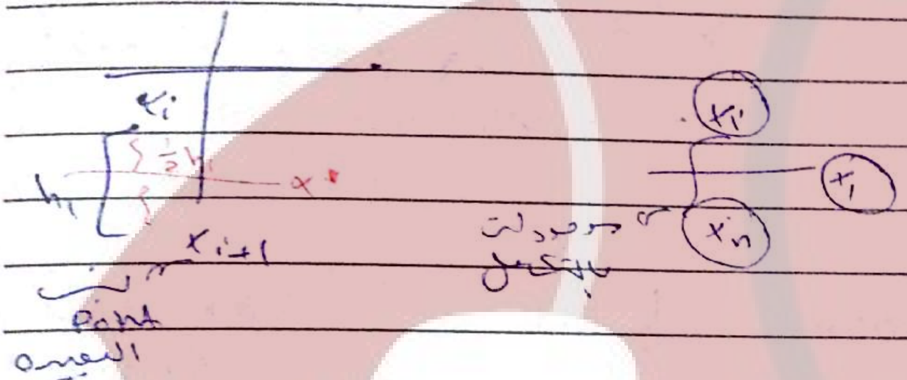
$$I\Big|_{best\ h_1,h_2} = I\Big|_{h_2} + \frac{1}{3}\left(I\Big|_{h_2} - I_{h_1}\right)$$

T.R only



Integration:-

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} (5 + 3\sin(x))\,dx$$

radian.

we well general Table.

$$5x - 3\cos x \Big]_0^{\pi} = 21.70796$$

analytically.

② single application of T.R $\int_{x_0}^{x_n} = \left\{\frac{f(x_n) + f(x_0)}{2}\right\} h.$

x	$f(x) = 5+3\sin(x)$
0	$5+3\sin 0 = 5$
π	$5+3\sin\pi = 5$

$$\Rightarrow \int_0^{\pi} f(x)\,dx = \left\{\frac{5+5}{2}\right\}\pi = 15.70796$$

③ Single Application of $\frac{1}{3}$ S.R $\int_{x_0}^{x_2} = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$

x_i	$f(x_i)$
0	5
$\pi/2$	8
π	5

$h = \frac{\pi}{2}$

$$= \frac{\pi/2}{3}\left[5 + 4\times 8 + 5\right]$$

$$= 21.99148$$

④ single application of $\frac{3}{8}$ S.R $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$ $\int_0^{\pi}$ $= \frac{3}{8} h \left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right]$

??

$h = \frac{\pi}{3}$

x_i	$f(x_i)$
0	5
$\frac{\pi}{3}$	7.598075
$\frac{2\pi}{3}$	7.598075
π	5

$= \frac{3}{8} \times \frac{\pi}{3} \left[5 + 3 \times 7.598075 + 3 \; 7.598075 + 5 \right]$

$= 21.8295$

Double. 5/63

⑤ ~~Multiple~~ application of $\frac{1}{3}$ S.R

Triple → 7 pw

6 sigment

$h = \frac{\pi}{4}$

x_i	$f(x_i)$	
0	5	①
$\frac{\pi}{4}$	7.1213	④
$\frac{\pi}{2}$	8	②
$\frac{3\pi}{4}$	7.1213	④
π	5	①

$\int_{D.S.R} = \frac{1}{3}\left(\frac{\pi}{4}\right) \left[5 + 4(7.1213 + 7.1213) + 2 \times 8 + 5 \right]$

⑥ Boal's Fourmula. 5 pw single application.

total Interval

$$\int_0^{\pi} = \left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4) \right] \frac{(x_n - x_0)}{90}$$

$$= \frac{\pi}{90} \left[7 \times 5 + 32(7.1213) + 12(8) + 32(7.1213) + 7(5) \right]$$

$$= 21.7036300 2$$

minim # form maximize # of accuracy of # of data point.

$\bar{e}^{x}$

x	$f(x)$
0	1
0.1	0.9048
0.3	0.7408
0.5	0.6065
0.7	0.4966
0.95	0.3867
1.2	0.3012

0.1 ; h = 0.2 ; 0.25

$\int_0^{0.1} T.R$

$\int_{0.1}^{0.7} \frac{3}{8} S.R$ ($\int_{0.3}^{0.5}$)

$\int_{0.7}^{1.2} \frac{1}{3} S.R$ (0.95)

$\int_0^{1.2} = \int_0^{0.1} + \int_{0.1}^{0.7} + \int_{0.7}^{1.2}$

$(T.R, \frac{3}{8}, \frac{1}{3})$

best answer.

(2)

~~0.09524~~ 0.09524 ~~0.4096~~ 0.4082475

$\frac{0.1}{2}[1+0.9048] + \frac{3}{8}(0.2)[0.9048+3(0.7408)+3(0.6065)+0.4966]$

$+\frac{1}{3}(0.25)[0.4966+4(0.3867)+0.3012]$

~~0.1964~~ 0.19533333

0.69887683

$= \boxed{\cancel{0.70124}}$ best answer. (Neumirical answer) (true answer)

(1) $\int_0^{1.2} e^{-x} = -\bar{e}^{x}\Big]_0^{1.2} = 0.6988$ (True answer)

(3) Trapzoial only $= \int_0^{0.1} + \int_{0.1}^{0.7} (0.3, 0.5) + \int_{0.7}^{1.2} 0.95$

$= .09524 + 0.4096 + 0.1964 = 0.70124$

Scanned by CamScanner

$O h^2$ min error

T.R تكرارية

integration عكس diff بس بدل Form نفس

$$\int_0^2 \left(x+\frac{1}{x}\right)^2 dx \quad (O h^8) \quad \frac{1}{3} \quad \frac{1}{15} \quad \frac{1}{63}$$

$O h^2$	$O h^4$	$O h^6$	$O h^8$
5.125			
4.969725			
4.852731275			

$h = 0.25$

x	$f(x)$
1	4
1.125	4.0557
1.25	4.2025
1.375	4.41955
1.5	4.9644
1.625	5.0193
1.75	5.38903
1.875	5.80066
2	6.25

$$I\Big|_{h_1=1} = \frac{1}{2}[4 + 6.25] = 5.125$$

$$I\Big|_{h_2=\frac{1}{2}h_1} = \frac{0.5}{2}\left[4 + 2(4.9644) + 6.25\right] = 4.969722$$

$$I\Big|_{h_3=\frac{1}{4}h_1} = \frac{0.25}{2}\left[4 + 2(4.20205 + 4.9694 + 5.38903) + 6.25\right]$$

$$= 4.852731275$$

$I_{h_4 = \frac{1}{8} h_1} = \frac{0.125}{2} [4 + 6.25 + 2 \ldots]$

$o(h^4)$ $\frac{1}{3}$

$4.9097222 + \frac{1}{3}[4.9097222 - 5.125] = 4.837962963$

$4.85273275 + \frac{1}{3}[4.85273275 - 4.9097222] = 4.83373626$

$o(h)^6$ $\frac{1}{15}$

$4.8337362 + \frac{1}{5}[4.83372626 - 4.837962963] = 4.83345448$

Gaussian Quadrature [Legendre]

$$\int_{x_0}^{x_n} f(x)\,dx \simeq C_0 f'(x_0) + c_1 f'(x_1) + c_2 f'(x_2) + \ldots$$

where $f(x)$ is not $f'(x)$

2 data point of Legendre

$$\int_{x_0}^{x_n} f(x)\,dx \simeq \overset{(1)}{C_0} \underset{f'(-0.577)}{f'(x_0)} + \overset{(1)}{C_1} \underset{f'(+0.577)}{f'(x_1)}$$

3 data point of legendre $\int_0^{x_n} f(x)\,dx = C_0 f(x_0) + C_1 f'(x_1) + C_2 f'(x_2)$

C's and X's are obtained from special tables.

points		
2	$C_0 = 1$	$x_0 = -0.57735$
	$C_1 = 1$	$x_1 = +0.577355$
3	C_0	x_0
	C_1	x_1
	C_2	x_2

$$f'(x) = f\left\{\frac{(b-a)t + b+a}{2}\right\}\left(\frac{b-a}{2}\right)$$

$a = x_0 \qquad b = x_n$

use n data point of legendre to evalu $\int_0 x e^{2x}$

$$\cong \int_0^3 f'(x)\,\partial x = (1) f'(0.577) + f'(-0.577)$$

$$f'(x) = f\left\{\frac{(3-0)t + 3+0}{2}\right\}\left\{\frac{3-0}{2}\right\}$$

$$f'(x) = f\left(\frac{3t+3}{2}\right)(1.5)\frac{3}{2}$$

$$f'(x) = f(1.5t+1.5) \times 1.5$$

x هعوضها محل

$$f'(x) = \left[(1.5t+1.5)\, e^{2(1.5t+1.5)}\right] \times 1.5$$

Differential Equation.

ordinery — Partial

Intial value — Boundry Value.

*start frome — $y(0)$, $y(2)$ — different point. — نفرض y' ونفرض — Integration: $y(2) = y(2)$ ✓ — $y(1) \neq y(2)$ ✗

second order (y, y') second and up. (y, y', y'') third.

shooting — Finite elment. — Trail and error Tecnick — error + error

* $y(0)$, $y'(0)$, $y''(0)$ — point نفس عند

$y(2)$, $y'(2)$, $y''(2)$

$i+1$ $i+2$ $i+3$ — $y(1)$ $y(2)$

First order, Intail Value

$$\frac{\partial y}{\partial x} = \frac{f(x_{i+1}) - f(x_i)}{h}$$

O.D.E (Intial Value)

First O.D.E — Higher O.D.E

Euller — Heun's — Polygon — Runge-Kutta (very efficient. step size كبير) — 2nd — 3rd — 4rt

single ✓

2 first O.D.E — 3 first O.D.E

معادلتين — syminTasiouly. (Parrlel)

First O.D.E عادية ترتيب condition

$y' = f(x, y)$ مشتقة صريحة function صريحة

Eulers method.

$y_{i+1} = y_i + f'(x_i, y_i)\, h$ $\quad y(0) - y(4)$ $\quad i \to x_i$

$y' = f(x,y)$

$y'(x_i) = f(x_i, y(x_i))$

$y(x_i) \to x_i \quad x_{i+1} \quad x_{i+2}$

$y_i \quad y(x_{i+1})$

$y_{next} = y(x_i) + \underbrace{f'(x_i, y_i)}_{\text{correction}}\, h \leftarrow$ step size.

بنبني ← $y(1)$
العتيم
كانو ببني
accumlation.

where $f'(x_i, y(x_i)) = y'(x_i, y(x_i))$

$y' = 4e^{0.8x} - 0.5y$, $y(0) = 2$ $\quad y' = f(x,y)$

integrate from $x=0$ to $x=4$, using step size of 1

$$\int_0^4 = \int_0^1 + \int_1^2 + \int_2^3 + \int_3^4$$

for $x = 1$ $\quad y(1) = y(0) + y'(0, y(0))\, h = 1$ $\quad y'(0,2)$

$y(1) = y(0) + y'(0,2)\, h$

$y(1) = 2 + (4e^0 - 0.5 \times 2) \times 1$

$\boxed{y(1) = 5}$

$y' + \underbrace{0.5}_{P(x)} y = \underbrace{4e^{0.8x}}_{r(x)}$

$$y = e^{-\int P(x)\, dx} \left[\int e^{\int P(x)\, dx}\, 4e^{0.8x}\, dx + C \right]$$

$$e^{-0.5x} \left[\int 4e^{0.5x} e^{0.8x}\, dx + C \right]$$

$$e^{-0.5x} \left[\frac{4}{1.3} e^{1.3x} + C \right]$$

$$y_g = \frac{4}{1.3} e^{0.8x} + C e^{-0.5x}$$

$y(0) = 2$

$2 = \frac{4}{1.3} + C \qquad C = 2 - \frac{4}{1.3}$

$y(1) = 6.1946314$ anyltical

$y(2) = 14.8439219$

$y(3) = 33.6771718$

$y(4) = 75.3389626$

$y(1) = 5$ initial gess

for $x=2$ $\quad y(2) = y(1) + f'(1,5)\,(1)$

$y'((1,5)) = 4e^{0.8(1)} - 0.5\times 5 = 6.4021637\,14$

$\boxed{y(2)} = 5 + 6.4021637\,14\,(1) = \boxed{11.4021637\,14}$

for $x=3$ $\quad f(3) = f(2) + f'(2, 11.4021637\,14)\,(1)$

$y'(2, 11.4\text{---}) = 4e^{0.8\times 2} - 0.5\times 11.4\text{---} = 14.1110478\,4$

$y(3) = 11.4021637\,14 + 14.1110478\,(1)$

$= 25.5132115\,5$

$y(3) = 25.5132115\,5$

for $x=4$

$y(4) = y(3) + y'(3, 25.513)\,(1)$

$y(4) = 56.849313$

2) Heun's method.

correction

avarge value.

predect valu

$$y(x_{i+1}) = y(x_i) + \left\{\frac{f'(x_i, y(x_i)) + f'(x_{i+1}, y(x_{i+1}))}{2}\right\} h$$

$y^P(x_{i+1}) = y(x_i) + f'(x_i, y(x_i))\, h.$ from Euler.

$y' = 4e^{0.8x} - 0.5y$, $y(0) = 2$ stepsize $= 1$

(نبدأ من هنا)

$$y(1) = y(0) + \left\{\frac{f'(0,2) + f'(1, y^P(1))}{2}\right\}(1)$$

$f'(0,2) = 4e^{0.8(0)} - 0.5\times 2 = 3$

$y^p_{(1)} = y(0) + f'(0,2)\, h^{(1)}$

$2 + 3 \times 1 = 5$

$f'(1,5) = \cancel{4f(1)} \cancel{+ f'c}$

$= 4e^{0.8(1)} - 0.5 \times 5 = 6.4021637$

$y(1) = 2 + \left\{ \frac{3 + 6.40216371}{2} \right\}(1)$

$y(1) = 6.70108186$ الأقرب analytical.

$y(2) = y(1) + \left\{ \frac{f'(1, 6.70108186) + f'(2, y^p(2))}{2} \right\}(1)$

$f'(1, 6.7) = 4e^{0.8(1)} - 0.5 \times 6.7 = 5.5516227.9$

$y^p(2) = y(1) + f'(1, 6.7)\, h$

$= \cancel{6.70}\ 12.25270464$

$f'(2, 12.2527) = 4e^{0.8(2)} = 13.6857774$

$y(2) = 6.7 + \left\{ \frac{5.55 + 13.686}{2} \right\}(1)$

$= 16.32$

حلولي

$\boxed{\begin{array}{l} y(3) = 37.1992489 \\ y(4) = 83.3377673 \end{array}}$

step size $\underline{0.5}$

Polygon.

mid (stepsize (النص)

$y(x_{i+1}) = y(x_i) + f'(x_{i+\frac{1}{2}}, y^P(x_{i+\frac{1}{2}}))(h)$

$y^P(x_{i+\frac{1}{2}}) = y(x_i) + f'(x_i, y(x_i)) \times \frac{h}{2}$ نضرب بنص h

accuracy

$y' = 4e^{0.8x} - 0.5y$, $y(0) = 2$ step = 1

for x = 1

$y(1) = y(0) + f'(0+\frac{1}{2}, y^P(\frac{1}{2}))(1)$

$y^P(\frac{1}{2}) = y(0) + f'(0, y(0)) \frac{1}{2}$

$= 2 + (3) \times (0.5)$

$= 3.5$

$y(1) = y(0) + f'(0.5, 3.5)(1)$

$f'(0.5, 3.5) = 4e^{0.8 \times 0.5} - 0.5 \times 3.5 = 4.21729879$

$y(1) = 2 + 4.21729879$

$= 6.21729879$