ \begin{array}{c} 3 \times 1 \\ 3 \times 1 \end{array} \right\} [A]^{-1}_{3 \times 3} X$$

$$[A^{-1}]_{3 \times 3} \left\{ \begin{array}{c} 3 \times 1 \\ 3 \times 1 \end{array} \right\} = \left\{ \begin{array}{c} 3 \times 1 \\ 3 \times 1 \end{array} \right\} \quad \text{solution.}$$

$$[A|B] \quad \begin{array}{cccc} 2 & 3 & -5 & 7 \\ 1 & -1 & 4 & 5.2 \\ -1 & 2.5 & 3.5 & -12 \end{array}$$

$$= \square / 8 \square$$

$$\square \times \text{Row 1} = \text{Row 2}$$

$$R_2 \left[\begin{array}{c} \square \\ \square \end{array} \right] \quad \text{sub 1}$$

$$\square / 8 \square$$

$$- 8 \square \times \text{Row 2}$$

$$\text{Row 1}$$

$$\text{Row 2}$$

$$x_1 = 3.22981$$

$$x_2 = -1.99825$$

$$x_3 = -0.50702$$

To find inverse.

3x3 matrix
find inverse

= minv () ctrl + shift + enter → 3x3

drag. matrix

$$\begin{bmatrix} 2 & 3 & -5 & 1 & 0 & 0 \\ 1 & -1 & 4 & 0 & 1 & 0 \\ -1 & 2.5 & 3.5 & 0 & 0 & 1 \end{bmatrix}$$

drag

= mmult (inv, origin matrix)

format cell → numbers, 2 digit

Copy invers

dp

original

matrix

→

get

Inverse.

ask : believe & recieve

③ Thomas method

Tri - Bandel.

[Tridiagonal, banded] matrix

No elimination

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}$$

upper
lower
main

upper
lower
main

one element
at least

slope > 0.4, 4.3

(a_1, a_2, a_3) → matrix (4, 4)

(b_2, b_3, b_4)

Row wise Row wise

for the 1st Row $A_1 = \frac{a_1}{d_1}$

$$R_1 = \frac{r_1}{d_1}$$

Start from Row 2 and Upwards

$$\text{For Row } i \Rightarrow A_i = \frac{a_i}{d_i - b_i A_{i-1}}$$

$$R_i = \frac{r_i - R_{i-1} b_i}{d_i - A_{i-1} \cdot b_i}$$

The last Row no A_4

$$\text{Row 2} \Rightarrow A_2 = \frac{a_2}{d_2 - b_2 A_1}$$

$$R_2 = \frac{r_2 - b_2 R_1}{d_2 - b_2 A_1}$$

$$\text{For the last row 4} \Rightarrow R_4 = \frac{r_4 - b_4 R_3}{d_4 - b_4 A_3}$$

Step 2: to evaluate the unknowns

$$x_4 = R_4$$

For all other x 's

$$x_i = R_i - A_i x_{i+1}$$

$$x_3 = R_3 - A_3 x_4$$

$$x_2 = R_2 - A_2 x_3$$

$$x_1 = R_1 - A_1 x_2$$

$$\begin{bmatrix} -1200 & 100 & 0 & 0 \\ 1100 & -1500 & 100 & 0 \\ 0 & 1100 & -1240 & 100 \\ 0 & 0 & 1100 & -1250 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$A_1 = \frac{100}{-1200} = -0.08\bar{3} \quad R_1 = \frac{-1000}{-1200} = 0.8\bar{3}$$

$$A_2 = \frac{100}{-1500 - 1100(-0.08\bar{3})} = -0.071005917$$

$$R_2 = \frac{-1000 - 1100(0.8\bar{3})}{-1500 - 1100(-0.08\bar{3})} = 0.670887574$$

$$A_3 = \frac{100}{-1240 - 1100(-0.071005917)} = -0.086666409$$

$$R_3 = \frac{0 - 1100(0.670887574)}{1240 - 1100(-0.08\bar{3})} = 0.616215115$$

$$A_4 = 0 \quad R_4 = \frac{0 - 1100(0.616215115)}{-1250 - 1100(-0.086666)} = 0.586705457$$

$$x_4 = R_4 = 0.586705457$$

$$x_3 = 0.616215115 - (-0.086666409)(0.586705457) = 0.666710747$$

$$x_2 = 0.698227982$$

$$x_1 = 0.891518999$$

④ L.U factorization.

$$[A][x] = [C]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$[L]$

$[U]$

Step ①

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

By Gauss elimination

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & a_{32}' & a_{33}' \end{bmatrix}$$

$$[L][U] = [A]$$

$$[U][L] = [A]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{bmatrix}$$

elimination

Pivot element
is sign.

Row 1 to find solution directly

Step ② Solve the following system

$$[L]\{D\} = \{C\}$$

Indeterminate variables

$$[I]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

Residual variable

$$= \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix}$$

Forward substitution

$$D_1 = C_1$$

$$l_{21}D_1 + D_2 = C_2$$

$$l_{31}D_1 + l_{32}D_2 + D_3 = C_3$$

D_1, D_2, D_3

Step ③ Solve the following system By Backward substitution

$$[U]\{x\} = [D]$$

to obtain x_1, x_2

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}$$

$$x_3 = \frac{D_3}{a_{33}''}$$

one cell,
bottom
(thru row)

$$\begin{bmatrix} 2 & 3 & -5 \\ 1 & -1 & 4 \\ -1 & 2.5 & 3.5 \end{bmatrix} \times \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & -5 & 1 & 0 & 0 \\ 0 & -2.5 & 6.5 & 1/2 & 1 & 0 \\ 0 & 4 & 1 & -1/2 & 0 & 1 \end{array} \right] \quad [L]$$

↓

$$\begin{bmatrix} 2 & 3 & -5 \\ 0 & -2.5 & 6.5 \\ 0 & 0 & 11.4 \end{bmatrix}$$

و6

(U) →

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [U]$$

sup

$$mmul(U \times L) \neq A$$

$$mmul(L \times U) = A$$

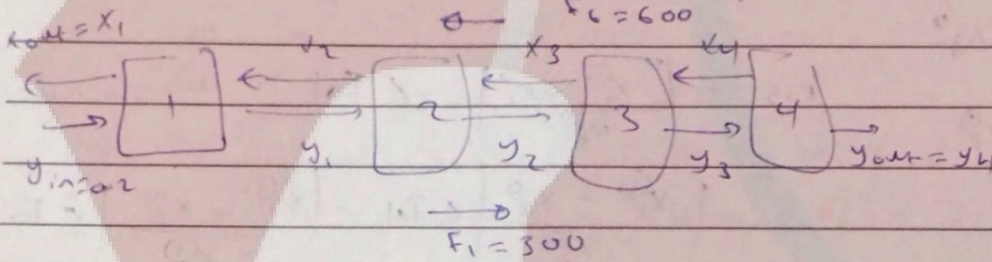
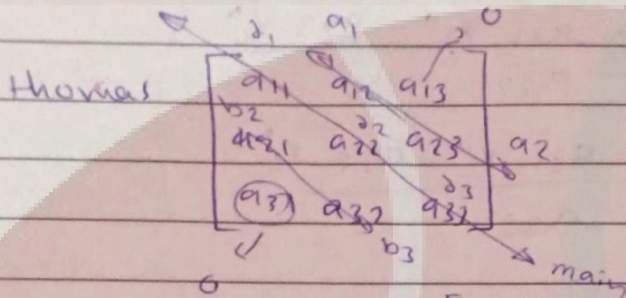
فقد

$$[L] \times \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 5.2 \\ -12 \end{Bmatrix}$$

$$mmul(mmvers(L) \rightarrow C)$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

matrix (min U & D)



$$600 y_2 + 0.2 \times 300 = 300 y_1 + 600 x_1 \quad (\text{Thomas})$$

$$2 = \frac{x_i}{y_i}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

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Iterative Methods.

Not analytical, expensive run time

"initial" guess Initial guess is required

1) Jacobi

2) Gauss Seidel

(n) initial guesses (x_1^0, x_2^0, x_3^0)

initial guess (n-1)

$$a_{11} x_1 + a_{21} x_2 + a_{31} x_3 = c_1$$

$$a_{12} x_1 + a_{22} x_2 + a_{32} x_3 = c_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = c_3$$

$$x_1 = \frac{c_1 - a_{12} x_2 - a_{13} x_3}{a_{11}}$$

$$x_2 = \frac{c_2 - a_{21} x_1 - a_{23} x_3}{a_{22}}$$

$$x_2 = \frac{c_2 - a_{21} x_1 - a_{23} x_3}{a_{22}}$$

$$x_3 = \frac{c_3 - a_{31} x_1 - a_{32} x_2}{a_{33}}$$