- Numerical method
- Summer 2020
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- Measuring Error
  - Why!
    1. to determaine the accurcy of numerical result
    2. to develop stopping Criteria for iterative algorithms

A: True Error

True Error = True value - Approximate value

B: Relative True Error

$$\text{Relative Error} = \frac{\text{True Error}}{\text{True value}}$$

C: Approximate Error

Approximate Error = present approximation - previous approximation

D: Relative Approximate Error

$$\text{Relative Approximate Error} = \frac{\text{Approximate Error}}{\text{present approximation}}$$

Part 1

⊛ Numerical methods

- algorithms that are used to obtain numerical Solution of mathmatical problem.

- why!
  - no analytical solution exists
  - analytical solution is difficult to obtain.

⊛ methods for solving nonlinear equations:-

- Bisection method
- Newton - Raphson method
- secant method

1. Bisection method:-

Theorem:-

An equation $f(x)=0$, where $f(x)$ is a real, contious function has at least one root between x_l and x_u if $f(x_l) * f(x_u) < 0$

Notes:

① at least one root exist between the ~~root~~ two points if the function is real, contious and chang sign

② if $f(x)$ dosen't change sign between two points, roots of the equation may still exist between the two point

③ if the function doesn't chang sign between points there maynot be any roots for the equation

④ if the sign between two point change, more than one root for equation may exist.

How to solve by Bisection method :-

1. specific x_l and x_u
2. evaluate $f(x_l)$ and $f(x_u)$
3. check sign $f(x_l) \neq f(x_u)$
4. evaluate $x_3 \rightarrow x_3 = \frac{x_l + x_u}{2}$
5. if $f(x_3) * f(x_l) > 0 \quad x_l = x_3$

   $f(x_3) * f(x_u) < 0 \quad x_u = x_3$
6. Check the error $\leq$ tol stop else repeat from $\boxed{4}$

Coding by matlab :-

```
y = inline ('f(x)');
x1 = a ;
x2 = b ;
x3 = (x1 + x2)/2 ;
tol = c ;
  While abs ((x2 - x1)/x3) > tol

     if  y(x1) * y(x2) > 0
             x1 = x3
             x2 = x2
       else
             x2 = x3
   end
4:   x3 = (x1 + x2)/2
  end
```

3. Secant method

Steps of secant method

1. specific x_1 and x_2
2. evaluat $f(x_1)$ and $f(x_2)$
3. evaluat $x_3 \rightarrow$ $x_3 = x_1 - \frac{f(x_1) * (x_1 - x_2)}{f(x_1) - f(x_2)}$
4. evaluat $f(x_3)$
5. evaluat absolut error
6. abs error < tol $\quad$ x_3 is the root

   else $x_1 = x_2$

   $x_2 = x_3$

Coding by matlab:-

```
y= inline(' f(x)    ');
x1 = a;
x2= b;
x3= x1 - (y(x1) * ( x2-x1)) / (y(x2) -y(x1))

while abs ( (x2-x1)/x3 ) >tol
      x1 = x2
      x2 = x3
   x3 = x1 - (y(x1) * (x2-x1)) / (y(x2) -y(x1))

end.
```


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3. Newton- Raphson method

- steps:-
  - Specific x_1
  - evaluat $f(x_1)$
  - evaluat $f'(x_1)$
  - $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$
  - evaluat abs error
  - abs error > tol x_2 is the root

    else $x_1 = x_2$ and repeat.

- Coding by matlab:-

```
y= inline(' f(x) ');
x1 = a;
x2= x1 - (y(x1)/D(x1));

while abs((x2-x1)/x2) >tol
      x2 = x1 -(y(x1)/D(x1))

      x1 = x2

end
```

* in new script:-

```
syms y(x)
y(x) = f(x);
D = diff(y(x))
```

4. False-position method

- Steps:-

- Specific x_1 , x_2
- evaluat $f(x_1)$, $f(x_2)$
- evaluat x_3 = $x_3 = x_1 - \frac{f(x_1)(x_1 - x_2)}{f(x_1) - f(x_2)}$
- evaluat $f(x_3)$
- if $f(x_3) * f(x_1) > 0$ $x_1 = x_3$

  else $f(x_3) * f(x_2) < 0$ $x_2 = x_3$

- Coding by matlab:-

```
y= inline(' f(x) ');
x1= a;
x2= b;
x3= x1 - (y(x1)(x1-x2))
         / y(x2)-y(x2)

while abs (x2-x1)/x2 >tol
 if y(x1)* y(x2) >0
      x1 = x3 ;
  else
      x2=x3 ;
 end.
```

5. fixed point iteration

- steps:-
- $f(X) = 0$
- $g(X) = X_1$
- evaluat $g(X_1)$
- $X_2 - g(X_1)$
- evaluat abs error
- if abs < tol X_2 = root   else $X_1 = X_2$
- Coding by matlab:-

```
y= inline('f(x)') ;
x1 = a ;
x2 = g(x1) ;
    while abs ((x2-x1)/x2) >tol
        x1 = x2
        x2 = g(x1)
    end.
```


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- Find root by matlab build-in function.

① Graph :-

x = [: :] interval

y = f(x)

plot(x,y)

② Root

y = [a b c] <=[معاملات]

r = root(y)

③ f zero

F = inline(' f(x) ')

y = fZero(y, f) f: intial giss from graph.

④ Syms

syms (y,x)

y = f(x)

f = f solve(y)

part 2.

methods to solve system of linear eq

1. Gaussian Elimenation
2. Gaussian seidel
3. LU method

- steps of Gaussian eleimnation:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- بنفرغ المعادلات بمصفوفة A
- بنفرغ نتائج المعادلات بمصفوفة B
- بنفرغ المتغيرات بمصفوفة X

$$B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$[A][X] = [B]$$

* Steps

① make upper matrix.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0^{(1)} & a_{22} & a_{23} & b_2 \\ 0^{(2)} & 0^{(3)} & a_{33} & b_3 \end{array}\right]$$

* $R_2 \rightarrow$ ① $-\left(\frac{a_{12}}{a_{11}}\right) * a_{11} + a_{12} = 0$ التعديل على السطر الثاني

وتصفير a_{12}

$\rightarrow -\left(\frac{a_{12}}{a_{11}}\right) * R_1 + R_2$

* $R_3 \rightarrow$ ② $-\left(\frac{a_{13}}{a_{11}}\right) * a_{11} + a_{13} = 0$ تصفير a_{13}

التعديل على السطر الثالث

$\rightarrow -\left(\frac{a_{13}}{a_{11}}\right) * R_1 + R_3$

* $R_3 \rightarrow$ ③ $-\left(\frac{a_{23}}{a_{22}}\right) * a_{22} + a_{23} = 0$ تصفير a_{23}

$\rightarrow -\left(\frac{a_{23}}{a_{22}}\right) * R_2 + R_3$

②

$$x_3 a_{33} = b_3$$
$$x_2 a_{22} + x_3 a_{23} = b_2$$
$$x_1 a_{11} + x_2 a_{12} + x_3 a_{13} = b_3$$

Gaussian seidl method

* steps:-

① $X_0 = [x_1, x_2, x_3]$ intial

$$x_1 = \frac{b_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

$$x_2 = \frac{b_2 + a_{21}x_1 - a_{23}x_3}{a_{22}}$$

$$x_3 = \frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

$itr^0 = [x_1^0, x_2^0, x_3^0]$ طريقة التعويض :

$$x_1^1 = \begin{cases} x_2^0 \\ x_3^0 \end{cases}$$

$$x_2^1 = \begin{cases} x_1^1 \\ x_3^0 \end{cases}$$

$$x_3^1 = \begin{cases} x_1^1 \\ x_2^1 \end{cases}$$

$itr\ 1 = [x_1^1, x_2^1, x_3^1]$

$$x_1^2 = \begin{cases} x_2^1 \\ x_3^1 \end{cases}$$

$$x_2^2 = \begin{cases} x_1^2 \\ x_3^1 \end{cases}$$

$$x_3^2 = \begin{cases} x_1^2 \\ x_2^2 \end{cases}$$

LU method

Steps:-

① $AX = b$

② $Ld = b$

③ $UX = d$

→ from $[A]$ → $[U]$ upper matrix page(11)

→ $[L]$ lower matrix.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{a_{12}}{a_{11}} & 1 & 0 \\ \frac{a_{13}}{a_{11}} & \frac{a_{23}}{a_{22}} & 1 \end{bmatrix}$$

→ $Ld = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{a_{12}}{a_{11}} & 1 & 0 \\ \frac{a_{13}}{a_{11}} & \frac{a_{23}}{a_{22}} & 1 \end{bmatrix} \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{?} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$d_1 = b_1$$

$$\frac{a_{12}}{a_{11}} d_1 + d_2 = b_2$$

$$\frac{a_{13}}{a_{11}} d_1 + \frac{a_{23}}{a_{22}} d_2 + d_3 = b_3$$

→ $UX = d$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{?} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$a''_{33} x_3 = d_3$$

$$a'_{22} x_2 + a'_{23} x_3 = d_2$$

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = d_1$$

* Solve matrix by matlab build-in function.

1- a = [ ]

b = [; ;]

x = a\b

2- a = [ ]

b = [; ;]

c = inv(a)

x = c.*b

3- syms x1 x2 ...

a = f1

b = f2

y = solve (f1 , f2 ..)

y. x1

y. x2

part 3

- Least squares Regression

1 curve fitting :-

* 1 Linearization

x	y	xy	x^2	y_{calc}	$(y_i-\bar{y})^2$	$(y_{calc}-\bar{y}_i)^2$

$$y = ax + b$$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b = \bar{y} - a\bar{x}$$

2 $$r^2 = \frac{\sum(y_i-\bar{y})^2 - \sum(y_{calc}-y_i)^2}{\sum(y_i-\bar{y})^2}$$

- Curve fitting by matlab:-

x = [ ] قيم

y = [ ]

after linearization
x` =
y` =

I = polyfit (x`, y`, n) → درجة الاقتران

a = I(1)

b = I(2)

* intepolation

1. Lagrange method

$$f(x) = L(x)\, f(x_n)$$

① Linear

$$f(x) = L_0(x) f(x_0) + L_1(x) f(x_1)$$

$$L_0 = \frac{x - x_1}{x_0 - x_1}$$

$$L_1 = \frac{x - x_0}{x_1 - x_0}$$

② quadratic

$$f(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$$

$$L_0 = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$L_1 = \left(\frac{x - x_0}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$L_2 = \left(\frac{x - x_0}{x_2 - x_1}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

③ Cubic

$$f(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2) + L_3(x) f(x_3)$$

$$L_0 = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)\left(\frac{x - x_3}{x_0 - x_3}\right)$$

$$L_1 = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)\left(\frac{x - x_3}{x_1 - x_3}\right)$$

$$L_2 = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)\left(\frac{x - x_3}{x_2 - x_3}\right)$$

$$L_3 = \left(\frac{x - x_0}{x_3 - x_0}\right)\left(\frac{x - x_1}{x_3 - x_1}\right)\left(\frac{x - x_2}{x_3 - x_2}\right)$$

- Newton method:
  - linear

$$f(x) = b_0 + b_1 (x - x_0)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

- quadratic

$$f(x) = b_0 + b_1 (x - x_0) + b_2 (x - x_0)(x - x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_0)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

- interpolation by matlab:

```
x = [          ];
y = [          ]
h = interp1 (x, y, g)
```

g: النقطة المطلوب الحل عندها

① Differentiation:-

steps:-

A- forward: $f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$

Backward: $f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$

Center: $f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$

② integration:-

1. Trapezoidal Rule

2. Simpson 1/3 Rule

1. Trapezoidal Rule:-

$$\int_a^b f(x)\,dx = b-a \left[\frac{f(a) + f(b)}{2}\right]$$

- multiple sigmant trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2(f(a+h)) + f(b)\right]$$

$$h = \frac{b-a}{n}$$

2. Simpson 1/3 Rule

- $\int_a^b f(x)\,dx = \frac{h}{3}\left(f(a) + 4\left(f\left(\frac{a+b}{2}\right)\right) + f(b)\right) \qquad h = \frac{b-a}{2}$

- multiple Simpson Rule :-

$$\int_a^b f(x)\,dx = \frac{b-a}{3n}\left[f(x_a) + \underset{i=\text{odd}}{4 f(x_i)} + \underset{i=\text{even}}{2 f(x_i)} + f(x_b) \right]$$

- integration by matlab

```
x = [          ]
y = [          ]
z = trapz (x,y)
```

Solving ODE

- Euler method
- Runge - kutta 2^{nd} method
- Runge - kutta 4^{th} method

- Euler method:

$$y_2 = y_1 + f'(x_1, y_1) \cdot h$$

$$h = x_2 - x_1$$

- Runge kutta 2^{nd}

$$y_2 = y_1 + \frac{h}{2}(k_1 + k_2)$$

$$k_1 = f(x_1, y_1)$$

$$k_2 = f(x_1 + h, y_1 + k_1 h)$$

- Runge kutta 4^{th}

$$y_2 = y_1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_1, y_1)$$

$$k_2 = f\left(\left(x_1 + \frac{h}{2}\right), \left(y_1 + \frac{k_1 h}{2}\right)\right)$$

$$k_3 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2 h}{2}\right)$$

$$k_4 = f(x_1 + h, y_1 + k_3 h)$$