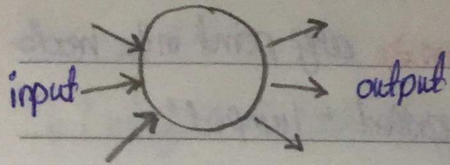
* Simulation.   1/2/2018

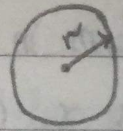
* Model (system):

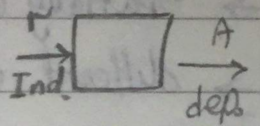


- system
- boundry
- surraunding

* Input → Independent Variable
* output → dependend.v

* circle:





* state of the goal ⇒ Analytical OR technical goal
* boundry ⇒ - Imajenary - Real.

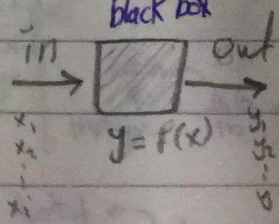
* Modeling:

- Discription
- formulation ⇒ (we need to write equation (Mathmatical))
- DOF ⇒ Dgree of freedom Analysis
- Solution ⇒ optimization (Mathmatical model + other Relationships)

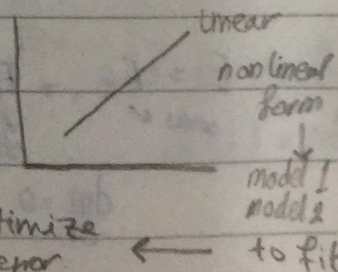
* Empirical modeling:

process modeling → Fundamental → (rigorous, theoretical)

process modeling → Empirical → (depends on expiermental data)

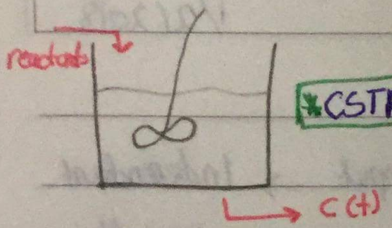


Box [illegible]
بس in و out
n data point



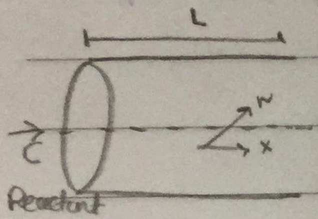
model 1
model 2

to optimize ← to fit
to error



* CSTR

* Not steady state
* C (space time) → position
* spatially uniform ⇒ any point in the reactor are the same, will called "lumped"



* PFR

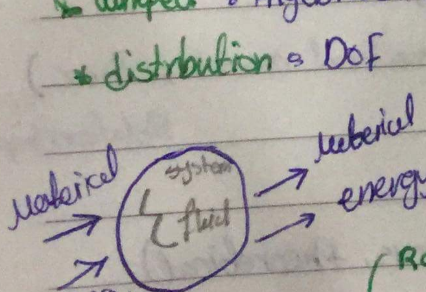
C → spatially uniform in r

C → different with x (distribution)

C (space time)

* Empirical
* Fundamental modeling → lumped / distribution ⇒ under unsteady state (Accumulate rate ≠ 0)

* lumped : Algebraic equation
* distribution : DoF (unsteady state ال عن بيفرق order) ⇒ under S.S



* Balance equation (Instantanous modeling)

$$\begin{pmatrix}\text{Rate of mass} \\ \text{or energy IN} \\ \text{To system}\end{pmatrix} - \begin{pmatrix}\text{Rate of mass} \\ \text{or energy out} \\ \text{of system}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass} \\ \text{or energy} \\ \text{accumlation} \\ \boxed{\text{with in}} \\ \text{the system}\end{pmatrix}$$

Instantanous.

$E = E_K + E_P + U$ → internal

undes, stal

mass → $\frac{dm}{dt}$ → mass of system

$\frac{dm}{dt} = 0$ $\frac{dE}{dt} = 0$

energy → $\frac{dE}{dt}$ → total energy of system

Simulation.

Macroscopic → tank full with water.

Microscopic → small elements (system like differential elements of the pipe)

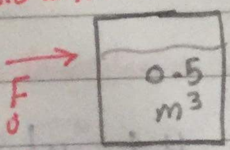
* Integral balance approach: هون باخذ فترة زمنية وبشوف يلي بصير
* Instentanous balance appreach لحين اعتبر مع الزمن لحظي.

* Integral Approach:-

$$\begin{bmatrix}\text{mass or energy INTO} \\ \text{system from } t \text{ to } t+\Delta t\end{bmatrix} - \begin{bmatrix}\text{mass or energy out} \\ \text{of system from } t \text{ to } t+\Delta t\end{bmatrix} =$$

$$\begin{bmatrix}\text{M OR E with in} \\ \text{the system at } t+\Delta t\end{bmatrix} - \begin{bmatrix}\text{M or E with in} \\ \text{the system at } t\end{bmatrix}$$

0.5 m^3/hr



0.1 m^3/hr

$0.1 \times 2 = 0.2 m^3$

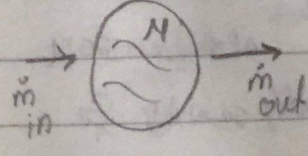
after 2hr

$2 \cdot 0.5 = 1 m^3$

$\Rightarrow 1m^3 - 0.2 = 0.8 m^3$

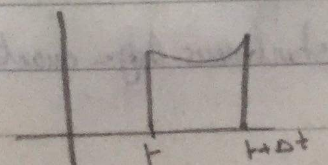
$0.5 + 0.8 = 1.3 m^3$

Accumulation of mass.



$$\int_t^{t+\Delta t} \dot{m}_{in}(t)\,dt - \int_t^{t+\Delta t} \dot{m}_{out}(t)\,dt = M\Big|_{t+\Delta t} - M\Big|_t$$

$$\int_t^{t+\Delta t} (\dot{m}_{in} - \dot{m}_{out})\,dt = M\Big|_{t+\Delta t} - M\Big|_t$$



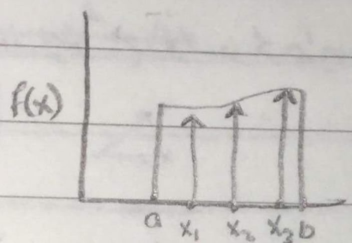
⇒ if not constant $\dot{m}(t)$ Δt, ⇒ if constant

$$\int_t^{t+\Delta t} \dot{m}(t)\,dt \qquad \dot{m}(t) \cdot \Delta t$$

$$\int_t^{t+\Delta t} (\dot m_{in} - \dot m_{out})\, dt = ??$$

* Remark :-

- Mean value Theorm of Integral $f(t)$



Area $= \int_a^b f(x)\, dx$ → Avg

. Mean $\bar f(b-a) = $ Area.

$$\int_a^b f(x)\, dx = \bar f\,(b-a), \quad \bar f = \int_a^b \frac{f(x)\,dx}{(b-a)}$$

* $Cp \rightarrow T \in [a,b]$

$$\bar{Cp} = \frac{\int_a^b Cp(T)}{b-a}$$

So, $\int_t^{t+\Delta t} (\dot m_{in} - \dot m_{out})\, dt = \overline{(\dot m_{in} - \dot m_{out})} \cdot \Delta t = M\big|_{t+\Delta t} - M\big|_t$

slope

$\overline{(\dot m_{in} - \dot m_{out})} = M\big|_{t+\Delta t} - M\big|_t = \frac{\Delta M}{\Delta t}$ $\cancel{\Delta t \approx 0}$

إذا كانت عند $\Delta t = 0$

بتكون نفس Avg quantity

if $\Delta t \approx 0 \Rightarrow \frac{\Delta M}{\Delta t} = \frac{dM}{dt}$

$\boxed{\frac{dM}{dt} = \dot m_{in} - \dot m_{out}}$ → In Instantenous Avg = quantity

in/out → $\dot v$ → $\dot m$; H_2O

system → M → V

Pin Pout → isothermal.

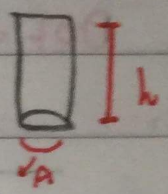
$\dot{m} = \rho \dot{V}$

$M = \rho V$

$$\rho \dot{V}_{in} - \rho \dot{V}_{out} = \frac{d(\rho V)}{dt}$$

cylindrical

$\dot{V}_{in} - \dot{V}_{out} = \frac{dV}{dt}$ → geometry ✓



$V = A\,h(t)$

A → constant, $h(t)$ → variable

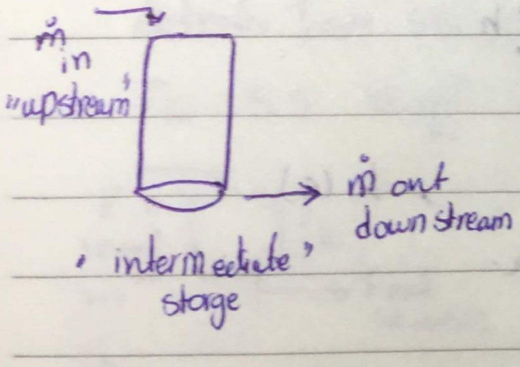
$\Downarrow$

$\frac{A\,dh}{dt}$

* Variable :

- state varible → $h = f(V_{in}, V_{out})$ (V_{in}: input, V_{out}: output)
- Input - output.

* Instentanous (differential) modeling:



$\rho \frac{dV}{dt} = m_{in} - m_{out}$

→ Accumulate in up stream

تأخير بـ down

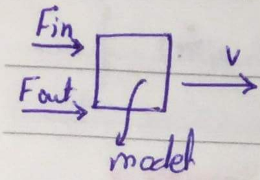
eq.(1) ⇒ $\rho \frac{dV}{dt} = \rho F_{in} - \rho F_{out}$

$\frac{dV}{dt} = F_{in} - F_{out}$

تغير

V → state variable

Fin, Fout → Input variable



$$\frac{dv}{dt} = F_{in} - F_{out} \Rightarrow \text{ODE}$$

So, I need Initial condition

" select input variable "

DOF = # of eq. (1)

\# of variable [V / Fin / Fout] = 3

DOF = 2 → Fin, Fout

depend variable V

state variable.

Fin(t), Fout(t), V(t)

$F_{in} = f(t)$, $F_{out} = f(t)$

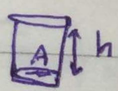
* specifical geometry of tank:

spherical — cylindrical

$V = Ah$

Ad

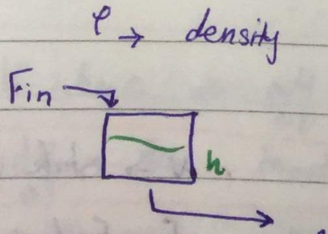
$h(t)$



$$A \frac{dh}{dt} = F_{in} - F_{out} \quad , h(0)$$

A → cross sectional Area

ρ → density



$F_{out} \propto h$

(Level of tank)

$x, y, y = f(x)$

$y = ax + b$

$F_{out} = \beta h(t) \rightarrow$ linear , in reality $F_{out} = \beta \sqrt{h(t)}$

$F_{out} \propto \sqrt{h(t)}$

$$A \frac{dh(t)}{dt} = F_{in}(t) - \beta h(t)$$

$$A \frac{dh(t)}{dt} + \beta h(t) = F_{in}(t)$$

→ linear ODE. i.c $h(0)$

* non liner → $F_{out} \propto \beta \sqrt{h}$

$$A \frac{dh(t)}{dt} + \beta \sqrt{h(t)} = F_{in}(t)$$

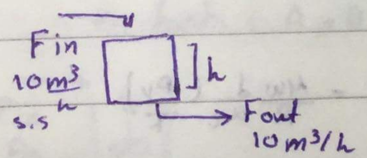
* need to spcify $F_{in}(t)$ at $\underline{\underline{S.S}}$ level ($\underline{\underline{h}}$) constant

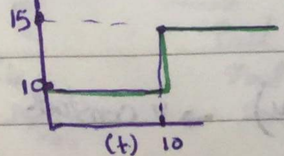
$F_{in} = F_{out}$

* time of change:

distrub. from S.S

F_{in} S.S $10 \frac{m^3}{h} \longrightarrow 15 \frac{m^3}{h}$



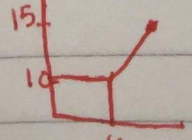


time of change when change system

(Step change) → $F_{in}(t) = \begin{cases} 10 \ m^3/h & t \leq 0^- \\ 15 \ m^3/h & t \geq 0^+ \end{cases}$

$$A \frac{dh}{dt} + \beta h(t) = F_{in}(t)$$

$$A \frac{dh}{dt}(t) + \beta h(t) = 15$$

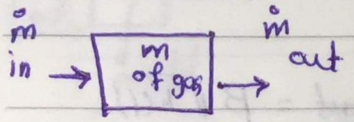


$F_{in} = \alpha t + \gamma$

$F_{in} = \gamma = F_{in}$ at S.S $= 10$

*

- gas intemediate storge tank :



- V Tank
- T Tank
- ρ Tank

⇒ overall material balance :-

$$\dot{m}_{in} - \dot{m}_{out} = \frac{d}{dt}(mg)$$

n moles , $m = n\, Mw$
mol . kg/mol

$$Mw\, n_{in} - Mw\, n_{out} = \frac{d}{dt}(mg)$$

gas behaviar → ideal
→ non ideal.

→ Assume ideal behavior:

$$PV = nRT$$

$$PV = \frac{mg}{Mw} RT$$

$$mg = \frac{Mw\, PV}{RT} \quad \text{-- **}$$

$$Mw\, n_{in} - Mw\, n_{out} = \frac{d}{dt}\left(\frac{Mw\, PV}{RT}\right) = \frac{Mw}{R}\frac{d}{dt}\left(\frac{PV}{T}\right)$$

→ assumptions :-

① V → constant

$$Mw\, n_{in} - Mw\, n_{out} = \frac{d}{dt}\left(\frac{Mw\, PV}{RT}\right) \rightarrow V \text{ constant}$$

$$\Rightarrow \frac{V\, Mw}{R}\frac{d}{dt}\left(\frac{P}{T}\right)$$

② isothermal (T constant)

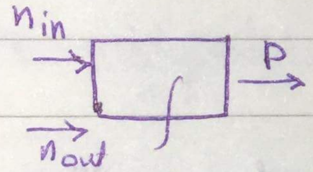
$$\frac{Mw\, V}{RT}\frac{dp}{dt} = Mw\, n_{in} - mw\, n_{out}$$

$$\left[\frac{dp}{dt} = \frac{RT}{V}(n_{in} - n_{out})\right]$$

→ P state variable $p(t)$

n_{in} , n_{out} , iput variable.

T, V, R , Parameters.



$$\frac{dp}{dt} = \frac{RT}{V}(n_{in} - n_{out})$$

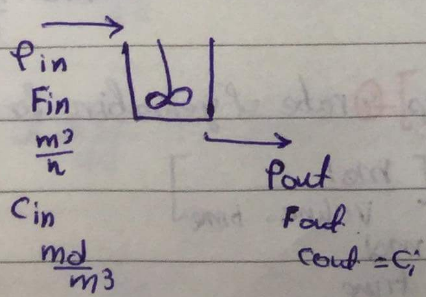
$\underline{i.c}$: $\frac{RT}{V}(n_{in} - n_{out})$, i.c $P(0)$

DoF ⇒ P, n_{in}, n_{out} → $3 - 1\,(eq) = \underline{\underline{2}}$ ① n_{in}, n_{out} ② Initial condition

material + energy balance not enough.   ③ حسب الـsystem بحتاج معادلة ثانية.

ex isothermal chemical Reaction. $A + B \rightarrow P$

Reactants : A + B , Product : P



$F_C = \left[\frac{m^3}{h} \cdot \frac{mol}{m^3}\right] = \frac{mol}{hr}$

* Rxn irrevar-sible
* T is constant 'not need energy balance'
* perfetly mixing

C (space, time) ← in general

C (time) depend only time "perfect mix."

① overall MB:

$\frac{dM}{dt} = m_{in} - m_{out}$

$\rho \rightarrow$ "density" function of consanbrabion.

Assume ρ in not funebion of concenbrabion

$\rho = aC + b$

$M = \rho V$

$$\left[\frac{dV}{dt} = F_{in} - F_{out}\right] \text{--- ①}$$

$\frac{dM}{dt} = \dot{m}_{in} + \text{rate of geration} - \dot{m}_{out} - \mathring{r}\text{ate of consumption.}$

$\frac{dM}{dt} = \mathring{m}_{in} - \mathring{m}_{out}$ "Total mass in - out"

consump. OR gēnrabion. لا يوجد

$* \; C_o V = mol$

$\frac{mol}{\cancel{m^3}} \cdot \cancel{m^3}$

For Ⓐ

$$\left[\frac{d(VC_A)}{dt} = F_{in} C_{A_o} - F_{out} C_A - \dot{r}_A\right] \text{--- ②}$$

$$\left[\frac{d(VC_B)}{dt} = F_{in} C_{B_o} - F_{out} C_B - \dot{r}_B\right] \text{--- ③}$$ rate of consuption for B $\frac{mol}{time}$

$$\left[\frac{d(VC_P)}{dt} = F_{in} C_{P_o} - F C_P + \dot{r}_P\right] \text{--- ④}$$ rate of genrabion for P

$\dot{r}$ rate of reation $\left[\frac{mol}{volume \cdot time}\right]$

$rV = \left[\frac{mol}{V \cdot time}\right] \cdot V = \frac{mol}{time}$

$\dot{r}_A = r_A V$

$\dot{r}_B = r_B V$

$\dot{r}_P = r_P V$

11/2/2018

simulation.

$$\frac{dv}{dt} = F_{in} - F \quad \text{---} ①$$ [assume $C_{P_0} = 0$]

A : $$\frac{d(VC_A)}{dt} = F_{in} C_{A_0} - F C_A - V r_A \quad \text{---} ②$$

B : $$\frac{d(VC_B)}{dt} = F_{in} C_{B_0} - F C_B - V r_B \quad \text{---} ③$$

P : $$\frac{d(VC_P)}{dt} = F_{in} C_{P_0} - F C_P - V r_P \quad \text{---} ④$$

$$\frac{d(VC_A)}{dt} = V \frac{dC_A}{dt} + C_A \frac{dV}{dt}$$ → From eq. ①

$$= V \frac{dC_A}{dt} + C_A (F_{in} - F_{out}) \quad \text{---} ⑤$$

substituted on ②

$$V \frac{dC_A}{dt} + \cancel{C_A} (F_{in} - F) = F_{in} C_{A_0} - \cancel{F C_A} - V r_A$$

$$V \frac{dC_A}{dt} = F_{in} (C_{A_0} - C_A) - V r_A$$

$$V \frac{dC_B}{dt} = F_{in} (C_{B_0} - C_B) - V r_B$$

$$V \frac{dC_P}{dt} = -F_{in} C_P + V r_P$$

$$\frac{dV}{dt} = F_{in} - F$$

→ 4 state variables,
V, C_A, C_B, C_P

→ initial condition
$V(0) = V(t=0)$ $C_A(0) = C_A(t=0)$
$C_B(0) = C_B(t=0)$ $C_P(0) = C_P(t=0)$

→ input variables
C_{A_0}, C_{B_0}, F_{in}

variables $\Rightarrow$ $F, F_{pn}, V, C_A, C_B, C_P, C_{A_0}, C_{B_0}$
$r_A, r_B; r_P$

need to Know $\rightarrow$ r_B, r_A, r_P Kinebices.

Reaction $\rightarrow$ irreversible

stioch. Eq 8

$\alpha A + \beta B = \gamma P$ $[\alpha, \beta, \gamma]$ stiochembry

A	B	P
α	β	γ
r_A	r_B	r_P

$r_B = \left(\frac{\beta}{\alpha}\right) r_A$ $r_P = \left(\frac{\gamma}{\alpha}\right) r_A$.

* need experision of $\underline{\underline{r_A}}$

$r_A = f(C_A, C_B)$ of reactants. - $r_A = \frac{\text{mol of } A}{V \cdot t}$

if $\longrightarrow$ $\alpha = 1$, $\beta = 2$, $\gamma = 3$

$A + 2B = P$

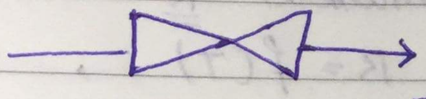
rate = 2nd order in (C_A & C_B)

$r_A = K C_A C_B$

$r_B = 2 K C_A C_B$

$r_P = \cancel{3} K C_A C_B$

Flow through valves :-



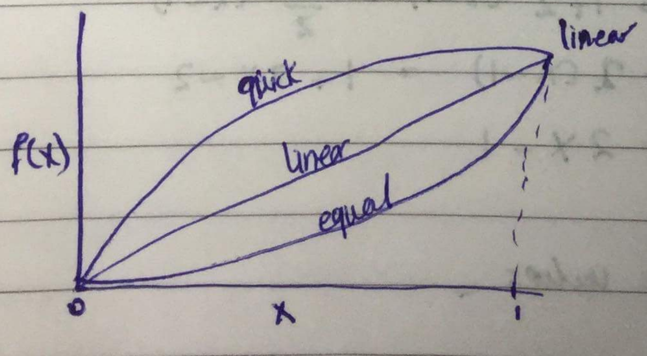
$$F = C_v\, f(x) \sqrt{\frac{\Delta P}{S.G}}$$

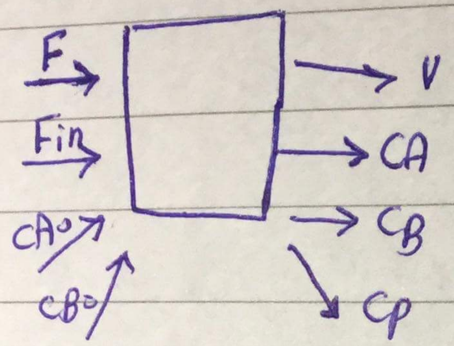
- $F \Rightarrow$ Volumetric flow rate
- $C_v \Rightarrow$ valve coefficient.
- $f(x) \Rightarrow$ flow characterstic.
- $x \Rightarrow$ valve opening
- $\Delta p \Rightarrow$ pressure drop
- SG $\rightarrow$ specific gravity , $SG = \frac{\rho}{\rho_{ref}}$

O → open
↓
1

C → close
↓
0

$$f(x) = \begin{cases} x & \text{linear} \\ \sqrt{x} & \text{quick opening} \\ \alpha^{x-1} & \text{equal percentage valve} \end{cases}$$





parameter $\underline{\underline{K}}$

$K = f(T)$,

function of Temperature

$K = K_o \, e^{-E/RT}$.

not linear eq., on graph is linear. $[\ln K = \ln K - \frac{E}{RT}]$

Draw $\underline{\underline{\ln K}}$ vs. $\frac{1}{T}$, slope = E/R.

* linearization.

$f(x) = f(x_o) + f'(x_o)(x - x_o) + \frac{f''(x_o)}{2}(x - x_o)^2$

$f(x) = x^2$, $x_o = 1$

$f'(x) = 2x$, $f''(x) = 2$

$f'\Big|_{x_o} = 2$, $f''\Big|_{x_o} = 2$ $f(x_o = 1) = 1$

$f(x) = x^2 = 1 + 2(x-1) + \frac{2}{2}(x-1)^2$

$x^2 = 1 + 2(x-1) = 1 + 2x - 2$

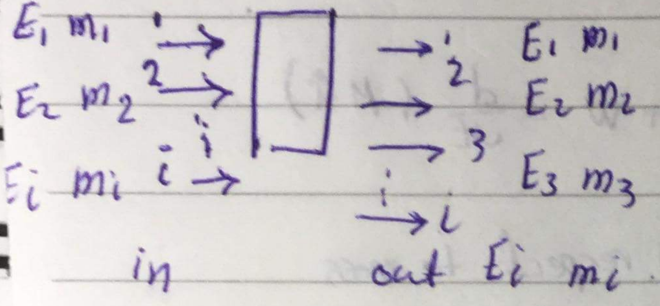
$= 2x - 1$

* Flow through valves

13/2/2018

→ Simulation.

Energy balance :-



ΣE_i ΣE_i

Rate of energy accumulation = [Rate of E by convection] − [Rate of E out by stream]

+ [Net rate of Heat in system from surrounding] + [Net Rate of work Performed by surrounding in system]

$U = m\hat{U}$

$[J] = [Kg]\,[J/kg]$

$$E = \dot{E}_K + \dot{E}_P + \dot{U}$$

$$= \frac{1}{2}\dot{m}v^2 + \dot{m}gz + \dot{U} \quad (\dot{U} \rightarrow \dot{m}\hat{U})$$

Go to eq. ① in prinsiple① CH.7

$$\Delta\dot{E} = \Delta\dot{H} + \Delta\dot{E}_K + \Delta\dot{E}_P = \frac{d}{dt}(\text{energy with system})$$

$$\Delta H = \Sigma m_j \hat{H}_j - \Sigma m_j \cdot \hat{H}_j$$

$$\Sigma m_i \hat{H}_i - \Sigma m_i H_i + Q + \dot{w} = \frac{d}{dt}(U)$$

$$\sum_i \dot{m}_i \hat{H}_i - \Sigma m_i \hat{H}_i + Q + W = \frac{d}{dt}(M\hat{u})$$

⇒ IF system at steady state with recpect to mass

$$\Sigma m_i - \Sigma m_i = \frac{d}{dt}(M\hat{u})$$

$$\Sigma m_i \underbrace{\hat{H}_i - \Sigma m_i \hat{H}_i}_{??} + Q + \dot{w} = M \frac{d\hat{u}}{dt}$$

in mixture ⇒ $\hat{H}$ mixture $= f(x_i, T, P)$

in pure component ⇒ $\hat{H}$ pure comp $= f(T, P) = \hat{H}_j$

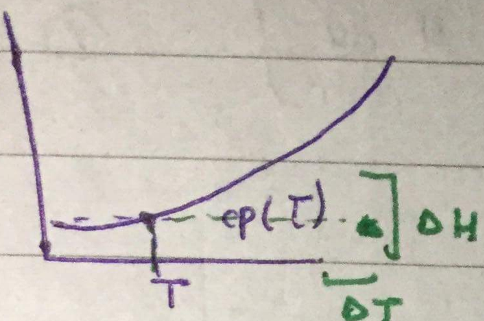
x_i → is mass fraction of j

$$\Sigma x_i = 1$$

$\hat{H}_{mix} = \Sigma x_j \hat{H}_j$ / m_j لو ضربنا كطاقة بـ mass

$$m \hat{H}_{mix} = \dot{m}(\Sigma x_j \hat{H}_j = \sum_j \dot{m} x_j \hat{H}_j = \Sigma m_j \hat{H}_j$$

$$= \Sigma m_j \hat{H}_j = m_1 + \hat{H}_1 + m_2 \hat{H}_2 + \cdots$$



$$\text{slope} = \frac{\Delta \hat{H}}{\Delta t} = cp(t) = \frac{d\hat{H}}{dT}$$

* seprable and integrable.

$$\int_{T_1}^{T_2} d\hat{H} = \int_{T_1}^{T_2} Cp(T)\, dT$$

Temp. Here, at referance.

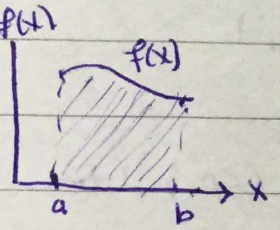
$$\Delta\hat{H} = \hat{H}_2 - \hat{H}_1 = \int_{T_1}^{T_2} Cp(T)\, dT$$

$$\Delta\hat{H} = \hat{H}_2(T_2) - \hat{H}_1(T_{ref.}) = \int_{T_{refs}}^{T_2} Cp(t)\, dt$$

$$\hat{H} = \Delta\hat{H} = \int_{T_{ref}}^{T} cp(T)\, dt$$

mean value ther of ∫ "integral":

$f(x)$:



$$\text{area} = \int_a^b f(x)\, dx.$$

$\bar{f}(b-a) = \text{Area}$

$$\bar{f} = \frac{\int_a^b f(x)\, dx}{(b-a)} \longleftarrow \bar{f}(b-a) = \int_a^b f(x)\, dx.$$

So ⇓

$$\bar{cp} = \frac{\int_{T_{ref}}^{T} cp\, dT.}{(T - T_{ref})} \longrightarrow \int_{T_{ref}}^{T} cp\, dt = \bar{cp}\,(T - T_{ref})$$

$$\left[\sum m_i \hat{H}_i - \sum m_i \hat{H}_j + \dot{Q} + \dot{W} = M \frac{du}{dt}\right] \text{---} ①$$

$$H_i = \int_{T_{ref}}^{T} Cp(T)\, dT$$

$$\left[H = \bar{Cp}\,(T - T_{ref})\right] \text{---} ②$$

$$\hat{H}_{mix.} = \sum x_i \hat{H}_j = \sum x_j \bar{Cp}_i \,(T - T_{ref})$$

$$= (T - T_{ref}) \sum_j x_j * \bar{Cp}_i$$

$$\bar{Cp}_{mix} = \frac{\sum x_j \bar{Cp}_i}{\sum x_j}$$

$$\bar{Cp}_{mi} = \sum x_i \bar{Cp}$$

so, $\hat{H}_{mix} = \bar{Cp}_{mix}\,(T - T_{ref})$

$$\sum_{in} m_i Cp_i\,(T_i - T_{ref}) = \sum_{out} m_i (Cp_i)(T_i - T_{ref}) + \dot{Q} + \dot{W} = M \frac{d\hat{u}}{dt}.$$

15/12/2018.

$$\sum_{\substack{\text{stream} \\ j}} m_i \, cp_i \, (T_j - T_{ref}) - \sum_{\substack{\text{stream} \\ \text{out}}} \dot{m} \, \bar{cp} \, (T_{ic} - T_{ref}) + Q + W$$

$$= \frac{d}{dt}(u) = \frac{d}{dt}(m \, \hat{u})$$

$$\frac{d(M)}{dt} = \sum_{in} m_j - \sum_{j} \dot{m}_{ic}$$

$$\sum_i m_i = \sum_{ic} m_{ic} \Rightarrow m_{in} = \dot{m}_{out} = \dot{m}$$

$$\rightarrow \quad M_{sys.} \frac{d\hat{u}}{dt}, \quad \hat{u}(T, p)$$

$$\hat{H} = \hat{u} + p\hat{V} \Rightarrow \hat{u} = \hat{H} - p\hat{V}$$

$$\frac{dV}{dt} = \frac{d}{dt}(H - pV)$$

$$\frac{dH}{dt} - \frac{d}{dt}(pV)$$

$$\frac{dH}{dt} - p \underbrace{\frac{dv}{dt}}_{V \text{ constant}} \rightarrow 0 \; - v \frac{dP}{dt}$$

$$= \frac{dH}{dt} - v \frac{dp}{dt}$$

$$= \frac{dH}{dt}$$ P constant * good approximation for liquid)

$$\frac{du}{dt} = \frac{dH}{dt} = \frac{d}{dt}(M_{system} \hat{H})$$

if M_{sys} constant $= M_{sys.} \frac{d\hat{H}}{dt} = \rho V \frac{d\hat{H}}{dt}$

$$M_{sys} = \rho V$$

$$\rho_{in} = \rho = \text{constant.}$$

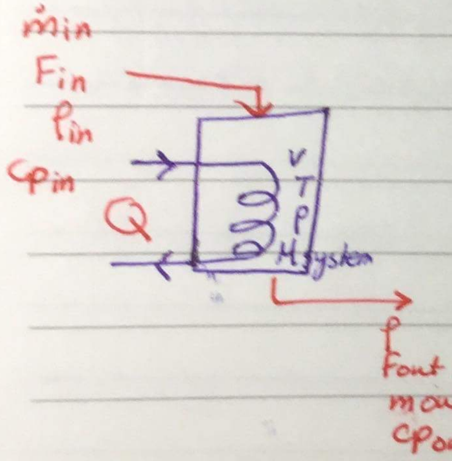
$$cp(T) = \frac{d\hat{H}}{dT} \longrightarrow d\hat{H} = C_p \, dT$$

$$\rho V \frac{d\hat{H}}{dt} = \rho V c_p \frac{dT}{dt}$$

$$\sum_{\substack{\text{stream} \\ i \\ \text{in}}} m_i \bar{c_p}_i (T_i - T_{ref}) - \sum_{\substack{\text{stream} \\ ic \\ \text{out}}} m_{ic} c_{p_{ic}} (T_{ic} - T_{ref})$$

$$+ Q + W = \rho V c_p \frac{dT}{dt}$$ "Temp. state variable"

Ex.: Stirred Tank Heaters



① MB :

$$\frac{dM_{system}}{dt} = m_{in} - m_{out}$$

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

* physical properities :-

$$\rho_{in} = \rho$$

$$c_{p_{in}} = c_p$$

change in avg $c_p \simeq$ negliable.

$$\dot{m}_{in} c_{p_{in}} (T_{in} - T_{ref}) - \dot{m}_{out} \bar{c_p}_{out} (T_{out} - T_{ref}) + Q + W = \rho V c_p \left(\frac{dT}{dt}\right)$$

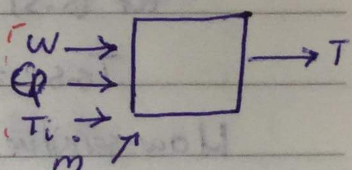
$T_{out} = T$

$$\dot{m} c_p (T_{in} - T_{ref}) - \dot{m} c_p (T - T_{ref}) + Q + W = \rho V c_p \frac{dT}{dt}$$

$$\dot{m} c_p (T_{in} - T) - \dot{m} c_p T_{ref} + \dot{m} c_p T_{ref} + Q + W = \rho V c_p \frac{dT}{dt}$$

$$\left(\frac{\rho V c_p}{\dot{m} c_p}\right) \frac{dT}{dt} = (T_{in} - T) + \frac{Q}{\dot{m} c_p} + \frac{W}{\dot{m} c_p}$$

$$\left[\left(\frac{\rho v c_p}{\dot m c_p}\right)\frac{dT}{dt} + T = T_{in} + \frac{Q}{\dot m c_p} + \frac{W}{\dot m c_p}\right]$$



[variable input

" Disturbances "

if the system :

① isolated ($Q=0$) → $\frac{\rho v c_p}{\dot m c_p}\frac{dT}{dt} + T = T_{in} + \frac{W}{\dot m c_p}$

② no viscous material ($W=0$) → $\frac{\rho v c_p}{\dot m c_p}\frac{dT}{dt} + T = T_{in}$

$$\left[\frac{\rho v c_p}{\dot m c_p}\right]$$

$\dot m = \rho F$

$m_{in} = \dot m_{out}$, $\rho_{in} = \rho$

$\Rightarrow F_{in} = F_{out} = F$

$$\left[\frac{\cancel{\rho}\, v\, \cancel{c_p}}{F\cancel{\rho}\,\cancel{c_p}}\right] = \left[\frac{v}{F}\right]\left[\frac{m^3}{\frac{m^3}{hr}}\right] = [hr]$$

$\left[\frac{\rho v c_p}{\dot m c_p}\right] \Rightarrow [\tau] \Rightarrow$ time constant.

$$\frac{dT}{dt} + (F/v)\,T = \underbrace{(F/v)}_{K_1} T_{in} + \frac{Q}{\rho v c_p}$$

⇐ اضرب الطرفين ب $\left(\frac{V}{F}\right)$

$$\overset{\tau}{\left(\frac{V}{F}\right)}\frac{dT}{dt} + T = T_{in} + K_2 Q \frac{V}{F}$$

$K_1 = 1$

$K_2 = \frac{v/F}{\rho v c_p} = \frac{1}{\rho F c_p} = \frac{1}{\dot m c_p}$

$\Rightarrow \tau \dfrac{dT}{dt} + T = K_1 T_{in} + K_2 Q$

at [s.s]

$T_{s.s} = K_1 (T_{in})_{s.s} + K_2 (Q)_{s.s}$

How sensitive $\underline{\underline{is}}$?

هنا نعرف تأثير على T_{ss}

$K_1 = \dfrac{T_{s.s}}{(T_{in})_{s.s}} \quad , \quad K_2 = \dfrac{T_{s.s}}{Q_{ss}}$

* الأكبر يكون تأثيره أكبر

18/2/2018.

Fittings $\to$ linear, non-linear] $\to$ optimization problem

$x \rightarrow \boxed{} \rightarrow y$ " black box "

input — output.

Independent — dependent. variable

"Don't care about the process"

x	y
x_1	y_1
x_2	y_2
x_n	y_n

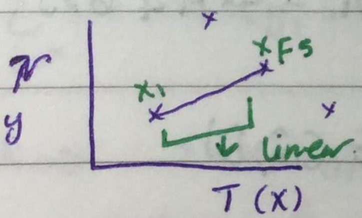
by experiment measure $\underline{y}$

(x_i, y_i) $\quad i = 1\,n$

ex: $\mu = f(T)$ "viscosity"

T	μ
25°c	μ_1
40°c	μ_2

$\downarrow$ plot



$y = f(x)$

$y = ax + b \qquad x[Y_1, X_{15}]$

$\underbrace{y}_{\text{data}} - \underbrace{Y}_{\text{model}} = \text{error}$

$y_i - Y_i = e_i$

$\boxed{Y_i = ax + b}$

$y_i - (\hat{a}x + \hat{b}) = e_i \longrightarrow$ error

$\therefore \quad y_i = \hat{a}x_i + \hat{b} + e_i$

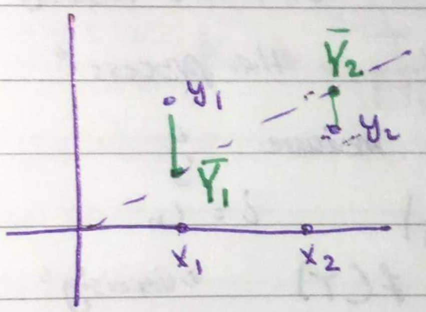
$e_i = y_i - (\hat{a}x_i + \hat{b})$

$i_1 \quad (x_1, y_1) \rightarrow \quad e_1 = y_1 - (\underbrace{\hat{a}x_1 + \hat{b}_1}_{\text{value}})$

$i_2 \quad (x_2, y_2) \rightarrow \quad e_2 = y_2 - (\hat{a}x_2 + b_2)$

$\vdots$

$e_i = y_i - (\hat{a}x_i + b_i)$



$y_2 - \hat{Y}_2 = e_2 \quad (-ve)$

$e_1 = y_1 - \bar{Y}_1 \quad (+ve)$

* بالاله بدي error أصغر
فاحطه بربع.

$\sum e_i^2 = \sum_{i=1} [y_i - (ax_i + b)]^2$

$$E(\hat{a}, \hat{b}) = \sum_{i=1}^{n} [y_i - (\hat{a}x_i + \hat{b})]^2$$

$\rightarrow$ minimize $E(\hat{a}, \hat{b})$

Remark:

$y = f(x_1, x_2)$

minimize $f(x_1, x_2)$
y_1, y_2

$y = (f(x_1)$

minimiz $f(x_1)$ How ?? by dirivatation.

$f(x) = x^2 \longrightarrow \min x^2$

$\frac{\partial f}{\partial x} = 2x = 0$

$\frac{\partial f'}{\partial x'} = \begin{matrix} -ve \rightarrow \text{max.} \\ +ve \rightarrow \text{min.} \end{matrix}$

$y = f(x_1, x_2)$

$$\begin{bmatrix} \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = 0 \Rightarrow \Delta f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E(\hat{a}, \hat{b}) = \sum_{i=1}^{n} [y_i - (\hat{a} x_i + \hat{b})]^2$$

$$\Delta E \begin{bmatrix} \frac{\partial E}{\partial \hat{a}} \\ \frac{\partial E}{\partial \hat{b}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

① أول مرة بشتق بالنسبة لـ a

الي برا العرض بعدين بس جوا

② بالنسبة لـ b

$$\frac{\partial E}{\partial \hat{a}} = \sum_{i=n}^{n} -2(y_i - \hat{a} - \hat{b}x_i) = 0$$

$$\frac{\partial E}{\partial \hat{b}} = \sum_{i=n}^{n} -2x_i (y_i - \hat{a} - \hat{b}x_i) = 0$$

* Remark $\Rightarrow$ $\sum (x_i + y_i) = \sum x_i + \sum y_i$

* sumation ($\sum$) is a linear opreator.

$\Rightarrow$ Eq. 1

$$\sum_{i=1}^{n} (y_i - \hat{a} - b\hat{x_i}) = 0 \rightarrow \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{a} - \sum_{i=1}^{n} b\hat{x_i} = 0$$

$$\Rightarrow \sum_{i=1}^{n} y_i - n\hat{a} - \hat{b} \sum_{i=1}^{n} x_i = 0$$

$\cancel{\sum_{i=1}^{n} (\ldots)}$

$$\hookrightarrow \sum_{i=1}^{n} y_i = n\hat{a} + \hat{b} \sum_{i=1}^{n} x_i \quad \text{linear.}$$

Eq. ② $\sum_{i=1}^{n} x_i (y_i - \hat{a} - \hat{b} x_i) = 0$

$$\sum x_i y_i - \hat{a} \sum x_i - \sum \hat{b} x_i^2 = 0$$

$$a \sum x_i + \hat{b} \sum x_i^2 = \sum x_i y_i$$

$$n \hat{a} + \hat{b} (\sum x_i) = \sum y_i \quad \text{--- ③}$$

$$(\sum x) \hat{a} + \hat{b} (\sum x_i^2) = \sum x_i y_i \quad \text{linear in } \hat{a} + \hat{b}$$

④

$\rightarrow Ax = b$

$$-\sum x_i \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

$$\hat{b} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - \sum (x_i)^2}$$

get $\hat{a}$ from eq ⑧

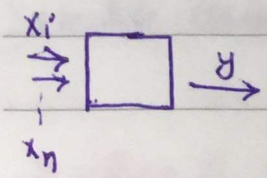
$$\hat{a} = \frac{\sum y_i - \hat{b} (\sum x_i)}{n}$$

$$\hat{a} = \underbrace{\frac{\sum y_i}{n}}_{\bar{y}} - \hat{b} \left(\underbrace{\frac{\sum x_i}{n}}_{\bar{x}} \right)$$

$$\boxed{\hat{a} = \bar{y} - \hat{b} \bar{x}}$$

20/2/2018.

Tues.



$$y = \alpha + \beta_1 X + \beta_2 x_2 + \dots \beta_i X_1 \dots + \beta_n x_n$$

$$y = \alpha + \beta_1 X + \beta_2 X^2 + \dots \beta_i X_i^n + \dots + \beta_n X^n$$

(y_i, x_1) "polynomial, linear in α and β"

$$y = \alpha + \tilde{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3 + \varepsilon \rightarrow \text{error}$$

true value

$$y = \bar{X}\hat{\beta} + \varepsilon$$

P = number of ~~values~~ variable.

n = number of data points.

$$\bar{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1p} \\ x_{21} & x_{22} & \dots & x_{2p} \\ \vdots & & & \\ x_{n0} & x_n & & x_{np} \end{bmatrix}$$

$(y_i, x_{1i}, x_{2i}, \dots x_{ii}, x_{ip})$

let $i=1$ $\cancel{x_{2i}}$ $\rightarrow (y_1, x_{11}, x_{21}, x_{i1}, x_{1p})$

$$y_2 = \hat{\alpha} + \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{12} + \dots \hat{\beta}_p x_{1p})$$

$$\hat{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}$$

$$\varepsilon = y - \bar{X}\hat{\beta}$$

minimum $\varepsilon^T \cdot \varepsilon = (y - \hat{\beta}\bar{X})(y - \bar{X}\hat{\beta})$ (T: Traspose)

$$\min E = (y - \hat{\beta}\bar{X})^T (y - \bar{X}\beta)$$

$$\left[\begin{array}{l} \frac{\partial E}{\partial \hat{\beta}_1} = 0 \\ \vdots \\ \frac{\partial E}{\partial \hat{\beta}_p} = 0 \end{array} \right.$$

$$\bar{X}\hat{\bar{X}}^T \hat{\beta} = \bar{X}^T y$$

$$X_{np} \quad X^T_{pn} = \bar{X}^T \bar{X} = C_{[p \times p]}$$

$$\hat{\beta}_{p \times 1}$$

$$[C_{p \times p}] [\hat{\beta}_{p \times 1}] \Rightarrow [p \times 1]$$

$$\bar{X}^T_{p \times n} y_{n \times 1} = b$$

$$X^T y \rightarrow , \; C\hat{\beta} = b$$

$$A^{-1} \quad Ax = b$$

$$\underbrace{A^{-1}A}_{\downarrow} x = A^{-1} b$$

$$I x = X = x^{-1} b$$

identity.

$A_{n \times n}$

$$n \left[A \right]_n = \left[\begin{array}{cc} \diagdown & 0 \\ L & \diagdown \end{array} \right] \cdot \left[\begin{array}{cc} \diagdown & 0 \\ 0 & \diagdown \end{array} \right]$$

$$A = LU$$

$$Ax = b$$

$$L\underbrace{Ux}_{C} = b$$

$Lc = b$ solve for $c \rightarrow Ux = C$

solve for x

* multiple linear Regragation.

$y = \cancel{\beta_0} + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ iF $x_1 = 1$

$y_i = (x_{1i}, x_{2i}, x_{3i})$ (model) 3 varible 3 point,

$i=1$

$y_1 = \beta_1 x_{11} + \beta_2 x_{21} + \beta_3 x_{13}$ True

$i=2$

$y_2 = \beta_1 x_{21} + \beta_2 x_{22} + \beta_3 x_{23}$

$i=3$

$y_3 = \beta_1 x_{31} + \beta_2 x_{32} + \beta_3 x_{33}$

$$\underset{\text{error}}{E} = \sum_{i=1}^{3} (y_i - \sum_{j=1}^{p} \beta_j x_{ij})^2$$

$$E = (y_1 - \beta_1 x_{11} - \beta_2 x_{12} - \beta_3 x_{13})^2 + (y_2 - \beta_1 x_{21} - \beta_2 x_{22} - \beta_3 x_{23})^2 + (y_3 - \beta_1 x_{31} - \beta_2 x_{32} - \beta_3 x_{33})^2$$ *

$\frac{\partial E}{\partial \beta_1} = 0 \Rightarrow$

$\beta_1 x_{11} x_{11} + \beta_2 x_{11} x_{12} + \beta_3 x_{11} x_{13} + \beta_1 x_{21} x_{21} + \beta_3 x_{21} x_{22}$

$+ \beta_3 x_{21} x_{23}$

$+ \beta_1 x_{31} x_{31} + \beta_2 x_{31} x_{32} + \beta_3 x_{31} x_{33}$

$= y_1 x_{11} + y_2 x_{21} + y_3 x_{31}$

$$\beta_1 (x_{11} x_{11} + x_{21} x_{21} + x_{31} x_{31}) + \beta_2 (x_{12} x_{12} + x_{22} x_{22} + x_{32} x_{32})$$
$$+ \beta_3 (x_{13} x_{13} + x_{23} x_{23} + x_{33} x_{33}) = y_1 x_{11} + y_2 x_{21} + y_3 x_{31}$$

$$\Rightarrow \beta_1 \left(\sum_{i=1}^{3} x_{i1} x_{i1} \right) + \beta_2 \left(\sum_{i=1}^{3} x_{i1} x_{i2} \right) + \beta_3 \left(\sum_{i=1}^{3} x_{i1} x_{i2} \right)$$
$$= \sum_{i=1}^{3} y_i x_{i1}$$

$$\begin{bmatrix} x_{11} & x_{21} & x_{31} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = x_{11} x_{11} + x_{21} x_{21} + x_{31} x_{31}$$

$$\underline{X}^T \underline{X} = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \\ x_{13} & x_{23} & x_{33} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{23} & x_{33} \end{bmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} X^T$$

$$X^T y = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \\ x_{13} & x_{23} & x_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= y_1 x_{11} + y_2 x_{21} + y_3 x_{31}$$