Central → Reduction of error 75%

$\Delta x = \Delta x$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + Oh^2$$

x_{i-2} x_{i-1} x_i x_{i+1} x_{i+2}

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h} + Oh^4$$

weakest → less # of data [illegible]

$$f(1)\Big|_{x=4} = \frac{f(5) - f(1)}{4} + Oh$$ less

1, 3, 5, 7, 9

$$f'(1)\Big|_{Dx=2} = \frac{f(3) - f(1)}{2} + Oh.$$ more

$$D\Big|_{better} = D_{more} + 1/(h_1/h_2)^2 - 1\left[D_{more} - D_{less}\right] + Oh^2$$

$$D_{better} = D_{more} + \frac{1}{3}\left[D_{mor} - D_{less}\right] + Oh^2$$

$h_2 = \frac{1}{2}h_1$

	$\frac{1}{3}$	$\frac{1}{15}$	$\frac{1}{63}$	
$O(h)$	$O(h)^2$	$O(h)^4$	Oh^6	Oh^8
h_1 o	o	o	o	o
$h_2 = \frac{1}{2}h_1$ o	o	o	o	
$h_3 = \frac{1}{4}h_1$ o	o			
$\frac{1}{8}h_1$ o	o	o		
$\frac{1}{16}h_1$ o				

$Dx = 2$

x_i	$f(1)$
1	
$1 + \frac{1}{16} \times 2$	
$1 + \frac{2}{16} \times 3$	
$1 + \frac{3}{16}(2)$	
$1 + \frac{4}{16}(2)$	

$Dx = 2$

3

oh → How much error.

$\boxed{*}$ if the error of $\boxed{O(h)}$ then halving the interval step size will reduce the error نصف by 50 %.

فترة

Interval

ناخد نص

$$f'(1) = \frac{f(5) - f(1)}{5 - 1} + O(h) \qquad \Delta x = 4$$

error 30% from true answer. $\Delta x = 4$

$$f'(1) = \frac{f(3) - f(1)}{3 - 1} + O(h) \qquad \Delta x = 2$$

1 ($\Delta x_2 = 2$)	f(1)
3	f(3)
5	f(5)
7	
9	

error = 15 %

$\boxed{*}$ if the error of $O(h)^2$, then halving the step size. the error will be reduce by 75 %.

$$f'(1) = \frac{-f(9) + 4f(5) - 2f(1)}{2 \times 4} + Oh^2 \qquad \Delta x = 4$$

error 20% $\Delta x = 4 = h$

$\Delta x = 4$	1 ($\Delta x = 2$)	f(1)
	3	f(3)
	5	f(5)
$\Delta x = 4$	7	f(7)
	9	f(9)

$$f'(1)\Big|_{\Delta x = 2} \frac{-f(5) + 4f(3) - 2f(1)}{2 \times 2} + Oh^2$$

error = 5 %

Backward

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$

الفرق بين خطوات

x_{i-2} x_{i-1} x_i

$f(x_{i-1})$ $f(x_i)$

x_{i-1} x_i

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_i - 2)}{2h} + Oh^2$$

$Oh \rightarrow$ How much error.

(*) if the error of $\boxed{O(h)}$ then halving the interval step size will reduce the error by 50 %.

هذا Interval نحوه للنص

$$f'(1) = \frac{f(5) - f(1)}{5 - 1} + O(h) \quad \Delta x = 4$$

error 30% from true answer. $\Delta x = 4$

1, 3, 5, 7, 9 — $f(1)$, $f(3)$, $f(5)$ — $\Delta x_2 = 2$

$$f'(1) = \frac{f(3) - f(1)}{3 - 1} + O(h) \quad \Delta x = 2$$

error = 15 %

(*) if the error of $O(h)^2$, then halving the step size. the error will be reduce by 75 %.

$$f'(1) = \frac{-f(9) + 4f(5) - 2f(1)}{2 \times 4} + Oh^2$$

error 20% $\Delta x = 4 = h$

1, 3, 5, 7, 9 — $f(1)$, $f(3)$, $f(5)$, $f(7)$, $f(9)$ — $\Delta x = 4$, $\Delta x = 2$

$$f'(1)\Big|_{\Delta x = 2} \frac{-f(5) + 4f(3) - 2f(1)}{2 \times 2} + Oh^2$$

error = 5 %

Backward.

$$f'(x_i) = \frac{f(x_i) - f(x_i - 1)}{h} + O(h)$$

الفرق بين نقطتين

x_i, x_{i-1} — $f(x_{i-1})$, $f(x_i)$

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h} + Oh^2$$

step Polymo 1×10^{-5} very small increment

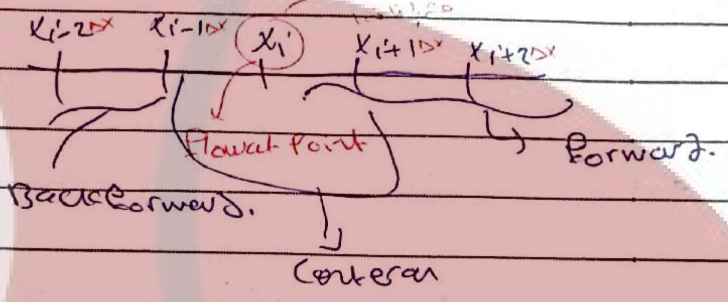
Defferentiation.

common step size data.

x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$.
x_3	$f(x_3)$.

Back, Forw, no central

x_i no Point → No central.



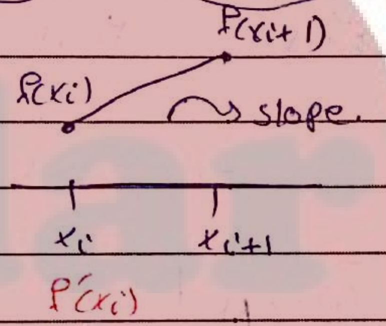
$$f(x_{i+1}) = f(x_i) + f'(x_i)\,h + \frac{f''(x_i)\,h^2}{2} + \frac{f'''(x_i)\,h^3}{6} + \cdots$$

$$\boxed{f'(x_i)\,h} = f(x_{i+1}) - f(x_i) - \frac{f(x_i)\,h^2}{2} + \frac{f'''(x_i)\,h^3}{6} - \cdots$$

$$f'(x_i) = \underbrace{\frac{f(x_{i+1}) - f(x_i)}{h}}_{=\text{slope.}} - \underbrace{\frac{f(x_i)\,h}{2} - \frac{f'''(x_i)\,h^2}{6}}_{\text{correct (error)}}$$

h_i

$DX = x_{i+1} - x_i$



$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_i) - 3f(x_i)}{2h} + O(h^2)$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O\,h$$

$1^{st} \to 2$ Data

$2^{n} \to 3$ Data

$$f''(x_i) = \frac{f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2} + O\,h^2$$

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step polyno 1×10^{-5}
very small
increment

Defferentiation.

common step size data.

→ no central

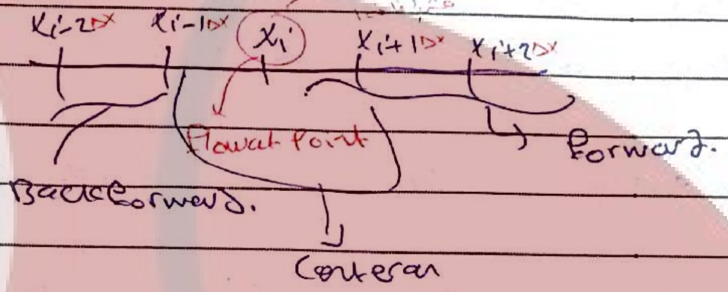
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$.
x_3	$f(x_3)$.

Back

Forw

x_i: no Point

→ No central.



$$f(x_{i+1}) = f(x_i) + f'(x_i)\,h + \frac{f''(x_i)\,h^2}{2} + \frac{f'''(x_i)\,h^3}{6} + \cdots$$

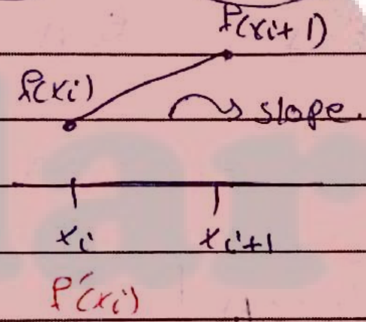
$$\boxed{f'(x_i)}\,h = f(x_{i+1}) - f(x_i) - \frac{f(x_i)\,h^2}{2} + \frac{f'''(x_i)\,h^3}{6} - \cdots$$

$$f'(x_i) = \underbrace{\frac{f(x_{i+1}) - f(x_i)}{h}}_{=\text{slope}} - \frac{f(x_i)\,h}{2} - \frac{f'''(x_i)\,h^2}{6}$$

correct (error)

h

$\Delta x = x_{i+1} - x_i$



$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_i) - 3f(x_i)}{2h} + O(h^2)$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O\,h$$

1^{st} → 2 Data

2^{n} → 3 Data

$$f''(x_i) = \frac{f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2} + O\,h^2$$

y	Z (0.8, y)
0	A_1
0.5	A_2 ??
1	A_3

0.7

		f(x)	1st D.D	2nd D.D
x_0	?	y_0	$\frac{y_2 - y_1}{x_1 - x_0}$	
x_1	?	y_1		
x_2	?	y_2		

Degree less by one

0.8 Z (0.8 (0.5) 9.0064

* Differentiation.

Equation → data

Function
لو فيه معادلة بنعمل .
مش لازم نعمل

data → Differentiation / Integration.

Differentiation

common size

Equally spaced → Forward, Backward, Central → Richardson Extrapolation,

Unequall spaced → fitting .

1
3
7
10.5
17

not Common size.

Integration.

$$\int_{x_0}^{x_n} f(x)\,dx \cong \int_{x_0}^{x_n} f_n(x)\,dx$$

Romberg Integration.

1
3
5
6
7
9

6 مينفعش استخدم D

D.D.T

بنقرأ a_0 ، a_1 (Position) ، مسا ﴿ x_0

	$Z(x, 0.5)$	1st D.D	2nd D.D
$x_0 = 0.5$	0.75 a_0	$\frac{10-7.51}{1.0-0.5}$ a_1	
$x_1 = 1.0$	10		

$f_1(x) = 0.75 + \frac{(10-7.51)}{1-0.5}(x - x_0)$

0.8 0.8 0.5

	a_0	1st a_1	2nd D.D a_2
$x_0 = 0.5$	7.51	$\frac{10-7.51}{1-0.5} = 4.98$ a_1	$\frac{5.02-4.98}{1.5-0.5} = 0.04$ a_2
$x_1 = 1.0$	10		
$x_2 = 1.5$	12.51	$\frac{12.51-10}{1.5-1} = 5.02$	

$f_2(x) = 7.51 + 4.98(x - x_0) + 0.04\,(x-0.5)(x-1)$

0.5

3D ⟸ x,y,z ⟸ degree نفس ⟸ # of Data بين $\boxed{y, x}$

2^{nd} degree in x و 2^{nd} degree in y

$Z(0.8, 0.7)$

		$f(x)$		
$x_0 = 0.5$	= 5	$y_0 = 0$		10.05
$x_1 = 1$	= 10	$y_1 = 0.5$		10
$x_2 = 1.5$	= 15	$y_2 = 1.0$		9.95

x	$Z(x, 0)$
0.5	5 A_1
1	10
1.5	15

0.8

$f_2(0.8) = \left(\frac{0.8-1}{0.5-1}\right)\left(\frac{0.8-1.5}{0.5-1.5}\right)5 + \left(\frac{0.8-0.5}{1-0.5}\right)\left(\frac{0.8-1.5}{1-1.5}\right)10 + \left(\frac{0.8-0.5}{1.5-0.5}\right)\left(\frac{0.8-1}{1.5}\right)15$

x	$Z(x, 0.5)$
0.5	7.51 A_2
1	10
1.5	12.51

0.8

x	$Z(x, 1)$
0.5	10.05 A_3
1	10
1.5	9.95

0.8

Newton's

$P_1(x) = a_0 + a_1(x - x_0)$

$$f(0.8) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$= 7.51 + \frac{(10 - 7.51)}{1 - 0.5}(0.8 - 0.5)$$

$$= 9.004$$

$x_0 = 0.5$	7.51
$x_1 = 1$	10
$x_2 = 1.5$	12.51

$$f(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) f(x_1) + \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) f(x_2)$$

0.8, 0.5, 0.8, 1, 7.51, 0.8, 0.5, 1, 1.5, 0.8, 10, 0.8, 1.5, 0.8, 1, 1.5, 12.51

تعوض كل ما ندلك

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \frac{\left\{\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right\} - \left\{\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right\}}{x_2 - x_0}\{x - x_0\}(x - x_1)$$

نحسب كل a لحال

0.8, 0.8

كل ما زاد order

accurate ~ يزيد

③ Divided difference table.

x_0	$f(x_0)$
x_1	$f(x_1)$

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

	a_0	1st D.D
x_0	$f(x_0)$	a_1
x_1	$f(x_1)$	$\frac{f(x_1) - f(x_0)}{x_1 - x_0}$

Ne ⇒ a لا تقرب first column stop

without skip — 1 skip — 2 skip

	a_0	1st D.D (n-1)	2nd D.D
1 → x_0	$f(x_0)$	a_1: $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$	a_2: $\frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$
3 x_1	$f(x_1)$	$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$	$\frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1}$
4 1.5 x_2	$f(x_2)$	$\frac{f(x_3) - f(x_2)}{x_3 - x_2}$	
6 x_3	$f(x_3)$		

2 2.25 2.25

3skip → 3rd D.D : بنأخذ extrap اكبر

x \ y	0	0.5	1.0	1.5	2	2.5
0		5				
0.5 x_0		7.51				
1.0 x_1		10				
1.5 y_1	15	12.51	9.45	7.32	4.33	0
2		15				

0.8 0.7

x_0, x_1, x_2 فوق تحت لتحت

	z(x, 0.5)
$x_0 = 0.5$	7.51
$x_1 = 1.0$	10

lagrage → newton → D.D.T

z(0.8, 0.5) → 1D اذا فش موجودين 2D اذا الها 1st 2nd

z(1.5, 0.7) → 1D → first, second lagrag, New, D.D

1.5

	y	z(1.5, y)
	0	15
y_0	0.5	12.5
0.7 → y_1	1	9.45
y_2	1.5	7.32
	2	4.33
	2.5	0

lagranege

$$f_1(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) f(x_0) + \frac{(x - x_0)}{x_1 - x_0} f(x_1)$$

0.8, 1, 7.51, 0.5, 0.8, 0.5, 1, 0.5, 10

7.5 < > 10 | 9.004 = ~~3.0~~ 64

conduc...

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