CHAPTER 2

Section 2-1

Provide a reasonable description of the sample space for each of the random experiments in Exercises 2-1 to 2-17. There can be more than one acceptable interpretation of each experiment. Describe any assumptions you make.

2-1. Each of three machined parts is classified as either above or below the target specification for the part. Let *a* and *b* denote a part above and below the specification, respectively.

$$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

2-2. Each of four transmitted bits is classified as either in error or not in error.

Let *e* and *o* denote a bit in error and not in error (*o* denotes okay), respectively.

$$S = \left\{\begin{matrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{matrix}\right\}$$

2-3. In the final inspection of electronic power supplies, either units pass, or three types of nonconformities might occur: functional, minor, or cosmetic. Three units are inspected.

Let *a* denote an acceptable power supply.
Let *f*, *m*, and *c* denote a power supply that has a functional, minor, or cosmetic error, respectively.

$$S = \{a, f, m, c\}$$

2-4. The number of hits (views) is recorded at a high-volume Web site in a day.

$S = \{0, 1, 2, ...\}$ = set of nonnegative integers

2-5. Each of 24 Web sites is classified as containing or not containing banner ads.

Let *y* and *n* denote a web site that contains and does not contain banner ads.
The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is $S = \{yynnynyyynnynynnnnnyynnyy\}$

2-6. An ammeter that displays three digits is used to measure current in milliamperes.

A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9.
The sample space *S* is 1000 possible three digit integers, $S = \{000, 001, ..., 999\}$

2-7. A scale that displays two decimal places is used to measure material feeds in a chemical plant in tons.

S is the sample space of 100 possible two digit integers.

2-8. The following two questions appear on an employee survey questionnaire. Each answer is chosen from the five point scale 1 (never), 2, 3, 4, 5 (always).

Is the corporation willing to listen to and fairly evaluate new ideas?
How often are my coworkers important in my overall job performance?

Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, ..., 55\}$

2-9. The concentration of ozone to the nearest part per billion.

$S = \{0,1,2,\ldots,1E09\}$ in ppb.

2-10. The time until a service transaction is requested of a computer to the nearest millisecond.

$S = \{0,1,2,\ldots,\}$ in milliseconds

2-11. The pH reading of a water sample to the nearest tenth of a unit.

$S = \{1.0,1.1,1.2,\ldots 14.0\}$

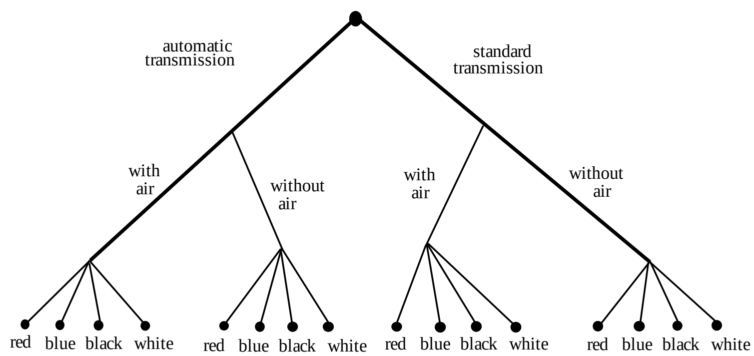
2-12. The voids in a ferrite slab are classified as small, medium, or large. The number of voids in each category is measured by an optical inspection of a sample.

Let *s*, *m*, and *l* denote small, medium, and large, respectively. Then *S* = {*s*, *m*, *l*, *ss*, *sm*, *sl*, ….}

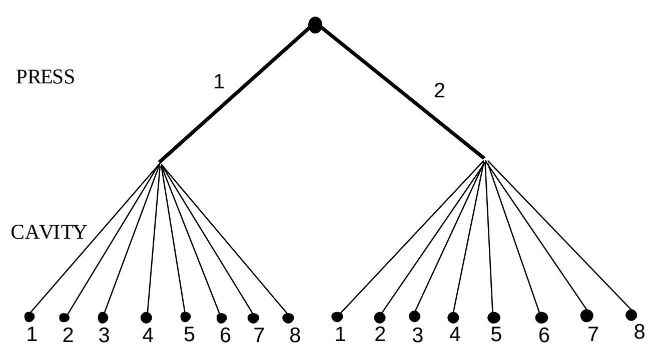
2-13 The time of a chemical reaction is recorded to the nearest millisecond.

$S = \{0,1,2,\ldots,\}$ in milliseconds.

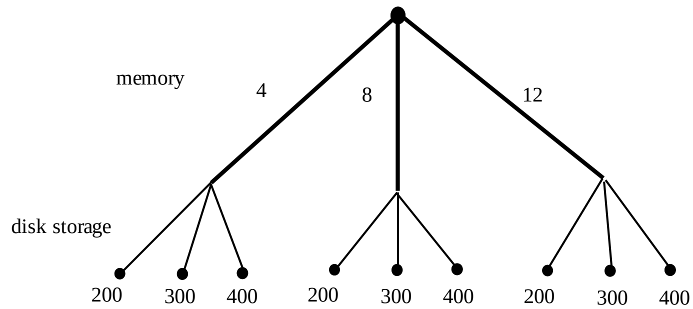
2-14. An order for an automobile can specify either an automatic or a standard transmission, either with or without air conditioning, and with any one of the four colors red, blue, black, or white. Describe the set of possible orders for this experiment.



2-15. A sampled injection-molded part could have been produced in either one of two presses and in any one of the eight cavities in each press.



2-16. An order for a computer system can specify memory of 4, 8, or 12 gigabytes and disk storage of 200, 300, or 400 gigabytes. Describe the set of possible orders.



2-17. Calls are repeatedly placed to a busy phone line until a connection is achieved.

Let c and b denote connect and busy, respectively. Then $S = \{c, bc, bbc, bbbc, bbbbc, \ldots\}$

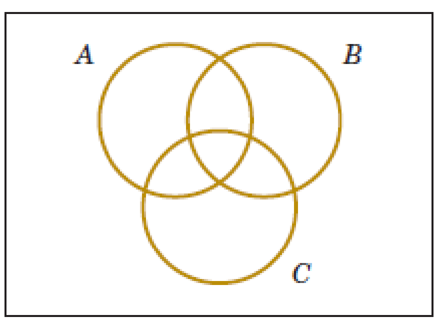
2-18. Four attempts are made to read data in a magnetic storage device before an error recovery procedure that repositions the magnetic head is used. The error recovery procedure attempts four repositionings before an "abort'' message is sent to the operator. Let

s denote the success of a read operation
f denote the failure of a read operation
S denote the success of an error recovery procedure
F denote the failure of an error recovery procedure
A denote an abort message sent to the operator

Describe the sample space of this experiment with a tree diagram.

$$S = \{s, fs, ffs, fffs, ffffS, ffffFS, ffffFFS, ffffFFFS, fffFFFA\}$$

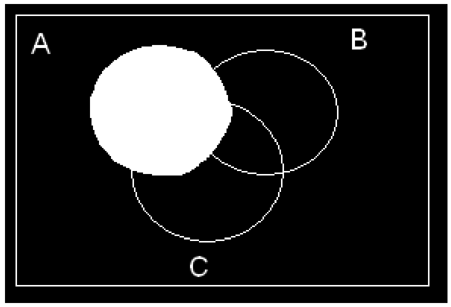
2-19. Three events are shown on the Venn diagram in the following figure:



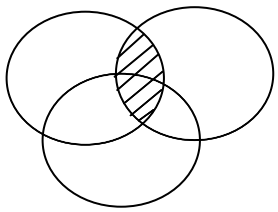
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $A \cap B$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $B' \cap C$

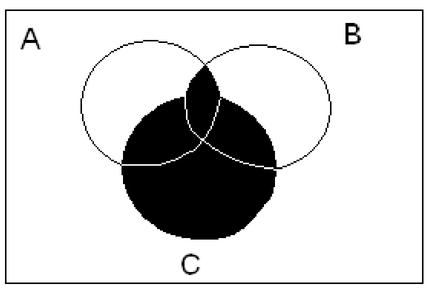
(a)



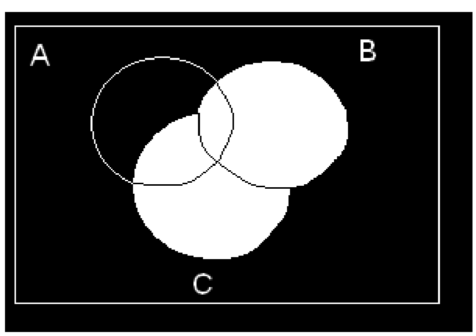
(b)



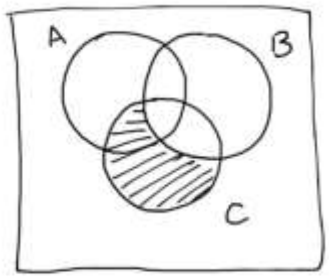
(c)



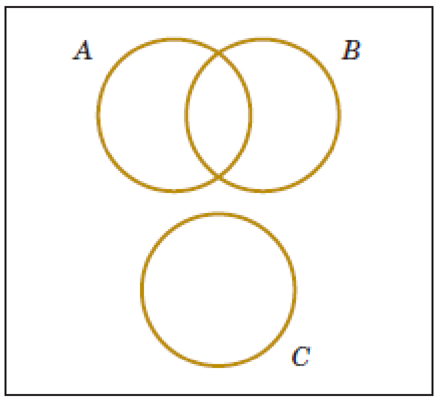
(d)



(e)



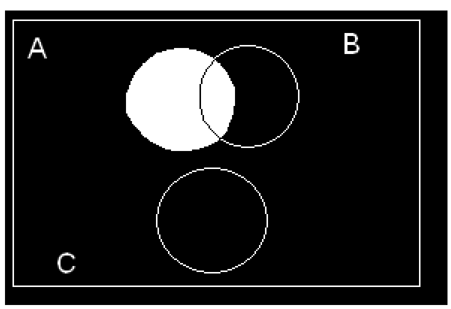
2-20. Three events are shown on the Venn diagram in the following figure:



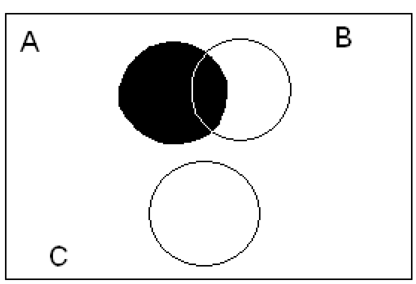
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $A \cap (B \cup C)'$ (e) $(A \cap B)' \cup C$

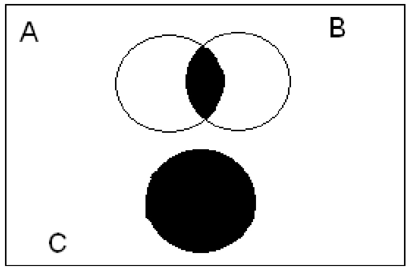
(a)



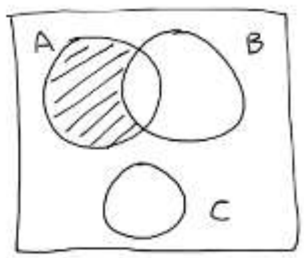
(b)



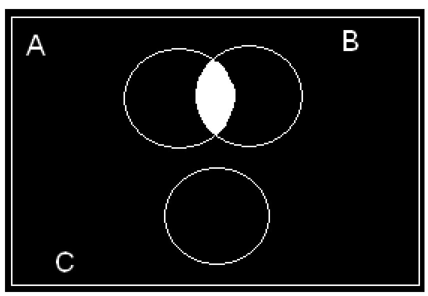
(c)



(d)



(e)



2-21. A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?
Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to 15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

(b) $A \cup B$ (c) $A \cap B$ (d) A' (e) $A \cup B \cup C$ (f) $A \cap B'$ (g) $A \cap B \cap C$ (h) $B' \cap C$ (i) $A \cup (B \cap C)$

(a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X denote the weight.

A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
$S = \{0, 1, 2, 3, \ldots\}$
(b) S
(c) $11 < X \le 15$ or $\{12, 13, 14, 15\}$
(d) $X \le 11$ or $\{0, 1, 2, \ldots, 11\}$
(e) S
(f) B' contains the values of X such that $X > 15$.
Thus $A \cap B'$ contains the values of X such that: $X > 15$ or $\{16, 17, 18, \ldots\}$

(g) ∅
(h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or ∅.
(i) $B \cap C$ is the event $8 \leq X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \geq 8$ or {8, 9, 10, …}

2-22. In an injection-molding operation, the length and width, denoted as X and Y, respectively, of each molded part are evaluated. Let

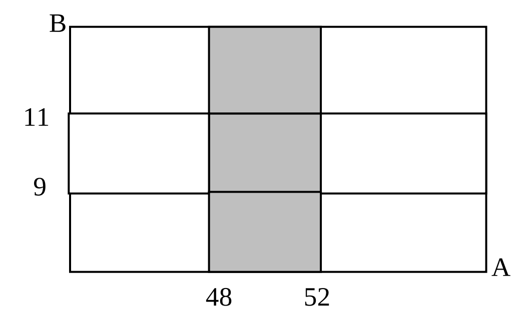
A denote the event of $48 < X < 52$ centimeters
B denote the event of $9 < Y < 11$ centimeters

Construct a Venn diagram that includes these events. Shade the areas that represent the following:

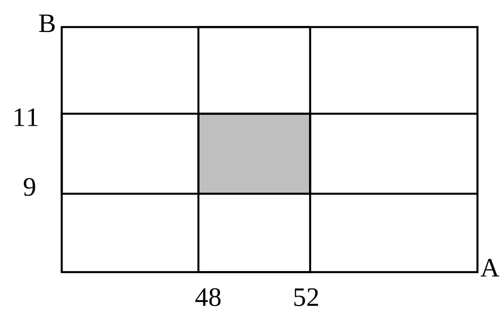
(a) A (b) $A \cap B$ (c) $A' \cup B$ (d) $A \cup B$

(e) If these events were mutually exclusive, how successful would this production operation be? Would the process produce parts with $X = 50$ centimeters and $Y = 10$ centimeters?

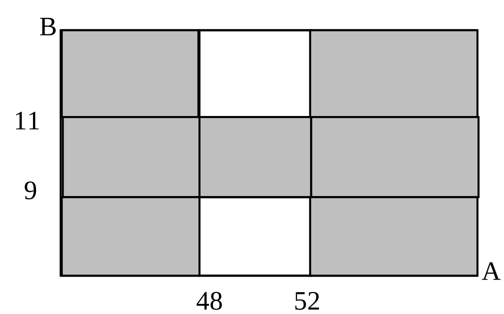
(a)



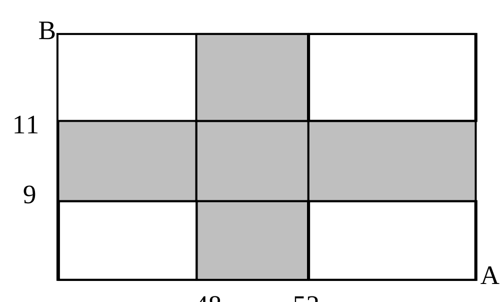
(b)



(c)



(d)



(e) If the events are mutually exclusive, then A∩B is the null set. Therefore, the process does not produce product parts with X = 50 cm and Y = 10 cm. The process would not be successful.

2-23. Let d and o denote a distorted bit and one that is not distorted (o denotes okay), respectively.

a) $$S = \begin{Bmatrix} ddddd, ddddo, dddod, ddodd, \\ doddd, odddd, dddoo, ddodo, \\ doddo, odddo, ddood, dodod, \\ oddod, doodd, ododd, ooddd, \\ ddooo, dodoo, ooddo, doood, \\ ooodd, ododo, oddoo, odood, \\ doodo, doooo, odooo, oodoo, \\ ooodo, ooood, oodod, ooooo \end{Bmatrix}$$

b) Ai's are not mutually exclusive.

$$A_1 \cap A_2 = \begin{Bmatrix} ddddd, ddddo, dddod, ddodd, \\ dddoo, ddodo, ddood, ddooo \end{Bmatrix}$$

c) $$A_1 = \begin{Bmatrix} ddddd, ddddo, dddod, ddodd, \\ doddd, dddoo, ddodo, doddo, \\ ddood, dodod, doodd, ddooo, \\ dodoo, doood, doodo, doooo \end{Bmatrix}$$

d) $$A_1' = \begin{Bmatrix} odddd \;\; odddo \;\; oddod \;\; ododd \\ ooddd \;\; ooddo \;\; ooodd \;\; ododo \\ oddoo \;\; odood \;\; odooo \;\; oodoo \\ ooodo \;\; ooood \;\; oodod \;\; ooooo \end{Bmatrix}$$

e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{ddddd, ddddo\}$

f) $$(A_1 \cap A_2) \cup (A_3 \cap A_4) = \begin{Bmatrix} ddddd, ddddo, dddod, ddodd, dddoo, \\ ddodo, ddood, ddooo, doddd, odddo, \\ odddd, doddo, ooddd, ooddo \end{Bmatrix}$$

2-24. In light-dependent photosynthesis, light quality refers to the wavelengths of light that are important. The wavelength of a sample of photosynthetically active radiations (PAR) is measured to the nearest nanometer. The red range is 675–700 nm and the blue range is 450–500 nm. Let A denote the event that PAR occurs in the red range, and let B denote the event that PAR occurs in the blue range. Describe the sample space and indicate each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $(A \cup B)'$

Let w denote the wavelength. The sample space is $\{w \mid w = 0, 1, 2, \ldots\}$
(a) $A=\{w \mid w = 675, 676, \ldots, 700 \text{ nm}\}$
(b) $B=\{ w \mid w = 450, 451, \ldots, 500 \text{ nm}\}$
(c) $A \cap B = \Phi$

(d) $(A \cup B)' = \{w \mid w = 0, 1, 2, \ldots, 448, 449, 701, 702, \ldots\}$

2-25. In control replication, cells are replicated over a period of two days. Not until mitosis is completed can freshly synthesized DNA be replicated again. Two control mechanisms have been identified—one positive and one negative. Suppose that a replication is observed in three cells. Let *A* denote the event that all cells are identified as positive, and let *B* denote the event that all cells are negative. Describe the sample space graphically and display each of the following events:

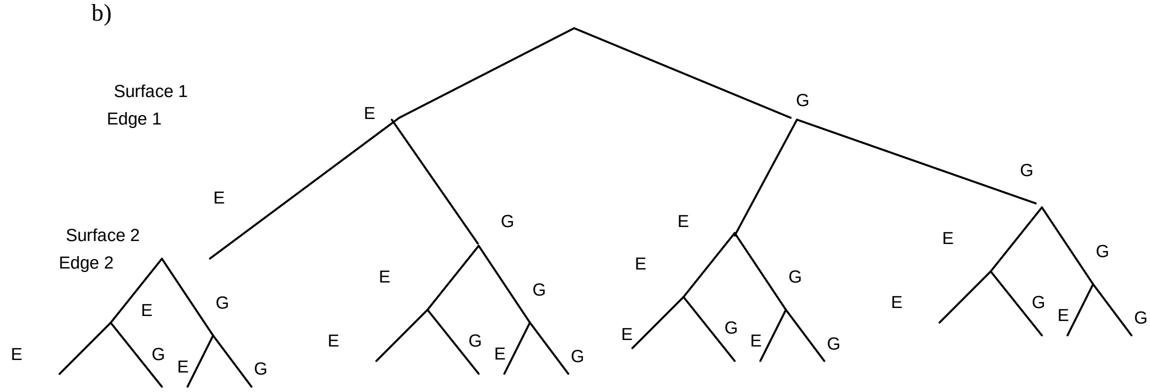
(a) A (b) B (c) $A \cap B$ (d) $(A \cup B)'$

Let *P* and *N* denote positive and negative, respectively.
The sample space is {*PPP*, *PPN*, *PNP*, *NPP*, *PNN*, *NPN*, *NNP*, *NNN*}.

(a) A={ *PPP* }
(b) B={ *NNN* }
(c) $A \cap B = \Phi$
(d) $(A \cup B)' =$ { *PPN*, *PNP*, *NPP*, *PNN*, *NPN*, *NNP* }

2-26. A ∩ B = 70, A′ = 15, A ∪ B = 95

2-27. a) $A' \cap B$ = 10, B' =10, $A \cup B$ = 94
b)



2-28. $A' \cap B$ = 55, B' =21, $A \cup B$ = 85

2-29. The rise time of a reactor is measured in minutes (and fractions of minutes). Let the sample space be positive, real numbers. Define the events *A* and *B* as follows:
$A = \{x \mid x < 65\}$ and $B = \{x \mid x > 45.5\}$.

Describe each of the following events:

(a) A' (b) B' (c) $A \cap B$ (d) $A \cup B$

(a) $A' = \{x \mid x \geq 65\}$
(b) $B' = \{x \mid x \leq 45.5\}$
(c) $A \cap B = \{x \mid 45.5 < x < 65\}$
(d) $A \cup B = \{x \mid x > 0\}$

2-30. a) {*ab*, *ac*, *ad*, *bc*, *bd*, *cd*, *ba*, *ca*, *da*, *cb*, *db*, *dc*}
b) {*ab*, *ac*, *ad*, *ae*, *af*, *ag*, *bc*, *bd*, *be*, *bf*, *bg*, *cd*, *ce*, *cf*, *cg*, *de*, *df*, *dg*, *ef*, *eg*, *fg*, *ba*, *ca*, *da*, *ea*, *fa*, *ga*, *cb*, *db*, *eb*, *fb*, *gb*, *dc*, *ec*, *fc*, *gc*, *ed*, *fd*, *gd*, *fe*, *ge*, *gf*}
c) Let *d* and *g* denote defective and good, respectively. Then *S* = {*gg*, *gd*, *dg*, *dd*}

d) S = {*gd, dg, gg*}

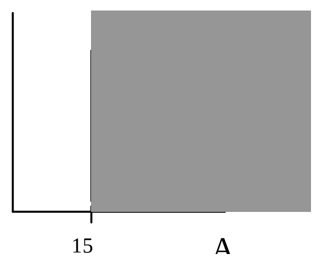
2-31. Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.
a) *S* = {*gg, gm, gj, mg, mm, mj, jg, jm, jj*}
b) *S* ={*gg,gm,gj,mg,mm,mj,jg,jm*}

2-32. Counts of the Web pages provided by each of two computer servers in a selected hour of the day are recorded. Let *A* denote the event that at least 15 pages are provided by server 1, and let *B* denote the event that at least 20 pages are provided by server 2. Describe the sample space for the numbers of pages for the two servers graphically in an *x* □ *y* plot. Show each of the following events on the sample space graph:

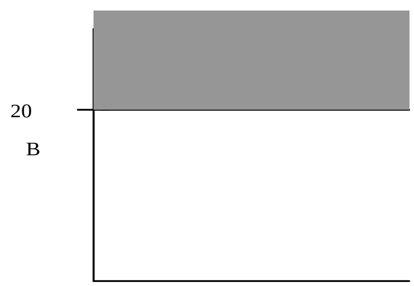
(a) *A* (b) *B* (c) $A \cap B$ (d) $A \cup B$

(a) The sample space contains all points in the nonnegative *X*-*Y* plane.

(b)



(c)



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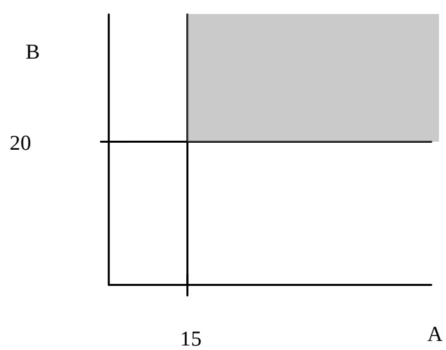

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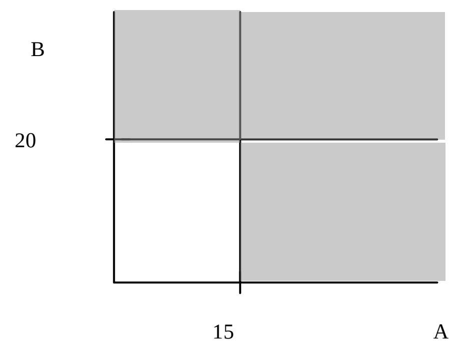
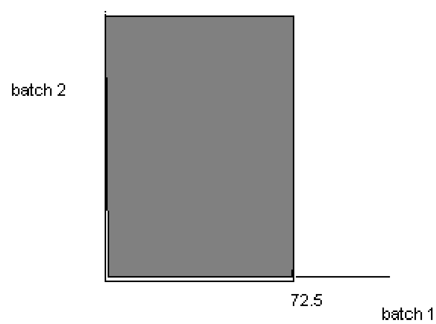


2-33. A reactor's rise time is measured in minutes (and fractions of minutes). Let the sample space for the rise time of each batch be positive, real numbers. Consider the rise times of *two* batches. Let *A* denote the event that the rise time of batch 1 is less than 72.5 minutes, and let *B* denote the event that the rise time of batch 2 is greater than 52.5 minutes.

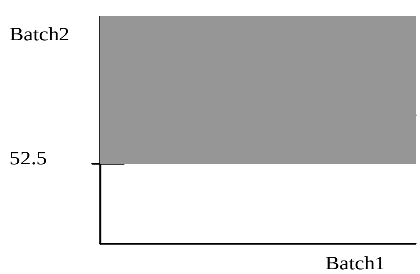
Describe the sample space for the rise time of two batches graphically and show each of the following events on a two dimensional plot:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

(a)



(b)



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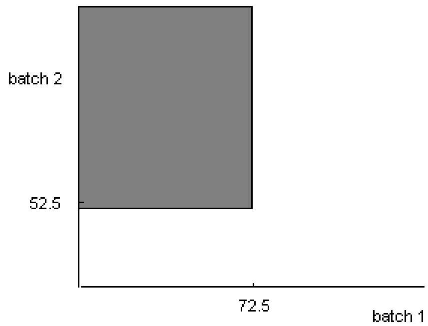

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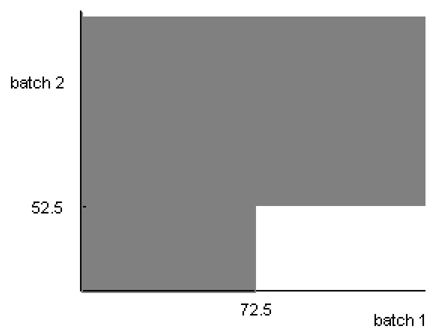


2-34. $2^{10} = 1024$

2-35. From the multiplication rule, the answer is $5 \times 3 \times 5 \times 2 = 150$

2-36. In a manufacturing operation, a part is produced by machining, polishing, and painting. If there are three machine tools, four polishing tools, and five painting tools, how many different routings (consisting of machining, followed by polishing, and followed by painting) for a part are possible?

From the multiplication rule, $3 \times 4 \times 5 = 60$

2-37. From the multiplication rule, $4 \times 4 \times 3 \times 4 = 192$

2-38. From equation 2-1, the answer is 15! = 1,307,674,368,000

2-39. A manufacturing operation consists of 8 operations. However, four machining operations must be completed before any of the remaining four assembly operations can begin. Within each set of four, operations can be completed in any order. How many different production sequences are possible?

From the multiplication rule and equation 2-1, the answer is 4!4! = 576

2-40. From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2-41. A batch of 100 semiconductor chips is inspected by choosing a sample of 5 chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?
(b) How many samples of five contain exactly one nonconforming chip?
(c) How many samples of five contain at least one nonconforming chip?

(a) From equation 2-4, the number of samples of size five is $\binom{100}{5} = \frac{100!}{5!95!} = 75{,}287{,}520$

(b) There are 10 ways of selecting one nonconforming chip and there are $\binom{90}{4} = \frac{90!}{4!86!} = 2{,}555{,}190$ ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{90}{4} = 25{,}551{,}900$

(c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{100}{5}$ minus the number of samples that contain no nonconforming chips $\binom{90}{5}$. That is

$$\binom{100}{5} - \binom{90}{5} = \frac{100!}{5!95!} - \frac{90!}{5!85!} = 31{,}338{,}252$$

2-42. a) If the chips are of different types, then every arrangement of 5 locations selected from the 15 results in a different layout. Therefore, $P_5^{15} = \frac{15!}{10!} = 360{,}360$ layouts are possible.

b) If the chips are of the same type, then every subset of 5 locations chosen from the 15 results in a different layout. Therefore, $\binom{15}{5} = \frac{15!}{5!10!} = 3003$ layouts are possible.

2-43. a) $\frac{8!}{2!6!} = 28$ sequences are possible.

b) $\frac{8!}{1!1!1!1!1!1!2!} = 20160$ sequences are possible.

c) 6! = 720 sequences are possible.

2-44. a) Every arrangement selected from the 10 different components comprises a different design. Therefore, $10! = 3{,}628{,}800$ designs are possible.

b) 7 components are the same, others are different, $\frac{10!}{7!1!1!1!1!1!} = 720$ designs are possible.

c) $\frac{10!}{3!4!} = 25200$ designs are possible.

2-45. Consider the design of a communication system.

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?

(b) As in part (a), how many three-digit phone prefixes are possible that do not start with 0, but contain 0 as the middle digit?

(c) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

(a) From the multiplication rule, $10^3 = 1000$ prefixes are possible

(b) From the multiplication rule, $9 \times 1 \times 10 = 90$ are possible

(c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.

$P_3^{10} = \frac{10!}{7!} = 720$ prefixes are possible.

2-46. A *byte* is a sequence of eight bits and each bit is either 0 or 1.

(a) How many different bytes are possible?

(b) If the first bit of a byte is a parity check, that is, the first byte is determined from the other seven bits, how many different bytes are possible?

(a) From the multiplication rule, $2^8 = 256$ bytes are possible

(b) From the multiplication rule, $2^7 = 128$ bytes are possible

2-47. In a chemical plant, 24 holding tanks are used for final product storage. Three tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?
(b) What is the probability that at least one tank in the sample contains high-viscosity material?
(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high-viscosity material and exactly one tank in the sample contains material with high impurities?

(a) The total number of samples possible is $\binom{24}{3} = \frac{24!}{3!21!} = 2{,}024.$ The number of samples in which exactly one tank has high viscosity is $\binom{6}{1}\binom{18}{2} = \frac{6!}{1!5!} \times \frac{18!}{2!16!} = 918$. Therefore, the probability is

$$\frac{918}{2024} = 0.454$$

(b) The number of samples that contain no tank with high viscosity is $\binom{18}{3} = \frac{18!}{3!15!} = 816.$ Therefore, the requested probability is $1 - \frac{816}{2024} = 0.597$.

(c) The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{1} = \frac{6!}{1!5!} \times \frac{4!}{1!3!} \times \frac{14!}{1!13!} = 336$.

Therefore, the probability is $\frac{336}{2024} = 0.166$

2-48. Plastic parts produced by an injection-molding operation are checked for conformance to specifications. Each tool contains 14 cavities in which parts are produced, and these parts fall into a conveyor when the press opens. An inspector chooses 3 parts from among the 14 at random. Two cavities are affected by a temperature malfunction that results in parts that do not conform to specifications.

(a) How many samples contain exactly 1 nonconforming part?
(b) How many samples contain at least 1 nonconforming part?

(a) The total number of samples is $\binom{14}{3} = \frac{14!}{3!11!} = 364.$ The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{12}{2} = \frac{2!}{1!1!} \times \frac{12!}{2!10!} = 132.$ Therefore, the requested probability is 90/220 = 0.409.

(b) The number of samples with no nonconforming part is $\binom{12}{3} = \frac{12!}{3!9!} = 220.$ The probability of at least one nonconforming part is $1 - \frac{220}{364} = 0.396$.

2-49. The number of ways to select two parts from 40 is $\binom{40}{2}$ and the number of ways to select two defective parts from the 5 defectives ones is $\binom{5}{2}$. Therefore the probability is $\frac{\binom{5}{2}}{\binom{40}{2}} = \frac{10}{78} = 0.01282$

2-50. The following table summarizes 204 endothermic reactions involving sodium bicarbonate.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is below target. Determine the number of reactions in each of the following events.

(a) $A \cap B$ (b) A' (c) $A' \cap B$ (d) $A' \cup B'$ (e) $A' \cap B'$

(a) $A \cap B = 56$
(b) $A' = 36 + 56 = 92$
c) $A' \cap B = 56$
d) $A' \cup B' = 56+40+16+36=148$
(e) $A' \cap B' = 36$

2-51. Total number of possible designs = $5 \times 3 \times 5 \times 3 \times 5 = 1125$

2-52. Consider the hospital emergency department data given below. Let A denote the event that a visit is to hospital 1, and let B denote the event that a visit results in not being admitted to any hospital.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the number of persons in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

a) $A \cap B = 3820$
b) $A' = 22252 - 5292 = 16960$
c) $A \cup B = 5292 + 16814 - 3820 = 18286$
d) $A \cup B' = 195 + 270 + 246 + 242 + 1277 + 1558 + 666 + 984 + 3820 = 9258$
e) $A' \cap B' = 270 + 246 + 242 + 1558 + 666 + 984 = 3966$

2-53. An article in *The Journal of Data Science* ["A Statistical Analysis of Well Failures in Baltimore County" (2009, Vol. 7, pp. 111–127)] provided the following table of well failures for different geological formation groups in Baltimore County.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Determine the number of wells in each of the following events.

(a) $A \cap B$ (b) B' (c) $A \cup B$ (d) $A \cap B'$ (e) $A' \cap B'$

(a) $A \cap B$ = 170 + 443 + 60 = 673
(b) B' = 1685 – 170 + 28 – 2 + 3733 – 443 + 363 – 14 + 309 – 29 + 1403 – 60 + 933 – 46 + 39 - 3= 7726
(c) $A \cup B$ = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915
(d) $A \cup B'$ = 1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3) = 8399
(e) $A' \cap B'$ = 28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3 = 1578

Section 2-2

2-54. Each of the possible five outcomes of a random experiment is equally likely. The sample space is {*a*, *b*, *c*, *d*, *e*}. Let *A* denote the event {*a*, *b*}, and let *B* denote the event {*c*, *d*, *e*}. Determine the following:

(a) P(A) (b) P(B) (c) P(B') (d) P(A∪B) (e) P(A∩B)

All outcomes are equally likely
(a) P(A) = 2/5
(b) P(B) = 3/5
(c) P(B') = 2/5
(d) P(A∪B) = 1
(e) P(A∩B) = P(∅)= 0

2-55. a) P(A) = 0.5
b) P(B) = 0.7
c) P(A') = 0.5
d) P(A∪B) = 1
e) P(A∩B) = 0.2

2-56. a) 0.6 + 0.2 = 0.8
b) 0.2 + 0.6 = 0.8

2-57. a) 1/10
b) 6/10

2-58. A part selected for testing is equally likely to have been produced on any one of seven cutting tools.

(a) What is the sample space?
(b) What is the probability that the part is from tool 1?
(c) What is the probability that the part is from tool 3 or tool 5?
(d) What is the probability that the part is not from tool 4?

(a) S = {1, 2, 3, 4, 5, 6,7}
(b) 1/7
(c) 2/7
(d) 5/7

2-59. An injection-molded part is equally likely to be obtained from any one of the twelve cavities on a mold.

(a) What is the sample space?

(b) What is the probability that a part is from cavity 1 or 2?
(c) What is the probability that a part is from neither cavity 3 nor 4?

a) S = {1,2,3,4,5,6,7,8,9,10,11,12}
b) 2/12
c) 10/12

2-60. In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL to the nearest mL. Assume that volumes are measured to the nearest mL and describe the sample space.

(a) What is the probability that equivalence is indicated at 100 mL?
(b) What is the probability that equivalence is indicated at less than 100 mL?
(c) What is the probability that equivalence is indicated between 98 and 102 mL (inclusive)?

The sample space is {95, 96, 97,…, 103, and 104}.

(a) Because the replicates are equally likely to indicate from 95 to 104 mL, the probability that equivalence is indicated at 100 mL is 0.1.
(b) The event that equivalence is indicated at less than 100 mL is {95, 96, 97, 98, 99}. The probability that the event occurs is 0.5.
(c) The event that equivalence is indicated between 98 and 102 mL is {98, 99, 100, 101, 102}. The probability that the event occurs is 0.5.

2-61. The sample space is {0, +2, +3, and +4}.
(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.36 + 0.34 + 0.15 = 0.85.
(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.15 + 0.36 + 0.34 = 0.85.

2-62. A credit card contains 16 digits between 0 and 9. However, only 100 million numbers are valid. If a number is entered randomly, what is the probability that it is a invalid number?

Total possible: 10^{16}, but only 10^8 are valid. Therefore, P(invalid) =1-P(valid) =1- $10^8/10^{16}$ = 1- 10^{-8}

2-63. Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between *A* and *Z*). If a license number is selected randomly, what is the probability that yours is the one selected?

3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10).
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26). The probability your license plate is chosen is then $(1/10^3)*(1/26^3)$ = 5.7 x 10^{-8}

2-64. A message can follow different paths through servers on a network. The sender's message can go to one of five servers for the first step; each of them can send to three servers at the second step; each of those can send to four servers at the third step; and then the message goes to the recipient's server.

(a) How many paths are possible?
(b) If all paths are equally likely, what is the probability that a message passes through the first of four servers at the third step?

a) 3*5*4 = 60
b) (3*5)/60 = 15/60=1/4

2-65. Magnesium alkyls are used as homogenous catalysts in the production of linear low-density polyethylene (LLDPE), which requires a finer magnesium powder to sustain a reaction. Redox reaction experiments using four different amounts of magnesium powder are performed. Each result may or may not be further reduced in a second step using two different magnesium powder amounts. Each of these results may or may not be further reduced in a third step using five different amounts of magnesium powder.

(a) How many experiments are possible?
(b) If all outcomes are equally likely, what is the probability that the best result is obtained from an experiment that uses all three steps?
(c) Does the result in part (b) change if five or six or seven different amounts are used in the first step? Explain.

(a) The number of possible experiments is 4 + 4 × 2 + 4 × 2 × 5 = 52
(b) There are 40 experiments that use all three steps. The probability the best result uses all three steps is 40/52 = 0.769.
(c) No, it will not change. With *k* amounts in the first step the number of experiments is *k* + 2*k* + 10*k* = 13*k*. The number of experiments that complete all three steps is 10*k* out of 13*k*. The probability is 10/13 = 0.769.

2-66. a) P(A) = 85/100 = 0.85
b) P(B) = 80/100 = 0.8
c) P(A') = 15/100 = 0.15
d) P(A∩B) = 70/100 = 0.7
e) P(A∪B) = (70 + 10 + 15)/100 = 0.95
f) P(A'∪B) = (70 + 10 + 5)/100 = 0.85

2-67. a) P(A) = 30/100 = 0.30
b) P(B) = 75/100 = 0.75
c) P(A') = 1 – 0.30 = 0.70
d) P(A∩B) = 20/100 = 0.2
e) P(A∪B) = 85/100 = 0.85
f) P(A'∪B) =90/100 = 0.9

2-68. An article in the *Journal of Database Management* ["Experimental Study of a Self-Tuning Algorithm for DBMS Buffer Pools" (2005, Vol. 16, pp. 1–20)] provided the workload used in the TPC-C OLTP (Transaction Processing Performance Council's Version C On-Line Transaction Processing) benchmark, which simulates a typical order entry application.

TABLE • 2E-1 Average Frequencies and Operations in TPC-C

Transaction	Frequency	Selects	Updates	Inserts	Deletes	Nonunique Selects	Joins
New order	43	23	11	12	0	0	0
Payment	44	4.2	3	1	0	0.6	0
Order status	4	11.4	0	0	0	0.6	0
Delivery	5	130	120	0	10	0	0
Stock level	4	0	0	0	0	0	1

The frequency of each type of transaction (in the second column) can be used as the percentage of each type of transaction. The average number of *selects* operations required for each type of transaction is shown. Let *A* denote the event of transactions with an average number of *selects* operations of 12 or fewer. Let *B* denote the event of transactions with an average number of *updates* operations of 12 or fewer. Calculate the following probabilities.

(a) P(A) (b) P(B) (c) P(A∩B) (d) P(A'∩B') (f) P(A∪B)

(a) The total number of transactions is 43+44+4+5+4=100

$$P(A) = \frac{44+4+4}{100} = 0.52$$

(b) $P(B) = \frac{100-5}{100} = 0.95$

(c) $P(A \cap B) = \frac{44+4+4}{100} = 0.52$

(d) $P(A' \cap B') = \frac{5}{100} = 0.05$

(d) $P(A \cup B) = \dfrac{100-5}{100} = 0.95$

2-69. Use the axioms of probability to show the following: $A \cup B$(d) $A \cap B'$

(a) For any event E, $P(E') = 1 - P(E)$. (b) $P(\varnothing) = 0$ (c) If A is contained in B, then $P(A) \le P(B)$.

(a) Because E and E' are mutually exclusive events and $E \cup E'$ = S
1 = P(S) = P($E \cup E'$) = P(E) + P(E'). Therefore, P(E') = 1 - P(E)
(b) Because S and $\varnothing$ are mutually exclusive events with S = $S \cup \varnothing$
P(S) = P(S) + P($\varnothing$). Therefore, P($\varnothing$) = 0
(c) Now, B = $A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,
P(B) = P(A) + P($A' \cap B$). Because P($A' \cap B$) ≥ 0 , P(B) $\geq$ P(A).

2.70. Consider the endothermic reaction's table given below. Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A' \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B)$ = (40 + 16)/204 = 0.2745
(b) $P(A')$ = (36 + 56)/204 = 0.4510
(c) $P(A \cup B)$ = (40 + 12 + 16 + 44 + 36)/204 = 0.7255
(d) $P(A' \cup B')$ = 56/204 =0.2745
(e) $P(A' \cap B')$ = 56/204 = 0.2745

2-71. Total number of possible designs is 1125. The sample space of all possible designs that may be seen on five visits. This space contains $(1125)^5$ outcomes.

The number of outcomes in which all five visits are different can be obtained as follows. On the first visit any one of 1125 designs may be seen. On the second visit there are 1124 remaining designs. On the third visit there are 1123 remaining designs. On the fourth and fifth visits there are 1122 and 1121 remaining designs, respectively. From the multiplication rule, the number of outcomes where all designs are different is 1125 × 1124 × 1123 × 1122 × 1121. Therefore, the probability that a design is not seen again is

$$\frac{1125 \times 1124 \times 1123 \times 1122 \times 1121}{(1125)^5} = 0.9911$$

2-72. Consider the hospital emergency room data is given below. Let A denote the event that a visit is to hospital 4, and let B denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B)$

(a) $P(B)$ = 953/22252 = 0.0428
(b) $P(A')$ = (5292+6991+5640)/22252 = 0.8055
(c) $P(A \cup B)$ = (195 + 270 + 246 + 242 + 984 + 3103)/22252 = 0.2265
(d) $P(A \cup B')$ = (4329 + (5292 – 195) + (6991 – 270) + 5640 – 246))/22252 = 0.9680
(e) $P(A' \cap B)$ = (195 + 270 + 246)/22252 = 0.0320

2-73. Consider the well failure data is given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Determine the following probabilities.

(a) $P(B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B)$

(a) P(B) = (170 +2 + 443 +14 +29 + 60 +46 +3)/8493 = 767/8493 = 0.0903
(b) P(A′) = (28 + 363 + 309 + 933 + 39)/8493 = 1672/8493 = 0.1969
(c) P(A ∪ B) = (1685+3733+1403+2+14+29+46+3)/8493 = 6915/8493 = 0.8142
(d) P(A ∪ B′) = (1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3))/8493 = 8399/8493 = 0.9889
(e) P(A′ ∩ B) = (2 + 14 + 29 + 46 + 3)/8493 = 94 /8493 = 0.0111

Section 2-3

2-74. a) P(A') = 1- P(A) = 0.6
b) P ($A \cup B$) = P(A) + P(B) - P($A \cap B$) = 0.4 + 0.2 - 0.1 = 0.5
c) P($A' \cap B$) + P($A \cap B$) = P(B). Therefore, P($A' \cap B$) = 0.2 - 0.1 = 0.1
d) P(A) = P($A \cap B$) + P($A \cap B'$). Therefore, P($A \cap B'$) = 0.4 - 0.1 = 0.3
e) P(($A \cup B$)') = 1 - P($A \cup B$) = 1 - 0.5 = 0.5
f) P($A' \cup B$) = P(A') + P(B) - P($A' \cap B$) = 0.6 + 0.2 - 0.1 = 0.7

2-75. If *A*, *B*, and *C* are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.2$, and $P(C) = 0.4$, determine the following probabilities:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$ (d) $P[(A \cup B) \cap C]$ (e) $P(A' \cap B' \cap C')$

(a) P($A \cup B \cup C$) = P(A) + P(B) + P(C), because the events are mutually exclusive. Therefore,
P($A \cup B \cup C$) = 0.2+0.2+0.4 = 0.8
(b) P ($A \cap B \cap C$) = 0, because $A \cap B \cap C = \varnothing$
(c) P($A \cap B$) = 0 , because $A \cap B = \varnothing$
(d) P($(A \cup B) \cap C$) = 0, because $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \varnothing$
(e) P($A' \cap B' \cap C'$) =1-[P(A) + P(B) + P(C)] = 1-(0.2+0.2+0.4) = 0.2

2-76. (a) P(Caused by sports) = P(Caused by contact sports or by noncontact sports)
= P(Caused by contact sports) + P(Caused by noncontact sports)
= 0.46 + 0.44 = 0.9
(b) 1- P(Caused by sports) = 0.1

2.77. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	10
Resistance	Low	15	5

(a) If a disk is selected at random, what is the probability that its scratch resistance is high and its shock resistance is high?
(b) If a disk is selected at random, what is the probability that its scratch resistance is high or its shock resistance is high?
(c) Consider the event that a disk has high scratch resistance and the event that a disk has high shock resistance. Are these two events mutually exclusive?

(a) 70/100 = 0.70
(b) (80+85-70)/100 = 0.95
(c) No, P($A \cap B$) $\neq$ 0

2-78. (a) P(High strength and high conductivity)= 74/100 =0.74
(b) P(Low strength or low conductivity)
= P(Low strength) + P(Low conductivity) – P(Low strength and low conductivity)
= (10 + 1)/100 + (15 + 1)/100 – 1/100
= 0.26
(c) No, they are not mutually exclusive. Because P(Low strength) + P(Low conductivity)
= (10 + 1)/100 + (15 + 1)/100
= 0.27, which is not equal to P(Low strength or low conductivity).

2-79. The analysis of shafts for a compressor is summarized by conformance to specifications.

		Roundness Conforms	
		Yes	No
Surface finish	Yes	334	15
Conforms	No	12	9

(a) If a shaft is selected at random, what is the probability that it conforms to surface finish requirements?
(b) What is the probability that the selected shaft conforms to surface finish requirements or to roundness requirements?
(c) What is the probability that the selected shaft either conforms to surface finish requirements or does not conform to roundness requirements?
(d) What is the probability that the selected shaft conforms to both surface finish and roundness requirements?

(a) 349/370
(b) $\frac{334+15+12}{370} = \frac{361}{370}$
(c) $\frac{334+15+9}{370} = \frac{358}{370}$
(d) 334/370

2-80. Cooking oil is produced in two main varieties: mono and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

		Type of oil	
		Canola	Corn
Type of Unsaturation	Mono	12	18
	Poly	88	82

(a) If a bottle of oil is selected at random, what is the probability that it belongs to the polyunsaturated category?

(b) What is the probability that the chosen bottle is monounsaturated canola oil?

(a) (88 + 82)/200 = 170/200 = 0.85
(b) 12 /200 = 0.06

2-81. A manufacturer of front lights for automobiles tests lamps under a high-humidity, high-temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 170 lamps:

		Useful life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	143	8
	Unsatisfactory	12	7

(a) Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.
(b) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

(a) P(unsatisfactory) = (12 + 7 + 8)/170 = 27/170
(b) P(both criteria satisfactory) = 143/170 = 0.841, No

2-82. A computer system uses passwords that are six characters, and each character is one of the 26 letters (*a*–*z*) or 10 integers (0–9). Uppercase letters are not used. Let *A* denote the event that a password begins with a vowel (either *a*, *e*, *i*, *o*, or *u*), and let *B* denote the event that a password ends with an even number (either 0, 2, 4, 6, or 8). Suppose a hacker selects a password at random. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A' \cup B')$

(a) 5/36
(b) 5/36
(c) $P(A \cap B) = P(A)P(B) = 25/1296$
(d) $P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B) = 1 - 25/1296 = 0.9807$

2-83. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or more. Let *B* denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

$P(A)$ = 152/204 = 0.7451, P(B) = 92/204 = 0.4510, $P(A \cap B)$ = (16+36)/204 = 0.2549
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7451 + 0.4510 - 0.2549 = 0.9412$
(b) $P(A \cap B') = (44 + 56)/204 = 0.4902$ and $P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.7451 + (1 - 0.4510) - 0.4902 = 0.8039$
(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.2549 = 0.7451$

2-84. A Web ad can be designed from three different colors, three font types, three font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

P(A) = 1/3, P(B) = 2/3, $P(A \cap B) = P(A)P(B) = (1/3)(2/3) = 2/9$

a) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 1/3 + 2/3 – 2/9 = 7/9
b) First P(A ∩ B′) = P(A)P(B′) = (1/3)(1/3) = 1/9. Then P(A ∪ B′) = P(A) + P(B′) – P(A ∩ B′) = 1/3 + 1/3 – 1/9= 5/9
c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 2/9 = 7/9

2-85. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 1, and let *B* denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Use the addition rules to calculate the following probabilities.

(a) *P*(*A* ∪ *B*) (b) *P*(*A* ∪ *B*′) (c) *P*(*A*′ ∪ *B*′)

P(A) = 5292/22252 = 0.2378, P(B) = 953/22252 = 0.0428, P(A ∩ B) = 195/22252 = 0.0088,
P(A ∩ B′) = (1277+3820)/22252 = 0.2291
(a) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.2378 + 0.0428 – 0.0088 = 0.2718
(b) P(A ∪ B′) = P(A) + P(B′) – P(A ∩ B′) = 0.2378 + (1 – 0.0428) – 0.2291 = 0.9659
(c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0088 = 0.9912

2-86. Consider the well failure data given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Use the addition rules to calculate the following probabilities.

(a) *P*(*A* ∪ *B*) (b) *P*(*A*′ ∪ *B*) (c) *P*(*A*′ ∪ *B*′)

P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903,
P(A ∩ B) = (170 + 443 + 60)/8493 = 0.0792, P(A ∩ B′) = (1515+3290+1343)/8493 = 0.7239
(a) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.8031 + 0.0903 – 0.0792 = 0.8142
(b) P(A′∪ B) = P(A′) + P(B) – P(A′∩ B) = (1- 0.8031) + 0.0903 – 0.0111 = 0.2761
(c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0792 = 0.9208

Section 2-4

2-87. a) P(A) = 85/100 b) P(B) = 80/100

c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{70/100}{80/100} = \frac{7}{8}$

d) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{70/100}{85/100} = \frac{7}{8.5}$

2-88. (a) $P(A) = \frac{9+30}{100} = 0.39$

(b) $P(B) = \dfrac{13+9}{100} = 0.22$

(c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{9/100}{22/100} = 0.409$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{9/100}{39/100} = 0.23$

2-89. The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by type of transformation completed:

		Total Textural Tranformation	
		Yes	No
Total color	Yes	238	30
Transformation	No	18	14

(a) If a leaf completes the color transformation, what is the probability that it will complete the textural transformation?
(b) If a leaf does not complete the textural transformation, what is the probability it will complete the color transformation?

Let *A* denote the event that a leaf completes the color transformation and let *B* denote the event that a leaf completes the textural transformation. The total number of experiments is 300.

(a) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{238/300}{(238+30)/300} = 0.888$

(b) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{30/300}{(30+14)/300} = 0.682$

2-90. a) 0.83
b) 0.90
c) 8/9 = 0.889
d) 80/83 = 0.964
e) 80/83 = 0.964
f) 3/10 = 0.3

2-91. The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		Coating Weight	
		High	Low
Surface	High	36	16
Roughness	Low	114	34

(a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?
(b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?
(c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

(a) 36/150 (b) 36/52 (c) 34/148

2-92. Consider the data on wafer contamination and location in the sputtering tool shown in Table 2-2. Assume that one wafer is selected at random from this set. Let *A* denote the event that a wafer contains four or more particles, and let *B* denote the event that a wafer is from the edge of the sputtering tool. Determine:

(a) $P(A)$ (b) $P(A \mid B)$ (c) $P(B)$ (d) $P(B \mid A)$ (e) $P(A \cap B)$ (f) $P(A \cup B)$

(a) P(A) = 0.05 + 0.10 = 0.15

(b) P(A|B) = $\dfrac{P(A\cap B)}{P(B)} = \dfrac{0.01+0.03}{0.28} = 0.143$

(c) P(B) = 0.28

(d) P(B|A) = $\dfrac{P(A\cap B)}{P(A)} = \dfrac{0.01+0.03}{0.15} = 0.267$

(e) P(A ∩ B) = 0.01 +0.03 = 0.04

(f) P(A ∪ B) = 0.15 + 0.28 – 0.04 = 0.39

2-93. The following table summarizes the number of deceased beetles under autolysis (the destruction of a cell after its death by the action of its own enzymes) and putrefaction (decomposition of organic matter, especially protein, by microorganisms, resulting in production of foul-smelling matter):

		Autolysis	
		High	Low
Putrefaction	High	14	47
	Low	30	9

(a) If the autolysis of a sample is high, what is the probability that the putrefaction is low?
(b) If the putrefaction of a sample is high, what is the probability that the autolysis is high?
(c) If the putrefaction of a sample is low, what is the probability that the autolysis is low?

Let *A* denote the event that autolysis is high and let *B* denote the event that putrefaction is high. The total number of experiments is 100.

(a) $P(B'|A) = \dfrac{P(A\cap B')}{P(A)} = \dfrac{30/100}{(14+30)/100} = 0.682$

(b) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{14/100}{(14+47)/100} = 0.230$

(c) $P(A'|B') = \dfrac{P(A'\cap B')}{P(B')} = \dfrac{9/100}{(30+9)/100} = 0.231$

2-94. A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		Evidence of Gas Leaks	
		Yes	No
Evidence of	Yes	70	19
electrical failure	No	38	5

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability
(a) That failure involves a gas leak
(b) That there is evidence of electrical failure given that there was a gas leak
(c) That there is evidence of a gas leak given that there is evidence of electrical failure

(a) P(gas leak) = (70 + 38)/132 = 0.818
(b) P(electric failure | gas leak) = (70/132)/(108/132) = 0.648
(c) P(gas leak | electric failure) = (70/132)/(89/132) = 0.787

2-95. a) 25/100
b) 24/99
c) (25/100)(24/99) = 0.0606
d) If the chips were replaced, the probability would be (25/100) = 0.25

2-96. a) 9/499 = 0.018

b) (10/500)(9/499) = 3.6×10^{-4}

c) (490/500)(489/499) = 0.96
d) 8/498 = 0.016
e) 9/498 = 0.018
f) $\left(\frac{10}{500}\right)\left(\frac{9}{499}\right)\left(\frac{8}{498}\right) = 5.795\times10^{-6}$

2-97. a) P = (8-1)/(300-1)=0.023
b) P = (8/300) $\times$ [(8-1)/(300-1)] = 6.243×10^{-4}
c) P = (292/300) $\times$ [(292-1)/(300-1)]=0.947

2-98. A computer system uses passwords that are exactly seven characters and each character is one of the 26 letters (*a*–*z*) or 10 integers (0–9). You maintain a password for this computer system. Let *A* denote the subset of passwords that begin with a vowel (either *a*, *e*, *i*, *o*, or *u*) and let *B* denote the subset of passwords that end with an even number (either 0, 2, 4, 6, or 8).
(a) Suppose a hacker selects a password at random. What is the probability that your password is selected?
(b) Suppose a hacker knows that your password is in event *A* and selects a password at random from this subset. What is the probability that your password is selected?
(c) Suppose a hacker knows that your password is in *A* and *B*and selects a password at random from this subset. What is the probability that your password is selected?

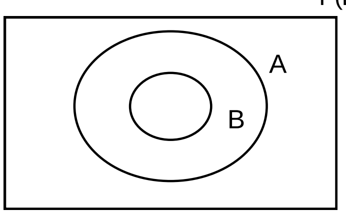
(a) $\dfrac{1}{36^7}$

(b) $\dfrac{1}{5(36^6)}$

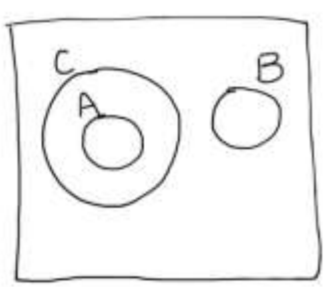
(c) $\dfrac{1}{5(36^5)5}$

2-99. If $P(A \mid B) = 1$, must $A = B$? Draw a Venn diagram to explain your answer.

No, if $B \subset A$, then P(A/B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$



2-100. Suppose *A* and *B* are mutually exclusive events. Construct a Venn diagram that contains the three events *A*, *B*, and *C* such that $P(C \mid A) = 1$ and $P(B \mid C) = 0$.



2-101. Consider the endothermic reactions given below. Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is below target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(12+44)/204}{(12+44+56)/204} = \dfrac{56}{112} = 0.5$

(b) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{56/204}{(12+44+56)/204} = \dfrac{56}{112} = 0.5$

(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{(40+16)/204}{(40+16+36)/204} = \dfrac{40}{92} = 0.435$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{(12+44)/204}{(12+44+40+16)/204} = \dfrac{56}{112} = 0.5$

2-102. Consider the hospital emergency room data given below. Let A denote the event that a visit is to hospital 1, and let B denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{195/22252}{953/22252} = \dfrac{195}{953} = 0.205$

(b) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{(270+246+242)/22252}{953/22252} = \dfrac{758}{953} = 0.795$

(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{(1277+3820)/22252}{(22252-953)/22252} = \dfrac{5097}{21299} = 0.239$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{195/22252}{5292/22252} = \dfrac{195}{5292} = 0.037$

2-103. Consider the well failure data given below.

Geological Formation Group	Wells Failed	Wells Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

(a) What is the probability of a failure given there are more than 1,000 wells in a geological formation?
(b) What is the probability of a failure given there are fewer than 500 wells in a geological formation?

(a) $P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{(170+443+60)/8493}{(1685+3733+1403)/8493}=\dfrac{673}{6821}=0.0987$

Also the probability of failure for fewer than 1000 wells is

$$P(B\mid A')=\frac{P(B\cap A')}{P(B')}=\frac{(2+14+29+46+3)/8493}{(28+363+309+933+39)/8493}=\frac{92}{1672}=0.0562$$

(b) Let *C* denote the event that fewer than 500 wells are present.

$$P(B\mid C)=\frac{P(A\cap C)}{P(C)}=\frac{(2+14+29+46+3)/8493}{(28+363+309+39)/8493}=\frac{48}{739}=0.0650$$

2-104. Let A denote the event that an egg survives to an adult
Let EL denote the event that an egg survives at early larvae stage
Let LL denote the event that an egg survives at late larvae stage
Let PP denote the event that an egg survives at pre-pupae larvae stage
Let LP denote the event that an egg survives at late pupae stage

a) $P(A)=31/421=0.0736$

b) $P(A\mid LL)=\dfrac{P(A\cap LL)}{P(LL)}=\dfrac{31/421}{306/421}=0.1013$

c) $P(EL)=412/421=0.9786$

$$P(LL\mid EL)=\frac{P(LL\cap EL)}{P(EL)}=\frac{306/421}{412/421}=0.7427$$

$$P(PP\mid LL)=\frac{P(PP\cap LL)}{P(LL)}=\frac{45/421}{306/421}=0.1471$$

$$P(LP\mid PP)=\frac{P(LP\cap PP)}{P(PP)}=\frac{35/421}{45/421}=0.7778$$

$$P(A\mid LP)=\frac{P(A\cap LP)}{P(LP)}=\frac{31/421}{35/421}=0.8857$$

The late larvae stage has the lowest probability of survival to the pre-pupae stage.

<u>Section 2-5</u>

2-105. a) $P(A\cap B)=P(A\mid B)P(B)=(0.3)(0.6)=0.18$

b) $P(A'\cap B)=P(A'\mid B)P(B)=(0.7)(0.6)=0.42$

2-106.

$$\begin{aligned}P(A) &= P(A\cap B)+P(A\cap B')\\ &= P(A|B)P(B)+P(A|B')P(B')\\ &= (0.25)(0.8)+(0.35)(0.2)\\ &= 0.2+0.07=0.27\end{aligned}$$

2-107. Let F denote the event that a connector fails and let W denote the event that a connector is wet.

$$\begin{aligned}P(F) &= P(F|W)P(W)+P(F|W')P(W')\\ &= (0.06)(0.10)+(0.02)(0.9)=0.024\end{aligned}$$

2-108. Let F denote the event that a roll contains a flaw and let C denote the event that a roll is cotton.

$$\begin{aligned}P(F) &= P(F|C)P(C)+P(F|C')P(C')\\ &= (0.01)(0.7)+(0.02)(0.3)=0.013\end{aligned}$$

2-109. Let R denote the event that a product exhibits surface roughness. Let N, A, and W denote the events that the blades are new, average, and worn, respectively. Then,

P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)
= (0.02)(0.25) + (0.03) (0.6) + (0.06)(0.15)
= 0.032

2-110. Let A denote the event that a respondent is a college graduate and let B denote the event that an individual votes for Bush.

P(B) = P(A)P(B|A) + P(A')P(B|A') = $(0.38 \times 0.52) + (0.62 \times 0.5) = 0.0613$

2-111. Computer keyboard failures are due to faulty electrical connects (20%) or mechanical defects (80%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (37%), improper connections (13%), or poorly welded wires (50%).

(a) Find the probability that a failure is due to loose keys.
(b) Find the probability that a failure is due to improperly connected or poorly welded wires.

(a) (0.8)(0.27) = 0.212
(b) (0.2)(0.13+0.50) = 0.126

2-112. Heart failures are due to either natural occurrences (85%) or outside factors (15%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

(a) Determine the probability that a failure is due to an induced substance.
(b) Determine the probability that a failure is due to disease or infection

(a)P = $0.15 \times 0.73 = 0.1095$
(b)P = $0.85 \times (0.27+0.17) = 0.374$

2-113. A batch of 30 injection-molded parts contains 6 parts that have suffered excessive shrinkage.

(a) If two parts are selected at random, and without replacement, what is the probability that the second part selected is one with excessive shrinkage?
(b) If three parts are selected at random, and without replacement, what is the probability that the third part selected is one with excessive shrinkage?

Let A and B denote the event that the first and second part selected has excessive shrinkage, respectively.

(a) P(B)= P(B|A)P(A) + P(B|A')P(A')
= (5/29)(6/30) + (6/29)(24/30) = 0.20

(b) Let C denote the event that the third part selected has excessive shrinkage.

$$P(C) = P(C|A \cap B)P(A \cap B) + P(C|A \cap B')P(A \cap B')$$
$$+P(C|A' \cap B)P(A' \cap B) + P(C|A' \cap B')P(A' \cap B')$$
$$= \frac{4}{28}\left(\frac{5}{29}\right)\left(\frac{6}{30}\right) + \frac{5}{28}\left(\frac{24}{29}\right)\left(\frac{6}{30}\right) + \frac{5}{28}\left(\frac{6}{29}\right)\left(\frac{24}{30}\right) + \frac{6}{28}\left(\frac{23}{29}\right)\left(\frac{24}{30}\right)$$
$$= 0.20$$

2-114. A lot of 100 semiconductor chips contains 10 that are defective.
(a) Two are selected, at random, without replacement, from the lot. Determine the probability that the second chip selected is defective.
(b) Three are selected, at random, without replacement, from the lot. Determine the probability that all are defective.

Let A and B denote the events that the first and second chips selected are defective, respectively.
(a) P(B) = P(B|A)P(A) + P(B|A')P(A') = (9/99)(10/100) + (10/99)(90/100) = 0.1
(b) Let C denote the event that the third chip selected is defective.

$$P(A \cap B \cap C) = P(C|A \cap B)P(A \cap B) = P(C|A \cap B)P(B|A)P(A)$$
$$= \frac{8}{98}\left(\frac{9}{99}\right)\left(\frac{10}{100}\right)$$
$$= 0.000742$$

2-115. An article in the *British Medical Journal* ["Comparison of treatment of renal calculi by operative surgery, percutaneous nephrolithotomy, and extracorporeal shock wave lithotripsy" (1986, Vol. 82, pp. 879–892)] provided the following discussion of success rates in kidney stone removals. Open surgery had a success rate of 78% (273/350) and a newer method, percutaneous nephrolithotomy (PN), had a success rate of 83% (289/350). This newer method looked better, but the results changed when stone diameter was considered. For stones with diameters less than 2 centimeters, 93% (81/87) of cases of open surgery were successful compared with only 83% (234/270) of cases of PN. For stones greater than or equal to 2 centimeters, the success rates were 73% (192/263) and 69% (55/80) for open surgery and PN, respectively. Open surgery is better for both stone sizes, but less successful in total. In 1951, E. H. Simpson pointed out this apparent contradiction (known as **Simpson's paradox**), and the hazard still persists today. Explain how open surgery can be better for both stone sizes but worse in total.

Open surgery

	success	failure	sample size	sample percentage	conditional success rate
large stone	192	71	263	75%	73%
small stone	81	6	87	25%	93%
overall summary	273	77	350	100%	78%

PN

	success	failure	sample size	sample percentage	conditional success rate
large stone	55	25	80	23%	69%
small stone	234	36	270	77%	83%
overall summary	289	61	350	100%	83%

The overall success rate depends on the success rates for each stone size group, but also the probability of the groups. It is the weighted average of the group success rate weighted by the group size as follows

P(overall success) = P(success| large stone)P(large stone)) + P(success| small stone)P(small stone).

For open surgery, the dominant group (large stone) has a smaller success rate while for PN, the dominant group (small stone) has a larger success rate.

2-116. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is below target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cup B')$ (d) Use the total probability rule to determine $P(A)$

P(A) = 112/204 = 0.5490, P(B) = 112/204 = 0.5490
(a) P(A ∩ B) = P(A | B)P(B) = (56/112)(112/204) = 0.2745
(b) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.5490 + 0.5490 – 0.2745 = 0.8235
(c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.2745 = 0.7255
(d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (56/112) (112/204) + (56/92)(92/204) = 112/204 = 0.5490

2-117. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 1 and let *B* denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cup B')$ (d)Use the total probability rule to determine $P(A)$

P(A) = 5292/22252 = 0.2378, P(B) = 953/22252 = 0.0428
(a) P(A ∩ B) = P(A | B)P(B) = (195/953)(953/22252) = 0.0088
(b) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.2378 + 0.0428 – 0.0088 = 0.2718
(c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0088 = 0.9912
(d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (195/953)(953/22252) + (5097/21299)(21299/22252) = 0.2378

2-118. Consider the hospital emergency room data given below. Suppose that three visits that resulted in LWBS are selected randomly (without replacement) for a follow-up interview.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

(a) What is the probability that all three are selected from hospital 3?
(b) What is the probability that all three are from the same hospital?

(a) $$P = \frac{\binom{246}{3}}{\binom{953}{3}} = 0.0170$$

(b) $$P = \frac{\binom{195}{3} + \binom{270}{3} + \binom{246}{3} + \binom{242}{3}}{\binom{953}{3}} = 0.0643$$

2-119. Consider the well failure data given below. Let A denote the event that the geological formation has more than 1000 wells, and let B denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Determine the following probabilities.

(a) $P(A\_ B)$ (b) $P(A\_ B)$ (c) $P(A\_\_ B\_)$ (d) Use the total probability rule to determine $P(A)$

P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903
(a) $P(A \cap B) = P(B \mid A)P(A) = (673/6821)(6821/8493) = 0.0792$
(b) $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.8031 + 0.0903 – 0.0792 = 0.8142$
(c) $P(A' \cup B') = 1 – P(A \cap B) = 1 – 0.0792 = 0.9208$
(d) $P(A) = P(A \mid B)P(B) + P(A \mid B')P(B') = (673/767)(767/8493) + (6148/7726)(7726/8493) = 0.8031$

2-120. Consider the well failure data given below. Suppose that two failed wells are selected randomly (without replacement) for a follow-up review.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

(a) What is the probability that both are from the gneiss geological formation group?
(b) What is the probability that both are from the same geological formation group?

(a) $$P = \frac{\binom{170}{2}}{\binom{767}{2}} = 0.0489$$

(b) $$P = \frac{\binom{170}{2} + \binom{2}{2} + \binom{443}{2} + \binom{14}{2} + \binom{29}{2} + \binom{60}{2} + \binom{46}{2} + \binom{3}{2}}{\binom{767}{2}} = 0.3934$$

2-121. A Web ad can be designed from three different colors, three font types, three font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Determine the probability that the ad color is red and the font size is not the smallest one.

Let R denote red color and F denote that the font size is not the smallest. Then P(R) = 1/3, P(F) = 2/3.

Because the Web sites are generated randomly these events are independent. Therefore, $P(R \cap F) = P(R)P(F) = (1/3)(2/3) = 0.222$

Section 2-6

2-122. If $P(A \mid B) = 0.6$, $P(B) = 0.8$, and $P(A) = 0.5$, are the events A and B independent?

Because $P(A \mid B) \neq P(A)$, the events are not independent.

2-123. $P(A') = 1 – P(A) = 0.7$ and $P(A' \mid B) = 1 – P(A \mid B) = 0.8$. Since $P(A' \mid B) \neq P(A')$, A' and B are not independent events.

2-124. If A and B are mutually exclusive, then $P(A \cap B) = 0$ and $P(A)P(B) = 0.09$.
As $P(A \cap B) \neq P(A)P(B)$, A and B are not independent.

2-125. a) $P(B \mid A) = 5/599 = 0.0067$
$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/599)(5/600) + (5/599)(595/600) = 0.0083$
As $P(B \mid A) \neq P(B)$, A and B are not independent.
b) A and B are independent.

2-126. $P(A \cap B) = 70/100$, $P(A) = 86/100$, $P(B) = 80/100$.
Then, $P(A \cap B) \neq P(A)P(B)$, so A and B are not independent.

2-127. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	11	4
Supplier	2	20	5
	3	24	6

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications.
(a) Are events A and B independent? (b) Determine $P(B \mid A)$.

(a) $P(A \cap B) = 11/70$, $P(A) = 15/70$, $P(B) = 55/100$, Then $P(A \cap B) \neq P(A)P(B)$, therefore, A and B are not independent.
(b) $P(B|A) = P(A \cap B)/P(A) = (11/70)/(15/70) = 0.733$

2-128. Redundant array of inexpensive disks (RAID) is a technology that uses multiple hard drives to increase the speed of data transfer and provide instant data backup. Suppose that the probability of any hard drive failing in a day is 0.002 and the drive failures are independent.
(a) A RAID 0 scheme uses two hard drives, each containing a mirror image of the other. What is the probability of data loss? Assume that data loss occurs if both drives fail within the same day.
(b) A RAID 1 scheme splits the data over two hard drives. What is the probability of data loss? Assume that data loss occurs if at least one drive fails within the same day.

(a) $P = (0.002)^2 = 4 \times 10^{-6}$
(b) $P = 1 - (0.998)^2 = 0.004$

2-129. The probability that a lab specimen contains high levels of contamination is 0.10. Four samples are checked, and the samples are independent.
(a) What is the probability that none contain high levels of contamination?
(b) What is the probability that exactly one contains high levels of contamination?
(c) What is the probability that at least one contains high levels of contamination?

It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the *i*th sample contains high levels of contamination.

(a) $P(H_1' \cap H_2' \cap H_3' \cap H_4') = P(H_1')P(H_2')P(H_3')P(H_4')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^4 = 0.6561$

(b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^3(0.1) = 0.0729$. Therefore, the answer is 4(0.0729) = 0.2916.

(c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.6561 = 0.3439.

2-130. In a test of a printed circuit board using a random test pattern, an array of 10 bits is equally likely to be 0 or 1. Assume the bits are independent.
(a) What is the probability that all bits are 1s?
(b) What is the probability that all bits are 0s?
(c) What is the probability that exactly 4 bits are 1s and 6 bits are 0s?

Let A_i denote the event that the ith bit is a one.

a) By independence $P(A_1 \cap A_2 \cap \ldots \cap A_{10}) = P(A_1)P(A_2)\ldots P(A_{10}) = (\frac{1}{2})^{10} = 0.000976$

b) By independence, $P(A_1' \cap A_2' \cap \ldots \cap A_{10}') = P(A_1')P(A_2')\ldots P(A_{10}^c) = (\frac{1}{2})^{10} = 0.000976$

c) The probability of the following sequence is

$P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5' \cap A_6' \cap A_7 \cap A_8 \cap A_9 \cap A_{10}) = (\frac{1}{2})^{10}$, by independence. The number of sequences consisting of four "1"'s, and six "0"'s is $\binom{10}{4} = \frac{10!}{4!6!} = 210$. The answer is $210\left(\frac{1}{2}\right)^{10} = 0.205$

2-131. (a) Let I and G denote an infested and good sample. There are 4 ways to obtain four consecutive samples showing the signs of the infestation: IIIIGGG, GIIIIGG, GGIIIIG, GGGIIII. Therefore, the probability is $4 \times (0.3)^4(0.7)^3 = 0.0111$.

(b) There are 14 ways to obtain three out of four consecutive samples showing the signs of infestation. The probability is $14 \times (0.3^3 \times 0.7^4) = 0.0908$

2-132. A player of a video game is confronted with a series of four opponents and an 70% probability of defeating each opponent. Assume that the results from opponents are independent (and that when the player is defeated by an opponent the game ends).
(a) What is the probability that a player defeats all four opponents in a game?
(b) What is the probability that a player defeats at least two opponents in a game?
(c) If the game is played three times, what is the probability that the player defeats all four opponents at least once?

(a) $P = (0.7)^4 = 0.2401$

(b) $P = 1 - 0.3 - 0.7 \times 0.3 = 0.49$

(c) Probability defeats all four in a game = 0.7^4 = 0.2401. Probability defeats all four at least once = $1 - (1 - 0.2401)^3$ = 0.5612

2-133. In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each

other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL, measured to the nearest mL. Assume that two technicians each conduct titrations independently.
(a) What is the probability that both technicians obtain equivalence at 100 mL?
(b) What is the probability that both technicians obtain equivalence between 98 and 104 mL (inclusive)?
(c) What is the probability that the average volume at equivalence from the technicians is 100 mL?

(a) The probability that one technician obtains equivalence at 100 mL is 0.1.
So the probability that both technicians obtain equivalence at 100 mL is $0.1^2 = 0.01$.
(b) The probability that one technician obtains equivalence between 98 and 104 mL is 0.7.
So the probability that both technicians obtain equivalence between 98 and 104 mL is $0.7^2 = 0.49$.
(c) The probability that the average volume at equivalence from the technician is 100 mL is $9(0.1^2) = 0.09$.

2-134. A credit card contains 16 digits. It also contains the month and year of expiration. Suppose there are 1 million credit card holders with unique card numbers. A hacker randomly selects a 16-digit credit card number.
(a) What is the probability that it belongs to a user?
(b) Suppose a hacker has a 10% chance of correctly guessing the year your card expires and randomly selects 1 of the 12 months. What is the probability that the hacker correctly selects the month and year of expiration?

(a) $P = \frac{10^6}{10^{16}} = 10^{-10}$

(b) $P = 0.1 \times (\frac{1}{12}) = 0.00833$

2-135. Eight cavities in an injection-molding tool produce plastic connectors that fall into a common stream. A sample is chosen every several minutes. Assume that the samples are independent.
(a) What is the probability that five successive samples were all produced in cavity 1 of the mold?
(b) What is the probability that five successive samples were all produced in the same cavity of the mold?
(c) What is the probability that four out of five successive samples were produced in cavity 1 of the mold?

Let A denote the event that a sample is produced in cavity one of the mold.

(a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

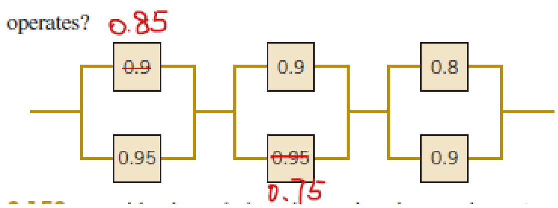
(b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup ... \cup B_8) = P(B_1) + P(B_2) + ... + P(B_8)$

From part (a), $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

(c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4 (\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4 (\frac{7}{8}) = 0.00107$.

2-136. Let A denote the upper devices function. Let B denote the lower devices function.
P(A) = (0.8)(0.7)(0.6) = 0.336
P(B) = (0.95)(0.95)(0.95) = 0.8574
$P(A \cap B)$ = (0.336)(0.8574) = 0.288
Therefore, the probability that the circuit operates = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.9054$

2-137. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



P = [1 – (0.15)(0.05)][1 – (0.1)(0.25)][1 – (0.2)(0.1)] = 0.9483

2-138. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target. Are these events independent?

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

P(A) = 112/204 = 0.5490, P(B) = 92/204 = 0.4510, P(A ∩ B) = 56/204 = 0.2745
Because P(A) P(B) = (0.5490)(0.4510) = 0.2476 ≠ 0.2745 = P(A ∩ B), A and B are not independent.

2-139. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B* denote the event that a visit results in LWBS (at any hospital). Are these events independent?

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, P(A ∩ B) = 242/22252 = 0.0109
Because P(A)*P(B) = (0.1945)(0.0428) = 0.0083 ≠ 0.0109 = P(A ∩ B), A and B are not independent.

2-140. Consider the well failure data given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Are these events independent?

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

P(A) = (1685+3733+1403)/8493 = 0.8031, P(B) = (170+2+443+14+29+60+46+3)/8493 = 0.0903,
P(A ∩ B) = (170+443+60)/8493 = 0.0792
Because P(A)*P(B) = (0.8031)(0.0903) = 0.0725 ≠ 0.0792 = P(A ∩ B), A and B are not independent.

2-141. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Are *A* and *B* independent events? Explain why or why not.

P(A) = (3*5*3*5)/(4*3*5*3*5) = 0.25, P(B) = (4*3*4*3*5)/(4*3*5*3*5) = 0.8,
P(A ∩ B) = (3*4*3*5) /(4*3*5*3*5) = 0.2
Because P(A)*P(B) = (0.25)(0.8) = 0.2 = P(A ∩ B), A and B are independent.

Section 2-7

2-142. Because, P(A|B) P(B) = P(A∩B) = P(B|A) P(A),

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.6(0.3)}{0.4} = 0.45$$

2-143.
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

$$= \frac{0.5 \times 0.7}{(0.5 \times 0.7) + (0.1 \times 0.3)} = 0.921$$

2-144. Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 35% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.02%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent?

Let F denote a fraudulent user and let T denote a user that originates calls from two or more metropolitan areas in a day. Then,

$$P(F|T) = \frac{P(T|F)P(F)}{P(T|F)P(F) + P(T|F')P(F')} = \frac{0.35(0.0002)}{0.35(0.0002) + 0.01(.9998)} = 0.0070$$

2-145. A new process of more accurately detecting anaerobic respiration in cells is being tested. The new process is important due to its high accuracy, its lack of extensive experimentation, and the fact that it could be used to identify five different categories of organisms: obligate anaerobes, facultative anaerobes, aerotolerant, microaerophiles, and nanaerobes instead of using a single test for each category. The process claims that it can identify obligate anaerobes with 97.8% accuracy, facultative anaerobes with 98.1% accuracy, aerotolerants with 95.9% accuracy, microaerophiles with 96.5% accuracy, and nanaerobes with 99.2% accuracy. If any category is not present, the process does not signal. Samples are prepared for the calibration of the process and 31% of them contain obligate anaerobes, 27% contain facultative anaerobes, 21% contain microaerophiles, 13% contain nanaerobes, and 8% contain aerotolerants. A test sample is selected randomly.

(a) What is the probability that the process will signal?
(b) If the test signals, what is the probability that microaerophiles are present?

(a) P=(0.31)(0.978)+(0.27)(0.981)+(0.21)(0.965)+(0.13)(0.992)+(0.08)(0.959)
= 0.97638

(b) $P = \frac{(0.21)(0.965)}{0.97638} = 0.207552$

2-146. In the 2012 presidential election, exit polls from the critical state of Ohio provided the following results:

	Obama	Romney
No college degree (60%)	52%	47%
College graduate (40%)	47%	50%

If a randomly selected respondent voted for Obama, what is the probability that the person has a college degree?

Let A denote the event that a respondent is a college graduate and let B denote the event that a voter votes for Obama.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A')P(B|A')} = \frac{(0.49)(0.48)}{(0.49)(0.48) + (0.51)(0.53)} = 46.5282\%$$

2-147. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received

good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 45% of products have been highly successful, 30% have been moderately successful, and 25% have been poor products.
(a) What is the probability that a product attains a good review?
(b) If a new design attains a good review, what is the probability that it will be a highly successful product?
(c) If a product does not attain a good review, what is the probability that it will be a highly successful product?

Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.
(a)

$$P(G) = P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P)$$
$$= 0.95(0.45) + 0.60(0.30) + 0.10(0.25)$$
$$= 0.6325$$

(b) Using the result from part a.,

$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.45)}{0.6325} = 0.6759$$

(c) $P(H|G') = \frac{P(G'|H)P(H)}{P(G')} = \frac{0.05(0.45)}{1-0.6325} = 0.0612$

2-148. An inspector working for a manufacturing company has a 98% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that 1% of the items its line produces are nonconforming.
(a) What is the probability that an item selected for inspection is classified as defective?
(b) If an item selected at random is classified as non defective, what is the probability that it is indeed good?

(a) P(D)=P(D|G)P(G)+P(D|G')P(G')=(.005)(.99)+(.98)(.01)=0.01475
(b) $P(G|D')=P(G\cap D')/P(D')=P(D'|G)P(G)/P(D')=(.995)(.99)/(1-.01475)=0.9998$

2-149. A new analytical method to detect pollutants in water is being tested. This new method of chemical analysis is important because, if adopted, it could be used to detect three different contaminants—organic pollutants, volatile solvents, and chlorinated compounds—instead of having to use a single test for each pollutant. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.
(a) What is the probability that the test will signal?
(b) If the test signals, what is the probability that chlorinated compounds are present?

Denote as follows: S = signal, O = organic pollutants, V = volatile solvents, C = chlorinated compounds
(a) P(S) = P(S|O)P(O)+P(S|V)P(V)+P(S|C)P(C) = 0.997(0.60) + 0.9995(0.27) + 0.897(0.13) = 0.9847
(b) P(C|S) = P(S|C)P(C)/P(S) = (0.897)(0.13)/0.9847 = 0.1184

2-150. Consider the endothermic reactions given below. Use Bayes' theorem to calculate the probability that a reaction's final temperature is 271 K or less given that the heat absorbed is below target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction final temperature is 271 K or less
Let B denote the event that the heat absorbed is below target

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B|A)}{P(A)P(B|A)+P(A')P(B|A')}$$
$$=\frac{(0.5490)(0.5)}{(0.5490)(0.5)+(0.4510)(0.6087)}=0.5000$$

2-151. Consider the hospital emergency room data given below. Use Bayes' theorem to calculate the probability that a person visits hospital 4 given they are LWBS.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Let L denote the event that a person is LWBS
Let A denote the event that a person visits Hospital 1
Let B denote the event that a person visits Hospital 2
Let C denote the event that a person visits Hospital 3
Let D denote the event that a person visits Hospital 4

$$P(D|L)=\frac{P(L|D)P(D)}{P(L|A)P(A)+P(L|B)P(B)+P(L|C)P(C)+P(L|D)P(D)}$$
$$=\frac{(0.0559)(0.1945)}{(0.0368)(0.2378)+(0.0386)(0.3142)+(0.0436)(0.2535)+(0.0559)(0.1945)}$$
$$=0.2540$$

2-152. Consider the well failure data given below. Use Bayes' theorem to calculate the probability that a randomly selected well is in the gneiss group given that the well has failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Let A denote the event that a well is failed
Let B denote the event that a well is in Gneiss
Let C denote the event that a well is in Granite
Let D denote the event that a well is in Loch raven schist
Let E denote the event that a well is in Mafic
Let F denote the event that a well is in Marble
Let G denote the event that a well is in Prettyboy schist
Let H denote the event that a well is in Other schist
Let I denote the event that a well is in Serpentine

$$P(B|A)=\frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|C)P(C)+P(A|D)P(D)+P(A|E)P(E)+P(A|F)P(F)+P(A|G)P(E)+P(A|G)P(G)+P(A|H)P(H)}$$
$$=\frac{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right)}{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right)+\left(\frac{2}{28}\right)\left(\frac{28}{8493}\right)+\left(\frac{443}{3733}\right)\left(\frac{3733}{8493}\right)+\left(\frac{14}{363}\right)\left(\frac{363}{8493}\right)+\left(\frac{29}{309}\right)\left(\frac{309}{8493}\right)+\left(\frac{60}{1403}\right)\left(\frac{1403}{8493}\right)+\left(\frac{46}{933}\right)\left(\frac{933}{8493}\right)+\left(\frac{3}{39}\right)\left(\frac{39}{8493}\right)}$$
$$=0.2216$$

2-153. Denote as follows: A = affiliate site, S = search site, B =blue, G =green

$$P(S \mid B) = \frac{P(B \mid S)P(S)}{P(B \mid S)P(S) + P(B \mid A)P(A)}$$
$$= \frac{(0.4)(0.7)}{(0.4)(0.7) + (0.8)(0.3)}$$
$$= 0.5$$

Section 2-8

2-154. Continuous: a, c, d, f, h, i; Discrete: b, e, and g

2-155. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The number of cracks exceeding one-half inch in 10 miles of an interstate highway.
(b) The weight of an injection-molded plastic part.
(c) The number of molecules in a sample of gas.
(d) The concentration of output from a reactor.
(e) The current in an electronic circuit.

(a) discrete (b) continuous (c) discrete, but large values might be modeled as continuous
(d) continuous (e) continuous

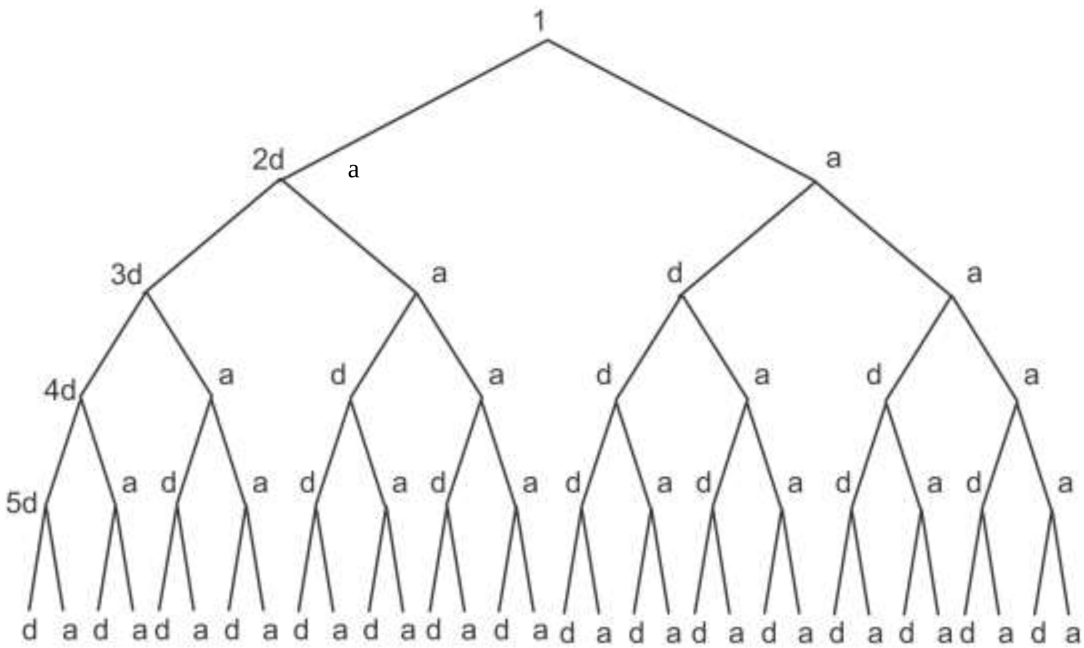
2-156. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The time for a computer algorithm to assign an image to a category.
(b) The number of bytes used to store a file in a computer.
(c) The ozone concentration in micrograms per cubic meter.
(d) The ejection fraction (volumetric fraction of blood pumped from a heart ventricle with each beat).
(e) The fluid flow rate in liters per minute.

(a) continuous (b) discrete, but large values might be modeled as continuous
(c) continuous (d) continuous (e) continuous

Supplemental Exercises

2-157. Let B denote the event that a glass breaks.
Let L denote the event that large packaging is used.
P(B)= P(B|L)P(L) + P(B|L')P(L')
= 0.02(0.60) + 0.03(0.40) = 0.024

2-158. Let "d" denote a defective calculator and let "a" denote an acceptable calculator



a) $S = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ daadd, daada, daaad, daaaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa, \\ aaddd, aadda, aadad, aadaa, \\ aaadd, aaada, aaaad, aaaaa \end{Bmatrix}$

b) $A = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ daadd, daada, daaad, daaaa \end{Bmatrix}$

c) $B = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa \end{Bmatrix}$

d) $A \cap B = \begin{Bmatrix} dddda, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa \end{Bmatrix}$

e) To determine $B \cup C$, we need C.

$C = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ daddd, dadda, dadad, dadaa, \\ adddd, addda, addad, addaa, \\ aaddd, aadda, aadad, aadaa \end{Bmatrix}$

$$B \cup C = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa, \\ aaddd, aadda, aadad, aadaa \end{Bmatrix}$$

2-159. Let A = excellent surface finish; B = excellent length
a) P(A) = 73/100 = 0.73
b) P(B) = 90/100 = 0.90
c) P(A') = 1 – 0.73 = 0.27
d) P(A∩B) = 70/100 = 0.70
e) P(A∪B) = 0.93
f) P(A′∪B) = 0.97

2-160. Shafts are classified in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

Tool 1		Roundness Conforms	
		Yes	No
Surface finish	Yes	173	1
Conforms	No	4	2

Tool 2		Roundness Conforms	
		Yes	No
Surface finish	Yes	152	4
Conforms	No	8	6

(a) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or to roundness requirements or is from tool 1?
(b) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or does not conform to roundness requirements or is from tool 2?
(c) If a shaft is selected at random, what is the probability that the shaft conforms to both surface finish and roundness requirements or the shaft is from tool 2?
(d) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or the shaft is from tool 2?

(a) (174+156+4+8+2)/350 = 0.9829
(b) (174+156+2+6+8)/350 = 0.9886
(c) (173+152+4+8+6)/350 = 0.98
(d) (173+1+152+4+8+6)/350 = 0.983

2-161. If *A*, *B*, and *C* are mutually exclusive events, is it possible for *P*(*A*) = 0.2, *P*(*B*) = 0.3, and *P*(*C*) = 0.6? Why or why not?

If A,B,C are mutually exclusive, then P(A∪B∪C) = P(A) + P(B) + P(C) = 0.2 + 0.3 + 0.6 = 1.1, which greater than 1. Therefore, P(A), P(B),and P(C) cannot equal the given values.

2-162. a) 350/367 b) 10/23

2-163. A researcher receives 100 containers of oxygen. Of those containers, 10 have oxygen that is not ionized, and the rest are ionized. Two samples are randomly selected, without replacement, from the lot.
(a) What is the probability that the first one selected is not ionized?
(b) What is the probability that the second one selected is not ionized given that the first one was ionized?
(c) What is the probability that both are ionized?
(d) How does the answer in part (b) change if samples selected were replaced prior to the next selection?

(a) P(the first one selected is not ionized)=10/100=0.1
(b) P(the second is not ionized given the first one was ionized) =10/99=0.101
(c) P(both are ionized)
= P(the first one selected is ionized) $\times$ P(the second is ionized given the first one was ionized)
= (90/100)$\times$(89/99)=0.809
(d) If samples selected were replaced prior to the next selection,
P(the second is not ionized given the first one was ionized) =10/100=0.1.
The event of the first selection and the event of the second selection are independent.

2-164. a) P(A) = 20/50 = 0.4
b) $P(B|A)$ = 19/49
c) $P(A \cap B)$ = P(A) P(B/A) = (0.4) (19/49) = 0.155
d) $P(A \cup B)$ = 1 – P(A' and B') = $1-\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)=0.645$
A = first is local, B = second is local, C = third is local
e) $P(A \cap B \cap C)$ = (20/50)(19/49)(18/48) = 0.058
f) $P(A \cap B \cap C')$ = (20/50)(19/49)(30/48) = 0.9496

2-165. a) P(A) = 0.02
b) P(A') = 0.98
c) P(B|A) = 0.5
d) P(B|A') = 0.04
e) $P(A \cap B) = P(B|A)P(A)$ = (0.5)(0.02) = 0.01
f) $P(A \cap B') = P(B'|A)P(A)$ = (0.5)(0.02) = 0.01
g) $P(B) = P(B|A)P(A) + P(B|A')P(A')$ = (0.5)(0.02) + (0.04)(0.98) = 0.0492

2-166. Incoming calls to a customer service center are classified as complaints (70% of calls) or requests for information (30% of calls). Of the complaints, 38% deal with computer equipment that does not respond and 57% deal with incomplete software installation; in the remaining 5% of complaints, the user has improperly followed the installation instructions. The requests for information are evenly divided on technical questions (50%) and requests to purchase more products (50%).
(a) What is the probability that an incoming call to the customer service center will be from a customer who has not followed installation instructions properly?
(b) Find the probability that an incoming call is a request for purchasing more products.

Let U denote the event that the user has improperly followed installation instructions.
Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.7)(0.05) = 0.035
b) P(P|R)P(R) = (0.5)(0.3) = 0.15

2-167. A congested computer network has a 0.005 probability of losing a data packet, and packet losses are independent events. A lost packet must be resent.
(a) What is the probability that an e-mail message with 100 packets will need to be resent?
(b) What is the probability that an e-mail message with 4 packets will need exactly 1 to be resent?
(c) If 10 e-mail messages are sent, each with 100 packets, what is the probability that at least 1 message will need some packets to be resent?

(a) $P = 1-(1-0.005)^{100} = 0.3942$
(b) $P = C_3^1(0.995^2)0.005 = 0.00149$
(c) $P = 1-[(1-0.005)^{100}]^{10} = 0.9933$

2-168. $P(A \cap B)$ = 80/100, P(A) = 82/100, P(B) = 90/100.

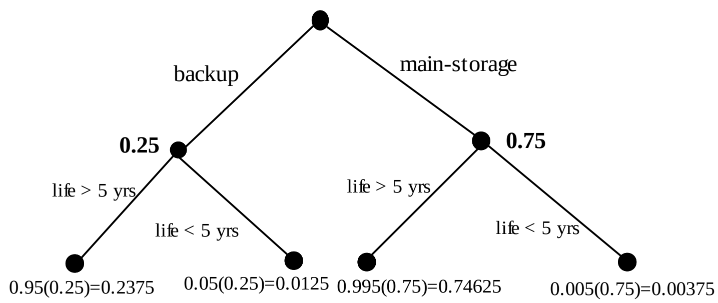
Then, $P(A \cap B) \neq P(A)P(B)$, so A and B are <u>not</u> independent.

2-169. Let A_i denote the event that the ith readback is successful. By independence
$P(A_1' \cap A_2' \cap A_3') = P(A_1')P(A_2')P(A_3') = (0.04)^3 = 0.000064$.

2-170. Semiconductor lasers used in optical storage products require higher power levels for write operations than for read operations. High-power-level operations lower the useful life of the laser. Lasers in products used for backup of higher-speed magnetic disks primarily write, and the probability that the useful life exceeds five years is 0.95. Lasers that are in products that are used for main storage spend approximately an equal amount of time reading and writing, and the probability that the useful life exceeds five years is 0.995. Now, 25% of the products from a manufacturer are used for backup and 75% of the products are used for main storage.
Let *A* denote the event that a laser's useful life exceeds five years, and let *B* denote the event that a laser is in a product that is used for backup.

Use a tree diagram to determine the following:

(a) $P(B)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$
(d) $P(A \cap B)$ (e) $P(A \cap B')$ (f) $P(A)$
(g) What is the probability that the useful life of a laser that failed before five years?
(h) What is the probability that a laser that failed before five years came from a product used for backup?



(a) $P(B) = 0.25$
(b) $P(A|B) = 0.95$
(c) $P(A|B') = 0.995$
(d) $P(A \cap B) = P(A|B)P(B) = 0.95(0.25) = 0.2375$
(e) $P(A \cap B') = P(A|B')P(B') = 0.995(0.75) = 0.74625$
(f) $P(A) = P(A \cap B) + P(A \cap B') = 0.95(0.25) + 0.995(0.75) = 0.98375$
(g) $P(A') = 1 - P(A) = 1 - 0.98375 = 0.01625$
(h)

$$P(B|A') = \frac{P(A'|B)P(B)}{P(A'|B)P(B) + P(A'|B')P(B')} = \frac{0.05(0.25)}{0.05(0.25) + 0.005(0.75)} = 0.769$$

2-171. Energy released from cells breaks the molecular bond and converts ATP (adenosine triphosphate) into ADP (adenosine diphosphate). Storage of ATP in muscle cells (even for an athlete) can sustain maximal muscle power only for less than five seconds (a short dash). Three systems are used to replenish ATP—phosphagen system, glycogen-lactic acid system (anaerobic), and aerobic respiration—but the first is useful only for less than 10 seconds, and even the second system provides less than two minutes of ATP. An endurance athlete needs to perform below the anaerobic threshold to sustain energy for extended periods. A sample of 100 individuals is described by the energy system used in exercise at different intensity levels.

	Primarily Aerobic	
Period	Yes	No
1	42	7
2	13	38

Let *A* denote the event that an individual is in period 2, and let *B* denote the event that the energy is primarily aerobic. Determine the number of individuals in
(a) $A' \cap B$ (b) B' (c) $A \cup B$

(a) $A' \cap B = 42$
(b) B'=45
(c) $A \cup B = 93$

2-172. A sample preparation for a chemical measurement is completed correctly by 30% of the lab technicians, completed with a minor error by 64%, and completed with a major error by 6%.
(a) If a technician is selected randomly to complete the preparation, what is the probability that it is completed without error?
(b) What is the probability that it is completed with either a minor or a major error?

(a) 0.3
(b) 0.7

2-173. In circuit testing of printed circuit boards, each board either fails or does not fail the test. A board that fails the test is then checked further to determine which one of five defect types is the primary failure mode. Represent the sample space for this experiment.

Let D_i denote the event that the primary failure mode is type i and let A denote the event that a board passes the test. The sample space is S = $\{A, A'D_1, A'D_2, A'D_3, A'D_4, A'D_5\}$.

2-174. The data from 200 machined parts are summarized as follows:

	Depth of Bore	
Edge Condition	Above Target	Below Target
Coarse	17	10
Moderate	25	15
Smooth	53	80

(a) What is the probability that a part selected has a moderate edge condition and a below-target bore depth?
(b) What is the probability that a part selected has a moderate edge condition or a below-target bore depth?
(c) What is the probability that a part selected does not have a moderate edge condition or does not have a below-target bore depth?

(a) 15/200 (b) 130/200 (c) 70/200

2-175. a) P(A) = 29/120 = 0.24
b) $P(A \cap B)$ = 20/120 = 0.1667
c) $P(A \cup B)$ = (29 + 105 – 20)/120 = 0.95
d) $P(A' \cap B)$ = 91/120 = 0.7583
e) $P(A|B) = P(A \cap B)/P(B)$ = (0.1667)(0.875) = 0.146

2-176. Let A_i denote the event that the *i*th order is shipped on time.
a) By independence, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = (0.96)^3 = 0.885$
b) Let

$B_1 = A_1' \cap A_2 \cap A_3$

$B_2 = A_1 \cap A_2' \cap A_3$

$B_3 = A_1 \cap A_2 \cap A_3'$

Then, because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3) = P(B_1) + P(B_2) + P(B_3)$$
$$= 3(0.96)^2(0.04)$$
$$= 0.11$$

c) Let

$B_1 = A_1' \cap A_2' \cap A_3$

$B_2 = A_1' \cap A_2 \cap A_3'$

$B_3 = A_1 \cap A_2' \cap A_3'$

$B_4 = A_1' \cap A_2' \cap A_3'$

Because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3 \cup B_4) = P(B_1)+P(B_2)+P(B_3)+P(B_4)$$
$$= 3(0.04)^2(0.96) + (0.04)^3$$
$$= 0.00467$$

2-177. Let E_1, E_2, and E_3 denote the samples that conform to a percentage of solids specification, a molecular weight specification, and a color specification, respectively. A total of 240 samples are classified by the E_1, E_2, and E_3 specifications, where *yes* indicates that the sample conforms.

***E*$_3$ yes**

		E_2		
		Yes	No	Total
E_1	Yes	200	1	201
	No	6	3	9
Total		206	4	210

***E*$_3$ no**

		E_2		
		Yes	No	Total
E_1	Yes	21	3	24
	No	6	0	6
Total		27	3	30

(a) Are E_1, E_2, and E_3 mutually exclusive events?
(b) Are E_1', E_2', and E_3' mutually exclusive events?
(c) What is $P(E_1'$ or E_2' or $E_3')$?
(d) What is the probability that a sample conforms to all three specifications?
(e) What is the probability that a sample conforms to the E_1 or E_3 specification?
(f) What is the probability that a sample conforms to the E_1 or E_2 or E_3 specification?

a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$
b) No, $E_1' \cap E_2'$ is not $\varnothing$
c) $P(E_1' \cup E_2' \cup E_3') = P(E_1') + P(E_2') + P(E_3') – P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3') + P(E_1' \cap E_2' \cap E_3') = 40/240$
d) $P(E_1 \cap E_2 \cap E_3) = 200/240$
e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) – P(E_1 \cap E_3) = 234/240$
f) $P(E_1 \cup E_2 \cup E_3) = 1 – P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2-178. Transactions to a computer database are either new items or changes to previous items. The addition of an item can be completed in less than 100 milliseconds 90% of the time, but only 20% of changes to a previous item can be completed in less than this time. If 35% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds?

(0.20)(0.35) +(0.65)(0.9) = 0.655

2-179. A steel plate contains 25 bolts. Assume that 5 bolts are not torqued to the proper limit. 3 bolts are selected at random, without replacement, to be checked for torque.
(a) What is the probability that all 3 of the selected bolts are torqued to the proper limit?
(b) What is the probability that at least 1 of the selected bolts is *not* torqued to the proper limit?

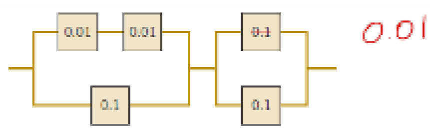
Let A_i denote the event that the *i*th bolt selected is not torqued to the proper limit.
a) Then,

$$P(A_1 \cap A_2 \cap A_3) = P(A_3 | A_1 \cap A_2)P(A_2 | A_1)P(A_1)$$
$$= \left(\frac{18}{23}\right)\left(\frac{19}{24}\right)\left(\frac{20}{25}\right) = 0.496$$

b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$$P(B) = 1 - P(B') = 1 - \left(\frac{20}{25}\right)\left(\frac{19}{24}\right)\left(\frac{18}{23}\right) = 0.504$$

2-180. The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit operates?



Let A, B denote the event that the first, second portion of the circuit operates. Then,
P(A) = (0.99)(0.99)+0.9-(0.99)(0.99)(0.9) = 0.998
P(B) = 0.99+0.9-(0.99)(0.9) = 0.999 and
P($A \cap B$) = P(A) P(B) = (0.998) (0.999) = 0.997

2-181. The probability that concert tickets are available by telephone is 0.9. For the same event, the probability that tickets are available through a Web site is 0.95. Assume that these two ways to buy tickets are independent. What is the probability that someone who tries to buy tickets through the Web and by phone will obtain tickets?

A_1 = by telephone, A_2 = website; $P(A_1) = 0.9$, $P(A_2) = 0.95$;
By independence $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.9 + 0.95 - 0.9(0.95) = 0.995$

2-182. The British government has stepped up its information campaign regarding foot-and-mouth disease by mailing brochures to farmers around the country. It is estimated that 97% of Scottish farmers who receive the brochure possess enough information to deal with an outbreak of the disease, but only 87% of those without the brochure can deal with an outbreak. After the first three months of mailing, 95% of the farmers in Scotland had received the informative brochure. Compute the probability that a randomly selected farmer will have enough information to deal effectively with an outbreak of the disease.

P(Possess) = 0.95(0.97) +(0.05)(0.87) = 0.965

2-183. In an automated filling operation, the probability of an incorrect fill when the process is operated at a low speed is 0.001. When the process is operated at a high speed, the probability of an incorrect fill is 0.02. Assume that 40% of the containers are filled when the process is operated at a high speed and the remainder are filled when the process is operated at a low speed.
(a) What is the probability of an incorrectly filled container?
(b) If an incorrectly filled container is found, what is the probability that it was filled during the high-speed operation?

Let D denote the event that a container is incorrectly filled and let H denote the event that a container is filled under high-speed operation. Then,

a) $P(D) = P(D|H)P(H) + P(D|H')P(H') = 0.02(0.40) + 0.001(0.60) = 0.0086$

b) $P(H|D) = \dfrac{P(D|H)P(H)}{P(D)} = \dfrac{0.02(0.40)}{0.0086} = 0.9302$

2-184. An encryption-decryption system consists of three elements: encode, transmit, and decode. A faulty encode occurs in 0.75% of the messages processed, transmission errors occur in 2% of the messages, and a decode error occurs in 0.1% of the messages. Assume the errors are independent.
(a) What is the probability of a completely defect-free message?
(b) What is the probability of a message that has either an encode or a decode error?

(a) $P(E' \cap T' \cap D') = (0.9925)(0.98)(0.999) = 0.972$
(b) $P(E \cup D) = P(E) + P(D) - P(E \cap D) = 0.00850$

2-185. D = defective copy

a) $P(D = 1) = \left(\frac{2}{80}\right)\left(\frac{78}{79}\right)\left(\frac{77}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{77}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{1}{78}\right) = 0.049$

b) $P(D = 2) = \left(\frac{2}{80}\right)\left(\frac{1}{79}\right)\left(\frac{78}{78}\right)+\left(\frac{2}{80}\right)\left(\frac{78}{79}\right)\left(\frac{1}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{1}{78}\right) = 0.00095$

c) Let A represent the event that the two items NOT inspected are not defective. Then, P(A) = (78/80)(77/79) = 0.95.

2-186. The tool fails if any component fails. Let F denote the event that the tool fails. Then, $P(F') = (0.99)^{20}$ by independence and $P(F) = 1 - (0.99)^{20} = 0.182$

2-187. An e-mail message can travel through one of two server routes. The probability of transmission error in each of the servers and the proportion of messages that travel each route are shown in the following table. Assume that the servers are independent.

		Probability of Error			
	Percentage of Messages	Server 1	Server 2	Server 3	Server 4
Route 1	30	0.01	0.025	—	—
Route 2	70	—	—	0.02	0.017

(a) What is the probability that a message will arrive without error?
(b) If a message arrives in error, what is the probability it was sent through route 1?

(a) (0.3)(0.99)(0.975) + (0.7)(0.98)(0.983) = 0.9639

(b) $P(route1|E) = \dfrac{P(E|route1)P(route1)}{P(E)} = \dfrac{0.03475(0.30)}{1-0.9639} = 0.2888$

2-188. A machine tool is idle 15% of the time. You request immediate use of the tool on five different occasions during the year. Assume that your requests represent independent events.
(a) What is the probability that the tool is idle at the time of all of your requests?
(b) What is the probability that the machine is idle at the time of exactly four of your requests?
(c) What is the probability that the tool is idle at the time of at least three of your requests?

(a) By independence, $0.15^5 = 7.59 \times 10^{-5}$

(b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

$$P(A_1 A_2 A_3 A_4 A_5' \;\; or \;\; A_1 A_2 A_3 A_4' A_5 \;\; or \;\; A_1 A_2 A_3' A_4 A_5 \;\; or \;\; A_1 A_2' A_3 A_4 A_5 \;\; or \;\; A_1' A_2 A_3 A_4 A_5)$$
$$= 5(0.15^4)(0.85^1)$$
$$= 0.00215$$

(c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A_4' A_5' \text{ or } A_1 A_2 A_3' A_4 A_5' \text{ or } A_1 A_2 A_3' A_4' A_5 \text{ or } A_1 A_2' A_3 A_4 A_5' \text{ or } A_1 A_2' A_3 A_4' A_5 \text{ or}$$
$$A_1 A_2' A_3' A_4 A_5 \text{ or } A_1' A_2 A_3 A_4 A_5' \text{ or } A_1' A_2 A_3 A_4' A_5 \text{ or } A_1' A_2 A_3' A_4 A_5 \text{ or } A_1' A_2' A_3 A_4 A_5)$$
$$= 10(0.15^3)(0.85^2)$$
$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2-189. A lot of 50 spacing washers contains 30 washers that are thicker than the target dimension. Suppose that 3 washers are selected at random, without replacement, from the lot.
(a) What is the probability that all 3 washers are thicker than the target?
(b) What is the probability that the third washer selected is thicker than the target if the first 2 washers selected are thinner than the target?
(c) What is the probability that the third washer selected is thicker than the target?

Let A_i denote the event that the ith washer selected is thicker than target.

(a) $\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{48}\right) = 0.207$

(b) 30/48 = 0.625

(c) The requested probability can be written in terms of whether or not the first and second washer selected are thicker than the target. That is,

$$\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{48}\right) + \left(\frac{30}{50}\right)\left(\frac{20}{49}\right)\left(\frac{29}{48}\right) + \left(\frac{20}{50}\right)\left(\frac{30}{49}\right)\left(\frac{29}{48}\right) + \left(\frac{20}{50}\right)\left(\frac{19}{49}\right)\left(\frac{30}{48}\right) = 0.60$$

2-190. Washers are selected from the lot at random without replacement.
(a) What is the minimum number of washers that need to be selected so that the probability that all the washers are thinner than the target is less than 0.10?
(b) What is the minimum number of washers that need to be selected so that the probability that 1 or more washers are thicker than the target is at least 0.90?

(a) If n washers are selected, then the probability they are all less than the target is $\frac{20}{50} \cdot \frac{19}{49} \cdots \frac{20-n+1}{50-n+1}$.

n	probability all selected washers are less than target
1	20/50 = 0.4
2	(20/50)(19/49) = 0.155
3	(20/50)(19/49)(18/48) = 0.058

Therefore, the answer is n = 3.

(b) Then event E that one or more washers is thicker than target is the complement of the event that all are less than target. Therefore, P(E) equals one minus the probability in part a. Therefore, n = 3.

2-191.

$a)\quad P(A \cup B) = \frac{112+60+254}{940} = 0.453$

$b)\quad P(A \cap B) = \frac{254}{940} = 0.27$

$c)\quad P(A' \cup B) = \frac{514+60+254}{940} = 0.881$

$d)\quad P(A' \cap B') = \frac{514}{940} = 0.547$

. e) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \left(\frac{254}{940}\right) \Big/ \left(\frac{314}{940}\right) = 0.809$

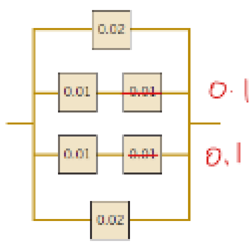
f) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \left(\frac{254}{940}\right) \Big/ \left(\frac{366}{940}\right) = 0.694$

2-192. The alignment between the magnetic media and head in a magnetic storage system affects the system's performance. Suppose that 10% of the read operations are degraded by skewed alignments, 5% of the read operations are degraded by off-center alignments, and the remaining read operations are properly aligned. The probability of a read error is 0.01 from a skewed alignment, 0.02 from an off-center alignment, and 0.001 from a proper alignment.
(a) What is the probability of a read error?
(b) If a read error occurs, what is the probability that it is due to a skewed alignment?

Let E denote a read error and let S,O,P denote skewed, off-center, and proper alignments, respectively.
Then,
(a) P(E) = P(E|S) P(S) + P(E|O) P (O) + P(E|P) P(P)
= 0.01(0.10) + 0.02(0.05) + 0.001(0.85)
= 0.00285

(b) $P(S|E) = \frac{P(E|S)P(S)}{P(E)} = \frac{0.01(0.10)}{0.00285} = 0.351$

2-193. The following circuit operates if and only if there is a path of functional devices from left to right. Assume that devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit does not operate?



Let A_i denote the event that the *i*th row operates. Then,

$P(A_1) = 0.98, P(A_2) = (0.99)(0.9) = 0.891, P(A_3) = 0.891, \ P(A_4) = 0.98.$

The probability the circuit does not operate is

$$P(A_1')P(A_2')P(A_3')P(A_4') = (0.02)(0.109)(0.109)(0.02) = 4.75 \times 10^{-6}$$

2-194. A company that tracks the use of its Web site determined that the more pages a visitor views, the more likely the visitor is to provide contact information. Use the following tables to answer the questions:

Number of pages viewed:	1	2	3	4 or more
Percentage of visitors	40	30	20	10
Percentage of visitors in each page-view category that provides contact information:	10	15	20	35

(a) What is the probability that a visitor to the Web site provides contact information?
(b) If a visitor provides contact information, what is the probability that the visitor viewed four or more pages?

(a) (0.4)(0.1) + (0.3)(0.15) +(0.2)(0.2) + (0.35)(0.1) = 0.16
(b) P(4 or more | provided info) = (0.35)(0.1)/0.16 = 0.219

2-195. An article in *Genome Research* ["An Assessment of Gene Prediction Accuracy in Large DNA Sequences" (2000, Vol. 10, pp. 1631–1642)], considered the accuracy of commercial software to predict nucleotides in gene sequences. The following table shows the number of sequences for which the programs produced predictions and the number of nucleotides correctly predicted (computed globally from the total number of prediction successes and failures on all sequences).

	Number of Sequences	Proportion
GenScan	177	0.93
Blastx default	175	0.91
Blastx topcomboN	174	0.97
Blastx 2 stages	175	0.90
GeneWise	177	0.98
Procrustes	177	0.93

Assume the prediction successes and failures are independent among the programs.
(a) What is the probability that all programs predict a nucleotide correctly?
(b) What is the probability that all programs predict a nucleotide incorrectly?
(c) What is the probability that at least one Blastx program predicts a nucleotide correctly?

(a) P=(0.93)(0.91)(0.97)(0.90)(0.98)(0.93)=0.67336
(b) P=(1-0.93)(1-0.91)(1-0.97)(1-0.90)(1-0.98)(1-0.93)=2.646×10^{-8}
(c) P=1-(1-0.91)(1-0.97)(1-0.90)=0.99973

2-196. A batch contains 36 bacteria cells. Assume that 12 of the cells are not capable of cellular replication. Of the cells, 6 are selected at random, without replacement, to be checked for replication.
(a) What is the probability that all 6 of the selected cells are able to replicate?
(b) What is the probability that at least 1 of the selected cells is not capable of replication?

(a) P=(24/36)(23/35)(22/34)(21/33)(20/32)(19/31)=0.069
(b) P=1-0.069=0.931

2-197. A computer system uses passwords that are exactly seven characters, and each character is one of the 26 letters (a–z) or 10 integers (0–9). Uppercase letters are not used.
(a) How many passwords are possible?
(b) If a password consists of exactly 6 letters and 1 number, how many passwords are possible?
(c) If a password consists of 5 letters followed by 2 numbers, how many passwords are possible?

(a) 36^7
(b) Number of permutations of six letters is 26^6. Number of ways to select one number = 10. Number of positions among the six letters to place the one number = 7. Number of passwords = $26^6 \times 10 \times 7$
(c) $26^5 10^2$

2-198. (a) $P(A)=\frac{15+35+40+7+40}{1200}=0.1142$

(b) $P(A\cap B)=\frac{35+7}{1200}=0.035$

(c) $P(A\cup B)=1-\frac{900}{1200}=0.25$

(d) $P(A'\cap B)=\frac{63+70+30}{1200}=\frac{163}{1200}=0.136$

(e) $P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}=\frac{0.035}{(35+63+30+7+70)/1200}=0.1708$

(f) $P=\frac{15}{1200}=0.0125$

2-199. Two suppliers each supplied 2000 parts that were evaluated for conformance to specifications. One part type was more complex than the other. The proportion of nonconforming parts of each type are shown in the table.

Supplier		Simple Component	Complex Assembly	Total
1	Nonconforming	2	10	12
	Total	1000	1000	2000
2	Nonconforming	4	6	10
	Total	1600	400	2000

One part is selected at random from each supplier. For each supplier, separately calculate the following probabilities:
(a) What is the probability a part conforms to specifications?
(b) What is the probability a part conforms to specifications given it is a complex assembly?
(c) What is the probability a part conforms to specifications given it is a simple component?
(d) Compare your answers for each supplier in part (a) to those in parts (b) and (c) and explain any unusual results.

(a) Let A denote that a part conforms to specifications and let B denote a simple component.
For supplier 1: P(A) = 1988/2000 = 0.994
For supplier 2: P(A)= 1990/2000 = 0.995
(b)
For supplier 1: P(A|B') = 990/1000 = 0.99
For supplier 2: P(A|B') = 394/400 = 0.985
(c)
For supplier 1: P(A|B) = 998/1000 = 0.998
For supplier 2: P(A|B) = 1596/1600 = 0.9975

(d) The unusual result is that for both a simple component and for a complex assembly, supplier 1 has a greater probability that a part conforms to specifications. However, supplier 1 has a lower probability of conformance overall. The overall conforming probability depends on both the conforming probability of each part type and also the probability of each part type. Supplier 1 produces more of the complex parts so that overall conformance from supplier 1 is lower.

Mind-Expanding Exercises

2-200. a) Let X denote the number of documents in error in the sample and let n denote the sample size.

$P(X \geq 1) = 1 - P(X = 0)$ and $P(X = 0) = \dfrac{\binom{2}{0}\binom{58}{n}}{\binom{60}{n}}$

Trials for n result in the following results

n	$P(X = 0)$	$1 - P(X = 0)$
30	0.4915	0.5085
40	0.2147	0.7853
45	0.1186	0.8814
46	0.1028	0.8972
47	0.0881	0.9119

Therefore $n = 47$.

b) A large proportion of the set of documents needs to be inspected in order for the probability of a document in error to be detected to exceed 0.9.

2-201. Suppose that a lot of washers is large enough that it can be assumed that the sampling is done with replacement. Assume that 60% of the washers exceed the target thickness.
(a) What is the minimum number of washers that need to be selected so that the probability that none is thicker than the target is less than 0.05?
(b) What is the minimum number of washers that need to be selected so that the probability that 1 or more washers are thicker than the target is at least 0.95?

Let n denote the number of washers selected.
a) The probability that none are thicker, that is, all are less than the target is 0.4^n by independence.
The following results are obtained:

n	0.4^n
1	0.4
2	0.16
3	0.064
4	0.0256

Therefore, $n = 4$

b) The requested probability is the complement of the probability requested in part a). Therefore, $n = 4$

2-202. A biotechnology manufacturing firm can produce diagnostic test kits at a cost of \$20. Each kit for which there is a demand in the week of production can be sold for \$100. However, the half-life of components in the kit requires the kit to be scrapped if it is not sold in the week of production. The cost of scrapping the kit is \$5. The weekly demand is summarized as follows:

Weekly Demand				
Number of units	0	50	100	200
Probability of demand	0.05	0.4	0.3	0.25

How many kits should be produced each week to maximize the firm's mean earnings?

Let x denote the number of kits produced.

Revenue at each demand

	0	50	100	200
$0 \le x \le 50$	-5x	100x	100x	100x
Mean profit = 100x(0.95)-5x(0.05)-20x				
$50 \le x \le 100$	-5x	100(50)-5(x-50)	100x	100x
Mean profit = [100(50)-5(x-50)](0.4) + 100x(0.55)-5x(0.05)-20x				
$100 \le x \le 200$	-5x	100(50)-5(x-50)	100(100)-5(x-100)	100x
Mean profit = [100(50)-5(x-50)](0.4) + [100(100)-5(x-100)](0.3) + 100x(0.25) - 5x(0.05) - 20x				

	Mean Profit	Maximum Profit
$0 \le x \le 50$	74.75 x	\$ 3737.50 at x=50
$50 \le x \le 100$	32.75 x + 2100	\$ 5375 at x=100
$100 \le x \le 200$	1.25 x + 5250	\$ 5500 at x=200

Therefore, profit is maximized at 200 kits. However, the difference in profit over 100 kits is small.

2-203. A steel plate contains 20 bolts. Assume that 5 bolts are not torqued to the proper limit. 4 bolts are selected at random, without replacement, to be checked for torque. If an operator checks a bolt,the probability that an incorrectly torqued bolt is identified is 0.95. If a checked bolt is correctly torqued, the operator's conclusion is always correct. What is the probability that at least one bolt in the sample of four is identified as being incorrectly torqued?

Let E denote the event that none of the bolts are identified as incorrectly torqued.
Let X denote the number of bolts in the sample that are incorrect. The requested probability is P(E').
Then,
P(E) = P(E|X=0)P(X=0) + P(E|X=1) P(X=1) + P(E|X=2) P(X=2) + P(E|X=3) P(X=3) + P(E|X=4)P(X=4)
and P(X=0) = (15/20)(14/19)(13/18)(12/17) = 0.2817.
The remaining probability for X can be determined from the counting methods. Then

$$P(X=1) = \frac{\binom{5}{1}\binom{15}{3}}{\binom{20}{4}} = \frac{\left(\frac{5!}{4!1!}\right)\left(\frac{15!}{3!12!}\right)}{\left(\frac{20!}{4!16!}\right)} = \frac{5!15!4!16!}{4!3!12!20!} = 0.4696$$

$$P(X=2) = \frac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}} = \frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{2!13!}\right)}{\left(\frac{20!}{4!16!}\right)} = 0.2167$$

$$P(X=3) = \frac{\binom{5}{3}\binom{15}{1}}{\binom{20}{4}} = \frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{1!14!}\right)}{\left(\frac{20!}{4!16!}\right)} = 0.0309$$

P(X = 4) = (5/20)(4/19)(3/18)(2/17) = 0.0010, P(E | X = 0) = 1, P(E | X = 1) = 0.05,
P(E | X = 2) = 0.05^2 = 0.0025, P(E|X=3) = 0.05^3 = 1.25x10^{-4}, P(E | X=4) = 0.05^4 = 6.25x10^{-6}.
Then, $P(E) = 1(0.2817) + 0.05(0.4696) + 0.0025(0.2167) + 1.25\times10^{-4}(0.0309) + 6.25\times10^{-6}(0.0010) = 0.306$
and P(E') = 0.694

2-204. If the events *A* and *B* are independent, show that *A*′ and *B*′ are independent.

$$\begin{aligned} P(A' \cap B') &= 1 - P([A' \cap B']') = 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B') \end{aligned}$$

2-205. Suppose that a table of part counts is generalized as follows:

		Conforms	
		Yes	No
Supplier	1	*ka*	*kb*
	2	*a*	*b*

where *a*, *b*, and *k* are positive integers. Let *A* denote the event that a part is from supplier 1, and let *B* denote the event that a part conforms to specifications. Show that *A* and *B* are independent events. This exercise illustrates the result that whenever the rows of a table (with *r* rows and *c* columns) are proportional, an event defined by a row category and an event defined by a column category are independent.

The total sample size is $ka + a + kb + b = (k + 1)a + (k + 1)b$. Therefore

$$P(A) = \frac{k(a+b)}{(k+1)a+(k+1)b}, P(B) = \frac{ka+a}{(k+1)a+(k+1)b}$$

and

$$P(A \cap B) = \frac{ka}{(k+1)a+(k+1)b} = \frac{ka}{(k+1)(a+b)}$$

Then,

$$P(A)P(B) = \frac{k(a+b)(ka+a)}{[(k+1)a+(k+1)b]^2} = \frac{k(a+b)(k+1)a}{(k+1)^2(a+b)^2} = \frac{ka}{(k+1)(a+b)} = P(A \cap B)$$