CHAPTER 3

Section 3-1

3-1. The range of X is $\{0,1,2,...,2000\}$

3-2. The range of X is $\{0,1,2,...,60\}$

3-3. The range of X is $\{0,1,2,...,999\}$

3-4. The range of X is $\{0,1,2,3,4,5,6,7,8,9,10\}$

3-5. The range of X is $\{1,2,...,591\}$. Because 590 parts are conforming, a nonconforming part must be selected in 591 selections.

3-6. The range of X is $\{0,1,2,\ldots,100\}$. Although the range actually obtained from lots typically might not exceed 10%.

3-7. The range of X is conveniently modeled as all nonnegative integers. That is, the range of X is {0, 1, 2, …}

3-8. The range of X is conveniently modeled as all nonnegative integers. That is, the range of X is {0, 1, 2, …}

3-9. The range of X is $\{0,1,2,...,20\}$

3-10. The possible totals for two orders are 0.3175 + 0.3175 = 0.635, 0.3175 + 0.635 = 0.9525, 0.3175 + 0.9525 = 1.27, 0.635 + 0.635 = 1.27, 0.635 + 0.9525 = 1.5875, 0.9525 + 0.9525 = 1.905.
Therefore the range of X is $\{0.635,\ 0.9525,\ 1.27,\ 1.5875,\ 1.905\}$

3-11. The range of X is {0, 1, 2, …, 7500}

3-12. The range of X is {10, 11, …, 100}

3-13. The range of X is {0,1,2, …, 50000)

Section 3-2

3-14.

$$f_X(0) = P(X = 0) = 1/6 + 1/6 = 1/3$$
$$f_X(1.5) = P(X = 1.5) = 1/3$$
$$f_X(2) = 1/6$$
$$f_X(3) = 1/6$$

a) $P(X = 2) = 1/6$
b) $P(0.6 < X < 2.7) = P(X = 1.5) + P(X = 2) = 1/3 + 1/6 = 1/2$
c) $P(X > 3) = 0$
d) $P(0 \le X < 2) = P(X = 0) + P(X = 1.5) = 1/3 + 1/3 = 2/3$
e) $P(X = 0 \text{ or } X = 2) = 1/3 + 1/6 = 1/2$

3-15. All probabilities are greater than or equal to zero and sum to one.
a) $P(X \le 1) = 1/8 + 2/8 + 2/8 + 2/8 = 7/8$
b) $P(X > -2) = 2/8 + 2/8 + 2/8 + 1/8 = 7/8$
c) $P(-1 \le X \le 1) = 2/8 + 2/8 + 2/8 = 6/8 = 3/4$
d) $P(X \le -1 \text{ or } X = 2) = 1/8 + 2/8 + 1/8 = 4/8 = 1/2$

3-16. All probabilities are greater than or equal to zero and sum to one.
a) $P(X \le 1) = P(X = 1) = 0.5714$

b) $P(X > 2) = 1 - P(X = 2) = 1 - 0.2857 = 0.7143$
c) $P(2 < X < 6) = P(X = 3) = 0.1429$
d) $P(X \le 1 \text{ or } X > 1) = P(X = 1) + P(X = 2) + P(X = 3) = 1$

3-17. Probabilities are nonnegative and sum to one.
a) $P(X = 3) = 7/25$
b) $P(X \le 1) = 1/25 + 3/25 = 4/25$
c) $P(2 \le X < 4) = 5/25 + 7/25 = 12/25$
d) $P(X > -10) = 1$

3-18. Probabilities are nonnegative and sum to one.
a) $P(X = 2) = 3/4(1/4)^2 = 3/64$
b) $P(X \le 2) = 3/4[1+1/4+(1/4)^2] = 63/64$
c) $P(X > 2) = 1 - P(X \le 2) = 1/64$
d) $P(X \ge 2) = P(X = 2) + P(X > 2) = 3/64 + 1/64 = 1/16$

3-19. All probabilities are greater than or equal to zero and sum to one.
a) $P(X \le 2)=1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$
b) $P(X <1.65) = 1/8 + 2/8 + 2/8 + 2/8 = 7/8$
c) $P(X >1) = 1/8$
d) $P(X < -1 \text{ or } X>1) = 1/8 +1/8 = 1/4$

3-20. X = the number of patients in the sample who are admitted
Range of X = {0,1,2}
A = the event that the first patient is admitted
B = the event that the second patient is admitted
A and B are independent events due to the selection with replacement.

$P(A)=P(B)=1277/5292=0.2413$
$P(X=0) = P(A'\cap B') = (1-0.2413)(1-0.2413) = 0.576$
$P(X=1) = P(A \cap B')+P(A'\cap B) = 0.2413(1-0.2413)+ (1-0.2413)(0.2413) = 0.366$
$P(X=2) = (A\cap B) = 0.2413\times 0.2413 = 0.058$

x	P(X=x)
0	0.576
1	0.366
2	0.058

3-21. X = number of successful surgeries.
$P(X=0)=0.09(0.33)=0.0297$
$P(X=1)=0.91(0.33)+0.09(0.67)=0.3606$
$P(X=2)=0.91(0.67)=0.6097$

3-22. $P(X = 0) = 0.05^3 = 1.25 \times 10^{-4}$
$P(X = 1) = 3[0.95(0.05)(0.05)] = 7.125 \times 10^{-3}$
$P(X = 2) = 3[0.95(0.95)(0.05)] = 0.1354$
$P(X = 3) = 0.95^3 = 0.8574$

3-23. X = number of wafers that pass
$P(X = 0) = (0.3)^3 = 0.027$
$P(X = 1) = 3(0.3)^2(0.7) = 0.189$
$P(X = 2) = 3(0.3)(0.7)^2 = 0.441$
$P(X = 3) = (0.7)^3 = 0.343$

3-24. X: the number of computers that vote for a left roll when a right roll is appropriate.
p = 0.0002.
$P(X = 0)=(1 - p)^4 = 0.9998^4 = 0.9992$
$P(X = 1) = 4*(1 - p)^3 p = 4 \times 0.9998^3 \times 0.0002 = 7.9952 \times 10^{-4}$
$P(X = 2) = C_4^2 (1 - p)^2 p^2 = 2.399 \times 10^{-7}$
$P(X = 3) = C_4^3 (1 - p)^1 p^3 = 3.1994 \times 10^{-11}$

$P(X = 4) = C_4^0 (1 - p)^0 p^4 = 1.6 \times 10^{-15}$

3-25. P(X = 50 million) = 0.4, P(X = 25 million) = 0.4, P(X = 10 million) = 0.2

3-26. P(X = 10 million) = 0.3, P(X = 5 million) = 0.65, P(X = 1 million) = 0.05

3-27. P(X = 15 million) = 0.5, P(X = 5 million) = 0.25, P(X = -0.5 million) = 0.25

3-28. X = number of components that meet specifications
$P(X = 0) = (0.07)(0.02) = 1.4 \times 10^{-3}$
P(X = 1) = (0.07)(0.98) + (0.93)(0.02) = 0.0872
P(X = 2) = (0.93)(0.98) = 0.9114

3-29. X = number of components that meet specifications
P(X=0) = (0.05)(0.02)(0.03) = 0.00003
P(X=1) = (0.95)(0.02)(0.03) + (0.05)(0.98)(0.03)+(0.05)(0.02)(0.97) = 0.00301
P(X=2) = (0.95)(0.98)(0.03) + (0.95)(0.02)(0.97) + (0.05)(0.98)(0.97) = 0.09389
P(X=3) = (0.95)(0.98)(0.97) = 0.90307

3-30. X = final temperature
P(X = 266) = 70/250 = 0.28
P(X = 271) = 80/250 = 0.32
P(X = 274) = 100/250 = 0.4

$$f(x) = \begin{cases} 0.28, & x = 266 \\ 0.32, & x = 271 \\ 0.4, & x = 274 \end{cases}$$

3-31. X = waiting time (hours)
P(X=1) = 19/500 = 0.038
P(X=2) = 47/500 = 0.094
P(X=3) = 86/500 = 0.172
P(X=4) = 102/500 = 0.204
P(X=5) = 87/500 = 0.174
P(X=6) = 62/500 = 0.124
P(X=7) = 44/500 = 0.088
P(X=8) = 18/500 = 0.036
P(X=9) = 14/500 = 0.028
P(X=10) = 11/500 = 0.022
P(X=15) = 10/500 = 0.020

$$f(x) = \begin{cases} 0.038, & x = 1 \\ 0.940, & x = 2 \\ 0.172, & x = 3 \\ 0.204, & x = 4 \\ 0.174, & x = 5 \\ 0.124, & x = 6 \\ 0.088, & x = 7 \\ 0.036, & x = 8 \\ 0.028, & x = 9 \\ 0.022, & x = 10 \\ 0.020, & x = 15 \end{cases}$$

3-32. X = days until change
P(X=1.5) = 0.15
P(X=3) = 0.25
P(X=4.5) = 0.30
P(X=5) = 0.20
P(X=7) = 0.10

$$f(x)=\begin{cases}0.15, & x=1.5\\ 0.25, & x=3\\ 0.30, & x=4.5\\ 0.20, & x=5\\ 0.10, & x=7\end{cases}$$

3-33. X = Non-failed well depth
P(X=255) = (1515+1343)/7726 = 0.370
P(X=218) = 26/7726 = 0.003
P(X=317) = 3290/7726 = 0.426
P(X=231) = 349/7726 = 0.045
P(X=267) = (280+887)/7726 = 0.151
P(X=217) = 36/7726 = 0.005

$$f(x)=\begin{cases}0.005, & x=217\\ 0.003, & x=218\\ 0.045, & x=231\\ 0.370, & x=255\\ 0.151, & x=267\\ 0.426, & x=317\end{cases}$$

Section 3-3

3-34.

$$F(x)=\left\{\begin{array}{cc}0, & x<0\\ 1/3 & 0\le x<1.5\\ 2/3 & 1.5\le x<2\\ 5/6 & 2\le x<3\\ 1 & 3\le x\end{array}\right\} \text{ where } \begin{array}{l}f_X(0)=P(X=0)=1/6+1/6=1/3\\ f_X(1.5)=P(X=1.5)=1/3\\ f_X(2)=1/6\\ f_X(3)=1/6\end{array}$$

3-35.

$$F(x)=\left\{\begin{array}{cc}0, & x<-2\\ 1/8 & -2\le x<-1\\ 3/8 & -1\le x<0\\ 5/8 & 0\le x<1\\ 7/8 & 1\le x<2\\ 1 & 2\le x\end{array}\right\} \text{ where } \begin{array}{l}f_X(-2)=1/8\\ f_X(-1)=2/8\\ f_X(0)=2/8\\ f_X(1)=2/8\\ f_X(2)=1/8\end{array}$$

a) $P(X \le 1.25) = 7/8$
b) $P(X \le 2.2) = 1$
c) $P(-1.1 < X \le 1) = 7/8 - 1/8 = 3/4$
d) $P(X > 0) = 1 - P(X \le 0) = 1 - 5/8 = 3/8$

3-36.

$$F(x)=\begin{cases}0 & x<1\\ 4/7 & 1\le x<2\\ 6/7 & 2\le x<3\\ 1 & 3\le x\end{cases}$$

a) $P(X < 2) = 4/7$
b) $P(X \le 3) = 1$
c) $P(X > 2) = 1 - P(X \le 2) = 1 - 6/7 = 1/7$
d) $P(1 < X \le 2) = P(X \le 2) - P(X \le 1) = 6/7 - 4/7 = 2/7$

3-37.

$$F(x)=\begin{cases}0, & x<0\\ 0.008, & 0\le x<1\\ 0.104, & 1\le x<2\\ 0.488, & 2\le x<3\\ 1, & 3\le x\end{cases}$$

.

$$\begin{aligned}f(0) &= 0.2^3 = 0.008,\\ f(1) &= 3(0.2)(0.2)(0.8) = 0.096,\\ f(2) &= 3(0.2)(0.8)(0.8) = 0.384,\\ f(3) &= (0.8)^3 = 0.512,\end{aligned}$$

3-38.

$$F(x)=\begin{cases}0, & x<0\\ 0.9996, & 0\le x<1\\ 0.9999, & 1\le x<3\\ 0.99999, & 3\le x<4\\ 1, & 4\le x\end{cases}$$

.

$$\begin{aligned}f(0) &= 0.9999^4 = 0.9996,\\ f(1) &= 4(0.9999^3)(0.0001) = 0.0003999,\\ f(2) &= 5.999*10^{-8},\\ f(3) &= 3.9996*10^{-12},\\ f(4) &= 1*10^{-16}\end{aligned}$$

3-39.

$$F(x)=\begin{cases}0, & x<10\\ 0.2, & 10\le x<25\\ 0.5, & 25\le x<50\\ 1, & 50\le x\end{cases}$$

where P(X = 50 million) = 0.5, P(X = 25 million) = 0.3, P(X = 10 million) = 0.2

3-40.

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.1, & 1 \le x < 5 \\ 0.7, & 5 \le x < 10 \\ 1, & 10 \le x \end{cases}$$

where P(X = 10 million) = 0.3, P(X = 5 million) = 0.6, P(X = 1 million) = 0.1

3-41. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: $f(1) = 0.5$, $f(3) = 0.5$
a) $P(X \le 3) = 1$
b) $P(X \le 2) = 0.5$
c) $P(1 \le X \le 2) = P(X = 1) = 0.5$
d) $P(X > 3) = 0$

3-42. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(1) = 0.7, f(4) = 0.2, f(7) = 0.1
a) $P(X \le 4) = 0.9$
b) $P(X > 5) = 1 - P(X \le 5) = 0.1$
c) $P(X \le 5) = 0.9$
d) $P(X > 7) = 0$
e) $P(X \le 2) = 0.7$

3-43. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(-10) = 0.3, f(30) = 0.4, f(50) = 0.3
a) $P(X \le 50) = 1$
b) $P(X \le 40) = 0.7$
c) $P(40 \le X \le 60) = P(X=50) = 0.3$
d) $P(X<0) = 0.3$
e) $P(0 \le X < 10) = 0$
f) $P(-10 < X < 10) = 0$

3-44. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(1/8) = 0.2, f(1/4) = 0.7, f(3/8) = 0.1
a) $P(X \le 1/18) = 0$
b) $P(X \le 1/4) = 0.9$
c) $P(X \le 5/16) = 0.9$
d) $P(X > 1/4) = 0.1$
e) $P(X \le 1/2) = 1$

3-45.

$$F(x) = \begin{cases} 0, & x < 266 \\ 0.28, & 266 \le x < 271 \\ 0.6, & 271 \le x < 274 \\ 1, & 274 \le x \end{cases}$$

where P(X = 266 K) = 0.28, P(X = 271 K) = 0.32, P(X = 274 K) = 0.40

3-46.

$$F(x)=\begin{cases} 0, & x<1 \\ 0.038, & 1\le x<2 \\ 0.140, & 2\le x<3 \\ 0.312, & 3\le x<4 \\ 0.516, & 4\le x<5 \\ 0.690, & 5\le x<6 \\ 0.814, & 6\le x<7 \\ 0.894, & 7\le x<8 \\ 0.930, & 8\le x<9 \\ 0.958, & 9\le x<10 \\ 0.980, & 10\le x<15 \\ 1 & 15\le x \end{cases}$$

where P(X=1) = 0.038, P(X=2) = 0.102, P(X=3) = 0.172, P(X=4) = 0.204, P(X=5) = 0.174, P(X=6) = 0.124, P(X=7) = 0.08, P(X=8) = 0.036, P(X=9) = 0.028, P(X=10) = 0.022, P(X=15) = 0.020

3-47.

$$F(x)=\begin{cases} 0, & x<1.5 \\ 0.05, & 1.5\le x<3 \\ 0.30, & 3\le x<4.5 \\ 0.65, & 4.5\le x<5 \\ 0.85, & 5\le x<7 \\ 1 & 7\le x \end{cases}$$

where P(X=1.5) = 0.05, P(X= 3) = 0.25, P(X=4.5) = 0.35, P(X=5) = 0.20, P(X=7) = 0.15

3-48.

$$F(x)=\begin{cases} 0, & x<217 \\ 0.005, & 217\le x<218 \\ 0.008, & 218\le x<231 \\ 0.053, & 231\le x<255 \\ 0.423, & 255\le x<267 \\ 0.574, & 267\le x<317 \\ 1, & 317\le x \end{cases}$$

where P(X=255) = 0.370, P(X=218) = 0.003, P(X=317) = 0.426, P(X=231) = 0.045, P(X=267) = 0.151, P(X=217) = 0.005

Section 3-4

3-49. Mean and Variance

$$\mu = E(X) = 1f(1)+2f(2)+3f(3)+4f(4)+5f(5)$$
$$= 1(0.2)+2(0.2)+3(0.2)+4(0.2)+5(0.2)=3$$
$$V(X)=1^2 f(1)+2^2 f(2)+3^2 f(3)+4^2 f(4)+5^2 f(5)-\mu^2$$
$$=1(0.2)+4(0.2)+9(0.2)+16(0.2)+25(0.2)-3^2=2$$

3-50. Mean and Variance for random variable in exercise 3-14

$$\mu = E(X) = 0f(0)+1.5f(1.5)+2f(2)+3f(3)$$
$$=0(1/3)+1.5(1/3)+2(1/6)+3(1/6)=1.333$$

$$V(X)=0^2 f(0)+1.5^2 f(1)+2^2 f(2)+3^2 f(3)-\mu^2$$
$$=0(1/3)+2.25(1/3)+4(1/6)+9(1/6)-1.333^2=1.139$$

3-51. Determine E(X) and V(X) for random variable in exercise 3-15

$$\mu=E(X)=-2f(-2)-1f(-1)+0f(0)+1f(1)+2f(2)$$
.
$$=-2(1/8)-1(2/8)+0(2/8)+1(2/8)+2(1/8)=0$$
$$V(X)=-2^2 f(-2)-1^2 f(-1)+0^2 f(0)+1^2 f(1)+2^2 f(2)-\mu^2$$
$$=4(1/8)+1(2/8)+0(2/8)+1(2/8)+4(1/8)-0^2=1.5$$

3-52. Determine E(X) and V(X) for random variable in exercise 3-16

$$\mu=E(X)=1f(1)+2f(2)+3f(3)$$
$$=1(0.5714286)+2(0.2857143)+3(0.1428571)$$
$$=1.571429$$
$$V(X)=1^2 f(1)+2^2 f(2)+3^2 f(3)+-\mu^2$$
$$=1.428571$$

3-53. Mean and variance for exercise 3-17

$$\mu=E(X)=0f(0)+1f(1)+2f(2)+3f(3)+4f(4)$$
$$=0(0.04)+1(0.12)+2(0.2)+3(0.28)+4(0.36)=2.8$$
$$V(X)=0^2 f(0)+1^2 f(1)+2^2 f(2)+3^2 f(3)+4^2 f(4)-\mu^2$$
$$=0(0.04)+1(0.12)+4(0.2)+9(0.28)+16(0.36)-2.8^2=1.36$$

3-54. $$E(X)=\frac{3}{4}\sum_{x=0}^{\infty}x\left(\frac{1}{4}\right)^x=\frac{3}{4}\sum_{x=1}^{\infty}x\left(\frac{1}{4}\right)^x=\frac{1}{3}$$

The result uses a formula for the sum of an infinite series. The formula can be derived from the fact that the series to sum is the derivative of $h(a)=\sum_{x=1}^{\infty}a^x=\frac{a}{1-a}$ with respect to *a*.

For the variance, another formula can be derived from the second derivative of *h*(*a*) with respect to *a*. Calculate from this formula

$$E(X^2)=\frac{3}{4}\sum_{x=0}^{\infty}x^2\left(\frac{1}{4}\right)^x=\frac{3}{4}\sum_{x=1}^{\infty}x^2\left(\frac{1}{4}\right)^x=\frac{5}{9}$$

Then $V(X)=E(X^2)-[E(X)]^2=\frac{5}{9}-\frac{1}{9}=\frac{4}{9}$

3-55.

$$\mu=E(X)=0f(0)+1f(1)+2f(2)$$
$$=0(0.033)+1(0.364)+2(0.603)$$
$$=1.57$$
$$V(X)=0^2 f(0)+1^2 f(1)+2^2 f(2)-\mu^2$$
$$=0(0.033)+1(0.364)+4(0.603)-1.57^2$$
$$=0.3111$$

3-56. Mean and variance for exercise 3-20

$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3)$$
$$= 0(1.25\times10^{-4}) + 1(7.125\times10^{-3}) + 2(0.1354) + 3(0.8574)$$
$$= 2.850125$$
$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) - \mu^2$$
$$= 0.1421125$$

3-57. Determine x where range is [1,2,3,4,x] and mean is 6.

$$\mu = E(X) = 6 = 1f(1) + 2f(2) + 3f(3) + 4f(4) + xf(x)$$
$$6 = 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2) + x(0.2)$$
$$6 = 2 + 0.2x$$
$$4 = 0.2x$$
$$x = 20$$

3-58. (a) F(0) = 0.15

Nickel Charge: X	CDF
0	0.15
2	0.15 + 0.37 = 0.52
3	0.15 + 0.37 + 0.33 = 0.85
4	0.15 + 0.37 + 0.33 + 0.15 = 1

(b)E(X) = 0 × 0.15 + 2 × 0.37 + 3 × 0.33 + 4 × 0.15 = 2.33

$$V(X) = \sum_{i=1}^{4} f(x_i)(x_i - \mu)^2 = 1.42096$$

3-59. X = number of computers that vote for a left roll when a right roll is appropriate.

μ = E(X)=0*f(0)+1*f(1)+2*f(2)+3*f(3)+4*f(4)

$= 0 + 7.995 \times 10^{-4} + 2 \times 2.399 \times 10^{-7} + 3 \times 3.19936 \times 10^{-11} + 4 \times 1.6 \times 10^{-15} = 0.0008$

$$V(X) = \sum_{i=1}^{5} f(x_i)(x_i - \mu)^2 = 7.9982 \times 10^{-4}$$

3-60. μ=E(X)=350*0.05+450*0.1+550*0.47+650*0.38=568

$$V(X) = \sum_{i=1}^{4} f(x_i)(x - \mu)^2 = 6476$$

$$\sigma = \sqrt{V(X)} = 80.47$$

3-61. (a)

μ = E(X) =

Transaction	Frequency	Selects: X	f(X)
New order	34	23	0.34
Payment	44	4.2	0.44
Order status	9	11.4	0.09
Delivery	9	130	0.09
Stock level	4	0	0.04
total	100		

23*0.34+4.2*0.44+11.4*0.09+130*0.09+0*0.04 =22.394

$$V(X) = \sum_{i=1}^{5} f(x_i)(x - \mu)^2 = 1218.83 \qquad \sigma = \sqrt{V(X)} = 34.91$$

(b)

Transaction	Frequency	All operation: X	f(X)
New order	34	23+11+12=46	0.34
Payment	44	4.2+3+1+0.6=8.8	0.44
Order status	9	11.4+0.6=12	0.09
Delivery	9	130+120+10=260	0.09
Stock level	4	0+1=1	0.04
total	100		

μ = E(X) = 46*0.34+8.8*0.44+12*0.09+260*0.09+1*0.04=44.032

V(X) = $\sum_{i=1}^{5} f(x_i)(x-\mu)^2$ =4911.70 $\sigma = \sqrt{V(X)} = 70.08$

3-62. μ = E(X) = 266(0.28) + 271(0.32) + 274(0.4) = 270.8

V(X) = $\sum_{i=1}^{5} f(x_i)(x-\mu)^2$ = 10.56

3-63. μ = E(X) = 1(0.038) + 2(0.102) + 3(0.172) + 4(0.204) + 5(0.174) + 6(0.124) + 7(0.08) + 8(0.036) + 9(0.028) + 10(0.022) +15(0.020)

= 4.808 hours

V(X) = $\sum_{i=1}^{5} f(x_i)(x-\mu)^2$ = 6.147

3-64. μ = E(X) = 1.5(0.05) + 3(0.25) + 4.5(0.35) + 5(0.20) + 7(0.15) = 4.45

V(X) = $\sum_{i=1}^{5} f(x_i)(x-\mu)^2$ = 1.9975

3-65. X = the depth of a non-failed well

x	f(x)	xf(x)	$(x-\mu)^2 f(x)$
217	0.0047=36/7726	1.011131245	19.5831
218	0.0034=26/7726	0.733626715	13.71039
231	0.0452=349/7726	10.43476573	116.7045
255	0.3699=(1515+1343)/7726	94.32953663	266.2591
267	0.1510=887/7726	40.32992493	33.21378
317	0.4258=3290/7726	134.9896454	526.7684

μ = E(X) = 255(0.370) + 218(0.003) + 317(0.426) + 231(0.045) + 267(0.151) + 217(0.005) = 281.83

V(X) = $\sum_{i=1}^{5} f(x_i)(x-\mu)^2$ = 976.24

<u>Section 3-5</u>

3-66. E(X) = (0 + 95)/2 = 47.5, V(X) = $[(95 - 0 + 1)^2 - 1]/12$ = 767.92

3-67. E(X) = (5 + 1)/2 = 3, V(X) = $[(5 - 1 + 1)^2 - 1]/12$ = 2

3-68. X=(1/100)Y, Y = 14, 15, 16, 17, 18, 19, 20

E(X) = (1/100) E(Y) = $\frac{1}{100}\left(\frac{14+20}{2}\right) = 0.17$ mm

$$V(X) = \left(\frac{1}{100}\right)^2 \left[\frac{(20-14+1)^2 - 1}{12}\right] = 0.0004 \text{ mm}^2$$

3-69. $$E(X) = 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) + 5\left(\frac{1}{5}\right) + 6\left(\frac{1}{5}\right) = 4$$

$$V(X) = (2)^2\left(\frac{1}{5}\right) + (3)^2\left(\frac{1}{5}\right) + (4)^2\left(\frac{1}{5}\right) + (5)^2\left(\frac{1}{5}\right) + (6)^2\left(\frac{1}{5}\right) - (4)^2 = 2$$

3-70. X = 640 + 0.1Y, Y = 0, 1, 2, ..., 9

E(X) = $640 + 0.1\left(\frac{0+9}{2}\right) = 640.45$ mm

$$V(X) = (0.1)^2\left[\frac{(9-0+1)^2 - 1}{12}\right] = 0.0825 \text{ mm}^2$$

3-71. a = 660, b = 685

a) μ = E(X) = (a + b)/2 = 672.5

V(X) = $[(b – a + 1)^2 – 1]/12$ = 56.25

b) a = 75, b = 100

μ = E(X) = (a + b)/2 = 87.5

V(X) = $[(b – a + 1)^2 – 1]/12$= 56.25

The range of values is the same, so the mean shifts by the difference in the two minimums (or maximums) whereas the variance does not change.

3-72. X is a discrete random variable because it denotes the number of fields out of 30 that are in error. However, X is not uniform because P(X = 0) ≠ P(X = 1).

3-73. The range of Y is 0, 5, 10, ..., 45, E(X) = (0 + 5)/2 = 2.5

E(Y) = 0(1/6) + 5(1/6) + 10(1/6) + 15(1/6) + 20(1/6) + 25(1/6)
= 5[0(1/6) + 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6)]
= 5(E(X))
= 5(2.5)
= 12.5

V(X) = 2.92, V(Y) = 5^2(2.92) = 73, σ_Y = 8.54

3-74. $$E(cX) = \sum_x cxf(x) = c\sum_x xf(x) = cE(X),$$

$$V(cX) = \sum_x (cx - c\mu)^2 f(x) = c^2 \sum_x (x-\mu)^2 f(x) = cV(X)$$

3-75. E(X) = (10 + 5)/2 = 7.5, V(X) = $[(10 – 5 + 1)^2 – 1]/12$ = 2.92, σ = 1.709

3-76. $$f(x_i) = \frac{3.5 \times 10^8}{10^9} = 0.35$$

<u>Section 3-6</u>

3-77. A binomial distribution is based on independent trials with two outcomes and a constant probability of success on each trial.

a) reasonable

b) independence assumption not reasonable

c) The probability that the second component fails depends on the failure time of the first component. The binomial distribution is not reasonable.
d) not independent trials with constant probability
e) probability of a correct answer not constant
f) reasonable
g) probability of finding a defect not constant
h) if the fills are independent with a constant probability of an underfill, then the binomial distribution for the number packages underfilled is reasonable
i) because of the bursts, each trial (that consists of sending a bit) is not independent
j) not independent trials with constant probability

3-78. (a) $P(X \le 3) = 0.411$
(b) $P(X > 8) = 1 - 0.9900 = 0.01$
(c) $P(X = 6) = 0.1091$
(d) $P(6 \le X \le 11) = 0.9999 - 0.8042 = 0.1957$

3-79. (a) $P(X \le 2) = 0.9298$
(b) $P(X > 8) = 0$
(c) $P(X = 5) = 0.0015$
(d) $P(5 \le X \le 7) = 1 - 0.9984 = 0.0016$

3-80. a) $P(X = 5) = \binom{10}{5} 0.5^5 (0.5)^5 = 0.2461$

b) $P(X \le 3) = \binom{10}{0} 0.5^0 0.5^{10} + \binom{10}{1} 0.5^1 0.5^9 + \binom{10}{2} 0.5^2 0.5^8 + \binom{10}{3} (0.5)^3 (0.5)^7$

$= 0.5^{10} + 10(0.5)^{10} + 45(0.5)^{10} + 120(0.5)^{10} = 0.1719$

c) $P(X \ge 9) = \binom{10}{9} 0.5^9 (0.5)^1 + \binom{10}{10} 0.5^{10} (0.5)^0 = 0.0107$

d) $P(3 \le X < 5) = \binom{10}{3} 0.5^3 0.5^7 + \binom{10}{4} 0.5^4 0.5^6$

$= 120(0.5)^{10} + 210(0.5)^{10} = 0.3223$

3-81. a) $P(X = 5) = \binom{10}{5} 0.01^5 (0.99)^5 = 2.40 \times 10^{-8}$

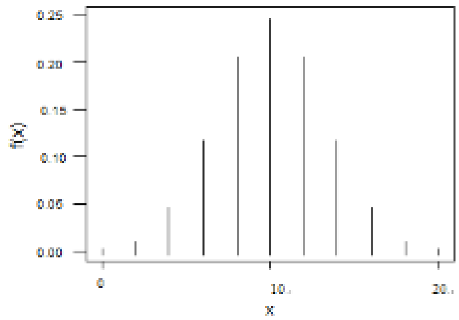
b) $P(X \le 3) = \binom{10}{0} 0.01^0 (0.99)^{10} + \binom{10}{1} 0.01^1 (0.99)^9 + \binom{10}{2} 0.01^2 (0.99)^8 + \binom{10}{3} (0.01)^3 (0.99)^7$

$= 0.999998$

c) $P(X \ge 9) = \binom{10}{9} 0.01^9 (0.99)^1 + \binom{10}{10} 0.01^{10} (0.99)^0 = 9.91 \times 10^{-18}$

d) $P(3 \le X < 5) = \binom{10}{3} 0.01^3 (0.99)^7 + \binom{10}{4} 0.01^4 (0.99)^6 = 1.138 \times 10^{-4}$

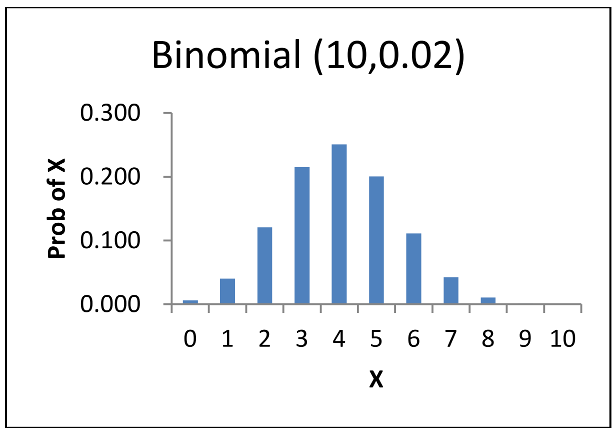
3-82.



a) $E(X) = np = 20(0.5) = 10$

b) Values x= 0 and x=20 are the least likely, the extreme values

3-83.



P(X = 0) = 0.817, P(X = 1) = 0.167, P(X = 2) = 0.015, P(X = 3) = 0.01. P(X = 4) = 0 and so forth.
Distribution is skewed with $E(X) = np = 10(0.02) = 0.2$

a) The most-likely value of X is 0.
b) The least-likely value of X is 10.

3-84. n = 3 and p = 0.5

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.03125 & 0 \le x < 1 \\ 0.1875 & 1 \le x < 2 \\ 0.5 & 2 \le x < 3 \\ 0.8125 & 3 \le x < 4 \\ 0.96875 & 4 \le x < 5 \\ 1 & 5 \le x \end{cases} \quad \text{where} \quad \begin{cases} f(0) = \left(\frac{1}{2}\right)^5 = \frac{1}{32} \\ f(1) = 5\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 = \frac{5}{32} \\ f(2) = 10\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{5}{16} \\ f(3) = 10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5}{16} \\ f(4) = 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) = \frac{5}{32} \\ f(5) = \left(\frac{1}{2}\right)^5 = \frac{1}{32} \end{cases}$$

3-85. n = 5 and p = 0.25

$$F(x)=\begin{cases} 0 & x<0 \\ 0.2373 & 0\le x<1 \\ 0.6328 & 1\le x<2 \\ 0.8964 & 2\le x<3 \\ 0.9843 & 3\le x<4 \\ 0.9989 & 4\le x<5 \\ 1 & 5\le x \end{cases} \quad \text{where} \quad \begin{cases} f(0)=\left(\frac{3}{4}\right)^5=\frac{243}{1024} \\ f(1)=5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4=\frac{405}{1024} \\ f(2)=10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3=\frac{135}{512} \\ f(3)=10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2=\frac{45}{512} \\ f(4)=5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)=\frac{15}{1024} \\ f(5)=\left(\frac{1}{4}\right)^5=\frac{1}{1024} \end{cases}$$

3-86. Let X denote the number of defective circuits.
Then, X has a binomial distribution with n = 40 and p = 0.02.

P(X = 0) = $\binom{40}{0}(0.02)^0(0.98)^{40}=0.4457$

3-87. Let X denote the number of times the line is occupied.
Then, X has a binomial distribution with n = 10 and p = 0.5

a) $P(\mathrm{X}=3)=\binom{10}{3}(0.5)^3(0.5)^7=0.1172$

b) Let Z denote the number of time the line is NOT occupied.

Then Z has a binomial distribution with n =10 and p = 0.5. $P(Z\ge 1)=1-P(Z=0)=1-\binom{10}{0}0.5^0\,0.5^{10}=0.9990$

c) $E(X)=10(0.5)=5$

3-88. Let X denote the number of questions answered correctly.
Then, X is binomial with n = 30 and p = 0.25.

a)
$$\begin{aligned} P(X\ge 20)=&\binom{30}{20}(0.25)^{20}(0.75)^{10}+\binom{30}{21}(0.25)^{21}(0.75)^{9} \\ &+\binom{30}{22}(0.25)^{22}(0.75)^{8}+\binom{30}{23}(0.25)^{23}(0.75)^{7}+\binom{30}{24}(0.25)^{24}(0.75)^{6} \\ &+\binom{30}{25}(0.25)^{25}(0.75)^{5}+\binom{30}{26}(0.25)^{26}(0.75)^{4}+\binom{30}{27}(0.25)^{27}(0.75)^{3} \\ &+\binom{30}{28}(0.25)^{28}(0.75)^{2}+\binom{30}{29}(0.25)^{29}(0.75)^{1}+\binom{30}{30}(0.25)^{30}(0.75)^{0} \\ &=1.821\times 10^{-6} \end{aligned}$$

b)
$$\begin{aligned} P(X<5)=&\binom{30}{0}(0.25)^{0}(0.75)^{30}+\binom{30}{1}(0.25)^{1}(0.75)^{29}+\binom{30}{2}(0.25)^{2}(0.75)^{28} \\ &+\binom{30}{3}(0.25)^{3}(0.75)^{27}+\binom{30}{4}(0.25)^{4}(0.75)^{26}=0.0979 \end{aligned}$$

3-89. Let X denote the number of mornings the light is green.

$$a) \quad P(X=1) = \binom{5}{1} 0.25^1 0.75^4 = 0.396$$

$$b) \quad P(X=4) = \binom{20}{4} 0.25^4 0.75^{16} = 0.190$$

$$c) \quad P(X>4) = 1 - P(X \le 4) = 1 - 0.415 = 0.585$$

3-90. X = number of samples mutated
X has a binomial distribution with p = 0.01, n = 20

(a) P(X = 0) = $\binom{20}{0} p^0 (1-p)^{20}$ = 0.8179

(b) P(X ≤ 1) = P(X = 0) + P(X = 1) = 0.9831

(c) P(X > 7) = P(X = 8) + P(X = 9) +…+ P(X = 20) = 0

3-91. (a) n=20, p=0.6122,
P(X≥3) = 1-P(X<3) = 1

(b)P(X≥7) = 1- P(X<7)= 0.995

(c) μ=E(X)= np=20*0.6122=12.244

V(X)=np(1-p) = 4.748

σ= $\sqrt{V(X)}$ =2.179

3-92. n=30, p=0.13

(a) P(X = 3) = $\binom{30}{3} p^3 (1-p)^{27}$ =0.208

(b) P(X ≥ 3) = 1-P(X<3)=1-0.233=0.767

(c) μ = E(X) = np = 30*0.13 = 3.9

V(X) = np(1-p) = 30*0.13*0.87=3.393

σ = $\sqrt{V(X)}$ = 1.842

3-93. (a) Binomial distribution, p =$10^4/36^7$ = 1.27609E-07, n = 1E09

(b) P(X=0) = $\binom{1E09}{0} p^0 (1-p)^{1E09}$ = 0

(c) μ = E(X) = np =1E09*1.27609E-07 = 127.6

V(X) = np(1-p) = 127.6

3-94. E(X) = 25 (0.01) = 0.25
V(X) = 25 (0.01) (0.99) = 0.248

$\mu_X + 3\sigma_X = 0.25 + 3\sqrt{0.248} = 1.74$

a) X is binomial with n = 25 and p = 0.01

$P(X > 1.74) = P(X \ge 2) = 1 - P(X \le 1)$

$$= 1 - \left[\binom{25}{0} (0.01)^0 (0.99)^{25} + \binom{25}{1} (0.01)^1 (0.99)^{24} \right] = 0.0258$$

b) X is binomial with n = 25 and p = 0.04

$$P(X > 1) = 1 - P(X \le 1)$$

$$= 1 - \left[\binom{25}{0}(0.04)^0(0.96)^{25} + \binom{25}{1}(0.04)^1(0.96)^{24}\right] = 0.2642$$

c) Let Y denote the number of times X exceeds 1 in the next five samples.
Then, Y is binomial with n = 5 and p = 0.190 from part b.

$$P(Y \ge 1) = 1 - P(Y = 0) = 1 - \left[\binom{5}{0}(0.2642)^0(0.7358)^5\right] = 0.7843$$

The probability is 0.651 that at least one sample from the next five will contain more than one defective

3-95. Let X denote the passengers with tickets that do not show up for the flight.
Then, X is binomial with n = 130 and p = 0.1.

a) $P(X \ge 10) = 1 - P(X \le 9)$

$$= 1 - \left[\begin{array}{l}\binom{130}{0}(0.1)^0(0.9)^{130} + \binom{130}{1}(0.1)^1(0.9)^{129} + \binom{130}{2}(0.1)^2(0.9)^{128} + \binom{130}{3}(0.1)^3(0.9)^{127} \\ + \binom{130}{4}(0.1)^4(0.9)^{126} + \binom{130}{5}(0.1)^5(0.9)^{125} + \binom{130}{6}(0.1)^6(0.9)^{124} + \binom{130}{7}(0.1)^7(0.9)^{123} \\ + \binom{130}{8}(0.1)^8(0.9)^{122} + \binom{130}{9}(0.1)^9(0.9)^{121}\end{array}\right]$$

$$= 0.8479$$

b) $P(X > 10) = 1 - P(X \le 10) = 0.7619$

3-96. Let X denote the number of defective components among those stocked.

a) $P(X = 0) = \binom{120}{0}(0.02)^0(0.98)^{120} = 0.0885$

b) $P(X \le 5) = \binom{125}{0}(0.02)^0(0.98)^{125} + \binom{125}{1}(0.02)^1(0.98)^{124} + \binom{125}{2}(0.02)^2(0.98)^{123} + \binom{125}{3}(0.02)^3(0.98)^{122}$

$$+ \binom{125}{4}(0.02)^4(0.98)^{121} + \binom{125}{5}(0.02)^5(0.98)^{120} = 0.9596$$

c) $P(X \le 10) = 0.9998$

3-97. $P(length\ of\ stay\ \le 4) = 0.508$.

a) Let *N* denote the number of people (out of five) that wait less than or equal to 4 hours.

$$P(N = 1) = \binom{5}{1}(0.508)^1(0.492)^4 = 0.149$$

b) Let *N* denote the number of people (out of five) that wait more than 4 hours.

$$P(N = 2) = \binom{5}{2}(0.492)^2(0.508)^3 = 0.307$$

c) Let *N* denote the number of people (out of five) that wait more than 4 hours.

$$P(N \ge 1) = 1 - P(N = 0) = 1 - \binom{5}{0}(0.508)^5(0.492)^0 = 0.971$$

3-98. Probability a person leaves without being seen (LWBS) = 195/5292 = 0.037

a) $P(X = 1) = \binom{5}{1}(0.037)^1(0.963)^4 = 0.159$

b) $P(X \ge 2) = 1 - P(X \le 1) = 1 - [P(X = 0) + P(X = 1)]$

$$= 1 - \binom{5}{0}(0.037)^0(0.963)^5 - \binom{5}{1}(0.037)^1(0.963)^4 = 0.012$$

c) $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.828 = 0.172$

3-99. $P(change\ < 4\ days) =$ 0.3. Let X = number of the 10 changes made in less than 4 days.

a) $P(X = 7) = \binom{10}{7}(0.4)^7(0.6)^3 = 0.042$

b) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \binom{10}{0}(0.4)^0(0.6)^{10} + \binom{10}{1}(0.4)^1(0.6)^9 + \binom{10}{2}(0.4)^2(0.6)^8$

$= 0.006 + 0.040 + 0.121 = 0.167$

c) $P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{10}{0}(0.4)^0(0.6)^{10} = 1 - 0.006 = 0.994$

d) E$(X) = np = 10(0.4) = 4$

3-100. $P(reaction < 272K) = 0.60$

a) $P(X = 12) = \binom{25}{12}(0.6)^{12}(0.4)^{13} = 0.076$

b) $P(X \geq 19) = P(X = 19) + P(X = 20)$

$= \binom{25}{19}(0.6)^{19}(0.4)^6 + \binom{25}{20}(0.6)^{20}(0.4)^5 + \binom{25}{21}(0.6)^{21}(0.4)^4 + \binom{25}{22}(0.6)^{22}(0.4)^3 + \binom{25}{23}(0.6)^{23}(0.4)^2$

$+ \binom{25}{24}(0.6)^{24}(0.4)^1 + \binom{25}{25}(0.6)^{25}(0.4)^0 = 0.0735$

c) $P(X \geq 18) = P(X = 18) + P(X = 19) + P(X = 20) + P(X = 21) + P(X = 22)$

$+ P(X = 23) + P(X = 24) + P(X = 25)$

$= \binom{25}{18}(0.6)^{18}(0.4)^7 + 0.0735 = 0.1535$

d) $E(X) = np = 25(0.6) = 15$

Section 3-7

3-101. a) $P(X = 1) = (1-0.5)^0 0.5 = 0.5$

b) $P(X = 4) = (1-0.5)^3 0.5 = 0.5^4 = 0.0625$

c) $P(X = 8) = (1-0.5)^7 0.5 = 0.5^8 = 0.0039$

d) $P(X \leq 2) = P(X = 1) + P(X = 2) = (1-0.5)^0 0.5 + (1-0.5)^1 0.5$

$= 0.5 + 0.5^2 = 0.75$

e) As $\mathrm{P}(X \leq 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 0.9375$,

$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.9375 = 0.0625$

3-102. E(X) = 2.5 = 1/p giving p = 0.4

a) $P(X = 1) = (1-0.4)^0 0.4 = 0.4$

b) $P(X = 4) = (1-0.4)^3 0.4 = 0.0864$

c) $P(X = 5) = (1-0.5)^4 0.5 = 0.05184$

d) $P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$

$= (1-0.4)^0 0.4 + (1-0.4)^1 0.4 + (1-0.4)^2 0.4 = 0.7840$

e) As $\mathrm{P}(X \leq 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 0.8704$,

$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.8704 = 0.1296$

3-103. Let X denote the number of trials to obtain the first success.
a) E(X) = 1/0.25 = 4
b) Because of the lack of memory property, the expected value is still 4.

3-104. a) E(X) = 4/0.25 = 16
b) P(X=16) = $\binom{15}{3}(0.75)^{12}0.25^4 = 0.0563$
c) P(X=15) = $\binom{14}{3}(0.75)^{11}0.25^4 = 0.0601$
d) P(X=17) = $\binom{16}{3}(0.75)^{13}0.25^4 = 0.0520$
e) The most likely value for X should be near μ_X. By trying several cases, the most likely value is x = 15.

3-105. Let X denote the number of trials to obtain the first successful alignment.
Then X is a geometric random variable with p = 0.7
a) $P(X=4) = (1-0.7)^3 0.7 = 0.3^3 0.7 = 0.0189$
b) $P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$= (1-0.7)^0 0.7 + (1-0.7)^1 0.7 + (1-0.7)^2 0.7 + (1-0.7)^3 0.7$
$= 0.7 + 0.3(0.7) + 0.3^2(0.7) + 0.3^3 0.7 = 0.9919$
c) $P(X \ge 4) = 1 - P(X \le 3) = 1 - [P(X=1) + P(X=2) + P(X=3)]$
$= 1 - [(1-0.7)^0 0.7 + (1-0.7)^1 0.7 + (1-0.7)^2 0.7]$
$= 1 - [0.7 + 0.3(0.7) + 0.3^2(0.7)] = 1 - 0.973 = 0.027$

3-106. Let X denote the number of people who carry the gene.
Then X is a negative binomial random variable with r = 2 and p = 0.15
a) $P(X \ge 4) = 1 - P(X < 4) = 1 - [P(X=2) + P(X=3)]$
$= 1 - \left[\binom{1}{1}(1-0.15)^0 0.15^2 + \binom{2}{1}(1-0.15)^1 0.15^2\right] = 1 - (0.0225 + 0.03825) = 0.9393$
b) $E(X) = r/p = 2/0.15 = 13.33$

3-107. Let X denote the number of calls needed to obtain a connection.
Then, X is a geometric random variable with p = 0.03.
a) $P(X=10) = (1-0.03)^9 0.03 = 0.97^9 0.03 = 0.0228$
b) $P(X > 5) = 1 - P(X \le 4) = 1 - [P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$
$= 1 - [0.03 + 0.97(0.03) + 0.97^2(0.03) + 0.97^3(0.03) + 0.97^4(0.03)]$
$= 1 - 0.1413 = 0.8587$
May also use the fact that P(X > 5) is the probability of no connections in 5 trials. That is,
$P(X > 5) = \binom{5}{0} 0.03^0 0.97^5 = 0.8587$
c) E(X) = 1/0.03 = 33.33

3-108. X = number of opponents until the player is defeated.
p = 0.7, the probability of the opponent defeating the player.
(a) $f(x) = (1 - p)^{x-1}p = 0.7^{(x-1)}*0.3$
(b) P(X > 2) = 1 – P(X = 1) – P(X = 2) = 0.49
(c) μ = E(X) = 1/p = 3.33
(d) P(X ≥ 4) = 1 – P(X = 1) – P(X = 2) – P(X = 3) = 0.363
(e) The probability that a player contests four or more opponents is obtained in part (d), which is $p_o = 0.363$.
Let Y represent the number of games played until a player contests four or more opponents.
Then, $f(y) = (1-p_o)^{y-1}p_o$.

$\mu_Y = E(Y) = 1/p_o = 2.75$

3-109. p=0.13
(a) $P(X=1) = (1-0.13)^{1-1}*0.13=0.13$.
(b) $P(X=4)=(1-0.13)^{4-1}*0.13 =0.086$
(c) $\mu=E(X)= 1/p=7.69\approx 8$

3-110. X = number of attempts before the hacker selects a user password.
(a) $p=9000/36^6=0.0000041$
$\mu=E(X) = 1/p= 241864$
$V(X)= (1-p)/p^2 = 5.850*10^{10}$

$\sigma=\sqrt{V(X)}$ =241864
(b) $p=100/36^3=0.00214$
$\mu=E(X) = 1/p= 467$
$V(X)= (1-p)/p^2 = 217892.39$

$\sigma=\sqrt{V(X)}$ =466.78
Based on the answers to (a) and (b) above, it is clearly more secure to use a 6 character password.

3-111. p = 0.005 , r = 9
a.) $P(X=9)=0.005^9=1.95x10^{-21}$
b). $\mu=E(X)=\dfrac{1}{0.005}=200$ days
c) Mean number of days until all 9 computers fail. Now we use $p=1.95x10^{-21}$

$$\mu=E(Y)=\frac{1}{1.95x10^{-21}}=5.12x10^{20}$$ days or 1.4 $x10^{18}$ years

3-112. Let Y denote the number of samples needed to exceed 1 in Exercise 3-66.
Then Y has a geometric distribution with p = 0.0169.
a) $P(Y = 8) = (1 – 0.0169)^7(0.0169) = 0.0150$
b) Y is a geometric random variable with p = 0.1897 from Exercise 3-66.
$P(Y = 8) = (1 – 0.1897)^7(0.1897) = 0.0435$
c) E(Y) = 1/0.1897 = 5.27

3-113. Let X denote the number of transactions until all computers have failed.
Then, X is negative binomial random variable with $p = 2\times10^{-8}$ and r = 3.
a) $E(X) = 1.5 \text{ x } 10^8$
b) $V(X) = [3(1–2\times10^{-8})]/(4\times10^{-16}) = 2.5 \text{ x } 10^{15}$

3-114. (a) $p^{6} = 0.7$, p = 0.942
(b) $0.7*p^{2} = 0.3$, p = 0.655

3-115. Negative binomial random variable f(x; p, r) = $\binom{x-1}{r-1}(1-p)^{x-r}p^r$

When r = 1, this reduces to $f(x) = (1–p)^{x-1}p$, which is the pdf of a geometric random variable.
Also, E(X) = r/p and $V(X) = [r(1–p)]/p^2$ reduce to E(X) = 1/p and $V(X) = (1–p)/p^2$, respectively.

3-116. $P(reaction < 272K) = 0.6$
a) $P(X=10)=0.4^9 0.6^1=0.000157$
b) $\mu=E(X)=\frac{1}{p}=\frac{1}{0.6}=1.67$
c) $P(X\le 3)=P(X=3)+P(X=2)+P(X=1)$

$= 0.4^2 0.6^1 + 0.4^1 0.6^1 + 0.4^0 0.6^1 = 0.936$

d) $\mu = E(X) = \frac{r}{p} = \frac{2}{0.6} = 3.33$

3-117. a) Probability that color printer will be discounted = 1/20 = 0.05

$\mu = E(X) = \frac{1}{p} = \frac{1}{0.05} = 20$ days

b) $P(X = 10) = 0.95^9 0.05 = 0.0315$

c) Lack of memory property implies the answer equals $P(X = 10) = 0.95^9 0.05 = 0.0315$

d) $P(X \leq 3) = P(X = 3) + P(X = 2) + P(X = 1) = 0.95^2 0.05 + 0.95^1 0.05 + 0.05 = 0.143$

3-118. $P(LWBS) = \frac{242}{4329} = 0.056$

a) $P(X = 5) = 0.944^4 0.056^1 = 0.044$

b) $P(X = 5) + P(X = 6) = 0.944^4 0.056^1 + 0.944^5 0.056^1 = 0.086$

c) $P(X \leq 4) = P(X = 4) + P(X = 3) + P(X = 2) + P(X = 1)$

$= 0.944^3 0.056^1 + 0.944^2 0.056^1 + 0.944^1 0.056^1 + 0.056 = 0.206$

d) $\mu = E(X) = \frac{r}{p} = \frac{3}{0.056} = 53.57$

<u>Section 3-8</u>

3-119. X has a hypergeometric distribution N = 100, n = 5, K = 20

a) $P(X=1) = \frac{\binom{20}{1}\binom{80}{4}}{\binom{100}{5}} = \frac{20(1581580)}{75287520} = 0.4201$

b) $P(X=6) = 0$, the sample size is only 5

c) $P(X=4) = \frac{\binom{20}{4}\binom{80}{1}}{\binom{100}{5}} = \frac{4845(80)}{75287520} = 0.005148$

d) $E(X) = np = n\frac{K}{N} = 5\left(\frac{20}{100}\right) = 1$

$V(X) = np(1-p)\left(\frac{N-n}{N-1}\right) = 5(0.2)(0.8)\left(\frac{95}{99}\right) = 0.7677$

3-120. a) $P(X=1) = \frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}} = \frac{(4\times16\times15\times14)/6}{(20\times19\times18\times17)/24} = 0.4623$

b) $P(X=3) = \frac{\binom{4}{3}\binom{16}{1}}{\binom{20}{4}} = \frac{4\times16}{(20\times19\times18\times17)/24} = 0.0132$

c)

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

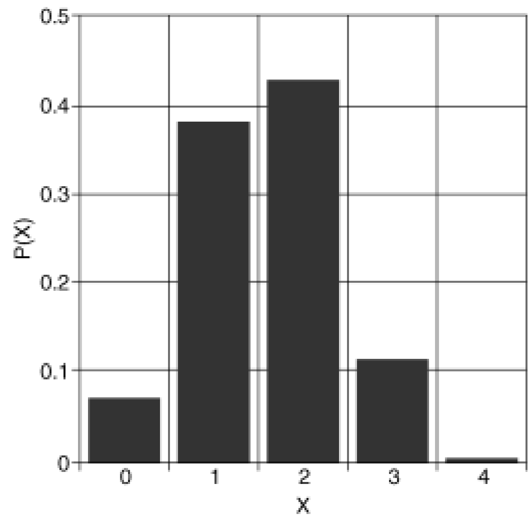
$$= \frac{\binom{4}{0}\binom{16}{4}}{\binom{20}{4}} + \frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}} + \frac{\binom{4}{2}\binom{16}{2}}{\binom{20}{4}}$$

$$= \frac{\left(\frac{16\times15\times14\times13}{24} + \frac{4\times16\times15\times14}{6} + \frac{6\times16\times15}{2}\right)}{\left(\frac{20\times19\times18\times17}{24}\right)} = 0.9866$$

d) E(X) = 4(4/20) = 0.8

V(X) = 4(0.2)(0.8)(16/19) = 0.539

3-121. N = 10, n = 4 and K = 4



3-122. (a) f(x) = $\binom{24}{x}\binom{12}{3-x} / \binom{40}{3}$

(b) μ = E(X) = np = 3*24/40 = 1.8

V(X)= np(1 – p)(N – n)/(N – 1) = 1.8*(1 – 24/40)(40 – 3)/(40 – 1) = 0.683

(c) P(X ≤ 2) = 1 – P(X = 3) =0.7951

3-123. Let X denote the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure. N = 900, K = 270, n = 10

a) n = 10

$$P(X=1)=\frac{\binom{270}{1}\binom{630}{9}}{\binom{900}{10}}=\frac{\left(\frac{270!}{1!269!}\right)\left(\frac{630!}{9!621!}\right)}{\frac{900!}{10!890!}}=0.1202$$

b) n = 10

$$P(X>1)=1-P(X\le 1)=1-[P(X=0)+P(X=1)]$$

$$P(X=0)=\frac{\binom{270}{0}\binom{630}{10}}{\binom{900}{10}}=\frac{\left(\frac{270!}{0!270!}\right)\left(\frac{630!}{10!620!}\right)}{\frac{900!}{10!890!}}=0.0276$$

$$P(X>1)=1-P(X\le 1)=1-[0.0276+0.1202]=0.8522$$

3-124. Let X denote the number of cards in the sample that are defective.

a) $P(X\ge 1)=1-P(X=0)$

$$P(X=0)=\frac{\binom{20}{0}\binom{130}{20}}{\binom{150}{20}}=\frac{\frac{130!}{20!10!}}{\frac{150!}{20!130!}}=0.04609$$

$$P(X\ge 1)=1-0.04609=0.95391$$

b) $P(X\ge 1)=1-P(X=0)$

$$P(X=0)=\frac{\binom{5}{0}\binom{145}{20}}{\binom{150}{20}}=\frac{\frac{145!}{20!125!}}{\frac{150!}{20!130!}}=\frac{145!130!}{125!150!}=0.4838$$

$$P(X\ge 1)=1-0.4838=0.5162$$

3-125. N=350

(a) K = 270, n = 3, P(X = 1)=0.120

(b) P(X≥1) = 0.988
(c) K = 34 + 21 = 55, P(X = 1)=0.337
(d) K = 350-7 = 343
P(X ≥ 1) = 0.99999506

3-126. Let X denote the count of the numbers in the state's sample that match those in the player's sample. Then, X has a hypergeometric distribution with N = 50, n = 6, and K = 6.

a) $P(X=6)=\frac{\binom{6}{6}\binom{44}{0}}{\binom{50}{6}}=\left(\frac{50!}{6!44!}\right)^{-1}=6.29\times10^{-8}$

b) $P(X=5)=\frac{\binom{6}{5}\binom{44}{1}}{\binom{50}{6}}=\frac{6\times44}{\binom{50}{6}}=1.66\times10^{-5}$

c) $P(X=4)=\frac{\binom{6}{4}\binom{44}{2}}{\binom{50}{6}}=0.00089$

d) Let Y denote the number of weeks needed to match all six numbers.

Then, Y has a geometric distribution with p = $\frac{1}{1,271,256}$ and

E(Y) = 1/p = $\frac{50!}{6!44!}=1,271,256$ weeks. This is more than 243 centuries!

3-127. Let X denote the number of blades in the sample that are dull.

a) $P(X\geq1)=1-P(X=0)$

$$P(X=0)=\frac{\binom{12}{0}\binom{36}{5}}{\binom{48}{5}}=\frac{\frac{36!}{5!31!}}{\frac{48!}{5!43!}}=\frac{36!43!}{31!48!}=0.2202$$

$$P(X\geq1)=1-P(X=0)=0.7798$$

b) Let Y denote the number of days needed to replace the assembly.

P(Y = 3) = $0.2202^2(0.7798)=0.0378$

c) On the first day, $P(X=0)=\frac{\binom{2}{0}\binom{46}{5}}{\binom{48}{5}}=\frac{\frac{46!}{5!41!}}{\frac{48!}{5!43!}}=\frac{46!43!}{48!41!}=0.8005$

On the second day, $P(X=0)=\frac{\binom{6}{0}\binom{42}{5}}{\binom{48}{5}}=\frac{\frac{42!}{5!37!}}{\frac{48!}{5!43!}}=\frac{42!43!}{48!37!}=0.4968$

On the third day, P(X = 0) = 0.2931 from part a). Therefore,
P(Y = 3) = 0.8005(0.4968)(1-0.2931) = 0.2811.

3-128. a) For Exercise 3-141, the finite population correction is 95/99.
For Exercise 3-142, the finite population correction is 16/19.
Because the finite population correction for Exercise 3-141 is closer to one, the binomial approximation to the distribution of X should be better in Exercise 3-141.

b) Assuming X has a binomial distribution with n = 5 and p = 0.2,

$$P(X=1)=\binom{5}{1}0.2^1 0.8^4=0.410$$

$$P(X=4)=\binom{5}{4}0.2^4 0.8^1=0.006$$

The results from the binomial approximation are close to the probabilities obtained in Exercise 3-141.

c) Assuming X has a binomial distribution with n = 4 and p = 0.2,

$$P(X=1)=\binom{4}{1}0.2^1 0.8^3=0.410$$
$$P(X=3)=\binom{4}{3}0.2^3 0.8^1=0.026$$

The results from the binomial approximation are close to the probabilities obtained in Exercise 3-142.

d) From Exercise 3-146, X is approximately binomial with n = 20 and p = 20/150 = 2/15.

$$P(X\geq 1)=1-P(X=0)=1-\binom{20}{0}\left(\frac{2}{15}\right)^0\left(\frac{13}{15}\right)^{20}=1-0.057=0.943$$

finite population correction is （150-20）/（150-1）=0.8725.

3-129. a) $P(X=4)=\dfrac{\binom{195}{4}\binom{953-195}{0}}{\binom{953}{4}}=0.0017$

b) $P(X=0)=\dfrac{\binom{195}{0}\binom{953-195}{4}}{\binom{953}{4}}=0.400$

c) Probability that all visits are from Hospital 1 $P(X=4)=\dfrac{\binom{195}{4}\binom{953-195}{0}}{\binom{953}{4}}=0.0017$

Probability that all visits are from Hospital 2 $P(X=4)=\dfrac{\binom{270}{4}\binom{953-270}{0}}{\binom{953}{4}}=0.0063$

Probability that all visits are from Hospital 3 $P(X=4)=\dfrac{\binom{246}{4}\binom{953-246}{0}}{\binom{953}{4}}=0.0044$

Probability that all visits are from Hospital 4 $P(X=4)=\dfrac{\binom{242}{4}\binom{953-242}{0}}{\binom{953}{4}}=0.0041$

Probability that all visits are from the same hospital
$$=.0017+.0063+.0044+.0041=0.0165$$

3-130. a) $P(X=2)=\dfrac{\binom{1343}{2}\binom{772613430}{4-2}}{\binom{7726}{4}}=0.124$

b) $P(X\geq 1)=1-P(X=0)=1-\dfrac{\binom{1343}{0}\binom{7726-1343}{4-0}}{\binom{7726}{4}}=1-0.466=0.531$

c) $\mu=E(X)=np=4\left(\frac{1343}{7726}\right)=0.695$

<u>Section 3-9</u>

3-131. a) $P(X=0)=\dfrac{e^{-5}5^0}{0!}=e^{-5}=0.0067$

b) $P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)$

$$=e^{-5}+\frac{e^{-5}5^1}{1!}+\frac{e^{-5}5^2}{2!}$$
$$=0.1247$$

c) $P(X=4)=\dfrac{e^{-5}5^{4}}{4!}=0.1755$

d) $P(X=8)=\dfrac{e^{-5}5^{8}}{8!}=0.0653$

3-132. a) $P(X=0)=e^{-0.7}=0.497$

b) $P(X\le 2)=e^{-0.7}+\dfrac{e^{-0.7}(0.7)}{1!}+\dfrac{e^{-0.7}(0.7)^{2}}{2!}=0.966$

c) $P(X=4)=\dfrac{e^{-0.7}(0.7)^{4}}{4!}=0.005$

d) $P(X=8)=\dfrac{e^{-0.7}(0.7)^{8}}{8!}=7.1\times 10^{-7}$

3-133. $P(X=0)=e^{-\lambda}=0.1$. Therefore, $\lambda = -\ln(0.1) = 2.303$.
Consequently, E(X) = V(X) = 2.303.

3-134. a) Let X denote the number of calls in one hour. Then, X is a Poisson random variable with $\lambda = 8$.

$P(X=5)=\dfrac{e^{-8}8^{5}}{5!}=0.0916$

b) $P(X\le 3)=e^{-8}+\dfrac{e^{-8}8}{1!}+\dfrac{e^{-8}8^{2}}{2!}+\dfrac{e^{-8}8^{3}}{3!}=0.0424$

c) Let Y denote the number of calls in two hours. Then, Y is a Poisson random variable with

$\lambda = 16$. $P(Y=15)=\dfrac{e^{-16}16^{15}}{15!}=0.0992$

d) Let W denote the number of calls in 30 minutes. Then W is a Poisson random variable with

$\lambda = 4$. $P(W=5)=\dfrac{e^{-4}4^{5}}{5!}=0.1563$

3-135. $\lambda=1$, Poisson distribution. $f(x) = e^{-\lambda}\lambda^{x}/x!$
(a) $P(X\ge 3)= 0.0803$
(b) In order that $P(X\ge 1) = 1-P(X=0)=1-e^{-\lambda}$ exceed 0.95, we need $\lambda=3$.
Therefore 3*16=48 cubic light years of space must be studied.

3-136. a) $\mu = 14.4$, $P(X = 0) = 6E10^{-7}$
b) $\lambda=14.4/6=2.4$, $P(X=0)=0.0907$
c) $\mu = 14.4(7)(28.35)/225 = 12.7$, $P(X \ge 1) = 0.999997$
d) $P(X \ge 28.8) = 1 - P(X \le 28) = 0.00046$. Unusual.

3-137. (a) $\lambda=0.61$. $P(X\ge 2)=0.125$
(b) $\lambda=0.61*10=6.1$, $P(X=0)= 0.0022$.

3-138. a) Let X denote the number of flaws in one square meter of cloth. Then, X is a Poisson random variable

with $\lambda = 0.1$. $P(X=3)=\dfrac{e^{-0.1}(0.1)^{3}}{3!}=0.00015$

b) Let Y denote the number of flaws in 10 square meters of cloth. Then, Y is a Poisson random variable

with $\lambda = 1$. $P(Y=1)=\dfrac{e^{-1}1^{1}}{1!}=e^{-1}=0.3679$

c) Let W denote the number of flaws in 15 square meters of cloth. Then, W is a Poisson random variable with λ = 1.5. $P(W=0)=e^{-1.5}=0.2231$

d) $P(Y\geq 2)=1-P(Y\leq 1)=1-P(Y=0)-P(Y=1)$

$$=1-e^{-1}-e^{-1}$$

$$=0.2642$$

3-139. a) $E(X)=\lambda=2$ errors per test area

b) $P(X\leq 2)=e^{-2}+\frac{e^{-2}2}{1!}+\frac{e^{-2}2^2}{2!}=0.677$

67.7% of test areas

3-140. a) Let X denote the number of cracks in 10 km of highway.
Then, X is a Poisson random variable with λ = 20.
$P(X=0)=e^{-20}=2.061\times10^{-9}$

b) Let Y denote the number of cracks in 1 km of highway.
Then, Y is a Poisson random variable with λ = 1.
$P(Y\geq 1)=1-P(Y=0)=1-e^{-1}=0.6321$

c) The assumptions of a Poisson process require that the probability of a event is constant for all intervals. If the probability of a count depends on traffic load and the load varies, then the assumptions of a Poisson process are not valid. Separate Poisson random variables might be appropriate for the heavy and light load sections of the highway.

3-141. a) Let X denote the number of flaws in 1 square meter of plastic panel.
Then, X is a Poisson random variable with λ = 0.5.
$P(X=0)=e^{-0.5}=0.6065$

b) Let Y denote the number of cars with no flaws,

$$P(Y=10)=\binom{10}{10}(0.6065)^{10}(0.3935)^{0}=0.0067$$

c) Let W denote the number of cars with surface flaws. Because the number of flaws has a Poisson distribution, the occurrences of surface flaws in cars are independent events with constant probability. From part (a), the probability a car contains surface flaws is 1 – 0.6065 = 0.3935. Consequently, *W* is binomial with *n* = 10 and *p* = 0.3935.

$$P(W=0)=\binom{10}{0}(0.3935)^{0}(0.6065)^{10}=0.0067$$

$$P(W=1)=\binom{10}{1}(0.3935)^{1}(0.6065)^{9}=0.0437$$

$$P(W\leq 1)=0.0067+0.0437=0.0504$$

3-142. a) Let X denote the failures in 8 hours. Then, X has a Poisson distribution with λ = 0.4.

$$P(X=0)=e^{-0.4}=0.670$$

b) Let Y denote the number of failure in 24 hours. Then, Y has a Poisson distribution with λ = 9.6.

$$P(Y\geq 5)=0.962$$

3-143. a) $P(X\geq 2)=1-[P(X=0)+P(X=1)]=1-\left[\frac{e^{-0.2}0.2^0}{0!}+\frac{e^{-0.2}0.2^1}{1!}\right]=0.0175$

b) $\lambda=0.2(5)=1$ per five days
$P(X=0)=e^{-1}=0.368$

c) $P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)$

$$=e^{-1}+\frac{e^{-1}1}{1!}+\frac{e^{-1}1^2}{2!}=0.920$$

3-144. a) $P(X=0)=e^{-1.7}=0.183$

b) $\lambda = 1.7(8) = 13.6$ per 8 minutes

$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)$

$=e^{-8}+\frac{e^{-8}8}{1!}+\frac{e^{-8}8^2}{2!}=0.000133$

c) No, if a Poisson distribution is assumed, the intervals need not be consecutive.

Supplemental Exercises

3-145. $E(X)=\frac{1}{5}\left(\frac{1}{3}\right)+\frac{2}{5}\left(\frac{1}{3}\right)+\frac{3}{5}\left(\frac{1}{3}\right)=\frac{2}{5}$,

$V(X)=\left(\frac{1}{5}\right)^2\left(\frac{1}{3}\right)+\left(\frac{2}{5}\right)^2\left(\frac{1}{3}\right)+\left(\frac{3}{5}\right)^2\left(\frac{1}{3}\right)-\left(\frac{2}{5}\right)^2=0.027$

3-146. a) $P(X=1)=\binom{1000}{1}(0.002)^1(0.998)^{999}=0.2707$

b) $P(X\ge 1)=1-P(X=0)=1-\binom{1000}{0}0.002^0(0.998)^{1000}=0.8649$

c) $P(X\le 2)=\binom{1000}{0}0.002^0(0.998)^{1000}+\binom{1000}{1}0.002^1(0.998)^{999}+\binom{1000}{2}0.002^2 0.998^{998}$

$=0.6767$

d) $E(X)=1000(0.002)=2$

$V(X)=1000(0.002)(0.998)=1.996$

3-147. a) n = 50, p = 5/50 = 0.1, since E(X) = 5 = np

b) $P(X\le 2)=\binom{50}{0}0.1^0(0.9)^{50}+\binom{50}{1}0.1^1(0.9)^{49}+\binom{50}{2}0.1^2(0.9)^{48}=0.112$

c) $P(X>47)=\binom{50}{48}0.1^{48}(0.9)^2+\binom{50}{49}0.1^{49}(0.9)^1+\binom{50}{50}0.1^{50}(0.9)^0=9.97\times 10^{-46}$

3-148. (a)Binomial distribution, p=0.02, n=12.

(b) P(X>1)=1-P(X≤1)= $1-\binom{12}{0}p^0(1-p)^{12}-\binom{12}{1}p^1(1-p)^{11}$ =0.0231

(c) μ =E(X)= np =12*0.02 = 0.24

V(X)=np(1-p) = 0.2352 σ= $\sqrt{V(X)}$ = 0.4850

3-149. (a) $(0.5)^{15}=0.0305\times 10^{-3}$

(b) $C_{15}^{7.5}(0.5)^{7.5}(0.5)^{7.5}$ = 0.5642 $\left[\because \left(n+\frac{1}{2}\right)!=\prod\left(n+\frac{1}{2}\right)=\sqrt{\pi}\prod_{k=0}^{n}\frac{2k+1}{2}\right]$

(c) $C_5^{15}(0.5)^5(0.5)^{10}+C_6^{15}(0.5)^6(0.5)^9=0.2443$

3-150. (a) Binomial distribution, n = 150, p = 0.01.
(b) P(X ≥ 1) = 0.634
(c) P(X ≥ 2) = 0.264
(d) μ = E(X) = np = 150 × 0.01 = 1.5

$V(X) = np(1 - p) = 1.485$
$\sigma = \sqrt{V(X)} = 1.2186$

(e) Let $p_d = P(X \geq 2) = 0.264$,
Y = number of messages that require two or more packets be resent.
Y is binomial distributed with n = 10, $p_m = p_d*(1/10) = 0.0264$
$P(Y \geq 1) = 0.235$

3-151. Let X denote the number of mornings needed to obtain a green light.
Then X is a geometric random variable with p = 0.30.
a) $P(X = 4) = (1-0.3)^3 0.3 = 0.1029$
b) By independence, $(0.7)^{10} = 0.0282$.

3-152. Let X denote the number of attempts needed to obtain a calibration that conforms to specifications.
Then, X is geometric with p = 0.8.
$P(X \leq 3) = P(X=1) + P(X=2) + P(X=3) = 0.8 + 0.2(0.8) + 0.2^2(0.8) = 0.992$.

3-153. Let X denote the number of fills needed to detect three underweight packages.
Then, X is a negative binomial random variable with p = 0.01 and r = 5.
a) $E(X) = 5/0.01 = 500$
b) $V(X) = [5(0.99)/0.01^2] = 49500$. Therefore, $\sigma_X = 222.486$.

3-154. Geometric with p=0.15
(a) $f(x)=(1-p)^{x-1}p=0.85^{(x-1)}0.15$
(b) $P(X=5) = 0.85^4*0.15=0.078$
(c) $\mu=E(X)= 1/p=6.67$
(d) $P(X\leq 10)=0.803$

3-155. (a) $\lambda=10*0.5=5$.
P(X=0) = 0.0067
(b) $P(X\geq 3)=0.875$
(c) $P(X\leq x) \geq 0.9$, x=8
(d) $\sigma^2= \lambda=10$. Not appropriate.

3-156. Let X denote the number of totes in the sample that do not conform to purity requirements. Then, X has a hypergeometric distribution with N = 15, n = 3, and K = 2.

$$P(X = 2) = \frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}} = \frac{13 \times 3!}{15 \times 14 \times 13} = 0.0286$$

3-157. Let X denote the number of calls that are answered in 30 seconds or less.
Then, X is a binomial random variable with p = 0.9.

a) $P(X = 9) = \binom{10}{9}(0.9)^9(0.1)^1 = 0.3874$

b) $P(X \geq 16) = P(X=16) + P(X=17) + P(X=18) + P(X=19) + P(X=20)$

$$= \binom{20}{16}(0.9)^{16}(0.1)^4 + \binom{20}{17}(0.9)^{17}(0.1)^3 + \binom{20}{18}(0.9)^{18}(0.1)^2$$

$$+ \binom{20}{19}(0.9)^{19}(0.1)^1 + \binom{20}{20}(0.9)^{20}(0.1)^0 = 0.9568$$

c) $E(X) = 20(0.9) = 18$

3-158. Let Y denote the number of calls needed to obtain an answer in less than 30 seconds.

a) $P(Y=4)=(1-0.9)^3 0.9=0.1^3 0.9=0.0009$
b) E(Y) = 1/p = 1/0.9 = 1.11

3-159. Let W denote the number of calls needed to obtain two answers in less than 30 seconds. Then, W has a negative binomial distribution with p = 0.8.

a) P(W=6) = $\binom{5}{1}(0.2)^4(0.8)^2 = 0.00512$

b) E(W) = r/p = 2/0.8 = 2.5

3-160. a) Let X denote the number of messages sent in one hour.

$$P(X=5)=\frac{e^{-10}10^5}{5!}=0.0378$$

b) Let Y denote the number of messages sent in 1.5 hours.
Then, Y is a Poisson random variable with λ =15.

$$P(Y=10)=\frac{e^{-15}(15)^{10}}{10!}=0.0486$$

c) Let W denote the number of messages sent in one-half hour.
Then, W is a Poisson random variable with λ = 5.

$$P(W<2)=P(W=0)+P(W=1)=0.0404$$

3-161. X is a negative binomial with r=4 and p=0.0001
$E(X)=r/p=3/0.0001=30000$ requests

3-162. X ~ Poisson(λ = 0.01), X ~ Poisson(λ = 1)

$$P(Y\le 2)=e^{-1}+\frac{e^{-1}(1)^1}{1!}+\frac{e^{-1}(1)^2}{2!}=0.9197$$

3-163. Let X denote the number of individuals that recover in one week. Assume the individuals are independent. Then, X is a binomial random variable with n = 25 and p = 0.1.
P(X ≥ 4) = 1 – P(X ≤ 3) = 1 – 0.7636 = 0.2364.

3-164. a.) P(X = 1) = 0 , P(X = 2) = 0.0025, P(X = 3) = 0.015, P(X = 4) = 0.0375, P(X = 5) = 0.065
P(X = 6) = 0.1275, P(X = 7) = 0.195, P(X = 8) = 0.175, P(X = 9) = 0.18, P(X = 10) = 0.2025
b.) P(X = 1) = 0.0025, P(X = 1.5) = 0.015, P(X = 2) = 0.0375, P(X = 2.5) = 0.065, P(X = 3) = 0.1275 P(X = 3.5) = 0.195,
P(X = 4) = 0.175, P(X = 4.5) = 0.18, P(X = 5) = 0.2025

3-165. Let X denote the number of assemblies needed to obtain 5 defectives.
Then, X is a negative binomial random variable with p = 0.01 and r=6.
a) E(X) = r/p = 600.
b) V(X) =(6* 0.99)/0.01^2 = 59400 and σ_X = 243.72.

3-166. Here n assemblies are checked. Let X denote the number of defective assemblies.
If P(X ≥ 1) ≥ 0.95, then P(X = 0) ≤ 0.05. Now,

P(X = 0) = $\binom{n}{0}(0.01)^0(0.99)^n = 99^n$ and $0.99^n \le 0.05$. Therefore,

$$n(\ln(0.99))\le \ln(0.05)$$

$$n\ge\frac{\ln(0.05)}{\ln(0.95)}=298.07$$

Therefore, n = 299

3-167. Require f(1) + f(2) + f(3) + f(4) + f(5)= 1. Therefore, *c*(1+2+3+4+5) = 1. Therefore, *c* = 1/15.

3-168. Let X denote the number of products that fail during the warranty period. Assume the units are independent. Then, X is a binomial random variable with n = 300 and p = 0.02.

a) P(X = 0) = $\binom{300}{0}(0.02)^0(0.98)^{500} = 0.0023$

b) E(X) = 300(0.02) = 6

c) P(X >2) = 1 – P(X ≤ 2) = 0.9398

3-169.
$$f_X(0) = (0.1)(0.65) + (0.3)(0.35) = 0.17$$
$$f_X(1) = (0.1)(0.65) + (0.4)(0.35) = 0.205$$
$$f_X(2) = (0.2)(0.65) + (0.2)(0.35) = 0.2$$
$$f_X(3) = (0.4)(0.65) + (0.1)(0.35) = 0.295$$
$$f_X(4) = (0.2)(0.65) + (0)(0.35) = 0.13$$

3-180. a) P(X =2.5) = 0

b) P(X > 2.5) = 0.4 + 0.3 + 0.1 = 0.8

c) P(2.7 < X < 5.1) = 0.4 + 0.3 = 0.7

d) E(X) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9

e) V(X) = $2^2(0.2) + 3^2(0.4) + 5^2(0.3) + 8^2(0.1) - (3.9)^2 = 3.09$

3-181.

x	2	5.7	6.5	8.5
f(x)	0.35	0.15	0.2	0.3

3-182. Let X and Y denote the number of bolts in the sample from supplier 1 and 2, respectively.
Then, X is a hypergeometric random variable with N = 100, n = 4, and K = 40.
Also, Y is a hypergeometric random variable with N = 100, n = 4, and K = 60.

a) P(X=4 or Y=4) = P(X = 4) + P(Y = 4)
$$= \frac{\binom{40}{4}\binom{60}{0}}{\binom{100}{4}} + \frac{\binom{40}{0}\binom{60}{4}}{\binom{100}{4}}$$
$$= 0.0233 + 0.1244$$
$$= 0.1477$$

b) P[(X=3 and Y=1) or (Y=3 and X = 1)]= $= \dfrac{\binom{40}{3}\binom{60}{1} + \binom{40}{1}\binom{60}{3}}{\binom{100}{4}} = 0.5003$

3-183. Let X denote the number of errors in a sector. Then, X is a Poisson random variable with λ = 0.30352.

a) P(X>1) = 1 – P(X≤1) = 1 – $e^{-0.30352}$ – $e^{-0.30352}$(0.30352) = 0.03772

b) Let Y denote the number of sectors until an error is found.
Then, Y is a geometric random variable and P = P(X ≥ 1) = 1 – P(X=0) = 1 – $e^{-0.30352}$ = 0.2618
E(Y) = 1/p = 3.82

3-184. Let X denote the number of orders placed in a week in a city of 800,000 people.

Then X is a Poisson random variable with $\lambda = 0.125(8) = 1$.
a) $P(X \ge 3) = 1 - P(X \le 2) = 1 - [e^{-1} + e^{-1}(1) + (e^{-1}1^2)/2!] = 1 - 0.9197 = 0.0803$.
b) Let Y denote the number of orders in 2 weeks. Then, Y is a Poisson random variable with $\lambda = 2$, and
$P(Y>2) = 1 - P(Y \le 2) = e^{-2} + (e^{-2}2^1)/1! + (e^{-2}2^2)/2! = 1 - 0.6767 = 0.3233$.

3-185. a) Hypergeometric random variable with N = 600, n = 5, and K = 150

$$f_X(0) = \frac{\binom{150}{0}\binom{450}{5}}{\binom{600}{5}} = 0.2359$$

$$f_X(1) = \frac{\binom{150}{1}\binom{450}{4}}{\binom{600}{5}} = 0.3968$$

$$f_X(2) = \frac{\binom{150}{2}\binom{450}{3}}{\binom{600}{5}} = 0.2646$$

$$f_X(3) = \frac{\binom{150}{3}\binom{450}{2}}{\binom{600}{5}} = 0.0874$$

$$f_X(4) = \frac{\binom{150}{4}\binom{450}{1}}{\binom{600}{5}} = 0.01431$$

$$f_X(5) = \frac{\binom{150}{5}\binom{450}{0}}{\binom{600}{5}} = 0.00093$$

b)

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0.0549	0.1868	0.2833	0.2524	0.1462	0.0576	0.0156	0.0029	0.0003	0.00002	0.0000008

3-186. Let X denote the number of totes in the sample that exceed the moisture content.
Then X is a binomial random variable with n = 10. We are to determine p.

If $P(X \ge 1) = 0.9$, then $P(X = 0) = 0.1$. Then $\binom{10}{0}(p)^0(1-p)^{10} = 0.1$, giving $10\ln(1-p) = \ln(0.1)$,

which results in p = 0.2057.

3-187. Let t denote an interval of time in hours and let X denote the number of messages that arrive in time t.
Then, X is a Poisson random variable with $\lambda = 5t$.
Then, $P(X=0) = 0.9$ and $e^{-5t} = 0.9$, resulting in t = 0.0211 hours = 75.86 seconds

3-188. a) Let X denote the number of flaws in 30 panels.
Then, X is a Poisson random variable with $\lambda = 30(0.02) = 0.6$.
$P(X = 0) = e^{-0.6} = 0.549$.

b) Let Y denote the number of flaws in one panel.

$P(Y \geq 1) = 1 - P(Y=0) = 1 - e^{-0.02} = 0.0198$.
Let W denote the number of panels that need to be inspected before a flaw is found.
Then W is a geometric random variable with p = 0.0198.
E(W) = 1/0.0198 = 50.51 panels.

c) $P(Y \geq 1) = 1 - P(Y=0) = 1 - e^{-0.02} = 0.0198$
Let V denote the number of panels with 1 or more flaws.
Then V is a binomial random variable with n = 50 and p = 0.0198

$$P(V \leq 2) = \binom{50}{0} 0.0198^0 (.9802)^{50} + \binom{50}{1} 0.0198^1 (0.9802)^{49} + \binom{50}{2} 0.0198^2 (0.9802)^{48} = 0.9234$$

Mind Expanding Exercises

3-189. The binomial distribution

$$P(X = x) = \frac{n!}{r!(n-r)!} p^x (1-p)^{n-x}$$

The probability of the event can be expressed as $p = \lambda/n$ and the probability mass function can be written as

$$P(X = x) = \frac{n!}{x!(n-x)!} [\lambda/n]^x [1 - (\lambda/n)]^{n-x}$$

$$P(X = x) \frac{n \times (n-1) \times (n-2) \times (n-3) \ldots\ldots \times (n-x+1)}{n^x} \frac{\lambda^x}{x!} (1 - (\lambda/n))^{n-x}$$

Now we can re-express as:
$[1 - (\lambda/n)]^{n-x} = [1 - (\lambda/n)]^n [1 - (\lambda/n)]^{-x}$

In the limit as $n \to \infty$

$$\frac{n \times (n-1) \times (n-2) \times (n-3) \ldots\ldots \times (n-x+1)}{n^x} \cong 1$$

As $n \to \infty$ the limit of $[1 - (\lambda/n)]^{-x} \cong 1$
Also, we know that as $n \to \infty$

$$(1 - \lambda/n)^n = e^{-\lambda}$$

Thus,

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

The distribution of the probability associated with this process is known as the Poisson distribution and we can express the probability mass function as

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

3-190. Show that $\sum_{i=1}^{\infty} (1-p)^{i-1} p = 1$ using an infinite sum.

To begin, $\sum_{i=1}^{\infty} (1-p)^{i-1} p = p \sum_{i=1}^{\infty} (1-p)^{i-1}$,
From the results for an infinite sum this equals

$$p\sum_{i=1}^{\infty}(1-p)^{i-1} = \frac{p}{1-(1-p)} = \frac{p}{p} = 1$$

3-191.

$$E(X) = [(a+(a+1)+...+b](b-a+1)$$

$$= \left[\sum_{i=1}^{b} i - \sum_{i=1}^{a-1} i\right] \Big/ (b-a+1) = \left[\frac{b(b+1)}{2} - \frac{(a-1)a}{2}\right] \Big/ (b-a+1)$$

$$= \left[\frac{(b^2-a^2+b+a)}{2}\right] \Big/ (b-a+1) = \left[\frac{(b+a)(b-a+1)}{2}\right] \Big/ (b-a+1)$$

$$= \frac{(b+a)}{2}$$

$$V(X) = \frac{\sum_{i=a}^{b}\left[i - \frac{b+a}{2}\right]^2}{b+a-1} = \frac{\left[\sum_{i=a}^{b} i^2 - (b+a)\sum_{i=a}^{b} i + \frac{(b-a+1)(b+a)^2}{4}\right]}{b+a-1}$$

$$= \frac{\frac{b(b+1)(2b+1)}{6} - \frac{(a-1)a(2a-1)}{6} - (b+a)\left[\frac{b(b+1)-(a-1)a}{2}\right] + \frac{(b-a+1)(b+a)^2}{4}}{b-a+1}$$

$$= \frac{(b-a+1)^2-1}{12}$$

3-192. Let X denote a geometric random variable with parameter p. Let $q = 1 - p$.

$$E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1}p = p\sum_{x=1}^{\infty} xq^{x-1} = p\sum_{x=1}^{\infty}\frac{d}{dq}q^x$$

$$= p \cdot \frac{d}{dq}\sum_{x=1}^{\infty} q^x = p \cdot \frac{d}{dq}\left(\frac{q}{1-q}\right) = p\left(\frac{1(1-q)-q(-1)}{(1-q)^2}\right)$$

$$= p\left(\frac{1}{p^2}\right) = \frac{1}{p}$$

$$\begin{aligned}
V(X) &= \sum_{x=1}^{\infty}(x-\tfrac{1}{p})^2(1-p)^{x-1}p = \sum_{x=1}^{\infty}\left(px^2-2x+\tfrac{1}{p}\right)(1-p)^{x-1} \\
&= p\sum_{x=1}^{\infty}x^2q^{x-1} - 2\sum_{x=1}^{\infty}xq^{x-1} + \tfrac{1}{p}\sum_{x=1}^{\infty}q^{x-1} \\
&= p\sum_{x=1}^{\infty}x^2q^{x-1} - \tfrac{2}{p^2} + \tfrac{1}{p^2} \\
&= p\sum_{x=1}^{\infty}x^2q^{x-1} - \tfrac{1}{p^2} \\
&= p\tfrac{d}{dq}\left[q+2q^2+3q^3+...\right] - \tfrac{1}{p^2} \\
&= p\tfrac{d}{dq}\left[q(1+2q+3q^2+...)\right] - \tfrac{1}{p^2} \\
&= p\tfrac{d}{dq}\left[\tfrac{q}{(1-q)^2}\right] - \tfrac{1}{p^2} = 2pq(1-q)^{-3} + p(1-q)^{-2} - \tfrac{1}{p^2} \\
&= \frac{\left[2(1-p)+p-1\right]}{p^2} = \frac{(1-p)}{p^2} = \frac{q}{p^2}
\end{aligned}$$

3-193. Let X = number of passengers with a reserved seat who arrive for the flight, n = number of seat reservations, p = probability that a ticketed passenger arrives for the flight.

a) In this part we determine n such that $P(X \geq 120) \geq 0.9$. By testing for n in Minitab the minimum value is $n = 131$.

b) In this part we determine n such that $P(X > 120) \leq 0.10$ which is equivalent to $1 - P(X \leq 120) \leq 0.10$ or $0.90 \leq P(X \leq 120)$.
By testing for n in Minitab the solution is $n = 123$.

c) One possible answer follows. If the airline is most concerned with losing customers due to over-booking, they should only sell 123 tickets for this flight. The probability of over-booking is then at most 10%. If the airline is most concerned with having a full flight, they should sell 131 tickets for this flight. The chance the flight is full is then at least 90%. These calculations assume customers arrive independently and groups of people that arrive (or do not arrive) together for travel make the analysis more complicated.

3-194. Let X denote the number of nonconforming products in the sample.
Then, X is approximately binomial with p = 0.01 and n is to be determined.
If $P(X \geq 1) \geq 0.90$, then $P(X = 0) \leq 0.10$.
Now, $P(X = 0) = \binom{n}{0}p^0(1-p)^n = (1-p)^n$. Consequently, $(1-p)^n \leq 0.10$, and
$n \leq \dfrac{\ln 0.10}{\ln(1-p)} = 229.11$. Therefore, $n = 230$ is required.

3-195. If the lot size is small, 10% of the lot might be insufficient to detect nonconforming product. For example, if the lot size is 10, then a sample of size one has a probability of only 0.2 of detecting a nonconforming product in a lot that is 20% nonconforming.

If the lot size is large, 10% of the lot might be a larger sample size than is practical or necessary. For example, if the lot size is 5000, then a sample of 500 is required. Furthermore, the binomial approximation to the hypergeometric distribution can be used to show the following. If 5% of the lot of size 5000 is nonconforming, then the probability of zero nonconforming products in the sample is approximately 7E-12. Using a sample of 100, the same probability is still only 0.0059. The sample of size 500 might be much larger than is needed.

3-196. Let X denote the number of acceptable components. Then, X has a binomial distribution with p = 0.98 and n is to be determined such that $P(X \geq 100) \geq 0.95$

n	$P(X \geq 100)$
102	0.666
103	0.848
104	0.942
105	0.981

Therefore, 105 components are needed.

3-197. Let X denote the number of rolls produced.

Revenue at each demand				
	0	1000	2000	3000
$0 \leq x \leq 1000$	0.05x	0.3x	0.3x	0.3x
mean profit = 0.05x(0.3) + 0.3x(0.7) - 0.1x				
$1000 \leq x \leq 2000$	0.05x	0.3(1000) + 0.05(x-1000)	0.3x	0.3x
mean profit = 0.05x(0.3) + [0.3(1000) + 0.05(x-1000)](0.2) + 0.3x(0.5) - 0.1x				
$2000 \leq x \leq 3000$	0.05x	0.3(1000) + 0.05(x-1000)	0.3(2000) + 0.05(x-2000)	0.3x
mean profit = 0.05x(0.3) + [0.3(1000)+0.05(x-1000)](0.2) + [0.3(2000) + 0.05(x-2000)](0.3) + 0.3x(0.2) - 0.1x				
$3000 \leq x$	0.05x	0.3(1000) + 0.05(x-1000)	0.3(2000) + 0.05(x-2000)	0.3(3000)+ 0.05(x-3000)
mean profit = 0.05x(0.3) + [0.3(1000)+0.05(x-1000)](0.2) + [0.3(2000)+0.05(x-2000)]0.3 + [0.3(3000)+0.05(x-3000)]0.2 - 0.1x				

	Profit	Max. profit
$0 \leq x \leq 1000$	0.125 x	\$ 125 at x = 1000
$1000 \leq x \leq 2000$	0.075 x + 50	\$ 200 at x = 2000
$2000 \leq x \leq 3000$	200	\$200 at x = 3000
$3000 \leq x$	-0.05 x + 350	\$200 at x = 3000

The bakery can produce anywhere from 2000 to 3000 and earn the same profit.