1-2 38/40

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Excellent

The University of Jordan
School of Engineering
Department of Chemical Engineering
Numerical Methods in Chemical Engineering (0935301)
Midterm Exam

CHE905301 Numerical Midterm Exam Summer Semester 2023\2024

Midterm Exam-July 30, 2024

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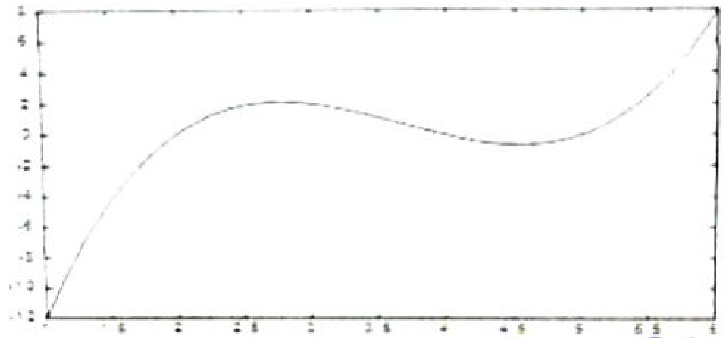
Q1. What is the sufficient and necessary condition for the bisection method to find a root of ***f(x)*** on the interval **[a, b]**. **[2 points]** we need two points X_{upper} and X_{lower} and ~~the~~ $f(x_u)$ and $f(x_L)$ should be different signs to have the root between them / and the functions should be real and continuous

Q2. The **bisection (Interval Halving) method** is applied to the function ***f(x) = (x − 2)(x − 4)(x − 5)*** (see graph). The initial interval used for the method is **[a, b] = [3.5, 4.5]**. Which root will the method converge to?
x_L x_u



[3 points]

x_L	x_u	x_m	$E_a\%$
3.5 (+)	4.5 (−)	4	—

$f(x_m) = 0 = f(4)$

$\therefore$ the root is $\boxed{4}$

$f(3.5) = 1.125$

$f(4.5) = -0.625$

$f(4) = 0$ $\therefore$ ✓

Q3. If $f(x)$ is a real continuous function in $[a,b]$, and $f(a)f(b)<0$, then for $f(x)=0$, there is (are) ___________ in the domain $[a,b]$. **[2 points]**

1. one root
2. no root
3. at least one root

Q4. The goal of forward elimination steps in **Naïve Gauss elimination** method is to reduce the coefficient matrix to. **[3 points]**

1. Diagonal matrix
2. lower triangular matrix
2. Identity matrix
4. Upper triangular matrix

Q5. A diagonally dominant matrix is? **[2 points]**

$$\underset{A}{\begin{bmatrix} 3 & 3 & 7 \\ 3 & 1 & 1 \\ -3 & 6 & 2 \end{bmatrix}} \quad \underset{B}{\begin{bmatrix} 3 & 1 & 1 \\ 3 & -3 & 7 \\ -3 & 6 & 2 \end{bmatrix}} \quad \underset{C}{\begin{bmatrix} 3 & 1 & 1 \\ -3 & 6 & 2 \\ 3 & 3 & 7 \end{bmatrix}} \quad \underset{D}{\begin{bmatrix} -3 & 6 & 2 \\ 3 & 1 & 1 \\ 3 & 3 & 7 \end{bmatrix}}$$

$3 > 2$ ✓
$6 > -1$ ✓
$7 > 6$ ✓

Q6. Using **Fixed-Point method (Successive Substitution)**, find the root of the function $f(x) = x^2 - 2x - 3$, using $g(x) = (2x+3)^{0.5}$ given $x_0 = 4$ and $x>0$. Perform **three iterations only.** **[4 points]**

	x_0	$g(x)$	$\epsilon_a\%$
1	4	3.3166	20.61%
2	3.3166	3.1037	6.859%
3	3.1037	3.0344	2.283%

$\boxed{\text{the root is } 3.0344}$

$f(x)$

Q7. Using **Newton's method**, perform **3 iterations** to find the root of **[4 points]**

$f(x) = (x/2)^2 - \sin(x)$. Given $x_0 = 1.5000$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$\boxed{\frac{1}{4}x^2 - \sin(x)}$

$f'(x) = \frac{1}{2}x - \cos(x)$

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x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$\epsilon_a\%$
1.5	0.5363	-0.2496	3.6486	58.88%
3.6486	3.2644	0.8263	-0.3020	1308.2%
-0.3020	-0.01753	-1.151	-0.3172	4.79%

2

Q8. Solve the following linear system of algebraic equations using **Naïve Gaussian elimination [4 points]**

$$-120x_1 + 40x_2 + 60x_3 = -500$$
$$-130x_2 + 30x_3 = -200$$
$$90x_2 - 90x_3 = 0$$

$-130x_2 + 30(2) = -200$

$-120x_1 + (40\times2) + (60\times2) = -500$

$$\begin{bmatrix} -120 & 40 & 60 \\ 0 & -130 & 30 \\ 0 & 90 & -90 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -500 \\ -200 \\ 0 \end{bmatrix}$$

$\frac{90}{-130} = -\frac{9}{13}$

$-\frac{9}{13} * (-130x_2 + 30x_3 = -200) \rightarrow 90x_2 - 20.77x_3 = 138.46$

$(90x_2 - 90x_3 = 0) - (90x_2 - 20.77x_3 = 138.46)$

$-69.23x_3 = -138.46$

$\boxed{x_3 = 2}$ $\boxed{x_2 = 2}$ $\boxed{x_1 = 5.83}$

Q9. Find the **3rd column of the inverse** of the matrix **A** using the **LU** factorization of the matrix: **[4 points]**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -9 \end{bmatrix}$$

$LZ = I$

$UX = Z$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \boxed{z_1 = 0} \; \boxed{z_2 = 0} \; \boxed{z_3 = 1}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$-9x_3 = 1 \rightarrow x_3 = -0.111$

$-3x_2 - 6x_3 = 0 \rightarrow -3x_2 - (6 * -0.111) = 0$

$-3x_2 = -0.666$

$x_2 = 0.222$

$x_1 + 2(0.222) + 3(-0.111) = 0$

$x_1 + 0.444 - 0.333 = 0$

$x_1 = -0.111$

$\therefore$ the third column of the inverse [A] is $\begin{bmatrix} -0.111 \\ 0.222 \\ -0.111 \end{bmatrix}$

Q10. Consider the matrix $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$ **[5 ponits]**

$L = \begin{bmatrix} 1 & 0 & 0 \\ d_{21} & 1 & 0 \\ d_{31} & d_{32} & 1 \end{bmatrix}$

$d_{21} = \frac{0}{2} = 0$

$d_{31} = \frac{6}{2} = 3$ $\quad d_{32} = 0$

1. Calculate the **LU** decomposition of **A**.

2. Use your answer to (1) to efficiently calculate the solution of:

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

$0 * (2x_1 + 3x_2 + 3x_3 = 2) \Rightarrow 0$

$3 * (2x_1 + 3x_2 + 3x_3 = 2) \rightarrow$

$6x_1 + 9x_2 + 9x_3 = 6$

$(6x_1 + 9x_2 + 8x_3 = 5) - (6x_1 + 9x_2 + 9x_3 = 6)$

$0x_2 - x_3 = -1$

$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$

$U = $

$\boxed{= UX}$ $\quad \boxed{LZ = C}$ $\quad \boxed{UX = Z}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$

$\boxed{z_1 = 2}$ $\quad \boxed{z_2 = 2}$

$(3 \times 2) + z_3 = 5$ $\quad \boxed{z_3 = -1}$

$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

$-x_3 = -1$ $\quad \boxed{x_3 = 1}$

$5x_2 + 7 = 2$

$5x_2 = -5$ $\quad \boxed{x_2 = -1}$

$2x_1 - 3 + 3 = 2$

$\boxed{x_1 = 1}$