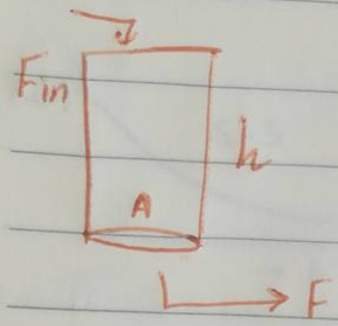
27/12/2018.



$$\boxed{F_{in} - F = A\frac{dh}{dt}}$$

$F \propto h \Rightarrow F = \alpha h = \frac{h}{R}$

Case 1 $F_{in} = 0 = -h\alpha A \frac{dh}{dt}$

$h(t) = h(0)e^{-t/\tau}$, $\tau = \frac{A}{\alpha}$

Case 2 $F_{in}(t) \neq 0$

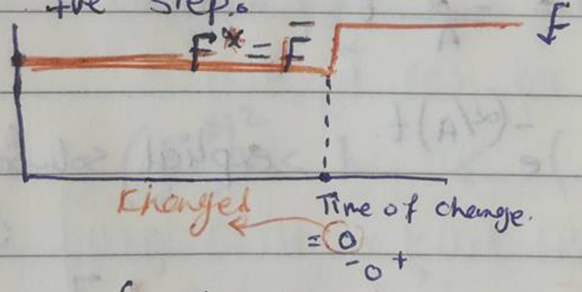
$F = \alpha h$

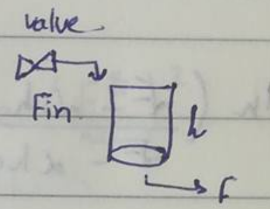
at S.S , $\bar{F}_{in} = \bar{F} = \alpha \bar{h}$ " $\bar{*}$ mean S.S"

$\bar{h} = h(0)$

$\bar{F} = F^*$

$F_{in}(t)$ +ve step:





$$F_{in}(t) = \begin{cases} F^* & t < 0 = 0^- \quad \text{, S.S} \\ F & t \geq 0 \end{cases}$$

$F_{in} - \alpha h = A\frac{dh}{dt}$

$F_{in} = F \quad t \geq 0$

$F - \alpha h = A\frac{dh}{dt}$ "seprabe and integrate" seprabe integral :-

$$+ \left(\frac{A}{\alpha}\right) \cdot \frac{dh}{dt} = -h$$

$$\left(\frac{A}{\alpha}\right) \int_{h(0)}^{h} \frac{dh}{h} = -\int_{0}^{t} dt \, .$$ need $\underline{\underline{i.c}}$ $h(0)$.

$$\left(\frac{A}{\alpha}\right) \ln\left(\frac{h}{h_0}\right) = -t.$$

[:. Remark :

$y = e^x$

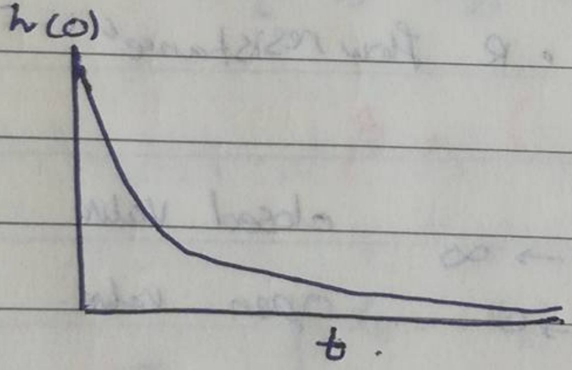
$\ln y = x$]

$$h = h(0)\, e^{\left(-\frac{\alpha}{A}\right)t}$$

let $\frac{1}{\tau} = \frac{\alpha}{A}$

$$h = h(0)\, e^{-t/\tau}$$

" اللازم لما أعوض عن 0 يعطيني i.c ".

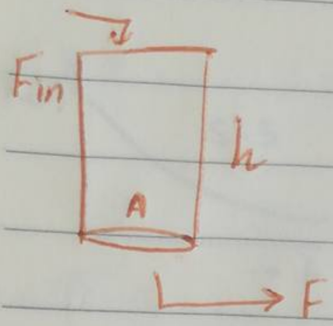


exponentially decay.

Hw => $\underline{\underline{CH.2}}$: 5 , 8 ; 9 , 13 , 14

11/3/2018
Sunday

27/12/2018.



$$\boxed{Fin - F = A\frac{dh}{dt}}$$

$$F \propto h \Rightarrow F = \alpha h = \frac{h}{R}$$

Case 1

$$Fin = 0 = -h\alpha A\frac{dh}{dt}$$

$$h(t) = h(0)e^{-t/\tau} \quad , \tau = \frac{A}{\alpha}$$

Case 2

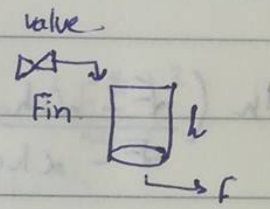
$$Fin(t) \neq 0$$

$$F = \alpha h$$

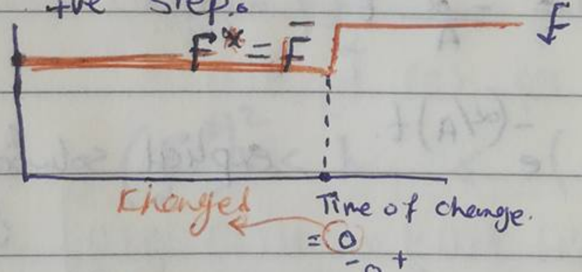
at S.S , $\bar{Fin} = \bar{F} = \alpha \bar{h}$ "$\bar{*}$ mean S.S"

$$\bar{h} = h(0)$$

$$\bar{F} = F^*$$



Fin (t) +ve step:



$$Fin(t) = \begin{cases} F^* & t < 0 \\ & = 0^- \\ F & t \geq 0 \end{cases} \quad \text{, S.S}$$

$$fin - \alpha h = A\frac{dh}{dt}$$

$$fin = F \quad t \geq 0$$

$$F - \alpha h = A\frac{dh}{dt}$$ "seprabe and integrate"

$$A\,dh = (F - \alpha h)\,dt$$

$$A\left(\frac{dh}{F - \alpha h}\right) = dt.$$

let $x = F - \alpha h$

$$dx = -\alpha\,dh$$

$$dh = \frac{-dx}{\alpha}$$

$$\int_{x(0)}^{x}\left[\frac{-A}{\alpha}\right]\frac{dx}{x} = \int_{0}^{t} dt \rightarrow \left(\frac{A}{\alpha}\right)(\ln x - \ln x_0) = -t.$$

$$x = F - \alpha h$$

$$x_0 = F - \alpha h(0).$$ moment of change level still $\underline{\underline{h(0)}}$

$$h(0) = h_0$$

$$\ln\left(\frac{F - \alpha h}{F - \alpha h_0}\right) = \frac{-\alpha}{A}t.$$

$$\frac{F - \alpha h}{F - \alpha h_0} = e^{-(\alpha/A)t}.$$, explicit solution in term of $h(t)$.

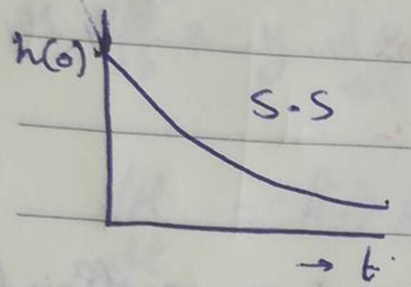
$$\left[\bar{F_{in}} = \bar{F} = \alpha\bar{h} = \alpha h(0) = F^*\right]$$

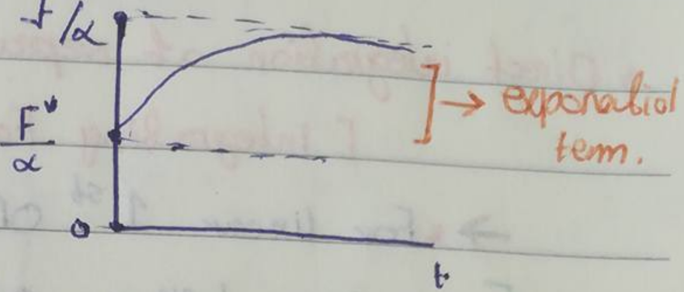
$$\boxed{h(t) = \left(\frac{F}{\alpha}\right) - \left(\frac{F - F^*}{\alpha}\right)e^{-\alpha/A\,t}}$$

at $t(0) \rightarrow t = 0 \rightarrow h = \frac{F^*}{\alpha} = h(0)$

$t \rightarrow \infty \longrightarrow h = \frac{F}{\alpha}$

اللازم يوصل output، عند ∞ حتى يصير
output عند s.s





if ⇒ $F = \alpha\sqrt{h}$, we cannot obtain an explicit solution with $h(t)$.

$$\left(\sqrt{h} - \frac{F^*}{\alpha}\right) + \left(\frac{F}{\alpha}\right) \ln\left(\frac{F - \alpha\sqrt{h}}{F - F^*}\right) = \left(\frac{-\alpha}{2A}\right) t$$

⇒ at ~~initial~~ Final S-S

$$\boxed{F - \alpha\sqrt{h} = A\frac{dh}{dt}}$$

$$F - \alpha\sqrt{h(0)} = 0$$

$$\sqrt{h(0)} = \frac{F}{\alpha} \Rightarrow h(0) = \left(\frac{F}{\alpha}\right)^2$$

at initial $F^* - \alpha\sqrt{h(0)}$

$$h(0) = \sqrt{\frac{F^*}{\alpha}} \Rightarrow h(0) = \left(\frac{F^*}{\alpha}\right)^2$$

$$\left(\frac{F^*}{\alpha}\right)^2 \le h \le \left(\frac{F}{\alpha}\right)^2$$

* Direct integration of seprarable equations
[integrating Factor method] :-

→ * For linear 1^{st} ODE

$$\left[a(t)\frac{dy}{dt} + b(t)\,y = c(t) \right] \text{---} (1)$$ " a, b, c coefficints

→ if $a(t) \neq 0$

Then $\left[\frac{dy(t)}{dt} + f(t)\,y(t) = g(t) \right]$ (2) , $\frac{b}{a} = f$, $\frac{c}{a} = g$

Eq (2) linear but not separable.

Eq (2) can be made seprablle by ~~intig~~ introducing an integrating factor

← coeff. of y

Integrating factor $= e^{\int f(t)\,dt} = \exp\left(\int f(t)\,dt\right) \xrightarrow{\text{call}} \mu(t)$

Multiply eq. (2) by $\mu(t)$

$$\left[\overbrace{\mu(t)\frac{dy}{dt} + \mu(t)\,f(t)\,y}^{(LHS)} = \mu(t)\;g(t) \right] \text{---} (3)$$

$$\mu(t) = e^{\int f(t)\,dt}$$

$$\frac{d\mu(t)}{d(t)} = f(t)\;e^{\int f(t)\,dt}$$

$$\frac{d\mu(t)}{d(t)} = \left(f(t)\;\mu(t)\right),$$

note : $\mu\frac{dy}{dt} + \mu f y = \mu g$

1/3/2018

$$\mu \frac{d\cancel{\mu}^{y}}{dt} + y\frac{d\mu}{dt} = \mu g$$

$$\frac{d\mu}{dt} = f(t)\,\mu(t)$$

$$\mu\frac{dy}{dt} + y\frac{d\mu}{dt} = \mu\frac{dy}{dt} + f(t)\,\mu(t) = \mu g \text{ --- } (*)$$

$$\mu\frac{dy}{dt} + y\frac{d\mu}{dt} = \frac{d}{dt}(\mu y) \text{ --- } (**)$$ "Chain Rule"

$$\left[\frac{d}{dt}(\mu y) = g\mu\right]$$ seprable ~~ind.~~ Equation --- $\boxed{***}$

seprate and integrate ***

$$\int d(\mu y) = \int g\mu\, dt$$

$$\mu y = \int g\mu\, dt + C$$ constant

$$y = \frac{1}{\mu}\int \mu g\, dt + \frac{1}{\mu}C \text{ ---- } *V$$

note → $\mu = e^{\int f(t)\,dt}$

$$\frac{1}{\mu} = e^{-\int f(t)\,dt} = \frac{1}{e^{\int f(t)\,dt}}$$

$$\boxed{y = e^{-\left[\int f(t)\,dt\right]} * \int \left[g(t)\,\mu(t)\right] dt + C e^{-\left[\int f(t)\,dt\right]}}$$

" genral solution " ↑

Ex: Tank.

$$A \frac{dh}{dt} + \alpha h = F_{in}(t)$$

specify, $F_{in}(t) = \begin{cases} F^{\alpha} & t < 0 \\ F & t \geq 0 \end{cases}$

use I.F.

$$\frac{dh}{dt} + \underbrace{\left(\frac{\alpha}{A}\right)}_{f(t)} h = \underbrace{\frac{F_{in}(t)}{A}}_{g(t)} \quad \text{— eq. 1}$$

constant

$$\mu = e^{\int f(t)\,dt} = e^{\int \frac{\alpha}{A} dt} = e^{\left(\frac{\alpha}{A}\right) t}$$

$$\text{Eq. 1} \rightarrow \int g(t)\, \mu(t)\, dt = \int \left(\frac{F_{in}(t)}{A}\right) e^{\left(\frac{\alpha}{A}\right)t} dt$$

$$\left[h(t) = e^{-\left(\frac{\alpha}{A}\right)t} \int_0^t \frac{(F_{in}(t))}{A} e^{\left(\frac{\alpha}{A}\right)t} dt + C\, e^{-\left(\frac{\alpha}{A}\right)t} \right] \text{— eq. 2}$$

at $t = 0$

$$h(0) = h_0 = 0 + C \Rightarrow C = h_0$$

$$h(t) = e^{\left(-\frac{\alpha}{A}\right)t} \int_0^t \left[\left(\frac{F_{in}(t)}{A}\right) e^{\left(\frac{\alpha}{A}\right)t} \right] dt + h_0\, e^{\left(-\frac{\alpha}{A}\right)t}$$

if $F_{in} = F$ constant.

$$h(t) = e^{-\left(\frac{\alpha}{A}\right)t} \left[\frac{F}{A} \int_0^t e^{\left(\frac{\alpha}{A}\right)t} dt \right] + h_0\, e^{-\left(\frac{\alpha}{A}\right)t}$$

$$h(t) = e^{\left(-\frac{\alpha}{A}\right)t}\left[\frac{F}{A}\cdot\frac{1}{(\alpha/A)}\, e^{(\alpha/A)t}\Big|_0^t\right] + h_0\, e^{\left(-\frac{\alpha}{A}\right)t}$$

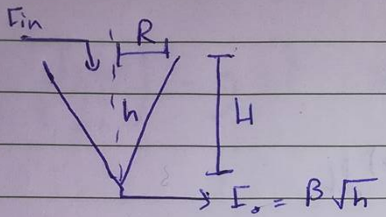
$$= e^{\left(-\frac{\alpha}{A}\right)t}\left[\left(\frac{F}{\alpha}\right)\left(e^{(\alpha/A)t} - 1\right)\right] + h_0\, e^{\left(-\frac{\alpha}{A}\right)t}$$

$$= \frac{F}{\alpha}\left(1 - e^{-(\alpha/A)t}\right) + h_0\, e^{(-\alpha/A)t}$$

$$h(t) = \frac{F}{\alpha}\left(1 - e^{\ \ }\right) + h_0 \cdot e^{(\alpha/A)t}$$

(1) * $F = \alpha h = \frac{h}{R} = \begin{cases} 0 & \text{if } R \to \infty \quad \text{fully closed value} \\ \infty & \text{if } R \to 0 \quad \text{fully open value} \end{cases}$

* (2) Conical



Cross-sectional Area changed, not constant.

$r = f(h)$

$$h = \left(\frac{r}{R}\right) H = \begin{cases} H & \text{if } r = R \\ 0 & r = 0 \end{cases}$$

$$F_i - F_o = \frac{d(v)}{dt}$$

$$F_i - \beta\sqrt{h} = \frac{dv}{dt}$$

$$V = \int_0^H \pi r^2 \, dh \quad \dots (1)$$

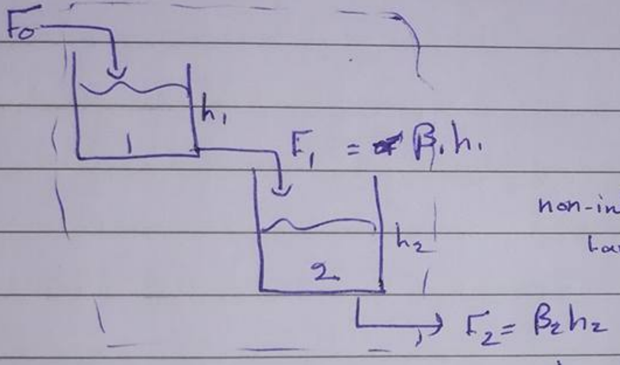
$$\Rightarrow V = \int_0^{H} \pi \left(\frac{R}{H}\right)^2 h^2\, dh = \pi\left(\frac{R}{H}\right)^2 \underbrace{\int_0^h h^2\, dh}_{\text{at any value}} = \alpha \frac{h^3}{3}\Big|_0^h = \alpha\frac{h^3}{3}$$

$$dv = \frac{\alpha}{3}\, 3h^2\, dh$$

$$\boxed{dv = \alpha h^2\, dh}$$

$$\Rightarrow F_0 - \beta\sqrt{h} = \alpha h^2 \frac{dh}{dt}$$

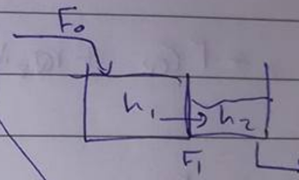
6 *



non-interacting tank

$h_2 = f(F_0)$

7



Interacting tank

$Tk_1 \longrightarrow Tk_2$

$h_1 > h_2$

$F_1 \propto \Delta h = h_1 - h_2$

$F_1 = \beta(h_1 - h_2) = \frac{h_1 - h_2}{R}$

Tk1 :- $F_0 - F_1 = A_1 \frac{dh_1}{dt}$

$F_1 = \beta_1 h_1$

$$F_0 - \beta_1 h_1 = A_1 \frac{dh_1}{dt}$$

Tk2 :- $F_1 - F_2 = A_2 \frac{dh_2}{dt}$

$$\beta_1 h_1 - \beta_2 h_2 = A_2 \frac{dh_2}{dt}$$

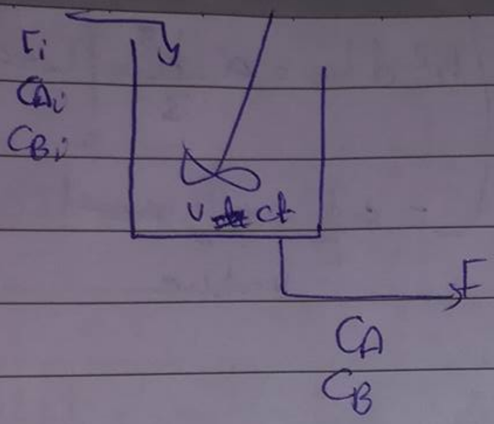
overall all :-

$$F_0 - \beta_1 h_1 = A_1 \frac{dh_1}{dt}$$

$$\beta_1 h_1 - \beta_2 h_2 = A_2 \frac{dh_2}{dt}$$

Day 4/3/ Simulation

4)



$A + B \longrightarrow 2P \qquad r_{A1} = k_1 C_A C_B$

$2A + P \longrightarrow Q \qquad r_{A2} = k_2 C_A C_P$

A:- $V \dfrac{dC_A}{dt} = F_i C_{Ai} - F C_A - r_{A1} V - r_{A2} V$

B:- $V \dfrac{dC_B}{dt} = F_i C_{Bi} - F C_B - r_{B1} V$

P:- $V \dfrac{dC_P}{dt} = - F C_P + r_{P1} V - r_{P2} V$

Q:- $V \dfrac{dC_Q}{dt} = - F C_Q + r_{Q2} V$

$\dfrac{r_{A1}}{1} = \dfrac{r_{B1}}{1} = \dfrac{r_{P1}}{2}$

$\dfrac{r_{A2}}{2} = \dfrac{r_{P2}}{1} = \dfrac{r_{Q2}}{1}$

6/3/2018.

$a(t)\frac{dy}{dt} + b(t)y = c(t)$. , $a(t)$, $b(t)$, $c(t)$ coefficien

* Integrating factor method:-

For Higher order D.E and constant coefficient.

$$a\frac{dy^2}{dt^2} + b\frac{dy}{dt} + cy = \begin{cases} 0 & \text{homogenous} \\ \neq 0 & \text{non homogenous.} \end{cases}$$

⇒ ① Homogenous case:-

$$a\frac{dy^2}{dt^2} + b\frac{dy}{dt} + cy = 0$$

Assume solution ⇒ $y(t) = e^{\lambda t}$

$y'(t) = \lambda e^{\lambda t}$, $y''(t) = \lambda^2 e^{\lambda t}$

if it's a solution it should satisfy equation:-

$$a\lambda^2 e^{\lambda t} + b\lambda e^{\lambda t} + ce^{\lambda t}$$

$e^{\lambda t}(a\lambda^2 + b\lambda + c) = 0$, $e^{\lambda t} \neq 0$

$a\lambda^2 + b\lambda + c = 0$, characteristic equation

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$, c_1 and c_2

from I.C

λ_1, λ_2 called $\rightarrow$ (eigen value) "pden: control"

+ or -] $\rightarrow$ stability of your solution

if $\rightarrow$ $\lambda_1 > 0$, $\lambda_2 > 0$

$$y(t) = c_1 e^{+\lambda_1 t} + c_2 e^{+\lambda_2 t} \quad , t \rightarrow \infty$$

n^{th} order :-

$$a_n + \frac{dy^n}{dt^n} + a_{n-1} \frac{dy^{n-1}}{dt^{n-1}} + a_1 \frac{dy}{dt} + a_0 y = 0$$

$$y = e^{\lambda t} \quad , \quad y^{(1)} = \lambda e^{\lambda t} \quad , \rightarrow y^{(n)} = \lambda^n e^{\lambda t}$$

$$\left[e^{\lambda t} \left(a_n \lambda^n + a_{n-1} \lambda^{n-1} + \cdots a_1 \lambda + a_0 \right) = 0 \right]$$

polynomial

roots of polynomial $\rightarrow$ real $\rightarrow$ distinct $\lambda_1 \neq \lambda_2$

real $\rightarrow$ repeated $\lambda_1 = \lambda_2 = \cdots \lambda_n$

roots of polynomial $\rightarrow$ complex $\rightarrow$ $a = bj$.

② non homogenous case:-

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = \underline{f(t)}$$ called Forced function " input "

Genral solution $y(t)$

$y(t) = y_c(t) + y_p(t)$.

$y_c(t) =$ homogenous [DE].

$y_p(t) =$ Assume a trail function.

$f(t)$	Trail function
A constant	B constant.
$Ae^{\alpha t}$	$Be^{\alpha t}$
$A\cos(\alpha t)$ or $A\sin(\alpha t)$	$B_1\cos(\alpha t) + B_2(\sin)(\alpha t)$
$A\,t^n$	$B_n t^n + B_{n-1}t^{n-1} + \dots B_1 t + B_0$

ex: $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = 4t^2$

$f(t) = 4t^2 \rightarrow f(t)^{\text{trail}} = At^2 + bt + c = y_b$

$y_b' = 2At + b \quad , y''b = 2A$

$2A - 6At - 3b - 4At^2 - 4bt - 4C = 4t^2$

$(2A - 3b - 4C) - (6A + 4b)t - (4A)t^2 = 4t^2$

$-4A = 4 \qquad A = 1$

$-6A - 4b = 0$

$+6 - 4b = 0 \rightarrow 4b = \frac{6 \div 2}{4 \div 2} \qquad b = \frac{3}{2}$

$2A - 3b - 4c = 0 \rightarrow 4c = -3b + 2A$

$4c = -3\left(\frac{3}{2}\right) + 2(-1)$

$4c = -\frac{9}{2} + \frac{2 \cdot 4}{2} = \frac{-1}{2} \cdot \frac{1}{4} = \boxed{-\frac{13}{8}}$

$$\boxed{y_p = -t^2 + \frac{3}{2} t - \frac{13}{8}}$$

→ homegenaus part :-

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y \; y$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda_1 = -1 \quad , \quad \lambda_2 = 4$$

$$\boxed{y_c = c_1 e^{-t} + c_2 e^{4t}}$$

$$y = y_c + y_b$$

i.c → at $y(0)$, $y'(0)$