11/3/2018

*characterstic eq.:

$$a_n \lambda^n + a_{n-1} \lambda^{n-1} \cdots + a_1 \lambda + a_0 = 0$$

complex conjecabe.:

sin and cos

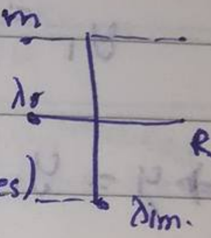
$$\boxed{y(t) = e^{\lambda r t} [c_1 \cos(\lambda in\, t) + c_2 \sin(\lambda in\, t)]}$$

complex roots:-

$$\lambda_{1,2} = \lambda_r \pm j \lambda_{im}$$

λ_{im} = Imajanary part

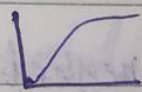
λr = repeated roots (r times)



$$\boxed{y(t) = e^{\lambda t} [c_1 + c_2 t + c_3 t^2 + \cdots c_r t^{r-1}]}$$

3 repeated roots:- $\boxed{r-1=2}$

$$y(t) = e^{\lambda t} [c_1 + c_2 t + c_3 t^3]$$

* if real points +ve (high oscillation)  goes to ∞
* if real points -ve (low ocillation)  goes to zero

$$\lambda_{1,2} = \lambda r \pm j \lambda_{im} \rightarrow y = c e^{\lambda t}$$

$$y = c_1 \underbrace{e^{(\lambda r + i \lambda m) t}}_{e^{\lambda r t} \cdot e^{i \lambda m t}} + c_2 \underbrace{e^{(\lambda r - j \lambda m) t}}_{e^{\lambda t} \cdot e^{-j \lambda t}}$$

$$y = e^{\lambda r t} [c_1 e^{\lambda m j t} + c_2 e^{-\lambda m j t}]$$

$$\begin{bmatrix} e^{-j\phi} = \cos\phi - j\sin\phi \\ e^{j\phi} = \cos\phi - j\sin\phi \end{bmatrix} \quad \phi = (\lambda_{im} j) t$$

$$y = e^{\lambda r t}\left[c_1 \left(\cos(\lambda_{im} t)\right) + j\sin(\lambda_{im} t)\right) + c_2 \left(\cos(\lambda_{im} t) - j\sin(\lambda_{im} t)\right)\right]$$

$$y_1 = e^{(\lambda r + \lambda_{im} j) t}, \quad y_2 = e^{(\lambda r - \lambda_{im} j) t}$$

$$\Rightarrow y = y_1 + y_2 = e^{\lambda r t}\left[\cos(\lambda_{im} t) + j\sin(\lambda_{im} t)\right] + e^{\lambda r t}\left[\cos(\lambda_{im} t) - j\sin(\lambda_{im} t)\right]$$

$$= \boxed{2e^{\lambda r t}\cos(\lambda_{im} t)}$$

$$y = y_1 - y_2 = e^{\lambda r}\left[\cos(\lambda_{im} t) + j\sin(\lambda_{im} t)\right] - e^{\lambda r}\left[\cos(\lambda_{im} t) - j\sin(\lambda_{im} t)\right]$$

$$= \boxed{2j e^{\lambda r}\sin(\lambda_{im} t)}$$

* neglect the constant:-

$$u(t) = e^{\lambda t}\cos(\lambda_{im} t)$$

$$u(t) = e^{\lambda t}\sin(\lambda_{im} t)$$

$$y = c_1 e^{\lambda r t}\cos(\lambda_{im} t) + c_2 e^{\lambda r t}\sin(\lambda_{im} t)$$

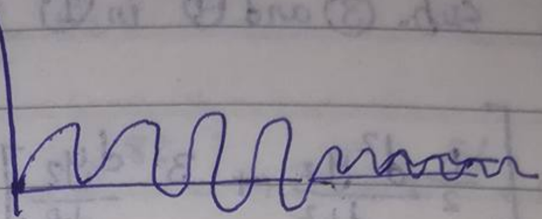
ex: $\boxed{\lambda_{1,2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} j}$ solution??

$$y(t) = e^{-\frac{1}{2}t} \cdot \left[c_1 \cos\left(\frac{\sqrt{3}}{2} t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2} t\right) \right]$$

plot → $t = 0 -- 100$

" 2 constant "

2nd order.



must have. I.C:

$y(0) = 1$, $y'(0) = 1$

$$y(0) = 1 = 1 \left[c_1 \underset{1}{\cos(0)} + c_2 \underset{0}{\sin(0)} \right]$$

$$\boxed{c_1 = 1}$$

$c_2 = ?$

system of DE :-

Ex: $$\begin{cases} \frac{dy_1}{dt} = 10 - 7y_1 - y_2 & (1) \\ \frac{dy_2}{dt} = -6y_2 + 2y_1 & (2) \end{cases}$$

Coupled ODE can't seprate to solve.

Solve eq (2) in (1)

$$\frac{dy_1}{dt} = \frac{1}{2} \frac{dy_2}{dt} + 3y_2 \quad — (3)$$

$$\frac{dy_1}{dt} = \frac{1}{2}\frac{d^2y}{dt^2} + 3\frac{dy_2}{dt} \quad \text{--- (4)}$$

sub. (3) and (4) in (1)

$$\left[\frac{1}{2}\frac{d^2y_2}{dt^2} + 3\frac{dy_2}{dt}\right] = 10 - 7\left[\frac{1}{2}\cdot\frac{dy_2}{dt} + 3y_2\right] + y_2$$

$$\frac{d^2y_2}{dt^2} + 13\frac{dy_2}{dt} + 40y_2 = 20$$

solution → $\boxed{y_2 = (y)_c + (y)_p}$ since DE is not homogenous.

(1) Homogenous part

$$\lambda^2 + 13\lambda + 40 = 0$$

$$(\lambda+5)(\lambda+8) = 0$$

$$\lambda_1 = -5, \quad \lambda_2 = -8$$

$$(y)_c = c_1 e^{-5t} + c_2 e^{-8t}$$

(2) non homogenous part

$(y)_p \Rightarrow$ constant $= 20$

$(y_2)_p = c$

$(y_2)'_p = 0$

$(y_2)''_p = 0$

$$40(y_2)_p = 20$$

$$(y_2)_p = \frac{1}{2}$$

$$\left[y_2 = c_1 e^{-5t} + c_2 e^{-8t} + \frac{1}{2}\right]$$

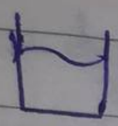
eq-3 $\rightarrow$ $y_1 = \frac{1}{2} \frac{dy_2}{dt} + 3y_2$

$$y_2' = -\frac{5}{2} c_1 e^{-5t} - \frac{8}{2} c_2 e^{-8t}$$

$$y_1 = \left[-\frac{5}{2} c_1 e^{-5t} - \frac{8}{2} c_2 e^{-8t}\right] + 3\left[c_1 e^{-5t} + c_2 e^{-8t} + \frac{1}{2}\right]$$

$$\left[y_1 = \frac{1}{2} c_1 e^{-5t} - c_2 e^{-8t} + \frac{3}{2}\right]$$

13/3/2018



CA_0
CB
C_c

$$\overset{(1)}{A \xrightarrow{k_1}} \overset{(2)}{B \xrightarrow{k_2}} C$$

$$r_1 = k_1 C_A$$

$$r_2 = k_2 C_B$$

(A) $$\frac{d V C_A}{dt} = + r_1 V$$

(B) $$+r_2 V + r_2 V = \frac{d(V C_B)}{dt}$$

(C) $$+r_2 V = \frac{d(V C_c)}{dt}$$

$$\rightarrow -k_1 C_A = \frac{dC_A}{dt} \quad \text{---} (1)$$

state variable :- C_A, C_B, C_c

$$\rightarrow k_1 C_A - k_2 C_B = \frac{dC_B}{dt} \quad \text{---} (2)$$

$$\rightarrow k_2 C_B = \frac{dC_c}{dt} \quad \text{---} (3)$$

$$\frac{dC_A}{dt} + k_1 C_A = 0$$ " homogenous system "

$$\rightarrow \int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} = \int -k\,dt \Rightarrow \boxed{C_A = C_{A0} e^{-k_1 t}}$$

mathmatical solution

13/2/2018.

solve by homogenous case :-

$$\frac{dCA}{dt} + KCA_1 = 0$$

Homogenous ODE :-

$$CA = y = ce^{\lambda t}$$

$$\frac{dCA}{dt} = c\lambda e^{\lambda t}$$

$$\lambda e^{\lambda t} + K_1 e^{\lambda t} = 0$$

$$e^{\lambda t} (\lambda_1 + K_1) = 0$$

$$e^{\lambda t} \neq 0 \quad , \quad \lambda = -K_1$$

$$\lambda + K_1 = 0 \qquad \lambda = -K_1$$

$$CA = c\, e^{-K_1 t}$$

$$I.C \ , \ t = 0 \ , \ CA = C$$

$$\boxed{CA = CA_0\, e^{-K_1 t}}$$

→ $\frac{dC_B}{dt} + K_2 C_B = K_1 CA_0\, e^{-K_1 t}$ → non Homogenous.

$$C_B = (C_B)_p + (C_B)_c$$

particular → non homogenous.

complement → homogenous part

* Homogenous part → $\frac{dC_B}{dt} + K_2 C_B = 0$

$$\boxed{C_{B_c} = C_1 e^{-K_2 t}}$$

* non homogenous part → $f(t) = K_1\, C_A\, e^{-K_1 t}$

trail function → $\boxed{C_2 e^{-K_2 t} = C_{Bp}}$

$$\frac{dc_B}{dt} = -c_2 K_1 e^{-K_1 t}.$$

$$-c_2 K_1 e^{-K_1 t} + K_2 c_2 e^{-K_1 t} = K_1 C_{A0} e^{-K_1 t}$$

→ solve for c_2 → $\left[c_2 = \frac{K_1 C_{A0}}{K_2 - K_1} \right]$

$$\boxed{C_B(t) = c_1 e^{-K_2 t} + \left(\frac{K_1 C_{A0}}{K_2 - K_1}\right) e^{-K_1 t}}$$

I.C → $C_{B0} = 0$

$$0 = c_1 + \frac{K_1 C_{A0}}{K_2 - K_1} \Rightarrow c_1 = -\frac{K_1 C_{A0}}{K_2 - K_1}$$

$$\left[C_B(t) = \overbrace{\left(\frac{K_1 C_{A0}}{K_2 - K_1}\right)}^{\alpha} \left(e^{-K_1 t} - e^{-K_2 t}\right) \right]$$

* when $t \to \infty$ → 0

maximum amount $\frac{dC_B}{dt} = 0$

$$\alpha = \frac{K_1 C_{A0}}{K_2 - K_1}, \quad C_B(t) = \alpha\left(e^{-K_1 t} - e^{-K_2 t}\right)$$

$$\frac{dC_B}{dt} = -\alpha K_1 e^{-K_1 t} + \alpha K_2 e^{-K_2 t} = 0$$

solve for t

$f(x) \to f'(x) \to x$

$$\frac{dC_B}{dt} + K_2 C_B = K_1 C_{A0}\, e^{-K_1 t}$$

**→ using Integrating factor :-

$$\mu = e^{\int K_2 dt} = e^{1/2 \int dt} = e^{K_2 t}$$

$$y = e^{-[\int f(t) dt]} \int g(t)\, \mu(t)\, dt + C e^{-\int f(t) dt}$$

$$C_B(t) = e^{-[\int K_2 dt]} \cdot \int \big(K_1 C_{A0} \underbrace{e^{-K_1 t} \cdot e^{K_2 t}}_{e^{(K_2 - K_1)t}}\big)\, dt + C e^{-K_2 t}$$

$$\int K_1 C_{A0}\, e^{(K_2-K_1)t}\, dt \Rightarrow \frac{K_1 C_{A0}}{K_2 - K_1} e^{(K_2-K_1)t}$$

$$\left(\frac{\frac{1}{2} - K_1}{K_2 - K_1}\right)$$

$$C_B(t) = e^{-K_2 t} \cdot \frac{K_1 C_{A0}}{K_2 - K_1} \cdot e^{(K_2 - K_1)t} + C e^{-K_2 t}$$

$$e^{1/2 t} \cdot e^{-K_1 t}$$

$$= e^{-K_2 t + K_2 t} \cdot e^{-K_1 t} \cdot \frac{K_1 C_{A0}}{K_2 - K_1} + C e^{-K_2 t}$$

$$\boxed{C_B = \left(\frac{K_1 C_{A0}}{K_2 - K_1}\right) e^{-K_1 t} + C e^{-K_2 t}}$$

$$C_B(0) = 0 \rightarrow \frac{K_1 C_{A0}}{K_2 - K_1} + C \Rightarrow \left[C = -\frac{K_1 C_{A0}}{K_2 - K_1} \right]$$

$$\frac{dC_A}{dt} = -K\,C_A \qquad (1)$$

$$\frac{dC_B}{dt} = K_1 C_A - K_2 C_B \qquad (2)$$

$$\frac{dC_c}{dt} = K_2 C_B \qquad (3)$$

solve eq. 2 for C_A :-

$$C_A = \frac{1}{K_1}\left[\frac{dC_B}{dt} + K_2 C_B\right]$$

$$C_A = \frac{1}{K_1}\frac{dC_B}{dt} + \frac{K_2}{K_1} C_B \qquad (4)$$

d.f. eq. (4)

$$\frac{dC_A}{dt} = \frac{1}{K_1}\frac{d^2C_B}{dt^2} + \frac{K_2}{K_1}\frac{dC_B}{dt} \qquad (5)$$

insert eq. (4) in eq. (1)

$$\frac{1}{K_1}\frac{d^2C_B}{dt^2} + \frac{K_2}{K_1}\frac{dC_B}{dt} = -K_1\left[\frac{1}{K_1}\frac{dC_B}{dt} + \frac{K_2}{K_1}C_B\right]$$

$$\boxed{\frac{d^2C_B}{dt^2} + (K_1+K_2)\frac{dC_B}{dt} + K_1K_2C_B = 0}$$

2nd order D.E "linear"

$$\lambda^2 + (k_1+k_2)\lambda + k_1 k_2 = 0$$

$$(\lambda + k_1)(\lambda + k_2) = 0$$

$\lambda_1 = -k_1$, $\lambda_2 = -k_2$ distinct and real

$$\left[C_B(t) = c_1 e^{-k_1 t} + c_2 e^{-k_2 t} \right] \quad (7)$$

need i.c

$$A \longrightarrow B \longrightarrow C$$

$C_{A_0} = C_A(0)$

$C_{B_0} = C_B(0) = 0$

$C_{C_0} = C_C(0) = 0$

at $t = 0$

$C_B(0) = 0 \rightarrow c_1 + c_2 \Rightarrow \boxed{c_1 = -c_2}$ *

eq. (2)

$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$ at $t=0$

$$\left[\frac{dC_B(0)}{dt} = k_1 C_A(0) - k_2 \cancelto{0}{C_B(0)} = k_1 C_A(0) \right] \quad (8)$$

Diff. eq. (7) "solution"

$$\frac{dC_B(t)}{dt} = -k_1 c_1 e^{-k_1 t} - c_2 k_2 e^{-k_2 t}$$

at $t = 0$ $\longrightarrow$ $\left[\frac{dC_{B_0}}{dt} = -k_1 c_1 - k_2 c_2 \right] \quad (9)$

H.w → P. 161 15/3/2018
CH.6 [1, 4, 5, 7, 14, 15, 18] [CH. 6]

combin ⑧ and ⑨ *

$$-K_1 C_1 - C_2 K_2 = K_1 CA_0$$
$$C_1 = -C_2$$
linear eq.

② equations with ② unknown C_1 and C_2

$$C_2 K_1 - C_2 K_2 = K_1 CA_0$$

$$C_2 = \frac{K_1}{(K_1 - K_2)} CA_0$$

$$C_1 = -\frac{K_1}{(K_1 - K_2)} CA_0$$

$$C_2 = -\left(\frac{K_1}{K_2 - K_1}\right) CA_0$$

$$C_1 = \left(\frac{K_1}{K_2 - K_1}\right) CA_0$$

con solution:-

$$C_B(t) = \left(\frac{K_1}{K_2 - K_1}\right) CA_0 \, e^{-K_1 t} - \left(\frac{K_1}{K_2 - K_1}\right) CA_0 \, e^{-K_2 t}$$

$$\boxed{C_B(t) = \left(\frac{K_1}{K_2 - K_1}\right) CA_0 \left(e^{-K_1 t} - e^{-K_2 t}\right)}$$

i.c at $t = 0$ $C_{B_0} = 0$

$$C_B(0) = \frac{K_1 CA_0}{K_2 - K_1} \underbrace{(1 - 1)}_{\text{zero}} = 0$$

15/3/2018

* linear and non linear algeberic equations :-

$$\begin{bmatrix} a_{11} x_1 + a_{12} x_2 = b_1 \\ a_{21} x_1 + a_{22} x_2 = b_2 \end{bmatrix}$$ 2 equation with 2 unknown x_1 and x_2

* in matrix :-

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{Ax=b} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

solution x :

* unique solution * infinite # of solution. [redundant equation]

→ A non singular → (indepenedent # of equations)
→ (uniqe solution)

$\det(A)$ $\neq 0$ non singuler. $\exists A^{-1}$ 'exist'
$= 0$ a singular = infinte # of solution

* for unique solution.

$A^{-1} \mid Ax = b$

$\underbrace{A^{-1} A}_{I \text{ unity}} x = A^{-1} b$

$I x = A^{-1} b$

$\boxed{x = A^{-1} b}$

⇒ $Ax = b$

$A = [L \backslash U]$

$Ax = LUx = b$ ①

let $Ux = c$ ②

eq - ①

$LC = b$ ⇒

$Ux = C$ solution of X

18/3/2018.

Matlab: $Ax = b$ and A^{-1} exists.

$\det(A) \neq 0 \rightarrow$ nonsi

$\underbrace{A^{-1} A}_{I} x = A^{-1} b$

$I x = A^{-1} b.$

command in matlab :- inv (A) $\longrightarrow$ ex :- X = inv (A) * b

LU Factorization $\Rightarrow [A] = [L\backslash 0][0 \backslash U] \Rightarrow C[\ ]$

$Ax = b$

$[A][x] = [b] \Rightarrow [L\backslash 0][0\backslash U][x] = [b]$

$Ax = b$

$L\underbrace{Ux}_{c} = b \quad \longrightarrow$ call $c = Ux$

$Lc = b \quad \longrightarrow$ solve for c

$Ux = c \quad \longrightarrow$ solve for x

* non linear - equations :-

$f(x) = 0 \qquad x^*$ (root)

DOF 1eq.

1 veriable] DOF = 0 we can solve it.

$ax^2 + bx + c = 0$

$DOF = 1 - 1 = 0 \quad , \quad x_{1,2} \Rightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

→ if
- $b^2 - 4ac > 0$ — x_1, x_2 real and distinct.
- $b^2 - 4ac = 0$ — x_1, x_2 real and repeated. $x_1 = x_2$
- $b^2 - 4ac < 0$ — complex root.

* nonlinear equation : polynomial of dgree ($\underline{n}$)

$$P_n = a_n x^n + \cdots + a_1 x + a_0 \quad [n \text{ roots}]$$

≫ $C = [\, a_n \quad a_{n-1} \quad a_{n-2} \quad \cdots \quad a_1 \quad a_0 \,]$

≫ solution = $\underline{\text{roots}}(c)$

* nonlinear eq. :

$f(x) = 0$ ...① find x^*

" Direct substitution method " Leleigstein "

" iterative in nature "

Add and substitu. x :

$g(x) = x + f(x) = x + 0$

① نضيف x على الطرفين

② نسمي $g(x) \Leftarrow f_x + x$

if x^* solution : $g(x^*) = x^* + f(x^*) = x^*$

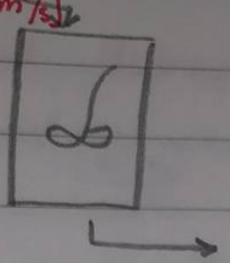
iterative formula.

$$x_{i+1} = g(x_i) \quad i = 1, 2, 3$$

C_{A0} [mol/m³]

F_{A0} [m³/s]

ex:-



C_A

F

assumption:-

① isothermal T is constant ∴ K is constant

② constant Volume ; overall balance

$F_{in} = F$

$r = K\,C_A^2$

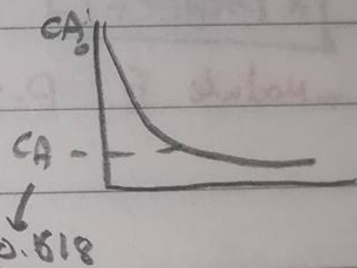
$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - K\,C_A^2$$

at S.S :

$$\frac{dC_A}{dt} = 0 \Rightarrow -K\,\bar{C_A}^2 - \frac{F}{V}\bar{C_A} - \frac{F}{V}\bar{C_{A0}} = 0$$

$\frac{F}{V} = 1\ min^{-1}$, $C_{A0} = 1\ \frac{mol}{L}$, $K = 1$ Liter/mol.min.

$$\bar{C_A}^2 + \bar{C_A} - 1 = 0$$



let $x = \bar{C_A}$

$$x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 * 1 * 1}}{2 * 1}$$

$$x_{1,2} = \frac{-1\sqrt{5}}{2} \Rightarrow x_{1,2} = \begin{cases} 0.618 \ ✓ \\ -1.618 \end{cases}$$

method(2) ⇒ $f(x) = x^2 + x - 1 = 0 \Rightarrow x^2 = -x + 1$

$$x = \sqrt{-x+1} \Leftarrow g(x)_{1}$$

$$x_{i+1} = g(x_i)$$

x_1 = (initial value)

$$x = -x^2 + 1 = g(x)_{2}$$

$x = \sqrt{-x+1} = g(x)$

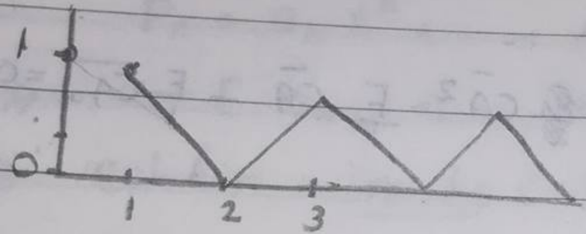
$x_1 = 1$

$x_{i+1} = g(x_i) = (-x+1)^{\frac{1}{2}}$

$x_i = 1$

$x_2 = (-x_1+1)^{1/2} = 0$

$x_3 = (0+1)^{1/2} = 1$, this will not converge or Divergre

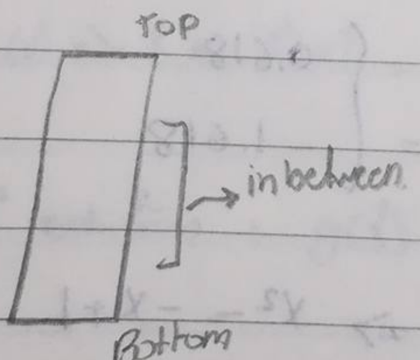


* Project :-

* Module 6 p. 490 . Absorption process.

- Stage-wise calculation.

- linear



→ Matlab code.

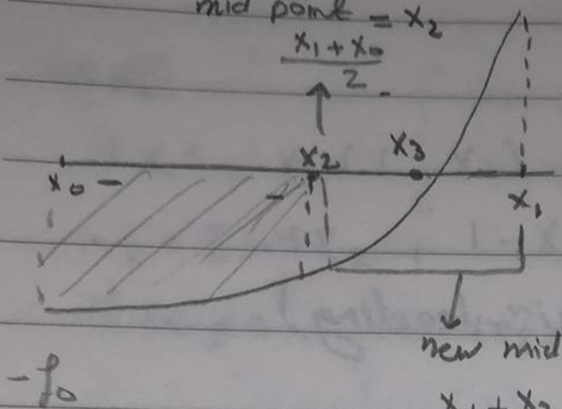
let $y_i = k_i V_i$

not linear

20/3/2018

* Interval Halving (Bisection) method:-

mid point $= x_2 = \frac{x_1 + x_0}{2}$



new mid point

$\frac{x_1 + x_2}{2} = x_3$

$x^* \in [x_0, x_1]$

$f_0 \cdot f_1 < 0$

if sign$(f_0 \cdot f_1)$ $\rightarrow$ $+$ $\rightarrow$ $f_0 (+)(-)$, $f_1 (+)(-)$

$\rightarrow$ $-$ $\rightarrow$ $f_0 > 0 < 0$, $f_1 < 0 > 0$

$f(x) = 0$, Find x^* "netwon method"

x_0 initial guess

Expand $f(x)$ at x_0 ; [Taylor series]

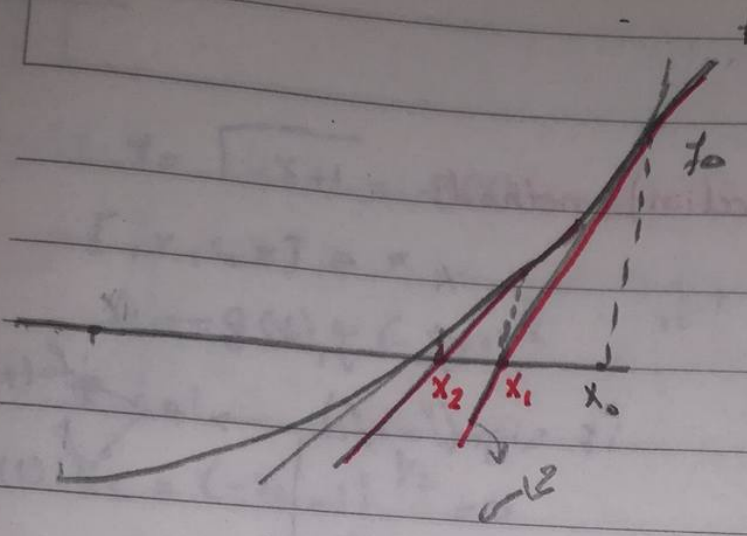
$$f(x) = \underbrace{f(x_0)}_{f_0} + \underbrace{f'(x_0)}_{f_0'}(x - x_0) = 0$$

$\therefore$ solve for $\underline{\underline{x}}$

$$f_0 + f_0' x - f_0' x_0 = 0$$

$$f_0' x = +f_0' x_0 - f_0$$

$$\left[x = x_0 - \frac{f_0}{f_0'} \right] , \left[x_{i+1} = x_i - \frac{f_i}{f_i'} \right]$$ iterative method



** Matlab → fzero (miss leading)

$$f_1(x_1, x_2) = x_1 - 4x_1^2 - x_1 x_2 = 0 \quad \text{--} ①$$
$$f_2(x_1, x_2) = 2x_2 - x_2^2 + 3x_1 x_2 = 0 \quad \text{--} ②$$

$$x_2 = 1 - 4x_1$$
$$f_2 = 2(1-4x_1) - (1-4x_1)^2 + 3x_1(1-4x_1) = 0$$
$$\Rightarrow 1 + 3x_1 - 28x_1^2 = 0$$

ution ① $x_1 = 0.25 \rightarrow x_2 = 0$

solution ② $\rightarrow x_1 = -0.1429 \rightarrow x_2 = 1.5714$

$f_1 \Rightarrow x_1 - 4x_1^2 - x_1 x_2 = 0$ --- ① $x_2 = 1 - 4x_1$

$f_2 \Rightarrow 2x_2 - x_2^2 + 3x\,x_1\,x_2 = 0$ — ②

eq. ①

$-4x_1^2 + x_1(1 - x_2) = 0$

$x_1 = [-4x_2 + 1 - x_2] = 0$

if $x_2 \neq 0$ $-4x_2 + 1 - x_2 = 0$

$x_2 = 1 - 4x_1$

$\begin{bmatrix} x_1 = 0 \\ x_2 = 0 \end{bmatrix}$ $\begin{matrix} x_1 = \\ x_2 = \end{matrix}\begin{bmatrix} 0.25 \\ 0.0 \end{bmatrix}$ $\begin{bmatrix} -0.1429 \\ 1.5714 \end{bmatrix}$

solution ①   solution ②

From eq. ② solve for x_2:-

$3x_1 x_2 = x_2^2 - 2x_2$

$\left(x_1 = \frac{1}{3} x_2 - \frac{2}{3} \right)$

$x_1 = \frac{1}{3}(x_2 - 2)$ substit in eq. ①

$\left[\frac{1}{3}(x_2 - 2) - 4\left(\frac{x_2}{3} - \frac{2}{3}\right)^2 - \frac{1}{3}(x_2 - 2)x_2 = 0 \right]$

$7x_2^2 - 25x_2 + 22 = 0$

$\Rightarrow$ Matlab:-

symbolic ToolBox $ax^2 + bx + c$

```
>> syms x a b c
>> f = ax^2 + bx + c
>> simplify (     )
```

```
>> P = [7 -25 22]
>> solution =roots(P)
```

$$(X_2)_{1,2} = \begin{Bmatrix} 2 \\ 1.5714 \end{Bmatrix}$$

$$X = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} -0.1429 \\ 1.5714 \end{bmatrix}$$

on matrix form :-

$$F(\underline{\bar{X}}) = 0 \qquad (n) \text{ eq. }, (n) \text{ unkwons}$$

$$\begin{bmatrix} f_1(x_1, x_2, \dots x_n) \\ f_2(x_1, x_2, x_3, \dots x_n) \\ \vdots \\ f_n(x_1, x_2, x_3, \dots x_n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$(X + \Delta X) \qquad x_0 \xleftrightarrow{\Delta X} x_1$$

$$\Delta X = \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_n \end{bmatrix} \qquad X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

$$X + \Delta X = \begin{bmatrix} X_1 + \Delta X_1 \\ X_2 + \Delta X_2 \\ \vdots \\ X_n + \Delta X_n \end{bmatrix}$$

$f_1(x_1+\Delta x_1, x_2+\Delta x_2, \dots x_n+\Delta x_n) = 0$ --(1)

$f_2(x_1+\Delta x_1, x_2+\Delta x_2, \dots x_n+\Delta x_n) = 0$ --(2)

$\vdots$

$f_n(x_1+\Delta x_1, x_2+\Delta x_2 \dots x_n+\Delta x_n) = 0$

* expand on $\underline{\underline{x}}$

* Remark $\Rightarrow$ $f(x, y, z) = 0$

$$\underline{X} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

$$f(x,y,z) = f(x_0, y_0, z_0) + \left(\frac{\partial f}{\partial x}\right)_{(x_0,y_0,z_0)}(x-x_0) + \left(\frac{\partial f}{\partial y}\right)_{(x_0,y_0,z_0)}(y-y_0) + \left(\frac{\partial f}{\partial z}\right)_{(x_0,y_0,z_0)}(z-z_0) \quad \text{--(3)}$$

expand (3) in (1)

$$\approx f_1(x_1, x_2 \dots x_n) + \left(\frac{\partial f_1}{\partial x_1}\right)_{x_0-x_0}(\overbrace{x_1+\Delta x_1-x_1}^{\Delta x_1}) + \frac{\partial f_1}{\partial x_2}(\overbrace{x_2+\Delta x_2-x_2}^{\Delta x_2}) + \dots + \frac{\partial f_1}{\partial x_n}(\Delta x_n)$$

$$f_1(x_2+\Delta x_1 \dots x_2+\Delta x_1) = f_1(x_1-x_n) + \left(\frac{\partial f_1}{\partial x_n}\right)_{x_1-x_n}\Delta x_1 + \left(\frac{\partial f_2}{\partial x_2}\right)_{x_1-x_n}\Delta x_2 + \dots + \left(\frac{\partial f_1}{\partial x_n}\right)\Delta x_n = 0$$

$$f_2(x_1+\Delta x_1+\dots x_n+\Delta x_n) = f_2(x_1-x_n) + \left(\frac{\partial f_2}{\partial x_1}\right)_{x_1-x_n}\Delta x_n$$

$\vdots$

$$f_n(x_1+\Delta x_1 \dots x_n+\Delta x_n) = f_n(x_1-x_n) + \left(\frac{\partial f_n}{\partial x_1}\right)_{x_1-x_n} x_1 + \left(\frac{\partial f_n}{\partial x_n}\right)_{x_1-x_n} x_n$$

Remark:-

$$\begin{bmatrix} a_{11} x_1 + a_{12} x_2 = b_1 \\ a_{21} x_1 + a_{22} x_2 = b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

27/3/2018

$$\left[f_1(x_1+\Delta x_1, \dots, x_n+\Delta x_n)\right] = \left[f_1(x_1, \dots x_n)\right] + \left(\frac{\partial f_1}{\partial x_1}\right)_x \Delta x_1 + \left(\frac{\partial f_2}{\partial x_2}\right)_x \Delta x_2 + \dots \left(\frac{\partial f_n}{\partial x_n}\right)_x \Delta x_n = 0$$

⋮

$$\left[f_1(x_1+\Delta x_1 \dots)\right] = \left[f_n(x_1 - x_n)\right] + \left(\frac{\partial f_n}{\partial x_1}\right)_x \Delta x_1 + \left(\frac{\partial f_n}{\partial x_2}\right)_x \Delta x_2 \dots + \left(\frac{\partial f_n}{\partial x_n}\right) \Delta x_n = 0$$

↳

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial Fn}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial fn}{\partial x_1} & \frac{\partial Fn}{\partial x_2} & & \frac{\partial Fn}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = - \begin{bmatrix} f_1(x_1 - x_n) \\ \vdots \\ f_n(x_n - x_n) \end{bmatrix}$$

→ $J(x)$ Jacobian. "solve of linear equation"

*note →

$x_1 \xrightarrow{\Delta x_1} (x_1)new$

⋮

$x_n \xrightarrow{\Delta x_n} (x_n)new$

$\Rightarrow$ $(x_1)_{new} = x_1 + \Delta x_1$

⋮

$(x_n)_{new} = x_n + \Delta x_n$.

$$f(X+\Delta X) = F(x) + J(X)\,\Delta X = 0$$ ⇒ Matrix form.

$F(x) + J(\bar{x})\,\Delta x = 0$

$\Delta x J(x) = -f(x) \rightarrow$ if $J^{-1}(\bar{x}) \Rightarrow$

$\underbrace{J^{-1}(\bar{x})\,J(x)}_{I}\,\Delta x = -J^{-1}(x)\,f(x)$

$I\,\Delta x = -J^{-1}(x)\,f(x) \Rightarrow \Delta x = -J^{-1}(x)\,f(x)$

For iteration $i \Rightarrow$

$\Delta x_i = -J^{-1}(x_i)\,f(x_i)$

$\Delta x_i = \bar{x}_{i+1} - x_i$

$x_{i+1} = x_i + \Delta x_i = \begin{bmatrix} x_1 + \Delta x_1 \\ \vdots \\ x_n + \Delta x_n \end{bmatrix}$

$$\boxed{\begin{array}{l} J(x_i)\,\Delta x_i = -f(x)_i \\ \downarrow \\ \Delta x_i = x_{i+1} - x_i \end{array}}$$

ex:- $f_1(x_1, x_2) = x_1 - 4x_1^2 - x_1 x_2 = 0$

$f_2(x_1, x_2) = 2x_2 - x_2^2 + 3x_1 x_2 = 0$

$\rightarrow$ solutions $x^* = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix}, \begin{bmatrix} -.1429 \\ 1.5714 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{x_2} \\ \frac{\partial f_2}{x_1} & \frac{\partial f_2}{x_2} \end{bmatrix} \cdot \begin{bmatrix} 1 - 8x_1 - x_2 & -x_1 \\ 3x_2 & 2 - 2x_2 + 3x_1 \end{bmatrix}$

$\Rightarrow$ need intial point $x_0 \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} = \begin{bmatrix} x_{0_1}(0) \\ x_{0_2}(0) \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$i = 1$

$$J(x_0)\,\Delta x_0 = -f(\underset{\circ}{x})$$

$$x_1 = x_0 + \Delta x_0$$

$$J(\underset{\circ}{x}) = \begin{bmatrix} 10 & 1 \\ -3 & 1 \end{bmatrix} \quad , \quad f(x_0) \begin{bmatrix} x_1 - 4x_1^2 - x_1 x_2 \\ 2x_2 - x_2^2 + 3x_1 x_2 \end{bmatrix}_{\text{at } [-1,\,-1]}$$

$$f(x_0) = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{(1)} = \begin{bmatrix} -0.5385 \\ 0.3846 \end{bmatrix} \dashrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{(7)} = \begin{bmatrix} -.1429 \\ 1.5714 \end{bmatrix}$$

(1) iteration

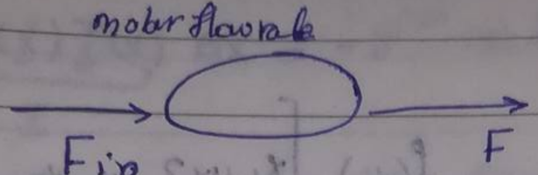
(7) iteration:

→ $\boxed{\text{Matlab}}$ f(zero)

fzero ("fi" , f2 , ——) , x_0) "evenburg-Marquardt" Method

How solution :-

P.8
CH.2.8



$\begin{bmatrix} H_2 = x \\ CH_4 = y \end{bmatrix}$

Feed
CH_4 $\quad x_{H_2} = x_i$
H_2 $\quad y_{CH_4} = y_i$

y_i mole fraction of comp. i.

$\frac{dP}{dt} = ?$

* ideal gas $PV = nRT$.

* mass balance

$$M F_i - M F = \frac{d(Mn)}{dt} \qquad n \text{ \# of moles.}$$

$$F_i - F = \frac{dn}{dt} \qquad n = \frac{PV}{RT}$$

$$F_i - F = \frac{d}{dt}\left(\frac{PV}{RT}\right) = \left(\frac{V}{RT}\right)\frac{dP}{dt}$$

$$\boxed{\frac{dP}{dt} = \left(\frac{RT}{V}\right)(F_i - F)} \quad (1)$$

* CH_4 balance :-

$$\boxed{y_i F_{in} - yF = \frac{d}{dt}(n_{CH_4})} \quad (2)$$

$$y = \frac{n_{CH_4}}{n} = \frac{P_{CH_4}}{P_T} \Rightarrow P_{CH_4} = yP$$

$$\boxed{n_{CH_4} = \frac{P_{CH_4} V}{RT}} \quad (3)$$

combining (2) and (3)

$$y_i F_{in} - yF = \frac{d}{dt}\left(P_{CH_4} \cdot \frac{V}{RT} \right)$$

$$\left[y_i F_{in} - yF = \left(\frac{V}{RT}\right) \frac{d P_{CH_4}}{dt} \right] \quad (4)$$

$P_{CH_4} \propto P_T$

$$y = \frac{P_{CH_4}}{P_T} \Rightarrow \boxed{P_{CH_4} = yP} \quad (5)$$

$$\boxed{y_i F_i - yF = \left(\frac{V}{RT}\right) \frac{d}{dt}(yP)} \quad (**)$$

$$\frac{dP}{dt} = \left(\frac{RT}{V}\right)(F_{in} - F) \quad (*)$$

$$\frac{d(yP)}{dt} = \left(\frac{RT}{V}\right) \cdot (y_i F_i - yF) \quad (**)$$

combing.

$$\frac{d(yP)}{dt} = y\frac{dP}{dt} + P\frac{dy}{dt} = \frac{RT}{V}(y_i F_i - yF)$$

$$y\left[\frac{RT}{V}(F_{in} - F)\right] + P\frac{dy}{dt} = \frac{RT}{V}(y_i F_i - yF)$$

$$P\frac{dy}{dt} = -\frac{RT}{V} y F_{in} + \cancel{\frac{RT}{V} yF} + \frac{RT}{V} y_i F_{in} - \cancel{\frac{RT}{V} yF}$$

$$\rightarrow P\frac{dy}{dt} = \frac{RT}{V}(y_i F_{in} - yF_{in})$$

$$\left[\begin{array}{l} \frac{dP}{dt} = \left(\frac{RT}{V}\right)(F_{in} - F) \\ \frac{dy}{dt} = \left(\frac{RT}{V}\right)\left(\frac{F_{in}}{P}\right)(y_i - y) \end{array}\right]$$

State variable ⇒ y, P inside tank.

# of variables (5) F_{in}, P, y, y_i, F.

# of equations = (2)

DOF = 3

specific , y_i, ~~F_i~~, F_{in}

at S.S

$$\underbrace{\left(\frac{RT}{V}\right)}_{\neq 0}(\bar{F}_{in} - \bar{F}) = 0$$

$$\bar{F}_{in} - \bar{F} = 0$$

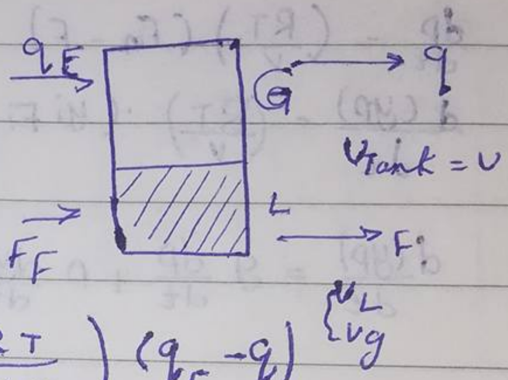
$$\bar{F}_{in} = \bar{F}$$

$$\underbrace{\left(\frac{RT}{V}\right)\left(\frac{\bar{F}_{in}}{P}\right)}_{\neq 0}(\bar{y}_i - \bar{y}) = 0$$

$$\bar{y}_i = \bar{y}$$

P2.14

q, F molar Flow rate.



$$\frac{dV_L}{dt} = F_F - F$$

$$\frac{dP}{dt} = \left(\frac{P}{V - V_L}\right)(F_F - F) + \left(\frac{RT}{V - V_L}\right)(q_F - q)$$

* liquid $\Rightarrow C F_F - C F = \frac{d}{dt}(C V_L)$

molar concentration $\left(\frac{mol}{L}\right)$, $\boxed{C_{in} = C}$

$$\boxed{F_F - F = \frac{d(V_L)}{dt}}$$

* gas $\Rightarrow q_F - q = \frac{d}{dt}(n_g)$

$$PV_g = n_g RT$$

$$n_g = \frac{PV_g}{RT}$$

$$q_F - q = \frac{d}{dt}\left(\frac{PV_g}{RT}\right) \Rightarrow q_F - q = \left(\frac{1}{RT}\right)\frac{d}{dt}(P V_g)$$

$$RT(q_F - q) = V_g \frac{dP}{dt} + P\frac{dV_g}{dt}$$ $\therefore V = V_L + V_g$

$V_g = V - V_L$

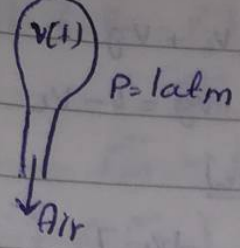
$$RT(q_F - q) = (V - V_L)\frac{dP}{dt} + P\frac{d(V - V_L)}{dt}$$

$$RT(q_F - q) = (V - V_L)\frac{dP}{dt} + P\left[\cancel{\frac{dV}{dt}}_{0} - \frac{dV_L}{dt}\right]$$

$$= (V - V_L)\frac{dP}{dt} - P\frac{dV_L}{dt} \longrightarrow (F_f - f)$$

1/4/2018

P.13 :-



mass balance:-

$$0 - Mq = \frac{d}{dt}(m_{air})$$

$$m(air) = \left(\frac{M}{RT}\right) P\, V_g$$

Equation of state for ideal gas:-

$$\left(PV = nRT = \frac{m}{M} RT\right)$$

$$-Mq = \left(\frac{M}{RT}\right)\frac{d}{dt}(\underbrace{P}_{1atm} V_g)$$

$$-q = \left(\frac{1}{RT}\right) P \frac{dV_g}{dt}$$

$$V(t) = V_g$$

q is constant. $\boxed{-q = \left(\frac{P}{RT}\right)\frac{dV}{dt}}$ -- (*) V : state variable

q : input.

approximate shape of bullon → sphere.

(i) q constant, seprate and integrate eq. (*)

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

$$\boxed{\frac{P}{RT}\left(4\pi r^2 \frac{dr}{dt}\right) = -q}$$

$$\int_{10}^{r}\left(\frac{P}{RT}\right)\cdot 4\pi \cdot r^2 dr = -\int_0^t q\, dt = -qt$$

$$\boxed{\frac{1}{3}(r^3 - 10^3) = -\left(\frac{RT}{4\pi P}\right) q t} \text{ -- (**)}$$

@ $t = 5\,min$ $\quad r = 7.5\ cm$

$$\frac{1}{3}(7.5^3 - 10^3) = -\left(\frac{RT}{4\pi P}\right) q \cdot (5)$$

$$\left(\frac{RT}{4\pi P}\right) q = \underline{\underline{38.54}}$$

Eq. ⊛⊛

$$\frac{1}{3}(r^3 - 10^3) = -(38.54)t.$$

$t = ?$ $\quad r = 5\ cm$

$$\boxed{t = 7.6}$$

q ~~not~~ ~~constant~~, $q \propto$ surface Area $= 4\pi r^2$

$$q \propto A_s$$

$$q = \alpha A_s \Rightarrow \boxed{q = \alpha 4\pi r^2}$$ نعوّض في المعادلة التي هي (*)

$$\boxed{\left(\frac{4\pi P}{RT}\right) r^2 \frac{dr}{dt} = -q} \quad \text{---} (*)$$

$$\frac{\cancel{4\pi} P}{RT} \cancel{r^2} \frac{dr}{dt} = -\alpha \cancel{4\pi r^2}$$

$$\left(\frac{P}{RT}\right)\frac{dr}{dt} = -\alpha \Rightarrow$$ seprable and integrable

$$\left(\frac{P}{RT}\right)\int_{10}^{r} dr = \int_{0}^{t} -\alpha\, dt$$

$$\boxed{(r-10) = -\left(\frac{RT\alpha}{P}\right)t}$$

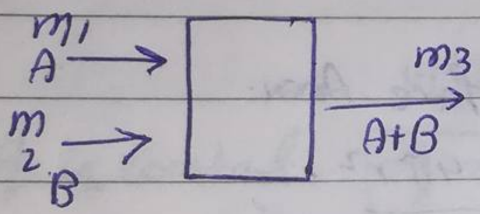
@ $t = 5$ min $r = 7.5$ "to estimate the value of $\left(\frac{\alpha RT}{P}\right)$"

$$\left(\frac{\alpha RT}{P}\right) = 0.5$$

$(r-10) = 0.5t$ at $r = 0.5$ cm

$t = 10$ min $t = ?$

* **Mixer** "dynamic"



Rate of in − Rate of out $= \frac{d}{dt}$ (m t)

$$m_1 + m_2 - m_3 = \frac{d}{dt}(m)$$

$$\frac{dm}{dt} = \begin{cases} + & \text{more } \underline{\text{in}} \text{ than } \underline{\text{out}} \\ 0 & \text{S.S} \\ - & \text{more } \underline{\text{out}} \text{ than } \underline{\text{in}} \end{cases}$$

HW CH.6

p. 5 $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 2\sin(x)$$

$$y = y_c + y_p$$

$y_c \Rightarrow \lambda^2 - 3\lambda - 4 = 0$

$\lambda_1 = -1$ $\lambda_2 = +4$

$$\boxed{y_c = c_1 e^{-t} + c_2 e^{4t}}$$

$y_p \Rightarrow f(t) = 2\sin(x)$

$$y_p = B_1 \sin(x) + B_2 \cos(x)$$

$y' = B_1 \cos(x) - B_2 \sin(x)$

$y'' = -B_1 \sin(x) - B_2 \cos(x)$

Substitute in general eq.

$(-5b_1 + 3b_2) \sin(x) - (5b_2 + 3b_1) \cos(x) = 2\sin(x)$

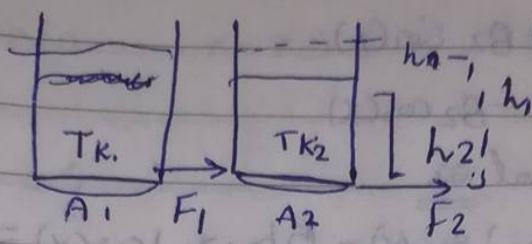
$-5b_1 + 3b_2 = 2$

$-(5b_2 + 3b_1) = 0$

$b_1 = -5/17 , \quad b_2 = 3/17$

3/4/2018

P. 14



$\Delta H = h_1 - h_2$

$F_2 =$

From Flow $TK_1 \longrightarrow TK_2$

$$h_1 > h_2$$

$$F \propto (h_1 - h_2)$$

$$h_1 > h_2$$

$$h_1 - h_2 > (+)$$

$$F_1 = \frac{\Delta h}{R} = \frac{h_1 - h_2}{R} = \beta (h_1 - h_2)$$

current voltage نفس فكرة $\left[I = \frac{V}{R} \right]$

at TK1 :
$$\frac{dh_1}{dt} = \frac{F_{A0}}{A_1} - \frac{\beta_1}{A_2} (h_1 - h_2) \quad \text{--- (1)}$$

$$\frac{dh_2}{dt} = \frac{\beta_1}{A_2} (h_1 - h_2) - \frac{\beta_2}{A_2} h_2 \quad \text{--- (2)}$$

I.C $\Rightarrow$ $h_1(0) = 6\ ft$

$h_2(0) = 5\ ft.$

* S.S condition :-

SS Flow rate $3\ ft^3/min$

h_1 7 ft

h_2 3 ft $\longrightarrow$ flow from $Tk_1 \rightarrow Tk_2$

Diff eq. 2

$$\frac{d^2h_2}{dt^2} = \left(\frac{B_1}{A_2}\right)\frac{dh_1}{dt} - \left(\frac{B_1}{A_2}\right)\frac{dh_2}{dt} - \left(\frac{B_2}{A_2}\right)\frac{dh_2}{dt}$$

$$\left(\frac{B_1}{A_2}\right)\frac{dh_1}{dt} = \frac{d^2h_2}{dt^2} + \frac{(B_1+B_2)}{A_2}\frac{dh_2}{dt} \quad \text{--- } 3$$

sub. eq 3 in eq. 1 :-

$$\left(\frac{B_1}{A_2}\right)\left(\frac{F_0}{A_1} - \underbrace{\frac{B_1}{A_1}(h_1-h_2)}_{\text{subst. this term from eq. 2}}\right) = \frac{d^2h_2}{dt^2} + \frac{(B_1+B_2)}{A_2}\frac{dh_2}{dt}$$

$$\left(\frac{B_1}{A_2}\right)\left(\frac{F_0}{A_1} - \frac{dh_2}{dt} + \frac{B_2}{A_2}h_2\right) = \frac{d^2h_2}{dt^2} + \left(\frac{B_1+B_2}{A_2}\right)\frac{dh_2}{dt}.$$

$$\frac{d^2h_2}{dt^2} + \left(\frac{B_1}{A_1} + \frac{B_1}{A_2} + \frac{B_2}{A_2}\right)\frac{dh_2}{dt} + \left(\frac{B_1B_2}{A_1A_2}\right)h_2 = \left(\frac{B_1}{A_1A_2}\right)F_0 \quad \text{--- } 4$$

$F_0 = 3\ ft^3/min$ (at S.S)

eq. 1 $\rightarrow$ $\frac{F_0}{A_1} - \frac{B_1}{A_1}(\bar{h}_1 - \bar{h}_2) = 0$

$$\bar{F}_1 = B_1(\bar{h}_1 - \bar{h}_2)$$

3/4/2018.

$$\frac{\bar{F_0}}{A_1} - (\bar{h_1} - \bar{h_2})\frac{\beta_1}{A_1} = \frac{\bar{F_1}}{A_1} \Rightarrow \bar{F_1} = \bar{F_0}$$

$$\left(\frac{\beta_1}{A_2}\right)(\bar{h_1} - \bar{h_2}) = \frac{\beta_2}{A_2}\bar{h_2}$$

$$\frac{\bar{F_1}}{A_2} = \frac{\bar{F_2}}{A_2} \Rightarrow \bar{F_1} = \bar{F_2} = \bar{F_0} = 3\ ft^3/min.$$

Find β ?

$$\bar{\beta_1} = \frac{\bar{F_1}}{\bar{h_1} - \bar{h_2}} = \frac{3}{7-3} = 0.75\ \frac{ft^2}{min}.$$

$$\bar{\beta_2} = \frac{\bar{F_2}}{\bar{h_2}} = \frac{3}{3} = 1\ \frac{ft^2}{min}$$

$*\ A_1 = A_2 = 5\ ft^2$

$h_1(0)\ \ 6ft$

$h_2(0)\ \ 5ft$

$\rightarrow$ subs. in eq. (4)

$$\frac{d^2h_2}{dt^2} + 0.5\frac{dh_2}{dt} + 0.03h_2 = \underbrace{0.03 * \bar{FA_0}}_{=0.09}$$

(1) homogenous part:-

$$[\lambda^2 + 0.5\lambda + 0.03] \Rightarrow \lambda_{1,2} = -0.07,\ = -0.43$$

$$h_2(t) = c_1 e^{-0.07t} + c_2 e^{-0.43t}$$

$h_2(p) = C,\ h_2'(p) = 0,\ h_2'' = 0$

$0.03(h_2p) = 0.09 \rightarrow h_2(p) = 3$

$$h_2(t) = c_1 e^{-0.7t} + c_2 e^{-0.43t} + 3$$

i.c $h_2(0) = 5$

$$5 = c_1 + c_2 + 3$$

$$5 - 3 = c_1 + c_2 \rightarrow 2 = c_1 + c_2$$

$$\frac{dh_2(t)}{dt} = -0.7 c_1 - 0.43 c_2 = 0$$

$$\begin{bmatrix} c_1 + c_2 = 2 \\ 0.7 c_1 + 0.43 c_2 \end{bmatrix}$$