CHAPTER 4

Section 4-2

4-1. a) $P(1 < X) = \int_1^{\infty} e^{-2x} dx = (-e^{-2x})\Big|_1^{\infty} = e^{-2} = 0.1353$

b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-2x} dx = (-e^{-2x})\Big|_1^{2.5} = e^{-2} - e^{-5} = 0.1286$

c) $P(X = 3) = \int_3^{3} e^{-2x} dx = 0$

d) $P(X < 4) = \int_0^{4} e^{-2x} dx = (-e^{-2x})\Big|_0^{4} = 1 - e^{-8} = 0.9997$

e) $P(3 \le X) = \int_3^{\infty} e^{-2x} dx = (-e^{-2x})\Big|_3^{\infty} = e^{-6} = 0.0025$

f) $P(x < X) = \int_x^{\infty} e^{-2x} dx = (-e^{-2x})\Big|_x^{\infty} = e^{-2x} = 0.10$

Then, 2x = –ln(0.10) = 2.3 ⇒ x = 1.15

g) $P(X \le x) = \int_0^{x} e^{-2x} dx = (-e^{-2x})\Big|_0^{x} = 1 - e^{-2x} = 0.10$

Then, x = $\dfrac{-\ln(0.9)}{2}$ = 0.0527

4-2. a) $P(X < 2) = \int_0^{2} \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_0^{2} = \left(\frac{3}{16} - \frac{1}{32}\right) - 0 = 0.1563$

b) $P(X < 9) = \int_0^{8} \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_0^{8} = (3 - 2) - 0 = 1$

c) $P(2 < X < 4) = \int_2^{4} \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_2^{4} = \left(\frac{3}{4} - \frac{1}{4}\right) - \left(\frac{3}{16} - \frac{1}{32}\right) = 0.3438$

d) $P(X > 6) = \int_6^{8} \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_6^{8} = (3 - 2) - \left(\frac{27}{16} - \frac{27}{32}\right) = 0.1563$

e) $P(X < x) = \int_0^{x} \frac{3(8u - u^2)}{256} du = \left(\frac{3u^2}{64} - \frac{u^3}{256}\right)\Big|_0^{x} = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right) - 0 = 0.95$

Then, $x^3 - 12x^2 + 243.2 = 0$, and $x = 6.9172$

4-3. a) $P(X<0)=\int_{-\pi/2}^{0}\cos x dx=\sin x\Big|_{-\pi/2}^{0}=0-(-1)=1$

b) $P(X<-\pi/4)=\int_{-\pi/2}^{-\pi/4}\cos x dx=\sin x\Big|_{-\pi/2}^{-\pi/4}=-0.7071-(-1)=0.2929$

c) $P(-\pi/4<X<\pi/4)=\int_{-\pi/4}^{\pi/4}\cos x dx=\sin x\Big|_{-\pi/4}^{\pi/4}=0.7071-(-0.7071)=1.4142$

d) $P(X>-\pi/4)=\int_{-\pi/4}^{\pi/2}\cos x dx=\sin x\Big|_{-\pi/4}^{\pi/2}=1-(-0.7071)=1.7071$

e) $P(X<x)=\int_{-\pi/2}^{x}\cos x dx=\sin x\Big|_{-\pi/2}^{x}=\sin x-(-1)=0.95$

Then, sin x = -0.05, and x = -0.05 radians.

4-4. a) $P(X<2)=\int_{1}^{2}\frac{2}{x^3}dx=(\frac{-1}{x^2})\Big|_{1}^{2}=(\frac{-1}{4})-(-1)=0.75$

b) $P(X>3)=\int_{3}^{\infty}\frac{2}{x^3}dx=(\frac{-1}{x^2})\Big|_{3}^{\infty}=0-(\frac{-1}{9})=0.11$

c) $P(4<X<8)=\int_{4}^{8}\frac{2}{x^3}dx=(\frac{-1}{x^2})\Big|_{4}^{8}=(\frac{-1}{64})-(\frac{-1}{16})=0.0469$

d) $P(X<4 or X>8)=1-P(4<X<8)$. From part (c), $P(4<X<8)$ = 0.0469. Therefore, $P(X<4 or X>8)$ = 1 – 0.0469 = 0.9531

e) $P(X<x)=\int_{1}^{x}\frac{2}{x^3}dx=(\frac{-1}{x^2})\Big|_{1}^{x}=(\frac{-1}{x^2})-(-1)=0.95$

Then, x^2 = 20, and x = 4.4721

4-5. a) $P(X<4)=\int_{3}^{4}\frac{x}{7}dx=\frac{x^2}{14}\Big|_{3}^{4}=\frac{4^2-3^2}{14}=0.5$ because $f_X(x)=0$ for x < 3.

b) $P(X>3.5)=\int_{3.5}^{5}\frac{x}{7}dx=\frac{x^2}{14}\Big|_{3.5}^{5}=\frac{5^2-3.5^2}{14}=0.9107$ because $f_X(x)=0$ for x > 5.

c) $P(4<X<5)=\int_{4}^{5}\frac{x}{7}dx=\frac{x^2}{14}\Big|_{4}^{5}=\frac{5^2-4^2}{14}=0.6429$

d) $P(X<4.5)=\int_{3}^{4.5}\frac{x}{7}dx=\frac{x^2}{14}\Big|_{3}^{4.5}=\frac{4.5^2-3^2}{14}=0.8036$

e) $P(X>4.5)+P(X<3.5)=\int_{4.5}^{5}\frac{x}{7}dx+\int_{3}^{3.5}\frac{x}{7}dx=\frac{x^2}{14}\Big|_{4.5}^{5}+\frac{x^2}{14}\Big|_{3}^{3.5}=\frac{5^2-4.5^2}{14}+\frac{3.5^2-3^2}{14}=0.5714$

4-6. a) $P(1<X)=\int_5^{\infty} e^{-(x-5)}dx=-e^{-(x-5)}\Big|_5^{\infty}=1$, because $f_X(x)=0$ for x < 5. This can also be obtained from the fact that $f_X(x)$ is a probability density function for 5 < x.

b) $P(2\le X\le 5)=\int_5^{5} e^{-(x-5)}dx=-e^{-(x-5)}\Big|_5^{5}=0$

c) $P(5<X)=1-P(X\le 5)$. From part b, $P(X\le 5)=0$. Therefore, $P(5<X)=1$.

d) $P(8<X<12)=\int_8^{12} e^{-(x-5)}dx=-e^{-(x-5)}\Big|_8^{12}=e^{-3}-e^{-7}=0.0489$

e) $P(X<x)=\int_5^{x} e^{-(x-5)}dx=-e^{-(x-5)}\Big|_5^{x}=1-e^{-(x-5)}=0.85$

Then, x = 5 – ln(0.15) = 6.897

4-7. a) $P(0<X)=0.5$, by symmetry.

b) $P(0.5<X)=\int_{0.5}^{1} 2x^2dx=\frac{2}{3}x^3\Big|_{0.5}^{1}=0.6667-0.0833=0.5834$

c) $P(-0.5\le X\le 0.5)=\int_{-0.5}^{0.5} 2x^2dx=\frac{2}{3}x^3\Big|_{-0.5}^{0.5}=0.167$

d) P(X < –2) = 0

e) P(X < 0 or X > –0.5) = 1

f) $P(x<X)=\int_x^{1} 2x^2dx=\frac{2}{3}x^3\Big|_x^{1}=0.667-0.667x^3=0.05$

Then, x = 0.925

4-8. a) $P(X>5000)=\int_{5000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000}dx=-e^{\frac{-x}{1000}}\Big|_{5000}^{\infty}=e^{-5}=0.0067$

b) $P(1000<X<2000)=\int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000}dx=-e^{\frac{-x}{1000}}\Big|_{1000}^{2000}=e^{-1}-e^{-2}=0.233$

c) $P(X<1000)=\int_{0}^{1000} \frac{e^{\frac{-x}{1000}}}{1000}dx=-e^{\frac{-x}{1000}}\Big|_{0}^{1000}=1-e^{-1}=0.6321$

d) $P(X<x)=\int_{0}^{x} \frac{e^{\frac{-x}{1000}}}{1000}dx=-e^{\frac{-x}{1000}}\Big|_{0}^{x}=1-e^{-x/1000}=0.10$.

Then, $e^{-x/1000}=0.9$, and x = –1000 ln 0.9 = 105.36.

4-9. a) $P(X > 25) = \int_{25}^{25.25} 2.5dx = 2.5x\Big|_{25}^{25.25} = 0.625$

b) $P(X > x) = 0.85 = \int_{x}^{25.25} 2.5dx = 2.5x\Big|_{x}^{25.25} = 63.125 - 2.5x$

Then, 2.5x = 62.275 and x = 31.14.

4-10. a) $P(X < 74.7) = \int_{74.6}^{74.7} 1.3dx = 1.3x\Big|_{74.6}^{74.7} = 0.13$

b) P(X < 74.8 or X > 75.2) = P(X < 74.8) + P(X > 75.2) because the two events are mutually exclusive.

$$P\left(X < 74.8\right) = \int_{74.6}^{74.8} 1.3dx = 1.3x\Big|_{74.6}^{74.8} = 1.3\times 0.2 = 0.26$$

and

$$P\left(X < 75.2\right) = \int_{75.2}^{75.4} 1.3dx = 1.3x\Big|_{75.2}^{75.4} = 1.3\times 0.2 = 0.26$$

The result is 0.26 + 0.26 = 0.52.

c) $P(74.7 < X < 75.3) = \int_{74.7}^{75.3} 1.3dx = 1.3x\Big|_{74.7}^{75.3} = 1.3(0.6) = 0.780$

4-11. a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

P(X > 2.75) = $\int_{2.75}^{2.9} 2dx = 2(0.15) = 0.30$.

b) If the probability density function is centered at 2.60 meters, then $f_X(x) = 2$ for 2.3 < x < 2.9 and all rods will meet specifications.

4-12. a) $P(X < 90) = 0$ because the pdf is not defined in the range $(-\infty, 90)$.

b)

$$P(100 < X \le 300) = \int_{100}^{300} (-5.56\times10^{-4} + 5.56\times10^{-6}x)dx = (-5.56\times10^{-4}x + 2.78\times10^{-6}x^2)\Big|_{100}^{300}$$

$$= (-5.56\times10^{-4}\times300 + 2.78\times10^{-6}\times300^2) - (-5.56\times10^{-4}\times100 + 2.78\times10^{-6}\times100^2)$$

$$= 0.111$$

c)

$$P(X > 800) = \int_{800}^{1000} (4.44\times10^{-3} - 4.44\times10^{-6}x)dx = (4.44\times10^{-3}x - 2.22\times10^{-6}x^2)\Big|_{800}^{1000}$$

$$= (4.44\times10^{-3}\times10^3 - 2.22\times10^{-6}\times10^6) - (4.44\times10^{-3}\times800 - 2.22\times10^{-6}\times800^2)$$

$$= 0.0888 \cong 0.09$$

d) Find *a* such that $P(X > a) = 0.1$

$$P(X > a) = \int_a^{1000} (4.44\times10^{-3} - 4.44\times10^{-6}x)dx = (4.44\times10^{-3}x - 2.22\times10^{-6}x^2)\Big|_a^{1000} = 0.1$$

$$(4.44\times10^{-3}\times10^3 - 2.22\times10^{-6}\times10^6) - (4.44\times10^{-3}\times a - 2.22\times10^{-6}\times a^2) = 0.1$$

$$(2.22) - (4.44\times10^{-3}\times a - 2.22\times10^{-6}\times a^2) = 0.1$$

Then, $a \cong 787.76$

Section 4-3

4-13. a) P(X<2.8) = P(X ≤ 2.8) because X is a continuous random variable.
Then, P(X<2.8) = F(2.8) = 0.3(2.8) = 0.84.

b) $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - 0.3(1.5) = 0.55$

c) $P(X < -2) = F_X(-2) = 0$

d) $P(X > 6) = 1 - F_X(6) = 0$

4-14. a) $P(X < 1.7) = P(X \le 1.7) = F_X(1.7)$ because X is a continuous random variable. Then,
$F_X(1.7) = 0.2(1.7) + 0.5 = 0.84$

b) $P(X > -1.5) = 1 - P(X \le -1.5) = 1 - 0.2 = 0.8$

c) P(X < -2) = 0.1

d) $P(-1 < X < 1) = P(-1 < X \le 1) = F_X(1) - F_X(-1) = 0.7 - 0.3 = 0.4$

4-15. Now, $f(x) = e^{-2x}$ for 0 < x and $F_X(x) = \int_0^x e^{-2x}dx = -e^{-2x}\Big|_0^x = 1 - e^{-2x}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-2x}, & x > 0 \end{cases}$

4-16. Now, $f(x) = \frac{3(8x - x^2)}{256}$ for 0 < x < 8 and

$$F_X(x) = \int_0^x \frac{3(8u - u^2)}{256}du = \left(\frac{3u^2}{64} - \frac{u^3}{256}\right)\Bigg|_0^x = \frac{3x^2}{64} - \frac{x^3}{256} \text{ for } 0 < x.$$

Then, $F_X(x) = \begin{cases} 0, \ x \le 0 \\ \frac{3x^2}{64} - \frac{x^3}{256}, 0 \le x < 8 \\ 1, \ x \ge 8 \end{cases}$

4-17. Now, $f(x) = \cos x$ for -π/2 < x < π/2 and

$$F_X(x) = \int_{-\pi/2}^{x} \cos u du = \sin x\Big|_{-\pi/2}^{x} = \sin x + 1$$

Then, $F_X(x) = \begin{cases} 0, \ x \le -\pi/2 \\ \sin x + 1, -\pi/2 \le x < \pi/2 \\ 1, \ x \ge \pi/2 \end{cases}$

4-18. Now, $f(x) = \dfrac{2}{x^3}$ for x > 1 and

$$F_X(x) = \int_1^x \frac{2}{u^3} du = \left(\frac{-1}{u^2}\right)\Bigg|_1^x = \left(\frac{-1}{x^2}\right) + 1$$

Then, $F_X(x) = \begin{cases} 0, \ x \le 1 \\ 1 - \dfrac{1}{x^2}, \ x > 1 \end{cases}$

4-19. Now, $f(x) = x/7$ for 3 < x < 5 and $F_X(x) = \int_3^x \frac{u}{7} du = \frac{u^2}{14}\Big|_3^x = \frac{x^2 - 9}{14}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, & x < 3 \\ \dfrac{x^2 - 9}{14}, & 3 \le x < 5 \\ 1, & x \ge 5 \end{cases}$

4.20. Now, $f(x) = \dfrac{e^{-x/1000}}{1000}$ for 0 < x and

$$F_X(x) = \frac{1}{1000} \int_0^x e^{-x/1000} dy = -e^{-y/1000}\Big|_0^x = 1 - e^{-x/1000}$$ for 0 < x.

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$

P(X>3000) = 1- P(X ≤ 3000) = 1- F(3000) = $e^{-3000/1000}$ = 0.5

4-21. Now, f(x) = 2 for 2.3 < x < 2.9 and $F(x) = \int_{2.3}^x 2dy = 2x - 4.6$

for 2.3 < x < 2.9. Then,

$$F(x) = \begin{cases} 0, & x < 2.3 \\ 2x - 4.6, & 2.3 \le x < 2.9 \\ 1, & 2.9 \le x \end{cases}$$

$P(X > 2.7) = 1 - P(X \le 2.7) = 1 - F(2.7) = 1 - 0.8 = 0.2$ because X is a continuous random variable.

4-22. Now, $f(x) = \dfrac{e^{-x/10}}{10}$ for 0 < x and

$$F_X(x) = 1/10 \int_0^x e^{-x/10} dx = -e^{-x/10} \Big|_0^x = 1 - e^{-x/10} \text{ for } 0 < x.$$

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/10}, & x > 0 \end{cases}$

a) P(X<30) = F(30) =1 - e^{-3} = 1 - 0.0498 = 0.9502

b) $1/10 \int_{15}^{30} e^{-x/10} dx = e^{-1.5} - e^{-3} = 0.173343$

c) P(X_1>40)+ P(X_1<40 and X_2>40)= e^{-4}+(1- e^{-4}) e^{-4}=0.0363

d) P(20<X<40) = F(40)-F(20) = e^{-2}- e^{-4}=0.117

4-23. $F(x) = \int_0^x 1.5x dx = \frac{1.5x^2}{2}\Big|_0^x = 0.75x^2$ for 0 < x < 2. Then,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.75x^2, & 0 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

4-24. $f_X(x) = 2e^{-3x}, \; x > 0$

4-25. $f(x) = \begin{cases} 0.25, & 0 < x < 4 \\ 0.04, & 4 \le x < 9 \end{cases}$

4-26. $f_X(x) = \begin{cases} 0.5, & -2 < x < 1 \\ 0.75, & 1 \le x < 1.5 \end{cases}$

<u>Section 4-4</u>

4-27. $E(X) = \int_0^4 0.3x dx = 0.3 \dfrac{x^2}{2}\Big|_0^4 = 2.4$

$$V(X) = \int_0^4 0.3(x-2.4)^2 dx = 0.3 \frac{(x-2.4)^3}{3}\Big|_0^4 = 0.4096 + 1.3824 = 1.792$$

4-28. $E(X)=\int_0^4 0.13x^2dx = 0.13\frac{x^3}{3}\Big|_0^4 = 2.7733$

$V(X)=\int_0^4 0.13x(x-\tfrac{8}{3})^2dx = 0.13\int_0^4 (x^3-\tfrac{16}{3}x^2+\tfrac{64}{9}x)dx$

$=0.13(\tfrac{x^4}{4}-\tfrac{16}{3}\tfrac{x^3}{3}+\tfrac{64}{9}\cdot\tfrac{1}{2}x^2)\Big|_0^4 = 0.92444$

4-29. $E(X)=\int_{-1}^1 2x^3dx = 2\frac{x^4}{4}\Big|_{-1}^1 = 0$

$V(X)=\int_{-1}^1 2x^2(x-0)^2dx = 2\int_{-1}^1 x^4dx$

$=2\frac{x^5}{5}\Big|_{-1}^1 = 0.8$

4-30. $E(X)=\int_3^5 x\frac{x}{4}dx = \frac{x^3}{12}\Big|_3^5 = \frac{5^3-3^3}{12} = 8.167$

$V(X)=\int_3^5 (x-8.167)^2\frac{x}{4}dx = \int_3^5\left(\frac{x^3}{4}-\frac{16.334x^2}{4}+\frac{66.6999x}{4}\right)dx$

$=\frac{1}{4}\left(\frac{x^4}{4}-\frac{18.334x^3}{3}+\frac{66.6999x^2}{2}\right)\Bigg|_3^5 = 34.0055$

4-31. $E(X)=\int_0^8 x\frac{3(8x-x^2)}{256}dx = (\frac{x^3}{32}-\frac{3x^4}{1024})\Big|_0^8 = (16-12)-0 = 4$

$V(X)=\int_0^8 (x-4)^2\,\frac{3(8x-x^2)}{256}dx = \int_0^8\left(\frac{-3x^4}{256}+\frac{3x^3}{16}-\frac{15x^2}{16}+\frac{3x}{2}\right)dx$

$V(X)=\left(\frac{-3x^5}{1280}+\frac{3x^4}{64}-\frac{5x^3}{16}+\frac{3x^2}{4}\right)\Bigg|_0^8 = (\frac{-384}{5}+192-160+48) = 3.2$

4-32. $E(X)=\int_{2.3}^{2.9} 2xdx = x^2\Big|_{2.3}^{2.9} = 2.9^2-2.3^2 = 3.12$

$V(X)=\int_{2.3}^{2.9} 2(x-3.12)^2dx = \int_{2.3}^{2.9}(2x^2-12.48x+19.4688)dx = (\frac{2}{3}x^3-6.24x^2+19.4688x)\Big|_{2.3}^{2.9} = 0.36$

4-33. $E(X)=\int_1^{\infty} x*\frac{3}{2}x^{-3}dx = -\frac{3}{2}x^{-1}\Big|_1^{\infty} = \frac{3}{2}$

4-34. a)

$$E(X) = \int_{1710}^{1720} x0.1dx = 0.05x^2\Big|_{1710}^{1720} = 1715$$

$$V(X) = \int_{1710}^{1720} (x-1715)^2 0.1dx = 0.1\frac{(x-1715)^3}{3}\Bigg|_{1710}^{1720} = 8.333$$

$$Therefore, \quad \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1705 < X < 1715) = P(1710 < X < 1715) = \int_{1710}^{1715} 0.1dx = 0.1x\Big|_{1710}^{1715} = 0.5$$

4-35. a) $E(X) = \int_{100}^{120} x\frac{500}{x^2}dx = 500\ln x\Big|_{100}^{120} = 91.16$

$$V(X) = \int_{100}^{120} (x-91.16)^2 \frac{500}{x^2}dx = 500\int_{100}^{120} 1 - \frac{2(91.16)}{x} + \frac{(91.16)^2}{x^2}dx$$

$$= 500(x - 182.32\ln x - 91.16^2 x^{-1}) \Big|_{100}^{120} = 304.688$$

b.) Average cost per part = \$0.50*91.16 = \$45.58.

4-36. (a) $E(X) = \int_{0.5}^{35} xf(x)dx = \int_{0.5}^{35} \frac{70}{69x}dx = \frac{70}{69}\ln x \Big| \begin{matrix} 35 \\ 0.5 \end{matrix} = 4.3101$

$$E(X^2) = \int_{0.5}^{35} x^2 f(x)dx = \int_{0.5}^{35} \frac{70}{69}dx = 35$$

Var(X) = $E(X^2) - (EX)^2 = 35 - \ 18.5770 = 16.423$

(b) 3*4.3101 = 12.9303

(c) $P(X > 25) = \int_{25}^{35} f(x)dx = 0.0116$

4-37. a) $E(X) = \int_{5}^{\infty} x10e^{-10(x-5)}dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_5^\infty + \int_5^\infty e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_5^\infty = 5.1$$

Now, $V(X) = \int_5^\infty (x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_5^\infty + 2\int_5^\infty (x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X) the integral above is recognized to equal 0. Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

Section 4-5

4-38. a) E(X) = (5.5 + 2.5)/2 = 4

$$V(X) = \frac{(5.5-2.5)^2}{12} = 0.75, \text{ and } \sigma_x = \sqrt{0.75} = 0.866$$

b) $P(X < 2.5) = \int_{2.5}^{2.5} 0.25dx = 0.25x\Big|_{2.5}^{2.5} = 0$

c) $F(x) = \begin{cases} 0, & x < 2.5 \\ 0.25x - 0.375, & 2.5 \le x < 5.5 \\ 1, & 5.5 \le x \end{cases}$

4-39. a) E(X) = (-2+3)/2 = 0.5,

$$V(X) = \frac{(3-(-2))^2}{12} = 2.083, \; and \; \sigma_x = 1.443$$

b) $P(-x < X < x) = \int_{-x}^{x} \tfrac{1}{5}dt = 0.2t\Big|_{-x}^{x} = 0.2(2x) = 0.4x$,

$0.4x = 0.9$. Therefore, x should equal 2.25.

c) $F(x) = \begin{cases} 0, & x < -2 \\ 0.2x + 0.4, & -2 \le x < 3 \\ 1, & 3 \le x \end{cases}$

4.40. a) $f(x)$ = 2.0 for 49.75 < x < 50.25.

E(X) = (50.25 + 49.75)/2 = 50.0,

$$V(X) = \frac{(50.25-49.75)^2}{12} = 0.0208, \text{ and } \sigma_x = 0.144.$$

b) $F(x) = \int_{49.75}^{x} 2.0dy$ for 49.75 < x < 50.25. Therefore,

$$F(x) = \begin{cases} 0, & x < 49.75 \\ 2x - 99.5, & 49.75 \le x < 50.25 \\ 1, & 50.25 \le x \end{cases}$$

c) $P(X < 50.1) = F(50.1) = 2(50.1) - 99.5 = 0.7$

4-41. a) The distribution of X is $f(x) = 6.67$ for $0.90 < x < 1.05$. Now,

$$F_X(x) = \begin{cases} 0, & x < 0.90 \\ 6.67x - 6, & 0.90 \le x < 1.05 \\ 1, & 1.05 \le x \end{cases}$$

b) $P(X > 1.02) = 1 - P(X \le 1.02) = 1 - F_X(1.02) = 0.2$

c) If P(X > x) = 0.90, then 1 – F(X) = 0.90 and F(X) = 0.10. Therefore, 6.67x – 6 = 0.10 and x = 0.915.

d) E(X) = (1.05 + 0.9)/2 = 0.975 and V(X) = $\frac{(1.05-0.9)^2}{12} = 0.00188$

4-42. $E(X) = \frac{(3.7+4.4)}{2} = 4.05\,\text{min}$

$V(X) = \frac{(4.4-3.7)^2}{12} = 0.0408\,\text{min}^2$

b) $P(X < 4) = \int_{3.7}^{4} \frac{1}{(4.4-3.7)} dx = \int_{3.7}^{4} (1/0.7)dx = (1/0.7)x\Big|_{3.7}^{4} = (1/0.7)(0.3) = 0.429$

c.) $F(X) = \int_{3.7}^{x} \frac{1}{(4.4-3.7)} dy = \int_{3.7}^{x} (1/0.7)dy = (1/0.7)y\,\Big|_{3.7}^{x}$ for 3.7 < x < 4.4. Therefore,

$$F(x) = \begin{cases} 0, & x < 3.7 \\ (1/0.7)x - 5.29, & 3.7 \le x < 4.4 \\ 1, & 4.4 \le x \end{cases}$$

4-43. a) The distribution of X is $f(x) = 100$ for 0.2050 < x < 0.2150. Therefore,

$$F(x) = \begin{cases} 0, & x < 0.2050 \\ 100x - 20.50, & 0.2050 \le x < 0.2150 \\ 1, & 0.2150 \le x \end{cases}$$

b) $P(X > 0.2125) = 1 - F(0.2125) = 1 - [100(0.2125) - 20.50] = 0.25$

c) If P(X > x) = 0.20, then 1 – F(X) = 0.20 and F(X) = 0.80.
Therefore, 100x - 20.50 = 0.80 and x = 0.213.

d) E(X) = (0.2050 + 0.2150)/2 = 0.2100 μm and

V(X) = $\frac{(0.2150-0.2050)^2}{12} = 8.33 \times 10^{-6}\,\mu\text{m}^2$

4-44. Let X denote the changed weight.
Var(X) = $2^2/12$

Stdev(X) = 0.5774

4-45. (a) Let X be the time (in minutes) between arrival and 8:30 am.

$$f(x) = \frac{1}{90}, \quad for \quad 0 \le x \le 90$$

So the CDF is $F(x) = \frac{x}{90}, \quad for \quad 0 \le x \le 90$

(b) $E(X) = 45$, Var(X) = 90^2/12 = 675

(c) The event is an arrival in the intervals 8:45-9:00 am or 9:15-9:30 am or 9:45-10:00 am so that the probability = 45/90 = 1/2

(d) Similarly, the event is an arrival in the intervals 8:30-8:40 am or 9:00-9:10 am or 9:30-9:40 am so that the probability = 30/90 = 1/3

4-46. a) E(*X*) = (746 + 752)/2 = 749

$$V(X) = \frac{(752-746)^2}{12} = 3, \; and \; \sigma_x = 1.7321$$

b) Let *X* be the volume of a shampoo (milliliters)

$$P(X < 750) = \int_{746}^{750} \frac{1}{6} dx = \frac{1}{6} x \Big|_{746}^{750} = \frac{1}{6}(4) = 0.667$$

c) The distribution of *X* is *f*(*x*) = 1/6 for $374 \le x \le 380$.

Now, $F_X(x) = \begin{cases} 0, & x < 746 \\ (x-746)/6, & 746 \le x < 752 \\ 1, & 752 \le x \end{cases}$

P(*X* > *x*) = 0.95, then 1 – F(*X*) = 0.95 and F(*X*) = 0.05.
Therefore, (*x* - 746)/6 = 0.05 and *x* = 746.3

d) Since E(*X*) = 749, then the mean extra cost = (749-750) x \$0.002 = -\$0.002 per container.

4-47. (a) Let *X* be the arrival time (in minutes) after 9:00 A.M.

$$V(X) = \frac{(120-0)^2}{12} = 1200 \; and \; \sigma_x = 34.64$$

b) We want to determine the probability the message arrives in any of the following intervals: 9:10-9:15 A.M. or 9:40-9:45 A.M. or 10:10-10:15 A.M. or 10:40-10:45 A.M.. The probability of this event is 20/120 = 1/6.

c) We want to determine the probability the message arrives in any of the following intervals: 9:15-9:30 A.M. or 9:45-10:00 A.M. or 10:15-10:30 A.M. or 10:45-11:00 A.M. The probability of this event is 60/120 = 1/2.

4-48. (a) Let X denote the measured voltage.

So the probability mass function is $P(X = x) = \frac{1}{8}, \quad for \quad x = 246,...,254$

(b) E(X)=250, Var(X)= $\frac{(254-246+1)^2-1}{12} = 6.67$

<u>Section 4-6</u>

4-49. a) $P(Z<1.32) = 0.90658$
b) $P(Z<2.0) = 0.97725$
c) $P(Z>1.45) = 1 - 0.92647 = 0.07353$
d) $P(Z > -2.15) = p(Z < 2.15) = 0.98422$
e) $P(-2.34 < Z < 1.76) = P(Z<1.76) - P(Z > 2.34) = 0.95116$

4-50. a) $P(-1 < Z < 1) = P(Z < 1) - P(Z > 1)$
$= 0.84134 - (1 - 0.84134)$
$= 0.68268$
b) $P(-2 < Z < 2) = P(Z < 2) - [1 - P(Z < 2)]$
$= 0.9545$
c) $P(-3 < Z < 3) = P(Z < 3) - [1 - P(Z < 3)]$
$= 0.9973$
d) $P(Z > 2) = 1 - P(Z < 2)$
$= 0.02275$
e) $P(0 < Z < 1) = P(Z < 1) - P(Z < 0)$
$= 0.84134 - 0.5 = 0.34134$

4-51. a) $P(Z < 1.28) = 0.90$
b) $P(Z < 0) = 0.5$
c) If $P(Z > z) = 0.1$, then $P(Z < z) = 0.90$ and $z = 1.28$
d) If $P(Z > z) = 0.9$, then $P(Z < z) = 0.10$ and $z = -1.28$
e) $P(-1.28 < Z < z) = P(Z < z) - P(Z < -1.28)$
$= P(Z < z) - 0.100273.$
Therefore, $P(Z < z) = 0.8 + 0.100273 = 0.900273$ and $z = 1.29$

4-52. a) Because of the symmetry of the normal distribution, the area in each tail of the distribution must equal 0.05. Therefore the value in Table III that corresponds to 0.95 is 1.65. Thus, $z = 1.65$.
b) Find the value in Table III corresponding to 0.995. $z = 2.58$.
c) Find the value in Table III corresponding to 0.85. $z = 1.04$
d) Find the value in Table III corresponding to 0.99865. $z = 3.0$.

4-53. a) $P(X < 12) = P(Z < (12-10)/2)$
$= P(Z < 1)$
$= 0.841345$
b) $P(X > 9) = 1 - P(X < 9)$
$= 1 - P(Z < (9-10)/2)$
$= 1 - P(Z < -0.5)$
$= 0.69146.$
c) $P(6 < X < 14) = P\left(\frac{6-10}{2} < Z < \frac{14-10}{2}\right)$
$= P(-2 < Z < 2)$
$= P(Z < 2) - P(Z < -2)]$
$= 0.9545.$
d) $P(2 < X < 4) = P\left(\frac{2-10}{2} < Z < \frac{4-10}{2}\right)$

= P(–4 < Z < –3)
= P(Z < –3) – P(Z < –4)
= 0.00132

e) P(–2 < X < 8) = P(X < 8) – P(X < –2)

$= P\left(Z < \frac{8-10}{2}\right) - P\left(Z < \frac{-2-10}{2}\right)$

= P(Z < –1) – P(Z < –6)
= 0.15866.

4-54. a) P(X > x) = $P\left(Z > \frac{x-10}{2}\right)$ = 0.5. Therefore, = 0 and x = 10.

b) P(X > x) = $P\left(Z > \frac{x-10}{2}\right)$ = 1 – $P\left(Z < \frac{x-10}{2}\right)$

= 0.95.

Therefore, $P\left(Z < \frac{x-10}{2}\right)$ = 0.05 and $\frac{x-10}{2}$ = –1.64. Consequently, x = 6.72.

c) P(x < X < 10) = $P\left(\frac{x-10}{2} < Z < 0\right)$ = $P(Z < 0) - P\left(Z < \frac{x-10}{2}\right)$

= 0.5 – $P\left(Z < \frac{x-10}{2}\right)$ = 0.2.

Therefore, $P\left(Z < \frac{x-10}{2}\right)$ = 0.3 and $\frac{x-10}{2}$ = –0.52. Consequently, x = 8.96.

d) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2) = 0.90.
Therefore, x/2 = 1.65 and x = 3.3

e) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2) = 0.99.
Therefore, x/2 = 2.58 and x = 5.16

4-55. a) P(X < 11) = $P\left(Z < \frac{11-5}{4}\right)$ = P(Z < 1.5) = 0.93319

b) P(X > 2) = $P\left(Z > \frac{2-5}{4}\right)$ = P(Z > –0.75) = 1 – P(Z < –0.75) = 1-0.226627 = 1-0.226627

c) P(3 < X < 7) = $P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$ = P(–0.5 < Z < 0.5)

= P(Z < 0.5) – P(Z < –0.5) = 0.38292

d) P(–2 < X < 9) = $P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$ = P(–1.75 < Z < 1)

= P(Z < 1) – P(Z < –1.75)] = 0.80128

e) P(2 < X < 8) = $P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$ =P(–0.75 < Z < 0.75)

= P(Z < 0.75) – P(Z < –0.75) = 0.54674

4-56. a) P(X > x) = $P\left(Z > \frac{x-5}{4}\right)$ = 0.5. Therefore, x = 5.

b) P(X > x) = $P\left(Z > \frac{x-5}{4}\right)$ = 0.95. Therefore, $P\left(Z < \frac{x-5}{4}\right)$ = 0.05

Therefore, $\frac{x-5}{4}$ = –1.64, and x = –1.56.

c) P(x < X < 7) = $P\left(\frac{x-5}{4} < Z < \frac{7-5}{4}\right)$ = 0.2.

Therefore, P(Z < $\frac{7-5}{4}$) – P(Z < $\frac{x-5}{4}$)= 0.2 where P(Z < 0.5) = 0.691462.

Thus P(Z < $\frac{x-5}{4}$) = 0.491462. Consequently, $\frac{x-5}{4}$ = -0.02 and x = 4.92.

d) P(3 < X < x) = $P\left(\frac{3-5}{4} < Z < \frac{x-5}{4}\right)$ = 0.95.

Therefore, $P\left(Z < \frac{x-5}{4}\right)$ – P(Z < –0.5) = 0.95 and $P\left(Z < \frac{x-5}{4}\right)$ – 0.30854 = 0.95

Consequently,

$P\left(Z < \frac{x-5}{4}\right)$ = 1.25854.

Because a probability cannot be greater than one, there is no solution for x. In fact, P(3 < X) = P(–0.5 < Z) = 0.69146. Therefore, even if x is set to infinity the probability requested cannot equal 0.95.

e) *P*(– x < X – 5 < x) = *P*(5 – x < X < 5 + x) = $P\left(\frac{5-x-5}{4} < Z < \frac{5+x-5}{4}\right)$

= $P\left(\frac{-x}{4} < Z < \frac{x}{4}\right)$ = 0.99

Therefore, x/4 = 2.58 and x = 10.32.

4-57. a) P(X < 6050) = $P\left(Z < \frac{6050-6000}{100}\right)$ = P(Z < 0.5) = 0.691462

b) P(5800 < X < 5900) = $P\left(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}\right)$

= P(–2 < Z < –1) = P(Z <– 1) – P(Z < –2) = 0.13591

c) P(X > x) = $P\left(Z > \frac{x-6000}{100}\right)$ = 0.95. Therefore, $\frac{x-6000}{100}$ = –1.65 and x = 5835.

4-58. (a) Let X denote the time.

$X \sim N(290, 60^2)$

$$P(X > 300) = 1 - P(X \le 300) = 1 - \Phi(\frac{300-290}{60}) = 1 - \Phi(0.17) = 1 - 0.5675 = 0.4325$$

(b) $\Phi^{-1}(0.25) \times 60 + 290 = (-0.67) \times 60 + 290 = 249.8$

$\Phi^{-1}(0.75)\times 60+290=0.68\times 60+290=330.8$

(c) c

4-59. (a) $1-\Phi(1.5)=1-0.9332=0.0668$

b) Let X denote the time.

X ~ N(129, 14^2)

$$P(X<100)=\Phi\left(\frac{100-129}{14}\right)=\Phi(-2.0714)=0.01916$$

c) $\Phi^{-1}(0.95)\times 14+129=152.0280$

Here 95% of the surgeries will be finished within 152.028 minutes.

d) 199 >> 152.028 so the volume of such surgeries is very small (less than 5%).

4-60. Let X denote the cholesterol level.

X ~ N(159.2, σ^2)

a) $P(X<200)=\Phi(\frac{200-159.2}{\sigma})=0.841$

$$\frac{200-159.2}{\sigma}=\Phi^{-1}(0.841)$$

$$\sigma=\frac{200-159.2}{\Phi^{-1}(0.841)}=40.8582$$

b) $\Phi^{-1}(0.25)\times 40.8528+159.2=131.6452$

$\Phi^{-1}(0.75)\times 40.8528+159.2=186.7548$

c) $\Phi^{-1}(0.9)\times 40.8528+159.2=211.5550$

d) $\Phi(2)-\Phi(1)=0.1359$

e) $1-\Phi(2)=0.0228$

f) $\Phi(-1)=0.1587$

4-61. a) P(X > 0.61) = $P\left(Z>\frac{0.61-0.6}{0.055}\right)$

= P(Z > 0.18)

= 1 – P(Z <0.18)

= 0.4286

b) P(0.47 < X < 0.63) = $P\left(\frac{0.47-0.6}{0.055}<Z<\frac{0.63-0.6}{0.055}\right)$

= P(–2.36 < Z < 0.55)

= P(Z < 0.55) – P(Z < –2.36)

= 0.70884 – 0.009137

= 0.6997

c) P(X < x) = $P\left(Z<\frac{x-0.6}{0.055}\right)$ = 0.90.

Therefore, $\frac{x-0.6}{0.055}$ = 1.28 and x = 0.6704.

4-62. a) P(X < 360) = $P\left(Z < \frac{360-370}{5}\right)$ = P(Z < –2) $\cong 0.02275$

b) P(X < 365) = $P\left(Z < \frac{365-370}{5}\right)$ = P(Z < –1) = 0.158655

and

P(X > 375) = $P\left(Z > \frac{375-370}{5}\right)$ = P(Z > 1) = 0.158655.

Therefore, the proportion of cans scrapped is 0.158655 + 0.158655 = 0.31731, or 31.73%

c) P(370 – x < X < 370 + x) = 0.99.

Therefore, $P\left(-\frac{x}{5} < Z < \frac{x}{5}\right)$ = 0.99

Consequently, $P\left(Z < \frac{x}{5}\right)$ = 0.995 and x = 5(2.58) = 12.9.

The limits are (357.1, 382.9).

4-63. a) If P(X > 365) = 0.999, then $P\left(Z > \frac{365-\mu}{5}\right)$ = 0.999.

Therefore, $\frac{365-\mu}{5}$ = –3.09 and μ = 380.45.

b) If P(X > 365) = 0.999, then $P\left(Z > \frac{365-\mu}{2}\right)$ = 0.999.

Therefore, $\frac{365-\mu}{2}$ = –3.09 and μ = 371.18.

4-64. a) P(X > 0.5) = $P\left(Z > \frac{0.5-0.5}{0.05}\right)$

= P(Z > 0)
= 1 – 0.5
= 0.5

b) P(0.45 < X < 0.55) = $P\left(\frac{0.45-0.5}{0.05} < Z < \frac{0.55-0.5}{0.05}\right)$

= P(–1 < Z < 1)
= P(Z < 1) – P(Z < –1)
= 0.68269

c) P(X > x) = 0.90, then $P\left(Z > \frac{x-0.5}{0.05}\right)$ = 0.90.

Therefore, $\frac{x-0.5}{0.05}$ = –1.28 and x = 0.436.

4-65. a) P(X > 68) = $P\left(Z > \frac{68-60}{4}\right)$

$$= 1 - P(Z < 2)$$
$$= 1 - 0.97725 = 0.02275$$

b) $P(X < 55) = P\left(Z < \frac{55-60}{4}\right)$
$$= P(Z < -1.25)$$
$$= 0.10565$$

c) $1{,}000{,}000 \text{ bytes} * 8 \text{ bits/byte} = 8{,}000{,}000 \text{ bits}$

$$\frac{8{,}000{,}000 \text{ bits}}{60{,}000 \text{ bits/sec}} = 133.33 \text{ seconds}$$

4-66. Let X denote the height.
$X \sim N(160, 5^2)$

(a) $P(145 < X < 175) = \Phi\left(\frac{175-160}{5}\right) - \Phi\left(\frac{145-160}{5}\right) = \Phi(3) - \Phi(-3) = 0.9973$

(b) $\Phi^{-1}(0.25) \times 5 + 160 = 156.6275$
$\Phi^{-1}(0.75) \times 5 + 160 = 163.3725$

(c) $\Phi^{-1}(0.05) \times 5 + 160 = 151.77575$
$\Phi^{-1}(0.95) \times 5 + 160 = 168.22425$

(d) $\left[1 - \Phi\left(\frac{170-160}{5}\right)\right]^5 = [1 - \Phi(2)]^5 = 6.0942 \times 10^{-9}$

4-67. Let X denote the height.
$X \sim N(1.41, 0.01^2)$

(a) $P(X > 1.42) = 1 - P(X \le 1.42) = 1 - \Phi(\frac{1.42-1.41}{0.01}) = 1 - \Phi(1) = 0.1587$

(b) $\Phi^{-1}(0.05) \times 0.01 + 1.41 = 1.3936$

(c) $P(1.38 < X < 1.44) = \Phi\left(\frac{1.44-1.41}{0.01}\right) - \Phi\left(\frac{1.38-1.41}{0.01}\right) = \Phi(3) - \Phi(-3) = 0.9973$

4-68. Let X denote the demand for water daily.
$X \sim N(1170, 170^2)$

(a) $P(X > 1320) = 1 - P(X \le 1320) = 1 - \Phi\left(\frac{1320-1170}{170}\right) = 1 - \Phi\left(\frac{150}{170}\right) = 0.1894$

(b) $\Phi^{-1}(0.99) \times 170 + 1170 = 1566.1$

(c) $\Phi^{-1}(0.05) \times 170 + 1170 = 891.2$

(d) $X \sim N(\mu,\ 170^2)$

$$P(X > 1320) = 1 - P(X \le 1320) = 1 - \Phi\left(\frac{1320-\mu}{170}\right) = 0.01$$

$$\Phi\left(\frac{1320-\mu}{170}\right)=0.99$$

$$\mu=1320-\Phi^{-1}(0.99)\times 170=923.9$$

4-69. a) P(X < 6000) = $P\left(Z<\frac{6000-7000}{600}\right)$ = P(Z < −1.67) = 0.04746.

b) P(X > x) = 0.95. Therefore, $P\left(Z>\frac{x-7000}{600}\right)$ = 0.95 and $\frac{x-7000}{600}$ = −1.64

Consequently, x = 6016

c) P(X > 7000) = $P\left(Z>\frac{7000-7000}{600}\right)=P(Z>0)=0.5$

P(Three lasers operating after 7000 hours) = $(1/2)^3$ =1/8

4-70. a) P(X > 0.0065) = $P\left(Z>\frac{0.0065-0.005}{0.001}\right)$

= P(Z > 1.5)

= 1-P(Z < 1.5)

= 0.06681.

b) P(0.0035 < X < 0.0065) = $P\left(\frac{0.0035-0.005}{0.001}<Z<\frac{0.0065-0.005}{0.001}\right)$

= P(−1.5 < Z < 1.5)

= 0.86638.

c) P(0.0035 < X < 0.0065) = $P\left(\frac{0.0035-0.005}{\sigma}<Z<\frac{0.0065-0.005}{\sigma}\right)$

= $P\left(\frac{-0.0015}{\sigma}<Z<\frac{0.0015}{\sigma}\right)$.

Therefore, $P\left(Z<\frac{0.0015}{\sigma}\right)$= 0.9975. Therefore, $\frac{0.0015}{\sigma}$ = 2.81 and σ= 0.000534.

4-71. a) P(X > 0.37) = $P\left(Z>\frac{0.37-0.35}{0.015}\right)$ = P(Z > 1.33) = 0.0918

b) If P(X < 0.37) = 0.999, then $P\left(Z<\frac{0.37-0.35}{\sigma}\right)$ = 0.999.

Therefore, 0.02/σ= 3.09 and $\sigma=\frac{0.02}{3.09}=0.0065$

c) If P(X < 0.37) = 0.999, then $P\left(Z<\frac{0.37-\mu}{0.015}\right)$ = 0.999.

Therefore, $\dfrac{0.37-\mu}{0.015}$ = 3.09 and μ = 0.3237

4-72. a) Let X denote the measurement error, X ~ N(0, 0.5^2)

$P(166.5 < 165.5 + X < 167.5) = P(1 < X < 2)$

$P(1 < X < 2) = \Phi\left(\dfrac{2}{0.5}\right) - \Phi\left(\dfrac{1}{0.5}\right) = \Phi(4) - \Phi(2) \approx 1 - 0.977 = 0.023$

b) $P(169.5 < 165.5 + X) = P(4 < X)$

$P(4 < X) = 1 - \Phi(4) = 1 - 1 = 0$

4-73. From the shape of the normal curve, the probability is maximized for an interval symmetric about the mean. Therefore *a* = 23.5 with probability = 0.1974. The standard deviation does not affect the choice of interval.

4-74. a) P(X > 9) = $P\left(Z > \dfrac{9-7.1}{1.5}\right)$ = P (Z > 1.2667) = 0.1026

b) $P(4 < X < 8) = P(X < 8) - P(X < 4) = P\left(Z < \dfrac{8-7.1}{1.5}\right) - P\left(Z < \dfrac{4-7.1}{1.5}\right)$

= 0.7257 – 0.0192 = 0.7065.

c) P(X > x) = 0.05, then $\Phi^{-1}(0.95) \times 1.5 + 7.1 = 1.6449 \times 1.5 + 7.1 = 9.5673$

d) P(X > 9) = 0.01, then P(X < 9) = 1 – 0.01 = 0.99

$P\left(Z < \dfrac{9-\mu}{1.5}\right) = 0.99$. Therefore, $\dfrac{9-\mu}{1.5} = 2.33$ and μ = 5.51.

4-75. a) P(X > 800) = $P\left(Z > \dfrac{80-50.9}{25}\right)$ = P (Z > 1.164) = 0.1230

b) P(X < 25) = $P\left(Z < \dfrac{25-50.9}{25}\right)$ = P (Z < -1.036) = 0.1501

c) P(X > x) = 0.05, then $\Phi^{-1}(0.95) \times 25 + 50.9 = 1.6449 \times 25 + 50.9 = 92.0213$

4-76. a) P(X > 6) = $P\left(Z > \dfrac{6-4.6}{2.9}\right)$ = P (Z > 0.48) = 0.3156

b) P(X > x) = 0.25, then $\Phi^{-1}(0.75) \times 2.9 + 4.6 = 0.6745 \times 2.9 + 4.6 = 6.5560$

c) P(X < 0) = $P\left(Z < \dfrac{0-4.6}{2.9}\right)$ = P (Z < -1.5862) = 0.0563

The normal distribution is defined for all real numbers. In cases where the distribution is truncated (because wait times cannot be negative), the normal distribution may not be a good fit to the data.

Section 4-7

4-77. a) E(X) = 200(0.3) = 60, V(X) = 200(0.3)(0.7) = 42 and $\sigma_X = \sqrt{42}$

Then, $P(X \le 50) \cong P\left(Z \le \frac{50.5-60}{\sqrt{42}}\right) = P(Z \le -1.47) = 0.0708$

b) $P(50 < X < 70) \cong P\left(\frac{50.5-60}{\sqrt{42}} < Z \le \frac{69.5-60}{\sqrt{42}}\right) = P(-1.47 < Z \le 1.47)$

$= 0.9292 - 0.0708 = 0.8584$

c) $P(59.5 < X \le 60.5) \cong P\left(\frac{59.5-60}{\sqrt{42}} < Z \le \frac{60.5-60}{\sqrt{42}}\right) = P(-0.07715 < Z \le 0.07715)$

$= 0.0614$

4-78. a) $P(X < 4) = \sum_{i=0}^{3} \frac{e^{-6}6^i}{i!} = 0.1512$

b) X is approximately $X \sim N(6,6)$

Then, $P(X < 4) \cong P\left(Z < \frac{4-6}{\sqrt{6}}\right) = P(Z < -0.82) = 0.206108$

If a continuity correction were used the following result is obtained.

$P(X < 4) = P(X \le 3) \cong P\left(Z \le \frac{3+0.5-6}{\sqrt{6}}\right) = P(Z \le -1.02) = 0.1539$

c) $P(9 < X < 12) \cong P\left(\frac{9-6}{\sqrt{6}} < Z < \frac{12-6}{\sqrt{6}}\right) = P(1.22 < Z < 2.45) = 0.1040$

If a continuity correction were used the following result is obtained.

$P(9 < X < 12) = P(10 \le X \le 11) \cong P\left(\frac{10-0.5-6}{\sqrt{6}} \le Z \le \frac{11+0.5-6}{\sqrt{6}}\right)$

$\cong P(1.43 < Z < 2.25) = 0.0642$

4-79. $Z = \frac{X-64}{\sqrt{64}} = \frac{X-64}{8}$ is approximately N(0,1).

(a) $P(X > 76) = 1 - P(X \le 76) = 1 - P\left(Z \le \frac{76-64}{8}\right)$

$= 1 - P(Z \le 1.5) = 1 - 0.9332 = 0.0668$

If a continuity correction were used the following result is obtained.

$P(X > 76) = P(X \ge 77) \cong P\left(Z \ge \frac{77-0.5-64}{8}\right)$

$\cong P(Z \ge 1.56) = 1 - 0.9406 = 0.0594$

b) 0.5

If a continuity correction were used, the following result is obtained.

$P(X < 64) = P(X \le 63) \cong P\left(Z \le \frac{63+0.5-64}{8}\right) = P(Z \le -0.06) = 0.4761$

c)

$$P(60 < X \le 68) = P(X \le 68) - P(X \le 60) = \Phi(\frac{68-64}{8}) - \Phi(\frac{60-64}{8})$$
$$= \Phi(0.5) - \Phi(-0.5) = 0.3829$$

If a continuity correction were used, the following result is obtained.

$$P(60 < X \le 68) = P(61 \le X \le 68) \cong P\left(\frac{61-0.5-64}{8} < Z \le \frac{68+0.5-64}{8}\right)$$
$$= P(-0.44 < Z \le 0.56) = 0.3823$$

4-80. Let X denote the number of defective chips in the lot.
Then, E(X) = 1000(0.02) = 20, V(X) = 1000(0.02)(0.98) = 19.6.

a) $P(X > 23) \cong P\left(Z > \frac{23.5-20}{\sqrt{19.6}}\right) = P(Z > 0.79) = 1 - P(Z \le 0.79) = 0.215$

b) $P(20 < X < 30) \cong P(20.5 < X < 29.5) = P(\frac{.5}{\sqrt{19.6}} < Z < \frac{9.5}{\sqrt{19.6}}) = P(0.11 < Z < 2.15)$

$= 0.9842 - 0.5438 = 0.44$

4-81. Let X denote the number of people with a disability in the sample.
X ~ Bin(1000, 0.193)

$Z = \frac{X - 1000 \times 0.193}{\sqrt{193(1-0.193)}} = \frac{X-193}{12.4800}$ is approximately N(0,1).

a)

$$P(X > 200) = 1 - P(X \le 200) = 1 - P(X \le 200 + 0.5) = 1 - \Phi(\frac{200.5-193}{12.48}) = 1 - \Phi(0.6) = 0.2743$$

(b)

$$P(170 < X < 220) = P(171 \le X \le 219) = \Phi\left(\frac{219.5-193}{12.48}\right) - \Phi\left(\frac{170.5-193}{12.48}\right)$$
$$= \Phi(2.12) - \Phi(-1.82) = 0.9486$$

4-82. Let X denote the number of accounts in error in a month.
X ~ BIN(370,000, 0.001)
(a) E(X) = 370
Stdev(X) =19.2258

(b) $Z = \frac{X - 370000 \times 0.001}{\sqrt{370(1-0.001)}} = \frac{X-370}{19.2258}$ is approximately N(0,1).

$$P(X < 350) = P(X \le 349 + 0.5) = \Phi\left(\frac{349.5-370}{19.2258}\right) = \Phi(-1.0663) = 0.1423$$

(c) $P(X \le v) = 0.95$

$v = \Phi^{-1}(0.95) \times 19.2258 + 370 = 400.61$

(d)

$$P(X > 400) = 1 - P(X \le 400) = 1 - P(X \le 400 + 0.5) = 1 - \Phi\left(\frac{400.5-370}{19.2258}\right) = 1 - \Phi(1.5864) = 0.0559$$

Then the probability is $0.0559^2 = 3.125 \times 10^{-3}$.

4-83. Let X denote the number of original components that fail during the useful life of the product. Then, X is a binomial random variable with p = 0.001 and n = 5000. Also, E(X) = 5000 (0.001) = 5 and V(X) = 5000(0.001)(0.999) = 4.995.

$$P(X \geq 8) \cong P\left(Z \geq \frac{7.5-5}{\sqrt{4.995}}\right) = P(Z \geq 1.12) = 1 - P(Z < 1.12) = 1 - 0.869 = 0.131$$

4-84. Let X denote the number of errors on a web site. Then, X is a binomial random variable with p = 0.025 and n = 100. Also, E(X) = 100 (0.025) = 2.5 and V(X) = 100(0.025)(0.975) = 2.4375

$$P(X \geq 1) \cong P\left(Z \geq \frac{0.5-2.5}{\sqrt{2.4375}}\right) = P(Z \geq -1.28) = 1 - P(Z < -1.28) = 1 - 0.1003 = 0.8997$$

4-85. Let X denote the number of particles in $10\,\text{cm}^2$ of dust. Then, X is a Poisson random variable with $\lambda = 5(1000) = 5{,}000$. Also, E(X) = λ = 5000 and V(X) = λ = 5000

$$P(X > 5000) = 1 - P(X \leq 5000) \cong 1 - P\left(Z \leq \frac{5000-5000}{\sqrt{5000}}\right) \cong 1 - P(Z \leq 0) \cong 0.5$$

If a continuity correction were used the following result is obtained.

$$P(X > 5000) = P(X \geq 5001) \cong P\left(Z > \frac{5001-0.5-5000}{\sqrt{5000}}\right) \cong P(Z > 0) \cong 0.5$$

4-86. X is the number of minor errors on a test pattern of 1000 pages of text. X is a Poisson random variable with a mean of 0.4 per page

a) The numbers of errors per page are random variables. The assumption that the occurrence of an event in a subinterval in a Poisson process is independent of events in other subintervals implies that the numbers of events in disjoint intervals are independent. The pages are disjoint intervals and the consequently the error counts per page are independent.

b) $P(X = 0) = \dfrac{e^{-0.4}\,0.4^0}{0!} = 0.670$

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.670 = 0.330$

The mean number of pages with one or more errors is 1000(0.330) = 330 pages

c) Let Y be the number of pages with errors.

$$P(Y > 360) \cong P\left(Z \geq \frac{360.5-330}{\sqrt{1000(0.330)(0.670)}}\right) = P(Z \geq 2.05) = 1 - P(Z < 2.05)$$
$$= 1 - 0.9798 = 0.0202$$

4-87. Let X denote the number of hits to a web site. Then, X is a Poisson random variable with a mean of 10,000 hits per day. Also, V(X) = 10,000.

a)

$$P(X > 20{,}000) = 1 - P(X \leq 20{,}000) \cong 1 - P\left(Z \leq \frac{20{,}000-10{,}000}{\sqrt{10{,}000}}\right)$$
$$= 1 - P(Z \leq 100) \approx 1 - 1 = 0$$

If a continuity correction were used, the following result is obtained.

$$P(X > 20{,}000) = P(X \ge 20{,}001) \cong P\left(Z \ge \frac{20{,}001 - 0.5 - 10{,}000}{\sqrt{10{,}000}}\right)$$
$$= P(Z \ge 100.005) \approx 1 - 1 = 0$$

b) $P(X < 9{,}800) = P(X \le 9{,}799) = P\left(Z \le \frac{9{,}799 - 10{,}000}{\sqrt{10{,}000}}\right) = P(Z \le -2.01) = 0.0222$

If a continuity correction were used the following result is obtained.

$$P(X < 9{,}800) = P(X \le 9{,}799) \cong P\left(Z < \frac{9{,}799 + 0.5 - 10{,}000}{\sqrt{10{,}000}}\right) \approx P(Z \ge -2.01) = 0.0222$$

c) If P($X > x$) = 0.01, then $P\left(Z > \frac{x - 10{,}000}{\sqrt{10{,}000}}\right) = 0.01$.

Therefore, $\frac{x - 10{,}000}{\sqrt{10{,}000}} = 2.33$ and $x = 10{,}233$

d) Let X denote the number of hits to a web site. Then, X is a Poisson random variable with a mean of 10,000 per day. Therefore, E(X) =10,000 and V(X) = 10,000

$$P(X > 10{,}200) \cong P\left(Z \ge \frac{10{,}200 - 10{,}000}{\sqrt{10{,}000}}\right) = P(Z \ge 2) = 1 - P(Z < 2)$$
$$= 1 - 0.97725 = 0.02275$$

If a continuity correction were used, we obtain the following result

$$P(X > 10{,}200) \cong P\left(Z \ge \frac{10{,}200.5 - 10{,}000}{\sqrt{10{,}000}}\right) = P(Z \ge 2.005) = 1 - P(Z < 2.005)$$

and this approximately equals the result without the continuity correction.

The expected number of days with more than 10,200 hits is (0.02275)*365 = 8.30 days per year.

e) Let Y denote the number of days per year with over 10,200 hits to a web site.
Then, Y is a binomial random variable with n = 365 and p = 0.02275.
E(Y) = 8.30 and V(Y) = 365(0.02275)(0.97725) = 8.28

$$P(Y > 15) \cong P\left(Z \ge \frac{15.5 - 8.30}{\sqrt{8.28}}\right) = P(Z \ge 2.56) = 1 - P(Z < 2.56)$$
$$= 1 - 0.9948 = 0.0052$$

4-88. Let X denotes the number of random sets that is more dispersed than the opteron.
Assume that X has a true mean = 0.5 x 1000 = 500 sets.

$$P(X \geq 750) \cong P\left(Z > \frac{750.5 - 1000(0.5)}{\sqrt{0.5(0.5)1000}}\right) = \left(Z > \frac{750.5 - 500)}{\sqrt{250}}\right)$$
$$= P(Z > 15.84) = 1 - P(Z \leq 15.84) \approx 0$$

4-89. With 10,500 asthma incidents in children in a 21-month period, then mean number of incidents per month is 10500/21 = 500. Let X denote a Poisson random variable with a mean of 500 per month. Also, E(X) = λ = 500 = V(X).

a) Using a continuity correction, the following result is obtained.

$$P(X > 550) \cong P\left(Z \geq \frac{550 + 0.5 - 500}{\sqrt{500}}\right) = P(Z \geq 2.2584) = 1 - 0.9880 = 0.012$$

Without the continuity correction, the following result is obtained

$$P(X > 550) \cong P\left(Z \geq \frac{550 - 500}{\sqrt{500}}\right) = P(Z \geq 2.2361)$$
$$= 1 - P(Z < 2.2361) = 1 - 0.9873 = 0.0127$$

b) Using a continuity correction, the following result is obtained.

$$P(500 < X < 550) = P(501 \leq X \leq 549) = P\left(\frac{500.5 - 500}{\sqrt{500}} \leq Z \leq \frac{549.5 - 500}{\sqrt{500}}\right)$$
$$= P(Z \leq 2.21) - P(Z \leq 0.02) = 0.9864 - 0.5080 = 0.4784$$
$$P(500 < X < 550) = P(X < 550) - P(X < 500) = P\left(Z \leq \frac{550 - 500}{\sqrt{500}}\right) - P\left(Z \leq \frac{500 - 500}{\sqrt{500}}\right)$$
$$= P(Z \leq 2.24) - P(Z \leq 0) = 0.9875 - 0.5 = 0.4875$$

c) $P(X \leq x) = 0.95$

$$x = \Phi^{-1}(0.95) \times \sqrt{500} + 500 = 536.78$$

d) The Poisson distribution would not be appropriate because the rate of events should be constant for a Poisson distribution.

Section 4-8

4-90. a) $P(X \leq 0) = \int_0^0 \lambda e^{-\lambda x} dx = 0$

b) $P(X \geq 2) = \int_2^\infty 3e^{-3x} dx = -e^{-3x}\Big|_2^\infty = e^{-6} = 0.0025$

c) $P(X \leq 1) = \int_0^1 3e^{-3x} dx = -e^{-3x}\Big|_0^1 = 1 - e^{-3} = 0.9502$

d) $P(1 < X < 2) = \int_1^2 3e^{-3x} dx = -e^{-3x}\Big|_1^2 = e^{-3} - e^{-6} = 0.0473$

e) $P(X \le x) = \int_0^x 3e^{-3t} dt = -e^{-3t} \Big|_0^x = 1 - e^{-3x} = 0.05$ and x = 0.0171

4-91. If E(X) = 10, then $\lambda = 0.1$.

a) $P(X > 10) = \int_{10}^{\infty} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_{10}^{\infty} = e^{-1} = 0.3679$

b) $P(X > 20) = -e^{-0.1x} \Big|_{20}^{\infty} = e^{-2} = 0.1353$

c) $P(X < 30) = -e^{-0.1x} \Big|_0^{30} = 1 - e^{-3} = 0.9502$

d) $P(X < x) = \int_0^x 0.1e^{-0.1t} dt = -e^{-0.1t} \Big|_0^x = 1 - e^{-0.1x} = 0.90$ and x = 23.03.

4-92. (a) $P(X < 4) = 1 - e^{-0.1\times 4} = 0.330$

(b) $$P(X < 14 \mid X > 10) = \frac{P(X < 14, X > 10)}{P(X > 10)} = \frac{P(X < 14) - P(X < 10)}{1 - P(X < 10)}$$
$$= \frac{(1 - e^{-0.1\times 14}) - (1 - e^{-0.1\times 10})}{e^{-0.1\times 10}} = 0.330$$

(c) They are the same.

4-93. Let X denote the time until the first count. Then, X is an exponential random variable with $\lambda = 2$ counts per minute.

a) $P(X > 1) = \int_1^{\infty} 2e^{-2x} dx = -e^{-2x} \Big|_1^{\infty} = e^{-2} = 0.1353$

b) $P(X < \frac{10}{60}) = \int_0^{1/6} 2e^{-2x} dx = -e^{-2x} \Big|_0^{1/6} = 1 - e^{-1/3} = 0.2835$

c) $P(1 < X < 2) = -e^{-2x} \Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

4-94. a) E(X) = $1/\lambda$ = 1/4 = 0.25 minutes
b) V(X) = $1/\lambda^2 = 1/4^2$ = 0.0625, σ = 0.25

c) $P(X < x) = \int_0^x 4e^{-4t} dt = -e^{-4t} \Big|_0^x = 1 - e^{-4x} = 0.95$, x = 0.749

4-95. Let X denote the time until the first call. Then, X is exponential and $\lambda = \frac{1}{E(X)} = \frac{1}{15}$ calls/minute.

a) $P(X > 30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{x}{15}} dx = -e^{-\frac{x}{15}} \Big|_{30}^{\infty} = e^{-2} = 0.1353$

b) The probability of at least one call in a 15-minute interval equals one minus the probability of zero calls in a 10-minute interval and that is P(X > 10).

$$P(X>15)=-e^{-\frac{x}{15}}\Big|_{15}^{\infty}=e^{-1}=0.368.$$

Therefore, the answer is 1- 0.368 = 0.632. Alternatively, the requested probability is equal to P(X < 15) = 0.632.

c) $P(5<X<10)=-e^{-\frac{x}{15}}\Big|_{5}^{10}=e^{-1/3}-e^{-2/3}=0.2031$

d) P(X < x) = 0.90 and $P(X<x)=-e^{-\frac{t}{15}}\Big|_{0}^{x}=1-e^{-x/15}=0.90$. Therefore, x = 34.54 minutes.

4-96. Let X be the life of regulator. Then, X is an exponential random variable with $\lambda=1/E(X)=1/5$

a) Because the Poisson process from which the exponential distribution is derived is memoryless, this probability is

$$P(X<5)=\int_0^5 \tfrac{1}{5}e^{-x/5}dx=-e^{-x/5}\Big|_0^5=1-e^{-1}=0.6321$$

b) Because the failure times are memoryless, the mean time until the next failure is E(X) = 5 years.

4-97. Let X denote the time to failure (in hours) of fans in a personal computer. Then, X is an exponential random variable and $\lambda=1/E(X)=0.0004$.

a) $P(X>10{,}000)=\int_{10,000}^{\infty}0.0004e^{-x0.0004}dx=-e^{-x0.0004}\Big|_{10,000}^{\infty}=e^{-4}=0.0183$

b) $P(X<7{,}000)=\int_{0}^{7,000}0.0004e^{-x0.0004}dx=-e^{-x0.0004}\Big|_{0}^{7,000}=1-e^{-2.8}=0.9392$

4-98. Let X denote the time until a message is received. Then, X is an exponential random variable and $\lambda=1/E(X)=1/3$.

a) $P(X>3)=\int_3^{\infty}\tfrac{1}{3}e^{-x/3}dx=-e^{-x/3}\Big|_3^{\infty}=e^{-1}=0.3679$

b) The same as part a.

c) E(X) = 3 hours.

4-99. Let X denote the time until the arrival of a taxi. Then, X is an exponential random variable with $\lambda=1/E(X)=0.1$ arrivals/ minute.

a) $P(X>60)=\int_{60}^{\infty}0.1e^{-0.1x}dx=-e^{-0.1x}\Big|_{60}^{\infty}=e^{-6}=0.0025$

b) $P(X<10)=\int_{0}^{10}0.1e^{-0.1x}dx=-e^{-0.1x}\Big|_{0}^{10}=1-e^{-1}=0.6321$

c) $P(X>x)=\int_{x}^{\infty}0.1e^{-0.1t}dt=-e^{-0.1t}\Big|_{x}^{\infty}=e^{-0.1x}=0.15$ and x = 18.97 minutes.

d) P(X < x) = 0.9 implies that P(X > x) = 0.1. Therefore, this answer is the same as part c).

e) $P(X<x)=-e^{-0.1t}\Big|_{0}^{x}=1-e^{-0.1x}=0.5$ and x = 6.93 minutes.

4-100. (a) 1/2.5 = 0.4 per year

(b) $\lambda = 2.5 \times 0.25 = 0.625$

P(X=0) = 0.5353

(c)Let T denote the time between sightings

$T \sim EXP(0.4)$

$P(X > 0.5) = 1 - P(X \le 0.5) = 0.2865$

(d) $\lambda = 2.5 \times 3 = 7.5$

P(X=0) = 0.000553

4-101. Let X denote the number of insect fragments per gram. Then

$X \sim POI(14.4/225)$

a) 225/14.4 = 15.625

b) $P(X = 0) = \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-\frac{14.4 \times 28.35}{225}} = 0.1629$

c) $(0.1629)^7 = 3 \times 10^{-6}$

4-102. Let X denote the distance between major cracks. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.1$ cracks/km.

a) P(X > 20) = $\int_{20}^{\infty} 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_{20}^{\infty} = e^{-2} = 0.1353$

b) Let Y denote the number of cracks in 20 km of highway. Because the distance between cracks is exponential, Y is a Poisson random variable with $\lambda = 20(0.1) = 2$ cracks per 20 km.

P(Y = 2) = $\dfrac{e^{-2}2^2}{2!} = 0.2707$

c) $\sigma_X = 1/\lambda = 0.5$

d) $P(24 < X < 30) = \int_{24}^{30} 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_{24}^{30} = e^{-2.4} - e^{-3} = 0.0409$

e) P(X > 10) = $-e^{-0.1x}\Big|_{10}^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process, the answer is $0.3679^2 = 0.1353$. Alternatively, the answer is P(X > 20) = $e^{-2} = 0.1353$. The probability does depend on whether or not the lengths of highway are consecutive.

f) By the memoryless property, this answer is P(X > 20) = 0.1353 from part e).

4-103. Let X denote the lifetime of an assembly. Then, X is an exponential random variable with $\lambda = 1/E(X) = 1/400$ failures per hour.

a) P(X < 200) = $\int_{0}^{200} \frac{1}{400}e^{-x/400}dx = -e^{-x/400}\Big|_{0}^{200} = 1 - e^{-0.5} = 0.393$

b) P(X > 500) = $-e^{-x/400}\Big|_{500}^{\infty} = e^{-5/4} = 0.2865$

c) From the memoryless property of the exponential, this answer is the same as part a.,

P(X < 100) = 0.2212.

d) Let U denote the number of assemblies out of 10 that fail before 100 hours. By the memoryless property of a Poisson process, U has a binomial distribution with n = 10 and p = 0.2212 from part a). Then,

$$P(U \geq 1) = 1 - P(U = 0) = 1 - \binom{10}{0} 0.2212^0 (1 - 0.2212)^{10} = 0.9179$$

e) Let V denote the number of assemblies out of 10 that fail before 800 hours. Then, V is a binomial random variable with n = 10 and p = P(X < 800), where X denotes the lifetime of an assembly.

Now, P(X < 800) = $\int_0^{800} \frac{1}{400} e^{-x/400} dx = -e^{-x/400} \Big|_0^{800} = 1 - e^{-2} = 0.8647$.

Therefore, P(V = 10) = $\binom{10}{10} 0.8647^{10} (1 - 0.8647)^0 = 0.2337$.

4-104. Let Y denote the number of arrivals in one hour. If the time between arrivals is exponential, then the count of arrivals is a Poisson random variable and $\lambda = 1$ arrival per hour.

a) P(Y > 2) = $1 - P(Y \leq 2) = 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} + \frac{e^{-1} 1^2}{2!} \right] = 0.0803$

b) From part a), P(Y > 2) = 0.0803. Let W denote the number of one-hour intervals out of 30 that contain more than 2 arrivals. By the memoryless property of a Poisson process, W is a binomial random variable with n = 30 and p = 0.0803.

P(W = 0) = $\binom{30}{0} 0.0803^0 (1 - 0.0803)^{30} = 0.0812$

c) Let X denote the time between arrivals. Then, X is an exponential random variable with $\lambda = 1$ arrivals per hour.

P(X > x) = 0.1 and $P(X > x) = \int_x^{\infty} 1 e^{-1t} dt = -e^{-1t} \Big|_x^{\infty} = e^{-1x} = 0.1$. Therefore, x = 2.3 hours.

4-105. Let X denote the number of calls in 30 minutes. Because the time between calls is an exponential random variable, X is a Poisson random variable with $\lambda = 1/E(X) = 0.1$ calls per minute = 3 calls per 30 minutes.

a) P(X > 3) = $1 - P(X \leq 3) = 1 - \left[\frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!} + \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!} \right] = 0.3528$

b) P(X = 0) = $\frac{e^{-3} 3^0}{0!} = 0.04979$

c) Let Y denote the time between calls in minutes. Then, $P(Y \geq x) = 0.02$ and

$P(Y \geq x) = \int_x^{\infty} 0.1 e^{-0.1t} dt = -e^{-0.1t} \Big|_x^{\infty} = e^{-0.1x}$. Therefore, $e^{-0.1x} = 0.02$ and x = 39.12 minutes.

d) P(Y > 120) = $\int_{120}^{\infty} 0.1 e^{-0.1y} dy = -e^{-0.1y} \Big|_{120}^{\infty} = e^{-12} = 6.14 \times 10^{-6}$.

e) Because the calls are a Poisson process, the numbers of calls in disjoint intervals are independent. From Exercise 4-90 part b), the probability of no calls in one-half hour is $e^{-3} = 0.04979$. Therefore, the answer is $\left[e^{-3}\right]^4 = e^{-12} = 6.14 \times 10^{-6}$. Alternatively, the answer is the probability of no calls in two hours. From part d) of this exercise, this is e^{-12}.

f) Because a Poisson process is memoryless, probabilities do not depend on whether or not intervals are consecutive. Therefore, parts d) and e) have the same answer.

4-106. X is an exponential random variable with $\lambda = 0.1$ flaws per meter.

a) E(X) = 1/ $\lambda = 10$ meters.

b) P(X > 10) = $\int_{10}^{\infty} 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_{10}^{\infty} = e^{-1} = 0.368$

c) No

d) P(X < x) = 0.90. Then, P(X < x) = $-e^{-0.1t}\Big|_{0}^{x} = 1 - e^{-0.1x}$.

Therefore, $1 - e^{-0.1x} = 0.9$ and x = 23.03.

P(X > 8) = $\int_{8}^{\infty} 0.1e^{-0.1x}dx = e^{-0.8} = 0.449$

The distance between successive flaws is either less than 8 meters or not. The distances are independent and P(X > 8) = 0.449. Let Y denote the number of flaws until the distance exceeds 8 meters. Then, Y is a geometric random variable with p = 0.449.

e) P(Y = 5) = $(1 - 0.449)^4 0.449 = 0.041$.

f) E(Y) = 1/0.449 = 2.23.

4-107. a) $P(X > \theta) = \int_{\theta}^{\infty} \frac{1}{\theta} e^{-x/\theta}dx = -e^{-x/\theta}\Big|_{\theta}^{\infty} = e^{-1} = 0.3679$

b) $P(X > 2\theta) = -e^{-x/\theta}\Big|_{2\theta}^{\infty} = e^{-2} = 0.1353$

c) $P(X > 3\theta) = -e^{-x/\theta}\Big|_{3\theta}^{\infty} = e^{-3} = 0.0498$

d) The results do not depend on θ.

4-108. $E(X) = \int_{0}^{\infty} x\lambda e^{-\lambda x}dx.$ Use integration by parts with u = x and dv = $\lambda e^{-\lambda x}$.

Then, $E(X) = -xe^{-\lambda x}\Big|_{0}^{\infty} + \int_{0}^{\infty} e^{-\lambda x}dx = \frac{-e^{-\lambda x}}{\lambda}\Big|_{0}^{\infty} = 1/\lambda$

V(X) = $\int_{0}^{\infty} (x - \frac{1}{\lambda})^2 \lambda e^{-\lambda x}dx.$ Use integration by parts with $u = (x - \frac{1}{\lambda})^2$ and

dv = $\lambda e^{-\lambda x}$. Then,

$V(X) = -(x - \frac{1}{\lambda})^2 e^{-\lambda x}\Big|_{0}^{\infty} + 2\int_{0}^{\infty} (x - \frac{1}{\lambda})e^{-\lambda x}dx = (\frac{1}{\lambda})^2 + \frac{2}{\lambda}\int_{0}^{\infty} (x - \frac{1}{\lambda})\lambda e^{-\lambda x}dx$

The last integral is seen to be zero from the definition of E(X). Therefore, V(X) = $\left(\frac{1}{\lambda}\right)^2$.

4-109. X is an exponential random variable with μ = 3.5 days.

a) P(X < 2) = $\int_0^2 \frac{1}{3.5} e^{-x/3.5} dx = 1 - e^{-2/3.5} = 0.435$

b) $P(X > 7) = \int_7^\infty \frac{1}{3.5} e^{-x/3.5} dx = e^{-7/3.5} = 0.135$

c) $P(X > x) = 0.9$ and $P(X > x) = e^{-x/3.5} = 0.9$
Therefore, x = –3.5ln(0.9) = 0.369

d) From the lack of memory property P(X < 10 | X > 3) = P(X < 7) and from part (b) this equals 1 – 0.135 = 0.865

4-110. a) $\mu = E(X) = \frac{1}{\lambda} = 4.6$, then $\lambda = 0.2174$

$\sigma = \frac{1}{\lambda} = 4.6$

b) $P(X > 10) = \int_{10}^\infty \frac{1}{4.6} e^{-x/4.6} dx = e^{-10/4.6}\Big| = 0.1137$

c) $P(X > x) = \int_x^\infty \frac{1}{4.6} e^{-u/4.6} du = e^{-x/4.6} = 0.25$
Then, x = -4.6ln(0.25) = 6.38

<u>Section 4-9</u>

4-111. a) $\Gamma(7) = 6! = 720$

b) $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\pi^{1/2} = 1.32934$

c) $\Gamma\left(\frac{9}{2}\right) = \frac{7}{2}\Gamma\left(\frac{7}{2}\right) = \frac{7}{2}\frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{105}{16}\pi^{1/2} = 11.6317$

4-112. X is a gamma random variable with the parameters $\lambda = 0.5$ and $r = 2$.
The mean is $E(X) = r/\lambda = 2/0.5 = 4$.
The variance is $Var(X) = r/\lambda^2 = 2/0.5^2 = 8$.

4-113. a) The time until the tenth call is an Erlang random variable with $\lambda = 6$ calls per minute and r = 12.
b) E(X) = 12/6= 2 minutes. V(X) = 12/36 = 0.33 minutes.
c) Because a Poisson process is memoryless, the mean time is 1/6=0.167 minutes or 10 seconds

Let Y denote the number of calls in one minute. Then, Y is a Poisson random variable with 6 calls per minute.

d) P(Y = 4) = $\dfrac{e^{-6}6^4}{4!} = 0.134$

e) P(Y > 2) = 1 - $P(Y \le 2) = 1 - \dfrac{e^{-6}6^0}{0!} - \dfrac{e^{-6}6^1}{1!} - \dfrac{e^{-6}6^2}{2!} = 0.938$

Let W denote the number of one minute intervals out of 10 that contain more than 2 calls. Because the calls are a Poisson process, W is a binomial random variable with n = 10 and p = 0.938.

Therefore, P(W = 10) = $\binom{10}{10} 0.938^{10}(1-0.938)^0 = 0.527$

4-114. Let X denote the kilograms of material to obtain 15 particles. Then, X has an Erlang distribution with r = 15 and $\lambda = 0.02$.

a) E(X) = $\dfrac{r}{\lambda} = \dfrac{15}{0.02} = 750$

b) V(X) = $\dfrac{15}{0.02^2} = 37500$ and $\sigma_X = \sqrt{37500} = 193.65$ kg

4-115. Let X denote the time between failures of a laser. X is exponential with a mean of 20,000.

a.) Expected time until the second failure $E(X) = r/\lambda = 2/0.00005 = 40{,}000$ hours

b.) N=no of failures in 40000 hours

$$E(N) = \frac{40000}{20000} = 2$$

$$P(N \le 2) = \sum_{k=0}^{2} \frac{e^{-2}(2)^k}{k!} = 0.6767$$

4-116. Let X denote the time until 5 messages arrive at a node. Then, X has an Erlang distribution with r = 5 and $\lambda = 25$ messages per minute.

a) E(X) = 5/25 = 1/5 minute = 12 seconds.

b)V(X) = $\dfrac{5}{25^2} = 0.008$ minute^2 = 0.4 second and $\sigma_X = 0.089$ minute = 5.37seconds.

c) Let Y denote the number of messages that arrive in 12 seconds. Then, Y is a Poisson random variable with $\lambda = 25$ messages per minute = 5 messages per 12 seconds.

$$P(Y \ge 5) = 1 - P(Y \le 4) = 1 - \left[\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!} + \frac{e^{-5}5^4}{4!}\right]$$

$$= 0.5595$$

d) Let Y denote the number of messages that arrive in 6 seconds. Then, Y is a Poisson random variable with $\lambda = 2.5$ messages per 6 seconds.

$$P(Y \ge 5) = 1 - P(Y \le 4) = 1 - 0.8912 = 0.1088$$

4-117. Let X denote the number of bits until three errors occur. Then, X has an Erlang distribution with r = 3 and $\lambda = 10^{-5}$ error per bit.

a) E(X) = $\dfrac{r}{\lambda} = 3 \times 10^5$ bits.

b) V(X) = $\dfrac{r}{\lambda^2} = 3 \times 10^{10}$ and $\sigma_X = \sqrt{3 \times 10^{10}} = 173205.1$ bits.

c) Let Y denote the number of errors in 10^5 bits. Then, Y is a Poisson random variable with

$\lambda = 1/10^5 = 10^{-5}$ error per bit = 1 error per 10^5 bits.

$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \left[\frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} + \frac{e^{-1}1^2}{2!}\right] = 0.0803$

4-118. $\lambda = 20 \quad r = 100$

a) $E(X) = r/\lambda = 100/20 = 5$ minutes

b) 4 min – 3 min = 1 min

c) Let Y be the number of calls before 15 seconds $\lambda = 0.25*20 = 5$

$$P(Y \geq 3) = 1 - P(X \leq 2) = 1 - \left[\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!}\right] = 1 - .1247 = 0.8753$$

4-119. a) Let X denote the number of customers that arrive in 10 minutes. Then, X is a Poisson random variable with $\lambda = 0.2$ arrivals per minute = 2 arrivals per 10 minutes.

$P(X > 2) = 1 - P(X \leq 2) = 1 - \left[\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}\right] = 0.3233$

b) Let Y denote the number of customers that arrive in 15 minutes. Then, Y is a Poisson random variable with $\lambda = 3$ arrivals per 15 minutes.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!}\right] = 0.1847$$

4-120. $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$$\Gamma(r) = -x^{r-1}e^{-x}\Big|_0^\infty + (r-1)\int_0^\infty x^{r-2}e^{-x}dx = (r-1)\Gamma(r-1).$$

4-121. $\int_0^\infty f(x;\lambda,r)dx = \int_0^\infty \frac{\lambda^r x^{r-1}e^{-\lambda x}}{\Gamma(r)}dx$. Let y = λx, then the integral is $\int_0^\infty \frac{\lambda\, y^{r-1}e^{-y}}{\Gamma(r)}\frac{dy}{\lambda}$. From the definition of $\Gamma(r)$, this integral is recognized to equal 1.

4-122. If X is a chi-square random variable, then X is a special case of a gamma random variable. Now,

E(X) = $\frac{r}{\lambda} = \frac{(7/2)}{(1/2)} = 7$ and $V(X) = \frac{r}{\lambda^2} = \frac{(7/2)}{(1/2)^2} = 14$

4-123. Let X denote the number of patients arrive at the emergency department. Then, X has a Poisson distribution with $\lambda = 6.5$ patients per hour.

a) $E(X) = r/\lambda = 10/6.5 =$ 1.539 hour.

b) Let Y denote the number of patients that arrive in 20 minutes. Then, Y is a Poisson random variable with E(Y) = 6.5/3 = 2.1667 arrivals per 20 minutes. The event that the third arrival exceeds 20 minutes is equivalent to the event that there are two or fewer arrivals in 20 minutes. Therefore,

$$P(Y \leq 2) = \left[\frac{e^{-2.1667}2.1667^0}{0!} + \frac{e^{-2.1667}2.1667^1}{1!} + \frac{e^{-2.1667}2.1667^2}{2!}\right] = 0.6317$$

The solution may also be obtained from the result that the time until the third arrival follows a gamma distribution with r = 3 and λ = 6.5 arrivals per hour. The probability is obtained by integrating the probability density function from 20 minutes to infinity.

4-124. a) $E(X) = r/\lambda = 18$, then $r = 18\lambda$

$Var(X) = r/\lambda^2 = 18/\lambda = 36$, then $\lambda = 0.5$

Therefore, the parameters are $\lambda = 0.5$ and $r = 18\lambda = 18(0.5) = 9$

b) The distribution of each step is exponential with λ = 0.5 and 9 steps produce this gamma distribution.

Section 4-10

4-125. β=0.5 and δ=80 hours

$$E(X) = 80\Gamma(1+\tfrac{1}{0.5}) = 80 \times 2! = 160$$

$$V(X) = 80^2\Gamma(1+\tfrac{2}{0.5}) - 80^2[\Gamma(1+\tfrac{1}{0.5})]^2 = 128{,}000$$

4-126. a) $P(X < 10000) = F_X(10000) = 1 - e^{-(100^{0.3})} = 1 - e^{-3.981} = 0.9813$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-(50^{0.3})} = 0.0394$

4-127. If X is a Weibull random variable with β=1 and δ=100, the distribution of X is the exponential distribution with λ=0.01.

$$f(x) = \left(\frac{1}{100}\right)\left(\frac{x}{100}\right)^0 e^{-\left(\frac{x}{100}\right)^1} \text{ for } x > 0$$

$$= 0.01e^{-0.01x} \text{ for } x > 0$$

The mean of X is E(X) = 1/λ = 100.

4-128. Let X denote lifetime of a bearing. β = 3 and δ = 10000 hours

a) $P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^3} = e^{-(0.8^3)} = 0.5993$

b) $E(X) = 10000\Gamma\left(1+\tfrac{1}{3}\right) = 10000\Gamma(1.33)$

$= 10000(0.33)\Gamma(0.33) = 3300(2.707) = 8933.1$

= 8933.1 hours

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a binomial random variable with n = 10 and p = 0.5273.

$$P(Y = 10) = \binom{10}{10}0.5273^{10}(1-0.5273)^0 = 0.00166.$$

4-129. a) $E(X) = \delta\Gamma\left(1+\dfrac{1}{\beta}\right) = 900\Gamma(1+1/5) = 900\Gamma(6/5) = 900(0.91817). = 826.35$ hours

b) $V(X)=\delta^2\Gamma\left(1+\frac{2}{\beta}\right)-\delta^2\left[\Gamma\left(1+\frac{2}{\beta}\right)\right]^2=900^2\Gamma\left(1+\frac{2}{5}\right)-900^2\left[\Gamma\left(1+\frac{1}{5}\right)\right]^2$

$=900^2(0.88726)-900^2(0.91817)^2=35821.32\text{ hours}^2$

c) $P(X<500)=F_X(500)=1-e^{-\left(\frac{500}{900}\right)^5}=0.0515$

4-130. Let X denote the lifetime.

a) $E(X)=\delta\Gamma(1+\frac{1}{0.5})=\delta\Gamma(3)=2\delta=400$. Then $\delta=200$. Now,

P(X > 500) = $e^{-(\frac{500}{200})^{0.5}}=0.206$

b) P(X < 400) = $1-e^{-(\frac{400}{200})^{0.5}}=0.757$

4-131. a) β = 1/2, δ = 500

$E(X)=500\Gamma(1+2)=500\Gamma(3)=500\times 2!=1000$ hours

b) $V(X)=500^2\Gamma(1+4)-500^2[\Gamma(1+2)]^2$

$=500^2\Gamma(5)-500^2[\Gamma(3)]^2$

$=500^2\times 4!-500^2(2!)^2$

$=5\times 10^6$

c) P(X < 250) = F(250) $=1-e^{-(\frac{250}{500})^{0.5}}=1-0.7788=0.507$

4-132. $E(X)=\delta\Gamma(1+\frac{1}{2})=2.5$

So $\delta=\frac{2.5}{\Gamma(1+\frac{1}{2})}=\frac{5}{\sqrt{\pi}}$

$Var(X)=\delta^2\Gamma(2)-(EX)^2=\frac{25}{\pi}-2.5^2=1.7077$

Stdev(X)= 1.3068

4-133. $\delta^2\Gamma(1+\frac{2}{\beta})=Var(X)+(EX)^2=10.3+4.9^2=34.31$

$\delta\Gamma(1+\frac{1}{\beta})=E(X)=10.3$

Requires a numerical solution to these two equations.

4-134. a) $P(X<10)=F_X(10)=1-e^{-(10/8.6)^2}=1-e^{-1.3521}=0.7413$

b) $P(X>10)=1-F_X(10)=e^{-(10/8.6)^2}=0.2587$

c)

$P(8<X<11)=F_X(11)-F_X(8)=(1-e^{-(11/8.6)^2})-(1-e^{-(8/8.6)^2})=0.8052-0.5791=0.2261$

d) $P(X > x) = 1 - F_X(x) = e^{-(x/8.6)^2} = 0.9$

Therefore, $-(x/8.6)^2 = \ln(0.9) = -0.1054$, and x = 2.7920

4-135. a) $P(X > 3000) = 1 - F_X(3000) = e^{-(3000/5000)^2} = 0.698$

b) $P(X > 6000 \mid X > 3000) = \dfrac{P(X > 6000, X > 3000)}{P(X > 3000)} = \dfrac{P(X > 6000)}{P(X > 3000)}$

$= \dfrac{1 - F_X(6000)}{1 - F_X(3000)} = \dfrac{e^{-(6000/5000)^2}}{e^{-(3000/5000)^2}} = \dfrac{0.237}{0.698} = 0.340$

b) If it is an exponential distribution, then β = 1 and

$P(X > 3000) = 1 - F_X(3000) = e^{-(3000/5000)} = 0.549$

$P(X > 6000 \mid X > 3000) = \dfrac{1 - F_X(6000)}{1 - F_X(3000)} = \dfrac{e^{-(6000/5000)}}{e^{-(3000/5000)}} = \dfrac{0.301}{0.549} = 0.549$

For the Weibull distribution (with β = 2) there is no lack of memory property so that the answers to parts (a) and (b) differ whereas they would be the same if an exponential distribution were assumed. From part (b), the probability of survival beyond 6000 hours, given the device has already survived 3000 hours, is lower than the probability of survival beyond 3000 hours from the start time.

4-136. a) $P(X > 3000) = 1 - F_X(3000) = e^{-(3000/5000)^{0.5}} = 0.461$

b) $P(X > 6000 \mid X > 3000) = \dfrac{P(X > 6000, X > 3000)}{P(X > 3000)} = \dfrac{P(X > 6000)}{P(X > 3000)}$

$= \dfrac{1 - F_X(6000)}{1 - F_X(3000)} = \dfrac{e^{-(6000/4000)^{0.5}}}{e^{-(3000/4000)^{0.5}}} = \dfrac{0.334}{0.461} = 0.725$

c) If it is an exponential distribution, then β = 1

$P(X > 3000) = 1 - F_X(3000) = e^{-(3000/5000)} = 0.549$

$P(X > 6000 \mid X > 3000) = \dfrac{1 - F_X(6000)}{1 - F_X(3000)} = \dfrac{e^{-(6000/5000)}}{e^{-(3000/5000)}} = \dfrac{0.301}{0.549} = 0.549$

For the Weibull distribution (with β = 0.5) there is no lack of memory property so that the answers to parts (a) and (b) differ whereas they would be the same if an exponential distribution were assumed. From part (b), the probability of survival beyond 6000 hours, given the device has already survived 3000 hours, is greater than the probability of survival beyond 3000 hours from the start time.

d) The failure rate can be increased or decreased relative to the exponential distribution with the shape parameter β in the Weibull distribution.

4-137. a) $P(X > 3500) = 1 - F_X(3500) = e^{-(3500/2000)^3} = 0.0047$

b) The mean of this Weibull distribution is (2000) $0.33(2.707)$ = 1786.62

If it is an exponential distribution with this mean then

$P(X > 3500) = 1 - F_X(3500) = e^{-(3500/1786.62)} = 0.1410$

c) The probability that the lifetime exceeds 3500 hours is greater under the exponential distribution than under this Weibull distribution model.

Section 4-11

4-138. X is a lognormal distribution with θ = 5 and $\omega^2 = 9$

a) $P(X < 13300) = P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13330) - 5}{3}\right)$

$= \Phi(1.50) = 0.9332$

b) Find the value for which P(X ≤ x) = 0.90

$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 5}{3}\right) = 0.90$

$\frac{\ln(x) - 5}{3} = 1.28 \quad x = e^{1.28(3)+5} = 6904.99$

c) $\mu = E(X) = e^{\theta + \omega^2/2} = e^{5+9/2} = e^{9.5} = 13359.7$

$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{10+9}(e^9 - 1) = e^{19}(e^9 - 1) = 1.45 \times 10^{12}$

4-139. a) X is a lognormal distribution with θ=-2 and ω^2=16

$P(500 < X < 1000) = P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000))$

$= \Phi\left(\frac{\ln(1000) + 2}{4}\right) - \Phi\left(\frac{\ln(500) + 2}{4}\right) = \Phi(2.23) - \Phi(2.05) = 0.0073$

b) $P(X < x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) + 2}{4}\right) = 0.1$

$\frac{\ln(x) + 2}{4} = -1.28 \qquad x = e^{-1.28(4)-2} = 0.00081$

c) $\mu = E(X) = e^{\theta + \omega^2/2} = e^{-2+16/2} = e^6 = 403.43$

$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{-4+16}(e^{16} - 1) = e^{12}(e^{16} - 1) = 1.45 \times 10^{12}$

4-140. a) X is a lognormal distribution with θ = 3 and $\omega^2 = 4$

$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500) - 3}{2}\right)$

$= \Phi(1.61) = 0.9463$

b) $P(X < 15000 \mid X > 1000) = \frac{P(1000 < X < 1500)}{P(X > 1000)}$

$$= \frac{\left[\Phi\left(\frac{\ln(1500)-3}{2}\right)-\Phi\left(\frac{\ln(1000)-3}{2}\right)\right]}{\left[1-\Phi\left(\frac{\ln(1000)-3}{2}\right)\right]}$$

$$= \frac{\Phi(2.16)-\Phi(1.95)}{1-\Phi(1.95)} = \frac{0.9846-0.9744}{1-0.9744} = \frac{0.0102}{0.0256} = 0.3984$$

c) The product has degraded over the first 1000 hours, so the probability of it lasting another 500 hours is very low.

4-141. X is a lognormal distribution with θ=0.5 and ω^2=2

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1-\Phi\left(\frac{\ln(10)-0.5}{\sqrt{2}}\right)$$

$$= 1-\Phi(1.27) = 1-0.8980 = 0.102$$

b) $P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-0.5}{\sqrt{2}}\right) = 0.50$

$\frac{\ln(x)-0.5}{\sqrt{2}} = 0 \quad x = e^{0(\sqrt{2})+0.5} = 1.65$ seconds

c) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{0.5+2/2} = e^{1.5} = 4.48$

$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{1+2}(e^2-1) = e^3(e^2-1) = 128.33$

4-142. Find the values of θ and ω^2 given that E(X) = 100 and V(X) = 85,000

$100 = e^{\theta+\omega^2/2} \qquad 85000 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$

Let $x = e^{\theta}$ and $y = e^{\omega^2}$ then

$100 = x\sqrt{y}$ and $85000 = x^2 y(y-1) = x^2y^2 - x^2y$

Square the first equation to obtain $10000 = x^2 y$ and substitute into the second equation

$85000 = 10000(y-1)$

$y = 9.5$

Substitute *y* into the first equation and solve for x to obtain

$x = \frac{100}{\sqrt{9.5}} = 32.444$

$\theta = \ln(32.444) = 3.48$ and $\omega^2 = \ln(9.5) = 2.25$

4-143. a) Find the values of θ and ω^2 given that E(X) = 10000 and σ = 20,000

$10000 = e^{\theta+\omega^2/2} \qquad 20000^2 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$

Let $x = e^{\theta}$ and $y = e^{\omega^2}$ then

$10000 = x\sqrt{y}$ and $20000^2 = x^2 y(y-1) = x^2y^2 - x^2y$

Square the first equation $10000^2 = x^2 y$ and substitute into the second equation

$$20000^2 = 10000^2(y-1)$$
$$y = 5$$

Substitute *y* into the first equation and solve for x to obtain $x = \frac{10000}{\sqrt{5}} = 4472.1360$

$\theta = \ln(4472.1360) = 8.4056$ and $\omega^2 = \ln(5) = 1.6094$

b)

$$P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000) - 8.4056}{1.2686}\right)$$
$$= 1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$$

c) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\frac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$

$$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \quad x = e^{-1.28(1.2686)+8.4056} = 881.65 \text{ hours}$$

4-144. $E(X) = \exp(\theta + \omega^2/2) = 120.87$

$$\sqrt{\exp(\omega^2) - 1} = 0.09$$

So

$\omega = \sqrt{\ln 1.0081} = 0.0898$ and

$\theta = \ln 120.87 - \omega^2/2 = 4.791$

4-145. Let X ~ N(μ, σ^2), then Y = e^X follows a lognormal distribution with mean μ and variance σ^2. By definition, $F_Y(y) = P(Y \le y) = P(e^X < y) = P(X < \log y) = F_X(\log y) = \Phi\left(\frac{\log y - \mu}{\sigma}\right)$.

Because Y = e^X and X ~ N(μ, σ^2), we can show that $f_Y(Y) = \frac{1}{y} f_X(\log y)$

Finally, $f_Y(y) = \frac{\partial F_Y(y)}{\partial y} = \frac{\partial F_X(\log y)}{\partial y} = \frac{1}{y} f_X(\log y) = \frac{1}{y} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\log y - \mu}{2\sigma}\right)^2}$.

4-146. X has a lognormal distribution with θ = 10 and $\omega^2 = 25$

a) $P(X < 2000) = P(e^W < 2000) = P(W < \ln(2000)) = \Phi\left(\frac{\ln(2000) - 10}{5}\right)$

$$= \Phi(-0.4798) = 0.3157$$

b) $P(X > 1500) = 1 - P(e^W < 1500) = 1 - P(W < \ln(1500)) = \Phi\left(\frac{\ln(1500) - 10}{5}\right)$

$$= 1 - \Phi(-0.5374) = 1 - 0.2955 = 0.7045$$

c) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = 1 - \Phi\left(\dfrac{\ln(x) - 10}{5}\right) = 0.7$

$$-0.5244 = \frac{\ln(x) - 10}{5}$$

Therefore, $x = 1600.39$

4-147. X has a lognormal distribution with θ = 1.5 and ω = 0.3

a) $\mu = E(X) = e^{\theta + (\omega^2/2)} = e^{1.5 + (0.09/2)} = e^{1.55} = 4.7115$

$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{3 + 0.09}(e^{0.09} - 1) = 2.0697$

b) $P(X < 8) = P(e^W < 8) = P(W < \ln(8)) = \Phi\left(\dfrac{\ln(8) - 1.5}{0.3}\right) = \Phi(1.9315) = 0.9733$

c) $P(X < 0) = 0$ for the lognormal distribution. If the distribution is normal, then

$$P(X < 0) = P\left(Z < \frac{0 - 4.7115}{\sqrt{2.0697}}\right) = 0.0005$$

Because waiting times cannot be negative the normal distribution generates some modeling error.

Section 4-12

4-148. The probability density is symmetric.

4-149. a) $P(X < 0.25) = \int_0^{0.25} \dfrac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}) x^{\alpha - 1}(1 - x)^{\beta - 1}$

$$= \int_0^{0.25} \frac{\Gamma(3)}{\Gamma(2)\Gamma(1)})x = \frac{2!}{1!0!}\frac{x^2}{2}\Bigg|_0^{0.25} = 0.25^2 = 0.0625$$

b) $P(0.25 < X < 0.75) = \int_{0.25}^{0.75} \dfrac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}) x^{\alpha - 1}(1 - x)^{\beta - 1}$

$$= \int_{0.25}^{0.75} \frac{\Gamma(3)}{\Gamma(2)\Gamma(1)})x = \frac{2!}{1!0!}\frac{x^2}{2}\Bigg|_{0.25}^{0.75} = 0.75^2 - 0.25^2 = 0.5$$

c) $\mu = E(X) = \dfrac{\alpha}{\alpha + \beta} = \dfrac{2}{2 + 1} = 0.667$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{2}{3^2 \times 4} = 0.056$$

4-150. a) $P(X < 0.75) = \int_0^{0.75} \dfrac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}) x^{\alpha - 1}(1 - x)^{\beta - 1}$

$$= \int_0^{0.75} \frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)})(1-x)^{3.2} = \frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2)}{(3.2)(2.2)(1.2)\Gamma(1.2)} \frac{(-1)(1-x)^{4.2}}{4.2}\Bigg|_0^{0.75} = -(0.25)^{4.2} + 1 = 0.997$$

b) $P(0.5 < X) = \int_{0.5}^{1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$= \int_{0.5}^{1} \frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)})(1-x)^{3.2} = \frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2)}{(3.2)(2.2)(1.2)\Gamma(1.2)} \frac{(-1)(1-x)^{4.2}}{4.2}\Bigg|_{0.5}^{1} = 0 + (0.5)^{4.2} = 0.0544$$

c) $\mu = E(X) = \frac{\alpha}{\alpha+\beta} = \frac{1}{1+4.2} = 0.1923$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{4.2}{(5.2)^2(6.2)} = 0.0251$$

4-151. a) Mode = $\frac{\alpha-1}{\alpha+\beta-2} = \frac{1}{2+1.2-2} = 0.8333$

$$\mu = E(X) = \frac{\alpha}{\alpha+\beta} = \frac{2}{2+1.2} = 0.625$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{2.4}{(3.2)^2(4.2)} = 0.0558$$

b) Mode = $\frac{\alpha-1}{\alpha+\beta-2} = \frac{9}{10+6.25-2} = 0.6316$

$$\mu = E(X) = \frac{\alpha}{\alpha+\beta} = \frac{10}{10+6.25} = 0.6154$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{62.5}{(16.25)^2(17.25)} = 0.0137$$

c) Both the mean and variance from part a) are greater than for part b).

4-152. a) $P(X > 0.9) = \int_{0.9}^{1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$= \int_{0.9}^{1} \frac{\Gamma(10)}{\Gamma(9)\Gamma(1)})x^8 = \frac{(9)\Gamma(9)}{\Gamma(9)} \frac{x^9}{9}\Bigg|_{0.9}^{1} = 1 - (0.9^9) = 0.613$$

b) $P(X < 0.5) = \int_0^{0.5} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$= \int_0^{0.5} \frac{\Gamma(10)}{\Gamma(9)\Gamma(1)})x^8 = \frac{(9)\Gamma(9)}{\Gamma(9)} \frac{x^9}{9}\Bigg|_0^{0.5} = 0.5^9 = 0.002$$

c) $\mu = E(X) = \frac{\alpha}{\alpha+\beta} = \frac{9}{9+1} = 0.9$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{9}{(10)^2(11)} = 0.008$$

4-153. Let X denote the completion proportion of the maximum time.
The exercise considers the proportion 2/2.5 = 0.8

$$P(X > 0.8) = \int_{0.8}^{1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$$

$$= \int_{0.8}^{1} \frac{\Gamma(5)}{\Gamma(2)\Gamma(3)})x(1-x)^2 = \frac{(4)(3)\Gamma(3)}{\Gamma(2)\Gamma(3)}(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4})\Big|_{0.8}^{1} = 12(0.0833 - 0.0811) = 0.0272$$

Supplemental Exercises

4-154. $f(x) = 0.02$ for 50< x <75

a) $P(X > 70) = \int_{70}^{75} 0.02dx = 0.02x\Big|_{70}^{75} = 0.1$

b) $P(X < 60) = \int_{50}^{60} 0.02dx = 0.02x\Big|_{50}^{60} = 0.2$

c) $E(X) = \frac{75+50}{2} = 62.5$ seconds

$V(X) = \frac{(75-50)^2}{12} = 52.0833$ seconds^2

4-155. a) P(X < 40) = $P\left(Z < \frac{275-240}{14}\right)$
= P(Z < 2.5)
= 0.99379

b) P(X < 30) = $P\left(Z < \frac{205-240}{14}\right)$
= P(Z < –2.5)
= 0.00621
0.621% are scrapped

4-156. a) P(X <55) = $P\left(Z < \frac{55-60}{5}\right)$ = P(Z < -1) = 0.1587

b) P(X > 65) = $P\left(Z > \frac{65-60}{5}\right)$ = P(Z >1) = 1- P(Z < 1)
= 1 - 0.841345= 0.158655

c) P(X < x) = $P\left(Z < \frac{x-60}{5}\right)$ = 0.99

Therefore, $\frac{x-60}{5}$ = 2.33 and x = 72

4-157. a) P(X > 90.6) + P(X < 90.0)

$$= P\left(Z > \frac{90.6-90.5}{0.1}\right) + P\left(Z < \frac{90.0-90.5}{0.1}\right)$$

= P(Z > 1) + P(Z < –5)
= 1 – P(Z < 1) + P(Z < –5)
= 1 – 0.84134 + 0
= 0.15866.

Therefore, the answer is 0.15866.

b) The process mean should be set at the center of the specifications; that is, at μ = 90.3.

c) P(90.0 < X < 90.6) = $P\left(\frac{90.0-90.3}{0.1} < Z < \frac{90.6-90.3}{0.1}\right)$

= P(–3 < Z < 3) = 0.9973.

The yield is 100*0.9973 = 99.73%

d) P(90 < X < 90.6) = $P\left(\frac{90-90.3}{0.1} < Z < \frac{90.6-90.3}{0.1}\right)$

= P(–3 < Z < 3)
= 0.9973

P(X=10) = $(0.9973)^{10}$ = 0.9733

e) Let *Y* represent the number of cases out of the sample of 10 that are between 90.0 and 90.6 ml. Then Y follows a binomial distribution with *n*=10 and *p*=0.9973. Thus, E(*Y*)= 9.973 or 10.

4-158. a) P(60 < X < 70) = $P\left(\frac{60-100}{20} < Z < \frac{70-100}{20}\right)$ = P(–2 < Z < -1.5) = P(Z < –1.5) – P(Z < –2)

= 0.0441.

b) P(X > x) = 0.10. Therefore, $P\left(Z > \frac{x-100}{20}\right)$ = 0.10 and $\frac{x-100}{20}$ = 1.28

Therefore, x = 125.6 hours

4-159. E(X) = 1000(0.2) = 200 and V(X) = 1000(0.2)(0.8) = 160

a)

$$P(X > 225) = P(X \ge 226) \cong 1 - P\left(Z \le \frac{225.5-200}{\sqrt{160}}\right) = 1 - P(Z \le 2.02) = 1 - 0.9783 = 0.0217$$

b) $P(175 \le X \le 225) \cong P\left(\frac{174.5-200}{\sqrt{160}}\right) \le Z \le \frac{225.5-200}{\sqrt{160}}) = P(-2.02 \le Z \le 2.02)$

$= 0.9783 - 0.0217 = .9566$

c) If P(*X* > *x*) = 0.01, then $P\left(Z > \frac{x-200}{\sqrt{160}}\right)$ = 0.01.

Therefore, $\dfrac{x-200}{\sqrt{160}}$ = 2.33 and x = 229.5

4-160. The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with 0.00005.

a) $P(X > 20{,}000) = \int_{20000}^{\infty} 0.00005 e^{-.0.00005x} dx = -e^{-0.00005x}\Big|_{20000}^{\infty} = e^{-1} = 0.368$

b) $P(X < 30{,}000) = \int_{30000}^{\infty} 0.00005 e^{-.0.00005x} dx = -e^{-0.00005x}\Big|_{0}^{30000} = 1 - e^{-1.5} = 0.777$

c)

$$P(20{,}000 < X < 30{,}000) = \int_{20000}^{30000} 0.00005 e^{-.0.00005x} dx$$

$$= -e^{-0.00005x}\Big|_{20000}^{30000} = e^{-1} - e^{-1.5} = 0.145$$

4-161. Let X denote the number of calls in 3 hours. Because the time between calls is an exponential random variable, the number of calls in 3 hours is a Poisson random variable. Now, the mean time between calls is 0.75 hours and $\lambda = 1/0.75 = 4/3$ calls per hour = 4 calls in 3 hours.

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \left[\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!}\right] = 0.567$$

4-162. Let X denote the time in days until the fourth problem. Then, X has an Erlang distribution with r = 6 and $\lambda = 1/30$ problem per day.

a) E(X) = $\dfrac{6}{30^{-1}} = 180$ days.

b) Let Y denote the number of problems in 120 days. Then, Y is a Poisson random variable with $\lambda = 4$ problems per 120 days.

$$P(Y < 4) = \left[\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!}\right] = 0.4335$$

4-163. Let X denote the lifetime

a) $E(X) = 700\Gamma(1+1) = 700$

b)

$$V(X) = 700^2\Gamma(3) - 700^2[\Gamma(2)]^2$$

$$= 700^2(2) - 700^2(1) = 490000$$

c) P(X > 620.4) = $e^{-\left(\frac{620.4}{700}\right)^1} = 0.412$

4-164. a) $E(X) = \exp(\theta + \omega^2/2) = 0.001$

$\sqrt{\exp(\omega^2) - 1} = 2$

So

$\omega = \sqrt{\ln 5} = 1.2686$

And

$\theta = \ln 0.001 - \omega^2 / 2 = -7.7124$

b)

$P(X > 0.007) = 1 - P(\exp(W) \le 0.007) = 1 - P(W \le \ln 0.007) = 1 - \Phi(\frac{\ln 0.007 + 7.7124}{1.2686}) = 0.0150$

4-165. a) $P(X < 5) = \int_4^5 (0.25x - 1)dx = \left(0.25\frac{x^2}{2} - x\right)\Big|_4^5 = 0.125$

b) $P(X > 7) = \int_7^8 (0.25x - 1)dx = 0.25\frac{x^2}{2} - x\Big|_7^8 = 0.815$

c) $P(5 < X < 6) = \int_5^6 (0.25x - 1)dx = 0.25\frac{x^2}{2} - x\Big|_5^6 = 0.375$

d) $F(x) = \int_4^x (0.25t - 1)dt = 0.25\frac{t^2}{2} - t\Big|_4^x = \frac{x^2}{8} - x + 2$. Then,

$$F(x) = \begin{cases} 0, & x < 4 \\ \frac{x^2}{8} - x + 2, & 4 \le x < 8 \\ 1, & 8 \le x \end{cases}$$

e) $E(X) = \int_4^8 x(0.25x - 1)dx = 0.25\frac{x^3}{3} - \frac{x^2}{2}\Big|_4^8 = \frac{128}{3} - 32 - \left(\frac{16}{3} - 8\right) = \frac{40}{3}$

$$V(X) = \int_4^8 \left(x - \frac{40}{3}\right)^2 (0.25x - 1)dx = \int_4^8 \left(x^2 - \frac{80}{3}x + \frac{1600}{9}\right)(0.25x - 1)dx$$

$$= \int_4^8 \left(0.25x^3 - \frac{23}{3}x^2 + \frac{640}{9}x - \frac{1600}{9}\right)dx = \frac{x^4}{16} - \frac{23}{9}x^3 + \frac{320}{9}x^2 - \frac{1600}{9}x\Big|_4^8$$

$$= 90.66$$

4-166. Let X denote the time between calls. Then, $\lambda = 1/E(X) = 0.2$ calls per minute.

a) $P(X < 5) = \int_0^5 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_0^5 = 1 - e^{-1} = 0.632$

b) $P(5 < X < 15) = -e^{-0.2x}\Big|_5^{15} = e^{-1} - e^{-3} = 0.318$

c) P(X < x) = 0.9. Then, $P(X < x) = \int_0^x 0.2e^{-0.2t}dt = 1 - e^{-0.2x} = 0.9$. Now, x = 46.05 minutes.

d) This answer is the same as part a).

$$P(X < 5) = \int_0^5 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_0^5 = 1 - e^{-1} = 0.632$$

e) This is the probability that there are no calls over a period of 5 minutes. Because a Poisson process is memoryless, it does not matter whether or not the intervals are consecutive.

$$P(X > 5) = \int_5^\infty 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_5^\infty = e^{-1} = 0.368$$

f) Let Y denote the number of calls in 30 minutes.
Then, Y is a Poisson random variable with $\lambda = 6$.

$$P(Y \le 2) = \frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} = 0.062.$$

g) Let W denote the time until the fifth call.
Then, W has an Erlang distribution with λ = 0.2 and r = 5.
E(W) = 5/0.2 = 25 minutes.

4-167. Let X denote the lifetime. Then $\lambda = 1/E(X) = 1/8$.

a) $P(X < 4) = \int_0^4 \frac{1}{8}e^{-x/8}dx = -e^{-x/8}\Big|_0^4 = 1 - e^{-0.5} = 0.3935$.

b) Let W denote the number of CPUs that fail within the next four years. Then, W is a binomial random variable with n = 10 and p = 0.3935. Then,

$$P(W \ge 1) = 1 - P(W = 0) = 1 - \binom{10}{0}0.3935^0(1 - 0.3935)^{10} = 0.9933.$$

4-168. X is a lognormal distribution with θ=0 and ω^2=9

a)

$$P(10 < X < 50) = P(10 < e^W < 50) = P(\ln(10) < W > \ln(50))$$

$$= \Phi\left(\frac{\ln(50) - 0}{3}\right) - \Phi\left(\frac{\ln(10) - 0}{3}\right)$$

$$= \Phi(1.30) - \Phi(0.77) = 0.1238$$

b) $P(X < x) = P(e^W < x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 0}{3}\right) = 0.05$

$$\frac{\ln(x) - 0}{3} = -1.64 \quad x = e^{-1.64(3)} = 0.0073$$

c) $\mu = E(X) = e^{\theta + \omega^2/2} = e^{0+9/2} = e^{4.5} = 90.02$

$$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{0+9}(e^9 - 1) = e^9(e^9 - 1) = 65651866$$

4-169. a) Find the values of θ and ω^2 given that E(X) = 50 and V(X) = 4000

$$50 = e^{\theta + \omega^2/2} \qquad 4000 = e^{2\theta + \omega^2}(e^{\omega^2} - 1)$$

Let $x = e^\theta$ and $y = e^{\omega^2}$ then

$50 = x\sqrt{y}$ and $4000 = x^2y(y-1) = x^2y^2 - x^2y$

Square the first equation $x = \frac{50}{\sqrt{y}}$ and substitute into the second equation

$$4000 = \left(\frac{50}{\sqrt{y}}\right)^2 y^2 - \left(\frac{50}{\sqrt{y}}\right)^2 y = 2500(y-1)$$

$$y = 2.6$$

Substitute y back into the first equation and solve for x to obtain $x = \frac{50}{\sqrt{2.6}} = 31$

$\theta = \ln(31) = 3.43$ and $\omega^2 = \ln(2.6) = 0.96$

b)

$$P(X < 150) = P(e^W < 150) = P(W < \ln(150)) = \Phi\left(\frac{\ln(150) - 3.43}{0.98}\right)$$

$$= \Phi(1.61) = 0.946301$$

4-170. Let X denote the number of fibers visible in a grid cell. Then, X has a Poisson distribution and $\lambda = 100$ fibers per cm^2 = 60,000 fibers per sample = 0.5 fibers per grid cell.

a) $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-0.5} 0.5^0}{0!} = 0.3935$.

b) Let W denote the number of grid cells examined until 10 contain fibers. If the number of fibers have a Poisson distribution, then the number of fibers in each grid cell are independent. Therefore, W has a negative binomial distribution with p = 0.3935. Consequently, E(W) = 10/0.3935 = 25.41 cells.

c) V(W) = $\frac{10(1-0.3935)}{0.3935^2}$. Therefore, $\sigma_W = 6.25$ cells.

4-171. Let X denote the height of a plant.

a) P(X>2.25) = $P\left(Z > \frac{2.25-2.6}{0.5}\right)$ = P(Z > –0.7) = 1 – P(Z ≤ –0.7) = 0.7580

b) P(2.0 < X < 3.0) = $P\left(\frac{2.0-2.6}{0.5} < Z < \frac{3.0-2.6}{0.5}\right)$ =P(–1.2 < Z < 0.8) = 0.673

c) P(X > x) = 0.90 = $P\left(Z > \frac{x-2.6}{0.5}\right)$ = 0.90 and $\frac{x-2.6}{0.5}$ = –1.28.

Therefore, x = 1.96.

4-172. a) $P(X > 3.5) = \int_{3.5}^{4} (0.5x - 1)dx = 0.5\frac{x^2}{2} - x \Big|_{3.5}^{4} = 0.4375$

b) Yes, because the probability of a plant growing to a height of 3.5 centimeters or more without irrigation is small.

4-173. Let X denote the thickness.

a) P(X > 5.25) = $P\left(Z > \frac{5.25-5}{0.2}\right)$ = P(Z > 1.25) = 0. 1057

b) P(4.75 < X < 5.25) = $P\left(\frac{4.75-5}{0.2} < Z < \frac{5.25-5}{0.2}\right)$ = P (-1.25 < Z < 1.25) = 0.7887

Therefore, the proportion that do not meet specifications is
1 – P(4.75 < X < 5.25) = 0.2113.

c) If P(X < x) = 0.95, then $P\left(Z > \frac{x-5}{0.2}\right)$ = 0.95. Therefore, $\frac{x-5}{0.2}$ = 1.65 and x = 5.33.

4-174. Let X denote the dot diameter. If P(0.0035 < X < 0.0065) = 0.9970, then

$$P\left(\frac{0.0035-0.005}{\sigma} < Z < \frac{0.0065-0.005}{\sigma}\right) = P\left(\frac{-0.0015}{\sigma} < Z < \frac{0.0015}{\sigma}\right) = 0.9970.$$

Therefore, $\frac{0.0015}{\sigma} = 2.75$ and σ = 0.0005.

4-175. If P(0.005 – x < X < 0.005 + x), then P(–x/0.001 < Z < x/0.001) = 0.9970. Therefore, x/0.001 = 2.75 and x = 0.0028. The specifications are from 0.0022 to 0.0078.

4-176. Let X denote the life.

a) $P(X < 6100) = P(Z < \frac{6100-7000}{600}) = P(Z < -1.5) = \Phi(-1.5) = 0.0668$

b) If P(X > x) = 0.9, then P(Z < $\frac{x-7000}{600}$) = -1.28. Consequently, $\frac{x-7000}{600}$ = -1.28 and x = 6232 hours.

c) If P(X > 10,000) = 0.99, then P(Z > $\frac{10{,}000-\mu}{600}$) = 0.99. Therefore, $\frac{10{,}000-\mu}{600}$ = -2.33 and $\mu = 11{,}398$

d) The probability a product lasts more than 10000 hours is $[P(X > 10000)]^3$, by independence.
If $[P(X > 10000)]^3$ = 0.99, then P(X > 10000) = 0.9967.
Then, P(X > 10000) = $P(Z > \frac{10000-\mu}{600}) = 0.9967$.
Therefore, $\frac{10000-\mu}{600}$ = -2.72 and $\mu = 11{,}632$ hours.

4-177. X is an exponential distribution with E(X) = 7000 hours

a) $P(X < 6100) = \int_0^{6100} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 1 - e^{-\left(\frac{6100}{7000}\right)} = 0.5816$

b) $P(X > x) = \int_x^{\infty} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 0.9$

Therefore, $e^{-\frac{x}{7000}} = 0.9$ and $x = -7000\ln(0.9) = 737.5$ hours

4-178. Find the values of θ and ω^2 given that E(X) = 7000 and σ= 600

$$7000 = e^{\theta+\omega^2/2} \qquad 600^2 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$$

Let $x = e^{\theta}$ and $y = e^{\omega^2}$ then

$$7000 = x\sqrt{y} \text{ and } 600^2 = x^2 y(y-1) = x^2y^2 - x^2 y$$

Square the first equation $7000^2 = x^2 y$ and substitute into the second equation

$$600^2 = 7000^2(y-1)$$

$$y = 1.0073$$

Substitute *y* into the first equation and solve for x to obtain $x = \dfrac{7000}{\sqrt{1.0073}} = 6974.6$

$\theta = \ln(6974.6) = 8.850$ and $\omega^2 = \ln(1.0073) = 0.0073$

a)

$$P(X < 6100) = P(e^W < 6100) = P(W < \ln(6100)) = \Phi\left(\frac{\ln(6100) - 8.85}{0.0854}\right)$$

$$= \Phi(-1.57) = 0.0582$$

b) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = 1 - \Phi\left(\dfrac{\ln(x) - 8.85}{0.0854}\right) = 0.9$

$\dfrac{\ln(x) - 8.85}{0.0854} = -1.28 \quad x = e^{-1.28(0.0854)+8.85} = 6252.20$ hours

4-179. a) Using the normal approximation to the binomial with n = 8 × 100 × 100 = 80,000, and p = 0.0002 we have: E(X) = 80000(0.0002) = 16

$$P(X \geq 1.2) \cong P\left(\frac{X - \text{np}}{\sqrt{\text{np}(1-\text{p})}} \geq \frac{0.7 - 16}{\sqrt{80000(0.0002)(0.9998)}}\right)$$

$$= P(Z > -3.83) = 1 - 0.000064 = 0.999936$$

b) $P(X \geq 4.8) \cong P\left(\dfrac{X - \text{np}}{\sqrt{\text{np}(1-\text{p})}} \geq \dfrac{4.3 - 16}{\sqrt{80000(0.0002)(0.9998)}}\right)$

$$= P(Z \geq -2.93) = 1 - 0.0017 = 0.9983$$

4-180. Using the normal approximation to the binomial with X being the number of people who will be seated. Then X ~Bin(200, 0.95).

a) P(X ≤ 195) $= P\left(\dfrac{X - np}{\sqrt{np(1-p)}} \leq \dfrac{195.5 - 190}{\sqrt{200(0.95)(0.05)}}\right) = P(Z \leq 1.78) = 0.9625$

b)

$$P(X < 195)$$

$$\approx P(X \leq 194.5) = P\left(\frac{X - np}{\sqrt{np(1-p)}} \leq \frac{194.5 - 190}{\sqrt{200(0.95)(0.05)}}\right) = P(Z \leq 1.46) = 0.9279$$

c) P(X ≤ 195) ≅ 0.98,

Successively trying various values of n: The number of reservations taken could be reduced to about 199.

n	Z_o	Probability $P(Z < Z_0)$
200	1.78	0.9625
199	**1.48**	**0.9817**

198	2.40	0.9918

4-181. Let X denote the survival time of AMI patients.

a) $\delta = 0.25$ (scale parameter); $\beta = 1.16$ (shape parameter)

$$\mu = E(X) = \delta\Gamma\left(1+\frac{1}{\beta}\right) = 0.237$$

$$\sigma^2 = V(X) = \delta^2\Gamma\left(1+\frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2 = 0.042$$

b) $P(X > 1) = 1 - P(X \le 1) = 1 - F(1) = 1 - \left[1 - \exp\left(-\left(\frac{1}{0.25}\right)^{1.16}\right)\right] = 0.0068$

c) Find a such that $P(X > a) = 0.90$

$$P(X > a) = 1 - P(X \le a) = 1 - F(a) = 1 - \left[1 - \exp\left(-\left(\frac{a}{0.25}\right)^{1.16}\right)\right] = 0.90$$

Then, $a = 0.036$.

<u>Mind-Expanding Exercises</u>

4-182. a) P(X > x) implies that there are r - 1 or less counts in an interval of length x. Let Y denote the number of counts in an interval of length x. Then, Y is a Poisson random variable with mena E(Y) $= \lambda x$. Then, $P(X > x) = P(Y \le r-1) = \sum_{i=0}^{r-1} e^{-\lambda x}\frac{(\lambda x)^i}{i!}$.

b) $P(X \le x) = 1 - \sum_{i=0}^{r-1} e^{-\lambda x}\frac{(\lambda x)^i}{i!}$

c) $f_X(x) = \frac{d}{dx}F_X(x) = \lambda e^{-\lambda x}\sum_{i=0}^{r-1}\frac{(\lambda x)^i}{i!} - e^{-\lambda x}\sum_{i=0}^{r-1}\lambda i\frac{(\lambda x)^i}{i!} = \lambda e^{-\lambda x}\frac{(\lambda x)^{r-1}}{(r-1)!}$

4-183. Let X denote the diameter of the maximum diameter bearing.
Then, P(X > 1.6) = 1 - $P(X \le 1.6)$.

Also, $X \le 1.6$ if and only if all the diameters are less than 1.6. Let Y denote the diameter of a bearing. Then, by independence

$$P(X \le 1.6) = [P(Y \le 1.6)]^{10} = \left[P(Z \le \tfrac{1.6-1.5}{0.025})\right]^{10} = 0.999967^{10} = 0.99967$$

Then, P(X > 1.6) = 0.0033.

4-184. a) Quality loss = $Ek(X-m)^2 = kE(X-m)^2 = k\sigma^2$, by the definition of the variance.

b)

$$\begin{aligned} Quality\ \ loss &= Ek(X-m)^2 = kE(X-\mu+\mu-m)^2 \\ &= kE[(X-\mu)^2 + (\mu-m)^2 + 2(\mu-m)(X-\mu)] \\ &= kE(X-\mu)^2 + k(\mu-m)^2 + 2k(\mu-m)E(X-\mu). \end{aligned}$$

The last term equals zero by the definition of the mean.

Therefore, quality loss = $k\sigma^2 + k(\mu - m)^2$.

4-185. Let X denote the event that an amplifier fails before 60,000 hours. Let A denote the event that an amplifier mean is 20,000 hours. Then A' is the event that the mean of an amplifier is 50,000 hours. Now, P(E) = P(E|A)P(A) + P(E|A')P(A') and

$$P(E \mid A) = \int_0^{60,000} \tfrac{1}{20,000} e^{-x/20,000} dx = -e^{-x/20,000}\Big|_0^{60,000} = 1 - e^{-3} = 0.9502$$

$$P(E \mid A') = -e^{-x/50,000}\Big|_0^{60,000} = 1 - e^{-6/5} = 0.6988.$$

Therefore, P(E) = 0.9502(0.10) + 0.6988(0.90) = 0.7239

4-186. $P(X < t_1 + t_2 \mid X > t_1) = \frac{P(t_1 < X < t_1 + t_2)}{P(X > t_1)}$ from the definition of conditional probability.

Now,

$$P(t_1 < X < t_1 + t_2) = \int_{t_1}^{t_1+t_2} \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_{t_1}^{t_1+t_2} = e^{-\lambda t_1} - e^{-\lambda(t_1+t_2)}$$

$$P(X > t_1) = -e^{-\lambda x}\Big|_{t_1}^{\infty} = e^{-\lambda t_1}$$

Therefore, $P(X < t_1 + t_2 \mid X > t_1) = \dfrac{e^{-\lambda t_1}(1 - e^{-\lambda t_2})}{e^{-\lambda t_1}} = 1 - e^{-\lambda t_2} = P(X < t_2)$

4-187. a) $1 - P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = 1 - P(-6 < Z < 6) = 1.97 \times 10^{-9} = 0.00197\,ppm$

b) $1 - P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = 1 - P(-7.5 < \frac{X - (\mu_0 + 1.5\sigma)}{\sigma} < 4.5) = 3.4 \times 10^{-6} = 3.4\,ppm$

c) $1 - P(\mu_0 - 3\sigma < X < \mu_0 + 3\sigma) = 1 - P(-3 < Z < 3) = .0027 = 2,700\,ppm$

d) $1 - P(\mu_0 - 3\sigma < X < \mu_0 + 3\sigma) = 1 - P(-4.5 < \frac{X - (\mu_0 + 1.5\sigma)}{\sigma} < 1.5)$
$= 0.0668106 = 66,810.6\,ppm$

e) If the process is centered six standard deviations away from the specification limits and the process mean shifts even one or two standard deviations there would be minimal product produced outside of specifications. If the process is centered only three standard deviations away from the specifications and the process shifts, there could be a substantial amount of product outside of the specifications.