Transport Phenomena1

1st
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∴ Introduction :-

Transport Phenomena
- mass transfer
- energy transfer
- momentum transfer

* Driving Force → السبب باي بحرك ظواهر الانتقال
- heat ⇒ ΔT
- mass ⇒ Δ Concentration
- fluid ⇒ ΔP

heat

A $100\,C^\circ$ $\xrightarrow{\text{heat transfer}}$ B $0\,C^\circ$

when $T_A = T_B$

↳ رح يتوقف الانتقال ظاهريًا

$\Delta T = $ zero

Driving Force ⇒ ΔT

momentum

$P = 10\,bar$ $\qquad$ $P = 5\,bar$

$\xrightarrow{\text{fluid}}$

$P = mV$ $\qquad$ P : mumentum

m : mass

V velocity

$\text{stress} = \dfrac{F}{\text{area}}$ (force)

$\therefore \dfrac{P}{t} = F$

Driving force $= \Delta P$ (prussure)

mass

تركيز ↑ → تركيز ↓

Driving Force ⇒ Δ concentration

Studied together for the following reasons
- ↳ occur simultaneously
- ↳ basic equation that describe the 3 transport phenomena
- ↳ mathematical tools very similar
- ↳ molecular mechanisms

Bernoulli's equation مراجعة

- $P + \frac{\rho v^2}{2} + \rho g h = P_2 + \frac{\rho v_2^2}{2} + \rho g h_2$
- $\frac{P_1}{\rho} + \frac{v_1^2}{\rho} + g h_1 + w_p = \frac{P_2}{\rho} + \frac{v_2^2}{2} + g h_2 + f + W$

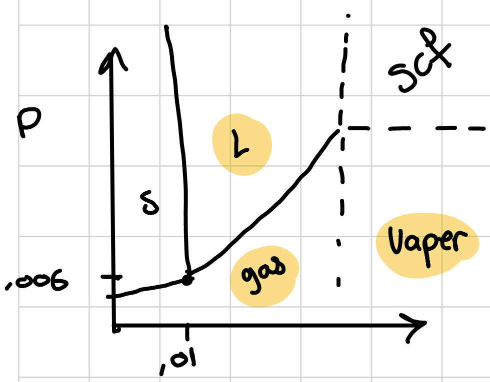
w_p → pump work (fan, Pump, ...)

f → frictional losses

W → work (turbine pump)

CAPTER 2:

* fluid → Vaper, gases, Liquids } does not permanently resist direction ↳ change its shape



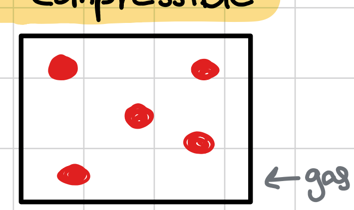
difference between vaper, gas
↳ ↑ prussure vaper ⇒ scf
gas ⇒ L

* Typical fluid : water, Air, CO_2, Oil, Slurries, Thick syrups
Slurries: mixture solid with Liquid
زي الطين
Thick syrups: زي العسل

Type of flud

غير قابل للضغط
"Incompressible"
* most Liquids.

قابل للضغط
"compressible"



Can get closer ≡ The density will change

* change density ≡ change pressure
if the change in density ↓ 5% ≡ طول التغير كانه ضل ثابت ≡ no change in prussure

$$\frac{|\rho_1 - \rho_2|}{\rho_2} \times 100\%$$

* mumentum Transfer → we treat fluid as a continuous distribution of mater $\boxed{\text{countinuum}}$
  - fluid statics ≡ rest
  - fluid Dynamics ≡ motion

static fluid

* stress σ = $\frac{\vec{F}}{\vec{A}}$
  - normal ≡ Pressure
  - shear

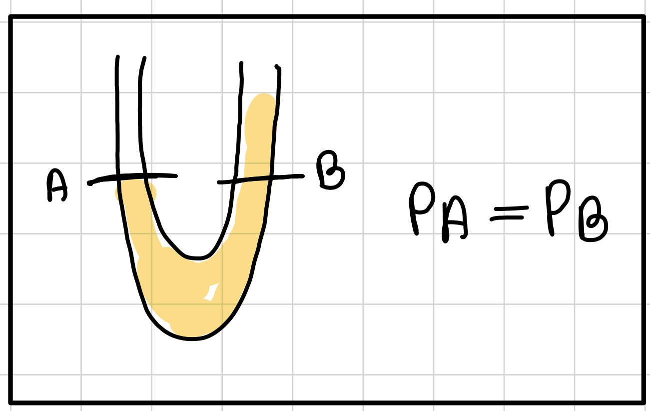
* Pressure $= \frac{\vec{F}_\perp}{A} = \rho g h$
  → scalar

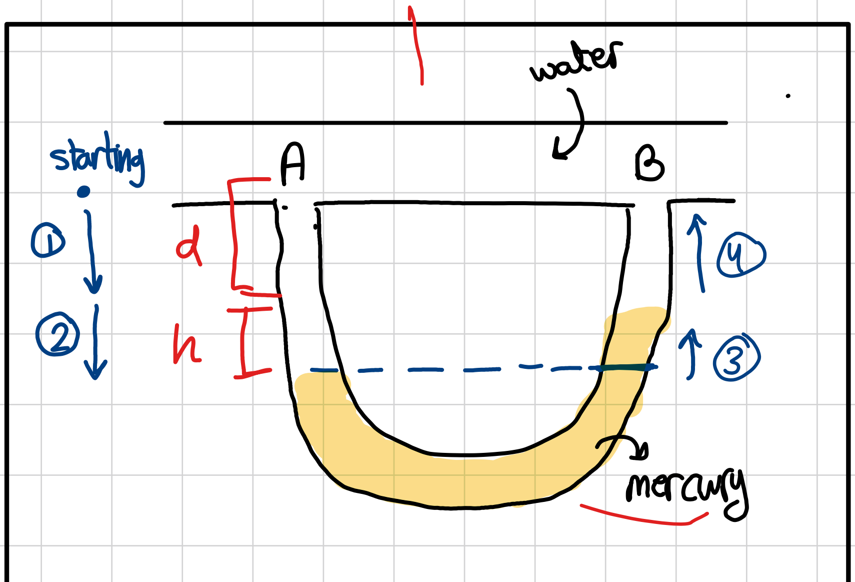
* force units
  - SI: $N \equiv kg \cdot \frac{m}{s^2}$ $\quad F = mg$
  - English: $F = lbf$ $\quad F = \frac{mg}{g_c}$
    $m = lbm$
    $g = 32.17 \frac{ft}{s^2}$
    $g_c = 32.17 \frac{lbm \cdot ft}{lbf \cdot s^2}$

* Dyne $= \frac{g \cdot cm}{s^2}$ $\quad$ * Poundal $= \frac{lbm \cdot ft}{s^2}$

* Pressure at two Point is equal when :-

1] same fluid
2] same height (horizontal)
3] Interconected (متصل)
4] at rest





* كيف احسب P_B ؟

$$P_A + \underset{①}{\rho_w g d} + \underset{②}{\rho_w h g} - \underset{③}{\rho_m g h} - \underset{④}{\rho_w g d} = P_B$$

(+) → نازل لتحت
(−) → فوق

EXAMPLE 2.2-1. ***Units and Dimensions of Force***

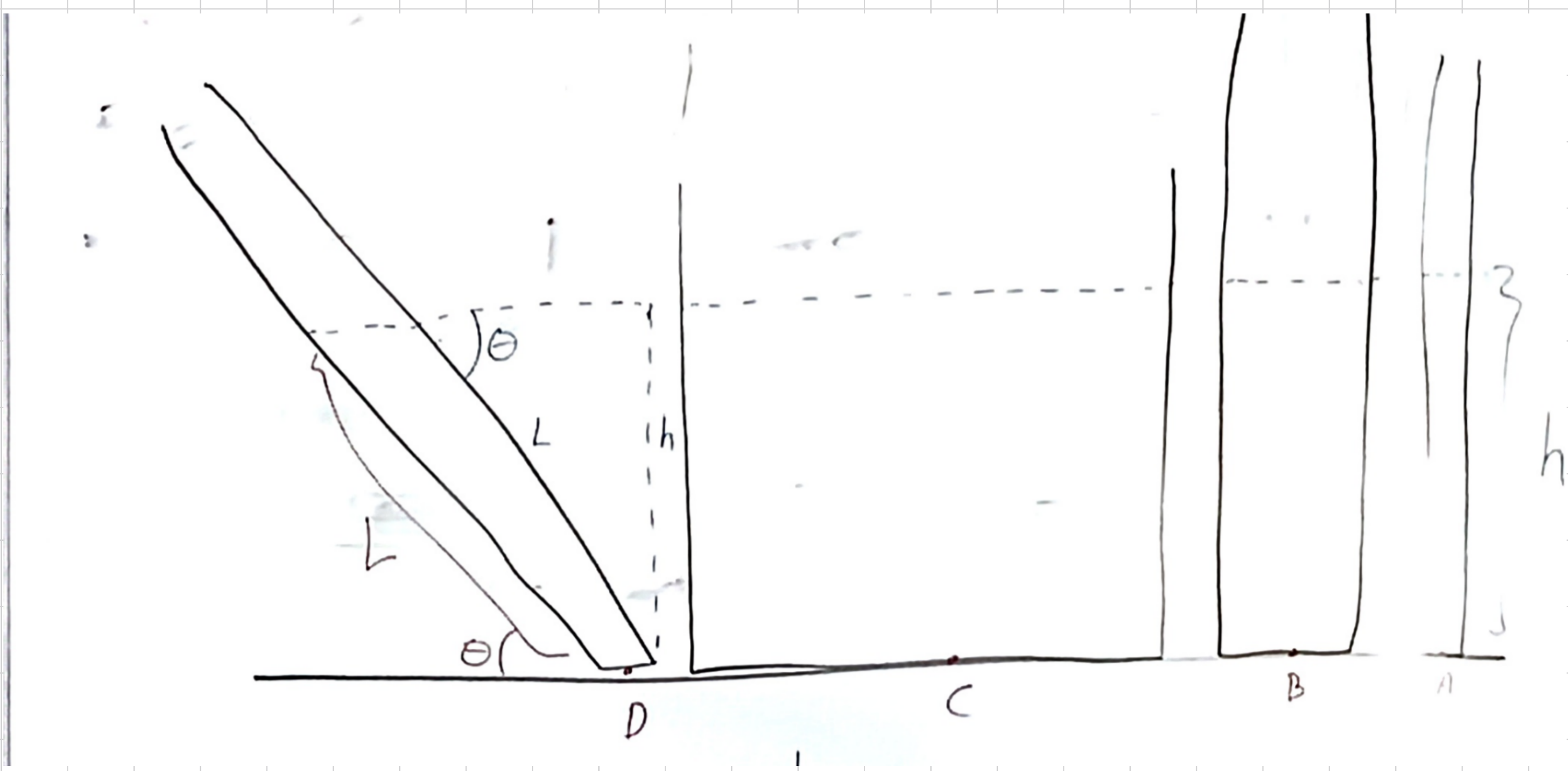
Calculate the force exerted by 3 lb mass in terms of the following.

(a) Lb force (English units).
(b) Dynes (cgs units).
(c) Newtons (SI units).

a) $F = \dfrac{mg}{g_c}$ $\quad \dfrac{3\ \cancel{lbm}}{} \Big| \dfrac{32.17\ \cancel{ft}}{\cancel{s^2}} \Big| \dfrac{lbf\ \cancel{s^2}}{32.17\ \cancel{lbm}\ \cancel{ft}} = 3\ lbf.$

b) $F = mg$ $\quad \dfrac{3\ \cancel{lbm}}{} \Big| \dfrac{453.59\ g}{1\ \cancel{lbm}} \Big| \dfrac{9.8\ \cancel{m}}{s^2} \Big| \dfrac{100\ cm}{\cancel{m}} = 1{,}332 \times 10^6\ dyn$

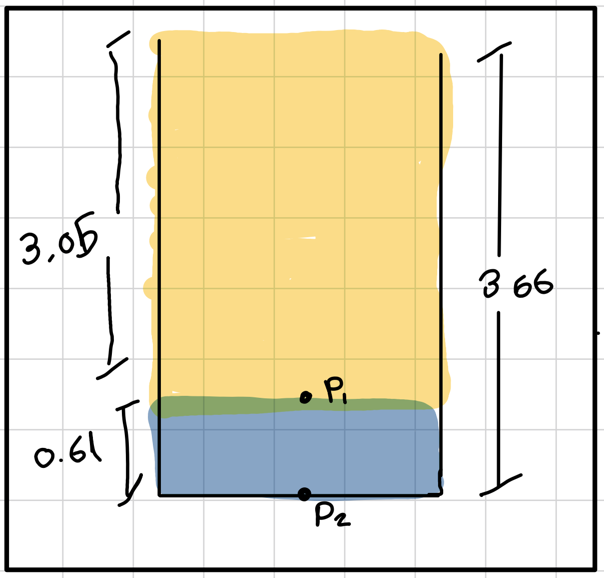
c) $F = mg$ $\quad \dfrac{3\ \cancel{lbm}}{} \Big| \dfrac{kg}{2.2046\ \cancel{lbm}} \Big| \dfrac{9.8\ m}{s^2} = 13.3\ N$



$P_A = P_B = P_C = P_D$ → كلهم نفس ال h
ونفس ال fluid

EXAMPLE 2.2-2. Pressure in Storage Tank

A large storage tank contains oil having a density of 917 kg/m^3 (0.917 g/cm^3). The tank is 3.66 m (12.0 ft) tall and is vented (open) to the atmosphere of 1 atm abs at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2.0 ft) of water in the bottom of the tank. Calculate the pressure in Pa and psia 3.05 m from the top of the tank and at the bottom. Also calculate the gage pressure at the tank bottom.



$P_1 = \rho_{oil}\, g\, h$

$$\frac{917\ kg}{m^3} \cdot \frac{9.8\ m}{s^2} \cdot 3.05\ m = \underline{27409.13}_{\text{gage}}$$

The abs $P_1 = P_{gage} + P_{atm}$

$27409.13 + 1.01325 \times 10^5 = 128729.13$ pa.

$*$ $\underline{\text{in Psia} = 18.68}$

تحويل وحدات

$P_{2\,gage} = \rho_{oil}\, g\, h_{oil} + \rho_{water}\, g\, h_{water}$

$917 \times 9.8 \times 3.05 + 1000 \times 0.61 \times 9.8 = 33387.13$ pa

$P_{2\,abs} = P_{gage} + P_{atm} = \boxed{134707.13\ \text{pa}} = 19.55$ psia

* head of a fluid → SI m ; English ft.

$P = \rho g h$

$h\,(\text{head}) = \dfrac{P}{\rho g}$

EXAMPLE 2.2-3. *Conversion of Pressure to Head of a Fluid*
Given the pressure of 1 standard atm as 101.325 kN/m² (Appendix A.1), do as follows.

رح احولها لـ SI = Pa عشان اسهل للوحدات

(a) Convert this pressure to head in m water at 4°C.
(b) Convert this pressure to head in m Hg at 0°C.

a) $1.0123 \times 10^5\ \text{pa} = \underbrace{1000}_{\rho_{water}} \times \underbrace{9.8}_{g} \times h$

$h = 10{,}33$ m water

b) $1.0123 \times 10^5 = \underbrace{13595.5}_{\rho_{Hg}\ \boxed{SI}} \times \underbrace{9.8}_{g} \times h$

$h_{Hg} = 0.760$ m Hg.

or

$P = P$

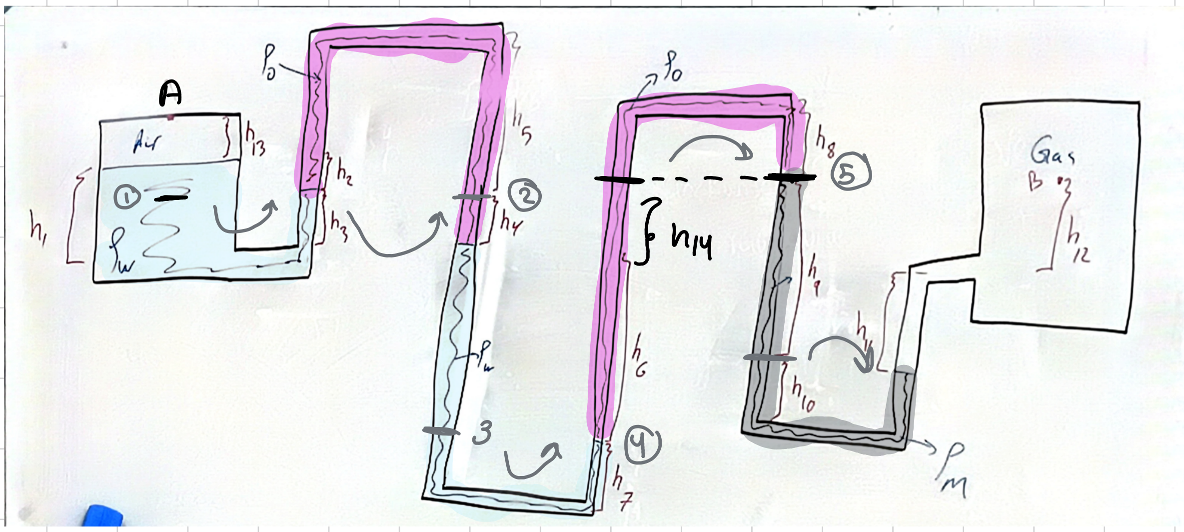
$\rho_w \cancel{g} h_w = \rho_{Hg} \cancel{g} h_{Hg}$

$\dfrac{\rho_w}{\rho_{Hg}} h_w = h_{Hg}$

Pressure measurement Devices

* manometer

↳ ΔP between two point.



$\rho(g) << \rho_L$

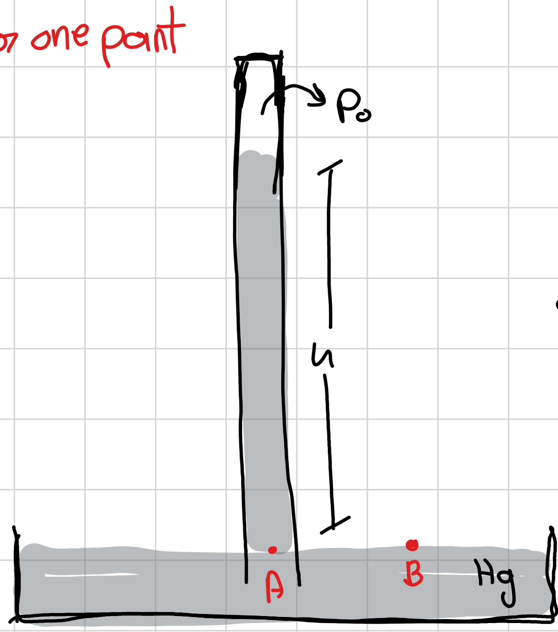
↳ لما يكون السؤال في g و L
يهمل ال g لانه تأثيره كتير اقل
لو السؤال كله غازات بهمهم دون
اهمال

$$P_A + \cancel{P_{Air}} + \rho_w g h_2 + \rho_o g h_4 + \rho_B g h_6 - \rho_o g (h_{14} + h_6) + \rho g h_9 = P_B$$

تحت يهمل لانه غاز لفوق

* Barometer

↳ one point



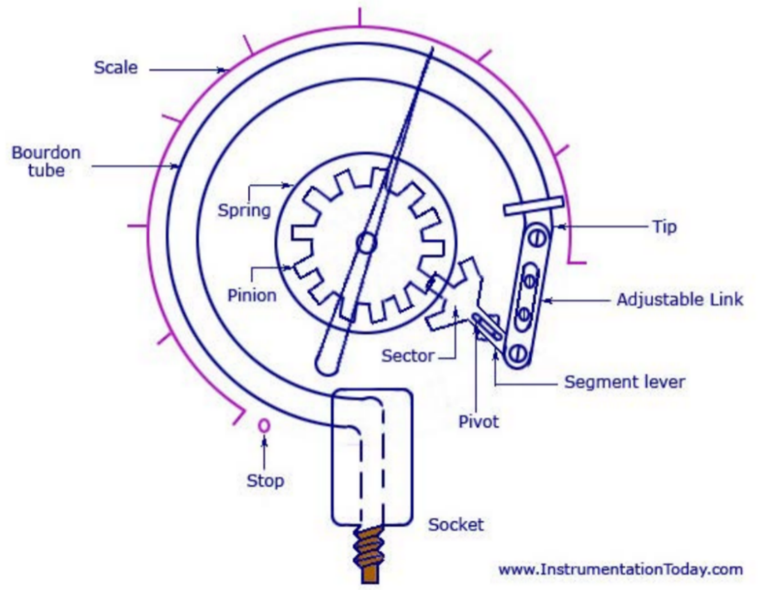
* $P_{atm} = P_B = P_A$
* $P_A = \cancel{P_o} + \rho_{Hg} g h$

↳ zero

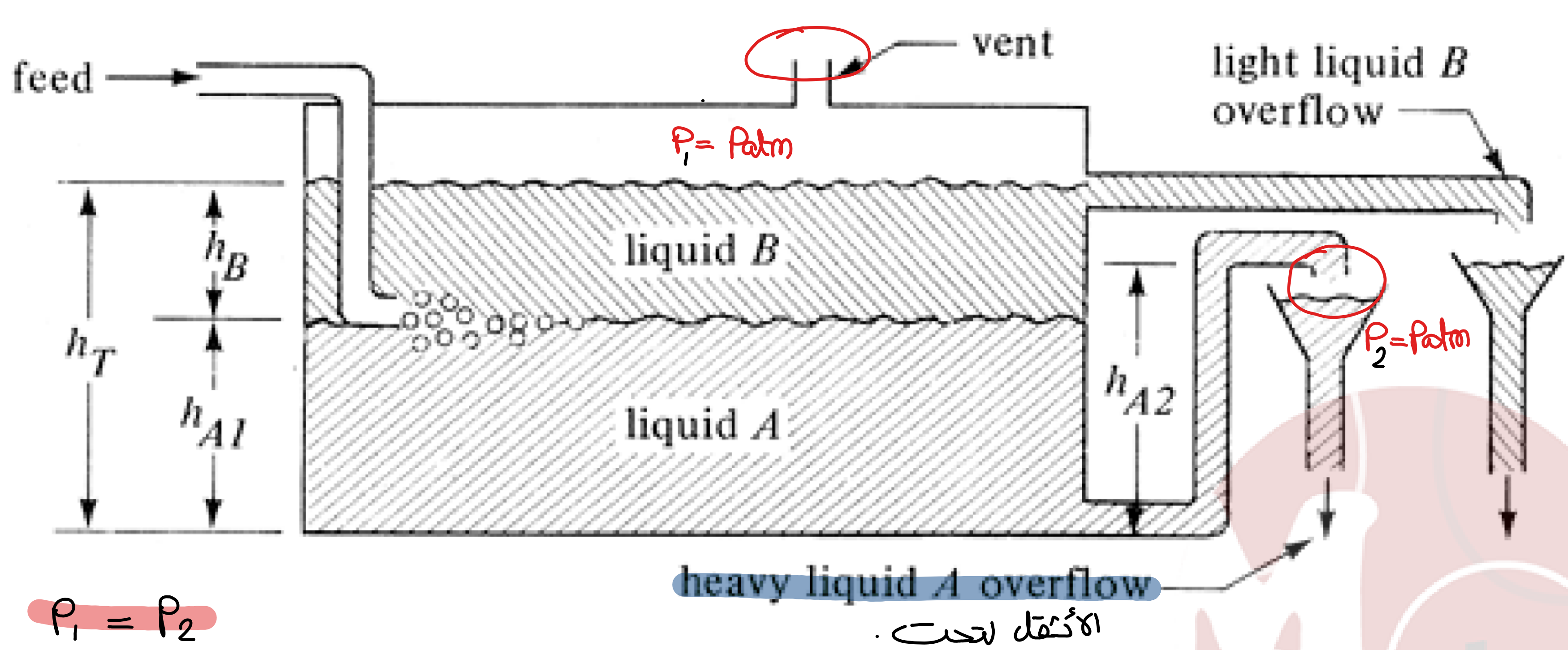
<< ↑ note >> بهاي الحالة

note P_o = zero
لما يكون الجهاز
تقيس ال Hg

- **Bourdon pressure gage**



* Gravity separator for two immiscible liquids
→ Two طريقة لعمل
fluids زي الزيت والماء



$P_1 = P_2$

$\rho_B h_B g + \rho_A h_{A1} g = h_{A2} \rho_A g$

① Ψ (flux) $= \dfrac{\text{ammunt}}{\text{Area . time}}$

$\hookrightarrow$ Ψ unit $\dfrac{\text{ammunt}}{m^2 . s}$

② Ψ(flux) $= \dfrac{\text{Driving force}}{\text{resistence}}$

- $\uparrow$ Driving Force $\uparrow$ flux
- $\downarrow$ resistence $\uparrow$ flux

③ $\Psi = -\delta \dfrac{\Delta \Gamma}{\Delta Z}$

Γ (ammunt of concentration) $= \dfrac{\text{amount of property}}{\text{Volume}}$

$\hookrightarrow$ unit $\dfrac{\text{amunt of property}}{m^3}$

Z (distance in the direction of flux)

$\hookrightarrow$ unit m

δ (proportionality constant, diffusivity)

$\hookrightarrow$ unit m^2/s

الوحده لازم تكون مثلك بال Balance

- Balanc amount $\equiv \dfrac{\text{amaunt}}{\text{time}}$

Input $-$ output $+$ Generation $-$ Consumption $=$ accumulation

Input: $\Psi_{in} A$

out put: $\Psi_{out} A$

Gen $= R_{Gen} A z$

reaction rate

$R_{unit} = \dfrac{\text{amont}}{\text{Time} . m^3}$

Conc $= R_{conc} A z$

Acc $= \dfrac{\Delta \Gamma}{\text{Time}} A \Delta z$

* Balance $\longrightarrow \dfrac{\partial \Gamma}{\partial t} - \delta \dfrac{\partial^2 \Gamma}{\partial z^2} = R$

* Balance with no generation $\longrightarrow \dfrac{\partial \Gamma}{\partial t} = \delta \dfrac{\partial^2 \Gamma}{\partial z^2}$

$$\Psi = \frac{\text{amount}}{\text{Area} \cdot \text{time}}$$

amount → heat
→ mass
→ momentum

$$\psi_z = -\delta \frac{d\Gamma}{dz}$$

القانون هاد ≡ لأي قانون من الجهة الثانية.

Introduction to Molecular Transport

- **Momentum transport**
  - Newton's law
  - $\tau_{zx} = -\underline{\underline{\nu}} \frac{\boxed{d(v_x \rho)}}{\boxed{dz}}$ — $d\Gamma$ / dz — kinematic viscosity
- **Heat transport**
  - Fourier's law
  - $\frac{q_z}{A} = -\alpha \frac{d(\rho c_p T)}{dz}$
- **Mass transport**
  - Fick's law
  - $J^*_{Az} = -D_{AB} \frac{dc_A}{dz}$

ح :

* momentum $\tau = -\nu \frac{d(v_x \rho)}{dz} \equiv \psi = -\delta \frac{d\Gamma}{dz}$

$\tau = -(\text{viscosity})(\text{velocity}) / dz$

$\tau = \frac{f}{A} = \frac{\text{mass}}{A} \frac{du}{dt} = \frac{\boxed{du \text{ mass}}}{A \ dt} \equiv \frac{\text{momentum}}{\text{Area . time}}$ #

ي بساوي صار . $\frac{\text{amount}}{\text{time area}}$ ل

$-\text{viscosity} \frac{(\text{velocity})}{dz} \times \frac{\rho}{\rho}$

$\frac{\text{viscosity}}{\rho} \quad \frac{\text{velocity} \times \rho}{dz}$

$\left.\frac{\frac{\cancel{kg}}{m.s}}{\frac{\cancel{kg}}{m^3}}\right\} \frac{m^2}{s} = \delta \qquad \frac{m}{s} \frac{kg}{m^3} \qquad \frac{\text{momentum}}{\text{Volume}} = \text{concentration} = \Gamma$

$\tau = -\mu \frac{\text{velocity}}{dz} \equiv \psi = -\delta \frac{d\Gamma}{dz}$

ثبات .

* Heat $\frac{q_z}{A} = \alpha \frac{d(\rho c_p T)}{dz}$

$\frac{\text{Jule}}{\text{Time. Area}} = \underbrace{\alpha}_{\frac{m^2}{s}} \frac{\cancel{kg} \ J \ \cancel{K}}{m^3 \ \cancel{kg} \ \cancel{K} \ m}$

معلومات استخدمناها عشان نثبت .

- viscosity $\mu = \frac{kg}{m.s}$
- force = mass . a
- $a = \frac{du}{dt}$
- q_z = heat flow $= \frac{J}{t}$

- shear stress τ

Gunite = Pa

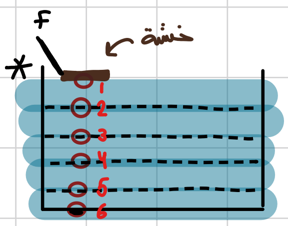
$\tau = \frac{f}{A}$, $\tau = -\mu \frac{\Delta U}{\Delta y}$

* viscosity of fluid

unite = $1cp = \frac{0.01\,g}{cm\,s} = 1\times10^{-3}\,\frac{kg}{m\,s}$

مقاومة ال flow

force
- Cohesive force — Between simelar melcules — $H_2O \approx H_2O$
- Adhesive force — ~ — Diffrent melcules — $H_2O \approx X$



انا بدي اعطي قوة للخشبة عشان تتحرك

نعتبر ال fluid نتحرك كطبقات

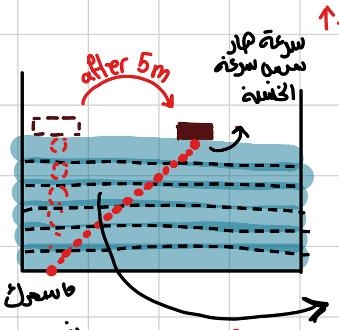
① force $H_2O \approx$ الجسم adhesive

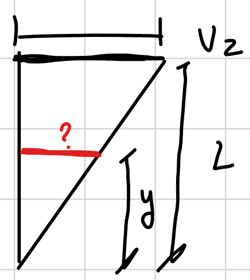
②③④⑤ $H_2O \approx H_2O$ cohesive

⑥ force $H_2O \approx$ الأرض adhesive

* اذا بدي ازيد سرعة الخشبة
↑ Force
↓ Area للخشب

↑f ①





$\frac{?}{V_2} = \frac{y}{L}$ تشابه المثلثات

Newtonian fluid

∴ slope ≡ viscosity

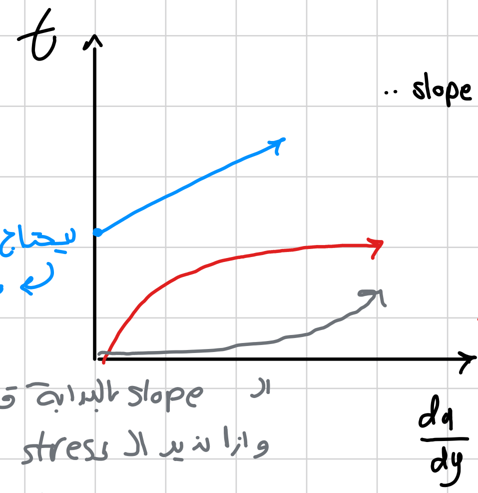
oil تحتاج اكثر احراك.

alcohol بمس سرعة

1) viscosity ⇒ constant لمس الفلوس

2) بلبشوا من الصفر

non-Newtonian fluid



∴ slope ≡ viscosity

يحتاج stress عشان يصير ال fluid
معجون الأسنان

بالبداية ال slope كبير
يعني مقاومة ال flow كبير
بعدين بصير ضعيف عادي
٧٨ ال slope يقل.
الدهان

ال slope بالبداية قليل
واذا نزيد ال stress تزيد
(slope) مقاومة
الشا والحا

مع τ قليل.

مع τ عالي

- Temp
  - Liquid $\downarrow \mu$ (viscosity)
  - gas $\uparrow \mu$ (viscosity)

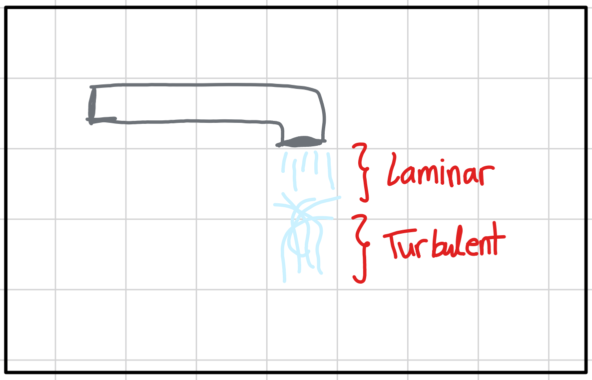
ليش بصير عنا أختلاف بتأثير الحرارة على ال μ بالسائل والغاز.

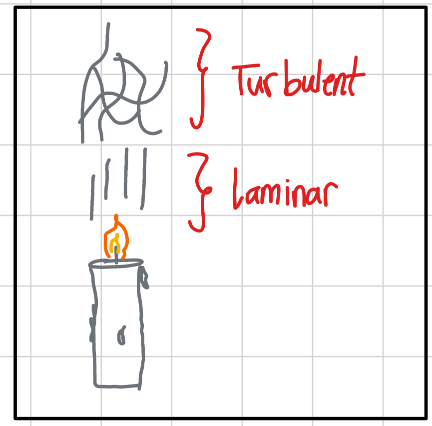
* μ in liquid جاي من cohesive force ، $\uparrow T$ $\uparrow KE$ cohesive forces $< KE$ فتقل ال μ .

* μ in gas جاي من التصادمات $\uparrow T$ $\uparrow KE$ $\uparrow$عدد التصادمات فتزيد μ .

- prasure
  - Liquid no effect
  - gas
    - μ viscosity $\uparrow$ شوي
    - kinematic viscosity $= \frac{\mu}{ص}$ $\downarrow$

ليش تقل ما تزيد زي ال μ؟
لأنه ص تتأثر بالضغط كثير فتأثر عليها

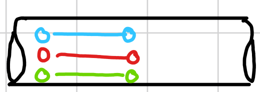




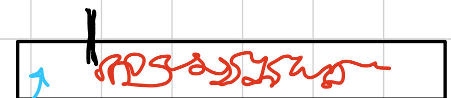
Flow

Laminar

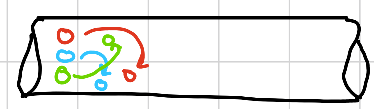
- smoth
- كل جزيء بطريقة
- low velocity
- without adidies or swirls
- newton low ✓



Turbulent



- velocity ↑
- with adidies or swirls
- all direction



- Laminar flow ⟶ critical velocity ⟶ Turbulent flow

* Reynolds number

↳ Inertial forces vs Viscous forces

↳ تقادم حركة الجسم زي الاحتكاك

$$\therefore Re = \frac{\text{Inertial forces}}{\text{viscous forces}} = \frac{\rho V^2}{\frac{\mu V}{D}} = \frac{\rho V D}{\mu}$$

Inertial forces: flow force

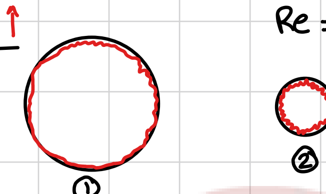
viscous forces: ↳ سيطرة على ال melc

always more than 1

نقدر اغيرهم (اعدلهم) ← بغير ال Temp اوبعمل الحادة .

* اغير ال D من دون ما تتغير السرعة ؟ -

$Re = \frac{\rho V D \uparrow}{\mu}$ ① $Re = \frac{\rho V D \downarrow}{\mu}$ ②



$Re_1 > Re_2$

لانه عندي بال D_2 كمية flow متأثرة بالاحتكاك اكثر من D_1

for a straight circular pipe, Internal Flow

Re ⟶ <2100 Laminar
⟶ 2100 ~ 4000 Transition
⟶ >4000 Turbulent

- Whole milk at 293 K having a density of 1030 kg/m³ and viscosity of 2.12 cp is flowing at the rate of 0.605 kg/s in a glass pipe having a diameter of 63.5 mm.

A) Re ?

$\rho = 1030 \text{ kg/m}^3$

viscosity = 2.12 cp = 2.12×10^{-3} kg/ms

mass flowrate = 0.605 kg/s

Diameter = 63.5 mm → 63.5×10^{-3} m

① velocity ← massflow rate بدي الطلع من ال

$$Re = \frac{\rho V D}{\mu}$$

mass flowrate → volume flow rate → velocity

ρ Q = VA

$$0.605 \frac{\text{kg}}{\text{s}} \left| \frac{\text{m}^3}{1030\text{kg}} \right. = \frac{5.87 \times 10^{-4} \text{ m}^3/\text{s}}{\frac{\pi}{4}(63.5 \times 10^{-3})^2 \text{ m}^2} = \boxed{0.185 \text{ m/s}}$$

$$Re = \frac{\rho V D}{\mu} = \frac{1030 \times 0.185 \times 0.0635}{2.12 \times 10^{-3}} = \boxed{5707}$$

Turbulent flow ↑4000

b) Calculate the flow rate in $\frac{m^3}{s}$ for a Re = 2100 and the velocity.

$$2100 = \frac{1030 \times V \times 0.0635}{2.12 \times 10^{-3}}$$

V = 0.068 m/s

↳ Q = VA

$$0.068 \times \frac{\pi}{4}(0.0635)^2 = 2.15 \times 10^{-4} \frac{m^3}{s}$$

acc = In − out + Gen − Con

- mass: acc = In − out
- compound: acc = In − out + $\underbrace{\text{Gen} - \text{Con}}_{\text{reaction}}$

$* \quad m = \rho V_1 A_1$ $\quad \frac{kg}{m^3} \Big| \frac{m}{s} \Big| m^2 \quad \to \frac{kg}{s}$

$*$ mass velocity / mass flux $\Rightarrow$ $\underline{G} = V\rho$ $\quad \frac{m}{s} \Big| \frac{kg}{m^3} \quad \to \frac{kg}{s.m^2}$

G velocity

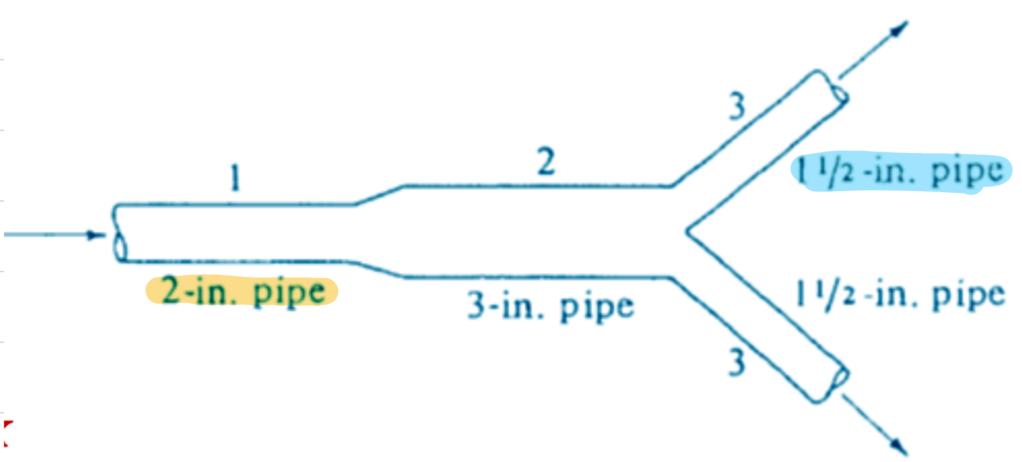
EXAMPLE 2.6-1. Flow of Crude Oil and Mass Balance

A petroleum crude oil having a density of 892 kg/m^3 is flowing through the piping arrangement shown in Fig. 2.6-2 at a total rate of 1.388 × 10^{-3} $\boxed{m^3/s}$ entering pipe 1.

Volume flow rate

The flow divides equally in each of pipes 3. The steel pipes are schedule 40 pipe (see Appendix A.5 for actual dimensions). Calculate the following using SI units.

(a) The total mass flow rate m in pipe 1 and pipes 3.
(b) The average velocity v in 1 and 3.
(c) The mass velocity G in 1.



Nominal Pipe Size (in.)	Outside Diameter		Schedule Number	Wall Thickness		Inside Diameter		Inside Cross-Sectional Area	
	in.	mm		in.	mm	in.	mm	ft²	m² × 10⁴ ↳ 10⁻⁴
1/8	0.405	10.29	40	0.068	1.73	0.269	6.83	0.00040	0.3664
			80	0.095	2.41	0.215	5.46	0.00025	0.2341
1/4	0.540	13.72	40	0.088	2.24	0.364	9.25	0.00072	0.6720
			80	0.119	3.02	0.302	7.67	0.00050	0.4620
3/8	0.675	17.15	40	0.091	2.31	0.493	12.52	0.00133	1.231
			80	0.126	3.20	0.423	10.74	0.00098	0.9059
1/2	0.840	21.34	40	0.109	2.77	0.622	15.80	0.00211	1.961
			80	0.147	3.73	0.546	13.87	0.00163	1.511
3/4	1.050	26.67	40	0.113	2.87	0.824	20.93	0.00371	3.441
			80	0.154	3.91	0.742	18.85	0.00300	2.791
1	1.315	33.40	40	0.133	3.38	1.049	26.64	0.00600	5.574
			80	0.179	4.45	0.957	24.31	0.00499	4.641
1 1/4	1.660	42.16	40	0.140	3.56	1.380	35.05	0.01040	9.648
			80	0.191	4.85	1.278	32.46	0.00891	8.275
1 1/2	1.900	48.26	40	0.145	3.68	1.610	40.89	0.01414	13.13
			80	0.200	5.08	1.500	38.10	0.01225	11.40
2	2.375	60.33	40	0.154	3.91	2.067	52.50	0.02330	21.65
			80	0.218	5.54	1.939	49.25	0.02050	19.05

① $\dot{m}_1 = \rho V_1 A_1$

↳ $V_1 A_1$ = volume flow rate

$= 892 \times 1388 \times 10^{-3} = \boxed{1{,}24 \frac{kg}{s}}$

mass in = mass out $\therefore \dot{m}_1 = \boxed{m_3 = 1{,}24 \frac{kg}{s}}$

② $\underline{Q_1} = V_1 A_1$ لازم عشان اعرف A1 و D1 ارجع ل AppendixA5

Volume flow rate

$\frac{1.388 \times 10^{-3}}{21\ 65 \times 10^{-4}} = \boxed{0.064 \frac{m}{s}}$

Q_2 رح اقسم Q_1 على 2 لانه

The flow divides equally in each

$\frac{1.388 \times 10^{3}}{2\,(1313 \times 10^{-4})} = \boxed{0.528 \frac{m}{s}}$

A_2 من ال Appendix

3) $G = V_1 \rho$

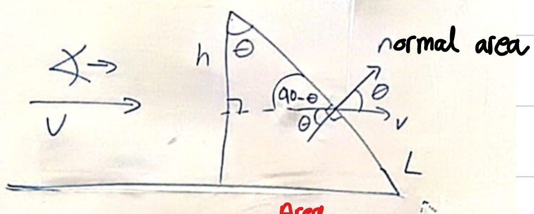
$0\ 064 \times 892 = \boxed{57\ 088 \frac{kg}{s.m^2}}$

surrounding

system الجزء اللي أنا مهتم أني ادرسه
3D

↳ boundary → real / imagenary
2D

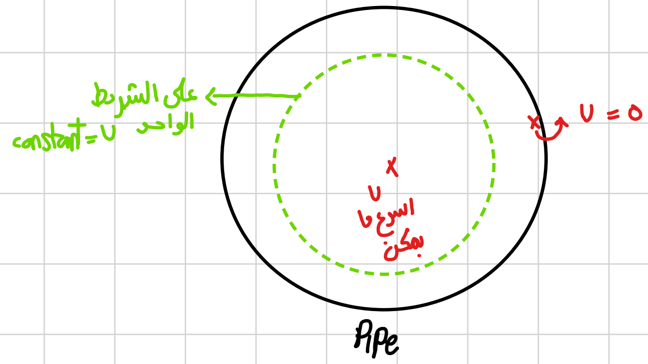
* control volume :- region fixed in space through which the fluid flows



$h = L\cos\theta$

$A = w L\cos\theta$

$V_{\perp} = V_n \cos\theta$



ال velocity مش متساوية بكل مكان.

$V_{total} = \Delta A_1 v_1 + \Delta A_2 v_2 + \ldots\ldots$

$\dot{V}_T = \sum_{i=0}^{n} dA_i v_i \equiv \oint_A v\, dA$

* $acc = \frac{dm}{dt} = \frac{\partial}{\partial t} \iiint_V \rho\, dV$

* net mass flux $= \iint_A v \rho \cos\theta\, dA$

* $V_{av} = \frac{1}{A} \iint v\, dA$

EXAMPLE 2.6-2. Overall Mass Balance in Stirred Tank

Initially, a tank contains 500 kg of salt solution containing 10% salt. At point (1) in the control volume in Fig. 2.6-5, a stream enters at a constant flow rate of 10 kg/h containing 20% salt. A stream leaves at point (2) at a constant rate of 5 kg/h. The tank is well stirred. Derive an equation relating the weight fraction w_A of the salt in the tank at any time t in hours.

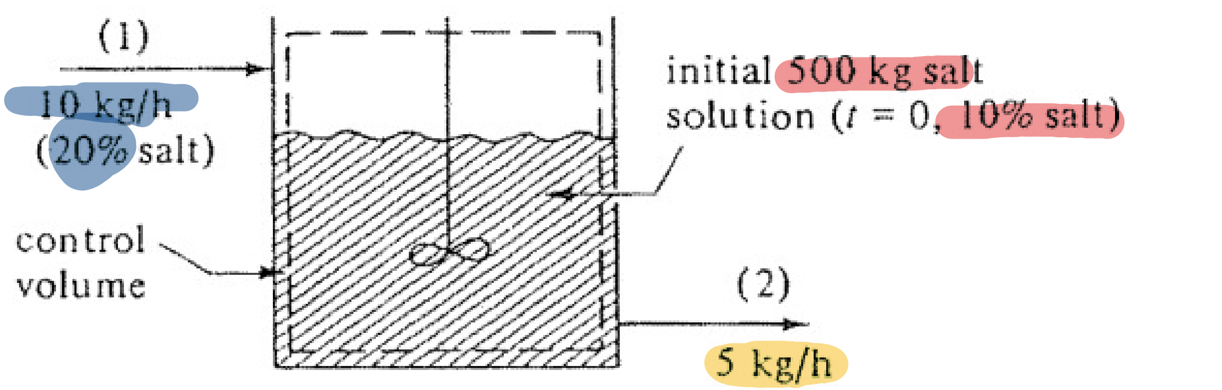
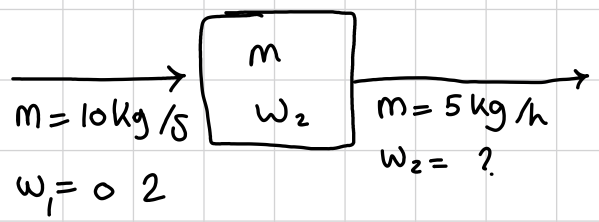


FIGURE 2.6-5. *Control volume for flow in a stirred tank for Example 2.6-2.*

$$\omega = \frac{m_s}{m_{tot}}$$



$$\therefore \int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$$

Total mass balance $= \frac{dm}{dt} = 10 - 5 = 5$

$$\int_{500}^{m} dm = \int_0^t 5\,dt$$

عند الزمن صفر

كم كانت ال mass ⇐ tank.

$$m - 500 = 5t$$

$$\boxed{m = 5t + 500}$$

Salt mass balance $= \frac{dwm}{dt} = 0\,2 \times 10 - w \times 5$

$$5t + 500\,\frac{dw}{dt} = 2 - 5w$$

$$\int_{0.1}^{w} \frac{dw}{2-10w} = \int_0^t \frac{dt}{5t+500}$$

$$\frac{-1}{10}\ln(2-5w)\Big]_{0.1}^{w} = \frac{1}{5}\ln|5t+500|\Big|_0^t$$

$$-\frac{1}{10}\left(\ln(2-10w) - \ln(2-10(0.1)\right) = -2\left(\ln(500+5t) - \ln(500+0)\right)$$

$$w = \frac{2 - \left(\frac{500}{500+5t}\right)^2}{10}$$

اشتقاق العلاقة لـ Vave و Vmax

- $\dot{V} = \int V\, dA$

$V_{ave}\, \underset{\text{total Area}}{A} = \int V\, dA$

$V_{ave} = \frac{1}{A} \int V\, dA$

$V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$

$$V_{ave} = \frac{1}{\pi R^2} \int_0^{2\pi}\int_0^{R} V_{max}\left(1 - \frac{r^2}{R^2}\right) r\, dr\, d\theta$$

$$V_{ave} = \frac{1}{\pi R^2} \int_0^{2\pi}\int_0^{R} V_{max}\left(r - \frac{r^3}{R^2}\right) dr\, d\theta$$

$$V_{ave} = \frac{1}{\pi R^2} \int_0^{2\pi} V_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)\Big]_0^{r} d\theta$$

$$V_{ave} = \frac{1}{\pi R^2} \int_0^{2\pi} V_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) - 0\ d\theta$$

$$V_{ave} = \frac{1}{\pi R^2} \int_0^{2\pi} V_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) \theta \Big]_0^{2\pi}$$

$$V_{ave} = \frac{1}{\pi R^2} V_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) 2\pi$$

$$V_{ave} = \frac{1}{\pi R^2} V_{max}\left(\frac{2R^4}{4R^2} - \frac{R^4}{4R^2}\right) 2\pi$$

$$V_{ave} = \frac{1}{\cancel{\pi} \cancel{R^2}} V_{max} \frac{\cancel{R^4}}{4\cancel{R^2}} 2\cancel{\pi}$$

$$\boxed{V_{ave} = \frac{V_{max}}{2}} \ \#$$

علاقة بين Vave و Vmax

• Area for O using double int

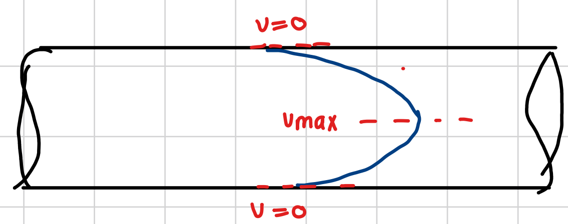
$dA = r\, dr\, d\theta$

- $V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$

قانون نقدر احسب منه ال velocity in laminar flow at any location (any r)

r → radius المكان يلي الاقي فيه ال v

R → R for the Pipe

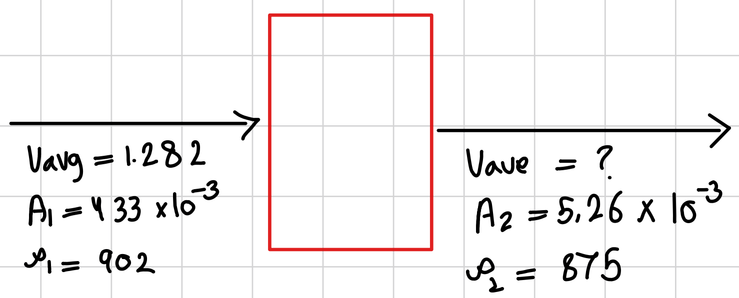


2.6-2. ***Flow of Liquid in a Pipe and Mass Balance.*** A hydrocarbon liquid enters a simple flow system shown in Fig. 2.6-1 at an average velocity of 1.282 m/s, where $A_1 = 4.33 \times 10^{-3}\ m^2$ and $\rho_1 = 902\ kg/m^3$. The liquid is heated in the process and the exit density is 875 kg/m³. The cross-sectional area at point 2 is $5.26 \times 10^{-3}\ m^2$. The process is steady state.

(a) Calculate the mass flow rate m at the entrance and exit.

(b) Calculate the average velocity v in 2 and the mass velocity G in 1.

Ans. (a) $m_1 = m_2 = 5.007$ kg/s, (b) $G_1 = 1156\ kg/s \cdot m^2$



note $\rho_1 \neq \rho_2$ so we can't use volume flow rate in the balance.

a) $\dot{m}_1 = \rho V_1 A_1$

$$= \frac{1.282\ \cancel{m}}{\boxed{s}} \cdot \frac{902\ \boxed{kg}}{\cancel{m^3}} \cdot 4.33\times10^{-3}\ \cancel{m^2} \rightarrow 5\ \frac{kg}{s}$$

steady state $\dot{m}_{in} = \dot{m}_{out}$

$\dot{m}_2 \rightarrow 5\ \frac{kg}{s}$

b) $\dot{m}_2 = V_2 A_2 \rho_2$

$$\frac{5\ \cancel{kg}}{\boxed{s}} \cdot \frac{m^3}{875\ \cancel{kg}} \cdot \frac{\boxed{m}}{5.26\times10^{-3}\ \cancel{m^2}} \rightarrow 1.086\ m/s$$

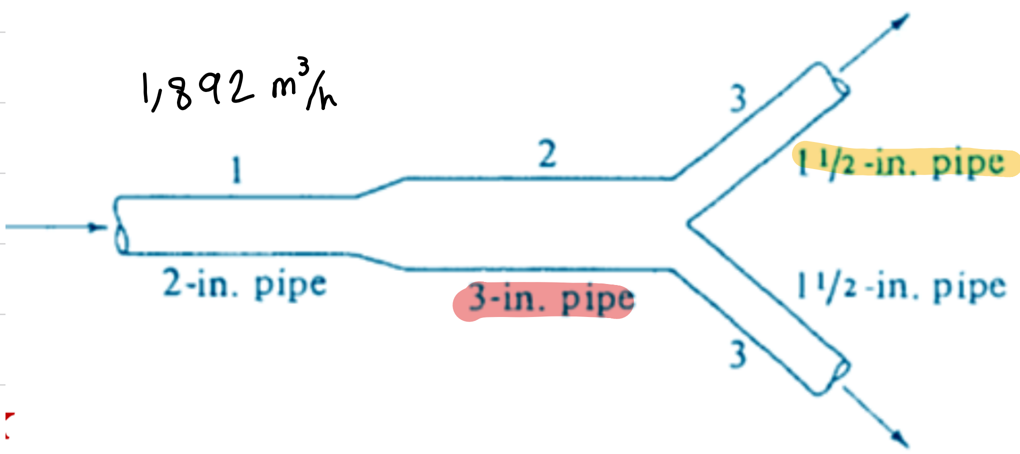
$G_1 = V_1 \rho_1$

$$\frac{\cancel{m}\ 1.282}{s} \cdot \frac{902\ kg}{\cancel{m^3}\ m^2} \rightarrow 1156.3$$

$G_2 = V_2 \rho_2$

$$\frac{1.086\ m}{s} \cdot \frac{875\ kg}{m^3} \rightarrow 950.2$$

2.6-7 ***Mass Balance for Flow of Sucrose Solution.*** A 20 wt % sucrose (sugar) solution having a density of 1074 kg/m³ is flowing through the same piping system as Example 2.6-1 (Fig. 2.6-2). The flow rate entering pipe 1 is 1.892 m³/h. The flow divides equally in each of pipes 3. Calculate the following:
(a) The velocity in m/s in pipes 2 and 3.
(b) The mass velocity G kg/m² · s in pipes 2 and 3.



Nominal Pipe Size (in.)	Outside Diameter in.	Outside Diameter mm	Schedule Number	Wall Thickness in.	Wall Thickness mm	Inside Diameter in.	Inside Diameter mm	Inside Cross-Sectional Area ft²	Inside Cross-Sectional Area m² × 10⁴
1/8	0.405	10.29	40	0.068	1.73	0.269	6.83	0.00040	0.3664
			80	0.095	2.41	0.215	5.46	0.00025	0.2341
1/4	0.540	13.72	40	0.088	2.24	0.364	9.25	0.00072	0.6720
			80	0.119	3.02	0.302	7.67	0.00050	0.4620
3/8	0.675	17.15	40	0.091	2.31	0.493	12.52	0.00133	1.231
			80	0.126	3.20	0.423	10.74	0.00098	0.9059
1/2	0.840	21.34	40	0.109	2.77	0.622	15.80	0.00211	1.961
			80	0.147	3.73	0.546	13.87	0.00163	1.511
3/4	1.050	26.67	40	0.113	2.87	0.824	20.93	0.00371	3.441
			80	0.154	3.91	0.742	18.85	0.00300	2.791
1	1.315	33.40	40	0.133	3.38	1.049	26.64	0.00600	5.574
			80	0.179	4.45	0.957	24.31	0.00499	4.641
1 1/4	1.660	42.16	40	0.140	3.56	1.380	35.05	0.01040	9.648
			80	0.191	4.85	1.278	32.46	0.00891	8.275
1 1/2	1.900	48.26	40	0.145	3.68	1.610	40.89	0.01414	13.13
			80	0.200	5.08	1.500	38.10	0.01225	11.40
2	2.375	60.33	40	0.154	3.91	2.067	52.50	0.02330	21.65
			80	0.218	5.54	1.939	49.25	0.02050	19.05
2 1/2	2.875	73.03	40	0.203	5.16	2.469	62.71	0.03322	30.89
			80	0.276	7.01	2.323	59.00	0.02942	27.30
3	3.500	88.90	40	0.216	5.49	3.068	77.92	0.05130	47.69
			80	0.300	7.62	2.900	73.66	0.04587	42.61
3 1/2	4.000	101.6	40	0.226	5.74	3.548	90.12	0.06870	63.79
			80	0.318	8.08	3.364	85.45	0.06170	57.35
4	4.500	114.3	40	0.237	6.02	4.026	102.3	0.08840	82.19
			80	0.337	8.56	3.826	97.18	0.07986	74.17
5	5.563	141.3	40	0.258	6.55	5.047	128.2	0.1390	129.1
			80	0.375	9.53	4.813	122.3	0.1263	117.5
6	6.625	168.3	40	0.280	7.11	6.065	154.1	0.2006	186.5
			80	0.432	10.97	5.761	146.3	0.1810	168.1
8	8.625	219.1	40	0.322	8.18	7.981	202.7	0.3474	322.7
			80	0.500	12.70	7.625	193.7	0.3171	294.7

ρ is constant ☺ I can use volume flow rate in Balance

a) $\dot{V}_1 = \dot{V}_2$

$\dot{V}_2 = \frac{1,892\ m^3}{h}$

$\dot{V}_2 = V_2 A_2 \Rightarrow \frac{1,892\ \cancel{m^3}}{h} \Big| \frac{1}{47,69 \times 10^{-4}\ \cancel{m^2}}$

$\boxed{V_2 = 396,7\ m/h}$

$\dot{V}_1 = \frac{\dot{V}_3}{2}$

divides equally in each of pipes 3.

$\dot{V}_3 = 0,946\ \frac{m^3}{h}$

$V_3 = \frac{0,946\ \cancel{m^3}}{h} \Big| \frac{1}{13,13\ \cancel{m^2} \times 10^{-4}} = \boxed{720\ m/h}$

b) $G_2 = \rho V_2$

$\frac{1074\ kg}{\cancel{m^3}\ m^2} \Big| \frac{396,7\ \cancel{m}}{h} \Rightarrow 4,26 \times 10^5\ \frac{kg}{m^2 \cdot h}$

$G_3 = \frac{1074\ kg}{\cancel{m^3}\ m^2} \Big| \frac{720\ \cancel{m}}{h} \Rightarrow 7,7\ \frac{kg}{m^2 \cdot h}$

2.7 Barnoulli's equation

$$\underbrace{\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_p}_{\text{energy بريد ال}} = \underbrace{\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + w_T + f}_{\text{energy بقلل ال}}$$

* momintum balance

· momintum $\equiv m \times v \equiv \frac{kg\ m}{s}$

$\Sigma f = ma$

$\Sigma f = m \frac{dv}{dt}$

$\Sigma f = \frac{d\, mv}{dt}$

$\Sigma f = \frac{dP}{dt}$ ← momintum : net force ≡ the rate change in momintum

$$\underset{acc}{\frac{dP}{dt}} = \underset{gen + con}{\Sigma f} + \underset{in}{\dot{m}_{in} V_{in}} - \underset{out}{\dot{m}_{out} V_{out}}$$

$$\frac{kg\ m}{s^2} = \frac{kg\ m}{s^2} + \frac{kg\ m}{s^2} + \frac{kg\ m}{s^2}$$

* steady state acc = zero

$$0 = \Sigma f + \dot{m}_{in} V_{in} - \dot{m}_{out} V_{out}$$

- β (correction factor)

flux $= \iint v\, m \cos\theta \overset{?}{=} m\, v_{avg}$

$\times \beta$ correction factor for assuming V is equle everywhere.

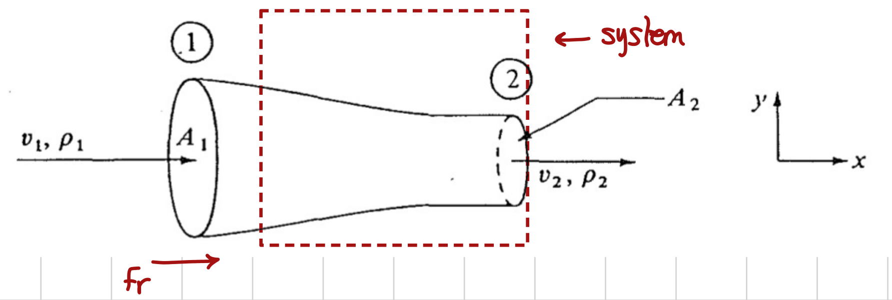
- note :- β_{in} مش شرط $= \beta_{out}$

اذا السؤال ما حطا عنها $\beta = 1$

- Force
  - Body force: Wight
  - Surface force: Pressure Force, Shear force

reaction force : action and reaction
not chemical reaction force

EXAMPLE 2.8-2. *Momentum Balance for Horizontal Nozzle*
Water is flowing at a rate of 0.03154 m^3/s through a horizontal nozzle shown in Fig. 2.8-1 and discharges to the atmosphere at point 2. The nozzle is attached at the upstream end at point 1 and frictional forces are considered negligible. The upstream ID is 0.0635 m and the downstream 0.0286 m. Calculate the resultant force on the nozzle. The density of the water is 1000 kg/m^3.



one direction flow
لو بيسوي G balanc X
and y balanc

Also if don't have information about the wight

① assuming the Fr direction to the right.

② Balance (stady state)

$$= P_1 A_1 - \cancel{P_2 A_2} + \dot{m}_1 V_1 + \dot{m}_2 V_2 + \Sigma F$$

$P_2 = 0$ gage

* ما تنسى اشارة اتجاه ال velocity

P_1 from B.E

$$\frac{P}{\rho} + \frac{V^2}{2} + \cancel{gz} + \cancel{W_p} = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + \cancel{gz} + \cancel{W_T} + \cancel{F}$$

$$P_1 = \frac{\rho(V_2^2 - V_1^2)}{2}$$

$Q_1 = V_1 A_1$

note $Q_1 = Q_2$
stady state and
constant ρ

$$V_1 = \frac{0.03154\ m^3}{s} \Big| \frac{1}{\frac{\pi}{4}(0.0635)^2 m^2} = \boxed{9.96\ m/s}$$

$$P_1 = \frac{\rho(V_2^2 - V_1^2)}{2}$$

$Q_2 = V_2 A_2$

$$V_2 = \frac{0.03154\ m^3}{s} \Big| \frac{1}{\frac{\pi}{4}(0.0286)^2} = \boxed{49.1\ m/s}$$

$$P_1 = \frac{1000(49.1^2 - 9.96^2)}{2}$$

$$\boxed{= 1155.8 \times 10^3\ Pa}$$

$\dot{m}_1 = ?$

$$\frac{0.03154\ m^3}{s} \Big| \frac{1000\ kg}{m^3} = \boxed{31.54\ \frac{kg}{s}}$$

steady state
$\dot{m}_1 = \dot{m}_2$ ✓

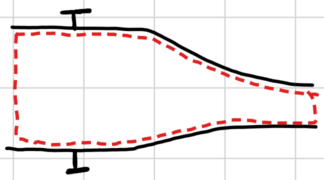
$$0 = P_1 A_1 - \cancel{P_2 A_2} + \dot{m}_1 V_1 - \dot{m}_2 V_2 + \Sigma F_x$$

$P_2 = 0$ gage

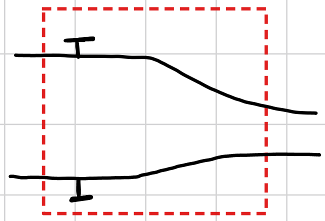
$$-\Sigma F = 1155.8 \times 10^3 \left(\frac{\pi}{4}(0.0635)^2\right) + 31.54(9.96) - 31.54(49.1)$$

$$-\Sigma F = 2425.9$$

عكس الاتجاه يلي فرضته بالأول $\Sigma F = -2425.9$ to the left direction

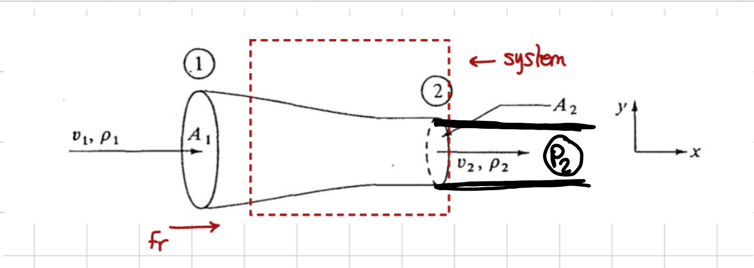


اذا حددت هيك ال System
تاني انا مضطرة احسب ال frection



ما بحس ال frection

* Example 2-8-2:- *تعديل معطيات من الدكتور*



$P_1 = 1155.8 \times 10^3$ Pa
$P_2 = 9.9 \times 10^5$ Pa
$A_1 = 3.12 \times 10^{-3}\ m^2$
$A_2 = 6.42 \times 10^{-4}\ m^2$
$V_1 = 9.96$ m/s
$V_2 = 49.1$
$\dot{m}_1 = 31.54 = \dot{m}_2$

$$\sum F = \dot{m}_1 V_1 - \dot{m}_2 V_2$$

$$\underset{\rightarrow}{Fr} + \underset{\rightarrow}{P_1 A_1} - \underset{\leftarrow}{P_2 A_2} = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

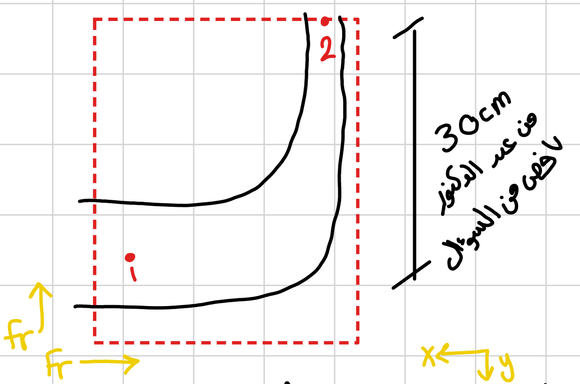
$$Fr + 1155.8 \times 10^3 \times 3.12 \times 10^{-3} - (9.9 \times 10^5) \times (6.42 \times 10^{-4}) = 31.54\,(49.1 - 9.96)$$

$fr = -1736$

fr to the left

*لما كان عندي P_2 كانت Fr اعلى

2.8-1. ***Momentum Balance in a Reducing Bend.*** Water is flowing at steady state through the reducing bend in Fig. 2.8-3. The angle $\alpha_2 = 90°$ (a right-angle bend). The pressure at point 2 is 1.0 atm abs. The flow rate is 0.020 m^3/s and the diameters at points 1 and 2 are 0.050 m and 0.030 m, respectively. ~~Neglect frictional and gravitational forces~~. Calculate the resultant forces on the bend in newtons and lb force. Use $\rho = 1000\ kg/m^3$.



$P_{1\,gage} = ?$

$P_{2\,gage} = $ zero

Volume flow rate $= 0.02\ \frac{m^3}{s}$ $\left(0.02\ \frac{m^3}{s} \times 1000\ \frac{kg}{m^3} = 20\ \frac{kg}{s}\right)$

$D_1 = 0.05\ m$, $A_1 = 1.96 \times 10^{-3}\ m^2$

$D_2 = 0.03\ m$, $A_2 = 7.07 \times 10^{-4}\ m^2$

given: that mass of the nozel and water = 2000g

من عند الدكتور

$$V_1 = \frac{Q}{Area} = \frac{0.02\ m^3/s}{1.96 \times 10^{-3}\ m^2} = 10.2\ m/s$$

$$V_2 = \frac{0.02\ m^3/s}{7.07 \times 10^{-4}} = 28.28\ m/s$$

PE to find P_1

$$\frac{P_1}{\omega} + \frac{V_1^2}{2} + gh_1 + \cancel{\omega_{Pump}} = \cancel{\frac{P_2}{\omega}} + \frac{V_2^2}{2} + gh_2 + \cancel{\omega_T} + \cancel{f}$$

$$P_1 = \left(\frac{V_2^2 - V_1^2}{2} + g(h_2 - h_1)\right)\rho$$

$$\left(\frac{(28.28)^2 - 10.2^2}{2} + 9.8 \times 0.3\right) \times 1000 = 350799.2$$

$$\Sigma F = \dot m_2 V_2 - \dot m_1 V_1$$

x balance. $-R_x - P_1 A_1 = 0 - \dot m_1(-V_1)$

$-R_x - (350799.2)(1.96 \times 10^{-3}) = -20(-10.2)$

$R_x = - 866.3$

to the left ←

y balance. $-R_y + g(\text{mass}_{n+w}) = \dot m_2(-V_2)$

$-R_y + 9.8(2) = 20(-28.28)$

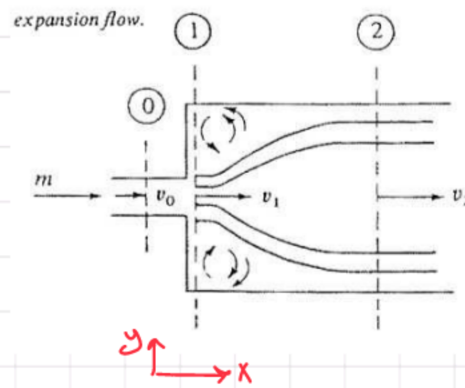
$R_y = 585.2$ ↑

$R_{net} = 1044.5$

$\tan^{-1} \frac{R_y}{R_x} = 34°$

EXAMPLE 2.8-4. Friction Loss in a Sudden Enlargement

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 2.8-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint:* Assume that $p_0 = p_1$ and $v_0 = v_1$. Make a mechanical-energy balance between points 0 and 2 and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)



- momentum balance 1,2

$\Sigma F = m_2 U_2 - m_1 V_1$

$P_1 A_1 - P_2 A_2 = m_2 V_2 - m_1 V_1$

$A_1 = A_2$, $m_2 = m_1 = m_0$

$A_2 (P_1 - P_2) = m_1 (U_2 - V_1)$

$A_2 (P_0 - P_2) = V_0 \wp A_0 \left(\frac{V_0 A_0}{A_2} - V_0\right)$

$\frac{P_2 - P_0}{\wp} = V_0^2 \frac{A_0}{A_2} \left(\frac{A_0}{A_2} - 1\right) \Rightarrow \boxed{\frac{P_2 - P_1}{\wp} = V_0^2 \left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right)}$

$\dot{m}_1 = \dot{m}_2$ steady state and constant $\wp$

$\dot{Q}_0 = \dot{Q}_2$ (volume flowrate)

$V_0 A_0 = V_2 A_2$

$V_2 = \frac{V_0 A_0}{A_2}$

$\dot{m} = \wp V_0 A_0$

- energy balance 0 – 2

$\left(\frac{P_0}{\wp} + \frac{V_0^2}{2} + \cancel{g z_0}\right) m_0 = \left(\frac{P_2}{\wp} + \frac{V_2^2}{2} + \cancel{g z_2}\right) m_2 + f$ $\quad \div m \quad$ note $\frac{f}{m} = F$

$\frac{P_2 - P_0}{\wp} = \frac{V_0^2 - V_2^2}{2} - F$

$\frac{P_2 - P_0}{\wp} = \frac{V_0^2 - \frac{V_0^2 A_0^2}{A_1^2}}{2} - F \Rightarrow \boxed{\frac{P_2 - P_1}{\wp} = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_1^2}\right) - F}$

$V_0^2 \left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right) = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2}\right) - F$

$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2}\right) - V_0^2 \left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right)$

$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2} - 2\frac{A_0}{A_2} + 2\frac{A_0^2}{A_2^2}\right)$

$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2} - 2\frac{A_0}{A_2} + 2\frac{A_0^2}{A_2^2}\right)$

if $X = \frac{A_0}{A_2}$ $\quad X^2 - 2X + 1$

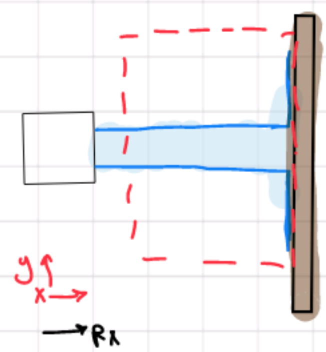
$(X-1)(X-1) = (X-1)^2$

$F = \frac{V_0^2}{2}\left(\frac{A_0}{A_2} - 1\right)^2$ #

2.8-3. *Force of Water Stream on a Wall.* Water at 298 K discharges from a nozzle and travels horizontally hitting a flat vertical wall. The nozzle has a diameter of 12 mm and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. Neglecting frictional resistance of the air on the jet, calculate the force in newtons on the wall.

Ans. $-R_x = 4.059$ N

note ρ water at 298K $= 996.9 \frac{kg}{m^3}$



momentumbalance X

$\Sigma f = m_2 v_2 - m_1 v_1$

$R_x = 0 - m_1 v_1$

$R_x = -m_1 v_1$

$m_1 = \rho v_1 A_1$

$\frac{6 \cancel{m}}{s} \left(996.9 \frac{kg}{\cancel{m^3}}\right)\left(\frac{\pi}{4}(12 \times 10^{-3})^2\right)\cancel{m^2}$

$= 676.5 \times 10^{-3} \frac{kg}{s}$

$R_x = -676.5 \times 6 \times 10^{-3}$

$= -4.06$

momentumbalance y

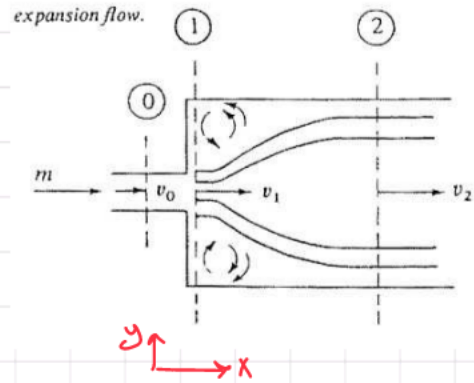
$\Sigma f = m_2 v_2 - m_1 v_1$

$F_y =$ zero

EXAMPLE 2.8-4. Friction Loss in a Sudden Enlargement

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 2.8-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint*: Assume that $p_0 = p_1$ and $v_0 = v_1$. Make a mechanical-energy balance between points 0 and 2 and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)

expansion flow.



- momentum balance 1,2

$$\Sigma F = m_2 U_2 - m_1 V_1$$

$$P_1 A_1 - P_2 A_2 = m_2 V_2 - m_1 V_1$$

$$A_1 = A_2 \quad , \quad m_2 = m_1 = m_0$$

$$A_2 (P_1 - P_2) = m_1 (V_2 - V_1)$$

$$A_2 (P_0 - P_2) = V_0 \rho A_0 \left(\frac{V_0 A_0}{A_2} - V_0\right)$$

$$\frac{P_2 - P_0}{\rho} = V_0^2 \frac{A_0}{A_2}\left(\frac{A_0}{A_2} - 1\right) \Rightarrow \boxed{\frac{P_2 - P_1}{\rho} = V_0^2\left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right)}$$

$\dot{m}_1 = \dot{m}_2$ steady state and constant ρ

$\dot{Q}_0 = \dot{Q}_2$ (volume flowrate)

$$V_0 A_0 = V_2 A_2$$

$$V_2 = \frac{V_0 A_0}{A_2}$$

$$\dot{m} = \rho V_0 A_0$$

- energy balance 0 – 2

$$\left(\frac{P_0}{\rho} + \frac{V_0^2}{2} + \cancel{g z_0}\right) m_0 = \left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}\right) m_2 + f \qquad \div m \qquad \text{note } \frac{f}{m} = F$$

$$\frac{P_2 - P_0}{\rho} = \frac{V_0^2 - V_2^2}{2} - F$$

$$\frac{P_2 - P_0}{\rho} = \frac{V_0^2 - V_0^2 \frac{A_0^2}{A_1^2}}{2} - F \Rightarrow \boxed{\frac{P_2 - P_1}{\rho} = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_1^2}\right) - F}$$

$$V_0^2\left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right) = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2}\right) - F$$

$$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2}\right) - V_0^2\left(\frac{A_0}{A_2} - \frac{A_0^2}{A_2^2}\right)$$

$$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2} - 2\frac{A_0}{A_2} + 2\frac{A_0^2}{A_2^2}\right)$$

$$F = \frac{V_0^2}{2}\left(1 - \frac{A_0^2}{A_2^2} - 2\frac{A_0}{A_2} + 2\frac{A_0^2}{A_2^2}\right)$$

if $X = \frac{A_0}{A_2}$

$$X^2 - 2X + 1$$

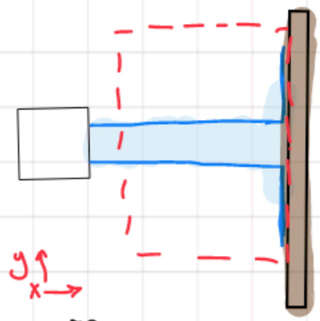
$$(X-1)(X-1) = (X-1)^2$$

$$F = \frac{V_0^2}{2}\left(\frac{A_0}{A_2} - 1\right)^2 \quad \#$$

2.8-3. *Force of Water Stream on a Wall.* Water at 298 K discharges from a nozzle and travels horizontally hitting a flat vertical wall. The nozzle has a diameter of 12 mm and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. Neglecting frictional resistance of the air on the jet, calculate the force in newtons on the wall.

Ans. $-R_x = 4.059$ N

note ρ water at 298 K $= 996.9 \frac{kg}{m^3}$



momentum balance x

$$\Sigma F = m_2 v_2 - m_1 v_1$$
$$R_x = 0 - m_1 v_1$$
$$R_x = -m_1 v_1$$

$$m_1 = \rho v_1 A_1$$
$$\frac{6\cancel{m}}{s}\left(996.9 \frac{kg}{\cancel{m^3}}\right)\left(\frac{\pi}{4}(12\times10^{-3})^2\right)\cancel{m^2}$$
$$= 676.5 \times 10^{-3} \frac{kg}{s}$$

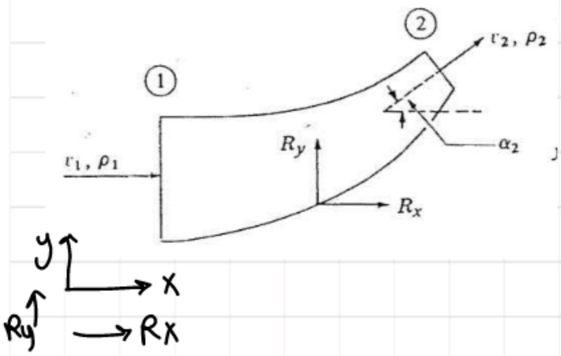
$$R_x = -676.5 \times 6 \times 10^{-3}$$
$$= -4.06$$

momentum balance y

$$\Sigma F = m_2 v_2 - m_1 v_1$$
$$F_y = \text{zero}$$

EXAMPLE 2.8-3. Momentum Balance in a Pipe Bend

Fluid is flowing at steady state through a reducing pipe bend, as shown in Fig. 2.8-3. Turbulent flow will be assumed with frictional forces negligible. The volumetric flow rate of the liquid and the pressure p_2 at point 2 are known as are the pipe diameters at both ends. Derive the equations to calculate the forces on the bend. Assume that the density ρ is constant.



momentum balance X:

$$R_x + P_1 A_1 - P_2 A_2 \cos\theta = \dot{m}_2 V_2 \cos\theta - \dot{m}_1 V_1$$

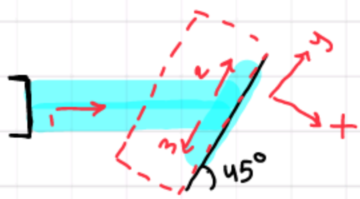
momentum balance y:

$$R_y - P_2 A_2 \sin\theta = \dot{m}_2 V_2 \sin\theta$$

$$R = \sqrt{R_y^2 + R_x^2} \quad , \quad \theta = \tan^{-1} \frac{R_x}{R_y}$$

2.8-5. *Force of Stream on a Wall.* Repeat Problem 2.8-3 for the same conditions except that the wall is inclined 45° with the vertical. The flow is frictionless. Assume no loss in energy. The amount of fluid splitting in each direction along the plate can be determined by using the continuity equation and a momentum balance. Calculate this flow division and the force on the wall.

Ans. $m_2 = 0.5774$ kg/s, $m_3 = 0.09907$ kg/s, $-R_x = 2.030$ N, $-R_y = -2.030$ N (force on wall).



حاله خاصه عشان مابدي Ry,Rx

mass balance $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

momentum balance X

$0 = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_1 V_1 \cos\theta$

$(\dot{m}_1 - \dot{m}_3)\cancel{V} - \dot{m}_3 \cancel{V_3} - m_1 \cancel{V_1} \cos\theta$

assume no loss energy

$V_1 = V_2 = V_3$

$\dot{m}_1 - \dot{m}_3 - \dot{m}_3 - \dot{m}_1 \cos\theta$

$-2\dot{m}_3 + \dot{m}_1(1-\cos\theta) = 0$

$\dot{m}_3 = \dfrac{\dot{m}_1(1-\cos\theta)}{2}$

$V = 6.0$ m/s

$D = 12 \times 10^{-3}$ m

$\rho = 996.9 \dfrac{kg}{m^3}$

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

$\dot{m}_1 = 996.9 \dfrac{kg}{m^3} \left| \dfrac{\pi (12\times10^{-3})^2 m^2}{4} \right| \dfrac{6\,m}{s}$

$\dot{m}_1 = \dot{m}_2 + \dfrac{\dot{m}_1(1-\cos\theta)}{2}$

$\dot{m}_1 = 0.68$ kg/s

$2m_1 = 2m_2 + m_1(1-\cos\theta)$

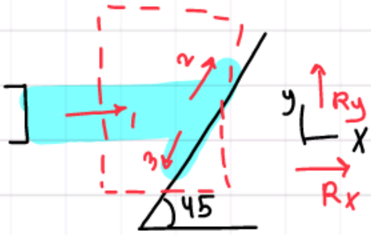
$2m_1 - m_1(1-\cos\theta) = 2\dot{m}_2$

$m_1(2-1+\cos\theta) = 2\dot{m}_2$

$\dot{m}_2 = \dfrac{m_1(1+\cos\theta)}{2}$

$\dot{m}_3 = \dfrac{0.68(1-0.71)}{2} = 0.0986$ kg/s

$\dot{m}_2 = \dfrac{0.68(1+0.71)}{2} = 0.5814$ kg/s



نختار ميلك ال system عشان
احسب Rx و Ry.

momentum balance

$$x: \; R_x = \dot{m}_2 V_2 \cos 45 - \dot{m}_3 V_3 \cos 45 - \dot{m}_1 V_1$$

↳ Direction velocity

$$R_x = 0.5814\,(6)\,\underbrace{\cancel{\cos 45}}_{0.71} - 0.0986\,(6)\,\underbrace{\cancel{\cos 45}}_{0.71} - 0.68\,(6)$$

$$R_x = -2.02 \text{ N}$$

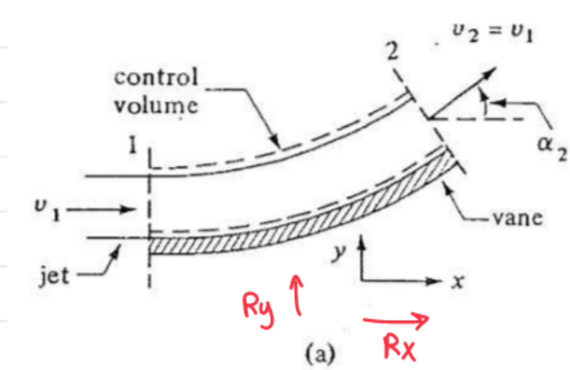
$$y: \; R_y = \dot{m}_2 V_2 \sin 45 - \dot{m}_3 V_3 \sin 45$$

$$0.5814\,(6)\,0.71 - 0.0986 \times 6 \times 0.71$$

$$R_y = 2.05$$

EXAMPLE 2.8-5. *Force of Free Jet on a Curved, Fixed Vane*

A jet of water having a velocity of 30.5 m/s and a diameter of 2.54×10^{-2} m is deflected by a smooth, curved vane as shown in Fig. 2.8-5a, where $\alpha_2 = 60°$. What is the force of the jet on the vane? Assume that $\rho = 1000\ kg/m^3$.



(a)

velocity = 30.5 m/s

D = 2.54×10^{-2} m

α = 60

$$\dot{m}_1 = \frac{30.5\ \cancel{m}}{s} \Bigg| \frac{(2.54\times10^{-2})^2\ \pi\ \cancel{m^2}}{4} \Bigg| \frac{1000\ kg}{\cancel{m^3}}$$

$$\dot{m}_1 = 15{,}5\ \frac{kg}{s} = \dot{m}_2$$

X:

$$R_x = \dot{m}_2 v_2 \cos 60 - \dot{m}_1 v_2$$

$$15{,}5\,(30{,}5)\cos 60 - 15{,}5\,(30{,}5) = -236.38$$

y:

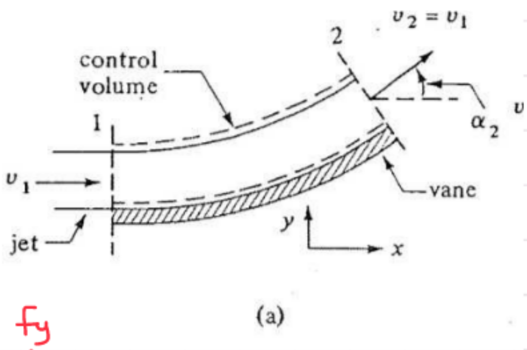
$$R_y = \dot{m}_2 v_2 \sin 60 - 0$$

$$15{,}5\,(30{,}5)\sin 60$$

$$R_y = 409{,}4$$

2.8-6. ***Momentum Balance for Free Jet on a Curved, Fixed Vane.*** A free jet having a velocity of 30.5 m/s and a diameter of 5.08×10^{-2} m is deflected by a curved, fixed vane as in Fig. 2.8-5a. However, the vane is curved downward at an angle of 60° instead of upward. Calculate the force of the jet on the vane. The density is 1000 kg/m³.

Ans. $-R_x = 942.8$ N, $-R_y = 1633$ N



Velocity = 30.5 m/s
Diameter = 5.08×10^{-2} m
α = 60
σ = 1000 kg/m³

F_y
F_x

mass balance

$\dot{m}_1 = \dot{m}_2$

$$\dot{m}_1 = \frac{1000\,\text{kg}}{m^3}\left|\frac{30.5\,m}{s}\right|\frac{(5.08\times10^{-2})^2\,\pi\,m^2}{4}$$

$\dot{m}_1 = 61.8$ kg/s

momentum balance

X:

$F_x = \dot{m}_2 V_2 \cos 60 - \dot{m}_1 V_1$

$61.8 \times 30.5 \times \cos 60 - 61.8 \times 30.5$

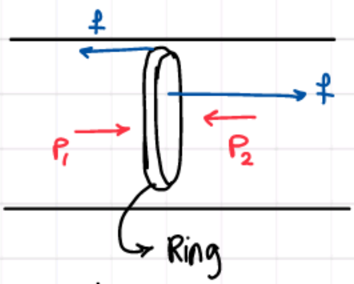
$F_x = -942.17$ N

y:

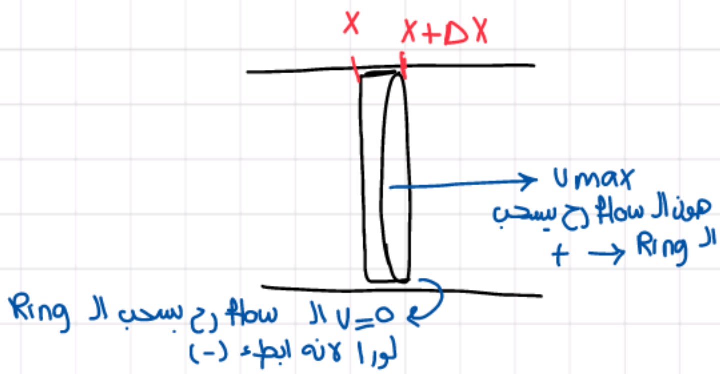
$F_y = \dot{m}_2 V_2 \sin 60 - 0$

$F_y = 1633$

* Shell balance



وصار ال system تبعنا
عشان احسب ال friction



Area $= \boxed{2\pi r\, \Delta X}$

balance

$$P_1(2\pi r\Delta r)\Big|_{X} - P_2(2\pi r\Delta r)\Big|_{X+\Delta X} + \tau(2\pi r_1\Delta X)\Big|_{r} - \tau\, 2\pi r_2\Delta X\Big|_{r+\Delta r} =$$

$$\underbrace{\cancel{\dot m_2 v_2 - \dot m_1 v_2}}_{= zero}$$ zero

$$\dot m_1 = \dot m_2$$

$\div\ \Delta X\, \Delta r$

$$\frac{\cancel{2\pi}(rP_1 - rP_2)}{\Delta X} + \cancel{2\pi}\,\frac{(\tau r|_{r} - \tau r|_{r+\Delta r})}{\Delta r}$$

تعريف المشتقة.

Pressure drop $P_1 - P_2$

$$\frac{\Delta P}{\Delta X}\, r = \frac{d\tau r}{dr}$$

$$\frac{\Delta P}{L} r = \frac{d\tau r}{dr}$$

$$\int \frac{\Delta P}{L} r \, dr = \int d\tau r$$

$$\frac{\Delta P}{L} \frac{r^2}{2} + C = \tau r$$

$$\tau = \frac{\Delta P}{L} \frac{r}{2} \qquad \tau = -\mu \frac{dV}{dr}$$

$$\int -\mu \, dV = \int \frac{\Delta P}{L} \frac{r}{2} \, dr$$

$$-\mu V = \frac{\Delta P}{L} \frac{r^2}{4}$$

$$V = \frac{\Delta P \, r^2}{L - \mu 4} + C$$

when $r = R$ $\quad V = 0$

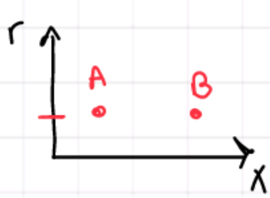
$$0 = \frac{-\Delta P \, R^2}{L \mu 4} + C$$

$$C = \frac{\Delta P \, R^2}{L \mu 4}$$

$$V = -\frac{\Delta P \, r^2}{L \mu 4} + \frac{\Delta P \, R^2}{L \mu 4}$$

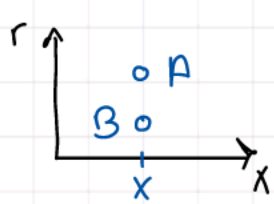
$$V = \frac{\Delta P \, R^2 (R^2 - r^2)}{L \mu 4} \times \frac{R^2}{R^2}$$

$$V = \frac{\Delta P \, R^2}{L \mu 4}\left(1 - \frac{r^2}{R^2}\right) \Longrightarrow \text{Velocity at any } r.$$



$V_A = V_B$

same r



velocity $A \neq B$

same X.

to find u_{max}

@ $r=0$

$$u = \frac{\Delta P R^2}{L\mu 4}\left(1 - \frac{r^2}{R^2}\right)$$

$$u_{max} = \frac{\Delta P R^2}{L\mu 4}$$

$$V_{avg} = \frac{1}{A}\int_A u \, dA$$

$$V_{avg} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \frac{\Delta P R^2}{L\mu 4}\left(1 - \frac{r^2}{R^2}\right) r \, dr \, d\theta$$

$$V_{avg} = \frac{\Delta P}{\pi \mu 4 L}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)(2\pi)$$

$$\frac{\Delta P}{\pi \mu 4 L}\left(\frac{2R^2}{4} - \frac{R^2}{4}\right) 2\pi$$

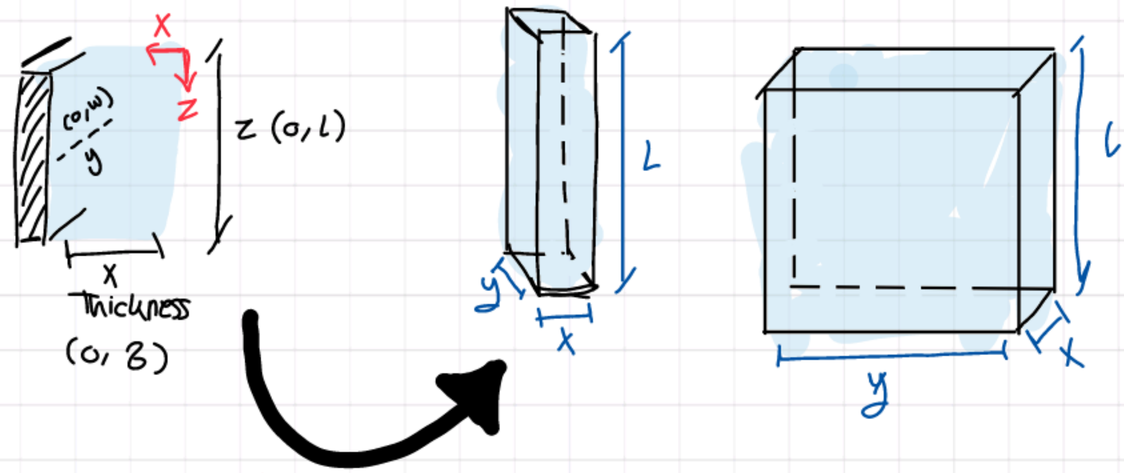
$$V_{avg} = \frac{\Delta P}{\cancel{\pi}\mu 4 L}\frac{R^2}{\cancel{4}\,2}\cancel{2\pi}$$

$$V_{avg} = \frac{V_{max}}{2} \quad \#$$

* falling film

↳ Thickness سُمك

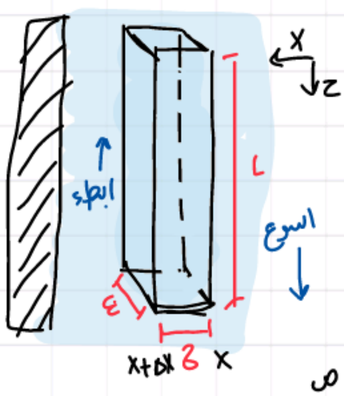
↳ application : cooling tour



note :- x start at Liquid- air.

* Balance momentum و اعمل ال system shell بدي اخذ

جزء من ال film



$f_g = mg$

$= \rho L w \delta g$

$f = \tau \text{ area}$

f_g

$\rho L w \delta g + \tau w L\Big|_{x} - \tau w L\Big|_{x+\Delta x} = 0 \Big) \div L w \Delta x$

$\rho g + \frac{\tau_x}{\Delta x} - \frac{\tau_{x+1}}{\Delta x}$

$\rho g = \frac{d\tau}{dx}$

$\int \rho g \, dx = \int d\tau$

$\rho g x + c = \tau$ at $x=0$ $\tau=0$ $c=0$

$$\tau = \rho g x$$

$$-\mu \frac{du}{dx} = \rho g x$$

$$\int du = \int \frac{\rho g x \, dx}{-\mu}$$

$$u = \frac{-\rho g}{\mu} \frac{x^2}{2} + c$$ at $x = \delta$ $u = 0$ $c = \frac{\rho g \delta^2}{2\mu}$

$$u = \frac{-\rho g}{\mu} \frac{x^2}{2} + \frac{\rho g \delta^2}{2\mu}$$

$$u = \frac{\rho g}{2\mu}(-x^2 + \delta^2)$$

$$u = \frac{\rho g \delta^2}{2\mu}\left(1 - \frac{x^2}{\delta^2}\right)$$ $\rightarrow$ velocity at any x

u_{max} $x = 0$

$$u_{max} = \frac{\rho g \delta^2}{2\mu}$$

$$V_{avg} = \frac{1}{A} \int_A u \, dA$$

$$V_{avg} = \frac{1}{w\delta} \int_0^w \int_0^\delta \frac{\rho g \delta^2}{2\mu}\left(1 - \frac{x^2}{\delta^2}\right) dx\,dy$$

$$V_{avg} = \frac{1}{\cancel{w}\delta}\left(\frac{\rho g \delta^2}{2\mu}\right)\left(\frac{3\delta^3}{3\delta^2} - \frac{\delta^3}{3\delta^2}\right)(\cancel{w})$$

$$V_{avg} = \frac{1}{\cancel{\delta}}\left(\frac{\rho g \delta^2}{2\mu}\right)\left(\frac{2\cancel{\delta^3}}{3\cancel{\delta^2}}\right)\cancel{\delta}$$

$$V_{avg} = V_{max} \times \frac{2}{3}$$

chapter 3 Differential equations of continuity

Type of time Derivatives and vector Notation

→ Partial time

→ fixed point x, y, z.

$$\frac{\partial \rho}{\partial t}$$

→ Total time

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \rho}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \rho}{\partial z}\underline{\frac{\partial z}{\partial t}}$$

→ velocity for the system

function of time and velocity

→ substantial time

derivative follows the motion

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}v_x + \frac{\partial \rho}{\partial y}v_y + \frac{\partial \rho}{\partial z}\underline{v_z}$$

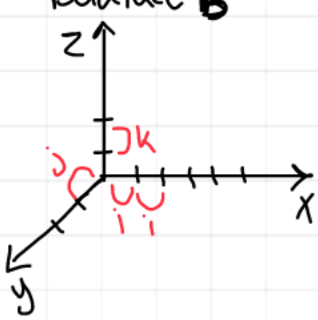
→ velocity for the shell.

→ scalars magnitude but no direction

conc, volume, time, temp, ...

→ Vectors magnitude and direction.

Boldface **B**



$$\vec{B} = iB_x + jB_y + kB_z$$

$$r.\vec{B} = \vec{B}.r$$

$$\vec{B}.\vec{A} = \vec{A}.\vec{B} = A.B\cos\phi_{AB}$$

$$(\vec{A}.\vec{B})\vec{C} \neq \vec{A}(\vec{B}.\vec{C})$$

Differential operation with scalar and vectors

↳ grad of a scalar

$$\nabla \rho = i\frac{\partial \rho}{\partial x} + j\frac{\partial \rho}{\partial y} + k\frac{\partial \rho}{\partial z}$$

↳ div of vector

$$\nabla \cdot v = \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \Big\} \text{ scaler}$$

↳ Laplacin of a scalar

$$\nabla^2 \rho = \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2}$$

* $\nabla rs = r\nabla s + s\nabla r$
* $(\nabla \cdot vs) = \nabla s \cdot v + s(\nabla \cdot v)$
* $v \cdot \nabla s = v_x \frac{\partial s}{\partial x} + v_y \frac{\partial s}{\partial y} + v_z \frac{\partial s}{\partial z}$

Differential eq of continuity

$$\frac{\partial \rho}{\partial t} = -\nabla\rho \cdot \vec{v} - \rho(\nabla \cdot v)$$

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial \rho}{\partial x} v_x + \frac{\partial \rho}{\partial y} v_y + \frac{\partial \rho}{\partial z} v_z\right) - \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot v)$$

at conc ρ

$$\frac{D\rho}{Dt} = 0$$

$$-\rho(\nabla \cdot v) = 0$$

$$\nabla \cdot V = 0$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Continuity eq in cylinderical coordinate

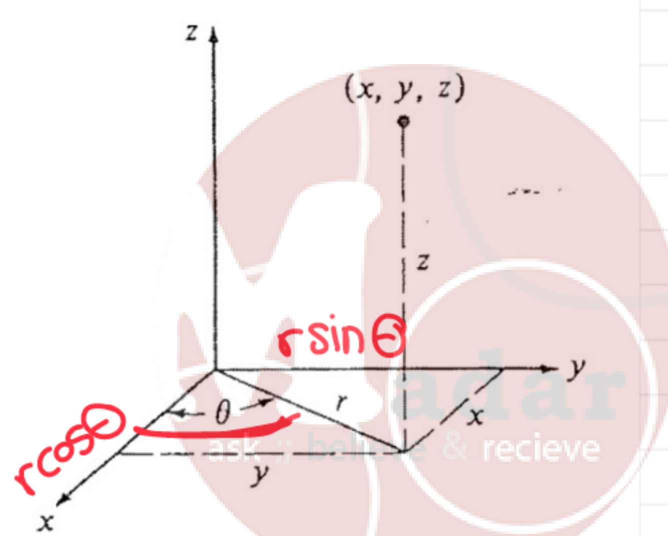
$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$\theta = \tan^{-1}\frac{y}{x}$

$r = +\sqrt{x^2+y^2}$



(a)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

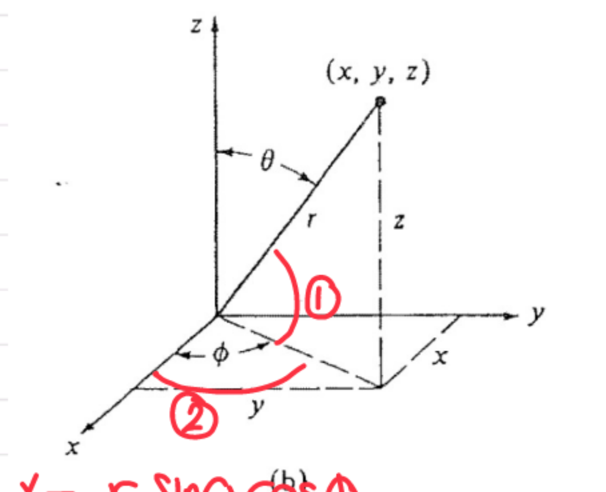
Continuity eq in spherical coordinates

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$r = +\sqrt{x^2+y^2+z^2}$$



$x = r\sin\theta\cos\phi$

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial\phi} = 0$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \qquad \phi = \tan^{-1}\frac{y}{z}$$

* momentum balance.

- **Equations of motion for the *x, y,* and *z* components are obtained:**

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)-\frac{\partial p}{\partial x}+\rho g_x \quad \text{x Direction}$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)-\frac{\partial p}{\partial y}+\rho g_y \quad \text{y Direction}$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right)-\frac{\partial p}{\partial z}+\rho g_z \quad \text{z Direction}$$

Equations of Motion for Newtonian Fluids

Shear-stress components for Newtonian fluids

- in cylindrical coordinates
- In rectangular coordinates
- in spherical coordinates

$$\tau_{xx} = -2\mu\frac{\partial v_x}{\partial x}+\frac{2}{3}\mu(\nabla\cdot\mathbf{v})$$

$$\tau_{yy} = -2\mu\frac{\partial v_y}{\partial y}+\frac{2}{3}\mu(\nabla\cdot\mathbf{v})$$

$$\tau_{zz} = -2\mu\frac{\partial v_z}{\partial z}+\frac{2}{3}\mu(\nabla\cdot\mathbf{v})$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z}+\frac{\partial v_z}{\partial y}\right)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x}+\frac{\partial v_x}{\partial z}\right)$$

$$(\nabla\cdot\mathbf{v}) = \left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)$$

constant ρ and μ, newton's low ⇒ Navier-stokes eq

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\underline{\underline{\rho g_x}}$$

X Direction

- **Similar equations are obtained for the y and z components.**

Conc ρ, μ, Newtonian fluid (Lamenar)

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\underline{\underline{\rho g_r}} \qquad (3.7\text{-}40)$$

r Direction

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\underline{\underline{\rho g_\theta}} \qquad (3.7\text{-}41)$$

θ Direction

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\underline{\underline{\rho g_z}} \qquad (3.7\text{-}42)$$

z Direction.

sphereical coordinate

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)=-\frac{\partial p}{\partial r}$$

$$+\mu\left(\nabla^2 v_r-\frac{2}{r^2}v_r-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}-\frac{2}{r^2}v_\theta\cot\theta-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_r \quad \textbf{(3.7-43)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\mu\left(\nabla^2 v_\theta+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta}{r^2\sin^2\theta}-\frac{2}{r^2}\frac{\cos\theta}{\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_\theta \quad \textbf{(3.7-44)}$$

$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_\phi v_r}{r}+\frac{v_\theta v_\phi}{r}\cot\theta\right)=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$

$$+\mu\left(\nabla^2 v_\phi-\frac{v_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right)+\rho g_\phi \quad \textbf{(3.7-45)}$$

EXAMPLE 3.8-1. Laminar Flow Between Horizontal Parallel Plates

Derive the equation giving the velocity distribution at steady state for laminar flow of a constant-density fluid with constant viscosity which is flowing between two flat and parallel plates. The velocity profile desired is

at a point far from the inlet or outlet of the channel. The two plates will be considered to be fixed and of infinite width, with the flow driven by the pressure gradient in the x direction.

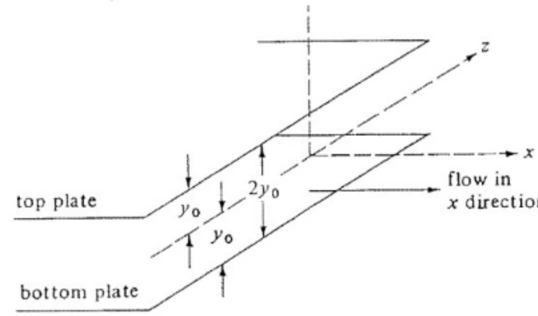


FIGURE 3.8-1. *Flow between two parallel plates in Example 3.8-1.*

steady state

no Vz

$$\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + \cancel{v_x \frac{\partial v_x}{\partial x}} + \cancel{v_y \frac{\partial v_x}{\partial y}} + \cancel{v_z \frac{\partial v_x}{\partial z}}\right) = \mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}\right) - \underbrace{\frac{\partial p}{\partial x}}_{\text{Driving force}} + \rho g_x$$

no change in Vx in X direction

↳ no Vy

$$0 = \mu\left(\frac{\partial^2 V_x}{\partial y^2}\right) - \frac{\partial P}{\partial x} + \rho g_x$$

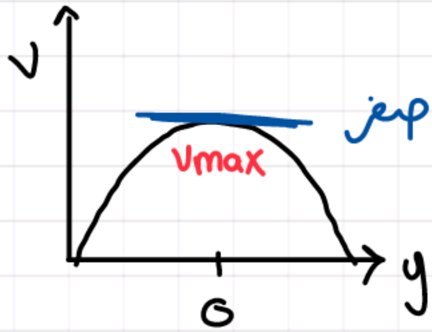
$$0 = \mu\left(\frac{\partial^2 V_x}{\partial y^2}\right) - \frac{dP}{dx}$$

$$\frac{dP}{dx}\frac{1}{\mu} = \frac{\partial^2 V_x}{\partial y^2}$$

$$\frac{dP}{dx}\frac{1}{\mu} = \frac{\partial}{\partial y}\left(\frac{dV_x}{\partial y}\right)$$

$$\int \frac{dP}{dx}\frac{dy}{\mu} = \int \partial\left(\frac{dV_x}{\partial y}\right)$$

$$\frac{dP}{dx}\frac{1}{\mu}y + C = \frac{\partial U_x}{\partial y}$$



المشتقة = صفر

at $y = 0$ $\quad \frac{\partial U_x}{\partial y} = 0$

$C = 0$

$$\frac{dP}{dx}\frac{1}{\mu}y = \frac{\partial U_x}{\partial y}$$

$$\int \frac{dP}{dx}\frac{1}{\mu}y\,\partial y = \int \partial U_x$$

$$V_x = \frac{dP}{dx}\frac{1}{\mu}\frac{y^2}{2} + C$$

at $y = y_0$

$V = 0$

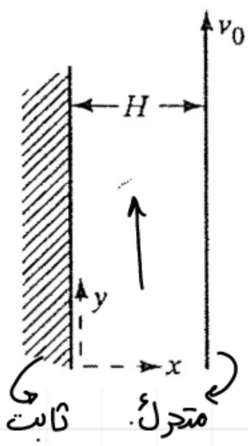
$$C = -\frac{dP}{dx}\frac{1}{\mu}\frac{y_0^2}{2}$$

$$\therefore V_x = -\frac{dP}{dx}\frac{1}{\mu}\frac{1}{2}\left(-y^2 + y_0^2\right)$$

$$\boxed{V_x = -\frac{dP}{dx}\frac{1}{2\mu}\left(1 - \frac{y^2}{y_0^2}\right)}$$

EXAMPLE 3.8-2. *Laminar Flow Between Vertical Plates with One Plate Moving*

A Newtonian fluid is confined between two parallel and vertical plates as shown in Fig. 3.8-2 (W6). The surface on the left is stationary and the other is moving vertically at a constant velocity v_0. Assuming that the flow is laminar, solve for the velocity profile.



$$\rho\left(\cancel{\frac{\partial v_y}{\partial t}} + \cancel{v_x}\cancel{\frac{\partial v_y}{\partial x}} + v_y\cancel{\frac{\partial v_y}{\partial y}} + \cancel{v_z}\cancel{\frac{\partial v_y}{\partial z}}\right) = \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \cancel{\frac{\partial^2 v_y}{\partial y^2}} + \cancel{\frac{\partial^2 v_y}{\partial z^2}}\right) - \frac{\partial p}{\partial y} + \rho g_y$$

steady state　no v_x　no change in V with dy　$v_z = 0$ no change in V with dz

$$0 = \mu\left(\frac{\partial^2 V_y}{\partial x^2}\right) - \frac{\partial P}{\partial y} + \rho g_y$$

$$\frac{dP}{dy}\frac{1}{\mu} = \frac{\partial^2 V_y}{\partial x^2}$$

$$\frac{dP}{dy}\frac{1}{\mu} = \frac{\partial}{\partial x}\left(\frac{V_y}{\partial x}\right)$$

$$\int \frac{dP}{dy}\frac{1}{\mu}\,dx = \int \partial\left(\frac{\partial V_y}{\partial x}\right)$$

$$\frac{dP}{dy}\frac{1}{\mu}x + C_1 = \frac{\partial V_y}{dx}$$

$$\frac{dP}{dy}\frac{1}{\mu}\frac{x^2}{2} + C_1 x + C_2 = V_y$$

at $X = 0$ $V = 0$
at $X = H$ $V = V_0$

- $0 + 0 + C_2 = 0$
- $\frac{dP}{dy}\frac{1}{\mu}\frac{H^2}{2} + C_1 H = V_0$

$$C_1 = \frac{V_0}{H} - \frac{dP}{dy}\frac{1}{\mu}\frac{H}{2}$$

$$\therefore V_y = \frac{dP}{dy}\frac{1}{\mu}\frac{x^2}{2} + \frac{xV_0}{H} - \frac{dP}{dy}\frac{1}{\mu}\frac{HX}{2}$$

EXAMPLE 3.8-3. Laminar Flow in a Circular Tube

Derive the equation for steady-state viscous flow in a horizontal tube of radius r_0, where the fluid is far from the tube inlet. The fluid is incompressible and μ is a constant. The flow is driven in one direction by a constant-pressure gradient.

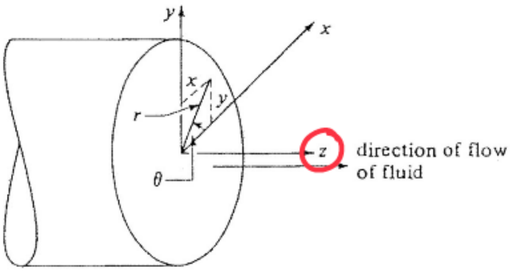


FIGURE 3.8-3. *Horizontal flow in a tube in Example 3.8-3.*

S.S $\quad v_r = 0 \quad v_\theta = 0$

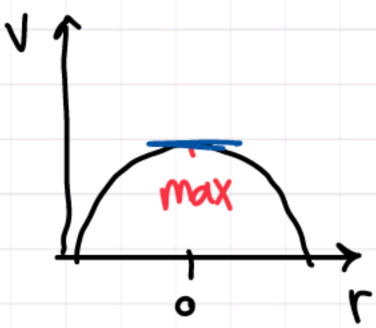
$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + v_r \cancel{\frac{\partial v_z}{\partial r}} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}} + v_z \cancel{\frac{\partial v_z}{\partial z}}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

$$0 = -\frac{\partial P}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right]$$

$$\frac{dP}{dz} = \mu \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial v_z}{\partial r}\right]$$

$$\int \frac{dP}{dz}\frac{r}{\mu}\,\partial r = \int \partial\left[r\frac{\partial v_z}{\partial r}\right]$$

$$\frac{dP}{dz}\frac{r^2}{2\mu} + C = r\frac{\partial v_z}{\partial r}$$



$r = 0 \quad \frac{dv}{dr} = 0 \quad \boxed{C = 0}$

$$\int \frac{dP}{dz}\frac{r}{2\mu}\,dr = \int dv_z$$

$$\frac{dP}{dz}\frac{r^2}{4\mu} + C = v_z$$

at $r = R \quad v = 0$

$$v_z = -\frac{dP}{dz}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

$$V_{avg} = \frac{1}{A} \iint_A v \, dA$$

$$V_{avg} = \frac{1}{\pi R} \int_0^{2\pi}\int_0^R \frac{-dP}{dz} \frac{1}{4\mu} (R^2 - r^2) \, r \, dr \, d\theta$$

$$\frac{1}{\cancel{\pi} R} \frac{-dP}{dz} \frac{1}{4\mu} \left(\frac{R^2 \cancel{R}^2}{2} - \frac{\cancel{R}^4}{4} \right) 2\cancel{\pi}$$

$$\frac{-dP}{dz} \frac{1}{4\mu \cancel{R}} \left(\frac{\cancel{R^4}\ 3}{\cancel{4}\ 2} \right) \times \cancel{2}$$

$$V_{avg} = \frac{-dP}{dz} \frac{R^3}{8\mu}$$

$$\int_{P_1}^{P_2} dP = \int_0^L \frac{-8\mu V_{avg}}{R^3} dz$$

$$P_2 - P_1 = -\frac{8\mu V_{avg}}{R^3} L$$

$$P_1 - P_2 = \frac{8\mu V_{avg} L}{R^3} \xrightarrow{R = \frac{D}{2}} P_1 - P_2 = \frac{32 \mu L V_{avg}}{D^3}$$

EXAMPLE 3.8-4. Laminar Flow in a Cylindrical Annulus

Derive the equation for steady-state laminar flow inside the annulus between two concentric horizontal pipes. This type of flow occurs often in concentric pipe heat exchangers.

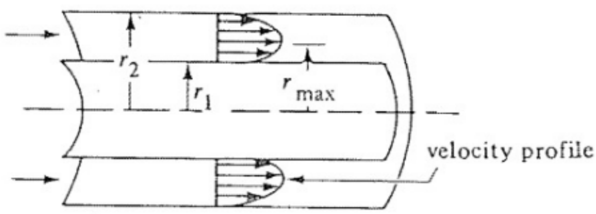


FIGURE 3.8-4. *Flow through a cylindrical annulus.*

steady state $\quad v_r = 0 \quad v_\theta = 0 \quad \frac{\partial v_z}{\partial z} \to 0$

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

$0 = \frac{dv_z}{d\theta} \qquad 0 = \frac{dv_z}{dz}$

$$\frac{-\partial P}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \rho g_z = 0$$

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = \frac{dP}{dz}$$

$$\int \partial\left(r\frac{\partial v_z}{\partial r}\right) = \int \frac{dP}{dz}\frac{r}{\mu}\,\partial r$$

$$r\frac{\partial v_z}{\partial r} = \frac{dP}{dz}\frac{r^2}{2\mu} + C$$

$\frac{dV}{dr} = 0 \quad V = r_{max}$

$$C = -\frac{dP}{dz}\frac{r_{max}^2}{2\mu}$$

$$r\frac{\partial v_z}{\partial r} = \frac{dP}{dz}\frac{1}{2\mu}\left(r^2 - r_{max}^2\right)$$

$$\int dv_z = \int \frac{dP}{dz}\frac{1}{2\mu}\left(r - \frac{r_{max}^2}{r}\right)dr$$

$$v_z = \frac{dP}{dz}\frac{1}{2\mu}\left(\frac{r^2}{2} - r_{max}^2 \ln r\right) + C \qquad \boxed{r = r_1 \quad v = 0}$$

$$C = -\frac{dP}{dz}\frac{1}{2\mu}\left(\frac{r_1^2}{2} - r_{max}^2 \ln r_1\right)$$

$$v_z = \frac{1}{2\mu}\frac{dP}{dz}\left(\frac{r^2 - r_1^2}{2} - r_{max}^2 \ln\frac{r}{r_1}\right)$$

Also $\Rightarrow$ $v_z = \frac{1}{2\mu}\frac{dP}{dz}\left(\frac{r^2 - r_2^2}{2} - r_{max}^2 \ln\frac{r}{r_2}\right)$ $\quad \boxed{r = r_2 \quad v = 0}$

$r_{max} = ?$

$$v_z = \frac{1}{2\mu}\frac{dP}{dz}\left(\frac{r^2 - r_1^2}{2} - r^2_{max}\ln\frac{r}{r_1}\right)$$

$$v_z = \frac{1}{2\mu}\frac{dP}{dz}\left(\frac{r^2 - r_2^2}{2} - r^2_{max}\ln\frac{r}{r_2}\right)$$

$$\left(\frac{r^2 - r_1^2}{2} - r^2_{max}\ln\frac{r}{r_1}\right) = \left(\frac{r^2 - r_2^2}{2} - r^2_{max}\ln\frac{r}{r_2}\right)$$

$$\frac{\cancel{r^2} - r_1^2 - \cancel{r^2} + r_2^2}{2} = r^2_{max}\left(-\ln\frac{r}{r_2} + \ln\frac{r}{r_1}\right)$$

$$\frac{r_2^2 - r_1^2}{2} = r^2_{max}\left(\ln\frac{\frac{\cancel{r}}{r_1}}{\frac{\cancel{r}}{r_2}}\right)$$

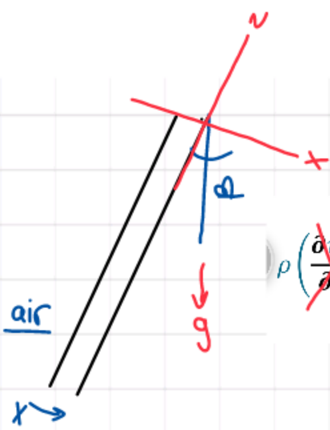
$$\frac{r_2^2 - r_1^2}{2} = r^2_{max}\ln\frac{r_2}{r_1})$$

$$r_{max} = \sqrt{\frac{r_2^2 - r_1^2}{2\ln(r_2/r_1)}}$$

3.8-4. ***Velocity Profile in Falling Film and Differential Momentum Balance.*** A Newtonian liquid is flowing as a falling film on an inclined flat surface. The surface makes an angle of β with the vertical. Assume that in this case the section being considered is sufficiently far from both ends that there are no end effects on the velocity profile. The thickness of the film is δ. The apparatus is similar to Fig. 2.9-3 but is not vertical. Do as follows.

(a) Derive the equation for the velocity profile of v_z as a function of x in this film using the differential momentum balance equation.
(b) What are the maximum velocity and the average velocity?
(c) What is the equation for the momentum flux distribution of τ_{xz}? [Hint: Can Eq. (3.7-19) be used here?]

Ans. (a) $v_z = (\rho g \delta^2 \cos\beta / 2\mu)[1 - (x/\delta)^2]$
(c) $\tau_{xz} = \rho g x \cos\beta$



no y

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + v_x \cancel{\frac{\partial v_z}{\partial x}} + v_y \cancel{\frac{\partial v_z}{\partial y}} + v_z \cancel{\frac{\partial v_z}{\partial z}}\right) = \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right) - \cancel{\frac{\partial p}{\partial z}} + \rho g_z$$ → Driving Force.

no v_x $\qquad$ $0 = \frac{\partial v_z}{\partial z}$ $\qquad$ open to atm

$g_z = g\cos\beta$

$$0 = \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z$$

$$-\frac{\rho}{\mu}(g\cos\beta) = \frac{\partial}{\partial x}\left(\frac{\partial v_z}{\partial x}\right)$$

$$C + \frac{-\rho}{\mu}(g\cos\beta)\, x = \frac{\partial v_z}{\partial x} \qquad \boxed{C = 0}$$

$$\int \partial v_z = \int \frac{-\rho}{\mu} g\cos\beta\, x\, dx$$

$$v_z = -\frac{\rho}{\mu} g\cos\beta \frac{x^2}{2} + c$$

$$c = +\frac{\rho}{\mu} g\cos\beta \frac{\delta^2}{2}$$

$$v_z = \frac{\rho}{2\mu} g\cos\beta(-x^2 + \delta^2)$$

$$v_{max} = \frac{\rho}{2\mu} g\cos\beta\, \delta^2$$

when $x = 0$

$J = \mu \frac{dv}{dx}$

0 air

open air

$\therefore \frac{dv}{dx} = 0$

when $x = \delta$ $\quad v_z = 0$

$$V_{avg} = \frac{1}{A} \iint_A v \, dA$$

$$V_{avg} = \frac{1}{w\delta} \frac{\rho g \cos\beta}{2\mu} \int_0^w \int_0^\delta \delta^2 - x^2 \, dx \, dy$$

$$V_{avg} = \frac{1}{\cancel{w}\,\cancel{\delta}} \frac{\rho g \cos\beta}{\cancel{2}\mu} \frac{\cancel{2}}{3} \delta^{\cancel{3}\,2} \cancel{w}$$

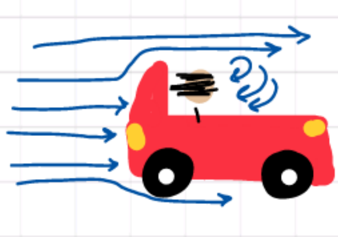
$$U_{max} = \frac{\rho}{2\mu} g \cos\beta \, \delta^2$$

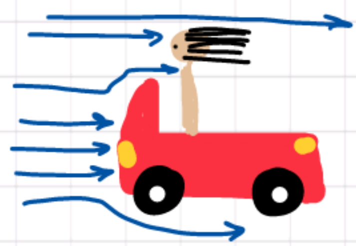
$$V_{avg} = \frac{2}{3} U_{max}.$$

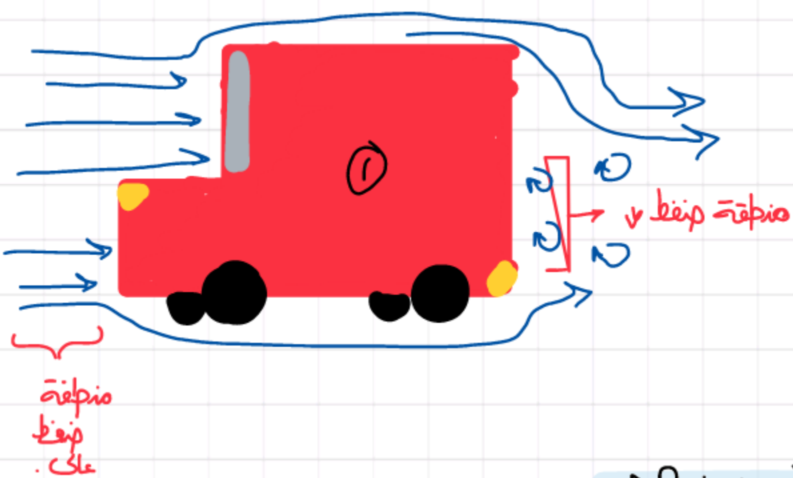
c) $\tau = \mu \dfrac{dU_z}{dx}$

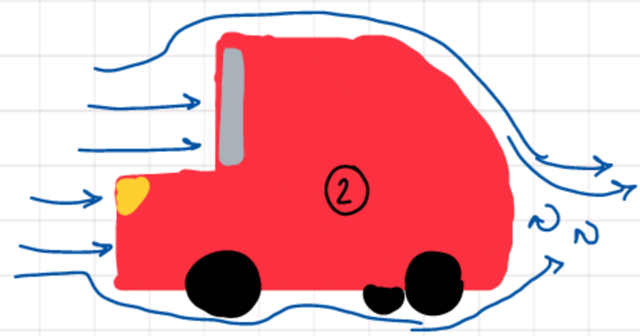
$$-\frac{\rho}{\mu} (g\cos\beta) x = \frac{\partial U_z}{\partial x}$$

$$\tau = -\rho g \cos\beta x$$





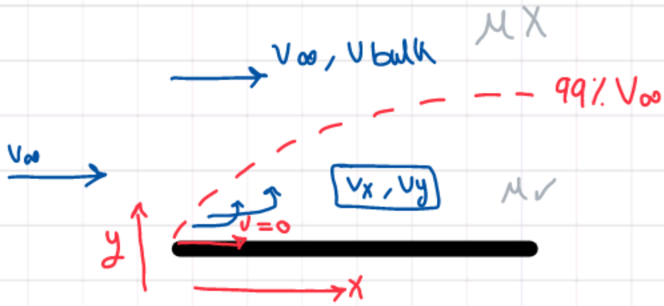




$$\Delta P \text{ in } \underline{\underline{1}} > \Delta P\ \underline{\underline{2}}$$

Boundar layer

↳ The region close to the solid surface, the fluid motion affected by the solid surface



$$Re = \frac{\rho V_\infty X}{\mu}$$

Re تتغير مع X ممكن اكون Laminar ← Turbulent

مو زي ال Pipe

Re → ↓ 5×10^5 Laminar
↳ ↑ 3×10^6 Turbulent

Re ↑ x ↑
δ ↑ $\sqrt{x}$ ↑

$$\delta \text{ (Thickness)} = \frac{5x}{\sqrt{Re}} = 5\sqrt{\frac{\mu x}{\rho V_\infty}}$$

B.L eq:-

constant
μ, ρ.
Laminar.

neg

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho}\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\cancel{v_x \frac{\partial v_y}{\partial x}} + \cancel{v_y \frac{\partial v_y}{\partial y}} = \cancel{\frac{\mu}{\rho}\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right)} - \cancel{\frac{1}{\rho}\frac{\partial p}{\partial y}} \to 0$$

الاشتقاق مش
مطلوب !!

$$\boxed{\begin{aligned} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} &= \frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2} \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0 \end{aligned}}$$

B.C ⇒ at $y=0$ $v_x=0$ $v_y=0$

at $y=\infty$ $v_x = v_\infty$

$$\tau = \frac{F}{A}$$

$$F_D = \tau A$$

$$F_D = \int_0^b \int_0^L \tau \, dx \, dz$$

$$F_D = 0.664 b \sqrt{\mu V_\infty^3 \rho L}$$

$$F_D = C_D \frac{V_\infty^2}{2} \rho A$$

$$C_D = 1.328 \sqrt{\frac{\mu}{L \rho V_\infty}}$$

$\mu = 1.005 \times 10^{-3}$

$\rho = 998.7$

3.10-1. ***Laminar Boundary Layer on Flat Plate.*** Water at 20°C is flowing past a flat plate at 0.914 m/s. The plate is 0.305 m wide.

(a) Calculate the Reynolds number 0.305 m from the leading edge to determine if the flow is laminar.

(b) Calculate the boundary-layer thickness at $x = 0.152$ and $x = 0.305$ m from the leading edge.

(c) Calculate the total drag on the 0.305-m-long plate.

Ans. (a) $N_{Re,L} = 2.77 \times 10^5$, (b) $\delta = 0.0029$ m at $x = 0.305$ m

A) $Re = \frac{V_\infty \rho x}{\mu}$ $\quad \frac{0.914 \times 998.7 \times 0.305}{1.005 \times 10^{-3}} = 277022 < 5 \times 10^5$ ✓ Laminar

B) Re at 0.152 $\quad Re = \frac{0.914 \times 0.152 \times 998.7}{1.005 \times 10^{-3}} = 138057$

$$\delta = \frac{5x}{\sqrt{Re}} = \frac{0.152 \times 5}{\sqrt{138057}} = 2.05 \times 10^{-3}$$

$$\delta = \frac{5 \times 0.305}{\sqrt{277022}} = 2.9 \times 10^{-3}$$

C) $F_D = 0.664 b \sqrt{\mu V_\infty^3 \rho L}$

$$0.664 \times 0.305 \sqrt{1.005 \times 10^{-3} \; 0.914^3 \times 988.7 \times 0.305}$$

$$= 9.742 \times 10^{-3}$$

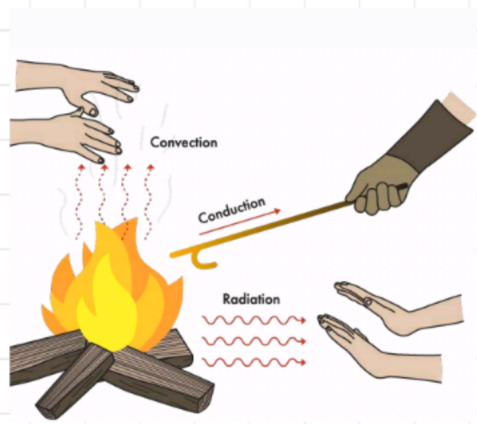
* Topic 2

* Drying
* alcohol distillation
* burning fuel
* evaporation.

heat transfer
- → Thermal energy transfer
- → Driving Force = ΔT
- → $\uparrow$ temp → $\downarrow$ temp

Thermal energy balance → find temp profile and heat flux.

Mechanism of heat transfer
- → Conduction
- → Convection
- → Radiation



* Conduction (Liquid, solid, gas)

→ transfer of energy of motion between adjacent molecules.

$\uparrow$ energy
$\uparrow$ motion

Hotter → adjacent molecules

transfer energy

Ex: ① through wall of exchangers, refrigerator
② freezing of ground during winter
③ heat treatment of steel forgings.

Convection (Liquid or gas)

↳ heat transfer by **bulk transport** and mixing of macroscopic elements of warmer portions to cooler portions of gas or liquid

* often → energy trans between fluid and solid surface

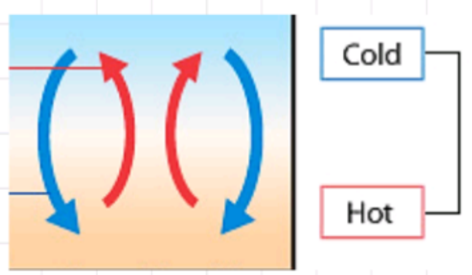
Convection → **Forced** : fluid forced to past a solid surface by pump, fan, other..

Ex: cooling cup of coffee by blowing over the surface
cooking in vessel being stirred

→ **Natural** : warmer or cooler fluid next to the solid surface → circulation becouse of the density diffrent. result from the temp diffrent

Temp ↑
ρ ↓

Temp ↓
ρ ↑



* **Radiation** (No physical medium)

↳ transfer energy throgh space by means of electromagnetic waves
same as electromagnatic light waves transfer light

*دائماً موجوده ولكن بهملها.

Ex: heat to the earth from sun
cooking of food by red-hat electric heaters

Fourier's law of heat conduction

$$\frac{q_x}{A} = -k\,\frac{dT}{dx}$$

heat flow + in a given direction the temp decrease in this direction

- q_x: heat transfer W
- A: cross-section area m^2
- T: temp K
- x: distanc m
- k: thermal conductivity $\frac{W}{m \cdot K}$

A.3-15 Thermal Conductivities of Building and Insulating Materials

Material	ρ $\left(\frac{kg}{m^3}\right)$	t (°C)	$k (W/m \cdot K)$		
Asbestos	577		0.151 (0°C)	0.168 (37.8°C)	0.190 (93.3°C)
Asbestos sheets	889	51	0.166		
Brick, building		20	0.69		
Brick, fireclay			1.00 (200°C)	1.47 (600°C)	1.64 (1000°C)
Clay soil, 4% H_2O	1666	4.5	0.57		
Concrete, 1:4 dry			0.762		
Corkboard	160.2	30	0.0433		
Cotton	80.1		0.055 (0°C)	0.061 (37.8°C)	0.068 (93.3°C)
Felt, wool	330	30	0.052		
Fiber insulation board	237	21	0.048		
Glass, window			0.52–1.06		
Glass wool	64.1	30	0.0310 (−6.7°C)	0.0414 (37.8°C)	0.0549 (93.3°C)
Ice	921	0	2.25		
Magnesia, 85%	271		0.068 (37.8°C)	0.071 (93.3°C)	0.080 (204.4°C)
	208		0.059 (37.8°C)	0.062 (93.3°C)	0.066 (148.9°C)
Oak, across grain	825	15	0.208		
Pine, across grain	545	15	0.151		
Paper			0.130		
Rock wool	192		0.0317 (−6.7°C)	0.0391 (37.8°C)	0.0486 (93.3°C)
	128		0.0296 (−6.7°C)	0.0395 (37.8°C)	0.0518 (93.3°C)
Rubber, hard	1198	0	0.151		
Sand soil					
4% H_2O	1826	4.5	1.51		
10% H_2O	1922	4.5	2.16		
Sandstone	2243	40	1.83		
Snow	559	0	0.47		
Wool	110.5	30	0.036		

EXAMPLE 4.1-1. *Heat Loss Through an Insulating Wall*

Calculate the heat loss per m²· of surface area for an insulating wall composed of 25.4-mm-thick fiber insulating board, where the inside temperature is 352.7 K and the outside temperature is 297.1 K.

$$\frac{q}{A} = -k\,\frac{\Delta T}{\Delta X}$$

الطول.

Fiber insulation board	237	21	0.048
Glass, window			0.52–1.06

$$\frac{q}{A} = -0.048\,\frac{(297.1 - 52.7)}{25.4}$$

$$\boxed{105.1\ \mathrm{w/m^2}}$$

Thermal conductivity $\underline{\underline{k}}$ ↳ Gases. Random motion ↗ colliding ↗ exchanging heat, momentum.

* ↓ size ↑ thermal conductivities. (moves faster)
* ↑ $\sqrt{T}$, ↑ k
* k is indepentant of P

but at vaccum k = 0

TABLE 4.1-1. *Thermal Conductivities of Some Materials at 101.325 kPa (1 Atm) Pressure (k in W/m · K)*

Substance	Temp. (K)	k	Ref.	Substance	Temp. (K)	k	Ref.
Gases				Solids			
Air	273	0.0242	(K2)	Ice	273	2.25	(C1)
	373	0.0316		Fire claybrick	473	1.00	(P1)
H_2	273	0.167	(K2)	Paper	—	0.130	(M1)
n-Butane	273	0.0135	(P2)	Hard rubber	273	0.151	(M1)
Liquids				Cork board	303	0.043	(M1)
Water	273	0.569	(P1)	Asbestos	311	0.168	(M1)
	366	0.680		Rock wool	266	0.029	(K1)
Benzene	303	0.159	(P1)	Steel	291	45.3	(P1)
	333	0.151			373	45	
Biological materials and foods				Copper	273	388	(P1)
					373	377	
Olive oil	293	0.168	(P1)	Aluminum	273	202	(P1)
	373	0.164					
Lean beef	263	1.35	(C1)				
Skim milk	275	0.538	(C1)				
Applesauce	296	0.692	(C1)				
Salmon	277	0.502	(C1)				
	248	1.30					

Thermal conductivity $\underline{\underline{k}}$

↳ Liquid ↑ energy molecules collide with lower energy

* ↑ Temp ↑ k linear ($k = aT + b$)
* k independent on P.

↳ Solid → metallic → by free electrons which moves through the solid

→ all other solid → vibration between adjacent atoms.

varies quite widely

* metallic solid ↑↑ k Cu, Al
* insulated non metallic ↓↓ k rock wool

Convective coefficient

$$q = hA(T_w - T_f)$$

Area

heat transfer

w

Temp

solid

k

avg , bulk temp fluid k

h: convective cofficient

↳ $W/(m^2 \cdot K)$

function of

→ system geometry

→ flow velocity

→ fluid properties

→ temperature diffrent

- many case empirical correlations

↳ cannot be theoretical

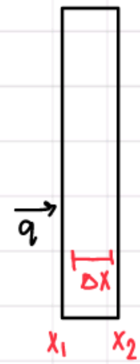
- h = film coefficient :- when fluid flow by a surface there is athin layer (film) of fluid adjacent to the wall presenting most of the resistance to heat transfer

codensing steam , condensilting organic h↑

air h↓

Conduction transfer

(A) Through a Flat Slab or Wall



- if A and k constant

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

↳ constant in solid

$$\int_{x_1}^{x_2} \frac{q_x}{A}\, dx = \int_{T_1}^{T_2} -k\, dT$$

$$\hookrightarrow \frac{q_x}{A}\Delta X = -k\,\Delta T$$

$$q = \frac{\Delta T}{R} \quad \text{↳ Driving Force.} \qquad q = \frac{(T_1 - T_2)kA}{\Delta X}$$

$$\boxed{R = \frac{\Delta X}{kA}}$$

- k not constant $k = AT + b$.

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

$$\int_{x_1}^{x_2} \frac{q_x}{A}\, dX = -\int_{x_1}^{x_2} a + bT\, dT$$

$$\frac{q_x}{A}\Delta X = \left(aT + \frac{bT^2}{2}\Big]_{T_1}^{T_2}\right) -$$

$$\frac{q_x}{A}(x_1 - x_2) = aT_2 + \frac{bT_2^2}{2} - aT_1 - \frac{bT_1^2}{2}$$

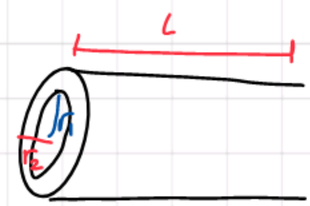
$$(x_1 - x_2)\frac{q}{A} = a(T_2 - T_1) + \frac{b}{2}(T_2 - T_1)(T_2 + T_1)$$

$$q = \frac{(T_2 - T_1)\left(a + \frac{b}{2}(T_2 + T_1)\right) \times A}{x_1 - x_2}$$

$$R = \frac{X_1 - X_2}{\left(a + \frac{b}{2}(T_1 + T_2)\right) \times A}$$

Through a Hollow Cylinder

- k constant, $A = 2\pi r L$



$$\frac{q}{A} = -k\frac{dT}{dr}$$

$$\int_{r_1}^{r_2} \frac{q}{2\pi r L}\, dr = \int_{T_1}^{T_2} -k\, dT$$

$$\frac{q}{2\pi L} \ln\frac{r_2}{r_1} = -k\,\Delta T$$

$$q = \frac{-2\pi L\,\Delta T}{\ln\frac{r_2}{r_1}}$$

- $A_{Lm} = \dfrac{A_2 - A_1}{\ln A_2/A_1}$

$$\frac{q}{A_{lm}} = -k\frac{dT}{dr}$$

$$q = kA_{lm}\frac{T_1 - T_2}{r_2 - r_1}$$

$R \to \dfrac{k}{w}$

(c) Through a Hollow sphere

- k constant $A = 4\pi r^2$

$$\frac{q}{A} = -k\frac{dT}{dr}$$

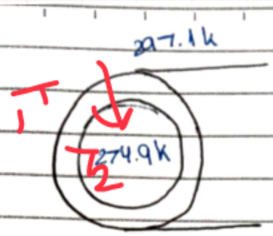
$$\frac{q}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -k\int_{T_1}^{T_2} dT$$

$$\frac{-q}{4\pi}\;\frac{1}{1/r_2 - 1/r_1} = k\,(T_1 - T_2)$$

$$q = \frac{k\,(T_1 - T_2)\;4\pi}{(1/r_1 - 1/r_2)}$$

EXAMPLE 4.2-1. Length of Tubing for Cooling Coil

A thick-walled cylindrical tubing of hard rubber having an inside radius of 5 mm and an outside radius of 20 mm is being used as a temporary cooling coil in a bath. Ice water is flowing rapidly inside and the inside wall temperature is 274.9 K. The outside surface temperature is 297.1 K. A total of 14.65 W must be removed from the bath by the cooling coil. How many m of tubing are needed?



$r_1 = 5mm$ $\quad r_2 = 20\,mm.$

$\boxed{L = ?}$

$Q = 14.65$

$A = 2\pi r L$

$$\frac{q}{2\pi r L} = -k\frac{dT}{dr}$$

solid A.3 Rubber $k = 0.151\,(W/m.K)$

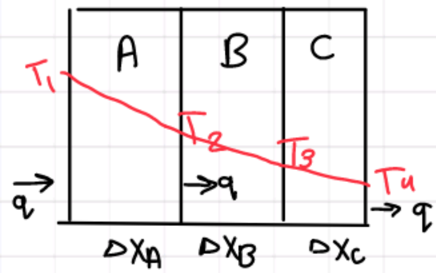
$$\frac{14.65\,w}{2\pi(L)\,r}\,dr = -k\,dT$$

$$\frac{14.65\,w}{2\pi L}\ln\frac{20}{5} = (0.151)(297.1 - 274.9)$$

$L = 0.964\ m$

Conduction Through Solid In series

* Plane walls in series ورا بعض مش جمب بعض



$$q_A = q_B = q_C = q$$

$$\frac{q}{A} = -k \frac{dT}{dx}$$

for A:

$$\int \frac{q_A}{A} \, dx = \int -k \, dT$$

$$\frac{q_A}{A} \Delta X_A = -k (T_2 - T_1) \quad \text{—— } 1$$

for B

$$\frac{q_B}{A} \Delta X_B = -k (T_3 - T_2) \quad \text{—— } 2$$

for C

$$\frac{q_C}{A} \Delta X_C = -k (T_4 - T_3) \quad \text{—— } 3$$

Σ eq 1 + eq 2 + eq 3:

$$T_1 - \cancel{T_2} = \frac{q_A}{A k_A} \Delta X_A$$

+

$$\cancel{T_2} - \cancel{T_3} = \frac{q_B}{A k_B} \Delta X_B$$

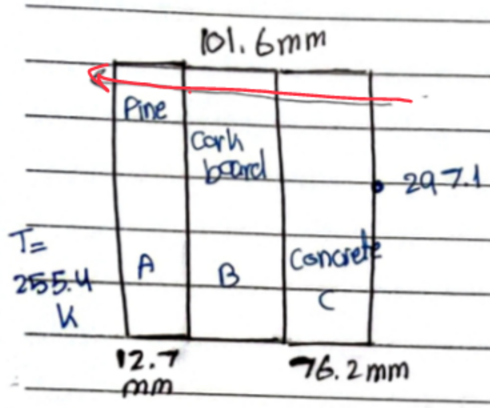
+

$$\cancel{T_3} - T_4 = \frac{q_C}{A k_C} \Delta X_C$$

لما طلعنا q.

$$= T_1 - T_4 = q (\Sigma R)$$

EXAMPLE 4.3-1. Heat Flow Through an Insulated Wall of a Cold Room
A cold-storage room is constructed of an inner layer of 12.7 mm of pine, a middle layer of 101.6 mm of cork board, and an outer layer of 76.2 mm of concrete. The wall surface temperature is 255.4 K inside the cold room and 297.1 K at the outside surface of the concrete. Use conductivities from Appendix A.3 for pine, 0.151; for cork board, 0.0433; and for concrete, 0.762 W/m · K. Calculate the heat loss in W for 1 m^2 and the temperature at the interface between the wood and cork board.



calualate 1) q
2) Temp's.

$* \; q = T_1 - T_4 \,/\, \Sigma R.$

$* \; R_A = \dfrac{\Delta X}{-k A} = \dfrac{12.7 \times 10^{-3}}{-0.151} = -0.084$ K/W

$* \; R_B = \dfrac{101.6 \times 10^{-3}}{-0.0433} = -2.346$ K/W

$* \; R_C = \dfrac{76.2 \times 10^{-3}}{-0.762} = -0.1$ K/W

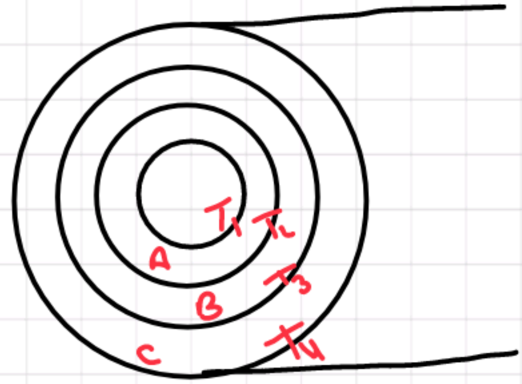
$q = \dfrac{-297.1 + 255.4}{-(\;2.53\;)} = \boxed{+16.48 \text{ W}}$

$q_A = 16.48 = \dfrac{255.4 - T_2}{-0.084}$

$T_2 = 256.78$ K

multilayer cylinders :. $q_A = q_B = q_c = q$.

$$\frac{q}{A} = -k\frac{dT}{dr}$$



$$\boxed{A = 2\pi r L.}$$

A:: $\int_{r_1}^{r_2} \frac{q}{2\pi r L}\, dr = \int_{T_1}^{T_2} -k\, dT \Rightarrow \frac{q}{2\pi L} \ln \frac{r_2}{r_1} = -k\, T_2 - T_1$

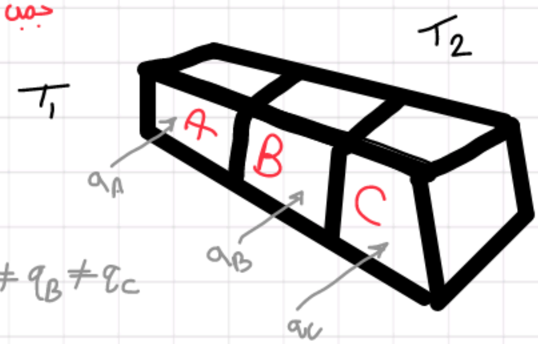
B: $\frac{q}{2\pi L} \ln \frac{r_3}{r_2} = -k\, T_3 - T_2$

C: $\frac{q}{2\pi L} \ln \frac{r_4}{r_3} = -k\, T_4 - T_3$

$$q = \frac{-k\,(T_4 - T_3)}{\ln \frac{r_4}{r_3}}\, 2\pi L$$

Conduction through Parallel

جنب بعض



$q_A \neq q_B \neq q_C$

$q_T = q_A + q_B + q_C$

زي كانه
الحيط في طوب وشباك، باب
ودرجة حرارة جوا الغرفة،
درجة حرارة برا الغرفة.

$$\frac{q_A}{A_A} = -k_A \frac{dT}{dz}$$

$$\int \frac{q_A}{A_A}\, dz = \int -k_A\, dT$$

$$\frac{q_A}{A_A} \Delta z = -k_A (T_2 - T_1)$$

$$\frac{q_B}{A_B} \Delta z = -k_B (T_2 - T_1)$$

$$\frac{q_C}{A_C} \Delta z = -k_C (T_2 - T_1)$$

$$q_A = -k_A (T_2 - T_1) \frac{A_A}{\Delta z}$$

$+$

$$q_B = -k_B (T_2 - T_1) \frac{A_B}{\Delta z}$$

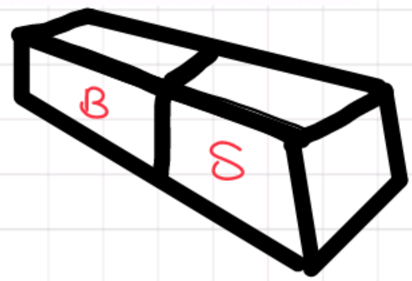
$+$

$$q_C = -k_C (T_2 - T_1) \frac{A_C}{\Delta z}$$

$$q_T = (T_1 - T_2) \left(\frac{A_A k_A}{\Delta z} + \frac{k_B A_B}{\Delta z} + \frac{A_C k_C}{\Delta z} \right)$$

$$q_T = (T_1 - T_2) \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)$$

The wall of a bakery oven is built of insulating brick 10 cm thick and thermal conductivity 0.22 $J \cdot m^{-1} \cdot s^{-1} \cdot {}^{\circ}C^{-1}$. Steel reinforcing members penetrate the brick, and their total area of cross-section represents 1% of the inside wall area of the oven. If the thermal conductivity of the steel is 45 $J \cdot m^{-1} \cdot s^{-1} \cdot {}^{\circ}C^{-1}$ calculate



$$q_S = -45 \frac{\Delta T}{10 \times 10^{-2}} (0.01A) \Rightarrow -4.5 A \Delta T$$

$$q_B = -\frac{0.22 \; \Delta T \; (0.99A)}{10 \times 10^{-2}} \Rightarrow -2.178 A \Delta T$$

Total area A

$A_S = 0.01A$

$A_B = 0.99A$

$q_T = q_S + q_B$

$\Delta T_S = \Delta T_B$

S Relative Total $\Rightarrow \dfrac{-4.5 \cancel{\Delta T} \cancel{A}}{-\cancel{\Delta T} A (4.5 + 2.17)} = 0.67$

B Relative Total $\Rightarrow \dfrac{-2.17 \Delta \cancel{T} A}{-\cancel{\Delta T} A (4.5 + 2.17)} = 0.33$

Total heat loss for each m^2 $\quad T_1 = 230 \quad T_2 = 25$

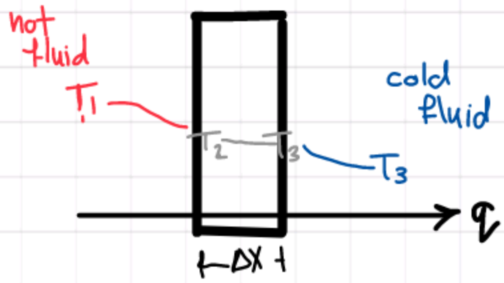
Take $A = 1 m^2$

$$q_A = \frac{(45)(205)(0.01)}{10 \times 10^{-2}} = 922.5 \; J/m^2$$

$$q_B = \frac{(.22)(205)(0.99)}{10 \times 10^{-2}} = 44649 \; J/m^2$$

$$q_T = 1.37 \; kW/m^2$$

* Combine convection and conduction and over all coefficient



$$h_i A (T_1 - T_2) = \frac{kA}{\Delta X_A} (T_2 - T_3) = h_c A (T_3 - T_4)$$

$$q_i = h_i A (T_1 - T_2)$$

$$q = \frac{k}{\Delta z} (T_2 - T_3) A$$

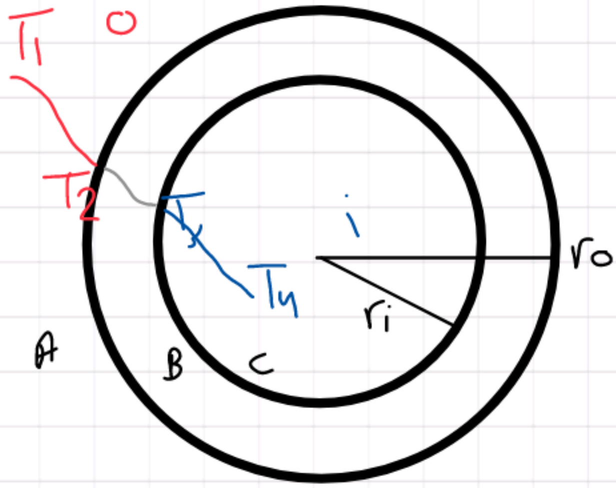
$$q_o = h_o A (T_3 - T_4)$$

$$T_1 - T_4 = q \left(\frac{1}{h_i A} + \frac{\Delta z}{k A} + \frac{1}{h_o A} \right)$$

$$q = \frac{T_1 - T_4}{\frac{1}{h_i A} + \frac{\Delta z}{k A} + \frac{1}{h_o A}}$$

$$q = A \, \Delta T_{overall} \, U$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{\Delta z}{k} + \frac{1}{h_o}$$



$$A_o = 2\pi r_o L$$

$$A_i = 2\pi r_i L$$

$$q_A = q_B = q_C$$

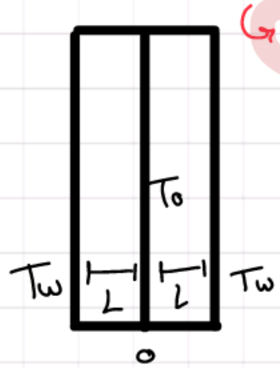
$$A_A h_A (T_1 - T_2) = \frac{k (T_2 - T_3)}{(r_3 - r_2)} A_{Blm} = h_i A_i (T_3 - T_4)$$

$$q = U_o A_o \Delta T_{overall}$$

$$q = U_i A_i \Delta T_{overall}$$

heat generation Palne wall

$\dot{Q}$ W/m³



$$Acc = In - out + gen - con$$

$$\frac{\Delta \rho A 2L c_p T}{\Delta T} = q_x - q_{x+\Delta x} + \dot{Q} A \Delta x$$

zero → S.S

$$\frac{q_{x+\Delta x} - q_x}{\Delta x} = \dot{Q} A$$

$$\frac{dq}{dx} = \dot{Q} A \qquad \frac{d\frac{q}{A}}{dx} = \dot{Q} \qquad \frac{q}{A} = -k\frac{dT}{dz}$$

$$\int d\left(-k\frac{dT}{dx}\right) = \int \dot{Q}\, dx$$

$$-k\frac{dT}{dx} = \dot{Q}x + C_1$$

at $x=0$ $T=T_0$

at $x=L$ $T=T_w$

at $x=-L$ $T=T_w$

$$\int -k\, dT = \int \dot{Q}x + C_1\, dx$$

$$-kT = \dot{Q}\frac{x^2}{2} + C_1 x + C_2$$

$$T = \frac{-\dot{Q}}{k}\frac{x^2}{2} + C_1 x + C_2 \qquad T_0 = C_2$$

① $$T_w = \frac{-\dot{Q}L^2}{k\,2} + C_1 L + \cancel{T_0}$$

② $$T_w = \frac{-\dot{Q}L^2}{k\,2} - C_1 L + \cancel{T_0} \qquad 0 = 2C_1 L \qquad \boxed{C_1 = 0}$$

$$T = \frac{-\dot{Q}x^2}{2k} + T_0$$

$$T = \frac{-\dot{q}\, x^2}{2k} + T_o$$

at $X = L$

$$T_w = \frac{-\dot{Q} L^2}{2k} + T_o$$ → T_w and T_o علاقة بين

* heat generation cylinder

$$T = \frac{\dot{q}}{4k}(R^2 - r^2) + T_w$$

$$T_o = \frac{\dot{q}}{4k}(R^2) + T_w$$

EXAMPLE 4.3-4. *Heat Generation in a Cylinder*

An electric current of 200 A is passed through a stainless steel wire having a radius R of 0.001268 m. The wire is $L = 0.91$ m long and has a resistance R of 0.126 Ω. The outer surface temperature T_w is held at 422.1 K. The average thermal conductivity is $k = 22.5$ W/m · K. Calculate the center temperature.

$q = -16.48$

$= \frac{255.4 - x}{-0.084} = -16.48$

$I = 200A$

$r = 0.001268\,m$

$L = 0.91\,m$

$R = 0.126\,\Omega$

$T_w = 422.1\,k$

$k = 22.5$

$T_o = ?$

$I^2R = \text{Watt}$

$200^2 \times 0.126 = 5040$

$\dot{Q} \times \text{Volume} = q$

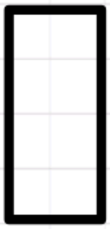
$\dot{q}\,\pi\,(0.001268)^2(0.91) = 5040$

$\dot{q} = 1.096\times10^9\ W/m^3$

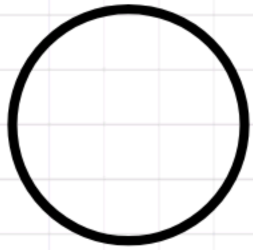
عشان اطلع $\dot{q}$

$T_o = \frac{1.096\times10^9}{4(22.5)}\ 0.001268^2 + 422.1$

$\boxed{T_o = 441.7\,k}$



كل ما ازيد
العازل بزيد العزل



مو شرط كل
ما ازيد العازل بزيد
العزل
في عندي r_{cr}.

$r_2 > r_{cr}$ ∴ ↑ insulating ↓ heat trans

$r_2 < r_{cr}$ ∴ ↑ insulating ↑ heat trans

Critical Thickness of Insulation for cylinder

$$q = \frac{2\pi L\,(T_1 - T_2)}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{r_2 h_o}}$$

$$\frac{dq}{dr_2} = \frac{-2\pi L\,(T_1 - T_0)\left(1/r_2 k - 1/r_2^2 h_o\right)}{\left[\frac{\ln(r_2/r_1)}{k} + \frac{1}{r_2 h_o}\right]^2} = 0$$

$$r_{cr} = \frac{k}{h_o}$$

EXAMPLE 4.3-5. *Insulating an Electrical Wire and Critical Radius*

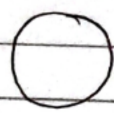
An electric wire having a diameter of 1.5 mm and covered with a plastic insulation (thickness = 2.5 mm) is exposed to air at 300 K and $h_o = 20\ W/m^2 \cdot K$. The insulation has a k of 0.4 W/m · K. It is assumed that the wire surface temperature is constant at 400 K and is not affected by the covering.

(a) Calculate the value of the critical radius.

(b) Calculate the heat loss per m of wire length with no insulation.

(c) Repeat (b) for the insulation present.

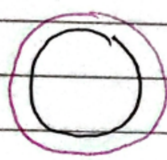
A) $r_{cr} = \frac{k}{h} = \frac{0.4}{20} = 0.02$

B) 

$q = hA(T-T)$

$20(\pi \times 2 \times 1 \times 7.5\times10^{-4})(400-300)$

$= \boxed{9.42\ W}$

c) 

هون دمج عني conduction و convection

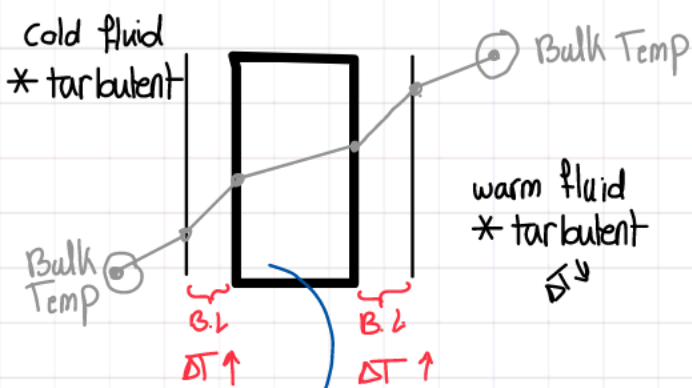
$$q = \frac{T-T_o\ (2\pi L)}{\frac{1}{r_2 h} + \frac{\ln r_2/r_1}{k}}$$

$$q = \frac{(100)(2\pi)(1)}{\frac{1}{3.25\times10^{-3}(20)} + \frac{\ln(3.25\times10^{-3}/0.75\times10^{-3})}{0.4}}$$

$= \boxed{32.98\ W}$

forced convection heat trans inside Pipes

* ↑ Temp , ↑ avg K.E



كل ما قلت
↑ conductivity الـ
↑ ΔT

* Laminar
* conduction بدال convection

h function of
empirical eq

- → system geometry
- → flow type and velocity   ↑ Turbulent, ↑ h_o
- → fluid physical properties
- → temp difference

Reynolds num :-

$$Re = \frac{\rho V D}{\mu}$$

Re < 2100 Laminar
2100 < Re < 6000 Transition
Re > 6000 Turbulant

Prantl number   gases ⇒ A3

$$Pr = \frac{\mu/\rho}{K_L/C_P\rho} = \frac{C_P\,\mu}{K_L}$$

↳ relative thickness hydrodynamic Layer and thermal B.L

$Pr(L) > Pr(g)$

Nusselt number

$$Nu = \frac{hD}{k}$$

* Laminar

↳ Re < 2100

$$Nu = 1.86 \left(Re\,Pr\,\frac{D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

* $Re\,Pr\,\frac{D}{L} > 100$ ✓

* $Re\,Pr\,\frac{D}{L} > 10$ ⇒ error 20%

* ha depent on heated lenghth

* Turbulent

↳ Re > 6000

$$Nu = 0.027\, Re^{0.8} \cdot Pr^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

* $0.7 > Pr > 16000$, $\frac{L}{D} > 60$

* h depent on the log mean driving force

- air at 1atm, Pipe, Turbulent

↳ $h_L = 3.52 \frac{V^{0.8}}{D^{0.2}}$ ⇒ SI

- water T = 4 to 105 c°

↳ $h_L = 1429(1 + 0.0146T) \frac{V^{0.8}}{D^{0.2}}$ ⇒ SI

- organic Liquid

↳ $h_L = 423 \frac{V^{0.8}}{D^{0.2}}$

EXAMPLE 4.5-1. ***Heating of Air in Turbulent Flow***

Air at 206.8 kPa and an average of 477.6 K is being heated as it flows through a tube of 25.4 mm inside diameter at a velocity of 7.62 m/s. The heating medium is 488.7 K steam condensing on the outside of the tube. Since the heat-transfer coefficient of condensing steam is several thousand $W/m^2 \cdot K$ and the resistance of the metal wall is very small, it will be assumed that the surface wall temperature of the metal in contact with the air is 488.7 K. Calculate the heat-transfer coefficient for an $L/D > 60$ and also the heat-transfer flux q/A.

EXAMPLE 4.5-1. *Heating of Air in Turbulent Flow*

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), SI Units

T (°C)	T (K)	ρ (kg/m³)	c_p (kJ/kg · K)	-5 $\mu \times 10^5$ (Pa · s, or kg/m · s)	k (W/m · K)	Pr N_{Pr}	$\beta \times 10^3$ (1/K)	$g\beta\rho^2/\mu^2$ (1/K · m³)
−17.8	255.4	1.379	1.0048	1.62	0.02250	0.720	3.92	2.79×10^8
0	273.2	1.293	1.0048	1.72	0.02423	0.715	3.65	2.04×10^8
10.0	283.2	1.246	1.0048	1.78	0.02492	0.713	3.53	1.72×10^8
37.8	311.0	1.137	1.0048	1.90	0.02700	0.705	3.22	1.12×10^8
65.6	338.8	1.043	1.0090	2.03	0.02925	0.702	2.95	0.775×10^8
93.3	366.5	0.964	1.0090	2.15	0.03115	0.694	2.74	0.534×10^8
121.1	394.3	0.895	1.0132	2.27	0.03323	0.692	2.54	0.386×10^8
148.9	422.1	0.838	1.0174	2.37	0.03531	0.689	2.38	0.289×10^8
176.7	449.9	0.785	1.0216	2.50	0.03721	0.687	2.21	0.214×10^8
204.4	477.6	0.740	1.0258	2.60	0.03894	0.686	2.09	0.168×10^8
232.2	505.4	0.700	1.0300	2.71	0.04084	0.684	1.98	0.130×10^8
260.0	533.2	0.662	1.0341	2.80	0.04258	0.680	1.87	0.104×10^8

هاد الجدول على 101.325 kPa فاخد منه الاشياء يلي ما بتتأثر بال Prussure

لانه بالسؤال P= 206.8 kPa

* $T_w = 488.7$

$T_B = 477.6 \rightarrow \mu_B = 2.60 \times 10^{-5}$

$U_B = 7.62$

$D = 25.4 \times 10^{-3}$

$Pr = 0.686 \Rightarrow A.3$

$Re = \frac{\rho U D}{\mu_{\hookrightarrow A.3}}$

$PV = \frac{m}{M.m} RT$

$\boxed{\rho} = \frac{m}{V} = \frac{\boxed{P} M m}{RT}$

$\frac{\rho_1}{\rho_2} = \frac{P_1}{P_2}$

$\frac{\rho_1}{0.714} = \frac{206.8}{101.325} \rightarrow A.3$

$\rho_1 = 1.51 \ kg/m^3$

$Re = 1.51 \times 7.62 \times 25.4 \times 10^{-3} / (2.60 \times 10^{-5}) = 11240.67$

> 6000

Turbulent

$Nu = 0.027 (11240.67)^{0.8} (0.686)^{1/3} \left(\frac{2.60 \times 10^{-5}}{2.64 \times 10^{-5}} \right)^{0.14}$

$\hookrightarrow$ Interpolation

$Nu = 41.35$

$Nu = \frac{h_L D}{k} = 41.35 = \frac{h_L (25.4 \times 10^{-3})}{0.03894}$

$h_L = 63.4 \ \frac{W}{m^2 . K}$ #

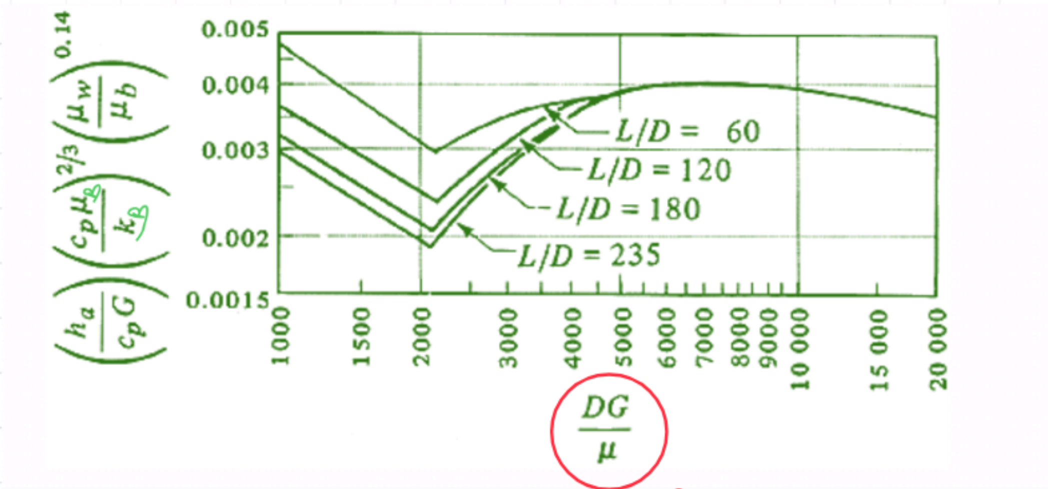
Five Appl

* flux $\frac{q}{A}$

$q = A (T_w - T) h_L$

$\frac{q}{A} = (488.7 - 477.6)(63.4) = 701.1 \ W/m^2.$

* Transition

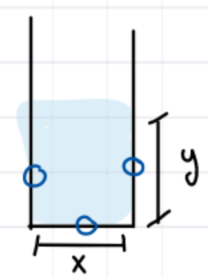


↳ Re

* non circular

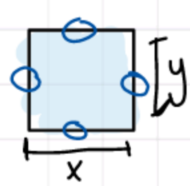
↳ equivelant diameter.

$$D_{eq} = 4 \, \frac{\text{cross sec area}}{\text{wetted perimeter}}$$



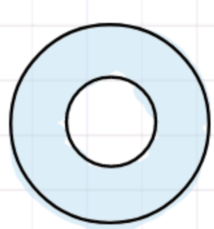
wetted perimeter $= 2y + X$

area $= xy$



wetted perimeter $= 2y + 2X$

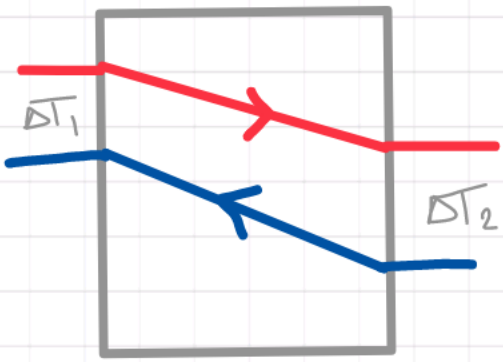
area $= xy$

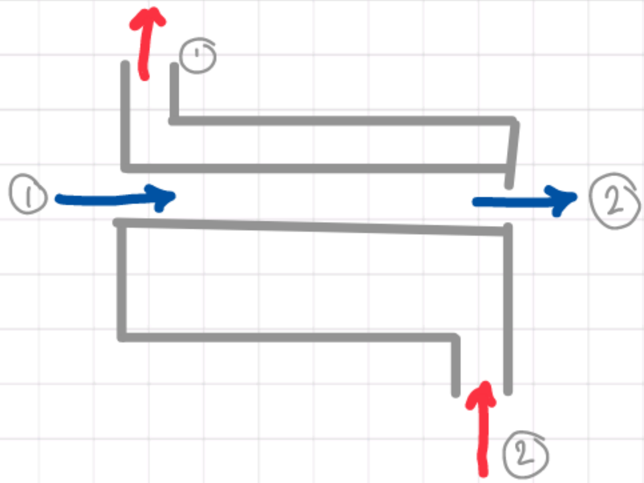


wetted perimeter $= \pi D_1 + \pi D_2$

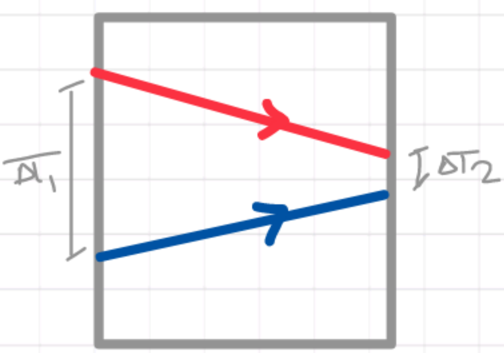
area $= \frac{\pi}{4} D_2^2 - \frac{\pi}{4} D_1^2$

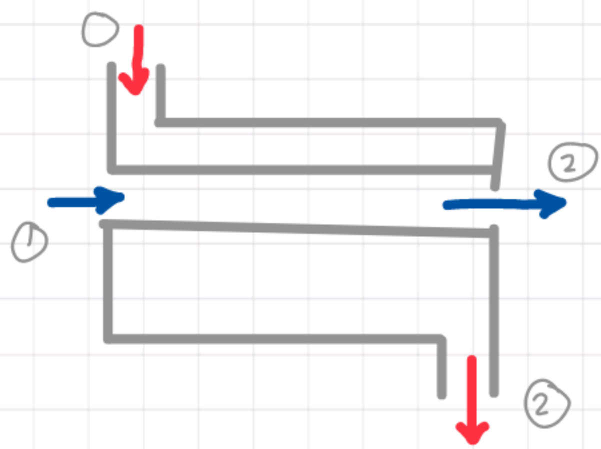
* Countercurrent





* Co-curent





$$q = UA\,\underline{\underline{\Delta T_m}}$$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 - \Delta T_1)}$$

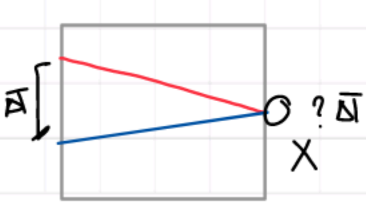
- $T_{c1} < T_{h1}$

  $T_{h2} > T_{c2}$

  $T_{c2} > T_{c1}$

  $T_{h1} > T_{h2}$

- لازم يضل عندي $\underline{\underline{\Delta T}}$



EXAMPLE 4.5-2. Water Heated by Steam and Trial-and-Error Solution
Water is flowing in a horizontal 1-in. schedule 40 steel pipe at an average temperature of 65.6°C and a velocity of 2.44 m/s. It is being heated by condensing steam at 107.8°C on the outside of the pipe wall. The steam side coefficient has been estimated as $h_o = 10\,500\ W/m^2 \cdot K$.

(a) Calculate the convective coefficient h_i for water inside the pipe.
(b) Calculate the overall coefficient U_i based on the inside surface area.
(c) Calculate the heat-transfer rate q for 0.305 m of pipe with the water at an average temperature of 65.6°C.

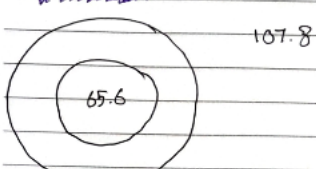
A.5-1 Dimensions of Standard Steel Pipe

Nominal Pipe Size (in.)	Outside Diameter in.	Outside Diameter mm	Schedule Number	Wall Thickness in.	Wall Thickness mm	Inside Diameter in.	Inside Diameter mm	Inside Cross-Sectional Area ft^2	Inside Cross-Sectional Area $m^2 \times 10^4$
1/8	0.405	10.29	40	0.068	1.73	0.269	6.83	0.00040	0.3664
			80	0.095	2.41	0.215	5.46	0.00025	0.2341
1/4	0.540	13.72	40	0.088	2.24	0.364	9.25	0.00072	0.6720
			80	0.119	3.02	0.302	7.67	0.00050	0.4620
3/8	0.675	17.15	40	0.091	2.31	0.493	12.52	0.00133	1.231
			80	0.126	3.20	0.423	10.74	0.00098	0.9059
1/2	0.840	21.34	40	0.109	2.77	0.622	15.80	0.00211	1.961
			80	0.147	3.73	0.546	13.87	0.00163	1.511
3/4	1.050	26.67	40	0.113	2.87	0.824	20.93	0.00371	3.441
			80	0.154	3.91	0.742	18.85	0.00300	2.791
1	1.315	33.40	40	0.133	3.38	1.049	26.64	0.00600	5.574
			80	0.179	4.45	0.957	24.31	0.00499	4.641
1 1/4	1.660	42.16	40	0.140	3.56	1.380	35.05	0.01040	9.648
			80	0.191	4.85	1.278	32.46	0.00891	8.275
1 1/2	1.900	48.26	40	0.145	3.68	1.610	40.89	0.01414	13.13
			80	0.200	5.08	1.500	38.10	0.01225	11.40

$L = 0.305$

$D_i = 0.0266 \qquad A_i = 0.0255$

$D_o = 0.0334 \qquad A_o = 0.032$



$q = A_i h_i (T_{wi} - 65.6)$

$q = A_o h_o (107.8 - T_{wo})$

$q = \dfrac{\Delta T}{\Sigma R}$

1) assuming T_{wi}

(علماً تكون بين 107.8 و 65.6

اقرب ل 65.6

$65.6 + \dfrac{107.8 - 65.6}{3} = \boxed{80C°} \quad \mu \rightarrow 3.56 \times 10^{-4}$

$Re = \dfrac{0.0266 \times 2.44 \times 980}{4.32 \times 10^{-4}} = 1.473 \times 10^{5}$ Turbulent.

$Nu = 0.027 \times (1.473 \times 10^5)^{0.8} (2.72)^{1/3} \left(\dfrac{4.32}{3.56}\right)^{0.14}$

$Nu = \dfrac{h_L (0.0266)}{0.663} = \boxed{h_L = 13157.86 = h_i}$

13324 الكتاب طالعة معاه *

$\Sigma R \rightarrow \dfrac{1}{h_o A_o} + \dfrac{1}{h_i A_i} + \dfrac{r_o - r_i}{k A_{lm}} = 0.008552$

$U_i = \dfrac{1}{R A_i} = 4586$

$q = U_i A_i (\Delta T_{overall}) = 4935$
$(107.6 - 65.6)$

$q = h_i A_i (T_{wi} - 65.6)$

$4935 = 13157.66 (T_{wi} - 65.6) \times 0.0255$

$T_{wi} = 80.3$ على حل الكتاب بتكون 80.1

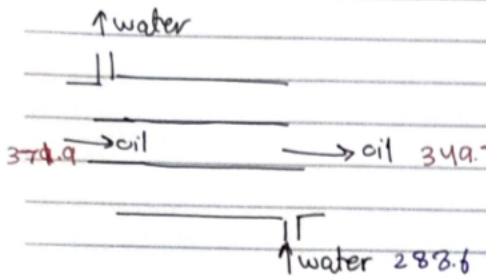
$error = \dfrac{80.3 - 80}{80.3} = 0.38\%$ ✓ اقل من 5%

EXAMPLE 4.5-2. *Water Heated by Steam and Trial-and-Error Solution*
Water is flowing in a horizontal 1-in. schedule 40 steel pipe at an average temperature of 65.6°C and a velocity of 2.44 m/s. It is being heated by condensing steam at 107.8°C on the outside of the pipe wall. The steam side coefficient has been estimated as $h_o = 10\,500\ W/m^2 \cdot K$.

(a) Calculate the convective coefficient h_i for water inside the pipe.
(b) Calculate the overall coefficient U_i based on the inside surface area.
(c) Calculate the heat-transfer rate q for 0.305 m of pipe with the water at an average temperature of 65.6°C.

4.5-4 >> oil C_{pm} = 2.30

حفظ * wate C_{pm} = 4.187 *



$q = \Delta T\ C_{pm}\ \dot{m}$

$(371.9 - 349.7)(2.30)(3630) =$ 185 347 kJ/h

$185\,347 = (T_1 - 288.6)(1450)(4.187)$

$T_1 = 319.1\ K$

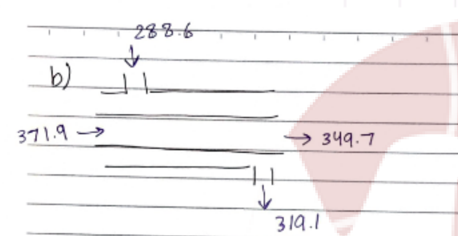
* $q = U_i A_i \Delta T_{lm}$

$\dfrac{61.1 - 52.8}{\ln 61.1/52.8} = 56.9\ K.$

51 490 W = 340 A x 56.9. $A = 2.66\ m^2$

G = 185347 kJ/h

b)



$\Delta T_{lm} = \dfrac{83.3 - 30.6}{\ln 83.3/30.6} = 52.6\ K$

51490 = 340 (A) (52.6) $A = 2.87\ m^2$

counter curent A < cocurent A
Driving Force > Driving Force