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Simulation

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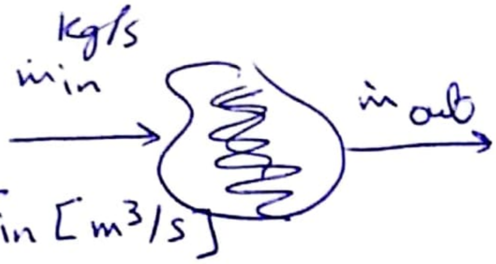
Simulation "previous lectures"



$F_{in}\,[m^3/s]$

$\rho\ [Kg/m^3]$

m_{sys} : mass within the system

$$\dot{m}_{in} - \dot{m}_{out} = \frac{d\,m_{sys}}{dt}$$

temp constant $\approx \rho$ constant.

$$\rho_{in}F_{in} - \rho_{out}F_{out} = \frac{d(V\rho)}{dt}$$

volume constant.

$$F_{in} - F_{out} = \frac{dV}{dt} \rightarrow (*)$$

if volume is constant then $F_{in} = F_{out}$.

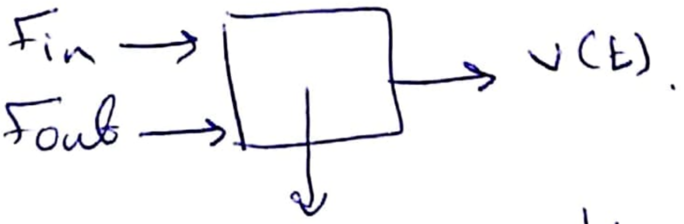
* state variable (independent variable)

Input $\dashv\vdash$ ((F_{in} , $\underline{\underline{F_{out}}}$))

1 eq (*)

3 unk F_{in}, F_{out}, V

DoF = 2

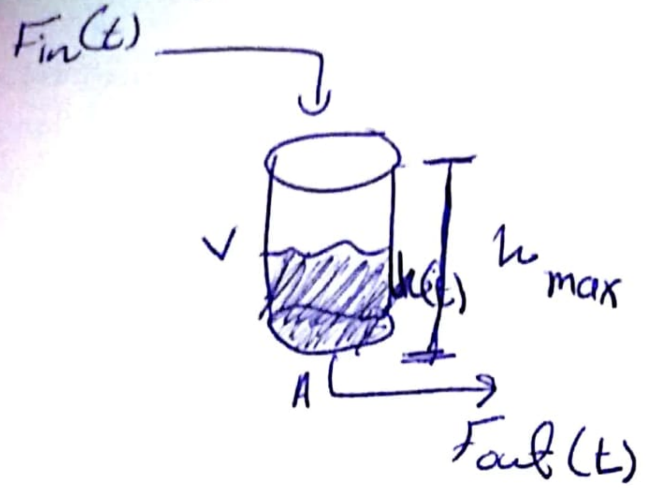


$$F_{in} - F_{out} = \frac{dV}{dt}$$

IC : $V(t=0) = \alpha$

initial condition

Geometry : cylindrical tank



$m = \rho V$

cylinder $\therefore V = A\,h(t)$

$\therefore F_{in}(t) - F_{out}(t) = \frac{dAh}{dt}$

① -- $F_{in}(t) - F_{out}(t) = A\frac{dh}{dt}$

DoF = 3 - 2 = 1

$\therefore F_{in}$ must be F(t)

② --- $F_{out} = f(h)$ -

$F_{out}(t) \propto h(t)$

$\frac{m^3}{s}$ m

$\boxed{F_{out} = \alpha\,h(t)}$ → not realistic

③ --- $\frac{m^3}{s} = \frac{m^2}{s} \cdot m$

Combine 1 with 3

$$F_{in}(t) = A\frac{dh(t)}{dt} + \alpha h$$

→ $F_{in}(t) \longrightarrow \boxed{A\frac{dh}{dt} + \alpha h = F_{in} \quad h(t=0) = \beta} \longrightarrow h(t)$

First order linear ODE
Non-homogenous

(2)

$$A\frac{dh}{dt} + \alpha h = f_{in}(t)$$

↳ At steady state: $\alpha h = \bar{F}_{in}$

$$\alpha = \frac{\bar{F}_{in}}{h}$$

- No inlet

$F_{in} = \text{zero}$

$$A\frac{dh}{dt} + \alpha h = 0$$ homogenous equation

Separate:

$$A\frac{dh}{dt} = -\alpha h$$

$$\frac{dh}{h} = \frac{-\alpha}{A}dt$$

$$\ln h = -\frac{\alpha}{A}t + c$$

$$h = e^{-\frac{\alpha}{A}t + c}$$

$$h = e^{\frac{-\alpha}{A}t} \cdot c_1$$

$$h = c_1 e^{-\frac{\alpha}{A}t}$$

$$h(t) = c_1 e^{-\frac{\alpha}{A}t}$$

$h(t=0) = 1$

$t = \infty \rightarrow e^{-\frac{\alpha}{A}t} = \frac{1}{\infty} = 0$

τ

$$\frac{A}{\alpha} = \frac{m^2/s}{m^2}$$

$\tau = [s]$ time constant

$\therefore h = c_1 e^{-t/\tau}$

* $A\frac{dh}{dt} + \alpha h = 0$

$$\frac{dh}{dt} + \frac{\alpha}{A}h = 0$$

$$\frac{dh}{dt} + \frac{1}{\tau}h = 0$$

$\tau\uparrow$ كل ما [illegible] level constant.

7/10/2019

No Inlet $\quad \frac{A}{\alpha} \frac{\partial h}{\partial t} + h = 0$

$\tau = \frac{A}{\alpha}$ time constant.

$\frac{\partial h}{\partial t} + \frac{1}{\tau} h = 0 \rightarrow \tau \frac{\partial h}{\partial t} + h = 0$ —— ①

$\frac{\partial h}{\partial t} + \frac{1}{\tau} h = 0$ —— ② $\quad h(t) = h_{(0)} e^{-\frac{1}{\tau} t}$

$\tau = \frac{A}{\alpha}$ ① $A <<<$ X-Section $\quad F_o = \alpha L$ linear

$\tau \rightarrow 0$

$\tau \frac{\partial h}{\partial t} \rightarrow 0 \quad h = 0$

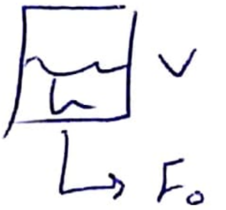
$\tau \frac{\partial h}{\partial t} \rightarrow 0 \quad h - 0$

$A >>>$

eq② $\quad \frac{1}{\tau} \rightarrow 0$

As time constant ↓ steady state ↑ will faster be reached.

No inlet



$$A \frac{dh}{dt} + F_o(t) = 0$$

$$F_o(t) \propto \sqrt{h}$$

$$Flow \propto \sqrt{\Delta P}$$ Bernoulli equation

$$F_o(t) = \beta \sqrt{h}$$

$$A \frac{dh}{dt} + \beta \sqrt{h} = 0$$ Non-linear DE

$$A \frac{dh}{dt} = -\beta \sqrt{h}$$ homogenous.

$$\frac{dh}{\sqrt{h}} = -\frac{\beta}{A} dt$$

$$\int \frac{dh}{\sqrt{h}} = \frac{-\beta}{A} \int dt + C$$

$$2\sqrt{h} = \frac{-\beta}{A} t + C$$

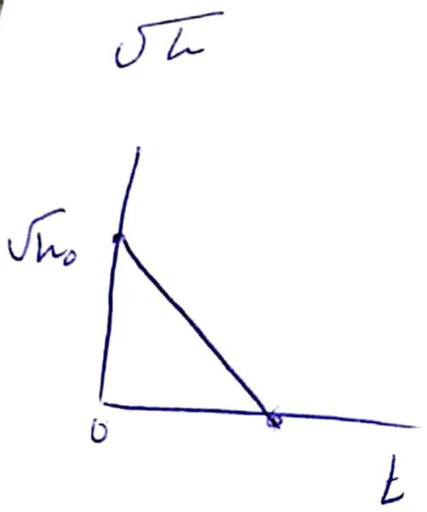
$$\sqrt{h} = -\left(\frac{\beta}{2A} t\right) + C$$

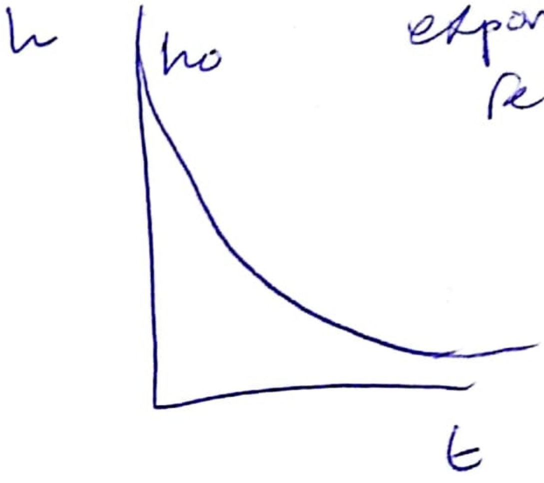
$$h(t=0) = h(0)$$

$$\sqrt{h_{(0)}} = C$$ Non-linear.

(5) $$\sqrt{h} = \frac{-\beta}{2A} t + \sqrt{h_0}$$

linear, decreasing.





exponantional relationship

No outlet flow.

no geometry.

$$m_{in} = \frac{\partial}{\partial t}(m_{system})$$

$$\rho F_{in} = \rho \frac{dV}{\partial t} \Rightarrow \frac{\partial v}{\partial t} = F_{in}.$$

Seperate and Integrate.

$$\partial v = F_{in}\, \partial t$$

$$V = \int F_{in}(t)\, dt + C$$

$V = F_{in}\, t + C$ if F_{in} is constant.

$t = 0$, $v(0) = 0$, $C = 0$

$$v = F_{in}(t)$$

Inlet and outlet Flows:

$$\frac{\partial h}{\partial t} + \frac{\alpha}{A} h = \frac{1}{A} F_{in}(t)$$

F_{in} = Constant (Ct)

$$\frac{\partial h}{\partial t} + \frac{\alpha}{A} h = \frac{1}{A} F_{in}$$ linear, 1^{st} order, non homogenous DE.

Integrating Factor method

Remark: 1^{st} ODE, linear

$y = F(x)$

Standard Form

$$\frac{\partial y}{\partial x} + p(x) y(x) = q(x) \quad \text{—— } (*)$$

Multiply eq $(*)$ by a magic function

$M(x)$ $\qquad$ $M'(x)$

$$M(x) \frac{\partial y}{\partial x} + M(x) p(x) y = M(x) q(x)$$

$$M'(x) = M(x) P(x)$$

$$M(x) \frac{dy}{dx} + M'(x) y = M(x) q(x)$$

$$\frac{\partial}{\partial x} \left(M(x) y(x) \right) = M(x) q(x)$$

$$\frac{\partial}{\partial x} (M(x) y(x) = M(x) q(x)$$

$$\partial(\mu(x)y(x)) = \int \mu(x)\, q(x)\, dx + C$$

$$\mu(x)\cdot y(x) = \int \mu(x)\, q(x)\, dx + C$$

$$y(x) = \frac{1}{\mu(x)} \int \mu(x)\, q(x)\, dx + \frac{C}{\mu(x)}$$

$$p(x) = \frac{\mu^2(x)}{\mu(x)} = \frac{1}{\mu(x)} * \mu^2(x)$$

$$\frac{\partial}{\partial x}\left[\ln \mu(x)\right] = p(x)$$

$$\left[\ln(y(x))\right]' = \frac{1}{y(x)} \cdot y'(x)$$

$$\partial(\ln \mu(x)) = \int p(x)\, dx + C = 0$$

$$\ln \mu(x) = \int p(x)\, dx$$

$$\mu(x) = e^{\int p(x)\, dx}$$

$$\frac{\partial h}{\partial t} + \frac{\alpha}{A} h = \frac{1}{A} F_i \quad — \quad (***)$$

$$\frac{\partial h}{\partial t} + \frac{1}{\tau} h = \frac{1}{A} F_i$$

$$\frac{\partial y}{\partial x} + p(x)\, y = q(x)$$

$$p(x) = \frac{1}{\tau}$$

$$M(x) = e^{\int p(x)\,dx} = e^{\int \frac{1}{\tau} dt}$$

$$M(x) = e^{\frac{1}{\tau} t}$$

Multiply (***) by $M(x)$

$$e^{\frac{1}{\tau}t} \frac{\partial h}{\partial t} + \frac{1}{\tau} e^{\frac{1}{\tau}t} h = e^{\frac{1}{\tau}t} \cdot \frac{1}{A} F_i$$

$$\frac{\partial}{\partial t}\left(e^{\frac{1}{\tau}t} \cdot h\right)$$

$$\frac{\partial}{\partial t}\left(e^{\frac{1}{\tau}t} h\right) = \tau^{\frac{1}{\tau}t} \cdot \frac{1}{A} F_i$$

Seperate and Integrate.

$$\int \partial\left(e^{\frac{1}{\tau}t} \cdot h\right) = \int e^{e\frac{1}{\tau}t} \frac{1}{A} F_i \, dt + C$$

$$e^{\frac{1}{t}t} \cdot h = \frac{F_i}{A} * \tau e^{\frac{1}{\tau}t} + C$$

$$h(t) = \frac{\tau}{A} F_{in} + C e^{-\frac{1}{\tau}t} \quad IC \quad h(t=0) = h(0)$$

$$h(0) = \frac{\tau F_{in}}{A} + C$$

(9) $$C = h(0) - \tau \frac{F_{in}}{A}$$

$$\tau = \frac{A}{\alpha} \rightarrow \alpha = \frac{A}{\tau}$$

$$\frac{1}{\tau} = \frac{\alpha}{A}$$

$$h(t) = \frac{\tau F_{in}}{A} + \left[h(0) - \frac{\tau F_{in}}{A}\right] e^{-\frac{1}{\tau} t}$$

$$h(t) = \frac{F_{in}}{\alpha} - \left(\frac{F_{in}}{\alpha} - h(0)\right) e^{-\frac{\alpha}{A} t}$$

Simulation 3/10/2019.

$$\Rightarrow \frac{dh}{dt} + \frac{1}{\tau}h = \frac{F_{in}}{A}$$

$$\frac{1}{\not{t}} \quad e^{\frac{1}{\tau}t}\frac{dh}{dt} + e^{\frac{1}{\tau}t} \cdot \frac{1}{\tau}h = e^{\frac{1}{\tau}t}\frac{F_{in}}{A}.$$

$$\frac{d}{dt}\left(e^{\frac{1}{\tau}t} \cdot h\right) = e^{\frac{1}{\tau}t}\frac{F_{in}}{A}.$$

Seperate and Integrate.

$$\int d\left(e^{\frac{1}{\tau}t}h\right) = \int e^{\frac{1}{\tau}t}\frac{F_{in}}{A}dt + \text{Constan}$$

$$e^{\frac{1}{\tau}t}h = \frac{\tau F_{in}}{A}e^{\frac{1}{\tau}t} + C$$

$$h(t) = \tau\frac{F_{in}}{A} + Ce^{-\frac{1}{\tau}t}$$

$\underline{I}C$: $h(t=0) = h(0).$

$$h(0) = \tau\frac{F_{in}}{A} + C \qquad C = h(0) - \tau\frac{F_{in}}{A}.$$

$$h(t) = \frac{\tau}{A}F_{in} + e^{-\frac{t}{\tau}}\left[h(0) - \frac{\tau}{A}F_{in}\right]$$

$$\tau = \frac{A}{\alpha} \qquad \alpha = \frac{A}{\tau} \qquad \frac{1}{\alpha}$$

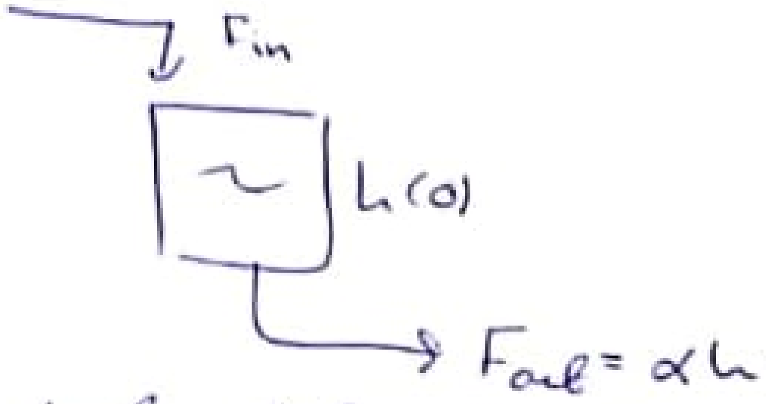
(11)

$$\frac{1}{\tau} = \frac{\alpha}{A}$$

$$\boxed{h(t) = \frac{F_{in}}{\alpha} - \left(\frac{F_{in}}{\alpha} - h(0)\right) e^{-\left(\frac{\alpha}{A}\right)t}}$$

this is the solution.

التأكد
when
$t = 0$
$t \to \infty$



at steady state $\bar{F}_{in} = \bar{F}_{out} = \alpha\, h(0) = F_{in}^*$

$$h(0) = \frac{F_{in}^*}{\alpha}$$

$$h(t) = \frac{F_{in}}{\alpha} - \left(\frac{F_{in}}{\alpha} - \frac{F_{in}^*}{\alpha}\right) e^{-\frac{\alpha}{A}t}$$

Step Function

القيمة
الافتتاحية
F_{in}^* to F_{in}

$$F_{in}(t) = \begin{cases} F_{in} & t \geq 0 \\ F_{in}^* & t < 0 \end{cases}$$

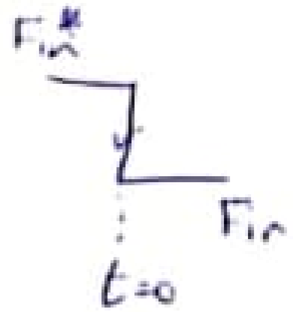
لحين تغير

كل ما بتغير ال F_{in} بتغير h

$$F_{in} \longrightarrow \boxed{} \longrightarrow h$$

$$\Delta F = F_{in} - F_{in}^* > 0$$

$$\Delta F = F_{in} - F_{in}^* < 0$$



$$h(t) = \frac{F_{in}}{\alpha} - \left(\frac{F_{in}}{\alpha} - h(0)\right) e^{-\left(\frac{\alpha}{A}\right) t}$$

Find t when $h = h_{max}$.

حتى لا يفيض ال tank

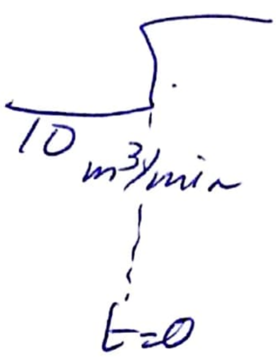
$F^{\alpha} = 10\ m^3/min$

$F_{in} = 15\ m^3/min$

$A = 1\ m^2$

~~calculate~~ α.

$$\alpha = \frac{F^{\alpha}}{h(0)}$$



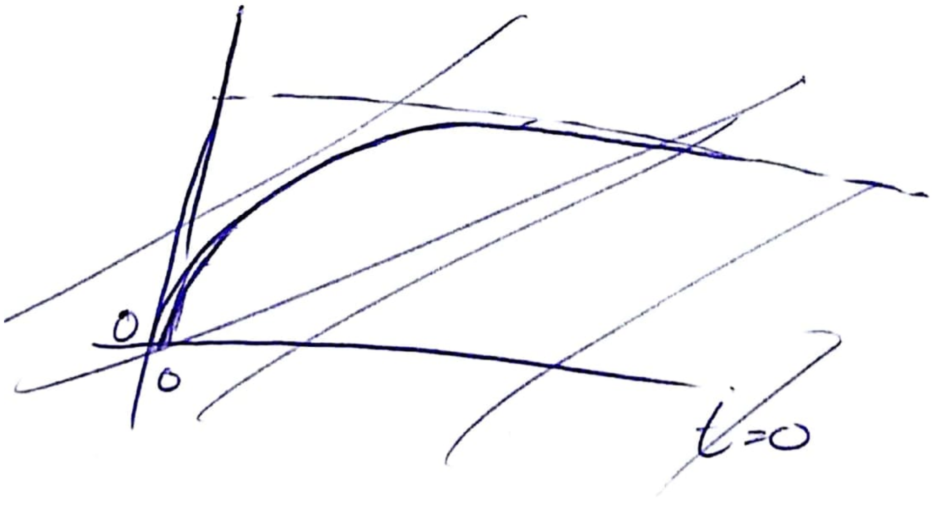
$\Delta F = 5\ m^3/min$

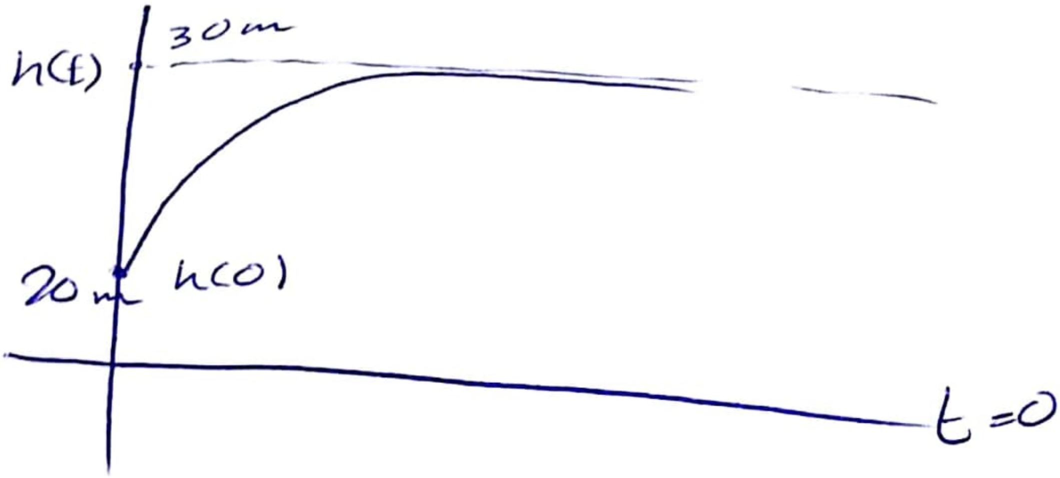
$$\alpha = 0.5\ m^2/min$$

$$h(0) = \frac{10}{0.5} = 20\,m$$

$$h(t) = \frac{15}{0.5} - \left(\frac{15-10}{0.5}\right)e^{-\frac{0.5}{1}t}$$

$$h(t) = 30 - 10e^{-.5t}$$



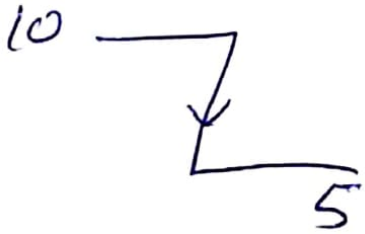


$F_{in}^{*} = 10\ m^3/min$

$F_{in} = 5\ m^3/min$

$A = 1\ m^2$

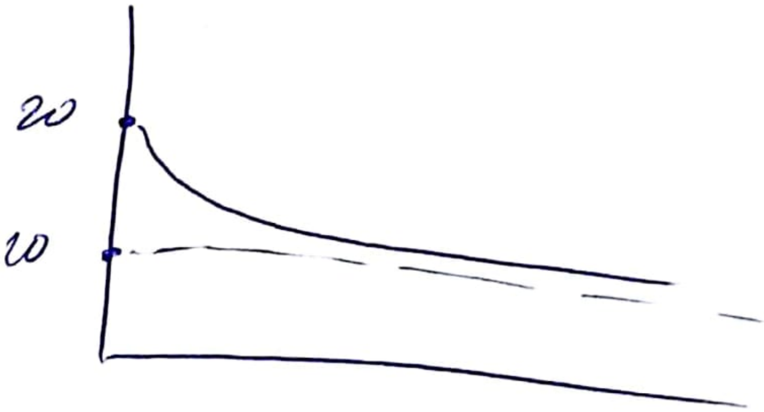
$\alpha = 0.5$



$$\boxed{h(t) = 10 + 10e^{-0.5t}}$$

$h(0) = 20\ m$

$h(\infty) = 10\ m$



بدنا نشوف تأثير ال τ على هاي العلاقات.

$\tau = \dfrac{A}{\alpha}$

كل ما كبر ال τ كبر ال cross sectional area

تقليل level بالتالي

$\rightarrow t = 1 : 0.1 : 20$

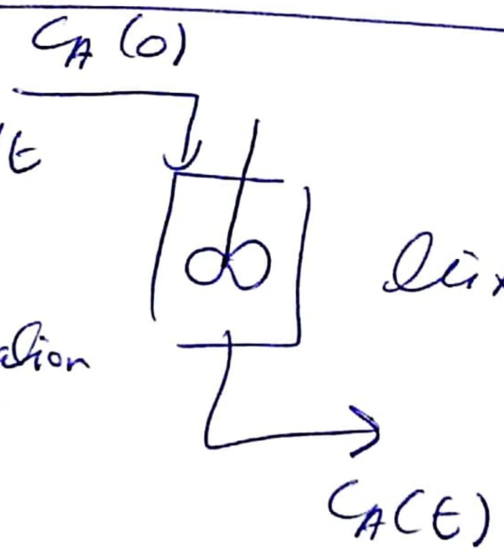
$h(t) = e^{-t_0} \propto \frac{1}{\tau}$

Valve coefficient $\rightarrow$ ~~β~~ α

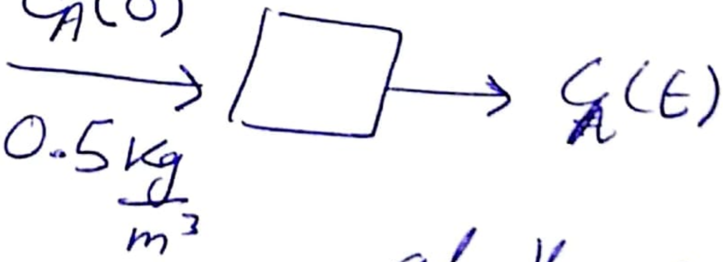
(ii). $A \frac{dh}{dt} + \beta \sqrt{h} = F_{in}$ constant.

nonlinear ODE.

$h(t=0) = h(0)$



الـ concentration تتغير مع الـ time just.



عم بيدخل salt و شو

at the end the concentration in should be = at ...

Simulation 6/10/2019

$$A \frac{dh}{dt} + \alpha \sqrt{h} = F_i$$

F_i ct

Let $x = \sqrt{h}$

$$\cancel{dx = 2\sqrt{h}\, dx}$$

$$dx = \frac{1}{2} \frac{1}{\sqrt{h}} dh$$

$$dh = 2\sqrt{h}\, dx$$

$$2Ax \frac{dx}{dt} + \alpha x = F_i$$

$$2Ax \frac{dx}{dt} = F_i - \alpha x$$

$$\frac{2Ax}{F_i - \alpha x} \frac{dx}{dt} = 1$$

$$\underbrace{\int \left(\frac{x}{F_i - \alpha x} \right) dx}_{LHS} = \int \frac{1}{2A} dt + C$$

$$LHS = \frac{1}{2A} t + C$$

$$\int \frac{x\,dx}{\alpha\left(\frac{F_i}{\alpha}-x\right)} = -\frac{1}{\alpha}\int \frac{-\;x\,dx}{\frac{F_i}{A}-x}$$

$$LHS = -\frac{1}{\alpha}\int \frac{\left(\frac{F_i}{\alpha}-x\right)-\frac{F_i}{\alpha}}{\frac{F_i}{\alpha}-x}\,dx$$

$$= -\frac{1}{\alpha}\int\left(\frac{\frac{F_i}{\alpha}-x}{\frac{F_i}{\alpha}-x}\right)dx - \frac{F_i}{\alpha}\int\frac{1}{\frac{F_i}{\alpha}-x}\,dx$$

$$= -\frac{1}{\alpha}\left[x - \frac{F_i}{\alpha}\int\left(\frac{1}{\frac{F_i}{\alpha}-x}\right)dx\right]$$

$$\int\left(\frac{1}{\frac{F_i}{\alpha}-x}\right)dx \qquad \text{let } u = \frac{F_i}{\alpha}-x$$

$$du = -dx$$

$$\int\frac{-du}{u} = -\ln u$$

$$\ln\frac{1}{u}$$

(18)

$$LHS = \frac{-1}{\alpha}x + \frac{1}{\alpha^2} F_i \ln \frac{1}{\frac{F_i}{\alpha} - x}$$

$$= -\frac{1}{\alpha}x + \frac{1}{\alpha^2}F_i\left[\cancel{\ln 1}^{0} - \ln \frac{F_i}{\alpha} - x\right]$$

~~LHS~~

$$-\frac{1}{\alpha}x - \frac{F_i}{\alpha^2}\ln\left(\frac{F_i}{\alpha} - x\right) = \frac{1}{2A}t + C$$

@ $t=0$

$$\frac{-1}{\alpha}\bar{x} - \frac{F_i}{\alpha^2}\ln\left(\frac{F_i}{\alpha} - \bar{x}\right) = C$$

~~LHS~~

$$+\frac{1}{\alpha}\bar{x} + \frac{F_i}{\alpha^2}\ln\left(\frac{F_i}{\alpha} - \bar{x}\right) + \frac{1}{\alpha}x + \frac{F_i}{\alpha^2}\ln\left(\frac{F_i}{\alpha} - x\right) = \frac{-1}{2A}t$$

$$\frac{F_i}{x^2}\left[\ln\left(\frac{F_i}{\alpha} - x\right) - \ln\left(\frac{F_i}{\alpha} - \bar{x}\right)\right] + \frac{1}{\alpha}(x - \bar{x}) = \frac{-1}{2A}t$$

$$\frac{1}{\alpha}(x - \bar{x}) + \frac{F_i}{\alpha^2}\left[\ln\left(\frac{\frac{F_i}{\alpha} - x}{\frac{F_i}{\alpha} - \bar{x}}\right)\right] = \frac{-1}{2A}t$$

$$\frac{1}{\alpha}(\sqrt{h} - \sqrt{\bar{h}}) + \frac{F_i}{\alpha^2}\left[\ln\left(\frac{\frac{F_i}{\alpha} - \sqrt{h}}{\frac{F_i}{\alpha} - \sqrt{\bar{h}}}\right)\right] = \frac{-1}{2A}t$$

implicit Solution

at steady state $\quad \bar{F}_{in} = \bar{F}_{out} = \alpha\sqrt{h_0}$

$$F^* = \alpha\sqrt{h_0} = \alpha\sqrt{\bar{h}}$$

$$\sqrt{\bar{h}} = \frac{F^*}{\alpha}$$

Remark

$$\ln \frac{\frac{F_i}{\alpha} - \sqrt{h}}{\frac{F_i}{\alpha} - \sqrt{\bar{h}}} = \ln \frac{\frac{F_i - \alpha\sqrt{h}}{\cancel{\alpha}}}{\frac{F_i - \alpha\sqrt{\bar{h}}}{\cancel{\alpha}}}$$

$$\frac{1}{\alpha}\left(\sqrt{h} - \frac{F^*}{\alpha}\right) + \frac{F_i}{\alpha^2}\left[\ln\left(\frac{F_i - \alpha\sqrt{h}}{F_i - \alpha\sqrt{\bar{h}}}\right)\right]$$

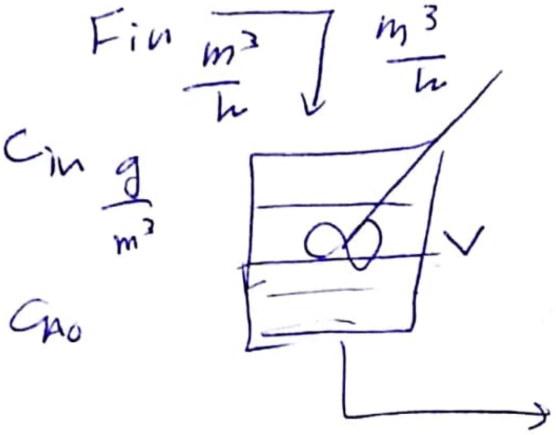
$$= -\frac{1}{2A}t$$

$$\left[\ln \frac{F_i - \alpha\sqrt{h}}{F_i - F^*}\right]$$

initial condition to be Realistic is to take initial steady state conditions.

This is implicit solution.

Mixing tank



$F_{out} = F(t)$

$C_{out} = c(t)$

C_A

$\rho_{in} = \rho_{out} = \rho$

بس عم نفرض

overall mass balance

$$F_{in}(t) - F(t) = \frac{dV}{dt}$$

Component mass balance for A:

$$[FC] = \frac{m^3}{h} \cdot \frac{g}{m^3} = \frac{g}{h}$$

$$F_{in} C_{A0} - F C_A = \frac{d}{dt}(V C_A)$$

$$= \frac{V\, dC_A}{dt} + C_A \frac{dV}{dt} \text{ ----}$$

$$F_{in} - F = \frac{dV}{dt} \text{ ---- (1)}$$

$$F_{in} C_{A0} - F C_A = V \frac{dC_A}{dt} + C_A (F_{in} - F)$$

$$\quad = \quad + C_A F_{in} - C_A F$$

$$F_{in} C_{A0} = V \frac{dC_A}{dt} + C_A F_{in}$$

$$F_{in} C_{A0} = \frac{dC_A}{dt} + \frac{C_A F_{in}}{V}$$

Simulation 8/10/2019

Mixing tank

MB A $F_{in}C_{A0} - FC_A = \frac{d}{dt}(VC_A)$

$$C_A(t=0) = C_A(0) \rightarrow 2$$

$$F_{in}C_{A0} - FC_A = V\frac{dC_A}{dt} + C_A\frac{dV}{dt}$$ from equation 2

$$F_{in}C_{A0} - FC_A = V\frac{dC_A}{dt} + C_A(F_{in}-F)$$

$$+ C_A F_{in} - C_A F$$

$$F_{in}C_{A0} = V\frac{dC_A}{dt} + C_A F_{in}$$

$$\frac{dC_A}{dt} = \frac{F_{in}}{V}(C_{A0} - C_A)$$

Volume not constant.

$$\frac{dC_A}{dt} + \frac{F_{in}}{V}C_A = \frac{F_{in}}{V}C_{A0}(t)$$

$$V(t=0) = V(0)$$

$$C_A(t=0) = C_A(0)$$

$F_i(t)$, $F(t)$ function of time.

the state variable $V(t)$, $C_A(t)$

the input F_{in}, F, $C_{A0}(t)$

(23) (should specify to solve the system)

* if volume constant (V ct)

From equation 1, $F_i - F = 0$, $\bar{F}_i = \bar{F} = F$

$$\frac{dC_A}{dt} + \underbrace{\left(\frac{F}{V}\right)(C_A) = \left(\frac{F}{V}\right)}_{\text{here are constant.}} C_{A_0}$$

$$\left[\frac{F}{V}\right] = \left[\frac{\frac{m^3}{hr}}{m^3}\right] = \left[\frac{1}{hr}\right] \Rightarrow \frac{1}{\tau} = \frac{F}{V}$$

time constant.

* نعطي أرقام

$C_{A_0} = 0.5 \frac{Kg}{m^3}$

$F = 2 \frac{m^3}{hr}$

$V = 100 m^3$

$C_A(0) = 0$

نكمل

$$\frac{dC_A}{dt} + \frac{2}{100} C_A = \frac{2}{100} \times .5$$

$$\frac{dC_A}{dt} + \frac{C_A}{50} = \frac{1}{100}$$

linear non homogenous

⇓

Integrating Factor.

$$\mu(x) = e^{\int \frac{1}{50} dt} = e^{t/50}$$

$$e^{t/50} \frac{dC_A}{dt} + \frac{e^{t/50}}{50} C_A = \frac{e^{t/50}}{100}$$

$$\frac{d}{dt}\left(e^{t/50} \cdot C_A\right) = \frac{1}{100} e^{50t}$$

$$e^{t/50} \cdot C_A = \frac{1}{100} \int e^{t/50} + k$$ ← constant of Integral

$$= \frac{50}{100} e^{t/50} + k$$

$$C_A(t) = \frac{1}{2} + K e^{-t/50}$$

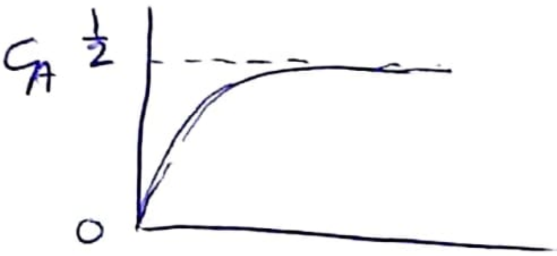
@ $t=0$, $C_A(0) = 0$, $0 = \frac{1}{2} + K$, $K = -\frac{1}{2}$

$$C_A(t) = \frac{1}{2}\left[1 - e^{-t/50}\right]$$

at time $= 0$, $C_A(0) = \cancel{\tfrac{1}{2}}\ 0$ اللي داخل ال tank

من اللي داخل على ال tank

time $= \infty$, $C_A(\infty) = \cancel{\tfrac{1}{2}}$, $\frac{1}{2}$ make ~~sinse~~ sense



$$\frac{dC_A}{dt} + \frac{1}{50} C_A = \frac{1}{100}$$

---

linear nonhomogenous

$$C_A = \overset{C_{A,c}}{C_{A,\text{complementary}}} + \overset{C_{A,P}}{C_{A,\text{Particular}}}.$$

$C_{A,c}$

$$\frac{dC_A}{dt} + \frac{1}{50} C_A = 0$$

Charasteristic equation: $r + \frac{1}{50} = 0$

$r = (-1/50)$

$$C_{A,C} = K e^{-1/50\, t}$$

<u>Particulate</u>

$$\frac{1}{50}\beta = \frac{1}{100}$$

$$\beta = \frac{1}{2}$$

$$C_{A,P} = \beta$$

$$C_{A,P} = 0$$

$$C_A(t) = \frac{1}{2}\left[1 - e^{-\frac{1}{50}t}\right]$$

$$C_A = K e^{-\frac{1}{50}} + \frac{1}{2}$$

$$C_A = K e^{-\frac{1}{50}t} + \frac{1}{2}$$

$$0 = K + \frac{1}{2} \qquad K = -\frac{1}{2}$$

More complicated now! there is a Rxn.

Continous Reactor (Rx) CSTR

$C_{A\,in}$

F_{in}

C_A

F

volume is constant

$F_{in} = F$

Mass balance on Reactant A

$$F_{in} C_{A\,in} - F C_A + r_A V = \frac{d(V C_A)}{dt}$$

I have to specify r_A : [mol / [time . Volume]]

need kinitics

First order rxn

$$r_A = -\frac{dC_A}{dt} = KC_A$$

$$F_{in} C_{A,in} - F C_A - V K C_A = \frac{d(VC_A)}{dt}$$

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A,in} - C_A) - KC_A$$

$$\frac{dC_A}{dt} + \left(\frac{F}{V} + K\right) C_A = \frac{F}{V} C_{A,in}$$

$$\frac{dC_A}{dt} + \left(\frac{F + KV}{V}\right) C_A = \frac{F}{V} C_{A,in}$$ with Rxn

When $K = 0$ (no Rxn)

we got the same equation

$$\frac{dC_A}{dt} + \frac{F}{V} C_A = \frac{F}{V} C_{A,in}$$ no Rxn

No Rxn

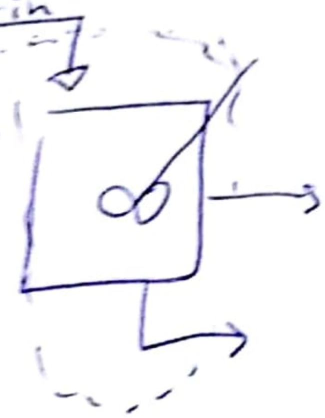
$$\tau = \frac{V}{F}$$

with Rxn

$$\tau_{rxn} = \frac{V}{F + VK}$$

$K > 0$ always positive

τ_{rxn} . for



نفس المثال اللي قبل

10/10/2019

Block box

DoF = 1

~~to~~ we should specify $C_{Ain}(t)$ as a function of t

could be → Constant (Step)

Ramp

$C_A = at + b$

$\frac{\Delta C_A}{\Delta t} = a$

بزيد مع الزمن بخط مستقيم

$$F_{in} - F = \frac{dV}{dt}$$

$$\frac{dC_A}{dt} + \left(\frac{F+VK}{V}\right) C_A = \frac{F_A}{V} C_{Ain}$$

If C_{Ain} is constant,

الحل

$$C_A(t) = A e^{-\left(\frac{F+VK}{V}\right)t} + \left(\frac{F}{F+VK}\right) C_{Ain}$$

$$C_A(t=0) = \bar{C}_A$$

$$A = -\frac{F}{(F+VK)} C_{Ain} + \bar{C}_A$$

$$A + 2B \rightleftharpoons P$$

$r_A = -$

$r_B = 2r_A$

$r_P = r_A$

$$F_{in} C_{Ain} - F C_A + V r_A = \frac{d}{dt}(V C_A)$$

$$F_{in} C_{Bin} - F C_B + V r_B = \frac{d}{dt}(V C_B)$$

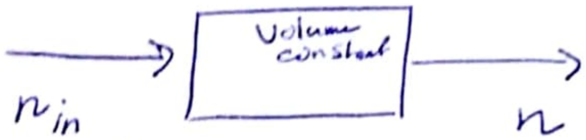
$$r_A = -K C_A C_B$$

$$r_B = -2 K_A C_A C_B$$

~~Sep~~
Simplify the rate of rxn, not C_B^2

13/10/2019.

* gas Surge Drum



Develop a dynamic model.

molar flow rate
[mol/h]

$$n_{in} - n = \frac{d}{dt}(n)_{\text{within the system}}$$

* Ideal gas

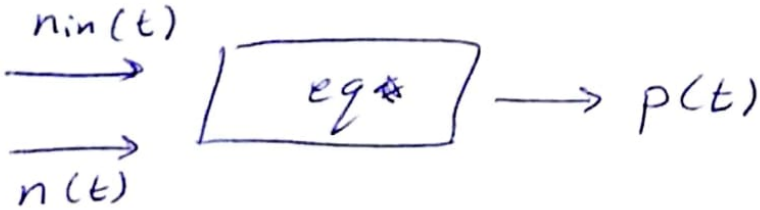
$$pV = nRT$$
$$n = \frac{pV}{RT}$$

* Isothermal
  T = constant

* V constant.

$$n_{in} - n = \left(\frac{V}{RT}\right)\frac{dP}{dt} \rightarrow \frac{dP}{dt} = \frac{RT}{V}(n_{in} - n)$$

State variable $p(t)$
Input variable n_{in}, n
Parameter : T, V



* Van der Waals

$$\left(P + \frac{a}{\hat{V}^2}\right)(\hat{V} - b) = RT$$

مثالي
من 1-5 bar
كل ما يزداد الضغط يبطل

$a, b = f$ (gas).

* Equilibrium relationship.

$$y_i = K_i x_i$$

$$\frac{y_i}{x_i} \quad \frac{V}{L} \Big\} K \qquad \underline{\underline{K_i}} = \frac{P_i^*(T)}{P} \ , \quad P_i^*(T) = A - \frac{B}{T + C}$$

vapor pressure

distrubution coefficient.

total pressure

Antoine equation.

Mathmatical model need

* Mass and energy balance
* Kinitics
* Equilibrium relationship.
* Heat Transfer: $Q = UA\Delta T$ correlation

when $K_i \ll$ almost all in ~~vapor~~ liquid.

* Flow through valves

$$\underset{\text{current}}{I} = \frac{V}{R} \qquad \frac{\text{driving Force}}{\text{Resistance}}.$$

Valve

$$F \propto \sqrt{h}$$

$$F \propto \sqrt{\Delta P} \rightarrow \rho g h$$

$\Delta P = P_1 - P_2$

$F \propto \sqrt{\Delta P}$

$$F = \frac{C_v \sqrt{\frac{\Delta P}{\text{sepecific gravity (SG)}}}}{R}$$

C_v → valve coefficient

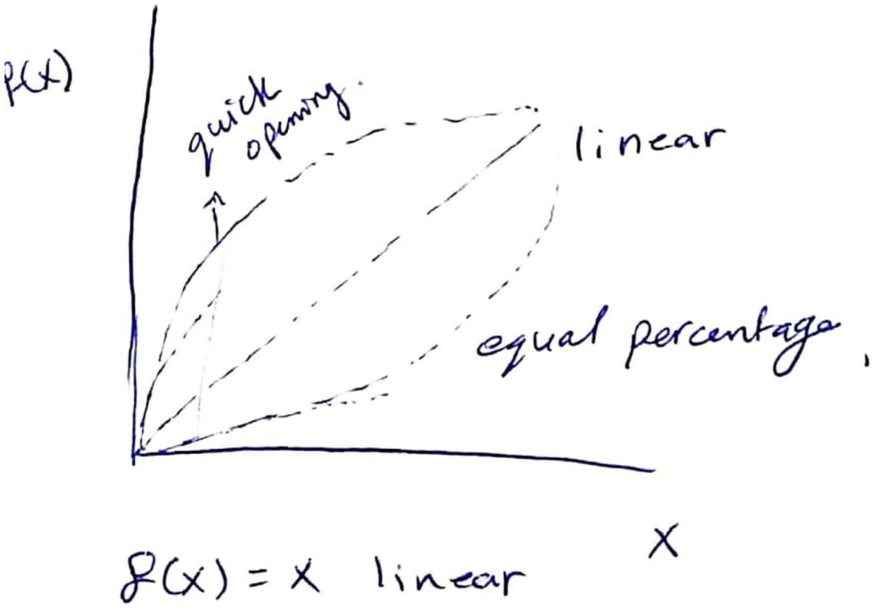
specific gravity (SG) → $\frac{\rho}{\rho_{H_2O}}$

$C_v = H$ (valve Type)

$$F = \frac{C_v\, f(x) \sqrt{\frac{\Delta P}{SG}}}{R}$$

$f(x)$ → flow charcteristic (0, 1)

x : Fraction of valve opening.

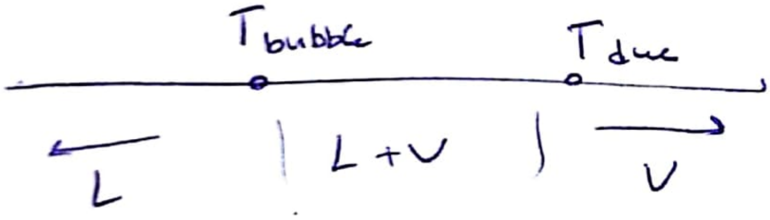


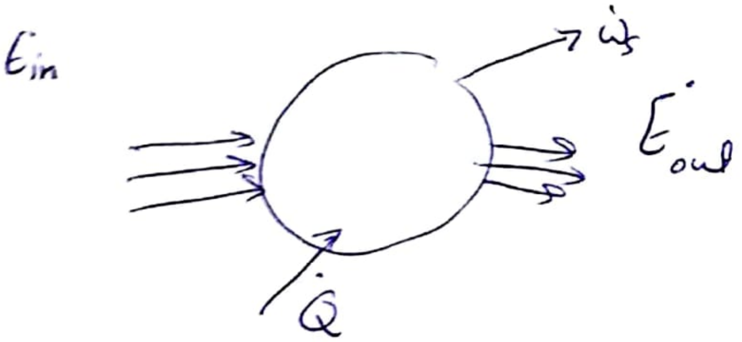
$f(x) = x$ linear

$f(x) = \alpha^{x-1}$ equal percentage ($\alpha = 50$)

$f(x) = \sqrt{x}$ quick opening.

* To seperate the Temprature should be between bubble and due points.





$$E = U + KE + PE$$

Internal energy

$$E_{in} - E_{out} + r_A V + \omega + Q = \frac{d\hat{U}}{dt} = \frac{d(m\hat{U})}{dt}$$

within the system

$$\hat{U} = \frac{U}{m} = \frac{KJ}{Kg}$$

if the system is a steady state in mass

$$m \frac{d\hat{U}}{dt}$$

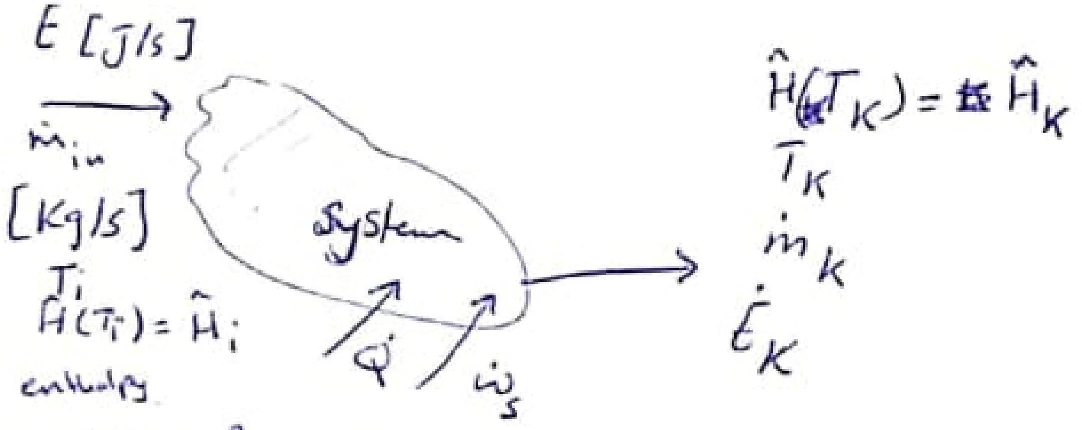
$$c_v = \frac{d\hat{U}}{dT}$$

15/10/2019

Energy balance. (EB)

$$\begin{bmatrix}\text{Rate of Energy}\\ \text{in}\\ \text{by convection}\end{bmatrix} - \begin{bmatrix}\text{Rate of energy}\\ \text{out}\\ \text{by convection}\end{bmatrix} + \begin{bmatrix}\text{Net rate}\\ \text{of heat}\\ \text{addition}\\ \text{from surroundings}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{Net Rate of}\\ \text{work performed}\\ \text{by surroundings}\\ \text{on the system}\end{bmatrix} = \begin{bmatrix}\text{Rate of energy}\\ \text{accumulation}\end{bmatrix}$$



كل اشي بدخل للسيستم اشارته positive.

$$\Delta \dot{H} + \Delta \dot{E}_P + \Delta \dot{E}_K = \dot{Q} + \dot{W}$$

$$\Delta \dot{H} = \sum_{\substack{out\\K}} \dot{m}_K \hat{H}_K - \sum_{\substack{in\\j}} \dot{m}_j \hat{H}_j$$

$$-\Delta \dot{H} + \dot{Q} + \dot{W} = \frac{d(U)}{dt} \text{ within the system}$$

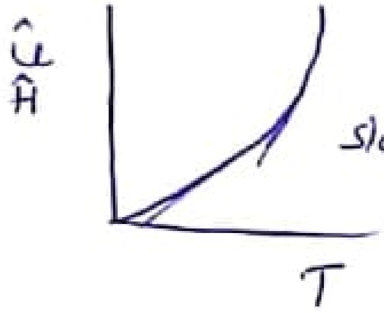
$$U_{System} = m_{sys} \hat{H}$$

steady state with respect to mass $\rightarrow$ $mass_{in} = mass_{out}$
$\Rightarrow m_{sys} = \text{Constant}$

$$\sum_{i}^{in} \dot m_i \hat H_i - \sum_{k}^{out} \dot m_k \hat H_k + \dot W + \dot Q = \frac{dU}{dt}$$

$$U = m_{sys}\hat U , \quad \frac{dU}{dt} = \frac{d}{dt}(m_{sys}\hat U) = m_{syst}\frac{d\hat U}{dt}.$$

$$\sum - - - - - - - = m_{sys}\frac{d\hat U}{dt}$$



slope $c_v(T) = \dfrac{d\hat U}{dt}$ $\qquad H = U + PV$

$c_p(T) = \dfrac{d\hat H}{dt}$ $\qquad \hat H = \Delta \hat H$ with respect to reference state.

In : $\Delta \hat H_i$, $\Delta \hat H_k$

$c_p = \dfrac{d\hat H}{dt}$ separate and integrate $\rightarrow$ $\displaystyle\int_{T_{ref}}^{T} d\hat H = \int_{T_{ref}}^{T} c_p \, dT$

$$\hat H(T) - \hat H(T_{ref}) = \bar c_p (T - T_{ref}) \leftarrow \bar c_p = \frac{\int_{T_{ref}}^{T} c_p \, dT}{T - T_{ref}}$$

if we have a mixture from 1 and 2

1	2
x_1	x_2
same T	
$\hat H_1$	$\hat H_2$ if pure compound
$\hat H_{mix}$	

$\hat H_{mix} = x_1 \hat H_1 + x_2 \hat H_2 = \sum_i x_i \hat H_i$ "ideal"

$\hat H_{mix}(x_i, T, P)$

$\dot m \hat H_{mix} = \dot m x_1 \hat H_1 + \dot m x_2 \hat H_2$

$= \dot m_1 \hat H_1 + \dot m_2 \hat H_2$

$$(c_p)_{mix} = \sum_j x_j c_{p_j}$$

$$\hat{H}_{mix}(T) = \sum x_j \hat{H}_j(T) = \sum_j x_j \bar{c}_{p_j} (T - T_{ref})$$

$$= (T - T_{ref}) \sum_j x_j c_{p_j}$$

$$\hat{H}_{mix}(T) = (T - T_{ref}) (\bar{c}_p)_{mix}.$$

$$\sum_{\substack{in \\ j}} m_j \bar{c}_{p_j} (T_j - T_{ref}) - \sum_{\substack{out \\ k}} \bar{c}_{p_k} (T_k - T_{ref}) + \dot{Q} + \dot{w}_s =$$

$$m_{sys} \frac{d\hat{u}}{dt} \rightarrow m_{sys} c_v \frac{dT}{dt}$$

$$c_v = \frac{d\hat{u}}{dT} = d\hat{u} = c_v \, dT$$

$$m_{sys} = \rho v$$

For liquid

$$c_v = c_p = c$$

$$\sum_{\substack{in \\ j}} m_j \bar{c}_{p_j} (T_j - T_{ref}) - \sum_{\substack{out \\ k}} \bar{c}_{p_k} (T_k - T_{ref}) + \dot{Q} + \dot{w}_s = \rho V c_v \frac{dT}{dt}$$

Energy balance for the system without Reaction.

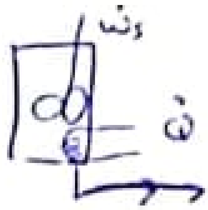
<u>Example</u> Stirred Heating Tank.

c_{pi}

$T_i(t)$

$\dot{m}_i(t)$

$F_i(t)$



$T_m / c_p / F_{out}$.

$$\left[\frac{\rho V c}{\dot{m} C}\right] = \left[\frac{\frac{kg}{m^3} \cdot m^3}{\frac{kg}{s}}\right] = [s]$$ time constant

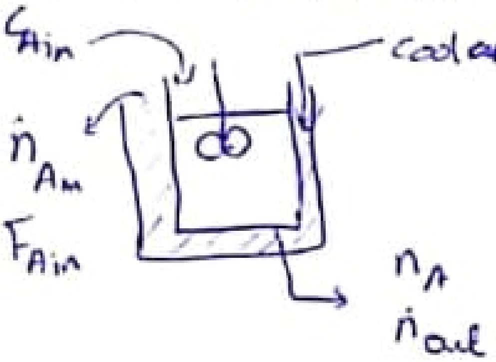
$$\tau = \frac{\rho V c}{\dot{m} C}$$

$$\tau \frac{dT}{dt} + T = \frac{1}{\dot{m} C} Q + \frac{1}{\dot{m} c} \dot{w}_s + T_i(t)$$

linear non homogenous Differenial equation.

17/10/2019

Non-Isothermal Reactor.



, volume is constant.

$A \longrightarrow$ products

$$\frac{dV}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\Rightarrow \dot{m}_{in} = \dot{m}_{out} = m$$

c_p const

MB on A

$$F_{Ain} C_{Ain} - F_A C_A - r_A V = V \frac{dC_A}{dt}$$

Remark: Q_{rxn} $\Delta \hat{H}_r \left[\frac{J}{mol} \right]$ $r_A \left[\frac{mol}{m^3 . h} \right] V \left[m^3 \right]$

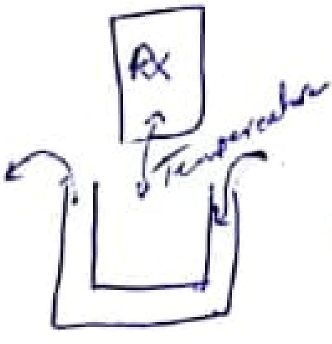
$\frac{J}{hr}$

EB

$$m c_{p_i} (T_i - T_{ref}) - c_p (T - T_{ref}) - Q - \Delta H_r r_A V$$

$$= \rho V c_v \frac{dT}{dt} \qquad Q = UA (T - T_c)$$

$c_v \approx c_p$



Mass balance in = out

Energy balance $Q = UA\Delta T = UA(\underset{\text{system}}{T} - \underset{\text{coolant}}{T_c})$

mass balance

at steady state $\quad mass_{in} - mass_{out} = \dfrac{d(m_{sys})}{dt} = 0$

$$m_{in} = m_{out}$$

Energy balance

$$\dot{m}\,\bar{c}_{p_i}(T_i - T_{ref}) - \dot{m}\,\bar{c}_p(T - T_{ref}) + \dot{Q} + \dot{\omega}_s = \rho \hat{V} c_v \frac{dT}{dt}$$

because it is liquid

then $c_v \approx c_p \approx \bar{c}_{p_i} \approx c$

$$\dot{m}\bar{c}(T_i - T_{ref}) - \dot{m}\bar{c}(T - T_{ref}) + \dot{Q} + \dot{\omega}_s = \rho \hat{V} c \frac{dT}{dt}$$

$$\dot{m}\bar{c}T_i \cancel{- \dot{m}cT_{ref}} - \dot{m}\bar{c}T + \cancel{\dot{m}cT_{ref}} + \dot{Q} + \dot{\omega}_s = \rho V c \frac{dT}{dt}$$

$$\dot{m}\bar{c}(T_i - T) + \dot{Q} + \dot{\omega}_s = \rho V c \frac{dT}{dt}$$

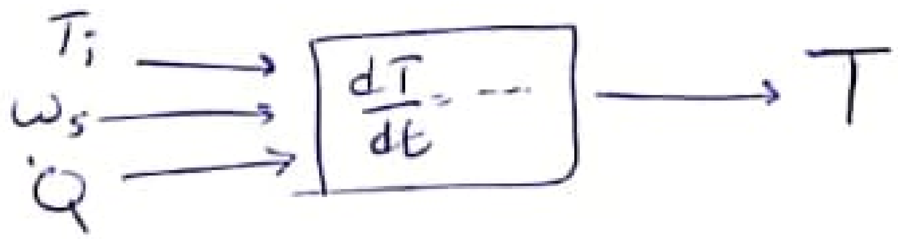
$$T_i - T + \frac{\dot{Q}}{\dot{m}\bar{c}} + \frac{\omega_s}{\dot{m}\bar{c}} = \left(\frac{\rho V c}{\dot{m}\bar{c}}\right)\frac{dT}{dt}$$

phesical properties

Geometry

$$\left(\frac{\rho V c}{\dot{m}\bar{c}}\right)\frac{dT}{dt} + T = \left(\frac{1}{\dot{m}\bar{c}}\right)\dot{Q} + \omega_s\left(\frac{1}{\dot{m}\bar{c}}\right) + T_i(t)$$

flow chalocteristic



Autocatalytic Reaction $A \rightarrow B$

Activated Rxn

$$\frac{dc_B}{dt} = K\, c_B(t)\, \underbrace{(c_{A0} - c_B)}_{c_A(t)}$$

$$\frac{dc_B}{dt} = -K c_{A0} c_B = -K c_B^2$$ Non-linear equation.

Bernouli equation

$$c_B^{-2} c_B' - K c_{A0} c_B^{-1} = -K$$

let $v = c_B^{-1}$

$$dv = -c_B^{-2}\, dc_B$$

$$c_B^{-2} \cdot c_B' = -v'$$ $c_B = \frac{1}{v}$

$$-v' - K c_{A0} v = -K$$ Linear, first order

$K c_{A0} \rightarrow \beta$

$$v' - \beta v = +K \Rightarrow$$ Integrating factor method.

$$\mu(t) = e^{\int \beta dt} = e^{\beta t}$$

$$e^{\beta t} v' - \beta v e^{\beta t} = K e^{\beta t}$$

$$\frac{d}{dt}(e^{\beta t} \cdot v) = K e^{\beta t} \rightarrow \int d(e^{\beta t} \cdot v) = \int K e^{\beta t} dt + c$$

$$e^{\beta t} \cdot v = \frac{K}{\beta} e^{\beta t} + c$$

$$\frac{K}{\beta} = \frac{K'}{K' c_{A0}} = \frac{1}{c_{A0}}$$

Bernouli equation.

$$\frac{dy}{dx} + P(x)\,y = g(x)\,y^{n}$$

$$y^{-n}y' + P\,y^{1-n} = g$$

$y^{1-n} \to v$

$$v = y^{1-n}$$

$$v' = 1-n\;\; y^{-n}.y'$$

$$y^{-n}.y' = \frac{1}{1-n}\,v'$$

then $\frac{1}{1-n}v' + Pv = g$

if $n = 0, 1$ then Integrating factor form.

Algebric equations

5/11/2019

$x_1, x_2, \dots x_n$ functions.

$x_1(t), x_2(t), x \dots x_n(t)$

$$\left[\begin{array}{l} x_1' = f_1(x_1, x_2 \dots, x_n) \\ x_2' = f_2(x_1, x_2 \dots x_n) \\ \vdots \\ x_n' = f_n(x_1, x_2 - x_n) \\ \text{IC}: \; x_1(0), x_2(0) \cdots x_n(0) \end{array}\right.$$

Differential system.

$\underline{x}' = \underline{f}[\underline{x}]$

at steady state.

$x_1' = \frac{dx_1}{dt} = 0$

$x_2' = \frac{dx_2}{dt} = 0$

$\vdots$

$\frac{dx_n}{dt} = 0$

at S.S

$f_1(\bar{x}_1, \bar{x}_2 \dots \bar{x}_n) = 0$

$f_2(\bar{x}_1, \bar{x}_2 \dots \bar{x}_n) = 0$

$\vdots$

$f_n(\bar{x}_1, \bar{x}_2, \dots \bar{x}_n) = 0$

$\underline{f}(\underline{x}) = \underline{0}$

* we need algabric system of equation to find the initial condition.

$\left.\begin{array}{l} x_1(0) = \text{—} \\ x_2(0) = \text{—} \end{array}\right\}$ IC

▶ Subject : .... / /

Linear system of ODE . Non - Homogeneous .

$$x_1' = \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n + \underline{g_1(t)}$$

$$x_2' = \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \underline{g_2(t)}$$

$\vdots$

$$x_n' = \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + \underline{g_n(t)}$$

$g_i \neq 0$ at least constant

$\underline{g}(t)$ are input variables.

$\bar{X}$ state variables.

$\Downarrow$

$$\begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix} + \begin{bmatrix} g_1(t) \\ g_2(t) \\ \vdots \\ g_n(t) \end{bmatrix}$$

$$x' = A\underline{x} + \underline{g}(t) \qquad n\times n \qquad n\times 1$$

at s.s. $A\underline{x} + \underline{g}(t) = 0$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + g_1 = 0$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + g_2 = 0$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + g_n = 0$$

$x_1 \ldots x_2 \ldots$

S. S. values

$$\underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$

$a_{11}x_1 + a_{12}x_2 + \cdots a_{nn}x_n = -g_1$

$= -g_2$

$= -g_n$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -g_1 \\ -g_2 \\ -g_n \end{bmatrix}$$

$A\underline{x} =$ vector.

$A\underline{x} = \underline{g}$ $\Rightarrow$ $\boxed{Ax = b}$ linear system of equations.

Methods of solution

LU Factorization

Gaussian Elimination.

$\boxed{A}\,\boxed{x} = \boxed{b}$

$\boxed{A} = \boxed{L}\cdot\boxed{U}$

$\boxed{A}\,\boxed{x} = \boxed{L}\,\boxed{U}\,\boxed{x} = \boxed{b}$

$Ax = LUx = b$

let $Ux = C$ $\Rightarrow$ solve for vector C

then solve for x

$Ux = C$ $\quad LC = b$

Subject :

$A x = b$

$A^{-1} A x = A^{-1} b$

$I x = A^{-1} b$

$x = A^{-1} b$ A^{-1} exists.

to check A^{-1}

$\det(A) \neq 0 \Rightarrow A^{-1}$ exist.

$\det(A) = 0 \Rightarrow A^{-1}$ is not exist.

rank$(A) < n$.

Example:

$x_1 + 2x_2 = 2$) 2 $\Rightarrow 2x_1 + 4x_2 = 4$

$2x_1 + 4x_2 = 4$ $\qquad 2x_1 + 4x_2 = 4$ } → Dependency equations

Dependent

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$x_1 + x_2 = 2 \Rightarrow$

Rank$(A) = 1$

det(1)(1)(4) $- (2)(2) = 0$

A^{-1} does not exist

rank $(A) < 2$.

if [A] Singular matrix $\Rightarrow$ we have infinite number of solution

while if $\det(A) \neq 0 \Rightarrow$ we have unique solution.

Subject

$X = A^{-1} b$ if A full Rank
$\det(A) \neq 0$

In MatLab:

~~X = inv(A) * b~~ ~~find inv(A)~~

detA = det(A) ⇒ ≠ 0 use

X = inv(A)*b

$AX = b$

X = b\A → LU factorization technique

fsolve: System of nonlinear eq.

$f(\bar{x}) = 0$

n equations is nonlinear

$$f(\underline{x}) = \begin{cases} f_1(x_1, x_2, \dots x_n) = 0 \\ f_2(x_1, x_2 \dots x_n) = 0 \\ \vdots \\ f_n(x_1, x_2 \dots x_n) = 0 \end{cases}$$

Newton's method.

$f(x) = 0$ 1 equation 1 variable

Expand using Taylor series around a given point x_0

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0^2)}{2!} + \dots$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) = 0$$

Solve for x:

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\text{Point } \underline{x} + \Delta \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} x_1 + \Delta x_1 \\ x_2 + \Delta x_2 \\ \vdots \\ x_n + \Delta x_n \end{bmatrix}$$

Expand around $\underline{x}$

$$\underline{\bar{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$\cancel{f(\bar{x}) = 0}$

$$f(\underline{\bar{x}} + \Delta \underline{x}) = \begin{bmatrix} f_1(\underline{x} + \Delta \underline{x}) \\ f_2(\underline{x} + \Delta x) \\ \vdots \\ f_n(\underline{\bar{x}} + \Delta \underline{x}) \end{bmatrix} = \begin{matrix} f_1(x_1 + \Delta x_1, x_2 + \Delta x_2, \ldots x_n + \Delta x_n) \\ f_2(x_1 + \Delta x_1, x_2 + \Delta x_2 \\ \vdots \\ f_n(x_1 + \Delta x_1, x_2 + \Delta x_2 \end{matrix}$$

Equal zeros

Expand around $(x_1, x_2, \ldots x_n)$

x_1 $\qquad$ $x_1 + \Delta x_1$

$$\overline{x_1 + \Delta x_1 - x_1} = \Delta x_1$$

$$f(x, y, z) = f(x_0, y_0, z_0) + \frac{\partial f}{\partial x}\Big|_{x_0, y_0, z_0} (x - x_0) + \frac{\partial f}{\partial y}\Big|_{x_0, y_0, z_0} (y - y_0) + \frac{\partial f}{\partial z}\Big|_{x_0, y_0, z_0} (z - z_0) + \cdots$$

(2)

$$(x_1 + \Delta x_1) - x_1 = \Delta x_1$$
$$(x_2 - \Delta x_2) - x_2 = \Delta x_2$$

Expand

$$f_1(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots) = f_1(x_1, x_2, \dots x_n) + \left.\frac{\partial f_1}{\partial x_1}\right|_{(x_1, x_2, \dots x_n)} \Delta x_1 + \left.\frac{\partial f_{1}}{\partial x_2}\right| \Delta x_2 + \dots + \left.\frac{\partial f_1}{\partial x_n}\right|_{\underline{x}} \Delta x_n = 0$$

$$f_2(x_1, x_2, \dots x_n) + \left.\frac{\partial f_2}{\partial x_1}\right|_{\underline{x}} \Delta x_1 + \left.\frac{\partial f_2}{\partial x_2}\right|_{\underline{x}} \Delta x_2 + \dots \frac{\partial f_2}{\partial x_n} \Delta x_n = 0$$

$$f_n($$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} =$$

$$\begin{bmatrix} f_1(\underline{x}) \\ f_2(\bar{\underline{x}}) \\ \vdots \\ f_n(\bar{\underline{x}}) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & & \\ \frac{\partial f_2}{\partial x_1} & & \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1} & & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Jac~~k~~obian

$$\underset{\text{given}}{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \\ & \frac{\partial f_n}{\partial x_n} \end{bmatrix}} \underset{\Delta x}{\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}} = - \underset{\substack{\text{given} \\ f(\bar{\underline{x}})}}{\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}}$$

$J(x_1, x_2, \dots x_n)$

$J(\bar{\underline{x}})$

$J(\bar{\underline{x}})\, \Delta \underline{x} = -f(\underline{x})$ Linear system

solve for $\Delta \bar{\underline{x}}$ $\Delta \underline{x} = (\underline{x} + \Delta\underline{x} - \underline{x})$

At iteration $\underline{\underline{K}}$:

$J(\underline{x}_K)\, \Delta \underline{\underline{x}}_K = -f(\underline{x}_K)$

solve for Δx_K

to get x_{K+1} : $x_{K+1} = x_K + \Delta x_K$

$\det J(x_K) = 0$

$\Delta x_K = -J(x_K)\, f(x_K)$

example

$$f_1(x_1, x_2) = x_1 - 4x_1^2 - x_1 x_2 = 0$$

$$f_2(x_1, x_2) = 2x_2 - x_2^2 + 3x_1 x_2 = 0$$

in vector form

$$\underline{f}(\underline{x}) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x_1 - 4x_1^2 - x_1 x_2 \\ 2x_2 - x_2^2 + 3x_1 x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Jacobian method.

$$J(x_1, x_2) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_1} \end{bmatrix}, \quad \frac{\partial f_1}{\partial x_1} = 1 - 8x_1 - x_2$$

$$\frac{\partial f_1}{\partial x_2} = -x_1$$

$$\frac{\partial f_2}{\partial x_1} = 3x_2$$

$$\frac{\partial f_2}{\partial x_2} = 2 - 2x_2 + 3x_1$$

$$J = \begin{bmatrix} 1 - 8x_1 - x_2 & -x_1 \\ 3x_2 & 2 - 2x_2 + 3x_1 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} x_1 = -1 \\ x_2 = -1 \end{bmatrix}$$

3 illeration حل by hand

At illeration 0:

$$J(x_0)\,\Delta x_0 = -f(x_0)$$

$$x_1 = x_0 + \Delta x_0 \leftarrow \Delta x_0$$

$$\tau(s) = K_c\left[1 + \frac{\tau_I}{s} + \Delta\tau_D\right]$$

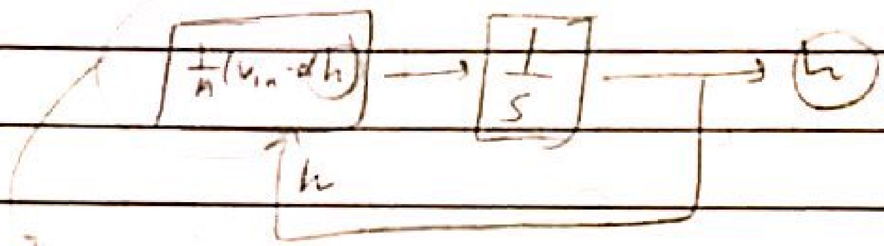
موضوع الدرس ________ اليوم ________ التاريخ / /

$$\frac{dh}{dt} = \frac{1}{A}\left(V_{in} - \alpha h\right)$$

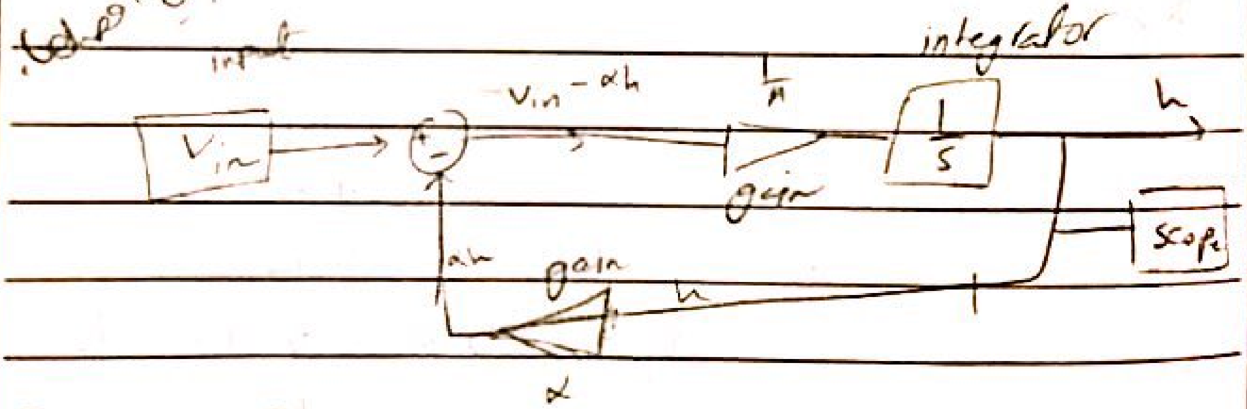
$$h = \int \frac{1}{A}\left(V_{in} - \alpha h\right) dt$$

input

$\left(\frac{1}{s}\right)$ integrator



بشكل اوضح

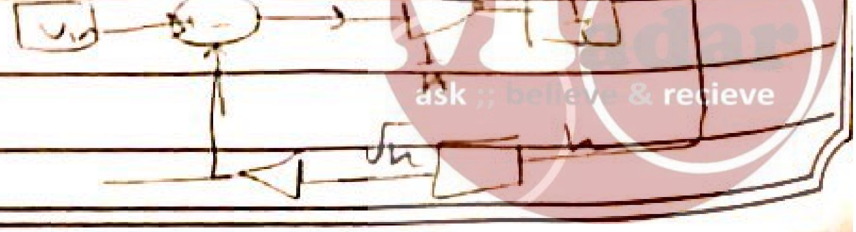


$D = 2\,m$

$V_{in} = 2\,m^3/min$

$\alpha = 0.4$

if $F = \alpha\sqrt{h}$ then ?



CH4: 2, 7

numerical
Simulank
Matlab command.

موضوع الدرس ______ اليوم ______ التاريخ / /

dsolve Analytical solution
if there is a solution.

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

eig(B)

B = [-3 -2 ; -2 -3]

[y1 y2] = dsolve('Dy1 = -3*y_1 +

$y_1' = 3y1 + 4y_2$

$y_2' = -4y_1 + 3y2$

symbolic → syms a b c x

x = solve(a*x^2 + b*x + c)

بعض الـ
حل analytically

x = -(b+(b^2 -4*ac)^(1/2)
-(b-

مرات بعرف
كيف احل
والقوانين

موضوع الدرس ______ اليوم ______ التاريخ / /

$3x^2 + 2x + 1 = 5x + 12$

syms x

x = solve(___).

بنقل الطرف كله يسا

double لجول لأرقام double(x)