

# CHEMICAL REACTION ENGINEERING I

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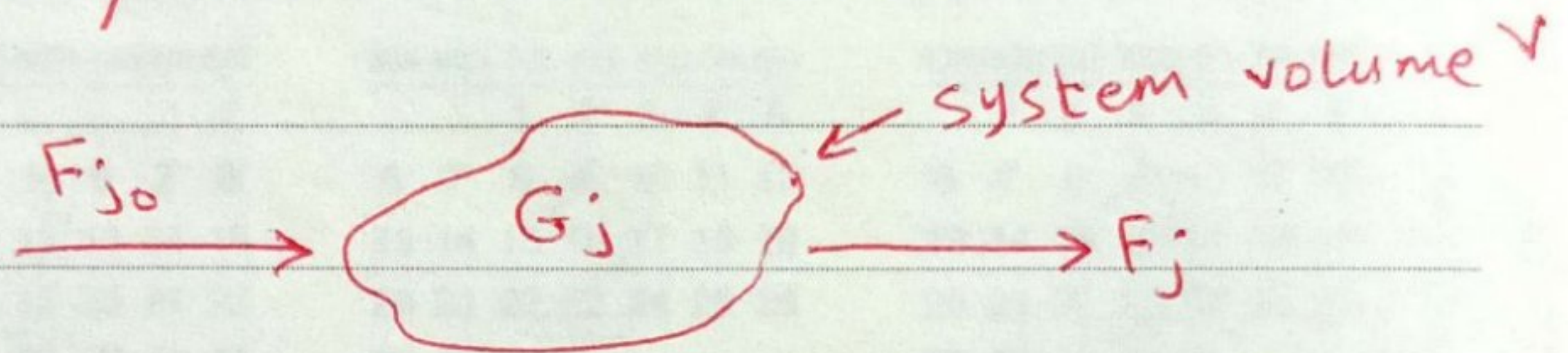
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Memo No. \_\_\_\_\_

Date 18 / 10 / 2021

## \* Ch 2 :- Mole balance.

\* Lec 2.1 :- (General mole balance equation) and Batch reactor.



$$\left[ \begin{array}{l} \text{molar Flow} \\ \text{rate of} \\ \text{species } j \text{ in} \end{array} \right] - \left[ \begin{array}{l} \text{molar Flow} \\ \text{rate of} \\ \text{species } j \\ \text{out} \end{array} \right] + \left[ \begin{array}{l} \text{molar rate} \\ \text{generation} \\ \text{of species} \\ j \end{array} \right] = \left[ \begin{array}{l} \text{molar rate} \\ \text{accumulation} \\ \text{of species} \\ j \end{array} \right]$$

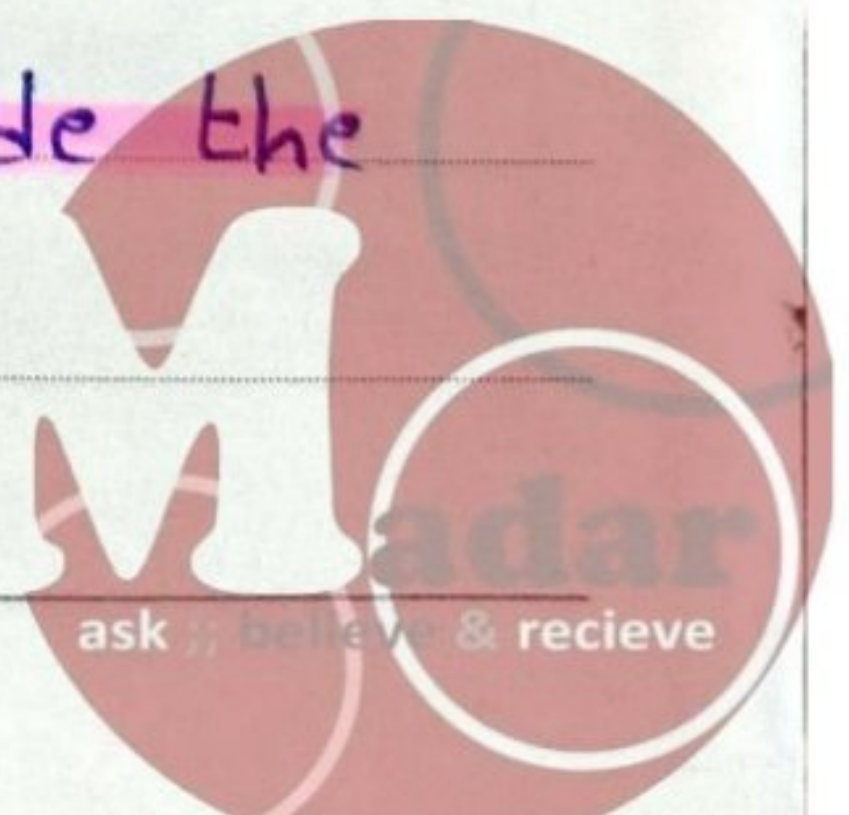
generation (+ve)  $\leftrightarrow$  (-ve) consumption

$$F_{j0} - F_j + G_j = \frac{dN_j}{dt}$$

المول  
الزمن

التغير مع الزمن في الموجودات  
system.

$N_j \rightarrow$  number of moles inside the system.



\* IF the system is uniform throughout its entire volume (spatially uniform), then

يعني اي نقطة جوال system يعبر فيها نفس الاشياء <sup>التركيز</sup>

generation of

rate of reaction  $\rightarrow f(c, T)$   
volume.

$$G_j = r_j V$$

+ve  $\rightarrow$  generation  
-ve  $\rightarrow$  consumption.

rate of reaction (r)  $\rightarrow$   $\frac{\text{mole}}{\text{time} \times \text{volume}}$   
consumption

$r_j$   $\rightarrow$  reactant  $\rightarrow$  generation/Formation.  
 $\rightarrow$  product  $\rightarrow$

$r_j$   $\rightarrow$  product  $\rightarrow$  +ve  
 $\rightarrow$  reactant  $\rightarrow$  -ve

Rate of Formation (generation) of species j.

- IF NOT spatially uniform:-

- ينقسم ال volume الكلي الـ system الى

rate  $\Delta V$  small volume  $\rightarrow$  يكونه الـ

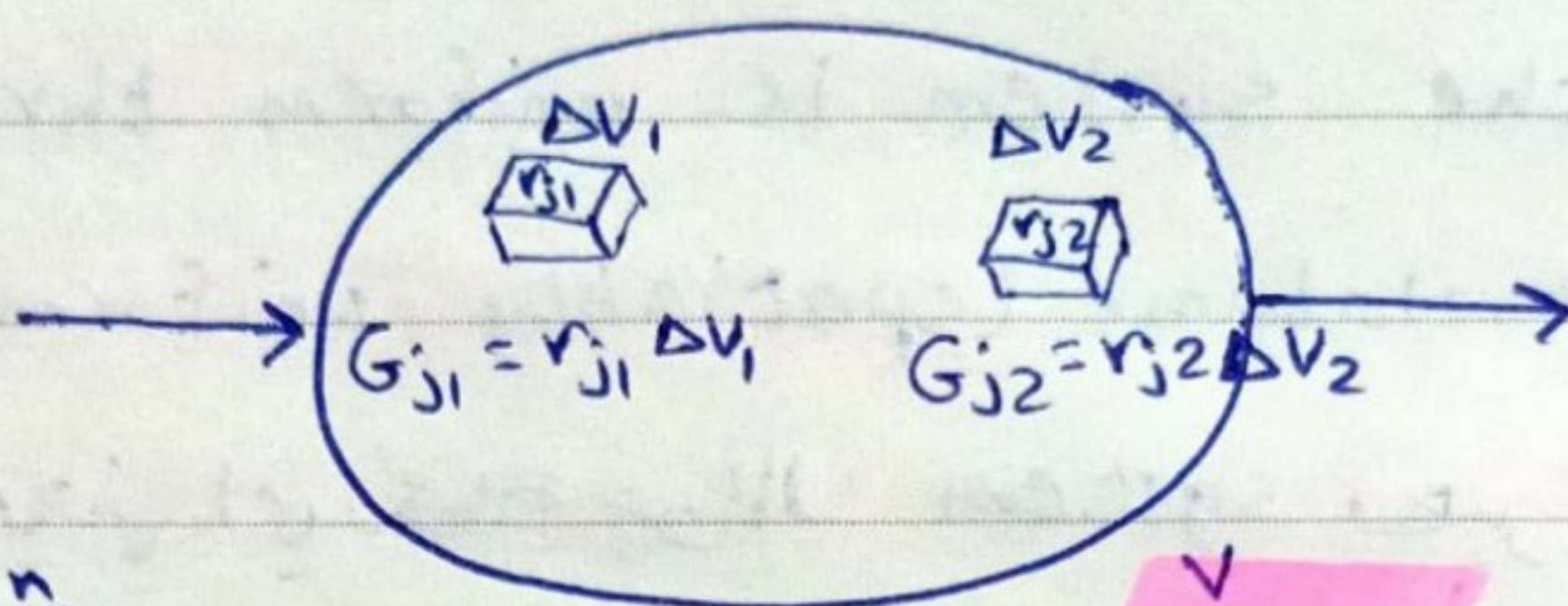
\* Rate of reaction (rA) :- tell us how fast a number of moles of one chemical species are being consumed to form another chemical species.



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$$G_J = \sum_{i=1}^n r_{ji} \Delta V_i \quad \lim_{\Delta V \rightarrow 0, n \rightarrow \infty} = \int_V r_J dv$$

- general mole balance on system volume  $V$  :-  
(المادة A بولن)

In - out + generation = Accumulation

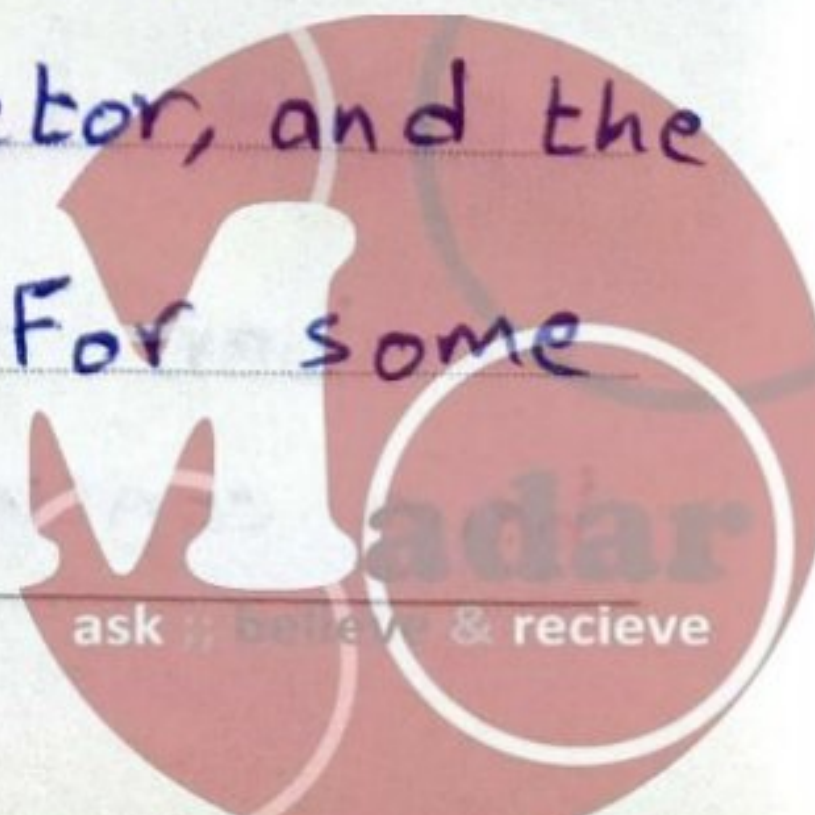
$$F_{A0} - F_A + \int_V r_A dv = \frac{dN_A}{dt}$$

وعلى حسب نوع  
ال reactor بتغير فيها

## [I] Batch Reactors properties.

يُدخل المواد بعدية بتشغل ال reactor لوقت محدد  
و بس تخلص بتخرج out للمواد.

\* Reactants are placed in the reactor, and the reaction is allowed to proceed For some amount of time.



\* closed system: no addition of reactants or removal of products during the reaction.

لا نه في عنا تغير في ال system - في عنا  
unsteady state condition

مرتبطة في التغير  $\rightarrow$  steady state, unsteady state  
مع ال time لي جوا ال reactor.

لا نه في reaction و كمية ال reactant  
يقتل مع الوقت و كمية ال product بتزيد مع الوقت.

- unsteady state conditions: - the  
composition changes with time.

(حسب ما هو في الممارسة) - Ideal batch reactor

- هو نه عنا assumption معينة

- في نفس اللحظة / الوقت و يكون التركيز والحرارة  
تساوية عن نقاط مختلفة جوا ال reactor.



|    |    |    |    |    |    |    |
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• vessel is perfectly mixed.

- Concentration and Temperature are spatially constant, but NOT constant in time.

$$F_{j0} - F_j + \sum r_j dv = dN_j / dt$$

$$\boxed{\sum r_j dv = \frac{dN_j}{dt}} \quad \text{Batch Reactor Design Equation.}$$

\* well-Mixed:-

If the reactor is perfectly mixed, the T, concentration, and therefore the reaction rate are spatially constant:

$$\boxed{\frac{dN_A}{dt} = r_A v} \rightarrow \text{Ideal Batch Reactor Design Equation.}$$

- احنا بنحس 2 نغرد الوقت لازم لنغير عدد مولان

الموارد المتفاعلة حتى نعطينا المواد الناتجة

لأنه من  $N_{A0}$  إلى  $N_A$

ask ; believe & recieve



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$$dt = \frac{dN_A}{r_A v}$$



$$\int_{t=0}^{t=t} dt = \int_{N_{A0}}^{N_A} \frac{dN_A}{r_A v}$$

$$t=0$$

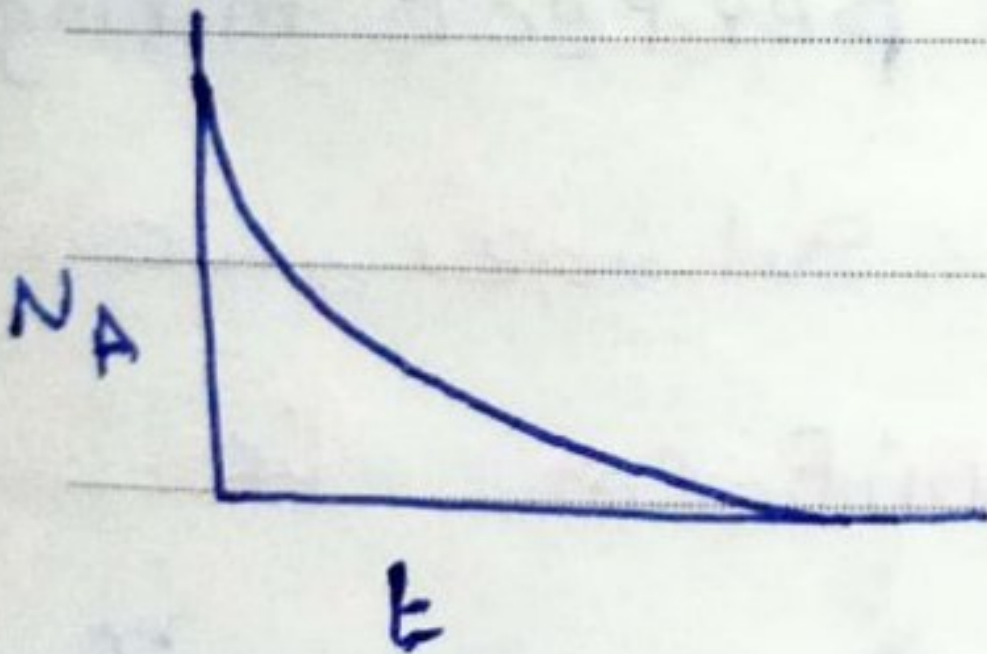
$$N_A = N_{A0}$$

$$t=t$$

$$N_A = N_A$$

$$t = \int_{N_{A0}}^{N_A} \frac{dN_A}{r_A v}$$

Time necessary to  
reduce the number of  
moles of A from  $N_{A0}$  to  
 $N_A$ :



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## 2 Continuous-Flow Reactors (CSTR)

\* Continuous stirred Tank Reactor (CSTR)

Properties:-

→ continuously add reactant and remove products (open system).

→ Inlet stream ~~is~~ instantaneously mixes with bulk of reactor volume.

<sup>main assumption</sup> → Ideal CSTR: assume perfect mixing occurs in vessel. → *الخليط متجانس*

• spatially uniform *لي*

→ • T and concentration are uniform throughout space.

• Composition of the exit stream is the same as that inside reactor.

$(C_{A, \text{outlet}} = C_{A, \text{tank}})$ .



## SS • Steady-state conditions :-

reaction rate is the same at every point and doesn't change with time.

• (بالنسبة للجو الـ Reactor)

- baffle → mixing <sup>perfect</sup> (ببساط في الـ mixing)

- Impeller → mixing (بالنسبة بعمل mixing)

- sparger → (إذا كان في Inlet gas stream)

- Jacket → heating or cooling (إذا احتجنا)

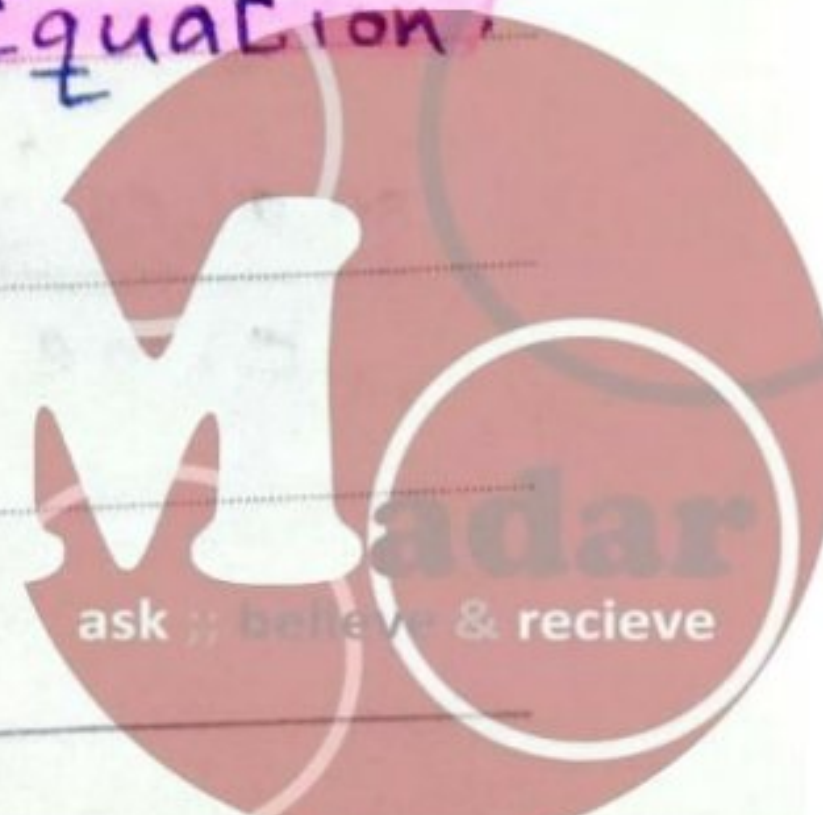
In - Out + generation = Accumulation.

$$F_{j0} - F_j + \int \dot{r}_j dv = \frac{dN_j}{dt}$$

CSTR is at steady state, so no change in moles  $j$  with time.

$$F_{j0} - F_j + \int \dot{r}_j dv = 0$$

steady state CSTR Design Equation





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A perfectly mixed CSTR has no spatial variations in reaction rate.

$$F_{j0} - F_j + r_j V = 0$$

rearrange to put in terms of  $V$ .

$$\boxed{\frac{F_{j0} - F_j}{-r_j} = V} \quad \text{(Ideal) steady state (CSTR) Design Equation.}$$

Ideal CSTR  $\equiv$  back mix reactor.

$\rightarrow r_j$  is measured at the outlet

because  $C_{j, \text{exit}} = C_{j, \text{tank}}$

\* $V$  :- Reactor volume required to reduce the entering flow rate of species  $j$  from  $F_{j0}$  to  $F_j$  at the outlet (and in the tank).

- How we determine the molar flow rate,  $F_j$  (units = mol/time)?

$$F_j = (C_j)(V)$$

$$\frac{\text{moles } j}{\text{time}} = \frac{\text{moles}}{\text{volume}} \times \frac{\text{volume}}{\text{time}}$$



$c_j$ : concentration of  $j$ .

$v$ : volumetric Flow rate.

\* Ideal SS CSTR design equation in terms of concentration:

$$V = \frac{C_{j0} V_0 - C_j V}{-r_j}$$

### [3] PLUG Flow Reactor (PFR).

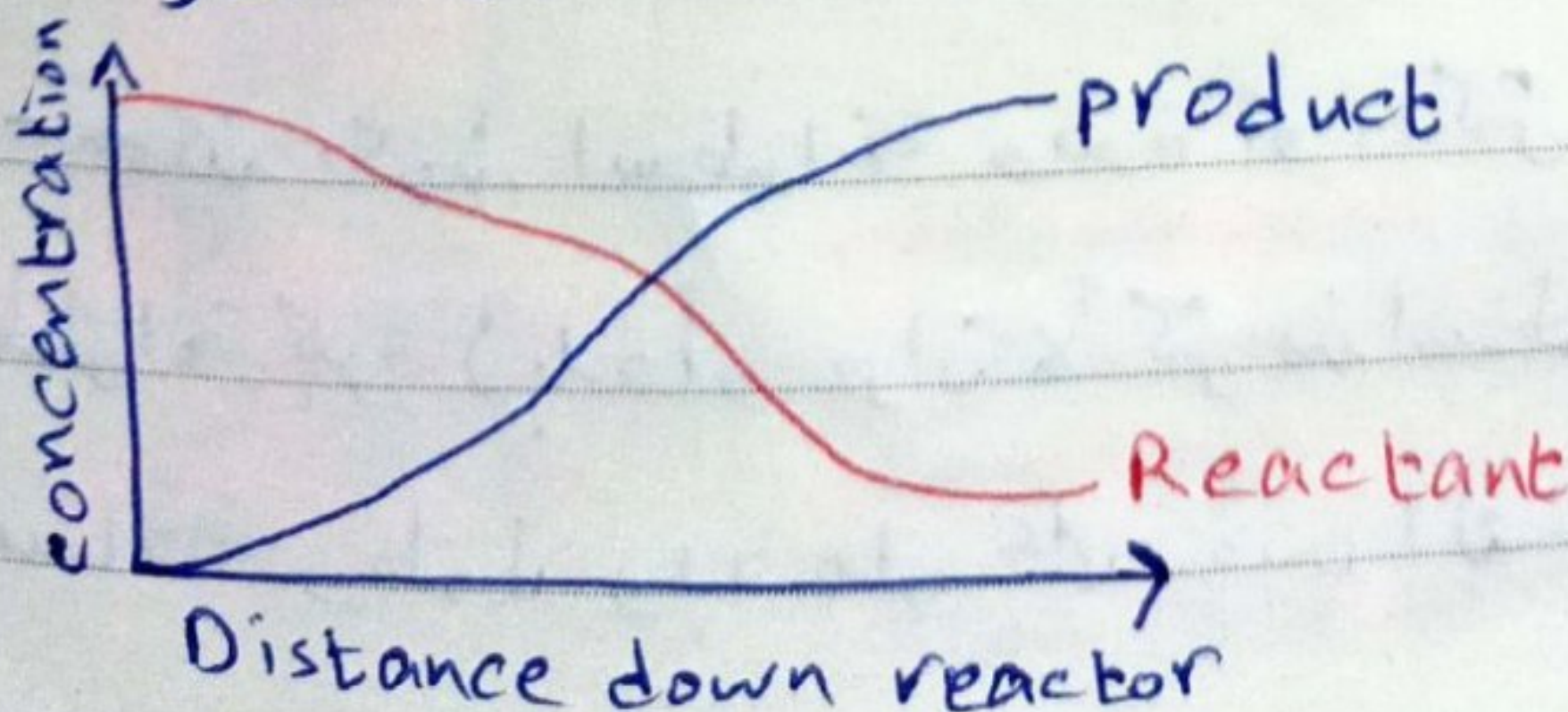
\* properties :- (يكونه Pipe مقسم الى plug).

→ Also called a <sup>↑ Ideal</sup> tubular reactor.

→ Cylindrical pipe with openings at both ends

→ Steady movement of material down

Length of reactor.

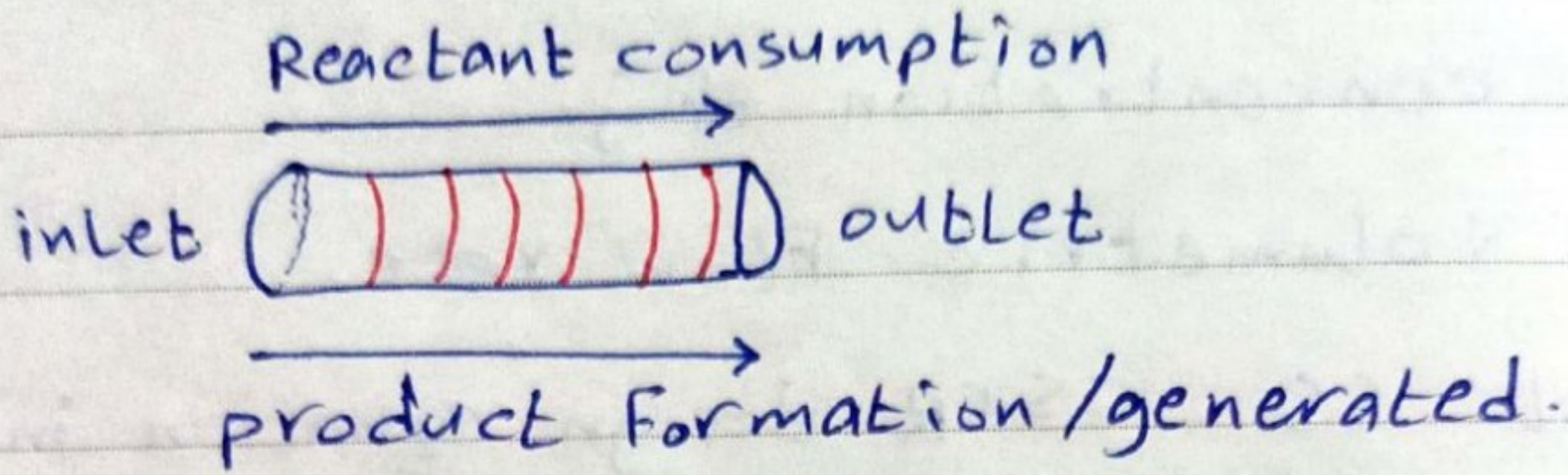




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→ Reactants are consumed as they flow down the length of the reactor.

→ Operated at steady state

main assumption ① No radial variation in  $T$ , concentration, or reaction rate. (لا يوجد اختلاف في  $T$ ، التركيز، أو معدل التفاعل عند نفس نصف القطر)

② All Fluid/gas elements have the same residence time.

هـ - الوقت الذي يتقضي به أول مجموعة جزيئات جوا الـ Reactor هو نفس الوقت الذي يتقضي به المجموعة الثانية، الثالثة، ...

- ممكن يكونه اسطوانة وحدة او اكثر من اسطوانة جوا

اسطوانة كبيرة (بلجاو مران لاكثر من اسطوانة بدل وحدة ليوفرنا مساحة وطول ويحطو على نفس النتيجة) (parallel arrangement)

\* In a plug Flow reactor the composition of the fluid varies from point to point along a flow path.

A diagram of a fluid element in a pipe. It is a horizontal cylinder of length  $\Delta V$ . The left face has area  $A$  and velocity  $v$ . The right face has area  $A$  and velocity  $v + \Delta v$ . Forces  $F_A$  are shown acting on both faces.

$$F_A|_V - F_A|_{V+\Delta V} + r_A \Delta V = 0$$

$$\lim_{\Delta v \rightarrow 0} \frac{F_A|_{v+\Delta v} - F_A|_v}{\Delta v} = r_A$$

$$r_A = \frac{dF_A}{dv}$$

### Ideal SS PFR Design Eqn.

$$V = \int_{F_{A0}}^{F_A} \frac{dF_A}{r_A}$$

Volume necessary to reduce the entering molar flow rate ( $\text{mol/s}$ ) from  $F_{A0}$  to the exit molar flow rate of  $F_A$ .



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- If we assume the PFR is ideal, the degree of completion isn't affected by PFR shape, only by PFR volume.

#### [4] Packed Bed Reactor (PBR).

- ببشء ال PFR في علو لكن هاد عودي غالباً.

- Cylindrical shell, vertically oriented.
- often gravity-driven Flow. (driving Force) <sup>ما في bomb</sup>
- Heterogeneous reaction? Fixed bed of catalyst inside.
- Reactants enter top and Flow through the packed bed of catalyst.
- Concentration gradient of reactant and product down the length of the reactor.
- Reaction occurs on the surface of the catalyst pellets.



الفروق  
بين  
PFR

Reaction Rate is based on the mass of the solid catalyst (w), not reactor volume (V).

number of moles of j reacting per unit time per unit mass of catalyst.

$$\frac{dF_j}{dw} = r'_j$$

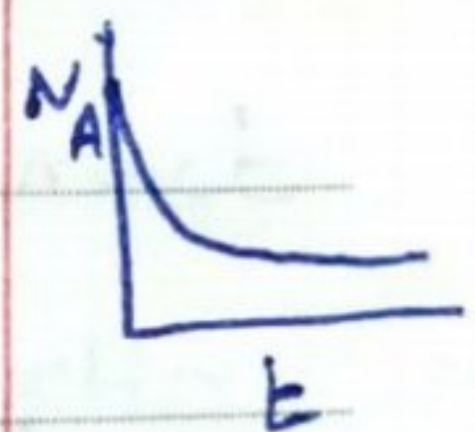
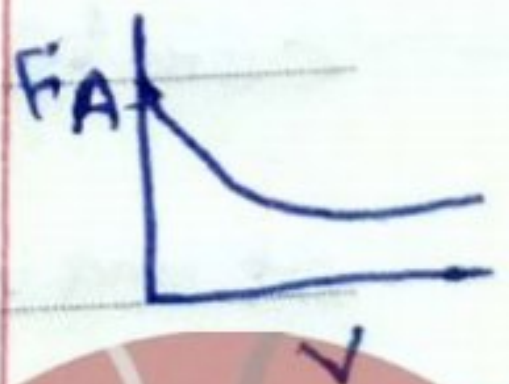
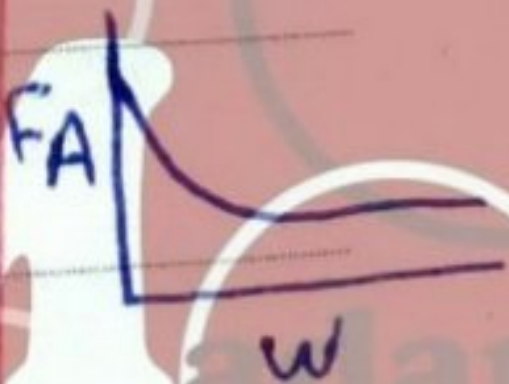
w → weight of the catalyst.

$r'_j$  → rate of catalytic rxn

↳ (mol / Kg catalyst \* s)

Summary



| Reactor | Differential              | Algebraic                       | Integral                                     |                                                                                       |
|---------|---------------------------|---------------------------------|----------------------------------------------|---------------------------------------------------------------------------------------|
| Batch   | $\frac{dN_A}{dt} = r_A V$ |                                 | $t = \int_{N_{A0}}^{N_A} \frac{dN_A}{r_A V}$ |  |
| CSTR    |                           | $V = \frac{F_{A0} - F_A}{-r_A}$ |                                              |                                                                                       |
| PFR     | $\frac{dF_A}{dV} = r_A$   |                                 | $V = \int_{F_{A0}}^{F_A} \frac{dF_A}{r_A}$   |  |
| PBR     | $\frac{dF_A}{dw} = r'_A$  |                                 | $w = \int_{F_{A0}}^{F_A} \frac{dF_A}{r'_A}$  |  |



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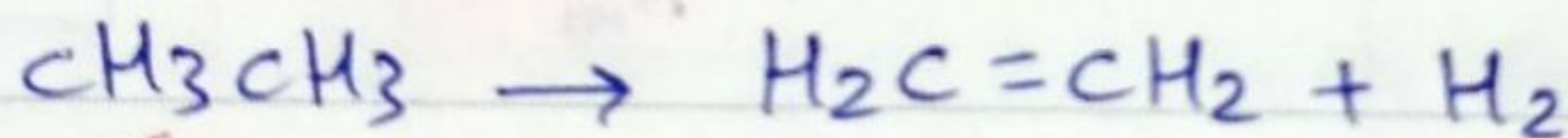
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$$W = \int_{F_{A0}}^{F_A} \frac{dF_A}{-r_A'} \quad \text{تابع (PBR)}$$

→ catalyst weight necessary to reduce the entering molar flow rate of species A ( $F_{A0}$ ) to a flow rate ( $F_A$ ).

\* There are 3 ways for a species to lose its identity:-

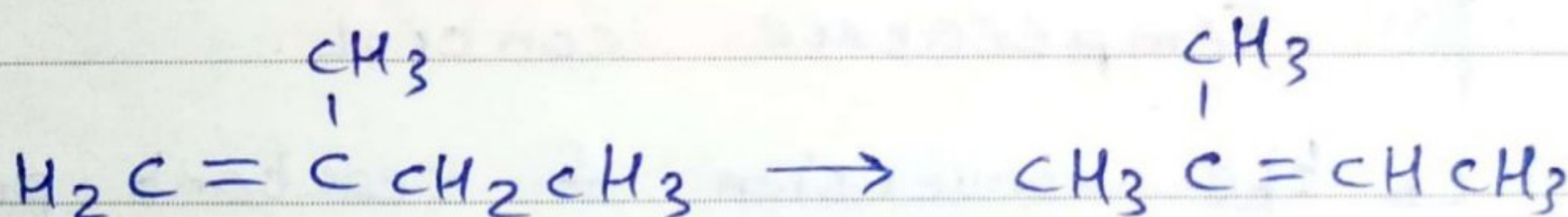
① <sup>يتفكك</sup> Decomposition → molecule broken down into smaller molecules.



② <sup>يتحد</sup> combination → molecules combination with another molecule or atom.



③ isomerization → compounds have the same number of atoms of each element, but ~~have~~ with different structure or configuration.



### \* Selection of Reactors :-

#### □ Batch

- small scale (مقياس صغير).
- high labour costs per batch.
- difficult for large-scale production.
- production of expensive products (e.g. pharmacy).





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## [2] CSTR : most homogenous Liquid-phase Flow reactors.

- when intense agitation is required.
- relatively easy to maintain good Temperature control.
- the conversion of reactant per volume of reactor is the smallest of the Flow reactors - very large reactors are necessary to obtain high conversions

\* كل انواع ال reactor بغير قسري homogenous reactions

ط ا عدا ① PBR ② Fluidized bed reactor

الفاعل غير متجانس heterogeneous.

$$W = \frac{F_{A0} - F_A}{-r_A}$$

بما ان معامل CSTR يس في Catalyst.

[3] PFR : most homogenous gas-phase Flow reactors.

→ relatively easy to maintain.

→ hot spots can occur.

→ difficult to control T within the reactor. (rate of reaction لا يتغير في نقطة أخرى)

→ usually produces the highest conversion per reactor volume (weight of catalyst if it is a packed-bed catalysed gas reaction) of any of the Flow reactors.

\* يوجد 3 أوراق عمل.

أول فيديوهين.





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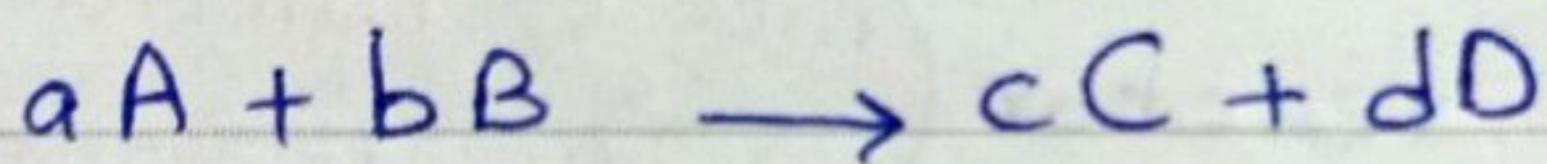
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## Ch② Conversion and reactor sizing.

(2.1) :- Mole balance in terms of conversion,  $X$ .

### \* Definition of Conversion

- generic equation

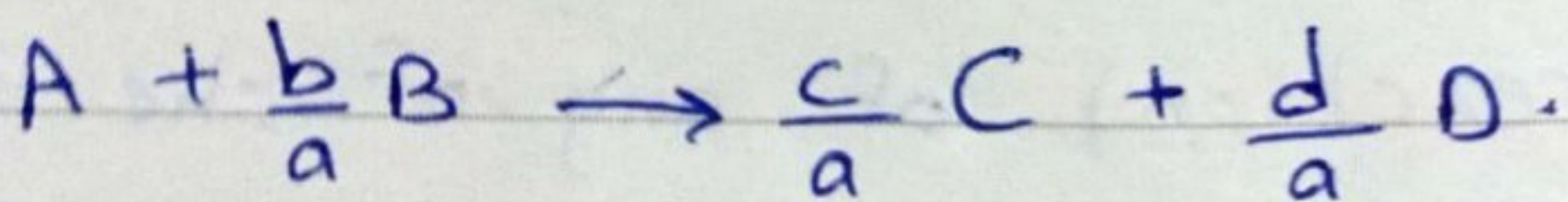


- Choose Limiting reactant A as a

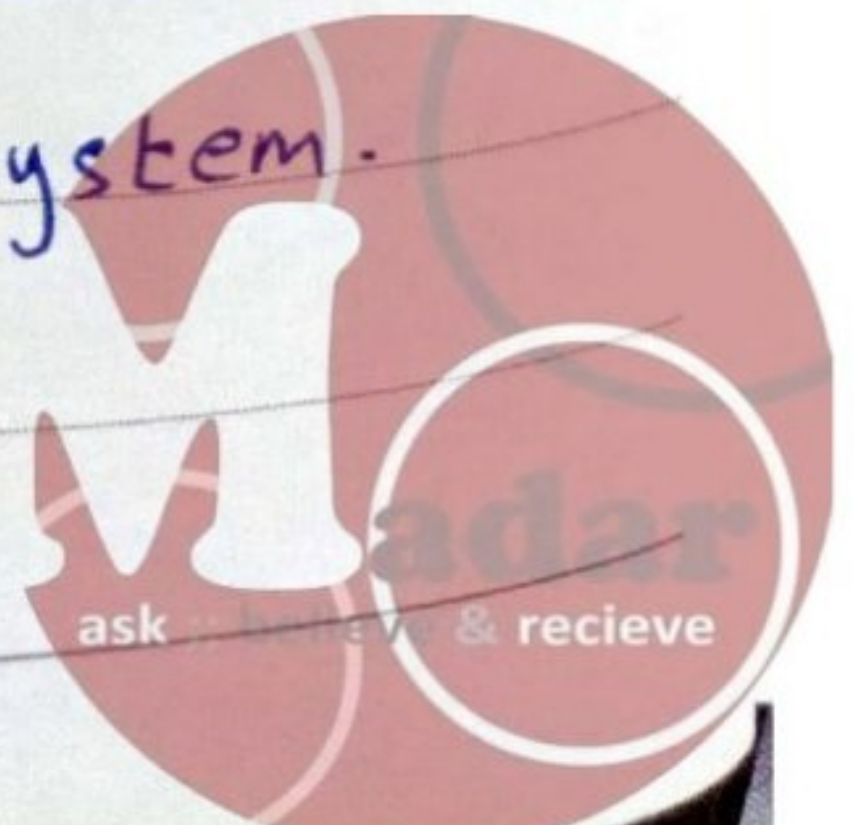
basis of calculations:-

$a, b, c, d \rightarrow$  Stoichiometric coefficient

$\Delta \cdot 1 = 1/| \text{Stoichiometric coefficient} |$  for



- Define Conversion,  $X_A$  :- the number of moles of A that have reacted per mole of A fed to the system.



$$X_A = \frac{\text{moles A reacted}}{\text{moles A Fed}}$$

- For Batch system : "Moles A Fed" is the amount of A at the start of the reactor (at  $t=0$ ).
- For Flow system : "Moles A Fed" is the amount of A entering the reactor.

\* Example :-  $A + 2B \rightarrow 2C$

- start with 1 mole of A and 1 mole of B.
- If A is the basis and at the end we have :-

1 mole A, 1 mole B  $\rightarrow X_A = 0/1 = 0$  (no reaction)

$\frac{1}{2}$  mole A, 0 mole B  $\rightarrow X_A = 0.5/1 = \frac{1}{2}$



Not possible!!

0 mole A, 1 mole B  $\rightarrow X_A = 1/1 = 1$  (complete reaction.)

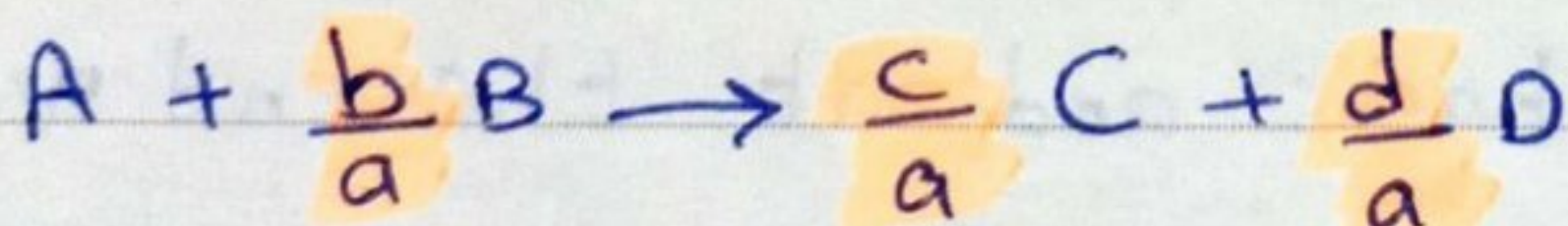
\* The correct approach is to take B as the basis because B is the Limiting reagent.

- At the end we have :-

1 mole A, 1 mole B  $\rightarrow X_B = 0/1 = 0$  (no reaction)

$\frac{1}{2}$  mole A, 0 mole B  $\rightarrow X_B = 1/1 = 1$  (complete reaction).

\* Expressing other components conversion in Terms of  $X_A$



\* Batch # System :- Longer reactant is in reactor, more reactant is converted to



product (until reactant is consumed or  
the reaction reaches equilibrium) (in reversible reaction).

∴ conversion ( $X_j$ ) is a function of time ( $t$ )  
in the batch reactor.

moles A fed

→ Moles A consumed.

$$N_A = N_{A0} - (N_{A0})(X_A)$$

↳ Moles A in reactor at time  $t$ .

$$N_A = N_{A0} (1 - X_A)$$

$$N_B = N_{B0} - \frac{b}{a} (N_{A0} X_A)$$

→ reactant.

→ product.

$$N_C = N_{C0} + \frac{c}{a} (N_{A0} X_A)$$

$$N_D = N_{D0} + \frac{d}{a} (N_{A0} X_A)$$

$$N_I = N_{I0} \quad (\text{inert component}).$$

لا يتفاعل





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## \* Batch Reactor.

(moles A remaining) = (moles A initially) - (A reacted)<sup>moles</sup>

$$N_A = N_{A0} - N_{A0}X \rightarrow dN_A = 0 - N_{A0}dX$$

↳ const. (  $0 = \text{constant}$  )

$$\frac{dN_A}{dt} = -N_{A0} \frac{dX}{dt} = r_A V.$$

$$\boxed{\frac{dX}{dt} = \frac{-r_A V}{N_{A0}}}$$

$$t=0 \quad X=0$$

$$t=t \quad X=X$$

Integrating.

$$\boxed{t = N_{A0} \int_0^X \frac{dX}{-r_A V}}$$

The necessary  $t$  to achieve conversion  $X$ .

## CSTR

- generic equation
- choose L.R as basis
- Define conversion,  $X$

المعادلات



steady state

$$\frac{dN_A}{dt} = 0$$

well mixed  $\rightarrow S_{rA} v = r_A v \rightarrow v = \frac{F_{A0} - F_A}{-r_A}$

↓ mol balance (conversion بالترتيب)

[moles A leaving] = [moles A entering] - [moles A reacted]

$$F_A = F_{A0} - F_{A0} X$$

$$v = \frac{F_{A0} - (F_{A0} - F_{A0} X)}{-r_A}$$

$$v = \frac{F_{A0} X}{-r_A}$$

CSTR volume necessary to achieve conversion X.





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**PFR**

steady state

$$\frac{dF_A}{dv} = r_A$$

$$F_A = F_{A0} - F_{A0} X \rightarrow \frac{dF_A}{dF_A} = 0 - F_{A0} dX$$

$$\frac{dX}{dv} = \frac{-r_A}{F_{A0}}$$

$$v=0$$

$$X=0$$

Integrating.

$$v=v$$

$$X=X$$

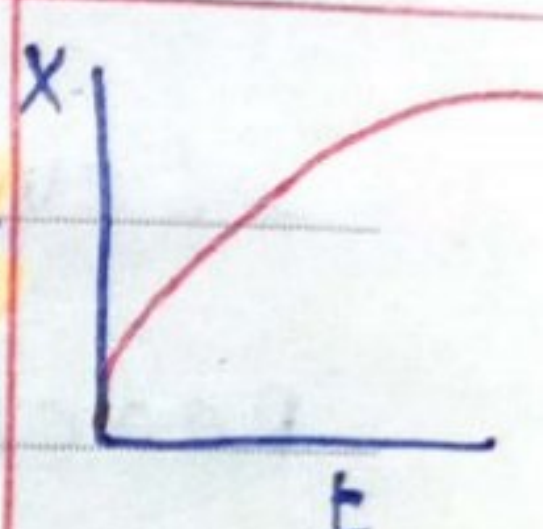
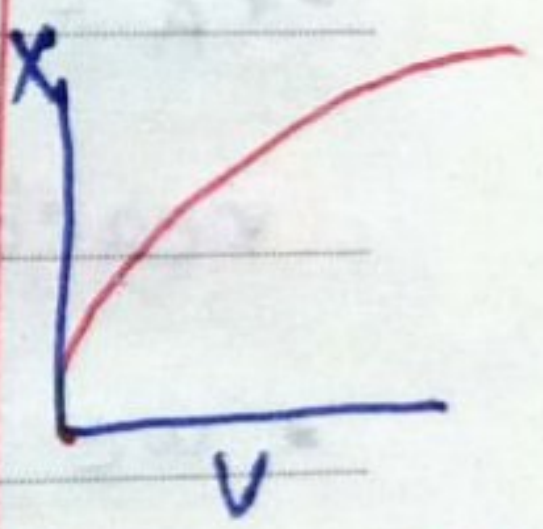
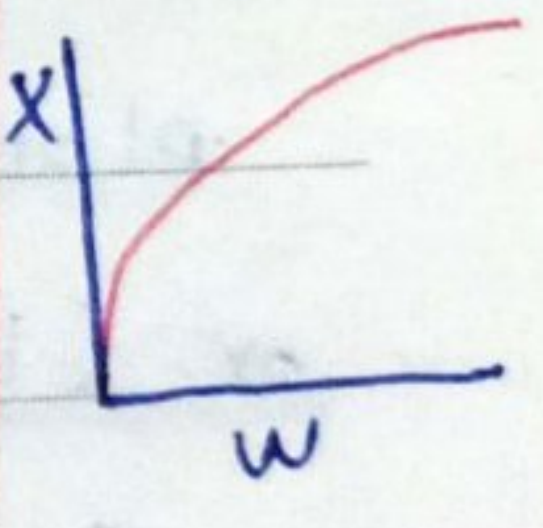
$$v = \int_0^X \frac{F_{A0}}{-r_A} dX$$

**PFR volume necessary to achieve conversion X.**

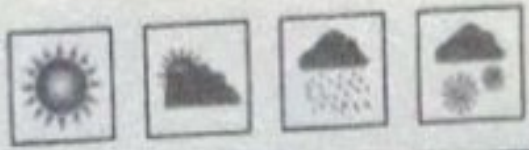
$F_{A0}$  const.  $\rightarrow$  <sup>المساكن</sup> ~~المساكن~~ بقدر زغالها برا



# \* Reactor Mole Balances Summary in terms of conversion, $X$ .

| Reactor | Differential                    | Algebraic                   | Integral                                                                                                                     |
|---------|---------------------------------|-----------------------------|------------------------------------------------------------------------------------------------------------------------------|
| Batch   | $N_{A0} \frac{dx}{dt} = -r_A V$ |                             | $t = N_{A0} \int_0^X \frac{dx}{-r_A V}$  |
| CSTR    |                                 | $V = \frac{F_{A0} X}{-r_A}$ |                                                                                                                              |
| PFR     | $F_{A0} \frac{dx}{dV} = -r_A$   |                             | $V = \int_0^X \frac{F_{A0} dx}{-r_A}$   |
| PBR     | $F_{A0} \frac{dx}{dW} = -r_A'$  |                             | $W = \int_0^X \frac{F_{A0} dx}{-r_A'}$  |





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## 2.2 :- Reactor sizing using Levenspiel plots.

"Applications of the Design Equations For Continuous-Flow Reactors".

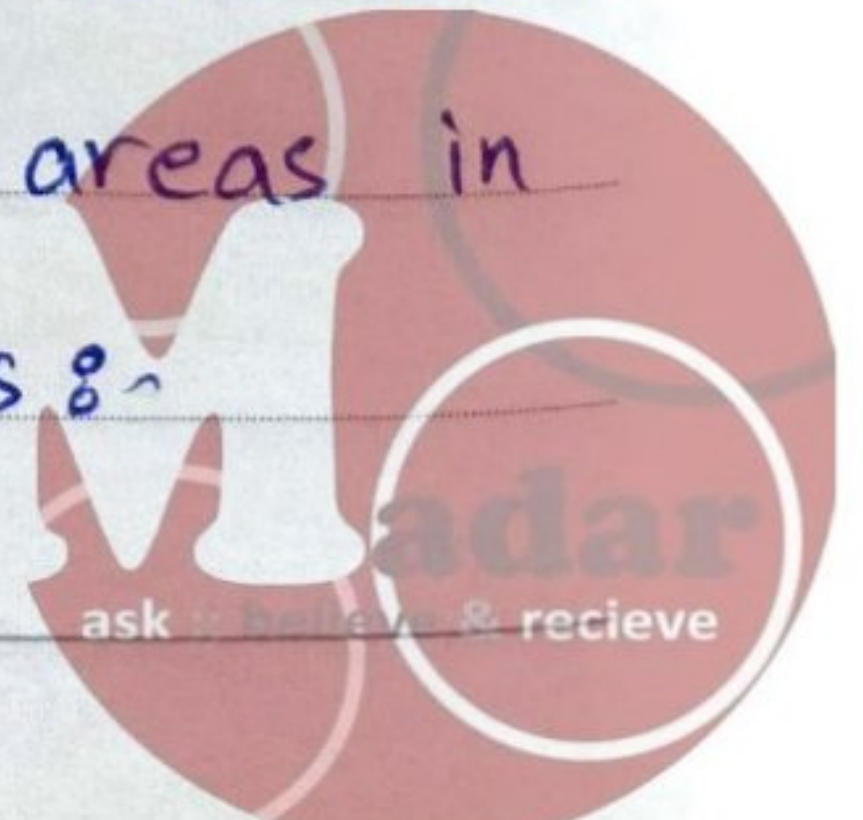
\* Levenspiel plots

Reactor Sizing.

- Given  $-r_A$  as a function of conversion,  $-r_A = F(X)$ , one can size any isothermal reaction system.

- we do this by constructing Levenspiel plot. Here we plot either  $(F_{A0}/-r_A)$  or  $(1/-r_A)$  as a function of  $X$ .

→ For  $(F_{A0}/-r_A)$  vs.  $X$ , the volume of a CSTR and the volume of a PFR can be represented as the shaded areas in the Levenspiel plot shown as:-



$$\frac{F_{A0}}{-r_A} = g(x).$$

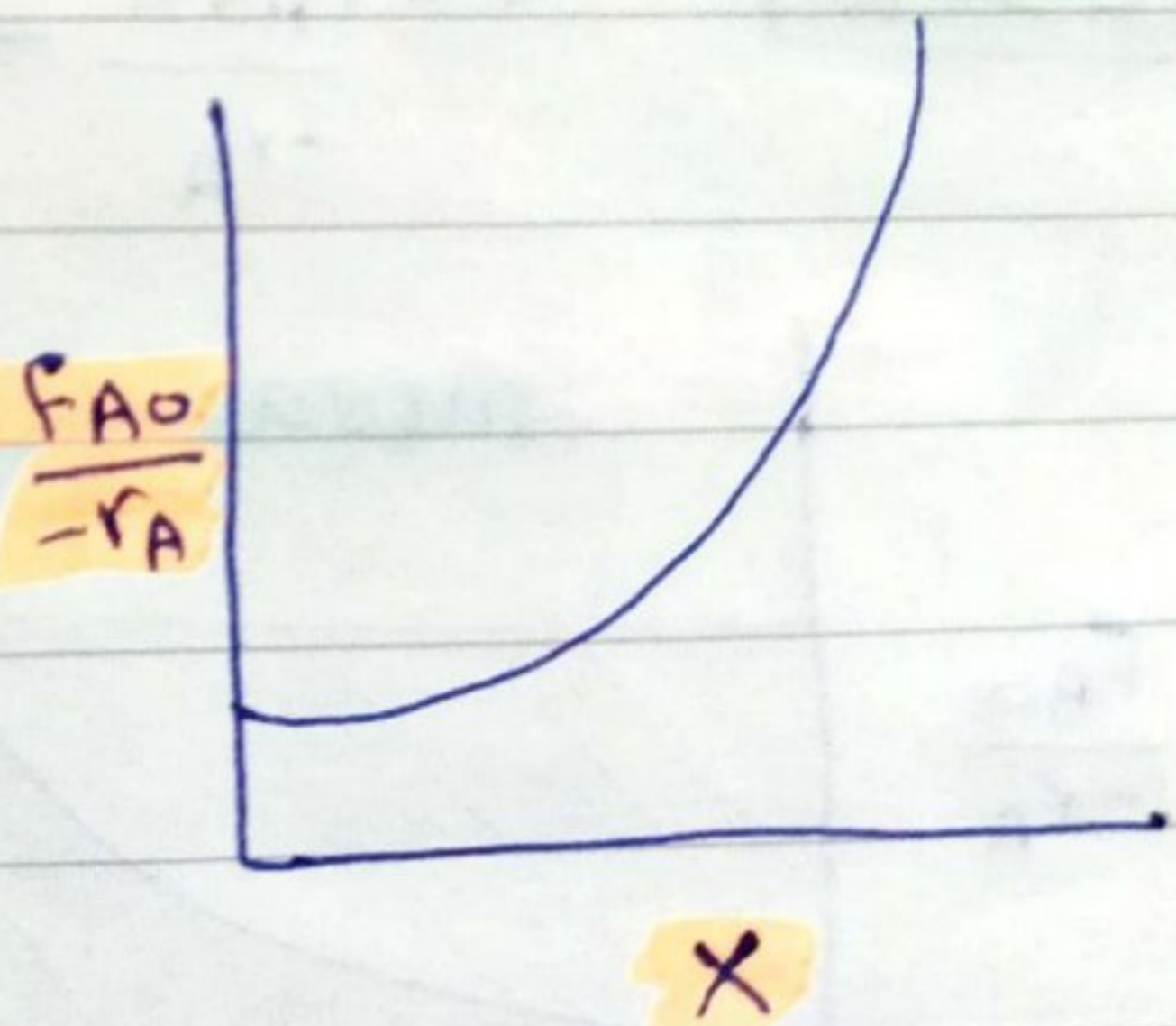
\* For all **irreversible reactions** of **order > zero**, as we approach **complete conversion** where **all the limiting reactant is used up**, i.e.  **$x=1$** , the <sup>معدل</sup>reciprocal rate approaches infinity as does the reactor volume.

~> As  **$x \rightarrow 1$** ,

**$-r_A \rightarrow 0$** , thus

**$\frac{1}{-r_A} \rightarrow \infty$** , therefore

**$V \rightarrow \infty$** .





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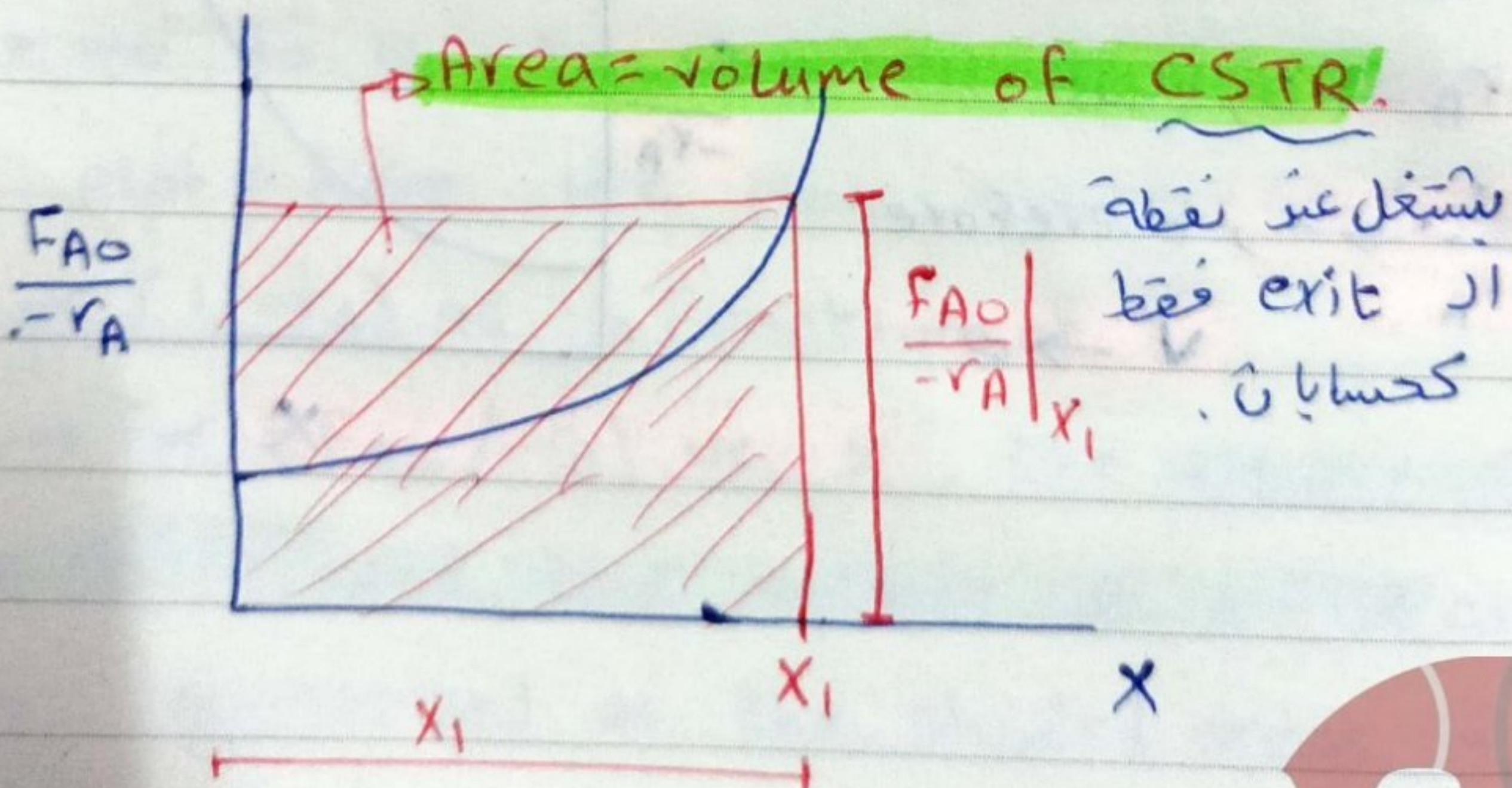
- For **reversible reactions**, the **maximum conversion** is the **equilibrium conversion**  $x_e$
- At **equilibrium**, the **reaction rate is zero** ( $-r_A = 0$ ).

Therefore: As  $x \rightarrow x_e$ ,  $-r_A \rightarrow 0$

Thus  $\frac{1}{-r_A} \rightarrow \infty$ , and therefore  $V \rightarrow \infty$

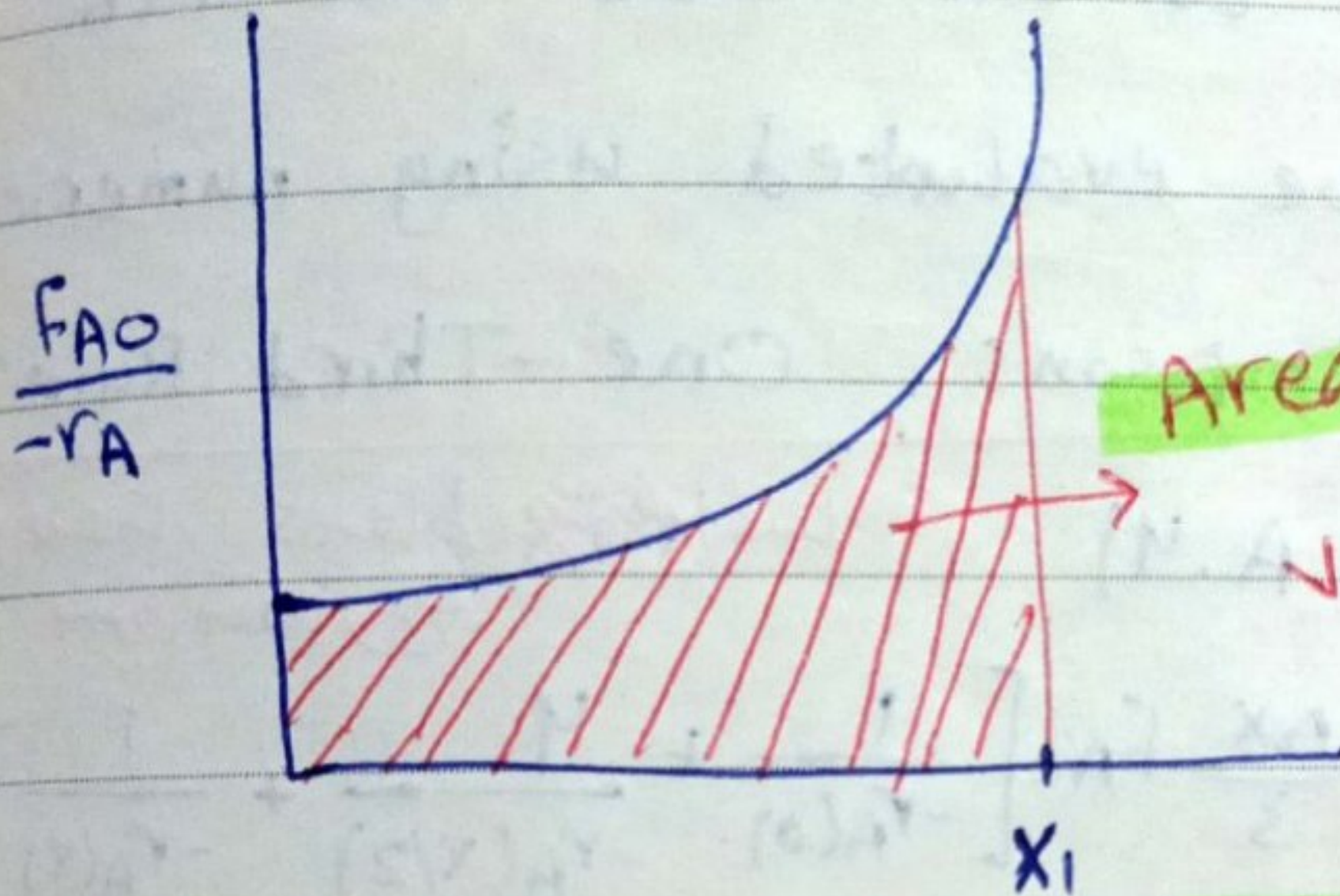
**CSTR**

$$V = \frac{F_{A0} x}{-r_A} \rightarrow V = \left( \frac{F_{A0}}{-r_A} \right)_{x_1} x_1$$



**PFR**

$$V = \int_0^X \frac{FA_0 dx}{-r_A}$$



Area = Volume of PFR

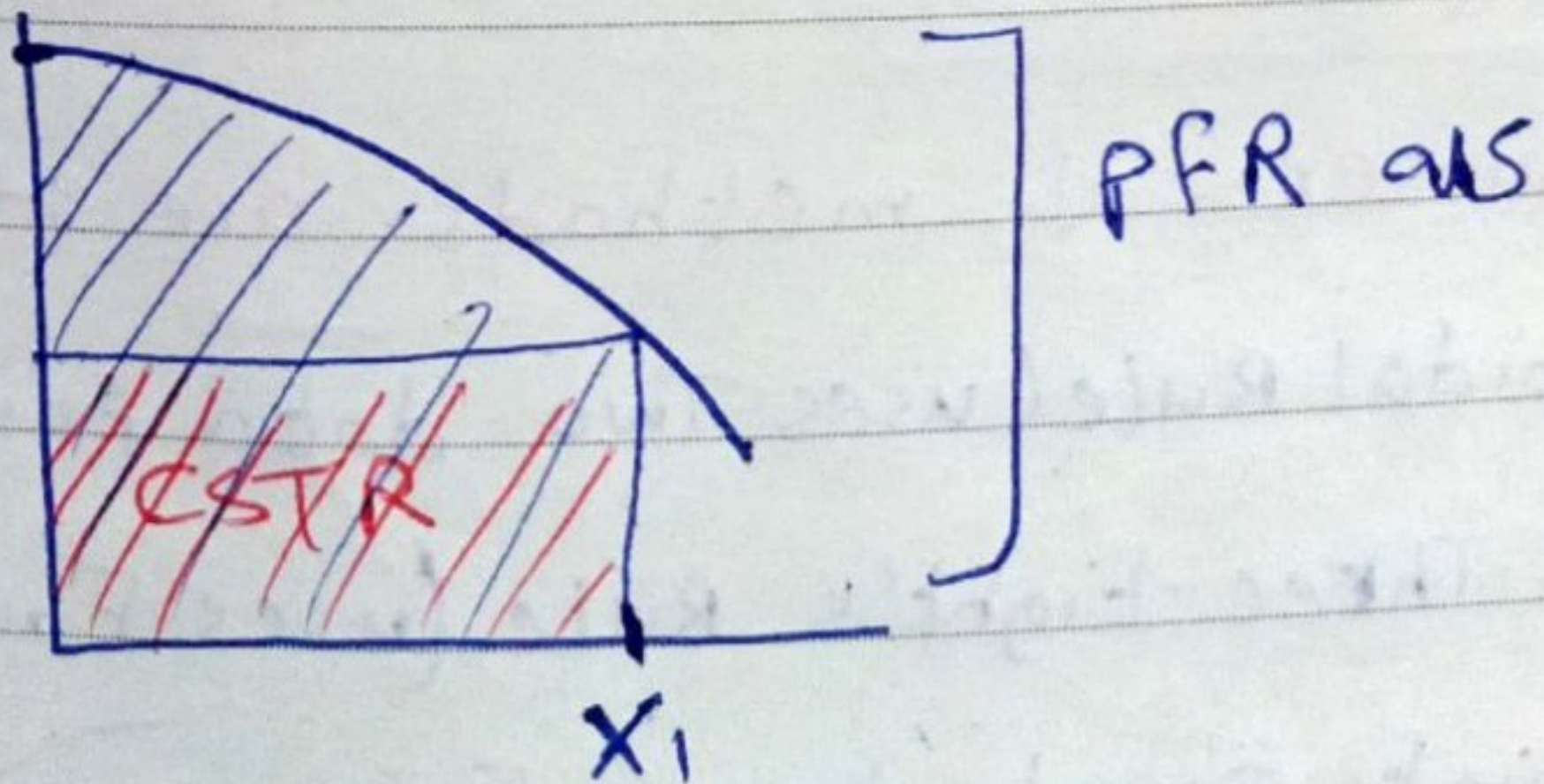
$$V = \int_0^{X_1} \left( \frac{FA_0}{-r_A} \right) dx$$

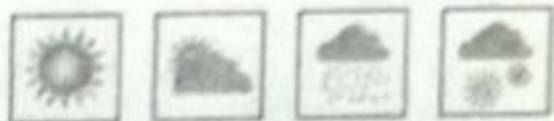
- Integration → Area under the curve

\* order  $\geq 1 \rightarrow V_{CSTR} > V_{PFR}$

\* order of rxn  $0 \leq < 1$

يكون حجم PFR اكبر من حجم CSTR





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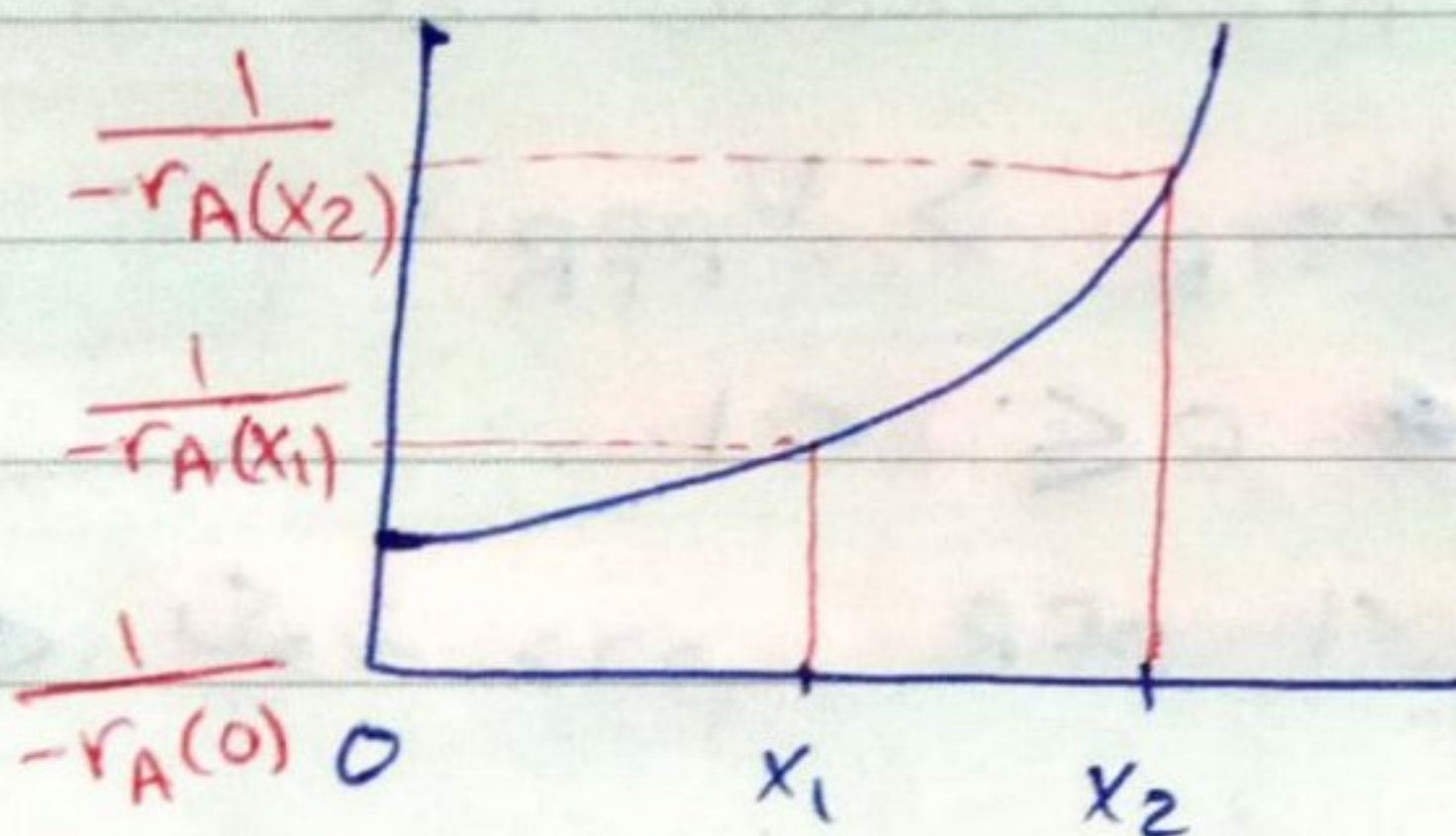
## \* Numerical Evaluations of Integrals

- The integral to calculate the PFR volume can be evaluated using numerical method as Simpson's One-Third Rule:

(See Appendix A.4)

بنتجا، 3 نقاط  
بشيم متساوية  
المساواة

$$V = \int_0^x \frac{F_{A0}}{-r_A} dx = \frac{\Delta x}{3} F_{A0} \left[ \frac{1}{-r_A(0)} + \frac{4}{-r_A(x/2)} + \frac{1}{-r_A(x)} \right]$$

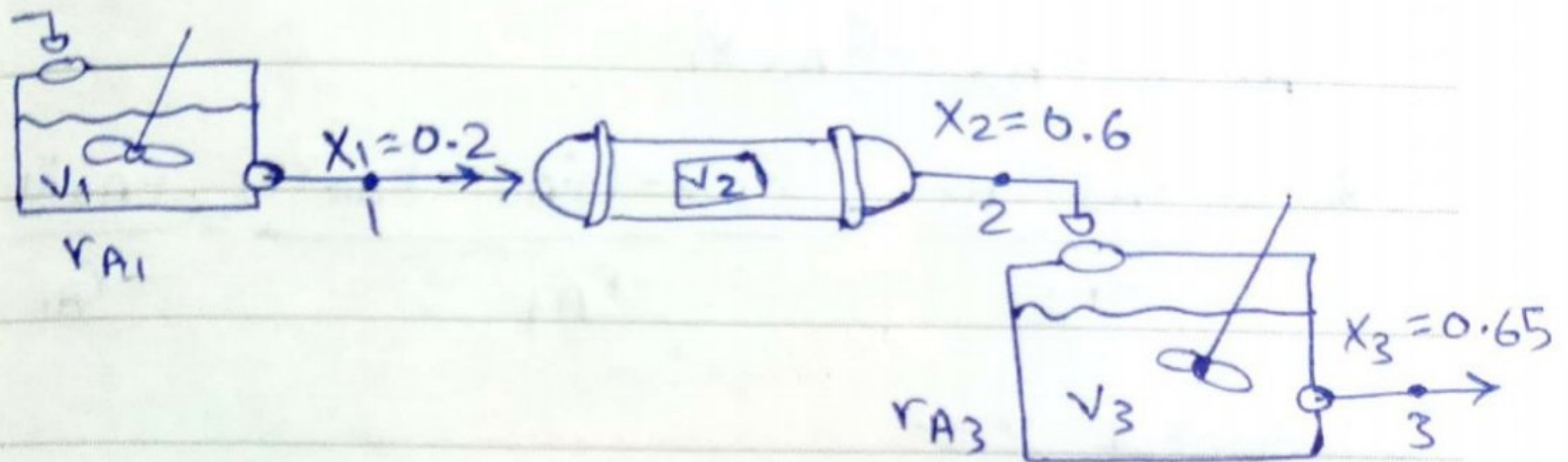


- Other numerical methods are :-

- Trapezoidal Rule (uses Two data points).
- Simpson's Three-Eight's Rule (uses Four data points)
- Five-point Quadrature Formula.



## \* Reactors in series



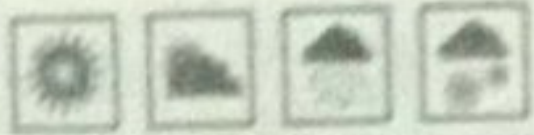
- Given:  $-r_A$  as a function of conversion, one can also design any sequence of reactors in series by defining  $x_i$ :

$x_i = \frac{\text{total moles of A reacted up to point } i}{\text{moles of A fed to first reactor.}}$

Only valid if there are no side streams. Inlet stream (يعني اذا ما في Inlet stream كافيّة).

Molar Flow rate of species A at point i:

$$F_{Ai} = F_{Ao} - F_{Ao} x_i$$



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\* Reactor 1 :-

$$F_{A1} = F_{A0} - F_{A0} X_1$$

$$V_1 = \frac{F_{A0} - F_{A1}}{-r_{A1}} = \frac{F_{A0} - (F_{A0} - F_{A0} X_1)}{-r_{A1}} = \frac{F_{A0} X_1}{-r_{A1}}$$

\* Reactor 2 :-

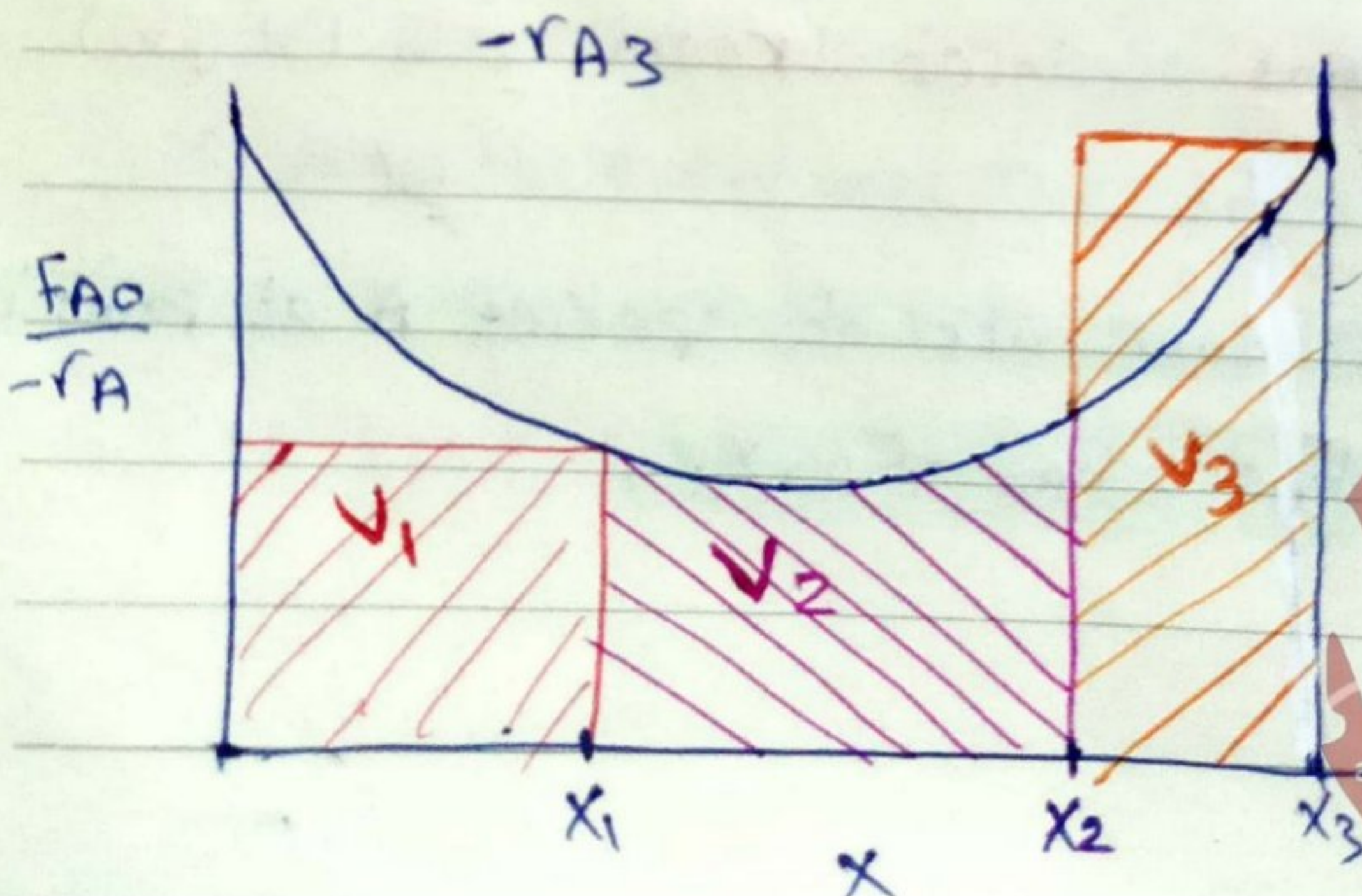
$$V_2 = \int_{X_1}^{X_2} \frac{F_{A0}}{-r_A} dx$$

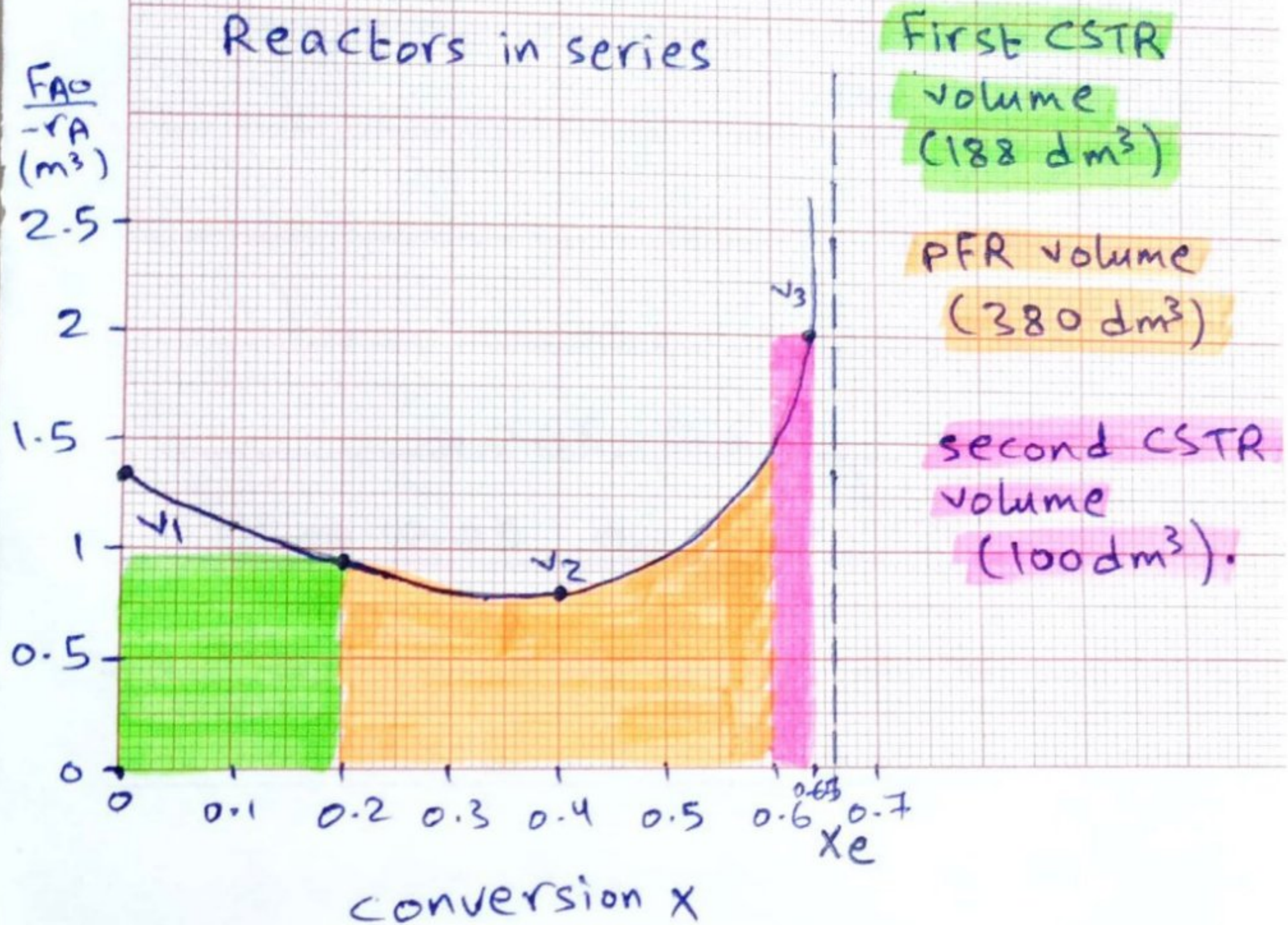
\* Reactor 3 :-

$$F_{A2} - F_{A3} + r_{A3} V_3 = 0$$

$$(F_{A0} - F_{A0} X_2) - (F_{A0} - F_{A0} X_3) + r_{A3} V_3 = 0$$

$$V_3 = \frac{F_{A0} (X_3 - X_2)}{-r_{A3}}$$



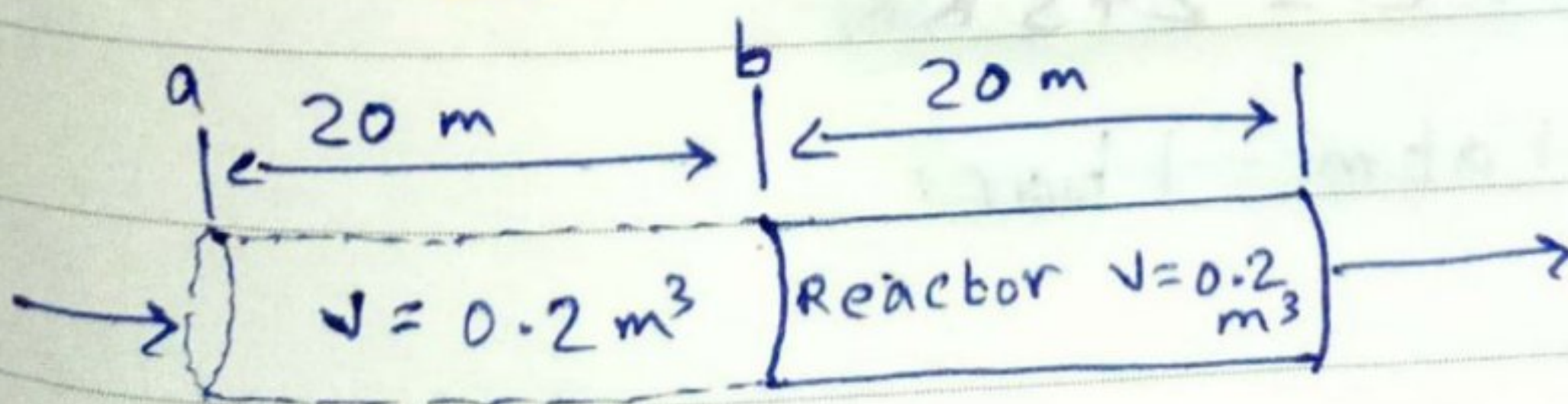


\* Space Time ( $\tau$ ) "residence time".

→ is the time necessary to process one reactor volume of fluid at entrance conditions.

$$\tau = \frac{V}{V_0}$$

$V \rightarrow$  volume of the reactor.  
 $V_0 \rightarrow$  volumetric flow rate.





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### \* Space Velocity :-

$$SV = \frac{V_0}{V}, \quad SV = \frac{1}{\tau}$$

• Two space velocities commonly used in industry are :-

- liquid-hourly space velocity, LHSV

$$LHSV = \frac{V_{0, \text{liquid}}}{V}$$

- gas-hourly space velocity, GHSV.

$$GHSV = \frac{V_{0, \text{STP}}}{V}$$

\* STP  $\rightarrow$  standard T and P.

$$T = 0^\circ\text{C} = 273\text{K}$$

$$P = 1\text{atm} = 1\text{bar}.$$

$$V_m = \frac{V}{n} = 22.41 \text{ L/mol} \\ \text{dm}^3/\text{mol}.$$

## ch③ Rate Laws and Stoichiometry.

### \*Types of Reactions:-

→ Homogeneous reaction :- involves only one phase.

→ Heterogeneous reaction :- involves more than one phase, and the reaction usually occurs at the interface between the phases.

→ irreversible reaction :- proceeds in only one direction and continues in that direction until the reactants are exhausted.

→ reversible reaction :- can proceed in either direction, depending on the concentrations of reactants and products relative to the corresponding equilibrium concentrations.



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rate of reaction = 0  
(نقطة التوقف)

→ reactants are exhausted.  
→ reach to equilibrium concentration.

colloide between molecules.  
تفاعل الجزيئات

### \* Molecularity of Reaction

- The molecularity of a reaction is the number of atoms, ions, or molecule involved (colliding) in a reaction step.
- The terms unimolecular, bimolecular and termolecular refer to reactions involving, respectively, one, two, or three atoms (or molecules) interacting or colliding in any one reaction step.



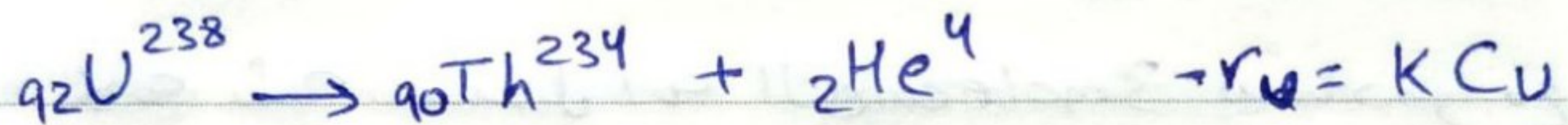


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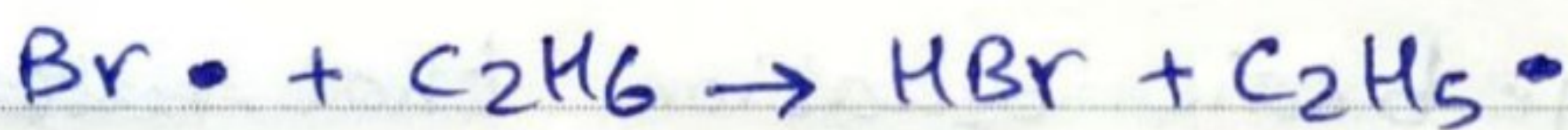
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- Most common unimolecular reaction examples

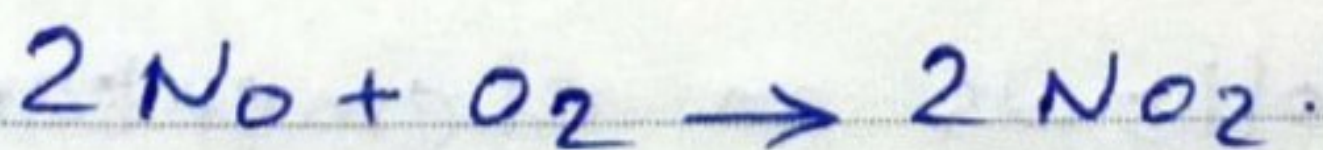


- The True bimolecular reactions that exist are reactions involving Free radical, such as



$$-r_{\text{Br} \cdot} = k C_{\text{Br} \cdot} C_{\text{C}_2\text{H}_6}$$

- probability of a termolecular reaction occurring is almost nonexistent, and in most instance the reaction pathway follows a series of bimolecular reactions as in the case of the reaction.





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- تفاعلات ال termolecular نقرىبا من موجودة  
لأنه شبه مستحيل أن ال 3 molecules يتصادم مع  
بعض في نفس اللحظة لتكوين ال product .

### Rate Laws

\* Rate Law (Kinetic expression) :-

→ the algebraic equation that ~~states~~ relates  $-r_A$  to the species concentrations<sup>is:-</sup>

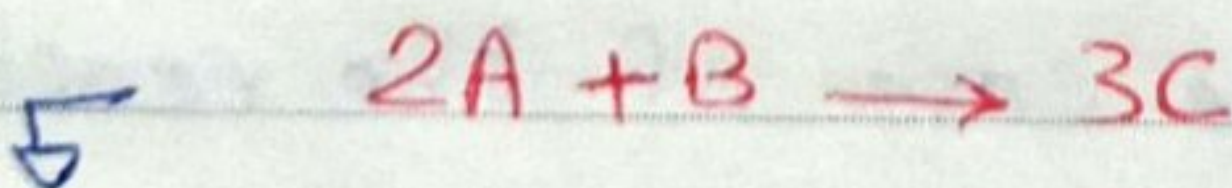
• power Law model:-

$$-r_A = k C_A^\alpha C_B^\beta$$

$\alpha$  order in A

$\beta$  order in B

overall Reaction order :  $n = \alpha + \beta$



- A reaction Follows an elementary rate Law if the reaction orders just

ask ; believe & recieve



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happens to agree with the stoichiometric coefficients for the reaction as written.

- e.g. If the above reaction follows an elementary rate law

$-r_A = k_A C_A^2 C_B \rightarrow 2^{\text{nd}}$  order in A,  
 $1^{\text{st}}$  order in B, overall  $3^{\text{rd}}$  order.

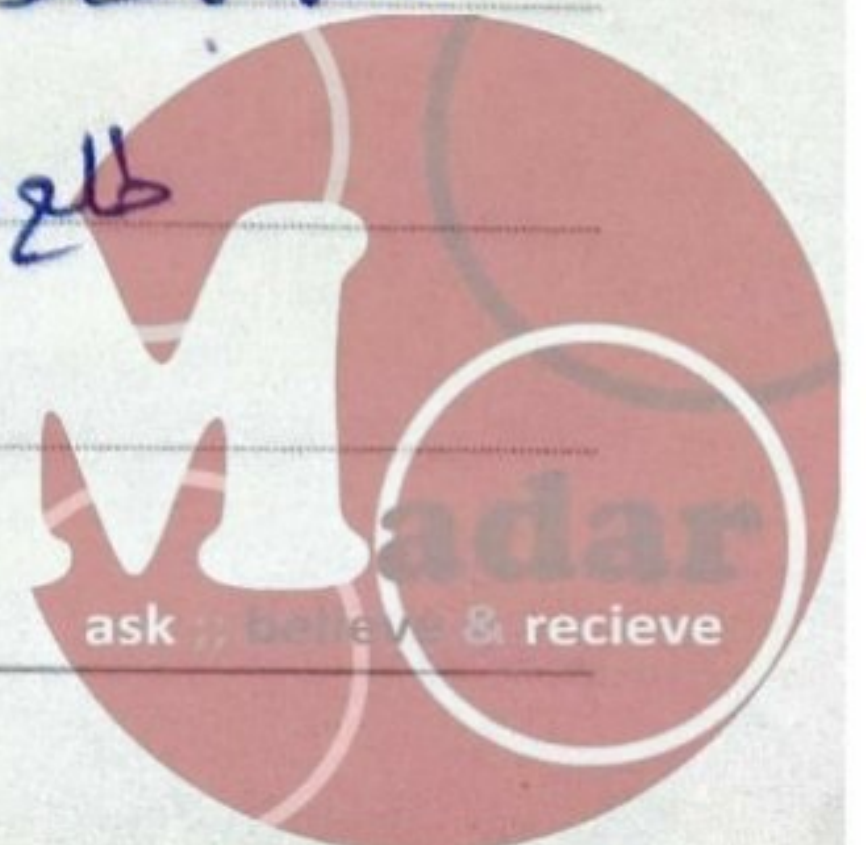
- Rate Laws are determined by experimental observation.

\* Elementary  $\rightarrow$  occurs in one step.

(order = stoichiometry) rate  $\rightarrow$   $\text{ويعبر الكتاب قانون ال rate}$

\* Follows Elementary  $\rightarrow$  doesn't occur in one step. (not Elementary).

لكن بالسرعة (بالسرعة كما اوجدنا قانون ال Rate)  
 $\rightarrow$   $\text{طاعة ال (order = stoichiometry)}$





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- Rate Laws could be non-elementary.

- For example, reaction could be :-

• second order in A  $-r_A = k_A C_A^2$

• zero order in B  $-r_B = k_B C_A^2$

• overall second order  $r_C = k_C C_A^2$

$k \rightarrow$  اذا ما حورد هي كمية

① اذا كل  $stoichi. = 1$  فهي بتكونه للكل

②  $stoichi.$  مختلفه بتكونه (L.R)  $\rightarrow$  Limiting reactant.



## **4.1** stoichiometry (constant volume Batch and Flow systems):-

\* How to find  $-r_A = f(x)$

step 1 → Rate Law  $-r_A = g(c_i)$

step 2 → stoichiometry  $(c_i) = h(x)$

step 3 → combine to get  $-r_A = f(x)$

\* **stoichiometry** :-

- We shall set up stoichiometry Tables using species A as our basis of calculation in the following reaction.
- We will use the stoichiometric tables to express the concentration as a function of conversion.
- We will combine  $c_i = f(x)$  with the appropriate rate law to obtain  $-r_A = f(x)$ .

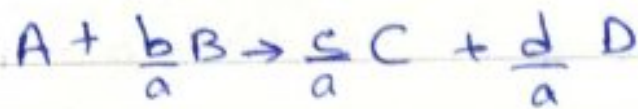




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A  $\rightarrow$  limiting reactant.

$$N_A = N_{A0} - N_{A0} X = N_{A0} (1 - X)$$

For every mole of A that reacts, (b/a) moles of B react. Therefore moles of B remaining:-

$$N_B = N_{B0} - \frac{b}{a} N_{A0} X = N_{A0} \left( \frac{N_{B0}}{N_{A0}} - \frac{b}{a} X \right)$$

Let  $\Theta_B = \frac{N_{B0}}{N_{A0}}$

Then:

$$N_B = N_{A0} \left( \Theta_B - \frac{b}{a} X \right)$$

$$N_C = N_{C0} + \frac{c}{a} N_{A0} X = N_{A0} \left( \Theta_C + \frac{c}{a} X \right)$$

$$\Theta_C = \frac{N_{C0}}{N_{A0}}$$

الطبعة 4 - Table (3-3) P. 101

Page.

4th Edition.



For liquid.

$$\Theta_i = \frac{N_{i0}}{N_{A0}} = \frac{C_{i0}V_0}{C_{A0}V_0} = \frac{C_{i0}}{C_{A0}} = \frac{y_{i0}}{y_{A0}}$$

\* Stoichiometry Constant volume

Batch System

- Note 1 (1) IF the reaction occurs in the liquid phase.

(2) IF a gas phase reaction occurs in a const. volume rigid (e.g. steel) batch reactor.

Then  $V = V_0$  <sup>contraction or expansion</sup> (2, b)

$$C_A = \frac{N_A}{V} = \frac{N_{A0}(1-x)}{V_0} = C_{A0}(1-x)$$

$$C_B = \frac{N_B}{V} = \frac{N_{A0}}{V_0} \left( \Theta_B - \frac{b}{a}x \right) = C_{A0} \left( \Theta_B - \frac{b}{a}x \right)$$

طبقة على كل ال species في ال reaction





suppose  $-r_A = k_A C_A^2 C_B$

Batch:  $V = V_0$

$$-r_A = k_A C_{A0}^2 (1-x)^2 C_{A0} \left( \Theta_B - \frac{b}{a}x \right)$$

$$-r_A = k_A C_{A0}^3 (1-x)^2 \left( \Theta_B - \frac{b}{a}x \right)$$

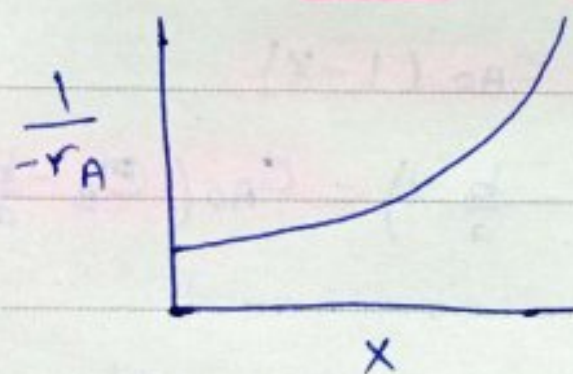
\* Equimolar Feed  $\rightarrow \Theta_B = 1$

\* stoichiometric Feed  $\rightarrow \Theta_B = \frac{b}{a}$

constant volume batch.

and we have  $-r_A = f(x)$

$x \uparrow$ ,  $-r_A \downarrow$ ,  $\frac{1}{-r_A} \uparrow$



## \*Batch Reactor - Equilibrium conversion - Example

calculate the equilibrium conversion for gas phase reaction,  $X_e$

- consider the following elementary reaction with  $K_c = 20 \text{ dm}^3/\text{mol}$  and  $C_{A0} = 0.2 \text{ mol/dm}^3$ .

Find  $X_e$  for a constant volume batch reactor.



$$-r_A = K_A \left[ C_A^2 - \frac{C_B}{K_c} \right]$$

**Solution:-** step 1:- "mole balance"

$$\frac{dX}{dt} = \frac{-r_A V}{N_{A0}}$$

step 2: rate Law  $-r_A = K_A C_A^2 - K_B C_B$

$$-r_A = K_A \left[ C_A^2 - \frac{C_B}{K_c} \right]$$

rate const. of Forward  $\rightarrow K_c = \frac{K_A}{K_B}$

" " " backward





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| <u>symbol</u> | <u>Initial</u>    | <u>Initial+change</u>               | <u>Remaining</u>          |
|---------------|-------------------|-------------------------------------|---------------------------|
| A             | $N_{A0}$          | $-N_{A0}X$                          | $N_{A0}(1-X)$             |
| B             | 0                 | $\frac{1}{2}N_{A0}X$                | $N_{A0} \frac{X}{2}$      |
| Totals:       | $N_{T0} = N_{A0}$ | $N_T = N_{A0} - N_{A0} \frac{X}{2}$ | $\Sigma \text{remaining}$ |

@ equilibrium :-  $-r_A = 0$   $0 = C_{Ae}^2 - \frac{C_{Be}}{K_c}$   
 stoichiometry :  $A \rightarrow B/2$  equi.

constant volume:  $V = V_0$

$$C_{Ae} = \frac{N_{Ae}}{V} = C_{A0}(1-X_e), \quad C_{Be} = C_{A0} \frac{X_e}{2}$$

$$K_c = \frac{C_{Be}}{C_{Ae}^2} = \frac{C_{A0} X_e / 2}{[C_{A0}(1-X_e)]^2} = \frac{X_e}{2 C_{A0} (1-X_e)^2}$$

$$2 K_c C_{A0} = \frac{X_e}{(1-X_e)^2} = (2)(20)(0.2) = 8$$

$X_{eb} = 0.703$   $\rightarrow$  max. conversion under these conditions.

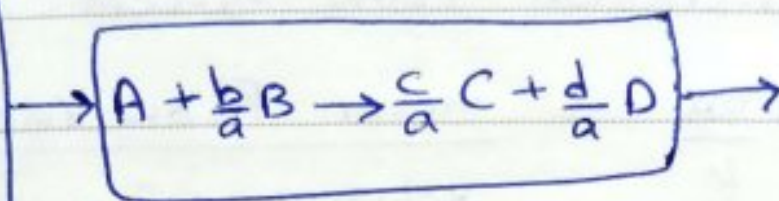


# \*Flow system

## stoichiometry Table.

Entering

$F_{A0}$   
 $F_{B0}$   
 $F_{C0}$   
 $F_{D0}$   
 $F_{I0}$



Leaving

$F_A$   
 $F_B$   
 $F_C$   
 $F_D$   
 $F_I$

| Species | symbol | Reactor Feed               | change                 | Reactor Effluent                        |
|---------|--------|----------------------------|------------------------|-----------------------------------------|
| A       | A      | $F_{A0}$                   | $-F_{A0}X$             | $F_A = F_{A0}(1-X)$                     |
| B       | B      | $F_{B0} = F_{A0} \Theta_B$ | $-\frac{b}{a} F_{A0}X$ | $F_B = F_{A0}(\Theta_B - \frac{b}{a}X)$ |

where  $\Theta_i = \frac{F_{i0}}{F_{A0}} = \frac{C_{i0}V_0}{C_{A0}V_0} = \frac{C_{i0}}{C_{A0}} = \frac{y_{i0}}{y_{A0}}$

|   |   |                            |                       |                                         |
|---|---|----------------------------|-----------------------|-----------------------------------------|
| C | C | $F_{C0} = F_{A0} \Theta_C$ | $\frac{c}{a} F_{A0}X$ | $F_C = F_{A0}(\Theta_C + \frac{c}{a}X)$ |
| D | D | $F_{D0} = F_{A0} \Theta_D$ | $\frac{d}{a} F_{A0}X$ | $F_D = F_{A0}(\Theta_D + \frac{d}{a}X)$ |

Inert I  $F_{I0} = F_{A0} \Theta_I$   $F_I = F_{A0} \Theta_I$

$F_{T0}$

$F_T = F_{T0} + \delta F_{A0}X$





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where:

$$\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$$

→  $\sum \text{stoi. of (product - reactant)}$

concentration - Flow system

$$C_A = \frac{F_A}{V} \equiv \frac{\text{molar Flow rate}}{\text{volumetric Flow rate}}$$

\* stoichiometry

- concentration Flow system:  $C_A = \frac{F_A}{V}$

- Liquid phase Flow system:  $V = V_0$

$$C_A = \frac{F_A}{V} = \frac{F_{A0}(1-X)}{V_0} = C_{A0}(1-X)$$

$$C_B = \frac{N_B}{V} = \frac{N_{A0}}{V_0} \left( \Theta_B - \frac{b}{a} X \right) = C_{A0} \left( \Theta_B - \frac{b}{a} X \right)$$



## \* Liquid Systems:-

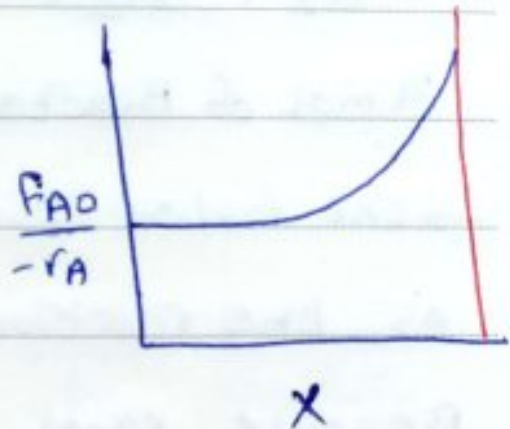
If the rate of reaction were  $-r_A = K C_A C_B$

then we would have

$$-r_A = C_{A0}^2 (1-x) \left( \Theta_B - \frac{b}{a} x \right)$$

This gives us

$$-r_A = f(x)$$



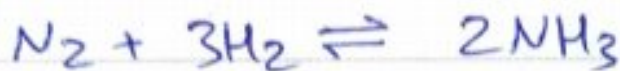


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## 4.2 Stoichiometry (Gas phase flow systems).



4 mol of Reactants gives 2 mol of product.

→ the molar Flow rate will be changing as the reaction progresses.

Because equal numbers of moles occupy equal volumes in the gas phase at the same T and p.

→ the volumetric Flow rate will also change.

- العلاقة بين  $(V, n)$  طردية  $\propto$  عند  $P, T$  ثابتة.

- Combining the compressibility factor equation of state with  $z = z_0$



stoichiometry:

$$C_T = \frac{P}{ZRT}$$

$$C_{T0} = \frac{P_0}{Z_0 R_0 T_0}$$

out  $\rightarrow F_T = C_T V$

in  $\rightarrow F_{T0} = C_{T0} V_0$

$$\rightarrow V = \frac{F_T}{C_T} = \frac{F_T ZRT}{P} = \frac{F_T \cancel{R} T}{P} \frac{P_0}{C_{T0} \cancel{R_0} T_0}$$

we obtain

$$V = V_0 \frac{F_T}{F_{T0}} \frac{P_0}{P} \frac{T}{T_0}$$

$$C_A = \frac{F_A}{V} = \frac{F_A}{V_0 \left( \frac{F_T}{F_{T0}} \right) \frac{P_0}{P} \frac{T}{T_0}} = \frac{F_{T0}}{V_0} \frac{F_A}{F_T} \frac{P}{P_0} \frac{T_0}{T}$$

since  $C_{T0} = F_{T0} / V_0$

$$C_A = \frac{F_A}{V} = C_{T0} \frac{F_A}{F_T} \frac{P}{P_0} \frac{T_0}{T}$$

- using the same method,





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$$C_B = C_{T0} \frac{F_B}{F_T} \frac{P}{P_0} \frac{T_0}{T}$$

\* The total molar Flow rate is:-

$$F_T = F_{T0} + F_{A0} \delta X$$

substituting  $F_T$  gives:  $v = v_0 \frac{F_T}{F_{T0}} \frac{P_0}{P} \frac{T_0}{T}$

$$v = v_0 \left( \frac{F_{T0} + F_{A0} \delta X}{F_{T0}} \right) \frac{T_0}{T} \frac{P_0}{P} = v_0 \left( 1 + \frac{F_{A0} \delta X}{F_{T0}} \right) \frac{T_0}{T} \frac{P_0}{P}$$

$$v = v_0 (1 + y_{A0} \delta X) \frac{T_0}{T} \frac{P_0}{P} = v_0 (1 + \epsilon X) \frac{T_0}{T} \frac{P_0}{P}$$

where  $\epsilon = y_{A0} \delta$ .

Concentration Flow system:  $C_A = \frac{F_A}{v}$



Gas phase Flow system:  $V = V_0(1 + \epsilon X) \frac{T}{T_0} \frac{P_0}{P}$

→ L.R

$$C_A = \frac{F_A}{V} = \frac{F_{A0}(1-X)}{V_0(1+\epsilon X) \frac{T}{T_0} \frac{P_0}{P}} = C_{A0} \frac{(1-X)}{(1+\epsilon X)} \frac{T_0}{T} \frac{P}{P_0}$$

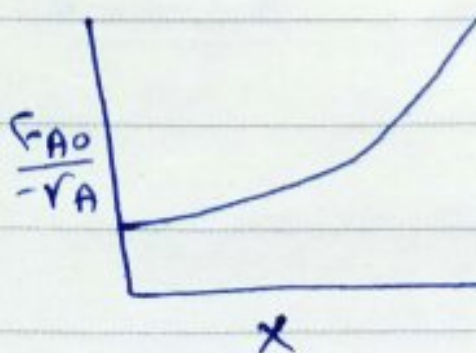
$$C_B = \frac{F_B}{V} = \frac{F_{A0}(\Theta_B - \frac{b}{a}X)}{V_0(1+\epsilon X) \frac{T}{T_0} \frac{P_0}{P}} = C_{A0} \frac{(\Theta_B - \frac{b}{a}X)}{(1+\epsilon X)} \frac{T_0}{T} \frac{P}{P_0}$$

→ For any species.

IF  $-r_A = K C_A C_B$

$$-r_A = K C_{A0}^2 \left[ \frac{(1-X)}{(1+\epsilon X)} \frac{(\Theta_B - \frac{b}{a}X)}{(1+\epsilon X)} \left( \frac{P}{P_0} \frac{T_0}{T} \right)^2 \right]$$

This gives us





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where

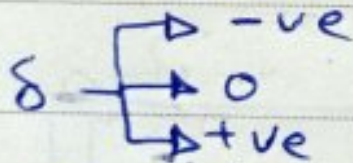
$$\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$$

$\delta = \frac{\text{change in total number of moles}}{\text{moles of A reacted}}$

$$\varepsilon = \left( \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) \frac{F_{A0}}{F_{T0}} = y_{A0} \delta$$

conversion

$\varepsilon = \frac{\text{change in total number of moles For complete}}{\text{total number of moles fed to the reactor.}}$



- في فرق بينة  $x_e$  في ال batch أو Flow



## \*Equilibrium Conversion ( $X_{ef}$ ) For Gas phase Flow Reaction System.

- Example :- "change in volume consideration في الحجم"

Consider the following elementary reaction



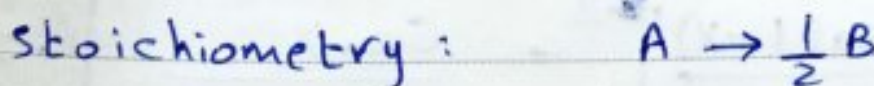
$$-r_A = k_A \left[ C_A^2 - \frac{C_B}{K_c} \right]$$

$K_c = 20 \text{ dm}^3/\text{mol}$  and  $C_{A0} = 0.2 \text{ mol/dm}^3$ .

- calculate Equilibrium Conversion For an isothermal isobaric Flow reactor ( $X_{ef}$ )??

- Solution :-

Rate Law :  $-r_A = k_A \left[ C_A^2 - \frac{C_B}{K_c} \right]$





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| species | Feed              | change       | Effluent                   |
|---------|-------------------|--------------|----------------------------|
| A       | $F_{A0}$          | $-F_{A0}X$   | $F_A = F_{A0}(1-X)$        |
| B       | 0                 | $+F_{A0}X/2$ | $F_B = F_{A0}X/2$          |
| Total   | $F_{T0} = F_{A0}$ |              | $F_T = F_{A0} - F_{A0}X/2$ |

Stoichiometry : Gas isothermal  $T = T_0$

Gas isobaric  $P = P_0$

$$V = V_0(1 + \epsilon X)$$

$$C_A = \frac{F_{A0}(1-X)}{V_0(1+\epsilon X)} = \frac{C_{A0}(1-X)}{1+\epsilon X}$$

$$C_B = \frac{F_B}{V} = \frac{F_{A0}X/2}{V_0(1+\epsilon X)} = \frac{C_{A0}X}{2(1+\epsilon X)}$$

$$-r_A = k_A \left[ \left( \frac{C_{A0}(1-X)}{1+\epsilon X} \right)^2 - \frac{C_{A0}X}{2(1+\epsilon X)K_c} \right]$$

pure A  $\rightarrow y_{A0} = 1$





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$$C_{A0} = y_{A0} P / RT_0 \rightarrow C_{A0} = P_0 / RT_0$$

$$\varepsilon = y_{A0} \delta = (1) \left( \frac{1}{2} - 1 \right) = -\frac{1}{2}$$

At equilibrium:  $-r_A = 0$

$$2K_c C_{A0} = \frac{x_e (1 + \varepsilon x_e)}{(1 - x_e)^2}, \quad \varepsilon = -\frac{1}{2}$$

$$2K_c C_{A0} = 2 \times \frac{20 \text{ dm}^3}{\text{mol}} \times \frac{0.2 \text{ mol}}{\text{dm}^3} = 8$$

$$8 = \frac{x_e - 0.5 x_e^2}{1 - 2x_e + x_e^2} \rightarrow 8.5 x_e^2 - 17 x_e + 8 = 0$$

$$x_{ef} = 0.757$$

For Batch system:

$$x_{eb} = 0.703$$

$\varepsilon$   $\begin{cases} \rightarrow -ve, \\ \rightarrow 0, \\ \rightarrow +ve. \end{cases}$

consideration      يختلف عن بعض الحالات في

change in volume.





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## \* Design of Continuous stirred Tank Reactors (CSTRs)

- Typically used For Liquid - phase reactions

$$V = \frac{F_{A0}X}{(-r_A)_{\text{exit}}}, \quad \tau = \frac{V}{V_0} = \frac{C_{A0}X}{-r_A}$$

### \* CSTRs in Parallel:-

- For equal sized reactors, and feed distributed equally:

$$V_i = \frac{F_{A0i} x_i}{-r_{Ai}}, \quad V_i = \frac{V}{n}$$

$$F_{A0i} = \frac{F_{A0}}{n}$$

$x_i = X$  and  $-r_{Ai} = -r_A$  (same in all reactors)

$$\rightarrow \boxed{V = \frac{F_{A0} X}{-r_A}}$$





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\* CSTRs in Series :-

$$V_1 = \frac{F_{A0} X_1}{-r_{A1}} \rightarrow C_{A1} = \frac{C_{A0}}{1 + k_1 \tau_1}$$

$$V_2 = \frac{F_{A0} (X_2 - X_1)}{-r_{A2}}$$

$$C_{A2} = \frac{C_{A1}}{1 + k_2 \tau_2} = \frac{C_{A0}}{(1 + k_1 \tau_1)(1 + k_2 \tau_2)}$$

• For equal sized reactors operating at the same T.

$$C_{An} = \frac{C_{A0}}{(1 + \tau k)^n} = \frac{C_{A0}}{(1 + Da)^n} \quad \text{and}$$

$$X = 1 - \frac{1}{(1 + Da)^n} = 1 - \frac{1}{(1 + \tau k)^n}$$





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## \* A single CSTR :-

• **Damköhler number (Da)** is a dimensionless number that can give us a quick estimate of the degree of conversion that can be achieved in **continuous flow reactors**.

if  **$Da < 0.1$** , then  **$X < 0.1$**

,  **$Da > 10$** , ,  **$X > 0.9$**

$$Da = \frac{-r_{A0}V}{F_{A0}}$$

- For **First order reaction**,  **$Da = \tau k$** .

- what is **Da** For a second order reaction?

• **First-order irreversible reaction**,

$$-r_A = kC_A$$

$$\tau = \frac{1}{k} \left( \frac{X}{1-X} \right) \rightarrow X = \frac{\tau k}{1 + \tau k}$$



$\tau_k = Da = \text{Damköhler number}$

• Damköhler number is the ratio of the rate of reaction of A to the rate of convective transport of A at the entrance to the reactor.

$$Da = \frac{-r_{A0} V}{F_{A0}} = \frac{\text{Rate of reaction at entrance}}{\text{Entering Flow rate of A}}$$



## 5.1 Isothermal Reactors Design

### \* Algorithm For Isothermal Reactor Design

1. Mole balances and Design Equation.

2. Rate Laws.

3. Stoichiometry.

4. Combine

5. Evaluate

- graphically.
- Numerically.
- Analytically.
- Software packages.

- الرجوع لصفحة [147] في الكتاب 4th Edition.

### \* Example ① :-

Liquid phase Undergraduate Laboratory Experiment





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## Feed

- Volumetric Flow rate  $v_0 = 0.0033 \text{ dm}^3/\text{s}$
- Acetic Anhydride 7.8% (1M)
- Water 92.2% (51.2M)
- Elementary with  $k' = 1.95 \times 10^4 \text{ dm}^3/(\text{mol}\cdot\text{s})$

Case 1 CSTR  $v = 1 \text{ dm}^3$

Case 2 PFR  $v = 0.311 \text{ dm}^3$

"موجود pdf و ال e-Learning"

و Example (2) أيضًا.



## 5.2 pressure Drop in PBR

Concentration Flow system:  $C_A = \frac{F_A}{\dot{V}}$

Gas phase Flow system  $\dot{V} = \dot{V}_0 (1 + \epsilon X) \frac{T}{T_0} \frac{P_0}{P}$

$$C_A = \frac{F_A}{\dot{V}} = \frac{F_{A0}(1-X)}{\dot{V}_0(1+\epsilon X) \frac{T}{T_0} \frac{P_0}{P}} = \frac{C_{A0}(1-X)}{1+\epsilon X} \frac{T_0}{T} \frac{P}{P_0}$$

$\frac{P}{P_0} \rightarrow$  تأثير على تركيز المتفاعلات والنواتج.

$$C_B = \frac{F_B}{\dot{V}} = \frac{F_{A0}(\theta_B - \frac{b}{a}X)}{\dot{V}_0(1+\epsilon X) \frac{T}{T_0} \frac{P_0}{P}} = \frac{C_{A0}(\theta_B - \frac{b}{a}X)}{1+\epsilon X} \frac{T_0}{T} \frac{P}{P_0}$$

Note:- press. drop doesn't affect Liquid phase reactions.

\* Sample Question:-

Analyze the Following second order gas phase reaction that occurs isothermally in a PBR:





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- Mole balance: Must use the differential form of the mole balance to separate variables:

$$F_{A0} \frac{dx}{dw} = -r_A'$$

- Rate laws: second order in A and irreversible

$$-r_A' = k C_A^2$$

- Stoichiometry

$$C_A = \frac{F_A}{V} = \frac{C_{A0}(1-x)}{(1+\epsilon x)} \frac{P}{P_0} \frac{T_0}{T}$$

Isothermal  $\rightarrow$   $T = T_0$

$$C_A = \frac{F_A}{V} = \frac{C_{A0}(1-x)}{(1+\epsilon x)} \frac{P}{P_0}$$

- Combine:-

$$\frac{dx}{dw} = \frac{k C_{A0}^2}{F_{A0}} \frac{(1-x)^2}{(1+\epsilon x)^2} \left(\frac{P}{P_0}\right)^2$$





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Need to find  $\frac{P}{P_0}$  as a function of  $W$   
(or  $v$  if you have a PFR).

$\frac{P}{P_0} \rightarrow$  (يحول في الـ PFR والـ PBR)

\* Ergun Equation:

$$\frac{dP}{dz} = \frac{-G}{\rho g_c D_p} \left( \frac{1-\phi}{\phi^3} \right) \left[ \frac{150(1-\phi)M}{D_p} + 1.75G \right]$$

LaminarTurbulent

$G \rightarrow$  mass Flux . ,  $\phi \rightarrow$  void

Constant mass Flow:

$$\dot{m} = \dot{m}_0 \rightarrow \rho = \rho_0 v_0 \rightarrow \rho = \rho_0 \frac{v_0}{v}$$

$$v = v_0 \frac{F_T}{F_{T0}} \frac{P_0}{P} \frac{T}{T_0} = v_0 (1 + \epsilon X) \frac{P_0}{P} \frac{T}{T_0}$$

\* Variable Density

$$\rho = \rho_0 \frac{P}{P_0} \frac{T_0}{T} \frac{F_{T0}}{F_T}$$





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\* in Gases.

\* variable density  $\rightarrow$  (change of  $\rho \rightarrow$  change of density)\*  $\rho \rightarrow f(T, P)$ 

$$\frac{dp}{dz} = \frac{-G}{\rho_0 g_c D_p} \left( \frac{1-\phi}{\phi^3} \right) \left[ \frac{150(1-\phi)M}{D_p} + 1.75G \right] \frac{P_0 T}{P T_0} \frac{F_T}{F_{T_0}}$$

Let:  $P_0 = \frac{G}{\rho_0 g_c D_p} \left( \frac{1-\phi}{\phi^3} \right) \left[ \frac{150(1-\phi)M}{D_p} + 1.75G \right]$

Catalyst Weight  $\rightarrow$  الوزن الكافى

$$W = Z A_c \rho_b = Z A_c (1-\phi) \rho_c$$

 $\rho_b$  = bulk density $\rho_c$  = solid catalyst density $\phi$  = porosity (void fraction) $(1-\phi)$  = solid fraction. $A_c \rightarrow$  cross sectional Area of the bed.



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$$\frac{dP}{dw} = \frac{-\beta_0}{A_c(1-\phi)\rho_c} \frac{P_0}{P} \frac{T}{T_0} \frac{F_T}{F_{T_0}}$$

$$\text{Let } \alpha = \frac{2\beta_0}{A_c(1-\phi)\rho_c} \frac{1}{P_0}$$

$$\frac{dy}{dw} = \frac{-\alpha}{2y} \frac{T}{T_0} \frac{F_T}{F_{T_0}}, \quad y = \frac{P}{P_0}$$

we will use this form for single reactions:

$$\frac{d(P/P_0)}{dw} = \frac{-\alpha}{2} \frac{1}{(P/P_0)} \frac{T}{T_0} (1 + \epsilon X)$$

$$\boxed{\frac{dy}{dw} = \frac{-\alpha}{2y} \frac{T}{T_0} (1 + \epsilon X)}$$

$$\boxed{\frac{dy}{dw} = \frac{-\alpha}{2y} (1 + \epsilon X)} \quad \text{Isothermal case.}$$



$$\frac{dx}{dw} = \frac{K_{CA_0}^2 (1-x)^2}{F_{A_0} (1+\epsilon x)^2} y^2$$

$$\frac{dx}{dw} = f(x, p) \text{ and } \frac{dp}{dw} = f(x, p) \text{ or } \frac{dy}{dw} = f(y, x)$$

→ For the special case of isothermal operations and  $\epsilon = 0$  we can obtain an analytical solution.

### - Packed bed Reactors

For  $\epsilon = 0$ , isothermal

$$\frac{dy}{dw} = \frac{-\alpha}{2y} (1 + \epsilon x)$$

when  $w = 0 \rightarrow y = 1 \xrightarrow{dx} (P_0 = P)$

$$dy^2 = -\alpha dw \rightarrow y^2 = 1 - \alpha w$$

$$y = (1 - \alpha w)^{\frac{1}{2}}$$



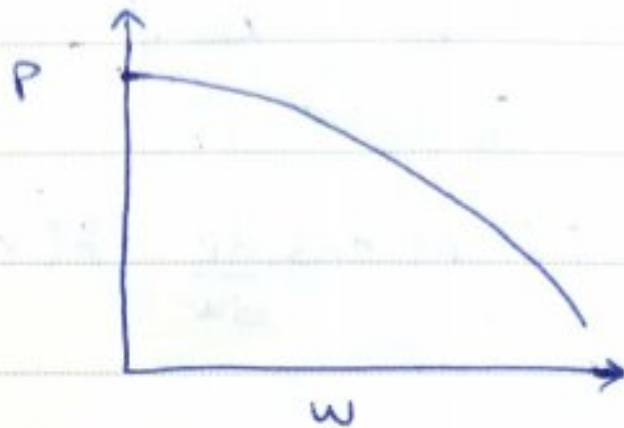


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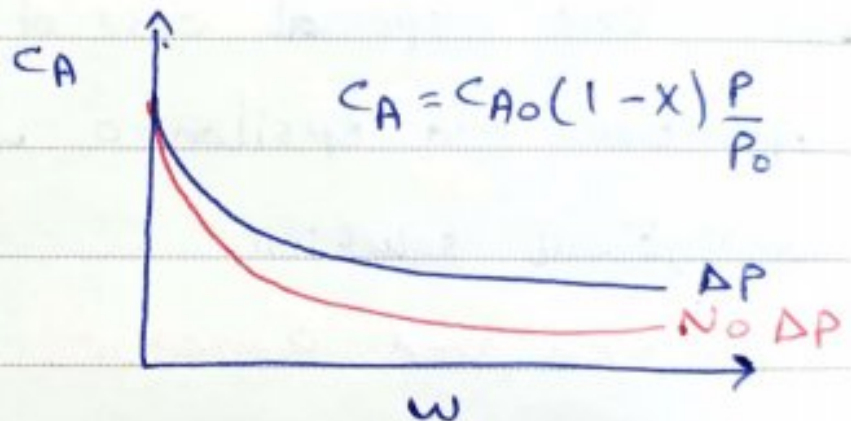
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① \* pressure drop  
in a PBR



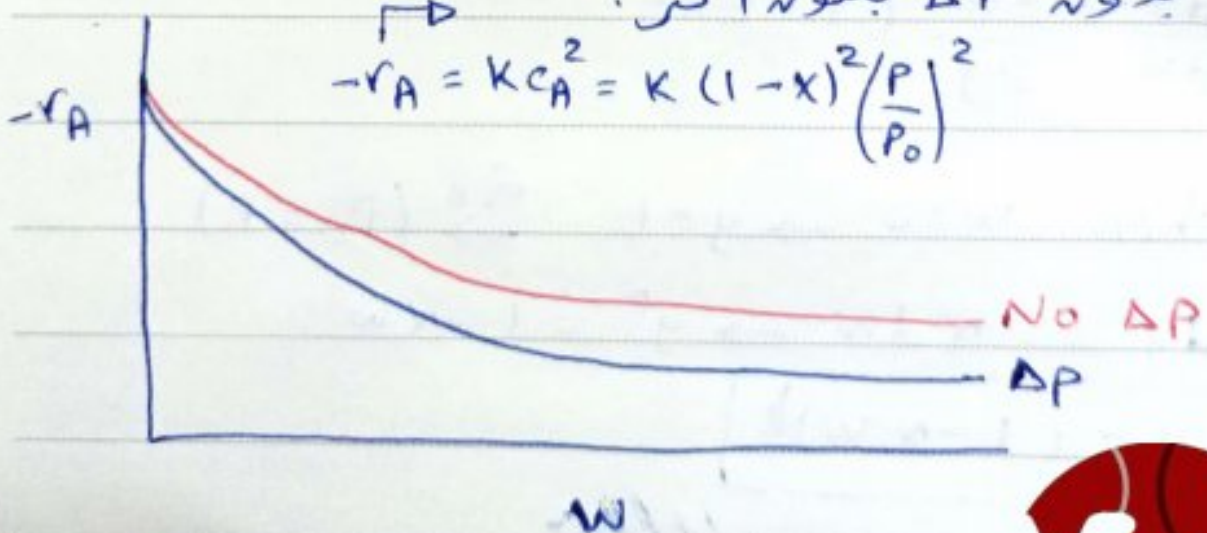
\* التركيز بالنهاية

بدون  $\Delta P$  يكون أقل  
منه لو في  $\Delta P$ .



② Concentration profile in a PBR.

بدون  $\Delta P$  يكون أكثر.

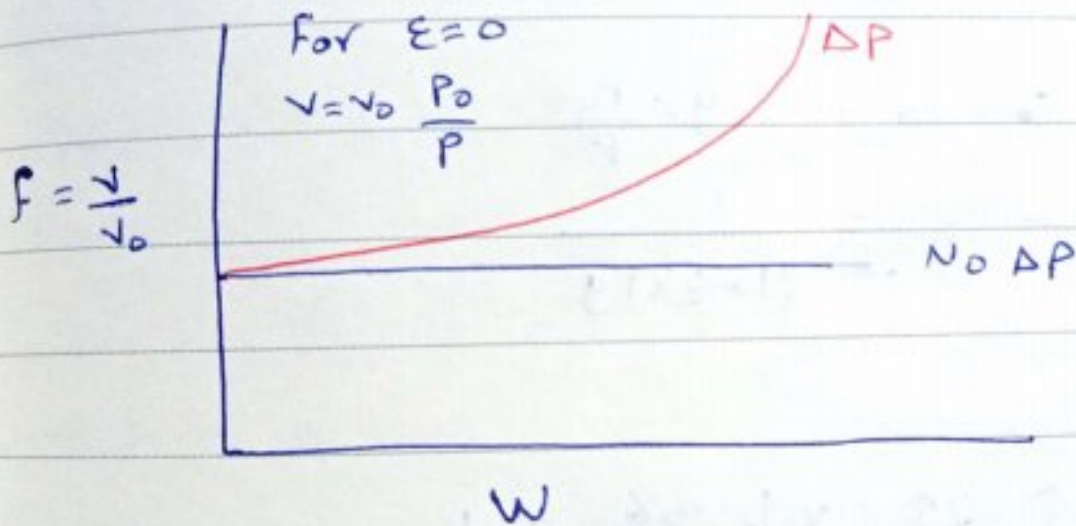
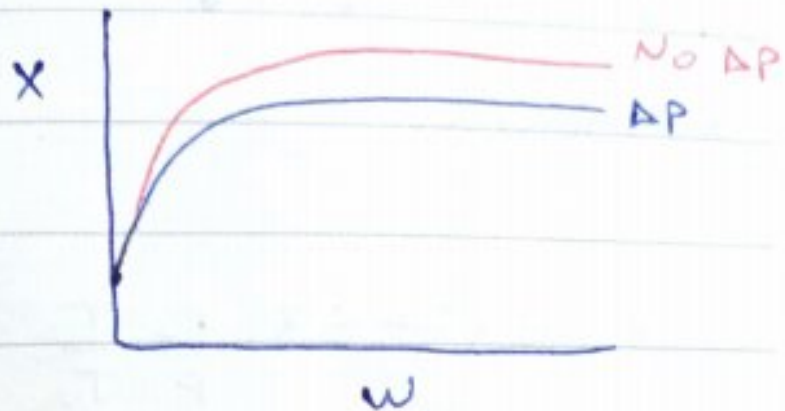


③ Reaction Rate in a PBR



#### ④ Conversion in a PBR.

- بدون  $\Delta P$  يكون أعلى.



#### ⑤ Flow Rate in a PBR.

- No  $\Delta P \rightarrow (\frac{v}{v_0} \rightarrow \text{constant})$ .

$\Delta P \rightarrow$  "إذا أي شيء ثابت"  $(P \downarrow \rightarrow v \uparrow)$

- كما يزداد الـ <sup>rate</sup> Volumetric Flow بقدر

Conversion قبل الـ Residence Time.





|    |    |    |    |    |    |    |
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 $P \downarrow \quad v \uparrow \quad \tau \downarrow \quad x \downarrow$ 

\* Summary :-

$$v = v_0 (1 + \epsilon x) \frac{P_0}{P} \frac{T}{T_0}$$

$$T = T_0 \quad y = \frac{P_0}{P}$$

$$f = \frac{v_0}{v} = \frac{1}{(1 + \epsilon x) y}$$

 $\epsilon \uparrow \quad v \uparrow \quad x \downarrow \quad \tau \downarrow$ 

### 5.3 Examples on pressure Drop.

- Example 1:- Gas phase Reaction in PBR For  $\delta=0$

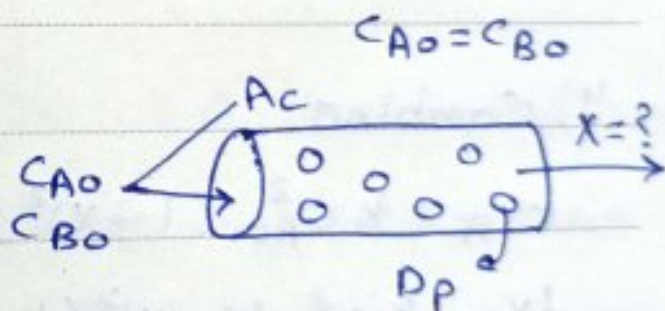
Gas phase reaction in PBR with  $\delta=0$   
(Analytical solution)  $A + B \rightarrow 2C$

Equimolar Feed of A and B and:

$$K_A = 1.5 \text{ dm}^3 / \text{mol} / \text{kg} / \text{min}$$

$$\alpha = 0.0099 \text{ kg}^{-1}$$

Find  $x$  at 100 kg



1) mole balance  $\frac{dx}{dw} = \frac{-r_A'}{F_{A0}}$

2] Rate Law  $-r_A' = K C_A C_B$





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3) stoichiometry  $C_A = C_{A0}(1-x)y$

$$C_B = C_{A0}(1-x)y$$

$$\frac{dy}{dw} = \frac{-\alpha}{2y}, \quad 2y dy = -\alpha dw$$

$$w=0, \quad y=1 \quad y^2 = 1 - \alpha w$$

$$y = (1 - \alpha w)^{\frac{1}{2}}$$

4) Combine

$$-r_A = k C_{A0}^2 (1-x)^2 y^2 = k C_{A0}^2 (1-x)^2 (1-\alpha w)$$

$$\frac{dx}{dw} = \frac{k C_{A0}^2 (1-x)^2 (1-\alpha w)}{F_{A0}}$$

$$\frac{dx}{(1-x)^2} = \frac{k C_{A0}^2 (1-\alpha w) dw}{F_{A0}}$$



$$\frac{X}{1-X} = \frac{k C_{A0}^2}{F_{A0}} \left( W - \frac{\alpha W^2}{2} \right)$$

@  $W=0$ ,  $X=0$  and @  $W=W$ ,  $X=X$

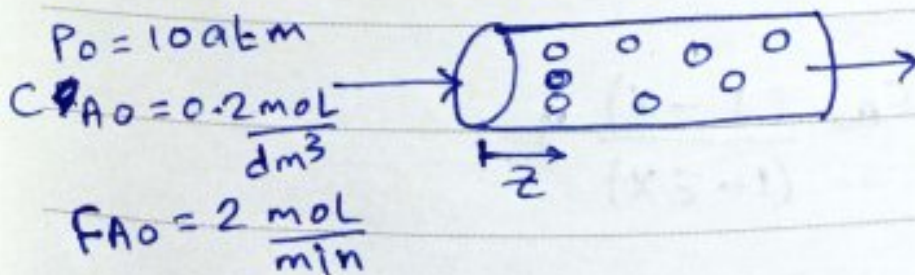
$X=0.6$  (with pressure drop).

$X=0.75$  (without pressure drop,  $\alpha=0$ ).

- لاحظ ان  $X$  اكثر بدون pressure drop.

Example (2) Gas phase Reaction in PBR for  $\delta \neq 0$ .

The reaction  $A + 2B \rightarrow C$  is carried out in a packed bed reactor in which there is pressure drop. The feed is stoichiometric in A and B.





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Plot the conversion and pressure ratio  $y = P/P_0$  as a function of catalyst weight up to 100 kg.

- Additional information

$$K_A = 6 \text{ dm}^3 / \text{mol}^2 / \text{kg} / \text{min}$$

$$\alpha = 0.02 \text{ kg}^{-1}$$



1) mole balance  $\frac{dx}{dw} = \frac{-r_A'}{F_{A0}}$

2) Rate Law  $-r_A' = K C_A C_B^2$

3) Stoichiometry : Gas, Isothermal

$$V = V_0 (1 + \epsilon X) \frac{P_0}{P}$$

$$C_A = C_{A0} \frac{(1 - X)}{(1 + \epsilon X)} y$$



$$4) C_B = C_{A0} \frac{(\theta_B - 2X)}{(1 + \epsilon X)} y$$

$$5) \frac{dy}{dw} = \frac{-\alpha}{2y} (1 + \epsilon X)$$

$$6) f = \frac{v}{v_0} = \frac{1 + \epsilon X}{y}$$

$$7) \epsilon = y_{A0} [1 - (1 - 2)] = \frac{1}{3} (-2) = -2/3$$

$$C_{A0} = 0.2, F_{A0} = 2, k = 6, \alpha = 0.02$$

Initial values  $w=0, x=0, y=1$

Final  $w=100$

Combine with polymath.

If  $\delta \neq 0$ , polymath must be used to solve.

• (في صورة لحل الـ Polymath)

• (رسم بياني (سلايد 60) +



# Example 2: Gas Phase Reaction in PBR for $\delta \neq 0$

## POLYMATH Results

POLYMATH Report 01-30-2006, Rev5.1.233

### Calculated values of the DEQ variables

| Variable | initial value | minimal value | maximal value | final value |
|----------|---------------|---------------|---------------|-------------|
| W        | 0             | 0             | 100           | 100         |
| X        | 0             | 0             | 0.8587763     | 0.8587763   |
| y        | 1             | 0.1148659     | 1             | 0.1148659   |
| eps      | -0.6666667    | -0.6666667    | -0.6666667    | -0.6666667  |
| Cao      | 0.2           | 0.2           | 0.2           | 0.2         |
| TheataB  | 2             | 2             | 2             | 2           |
| Cb       | 0.4           | 0.0151789     | 0.4           | 0.0151789   |
| Fao      | 2             | 2             | 2             | 2           |
| k        | 6             | 6             | 6             | 6           |
| Ca       | 0.2           | 0.0075895     | 0.2           | 0.0075895   |
| alpha    | 0.02          | 0.02          | 0.02          | 0.02        |
| ra       | -0.192        | -0.192        | -1.049E-05    | -1.049E-05  |

### ODE Report (RK45)

Differential equations as entered by the user

- [1]  $d(X)/d(W) = -ra/Fao$
- [2]  $d(y)/d(W) = -alpha*(1+eps*X)/2/y$

Explicit equations as entered by the user:

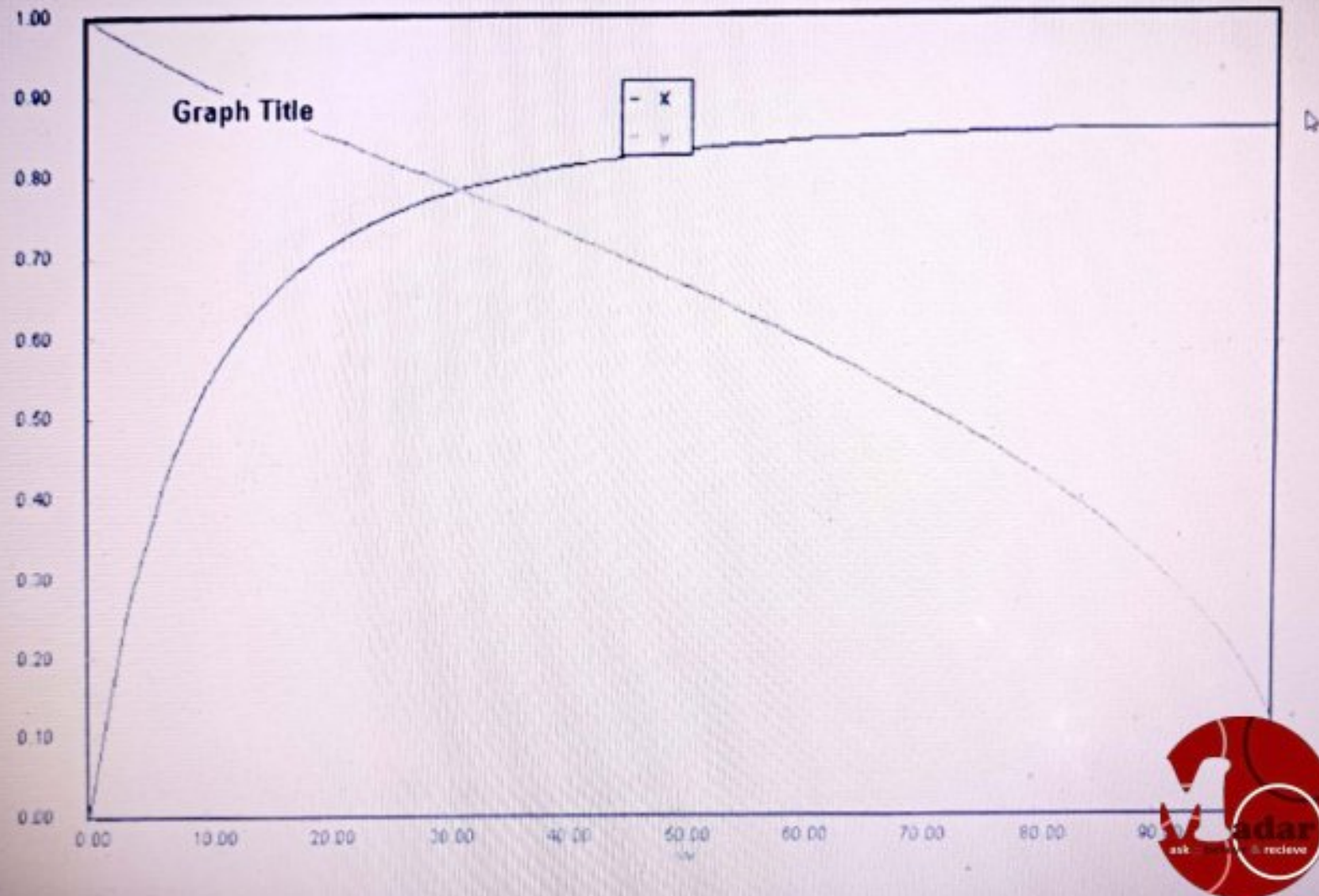
- [1]  $eps = (1-2-1)/3$
- [2]  $Cao = 0.2$
- [3]  $TheataB = 2$
- [4]  $Cb = Cao*(TheataB-2*X)/(1+eps*X)*y$
- [5]  $Fao = 2$
- [6]  $k = 6$
- [7]  $Ca = Cao*(1-X)/(1+eps*X)*y$
- [8]  $alpha = 0.02$
- [9]  $ra = -k*Ca*Cb^2$



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# Example 2: Gas Phase Reaction in **PBR** for $\delta \neq 0$





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- الرسم البياني - بنلاحظ انه  $X$  حارة  
ثابتة تقريباً عند  $w=80$  وبعد وبالتالي  
ما في داعي نستخدم  $w$  (catalyst) اكثر من  $80 \text{ Kg}$   
وبقدر نعرف لما  $w=80$ , كم ال  
pressure ratio هو الرسم.

\* Gas phase reaction in PBR with  
pressure Drop  $T=T_0$

mole balance (1)  $\frac{dx}{dw} = \frac{-r_A'}{F_{A0}}$

Rate Law (2)  $-r_A' = k C_A$

stoichiometry Gas  $T=T_0$

(3)  $C_A = \frac{C_{A0}(1-X)}{(1+\epsilon X)}$  y





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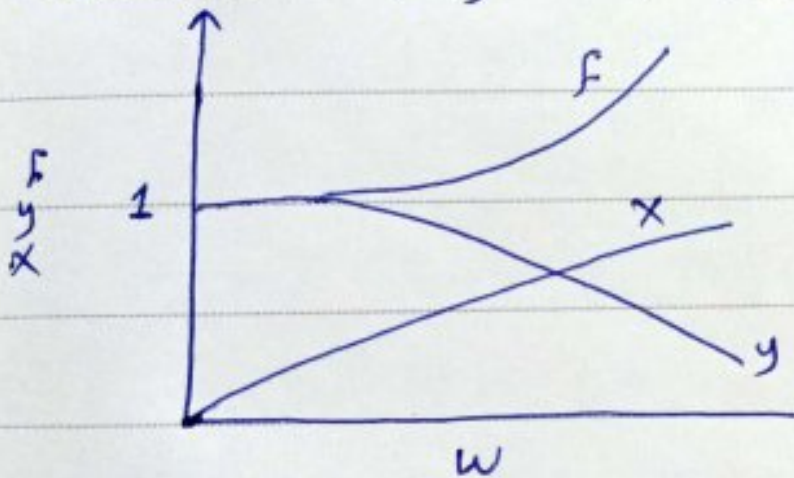
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$$(4) \quad \frac{dy}{dw} = \frac{-\alpha(1+\epsilon_X)}{2y}$$

(5)-(9) parameters,  $\epsilon, \alpha, \dots$

combine : polymath with combine For you





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## \* Collection and analysis of Rate Data:-

① power law models For homogeneous reaction  $-r_A = k C_A^\alpha$  ,  $-r_A = k C_A^\alpha C_B^\beta$

② Select reactor type and corresponding mole balance.

- batch reactor

$$-r_A = -\frac{dC_A}{dt}$$

- differential PBR (mole balance on product P)

$$-r_A = \frac{F_P}{\Delta W} = \frac{v_0 C_P}{\Delta W}$$

③ process your data in terms of measured variable ( $N_A$ ,  $C_A$  or  $P_A$ ), rewrite your mole balance in terms of the measured variable.

ask :: believe &amp; recieve



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④ Simplification : - if one of the reactants is in excess, assume its concentration is constant.

### \* Batch reactor

#### A - Differential analysis

$$-r_A = k C_A^\alpha = -dC_A/dt$$

$$\rightarrow \ln \left( \frac{-dC_A}{dt} \right) = \ln k + \alpha \ln C_A$$

↳ slope

① Find  $\frac{-dC_A}{dt}$  from  $C_A$  versus  $t$  data by

1) graphical method.

2) Finite differential method.

3) polynomial.

② plot  $\ln(-dC_A/dt)$  vs  $\ln(C_A)$ , reaction order ( $\alpha$ )  $\rightarrow$  slope.

$k \rightarrow$  From intercept.



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**[B] Integral method :- (trial and error).**

→ **Zero order ( $\alpha=0$ )**

$$\frac{dC_A}{dt} = r_A = -K$$

at  $t=0$  ,  $C_A = C_{A0}$

$$C_A = C_{A0} - Kt$$

→ **First order ( $\alpha=1$ )**

$$\frac{dC_A}{dt} = r_A = -K C_A$$

at  $t=0$  ,  $C_A = C_{A0}$

$$\ln\left(\frac{C_{A0}}{C_A}\right) = Kt \rightarrow \ln C_A = \ln C_{A0} - Kt$$

→ **second order ( $\alpha=2$ )**

$$dC_A/dt = r_A = -K C_A^2$$

at  $t=0$  ,  $C_A = C_{A0}$

$$\frac{1}{C_A} = \frac{1}{C_{A0}} + Kt$$

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### \* Method of Initial Rates

slope =  $-r_{A0}$  on  $([C] \text{ vs } t)$  graph, i.e. ①  
→ at  $C_{A0}$   
→  $C_{A0}, -r_{A0}$  بكونه نقطتين

②  $-r_{A0} = k C_{A0}^\alpha$

$$\ln(-r_{A0}) = \ln k + \alpha \ln C_{A0}$$

$(\ln(-r_{A0}) \text{ vs. } \ln C_{A0}) \rightarrow \text{slope} = \alpha$

### \* Method of half life :-

$$-r_A = k C_A^\alpha = \frac{-dC_A}{dt}$$

$$t_{\frac{1}{2}} = \frac{2^{\alpha-1} - 1}{k(\alpha-1)} \left( \frac{1}{C_{A0}^{\alpha-1}} \right)$$

$$t_{\frac{1}{n}} = \frac{n^{\alpha-1} - 1}{k(\alpha-1)} \left( \frac{1}{C_{A0}^{\alpha-1}} \right)$$

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$$\ln \frac{t_1}{2} = \ln \frac{2^{\alpha-1} - 1}{k(\alpha-1)} + (1-\alpha) \ln C_{A0}$$

$(\ln t_1/2 \text{ vs } \ln C_{A0}) \rightarrow \text{slope} = 1 - \alpha$

method of excess:-  $A + B \rightarrow P$

$$-r_A = k_A C_A^\alpha C_B^\beta$$

- reaction first run in an excess of B so that  $C_B$  remain unchanged during

the course  $-r_A = k' C_A^\alpha$

$$k' = k C_B^\beta \approx k_A C_{B0}^\beta$$

After determining  $\alpha$ , the reaction is carried out in an excess of A

$$-r_A = k'' C_B^\beta$$

$$, k'' = k_A C_A^\alpha \approx k_A C_{A0}^\alpha$$

ask :: believe &amp; recieve



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## \* Differential Reactors :-

$$-r_A' = \frac{F_{A0} - F_{Ae}}{\Delta w} = \frac{V_0 C_{A0} - V C_{Ae}}{\Delta w}$$

- in terms of conversion or product

Flow rate  $F_p$  :-

$$-r_A' = \frac{F_{A0} X}{\Delta w} = \frac{F_p}{\Delta w} = \frac{V_0 C_p}{\Delta w} = \frac{V_0 (C_{A0} - C_{Ae})}{\Delta w}$$

For constant volumetric Flow?