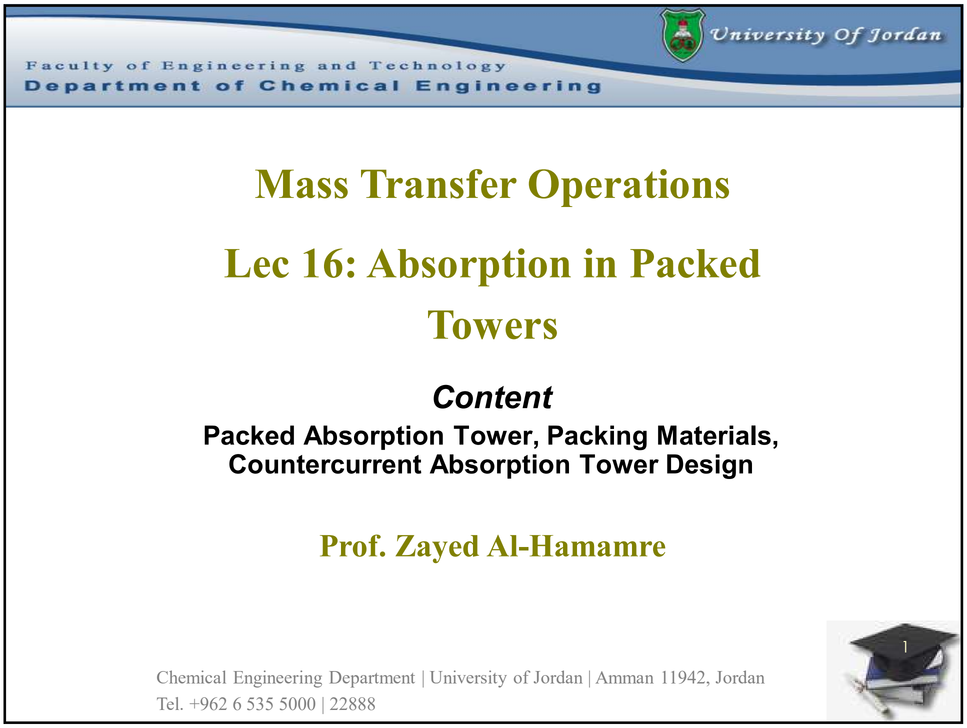


University Of Jordan
Faculty of Engineering and Technology
Department of Chemical Engineering

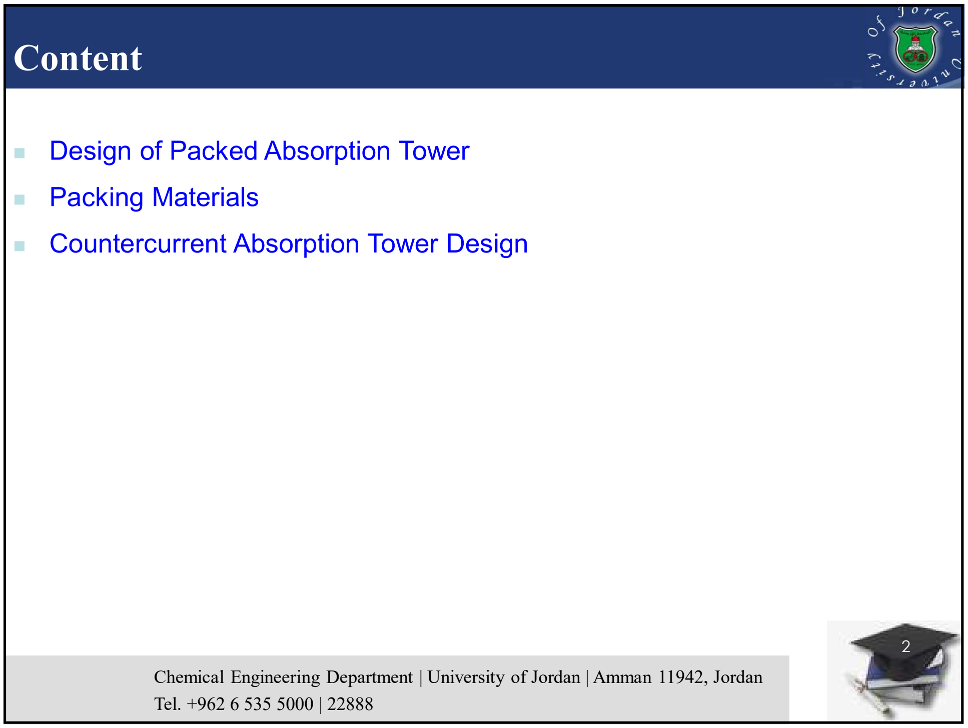
Mass Transfer Operations

Lec 16: Absorption in Packed Towers

Content

Packed Absorption Tower, Packing Materials,
Countercurrent Absorption Tower Design

Prof. Zayed Al-Hamamre



Content

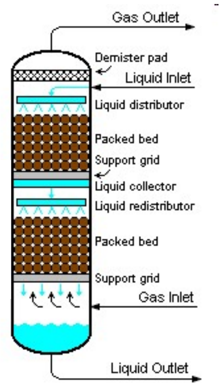
- Design of Packed Absorption Tower
- Packing Materials
- Countercurrent Absorption Tower Design

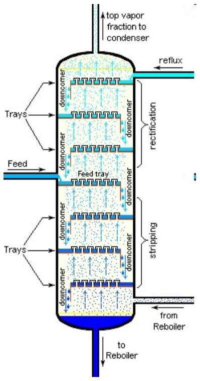
Introduction

- In continuous contacting equipment, the up-flowing gas remains in contact with down-flowing liquid throughout the packing, at every point of the tower.
- Therefore, packed tower is known as “continuous differential contact equipment It is different from the stage-wise contact equipment .

Continuous differential column

Requirement: Calculate the height of packing in a packed column where the separation process takes place.





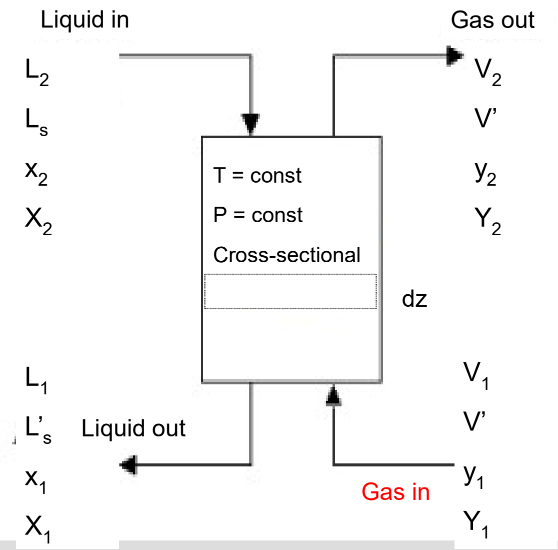
Stage wise contact is there in a plate column

Requirement: Calculate the number of plates required for the desired separation.

Design of packed absorption tower

- Packed bed is a hollow tube, pipe, or other vessel that is filled with a packing material.
- In tray column, the gas and the liquid phases come in contact in several discrete stages. Thus, a stage wise contact is there in a plate column.
- But in packed tower, the up-flowing gas remains in contact with down-flowing liquid throughout the packing, at every point of the tower.
- Therefore, packed tower is known as “continuous differential contact equipment It is different from the stage-wise distillation column.
- In the stage distillation column the equilibrium in each stage will vary not in a continuous fashion whereas in the packed column the equilibrium is changed point wise in each axial location

Mass Balances and the Operating Line for Packed Towers



V molar total gas flow rate (carrier gas + pollutant)

V’ molar inert carrier gas flow rate

L molar total solvent flow rate (solvent + absorbed pollutant)

L’ molar solvent flow rate

x is the liquid mole fraction of pollutant, y is the gas phase mole fraction of the pollutants, X is the liquid phase mole ratio and

Y is the gas phase mole ratio

Design of Packed Absorption Tower

- Steps for the design of packed tower

(A) Selection of solvent

(B) Selection of packing

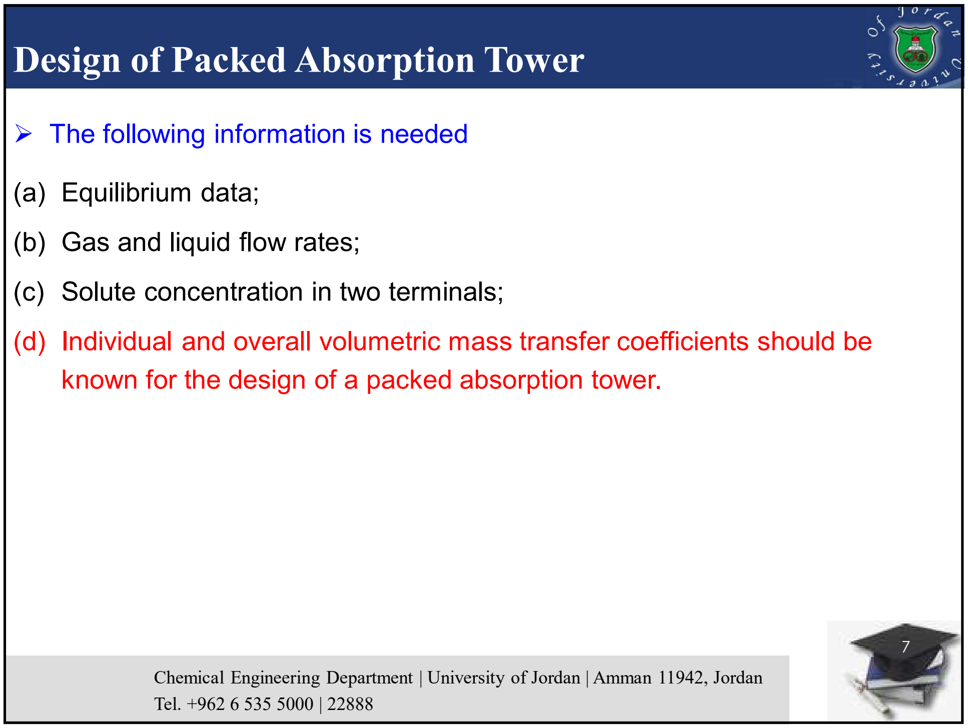
(C) Calculation of minimum solvent flow rate as well as actual solvent flow rate

(D) Column diameter

(E) Height of column

(F) Design of solvent distributors and redistributors (if needed)

(G) Design of gas distributor, packing support, shell, nozzles, column support



Design of Packed Absorption Tower

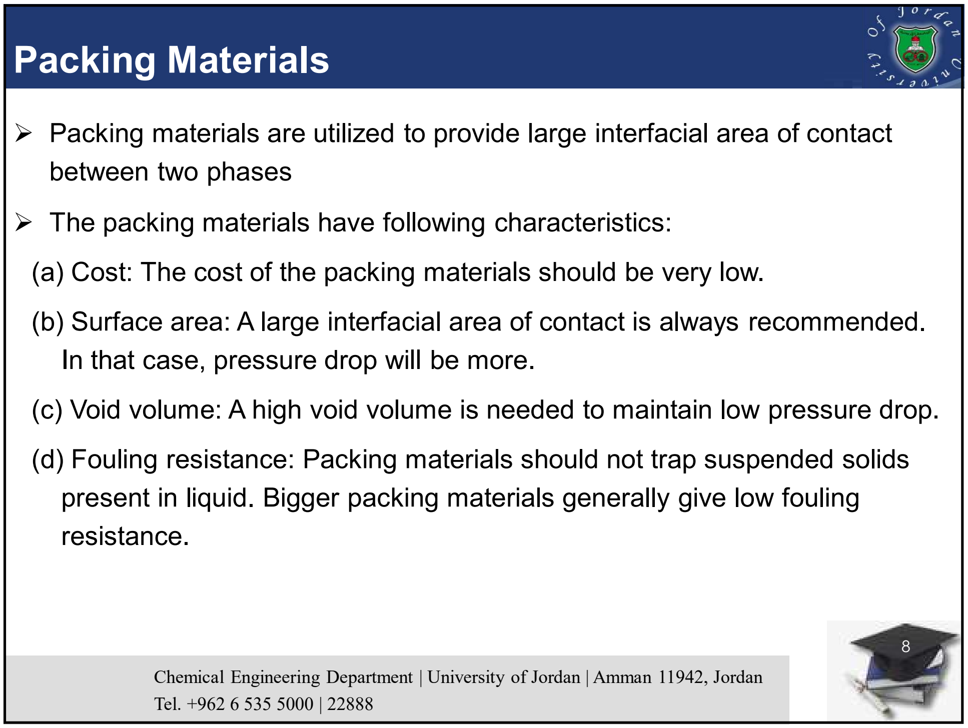
- ➢ The following information is needed

(a) Equilibrium data;

(b) Gas and liquid flow rates;

(c) Solute concentration in two terminals;

(d) Individual and overall volumetric mass transfer coefficients should be known for the design of a packed absorption tower.



Packing Materials

- ➢ Packing materials are utilized to provide large interfacial area of contact between two phases
- ➢ The packing materials have following characteristics:

(a) Cost: The cost of the packing materials should be very low.

(b) Surface area: A large interfacial area of contact is always recommended. In that case, pressure drop will be more.

(c) Void volume: A high void volume is needed to maintain low pressure drop.

(d) Fouling resistance: Packing materials should not trap suspended solids present in liquid. Bigger packing materials generally give low fouling resistance.

Packing Materials

(e) Mechanical strength: Good mechanical strength is desired for choosing packing materials as this will not break or deform during filling or operation.

(f) Uniform flow of streams: Stack of packing materials should have uniform void spaces through which both the streams (gas and liquid) can flow uniformly. Non-uniform flow of streams leads to stagnant liquid pool which in turn gives low mass transfer.

Packing Materials

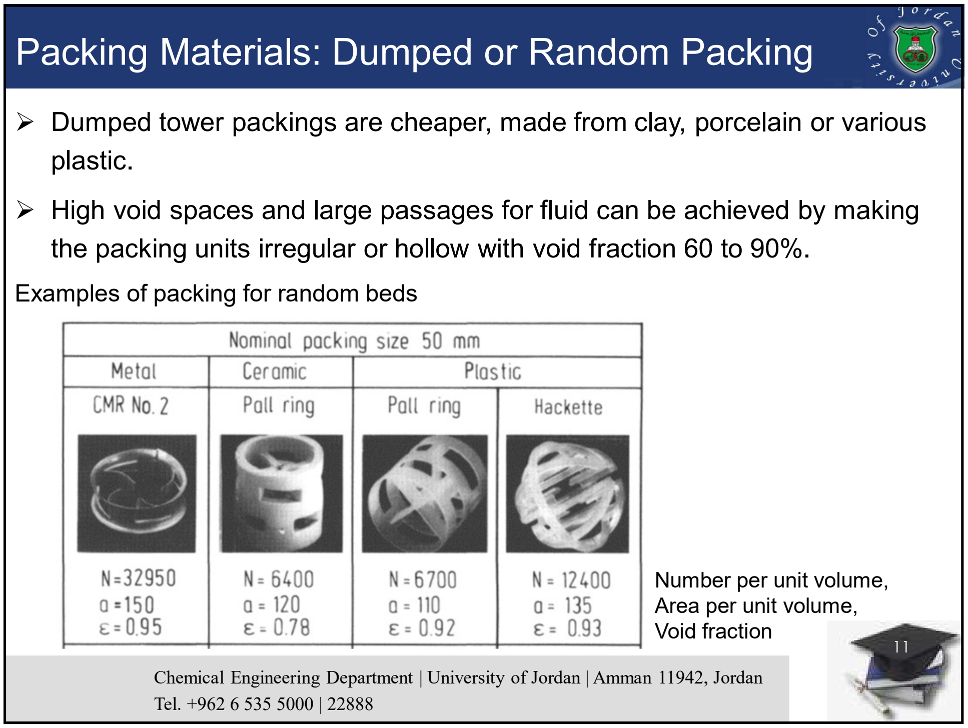
- There are tow types of packing:

1) Random packing
2) structured packing

- Random packing has a many types like (raschig rings, metal ball rings…etc).

Some of Raschig rings type

Structured packing



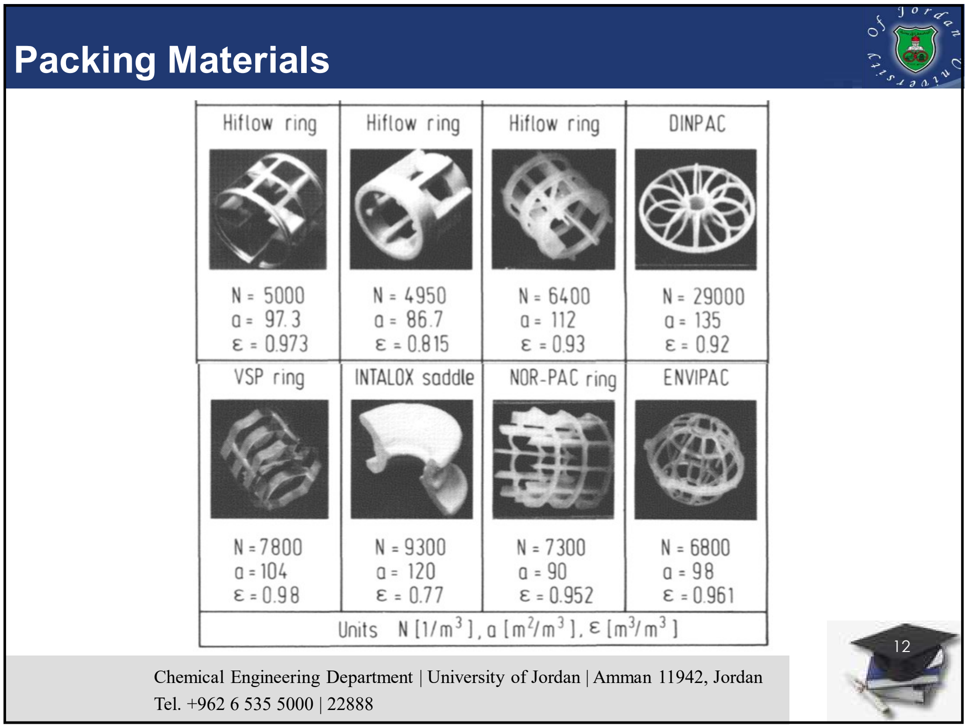
Packing Materials: Dumped or Random Packing

- Dumped tower packings are cheaper, made from clay, porcelain or various plastic.
- High void spaces and large passages for fluid can be achieved by making the packing units irregular or hollow with void fraction 60 to 90%.

Examples of packing for random beds

Nominal packing size 50 mm			
Metal	Ceramic	Plastic	
CMR No. 2	Pall ring	Pall ring	Hackette
N=32950 a=150 ε=0.95	N=6400 a=120 ε=0.78	N=6700 a=110 ε=0.92	N=12400 a=135 ε=0.93

Number per unit volume,
Area per unit volume,
Void fraction

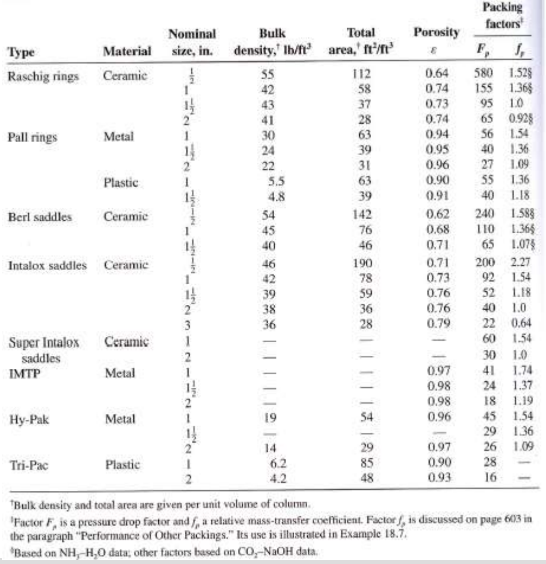


Packing Materials

Hiflow ring	Hiflow ring	Hiflow ring	DINPAC
N = 5000 a = 97.3 ε = 0.973	N = 4950 a = 86.7 ε = 0.815	N = 6400 a = 112 ε = 0.93	N = 29000 a = 135 ε = 0.92
VSP ring	INTALOX saddle	NOR-PAC ring	ENVIPAC
N = 7800 a = 104 ε = 0.98	N = 9300 a = 120 ε = 0.77	N = 7300 a = 90 ε = 0.952	N = 6800 a = 98 ε = 0.961

Units N $[1/m^3]$, a $[m^2/m^3]$, ε $[m^3/m^3]$

Dumped Packing Material Properties



Type	Material	Nominal size, in.	Bulk density,† lb/ft³	Total area,† ft²/ft³	Porosity ε	Packing factors‡ F_p	Packing factors‡ f_p
Raschig rings	Ceramic	$\frac{1}{2}$	55	112	0.64	580	1.52§
		1	42	58	0.74	155	1.36§
		$1\frac{1}{2}$	43	37	0.73	95	1.0
		2	41	28	0.74	65	0.92§
Pall rings	Metal	1	30	63	0.94	56	1.54
		$1\frac{1}{2}$	24	39	0.95	40	1.36
		2	22	31	0.96	27	1.09
	Plastic	1	5.5	63	0.90	55	1.36
		$1\frac{1}{2}$	4.8	39	0.91	40	1.18
Berl saddles	Ceramic	$\frac{1}{2}$	54	142	0.62	240	1.58§
		1	45	76	0.68	110	1.36§
		$1\frac{1}{2}$	40	46	0.71	65	1.07§
Intalox saddles	Ceramic	$\frac{1}{2}$	46	190	0.71	200	2.27
		1	42	78	0.73	92	1.54
		$1\frac{1}{2}$	39	59	0.76	52	1.18
		2	38	36	0.76	40	1.0
		3	36	28	0.79	22	0.64
Super Intalox saddles	Ceramic	1	—	—	—	60	1.54
		2	—	—	—	30	1.0
IMTP	Metal	1	—	—	0.97	41	1.74
		$1\frac{1}{2}$	—	—	0.98	24	1.37
		2	—	—	0.98	18	1.19
Hy-Pak	Metal	1	19	54	0.96	45	1.54
		$1\frac{1}{2}$	—	—	—	29	1.36
		2	14	29	0.97	26	1.09
Tri-Pac	Plastic	1	6.2	85	0.90	28	—
		2	4.2	48	0.93	16	—

† Bulk density and total area are given per unit volume of column.

‡ Factor F_p is a pressure drop factor and f_p a relative mass-transfer coefficient. Factor f_p is discussed on page 603 in the paragraph "Performance of Other Packings." Its use is illustrated in Example 18.7.

§ Based on NH_3–H_2O data; other factors based on CO_2–NaOH data.

Packing Materials: Structured packing materials

Mellapak

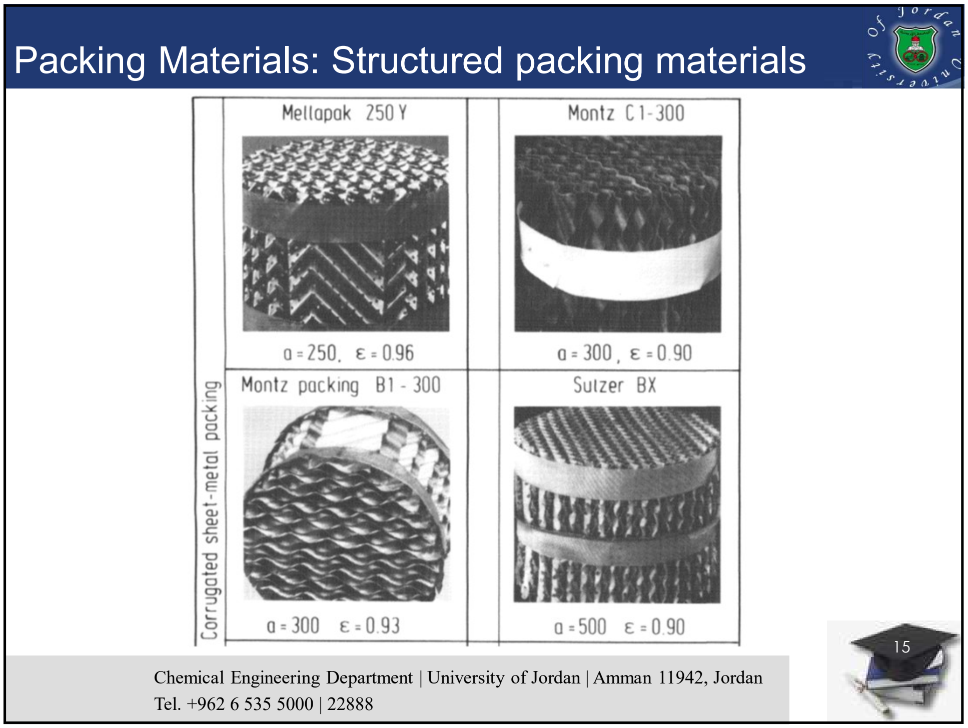
Flexipak

Montez
Corrugated metal
sheet

Wire mesh packing

Structured packing materials.

- These materials are used widely as packing materials in packed tower due to low gas pressure drop and improved efficiency.



Packing Materials: Structured packing materials

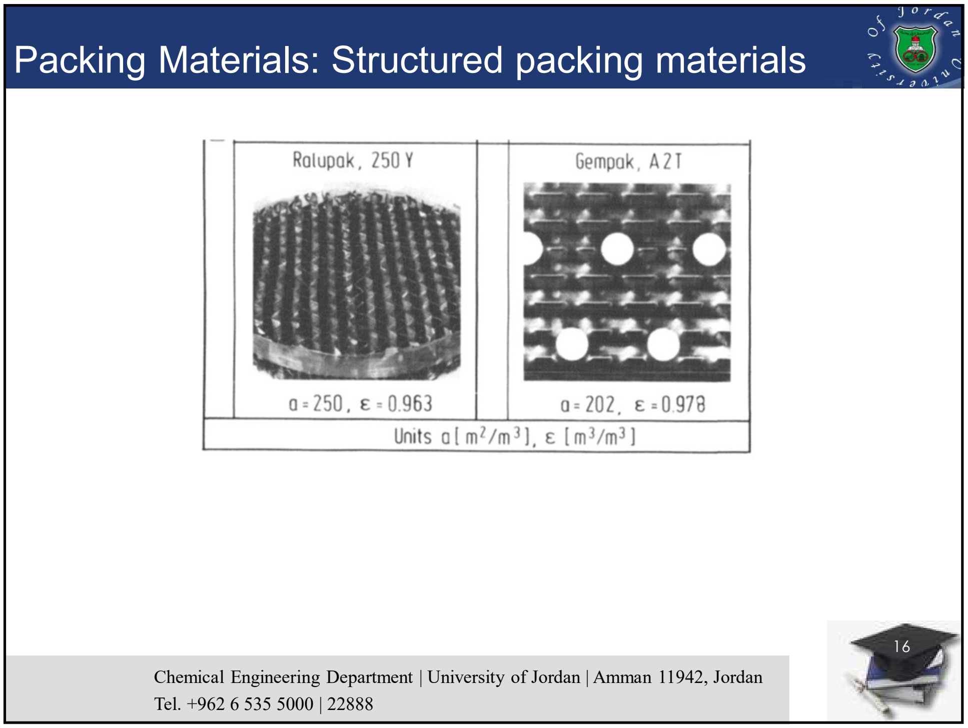
Corrugated sheet-metal packing

Mellapak 250 Y
$a = 250, \ \varepsilon = 0.96$

Montz C1-300
$a = 300, \ \varepsilon = 0.90$

Montz packing B1 - 300
$a = 300 \quad \varepsilon = 0.93$

Sulzer BX
$a = 500 \quad \varepsilon = 0.90$



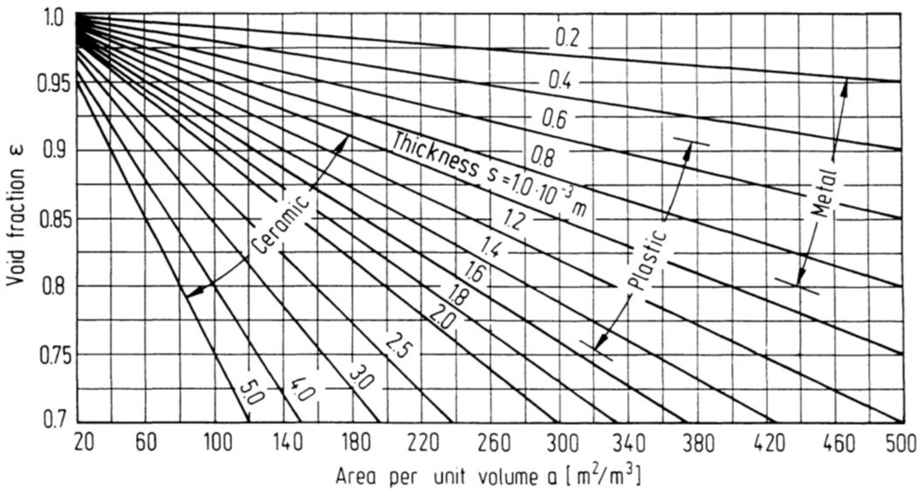
Packing Materials: Structured packing materials

Ralupak, 250 Y
$a = 250, \ \varepsilon = 0.963$

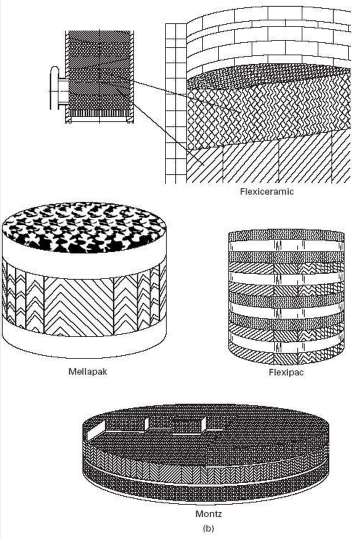
Gempak, A2T
$a = 202, \ \varepsilon = 0.978$

Units $a\,[m^2/m^3]$, $\varepsilon\,[m^3/m^3]$

Packing Materials: Structured packing materials



Packing Materials



Structured packing

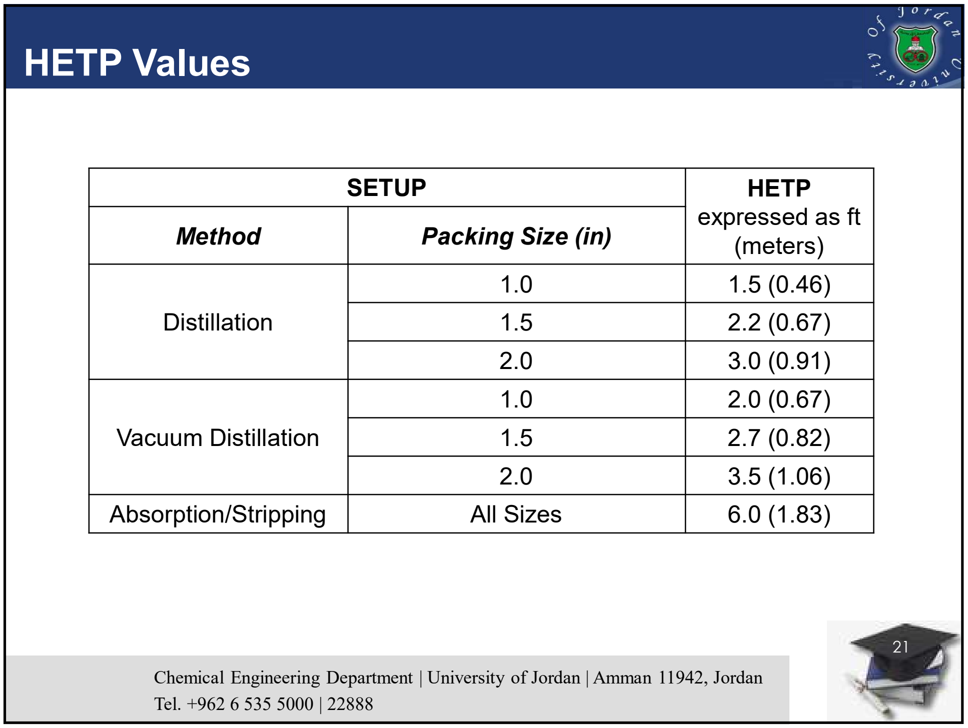
Comparison of Types of Packing

Table 6.3 Comparison of Types of Packing

	Random		
	Raschig Rings and Saddles	"Through Flow"	Structured
Relative cost	Low	Moderate	High
Pressure drop	Moderate	Low	Very low
Efficiency	Moderate	High	Very high
Vapor capacity	Fairly high	High	High
Typical turndown ratio	2	2	2

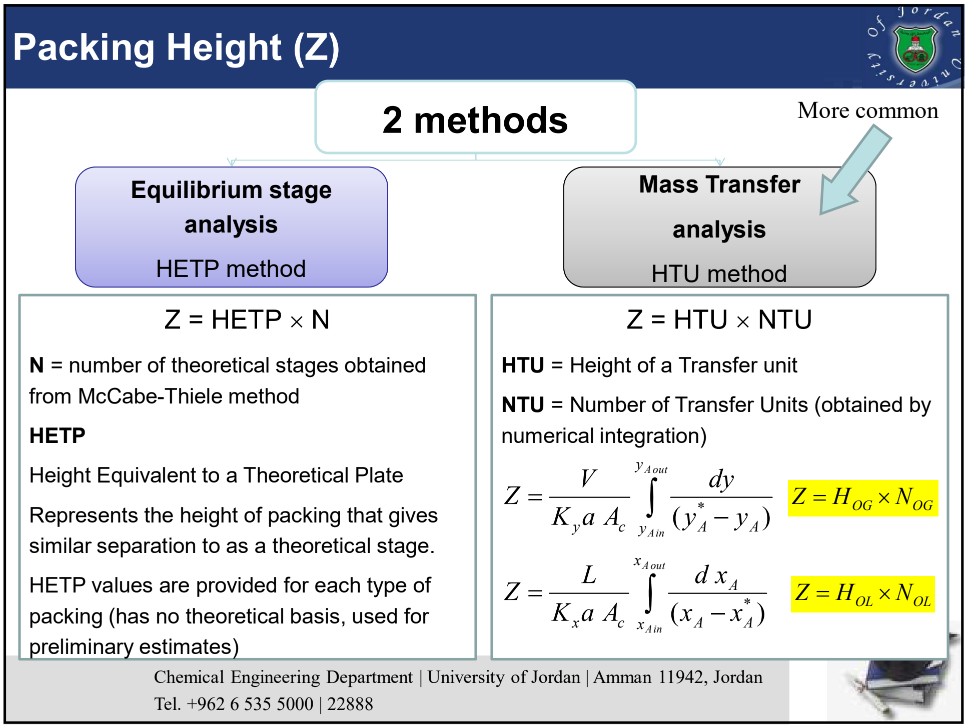
Packing Height (Z)

- Packed columns are continuous, differential-contacting devices that do not have physically distinguishable, discrete stages.
- Thus, packed columns are better analyzed by mass-transfer models than by equilibrium-stage concepts.
- However, in practice, packed-tower performance is often presented on the basis of equivalent equilibrium stages using a packed-height equivalent to a theoretical (equilibrium) plate (stage), called the HETP or HETS



HETP Values

SETUP		HETP expressed as ft (meters)
Method	***Packing Size (in)***	
Distillation	1.0	1.5 (0.46)
	1.5	2.2 (0.67)
	2.0	3.0 (0.91)
Vacuum Distillation	1.0	2.0 (0.67)
	1.5	2.7 (0.82)
	2.0	3.5 (1.06)
Absorption/Stripping	All Sizes	6.0 (1.83)



Packing Height (Z)

2 methods

More common

Equilibrium stage analysis

HETP method

Z = HETP × N

N = number of theoretical stages obtained from McCabe-Thiele method

HETP

Height Equivalent to a Theoretical Plate

Represents the height of packing that gives similar separation to as a theoretical stage.

HETP values are provided for each type of packing (has no theoretical basis, used for preliminary estimates)

Mass Transfer analysis

HTU method

Z = HTU × NTU

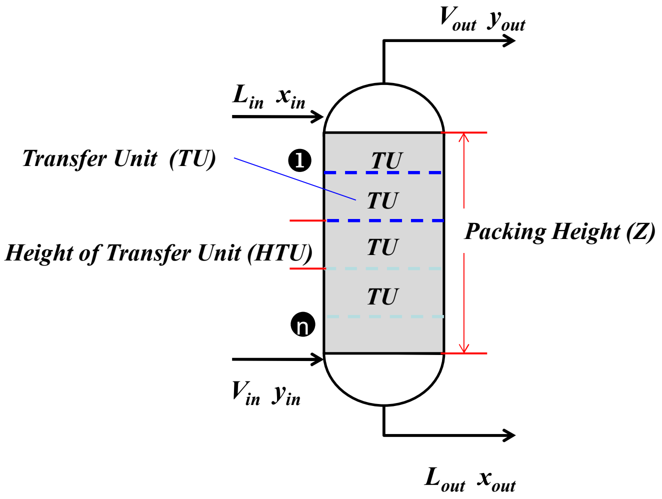
HTU = Height of a Transfer unit

NTU = Number of Transfer Units (obtained by numerical integration)

$$Z = \frac{V}{K_y a \, A_c} \int_{y_{A\,in}}^{y_{A\,out}} \frac{dy}{(y_A^* - y_A)} \qquad Z = H_{OG} \times N_{OG}$$

$$Z = \frac{L}{K_x a \, A_c} \int_{x_{A\,in}}^{x_{A\,out}} \frac{d\,x_A}{(x_A - x_A^*)} \qquad Z = H_{OL} \times N_{OL}$$

Packing Height (Z)



Packing Height (Z) = height of transfer unit (HTU) × number of transfer units (n)

Packing Height (Z)

Typical values for the HTU of random packings are:

25 mm (1 in.)	0.3 to 0.6 m (1 to 2 ft)
38 mm ($1\frac{1}{2}$ in.)	0.5 to 0.75 m ($1\frac{1}{2}$ to $2\frac{1}{2}$ ft)
50 mm (2 in.)	0.6 to 1.0 m (2 to 3 ft)

Packing Height (Z)

Table 11.3. Typical packing efficiencies

System	Pressure kPa	Column dia, m	Packing type	Packing size, mm	HTU m	HETP m
Absorption						
Hydrocarbons	6000	0.9	Pall	50		0.85
NH_3-Air-H_2O	101	—	Berl	50	0.50	
Air-water	101	—	Berl	50	0.50	
Acetone-water	101	0.6	Pall	50		0.75
Distillation						
Pentane-propane	101	0.46	Pall	25		0.46
IPA-water	101	0.46	Int.	25	0.75	0.50
Methanol-water	101	0.41	Pall	25	0.52	
	101	0.20	Int.	25		0.46
Acetone-water	101	0.46	Pall	25		0.37
	101	0.36	Int.	25		0.46
Formic acid-water	101	0.91	Pall	50		0.45
Acetone-water	101	0.38	Pall	38	0.55	0.45
	101	0.38	Int.	50	0.50	0.45
	101	1.07	Int.	38		1.22
MEK-toluene	101	0.38	Pall	25	0.29	0.35
	101	0.38	Int.	25	0.27	0.23
	101	0.38	Berl	25	0.31	0.31

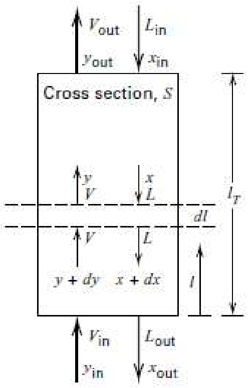
Pall = Pall rings, Berl = Berl saddles, Int. = Intalox saddles

Operating Line for Packed Absorption Towers

- A packed tower with countercurrent flow

- Mass balance around the cross-section A-A:

$$Vy_A + Lx_A = V(y_A - dy_A) + L(x_A + dx_A)$$



Therefore,

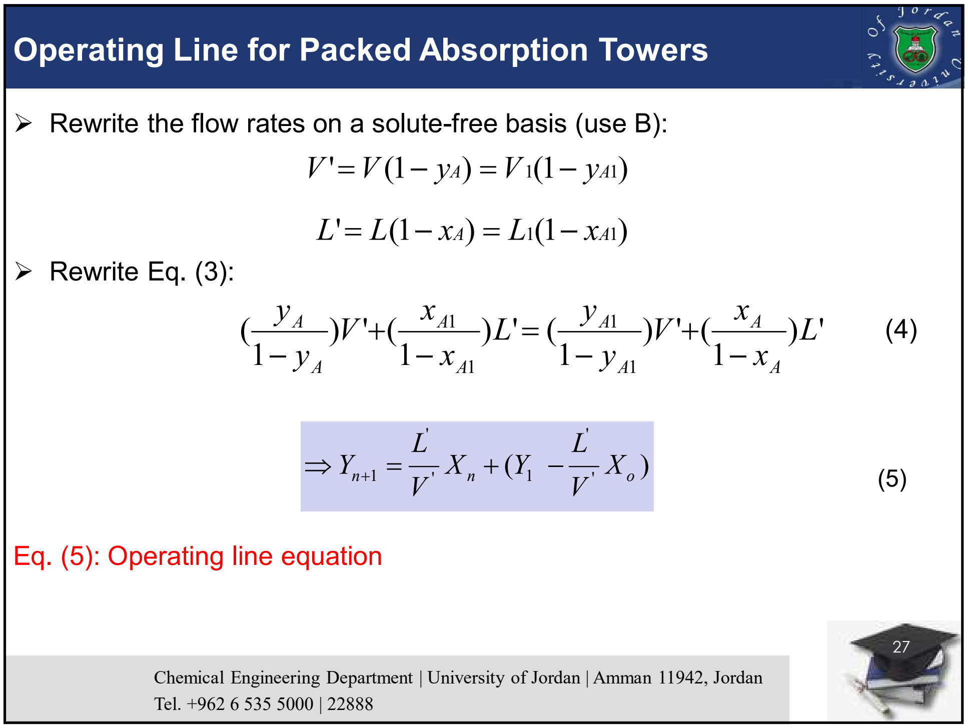
$$V\,dy_A = L\,dx_A$$

However, the flow rates do not remain constant through the column:

$$d(Vy_A) = d(Lx_A)$$

➢ Integrate from the top of the column down to the cross-section A-A:

$$Vy_A - V_1 y_{A1} = Lx_A - L_1 x_{A1} \quad \text{or} \quad x_{in}L_{in} + yV_l = xL_l + y_{out}V_{out} \qquad (3)$$



Operating Line for Packed Absorption Towers

- Rewrite the flow rates on a solute-free basis (use B):

$$V' = V(1-y_A) = V_1(1-y_{A1})$$

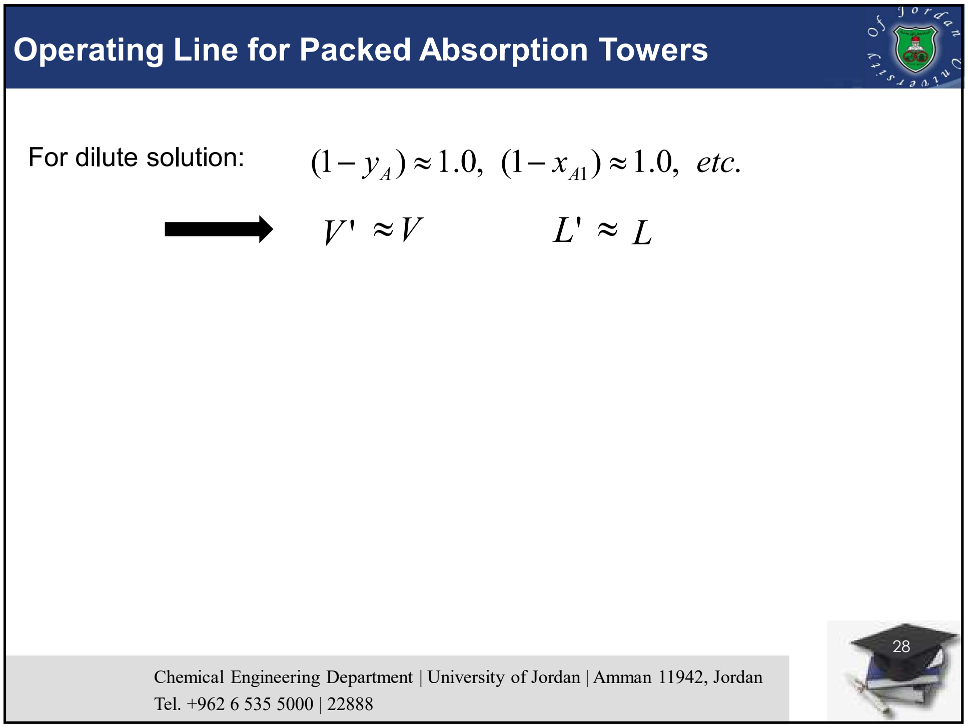
$$L' = L(1-x_A) = L_1(1-x_{A1})$$

- Rewrite Eq. (3):

$$\left(\frac{y_A}{1-y_A}\right)V' + \left(\frac{x_{A1}}{1-x_{A1}}\right)L' = \left(\frac{y_{A1}}{1-y_{A1}}\right)V' + \left(\frac{x_A}{1-x_A}\right)L' \qquad (4)$$

$$\Rightarrow Y_{n+1} = \frac{L'}{V'}X_n + \left(Y_1 - \frac{L'}{V'}X_o\right) \qquad (5)$$

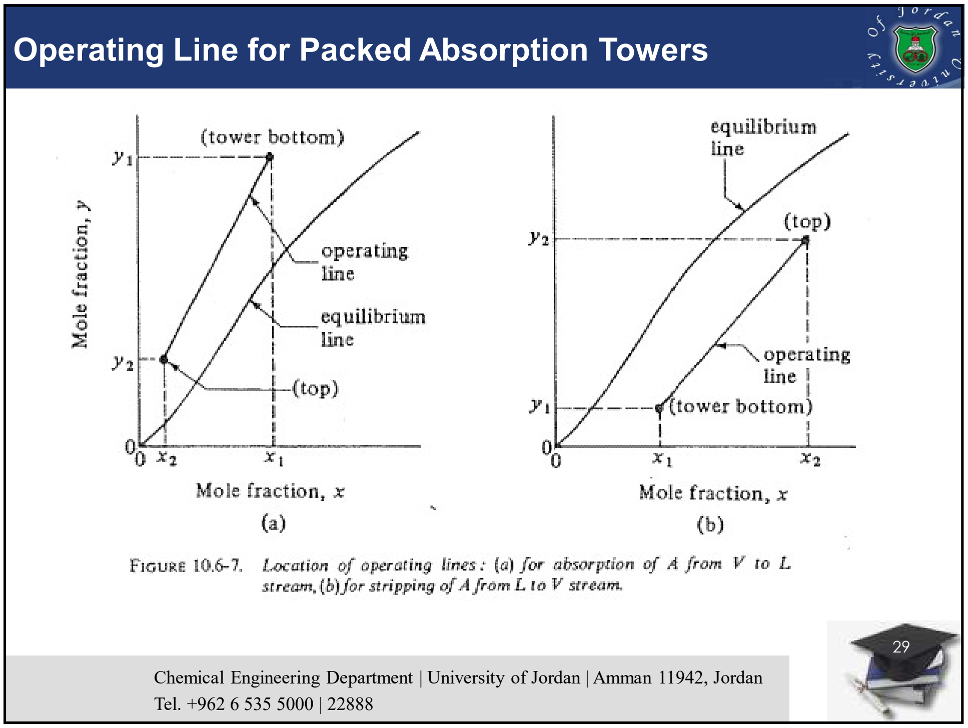
Eq. (5): Operating line equation



Operating Line for Packed Absorption Towers

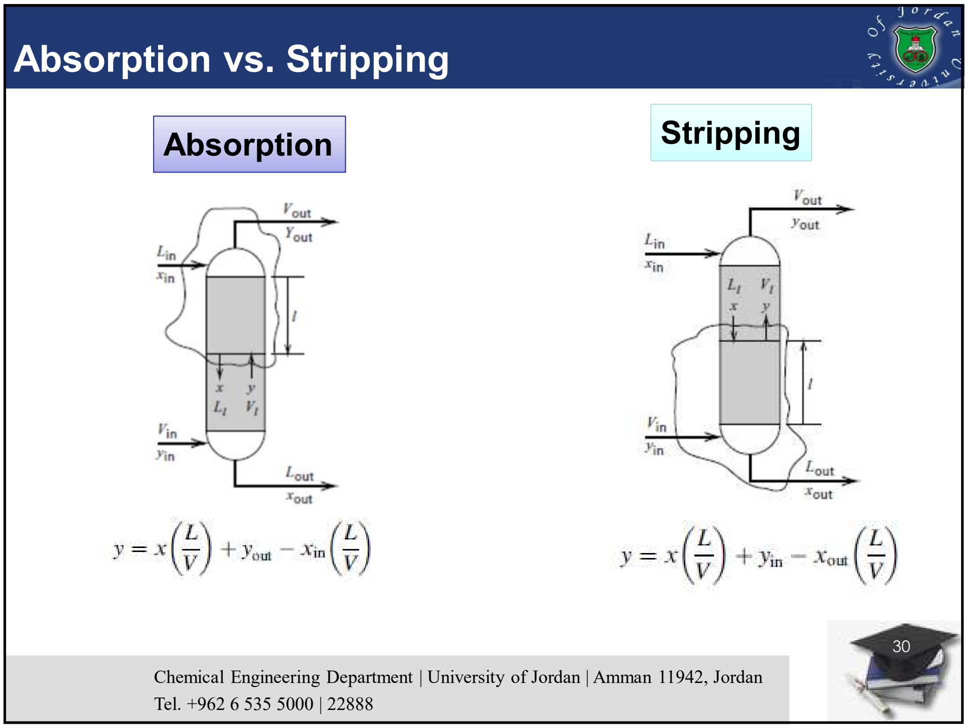
For dilute solution: $(1-y_A) \approx 1.0, \ (1-x_{A1}) \approx 1.0, \ etc.$

$$\Longrightarrow \quad V' \approx V \qquad L' \approx L$$



Operating Line for Packed Absorption Towers

FIGURE 10.6-7. *Location of operating lines: (a) for absorption of A from V to L stream, (b) for stripping of A from L to V stream.*



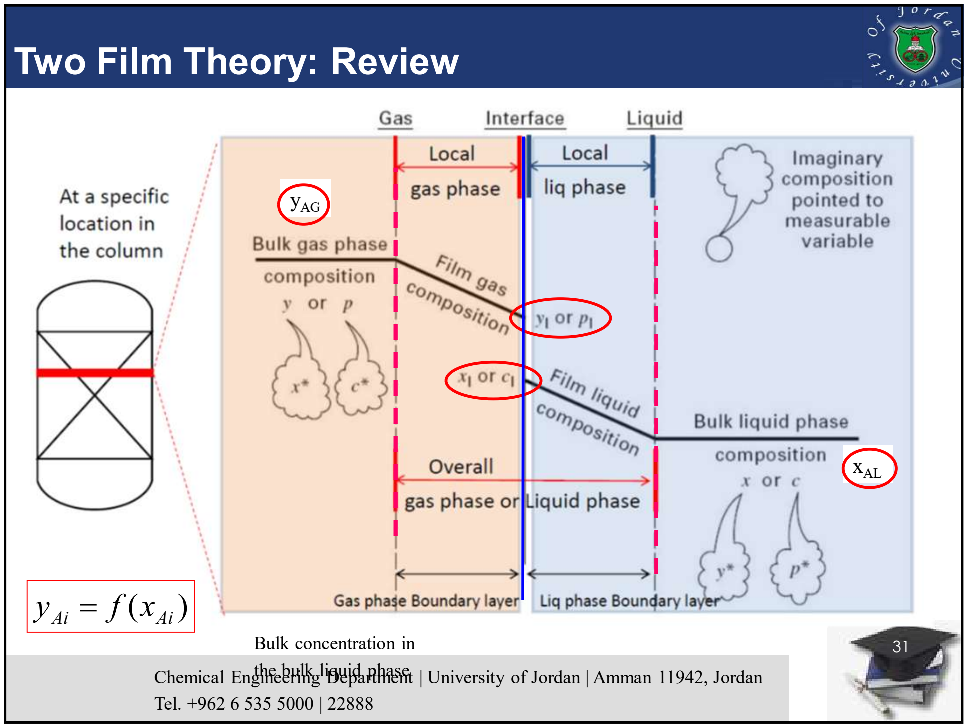
Absorption vs. Stripping

Absorption

$$y = x\left(\frac{L}{V}\right) + y_{out} - x_{in}\left(\frac{L}{V}\right)$$

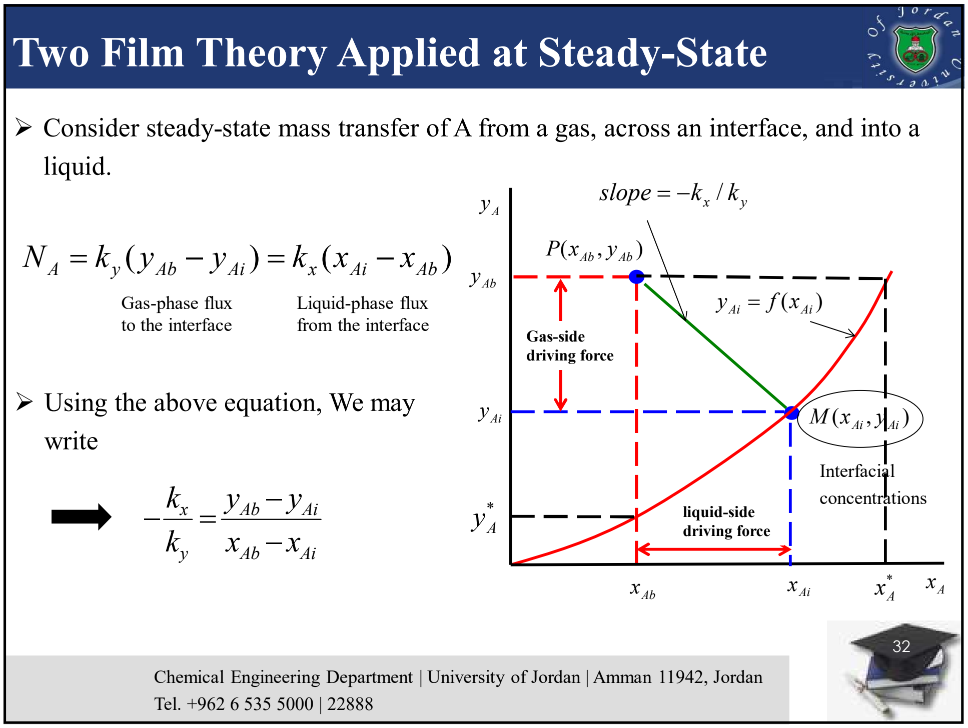
Stripping

$$y = x\left(\frac{L}{V}\right) + y_{in} - x_{out}\left(\frac{L}{V}\right)$$



Two Film Theory: Review

$$y_{Ai} = f(x_{Ai})$$



Two Film Theory Applied at Steady-State

- Consider steady-state mass transfer of A from a gas, across an interface, and into a liquid.

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab})$$

Gas-phase flux to the interface

Liquid-phase flux from the interface

- Using the above equation, We may write

$$\Rightarrow \quad -\frac{k_x}{k_y} = \frac{y_{Ab} - y_{Ai}}{x_{Ab} - x_{Ai}}$$

Two Film Theory: Review

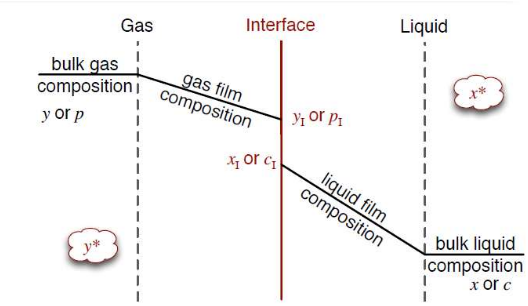
$$N_A = k_y a(y - y_I) = k_x a(x_I - x)$$

$$y = y_I - \frac{k_x a}{k_y a}(x - x_I)$$

relative resistance of mass transfer between the two phases

a = surface area per unit volume of packing

$$= \frac{interfacial\ area}{unit\ volume}$$



Overall mass transfer coefficient approach:

$$N_A = K_x a(x_A^* - x_A) = K_y a(y_A - y_A^*)$$

$$\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m}{k_x a}$$

$$\frac{1}{K_x a} = \frac{1}{k_x a} + \frac{1}{m k_y a}$$

Two Film Theory: Review

Where

y_A^* = equilibrium mole fraction of the solute in the vapor corresponding to the mole fraction x_A in the liquid

x_A^* = equilibrium mole fraction of the solute in the liquid corresponding to the mole fraction y_A in the vapor

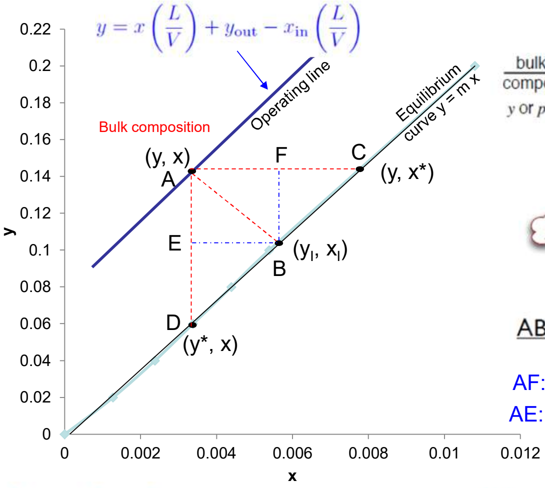
$K_y a$, $K_x a$: volumetric mass transfer coefficients

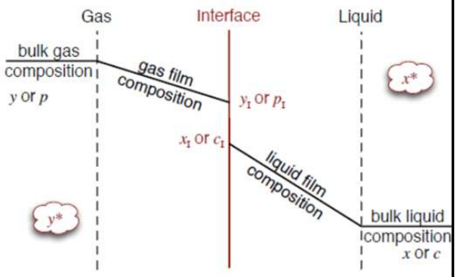
$$k'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \qquad k'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(SI)}$$

$$K'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \qquad K'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(SI)}$$

$$k'_y a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \qquad k'_x a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(English)}$$

Two Film Theory: Review





AB line: $y = y_I - \frac{k_x a}{k_y a}(x - x_I)$

AF: Deriving force in the liquid side

AE: Deriving force in the gas side

$$N_A = K_x(x_A^* - x_{A_b}) \\ = K_y(y_{A_b} - y_A^*)$$

$$x_A^* = \frac{y_{A_b}}{m}$$

$$y_A^* = x_{A_b} m$$

Volume-Based Mass-Transfer Coefficients

- If A is the absorption tower crc $K_y a$ ectio $K_x a$ area, and Z the packing height, then AZ is the tower packing volume.
Defining A_i as the total interfacial area:

$$A_i = aAz$$

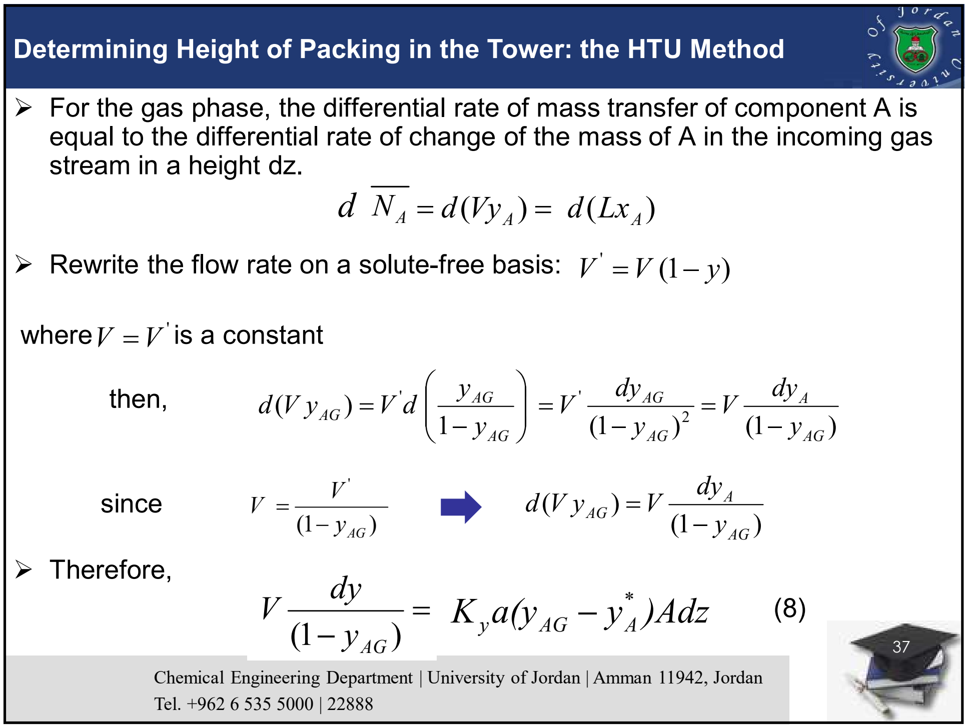
or in differential form:

$$dA_i = aAdz$$

- For constant mass flux, with units (moles/unit time) = (moles/unit time·area)(area):

$$d\,\overline{N}_A = N_A\, dA_i = K_y(y_{AG} - y_A^*)aAdz$$

$$\longrightarrow d\,\overline{N}_A = K_y a(y_{AG} - y_A^*)Adz = K_x a(x_A^* - x_{AL})Adz$$



Determining Height of Packing in the Tower: the HTU Method

- For the gas phase, the differential rate of mass transfer of component A is equal to the differential rate of change of the mass of A in the incoming gas stream in a height dz.

$$d\,\overline{N_A} = d(Vy_A) = d(Lx_A)$$

- Rewrite the flow rate on a solute-free basis: $V' = V(1-y)$

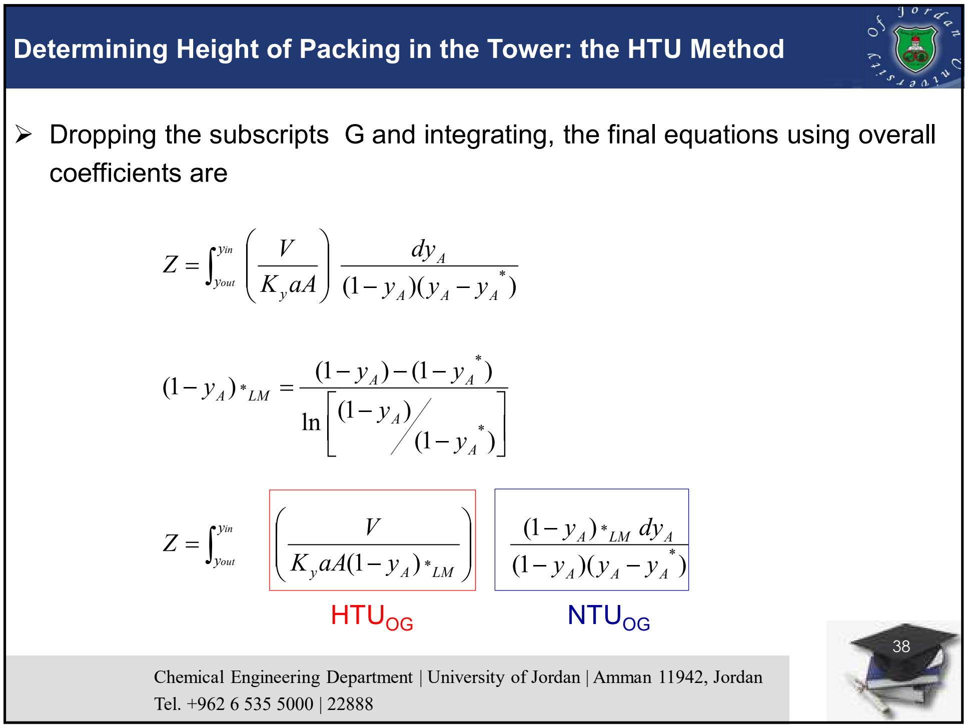
where $V = V'$ is a constant

then, $$d(V\,y_{AG}) = V'd\left(\frac{y_{AG}}{1-y_{AG}}\right) = V'\frac{dy_{AG}}{(1-y_{AG})^2} = V\frac{dy_A}{(1-y_{AG})}$$

since $$V = \frac{V'}{(1-y_{AG})} \Rightarrow d(V\,y_{AG}) = V\frac{dy_A}{(1-y_{AG})}$$

- Therefore,

$$V\frac{dy}{(1-y_{AG})} = K_y a(y_{AG} - y_A^*)Adz \qquad (8)$$



Determining Height of Packing in the Tower: the HTU Method

- Dropping the subscripts G and integrating, the final equations using overall coefficients are

$$Z = \int_{y_{out}}^{y_{in}} \left(\frac{V}{K_y aA}\right)\frac{dy_A}{(1-y_A)(y_A-y_A^*)}$$

$$(1-y_A)_{*LM} = \frac{(1-y_A)-(1-y_A^*)}{\ln\left[(1-y_A)\Big/(1-y_A^*)\right]}$$

$$Z = \int_{y_{out}}^{y_{in}} \underbrace{\boxed{\left(\frac{V}{K_y aA(1-y_A)_{*LM}}\right)}}_{HTU_{OG}} \underbrace{\boxed{\frac{(1-y_A)_{*LM}\,dy_A}{(1-y_A)(y_A-y_A^*)}}}_{NTU_{OG}}$$

Determining Height of Packing in the Tower: the HTU Method

- The term H_{OG} is called the overall **H**eight of a **T**ransfer **U**nit (*HTU*) based on the gas phase.
- Experimental data show that the *HTU* varies less with *V* than with $K_y a$.
- The smaller the *HTU*, the more efficient is the contacting.
- The term N_{OG} is called the overall **N**umber of **T**ransfer **U**nits (*NTU*) based on the gas phase.
- It represents the overall change in solute mole fraction divided by the average mole fraction driving force.
- The larger the *NTU*, the greater is the extent of contacting required.

Determining Height of Packing in the Tower: the HTU Method

But $K_y^{'} = K_y(1-y_A)_{LM}$

$$\Rightarrow \quad Z = \int_{y_{out}}^{y_{in}} \left(\frac{V}{K_y^{'} aA}\right) \frac{(1-y_A)_{*LM}}{(1-y_A)(y_A - y_A^{*})} dy_A$$

- Graphical integration of right hand side of the equation is performed to find NTU_{OG}
- For dilute system (i.e. when the mole fractions y and x in the gas and liquid streams are less than about 0.10, i.e., 10%.)

$$Z = \left(\frac{V(1-y_A)_{*LM}}{K_y^{'} aA(1-y_A)}\right) \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^{*})} dy_A$$

- Also , for dilute solutions: $(1-y_A) \approx (1-y_A)_{*LM} \approx 1.0$ and $K_y^{'} = K_y$

Determining Height of Packing in the Tower: the HTU Method

$$\Rightarrow \quad Z = \left(\frac{V}{K_y' aA}\right)\int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)}\,dy_A$$

- The ratio of flow rate to mass transfer has been designated as the height of a transfer unit (HTU), or, for the gas phase, H_{OG}

$$H_{OG} = \left(\frac{V}{K_y' aA}\right)$$

- Therefore, H_{OG} has been defined in such a way that it remains constant through the absorption column.

Determining Height of Packing in the Tower: the HTU Method

- The number of overall mass transfer units (NTU), or for the gas phase, N_{OG} has been defined:

$$N_{OG} = \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)}\,dy_A$$

- The height of the column may now be calculated from:

$$z = H_{OG} N_{OG}$$

Determining Height of Packing in the Tower: the HTU Method

- In similar manner, the analysis can be performed based on the liquid phase which would be useful in stripping calculations to give

$$\Rightarrow \quad Z = \left(\frac{L}{K_x^{'}aA}\right)\int_{xin}^{xout} \frac{(1-x_A)_{*LM}}{(1-x_A)(x_A^{*}-x_A)}dx_A$$

Hence, $H_{OL} = \left(\frac{L}{K_x^{'}aA}\right)$ and $N_{OL} = \int_{xin}^{xout} \frac{(1-x_A)_{*LM}}{(1-x_A)(x_A-x_A^{*})}dx_A$

$$\Rightarrow \quad z = H_{OL}N_{OL}$$

- For dilute solutions: $(1-x_A) \approx (1-x_A)_{LM} \approx 1.0$ and $K_x^{'} = K_x$

$$\Rightarrow \quad N_{OL} = \int_{Xin}^{Xout} \frac{1}{(x_A^{*}-x_A)}dx_A$$

Summary

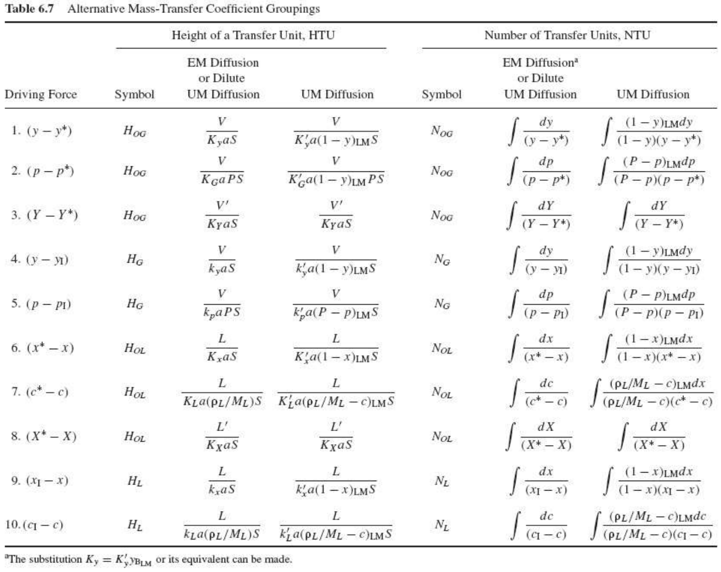


Table 6.7 Alternative Mass-Transfer Coefficient Groupings

	Height of a Transfer Unit, HTU			Number of Transfer Units, NTU		
Driving Force	Symbol	EM Diffusion or Dilute UM Diffusion	UM Diffusion	Symbol	EM Diffusion[a] or Dilute UM Diffusion	UM Diffusion
1. $(y-y^*)$	H_{OG}	$\frac{V}{K_y aS}$	$\frac{V}{K_y' a(1-y)_{LM}S}$	N_{OG}	$\int\frac{dy}{(y-y^*)}$	$\int\frac{(1-y)_{LM}dy}{(1-y)(y-y^*)}$
2. $(p-p^*)$	H_{OG}	$\frac{V}{K_G aPS}$	$\frac{V}{K_G' a(1-y)_{LM}PS}$	N_{OG}	$\int\frac{dp}{(p-p^*)}$	$\int\frac{(P-p)_{LM}dp}{(P-p)(p-p^*)}$
3. $(Y-Y^*)$	H_{OG}	$\frac{V'}{K_Y aS}$	$\frac{V'}{K_Y aS}$	N_{OG}	$\int\frac{dY}{(Y-Y^*)}$	$\int\frac{dY}{(Y-Y^*)}$
4. $(y-y_I)$	H_G	$\frac{V}{k_y aS}$	$\frac{V}{k_y' a(1-y)_{LM}S}$	N_G	$\int\frac{dy}{(y-y_I)}$	$\int\frac{(1-y)_{LM}dy}{(1-y)(y-y_I)}$
5. $(p-p_I)$	H_G	$\frac{V}{k_p aPS}$	$\frac{V}{k_p' a(P-p)_{LM}S}$	N_G	$\int\frac{dp}{(p-p_I)}$	$\int\frac{(P-p)_{LM}dp}{(P-p)(p-p_I)}$
6. (x^*-x)	H_{OL}	$\frac{L}{K_x aS}$	$\frac{L}{K_x' a(1-x)_{LM}S}$	N_{OL}	$\int\frac{dx}{(x^*-x)}$	$\int\frac{(1-x)_{LM}dx}{(1-x)(x^*-x)}$
7. (c^*-c)	H_{OL}	$\frac{L}{K_L a(\rho_L/M_L)S}$	$\frac{L}{K_L' a(\rho_L/M_L-c)_{LM}S}$	N_{OL}	$\int\frac{dc}{(c^*-c)}$	$\int\frac{(\rho_L/M_L-c)_{LM}dx}{(\rho_L/M_L-c)(c^*-c)}$
8. (X^*-X)	H_{OL}	$\frac{L'}{K_X aS}$	$\frac{L'}{K_X aS}$	N_{OL}	$\int\frac{dX}{(X^*-X)}$	$\int\frac{dX}{(X^*-X)}$
9. (x_I-x)	H_L	$\frac{L}{k_x aS}$	$\frac{L}{k_x' a(1-x)_{LM}S}$	N_L	$\int\frac{dx}{(x_I-x)}$	$\int\frac{(1-x)_{LM}dx}{(1-x)(x_I-x)}$
10. (c_I-c)	H_L	$\frac{L}{k_L a(\rho_L/M_L)S}$	$\frac{L}{k_L' a(\rho_L/M_L-c)_{LM}S}$	N_L	$\int\frac{dc}{(c_I-c)}$	$\int\frac{(\rho_L/M_L-c)_{LM}dc}{(\rho_L/M_L-c)(c_I-c)}$

[a]The substitution $K_y = K_y' y_{B_{LM}}$ or its equivalent can be made.

STEP-BY-STEP PROCEDURE

(1) For a particular gas-liquid system, draw equilibrium curve on *X*-*Y* plane.

(2) Draw operating line in *X*-*Y* plane (*PQ*) using material balance Equation. Lower terminal *Q* (X_2, Y_2) and upper terminal *P* (X_1, Y_1) are placed in *x*-*y* plane.

Overall mass balance Equation for the absorption tower is as follows:

$$L'(\frac{x_{A0}}{1-x_{A0}})+V'(\frac{y_{A2}}{1-y_{A2}})=L'(\frac{x_{A1}}{1-x_{A1}})+V'(\frac{y_{A1}}{1-y_{A1}})$$

If liquid mass flow rate, L' is not known, minimum liquid mass flow rate $L'_{\min}$ is to be determined. L' is generally 1.2 to 2 times the $L'_{\min}$.

STEP-BY-STEP PROCEDURE

In the figure, lower terminal of absorption tower is represented by Q (X_2, Y_2); i.e., bottom of the tower. Operating line is *PQ*. If liquid rate is decreased, slope of operating line (L'/V') also decreases and operating line shifts from *PQ* to $P^{/}Q$, when touches equilibrium line. This operating line is tangent to equilibrium line.

The driving force for absorption is zero at $P^{/}$ and is called "PINCH POINT".

(3) A point *A* (*x*, *y*) is taken on the operating line. From the known value of k_x and k_y or $k_x\bar{a}$ and $k_y\bar{a}$, a line is drawn with slope of k_x/k_y to equilibrium line, $B(x_i, y_i)$. Line *AB* is called "TIE LINE" and x_i and y_i are known for a set of values of *x* and *y*.

(4) Step (3) is repeated for other points in the operating line to get several (x_i, y_i) sets for $y_1 \geq y \geq y_2$.

STEP-BY-STEP PROCEDURE

(5) Calculate flow rate of gas $V = V'/(1-y_A)$ at each point

(6) Calculate height of the packing graphically or numerically.

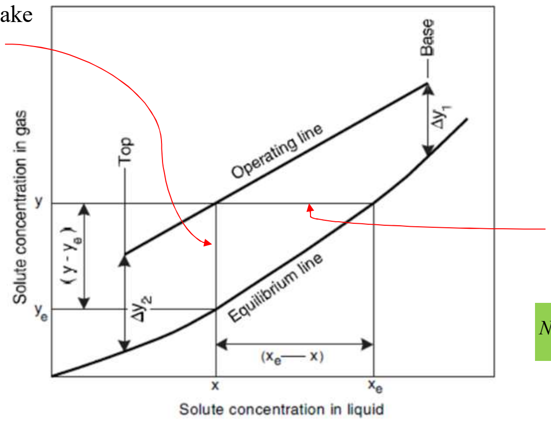
$$Z = \left(\frac{V}{K'_y aA}\right)\int_{y_{out}}^{y_{in}} \frac{(1-y_A)_{*LM}}{(1-y_A)(y_A-y_A^{*})} dy_A$$

Summary

At a given x take

$\Delta y = (y-y_e)$

$$\int_{y_{A\,out}}^{y_{A\,in}} \frac{dy}{(y_A - y_A^{*})}$$

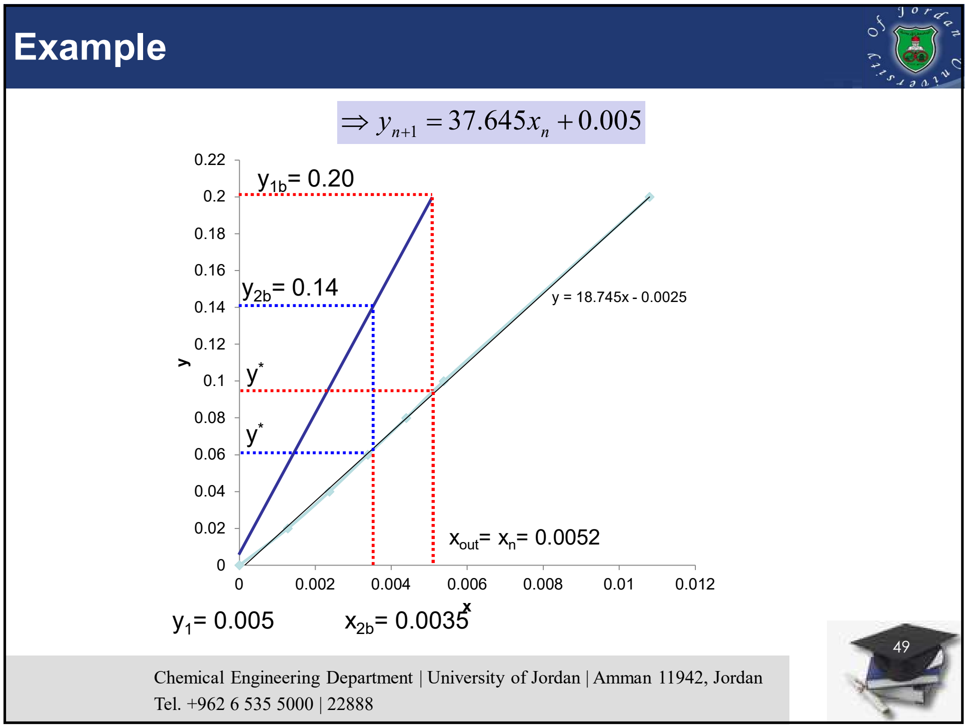


At a given y take

$\Delta x = (x_e-x)$

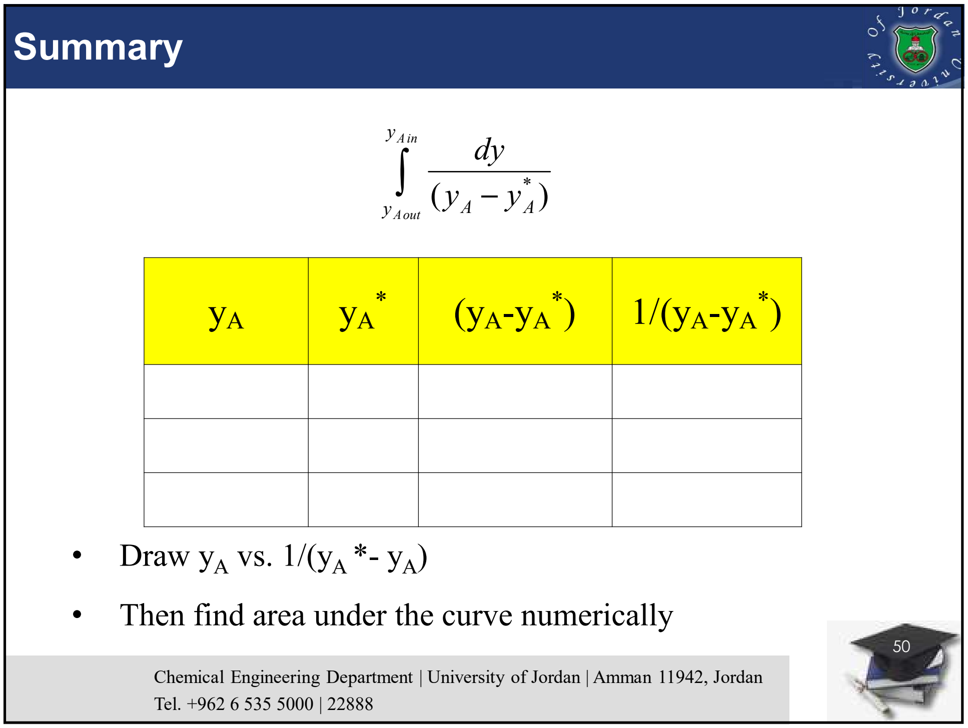
$$N_{OL} = \int_{x_{in}}^{x_{out}} \frac{1}{(x_A^{*} - x_A)} dx_A$$

Gas absorption concentration relationships



Example

$$\Rightarrow y_{n+1} = 37.645 x_n + 0.005$$



Summary

$$\int_{y_{A\,out}}^{y_{A\,in}} \frac{dy}{(y_A - y_A^*)}$$

y_A	y_A^*	$(y_A - y_A^*)$	$1/(y_A - y_A^*)$

- Draw y_A vs. $1/(y_A{}^* - y_A)$
- Then find area under the curve numerically

Summary

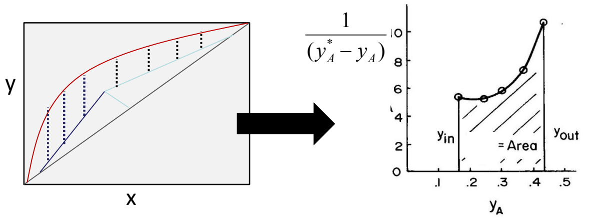
$$Z = \left(\frac{V}{K_y aA}\right)\int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^{*})} dy_A$$

H_{OG}

Substitute values to calculate H_{OG}

Integration = N_{OG}

- N_{OG} is evaluated graphically by numerical integration using the equilibrium and operating lines.
- Draw $1/(y_A^{*} - y_A)$ (on y-axis) vs. y_A (on x-axis). Area under the curve is the value of integration.



Evaluate area under the curve by numerical integration

Area = N

Numerical Evaluation of Integrals

Trapezoidal rule (2-point):

$$\int_0^{X_1} f(x)dx = \frac{h}{2}[f(X_0) + f(X_1)]$$

$$h = X_1 - X_0$$

Simpson's one-third rule (3-point):

$$\int_0^{X_2} f(x)dx = \frac{h}{3}[f(X_0) + 4f(X_1) + f(X_2)]$$

$$h = \frac{X_2 - X_0}{2} \qquad X_1 = X_0 + h$$

Simpson's three-eights rule (4-point):

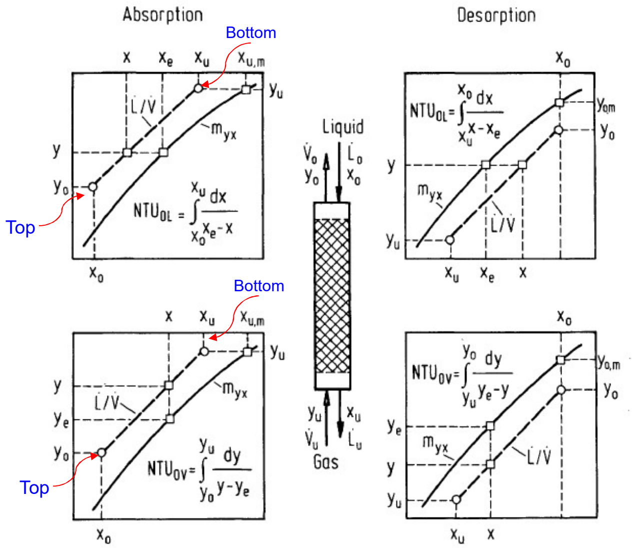
$$X_1 = X_0 + h \quad X_2 = X_0 + 2h$$

$$\int_0^{X_3} f(x)dx = \frac{3}{8}h[f(X_0) + 3f(X_1) + 3f(X_2) + f(X_3)] \qquad h = \frac{X_3 - X_0}{3}$$

Simpson's five-point quadrature :

$$\int_0^{X_4} f(x)dx = \frac{h}{3}[f(X_0) + 4f(X_1) + 2f(X_2) + 4f(X_3) + f(X_4)] \qquad h = \frac{X_4 - X_0}{4}$$

Summary



Summary

x,y	Mole fraction in liquid, gas or vapour
m_{yx}	Slope of the equilibrium line ———
$\dot{L}/\dot{V}$	Molar liquid-to-gas ratio or slope of the operating line – – –
X,Y	Load fraction of transfer component in liquid/gas
L,V	Carrier stream of liquid/gas phase

$$X_u = X_o + \frac{1}{L/V}(Y_u - Y_o) \qquad \frac{L}{V} = \frac{Y_o - Y_u}{X_o - X_u}, \qquad Y_o = Y_u + \frac{L}{V}(X_o - X_u)$$

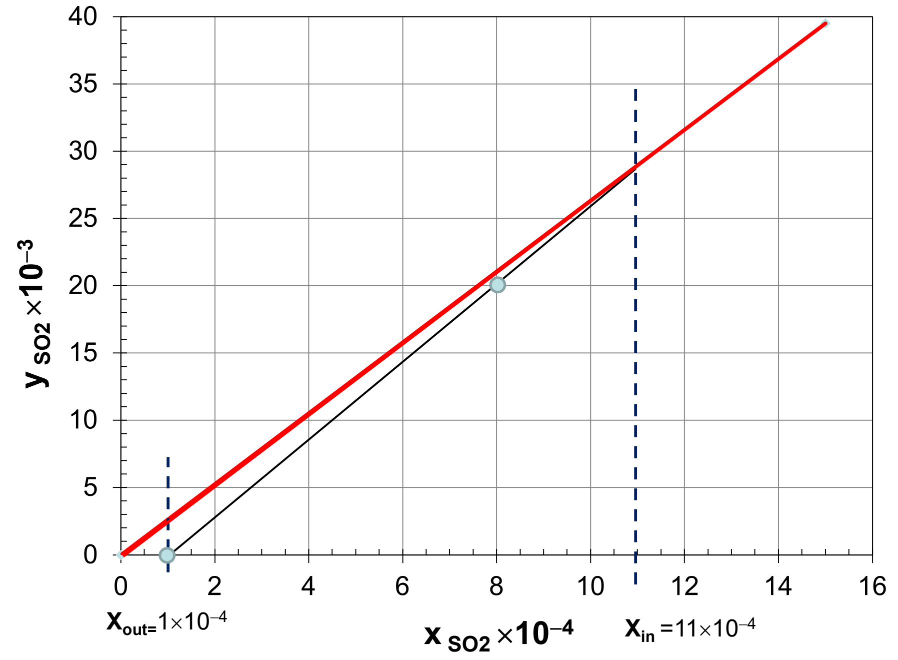
Example

We wish to strip SO_2 from water using air at 20C. The inlet air is pure. The outlet water contains 0.0001 mole fraction SO_2, while the inlet water contains 0.0011 mole fraction SO_2. Operation is at 855 mmHg and L/V = 0.9×$(L/V)_{max}$. Assume H_{OL} = 2.76 feet and that the Henry's law constant is 22,500 mmHg/mole frac SO_2.

Calculate the packing height required.

$$Z = \left(\frac{L}{K_x^{'}aA}\right)\int_{xin}^{xout}\frac{(1-x_A)_{LM}}{(1-x_A)(x_A^{*}-x_A)}dx_A$$

Example contd.



Example contd.

- $V(y_{out} – y_{out}) = L(x_{in}-x_{out}) \Rightarrow V(y_{out} – 0) = L(11x10^{-4}-1x10^{-4})$

$y_{out}= 10x10^{-4} (L/V)$

$(L/V) = 0.9 (L/V)_{max}$

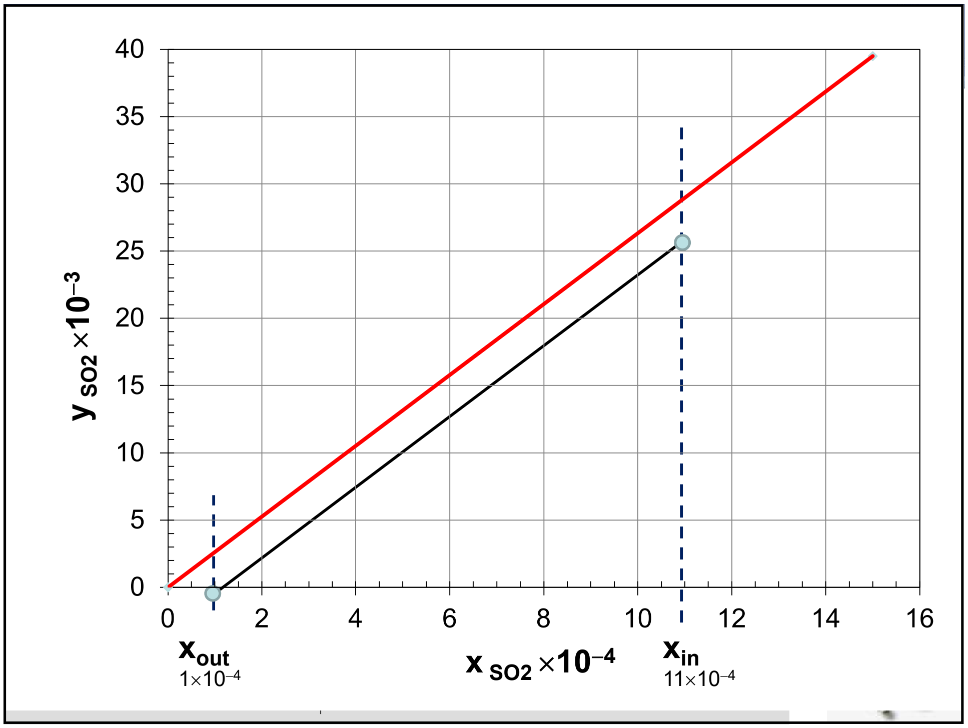
From pinch point and darwing, $(L/V)_{max}$ = slope= 29.29

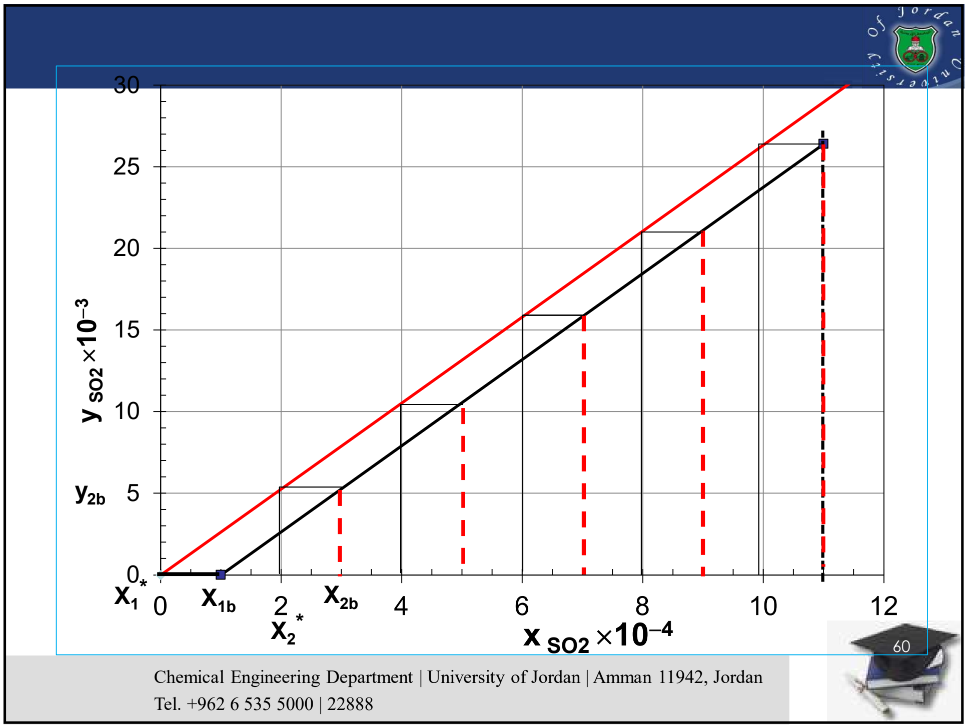
$\Rightarrow (L/V) = 0.9 \times 29.29 = 26.36$

$y_{out}= 10x10^{-4} (L/V) = 10x10^{-4} \times 26.36$

$\Rightarrow$ y_{out} = 0.02636 = 26.36×10^{-3}

Draw actual operating line





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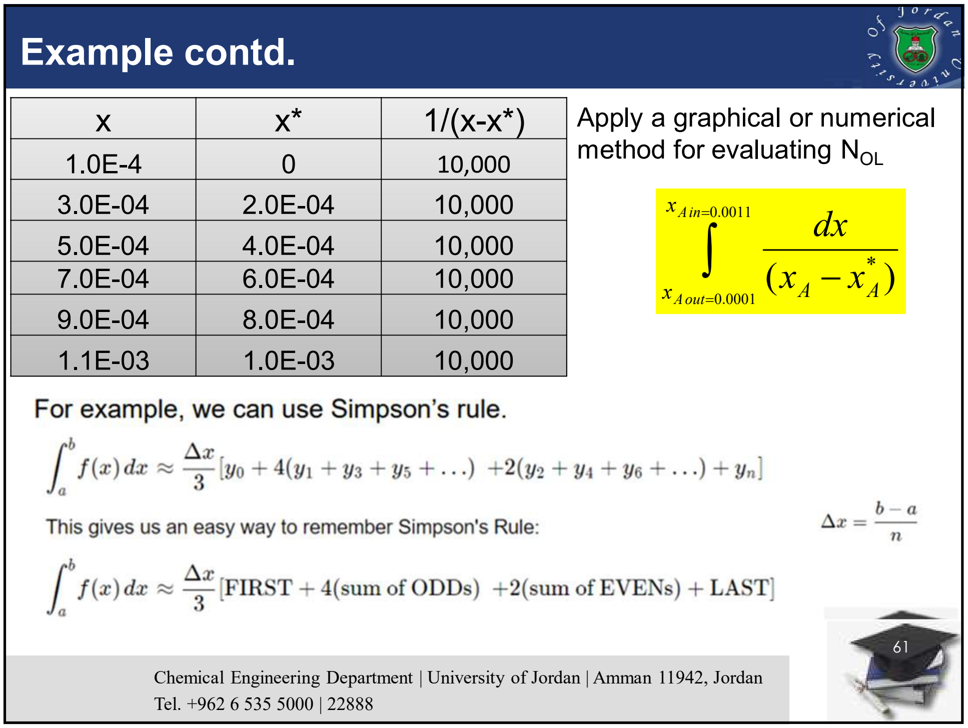


Example contd.

x	x*	1/(x-x*)
1.0E-4	0	10,000
3.0E-04	2.0E-04	10,000
5.0E-04	4.0E-04	10,000
7.0E-04	6.0E-04	10,000
9.0E-04	8.0E-04	10,000
1.1E-03	1.0E-03	10,000

Apply a graphical or numerical method for evaluating N_{OL}

$$\int_{x_{A\,out}=0.0001}^{x_{A\,in}=0.0011} \frac{dx}{(x_A - x_A^*)}$$

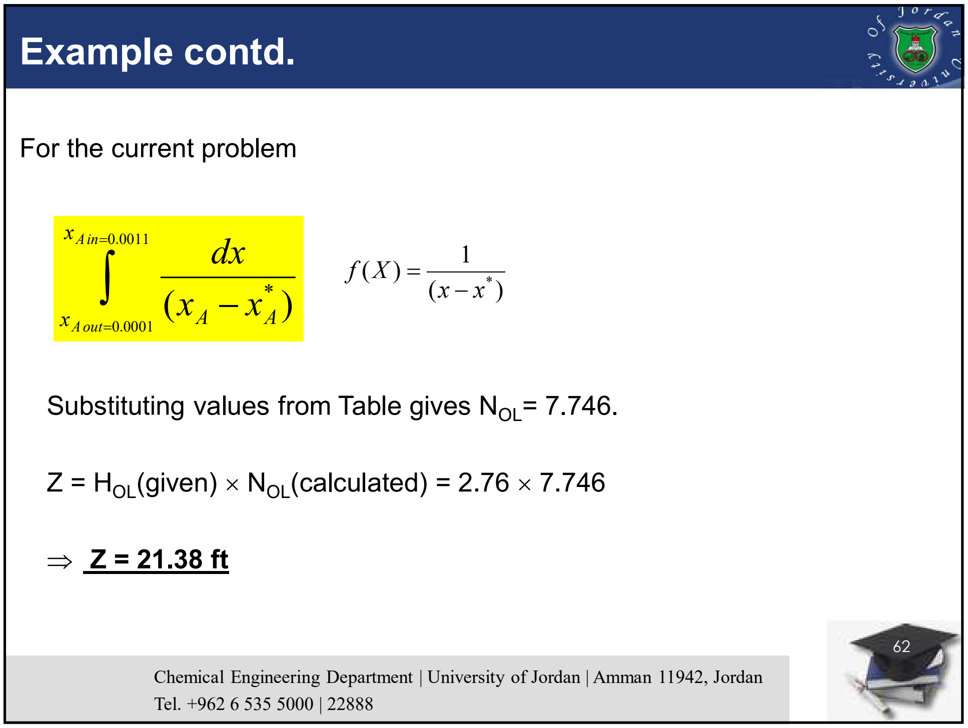
For example, we can use Simpson's rule.

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[y_0 + 4(y_1 + y_3 + y_5 + \ldots) \ +2(y_2 + y_4 + y_6 + \ldots) + y_n]$$

This gives us an easy way to remember Simpson's Rule:

$$\Delta x = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[\text{FIRST} + 4(\text{sum of ODDs}) \ +2(\text{sum of EVENs}) + \text{LAST}]$$



Example contd.

For the current problem

$$\int_{x_{A\,out}=0.0001}^{x_{A\,in}=0.0011} \frac{dx}{(x_A - x_A^*)}$$

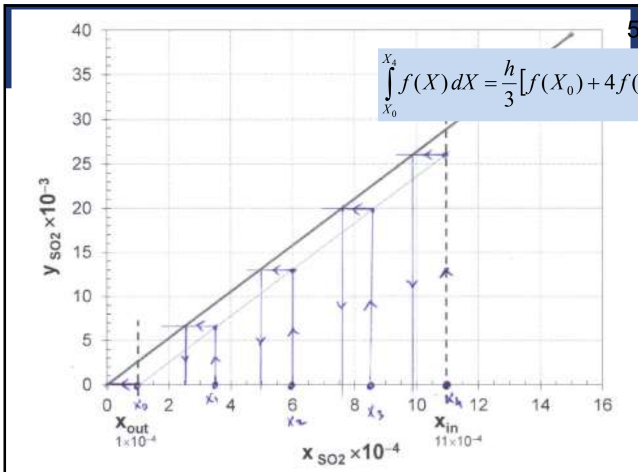
$$f(X) = \frac{1}{(x - x^*)}$$

Substituting values from Table gives N_{OL}= 7.746.

Z = H_{OL}(given) × N_{OL}(calculated) = 2.76 × 7.746

⇒ **Z = 21.38 ft**

5-point method



$$\int_{X_0}^{X_4} f(X)\,dX = \frac{h}{3}\left[f(X_0)+4f(X_1)+2f(X_2)+4f(X_3)+f(X_4)\right]$$

$$h = \frac{(X_4 - X_0)}{4}$$

Pay attention to accuracy of drawing and obtaining data.

	x	x*	f(x) = 1/(x-x*)
0	1.0E-4	0	10,000
1	3.5 x 10^-4	2.5 x 10^-4	10,000
2	6 x 10^-4	5 x 10^-4	10,000
3	8.5 x 10^-4	7.5 x 10^-4	10,000
4	11E-4	10.9 x 10^-4	10,000

Analytical Solution: Dilute Solution Case

- Also, for dilute solutions, Henry's Law is usually a good choice for an equilibrium relationship.

$$y_{Ai} = mx_{Ai}$$

And $(1\text{-}y_A) \approx (1\text{-}y_A)_{LM} \approx 1.0$

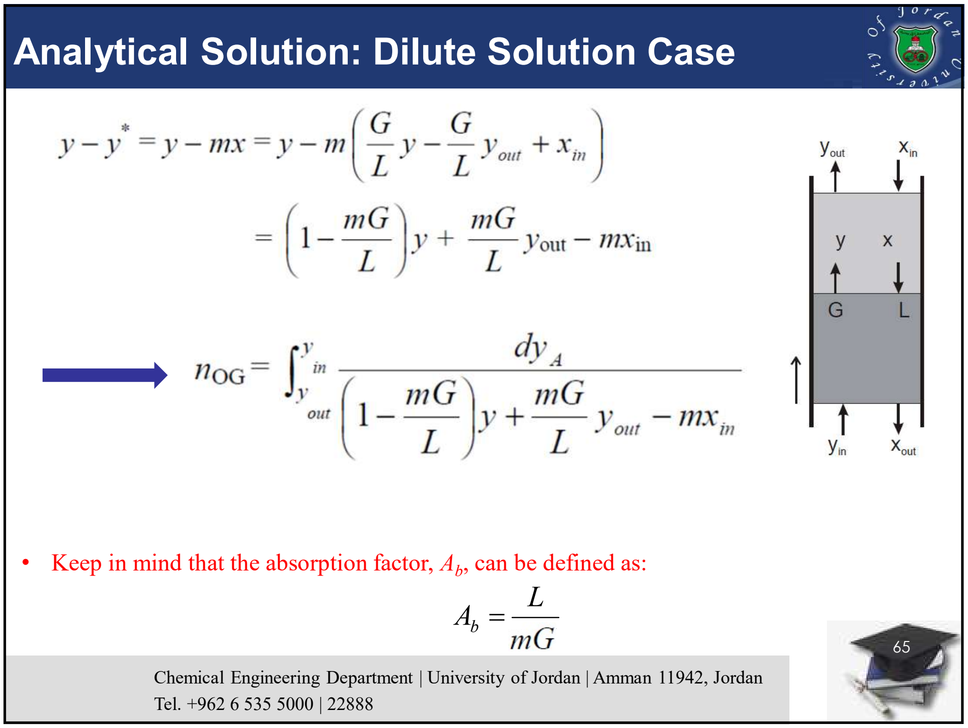
Therefore,

$$N_{OG} = \int_{y_{out}}^{y_{in}} \frac{(1-y_A)_{LM}}{(1-y_A)(y_A - y_A^{\ *})} dy_A = \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^{\ *})} dy_A$$

The operating line,

$$y = \frac{L}{G}x + y_{\text{out}} - \frac{L}{G}x_{\text{in}}$$

$$y_A = \frac{L}{G}(x_A - x_{\text{in}}) + y_{A1} = \frac{L}{mG}(mx_A - mx_{\text{in}}) + y_{\text{out}}$$



Analytical Solution: Dilute Solution Case

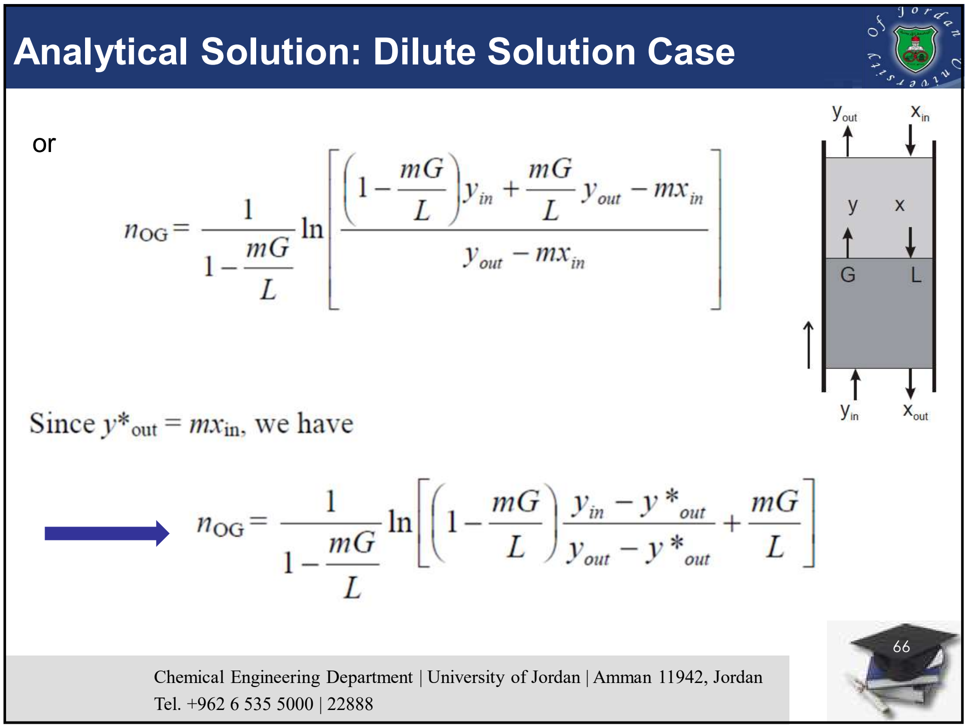
$$y - y^* = y - mx = y - m\left(\frac{G}{L}y - \frac{G}{L}y_{out} + x_{in}\right)$$

$$= \left(1 - \frac{mG}{L}\right)y + \frac{mG}{L}y_{\text{out}} - mx_{\text{in}}$$

$$\Rightarrow \quad n_{\text{OG}} = \int_{y_{out}}^{y_{in}} \frac{dy_A}{\left(1 - \frac{mG}{L}\right)y + \frac{mG}{L}y_{out} - mx_{in}}$$

- Keep in mind that the absorption factor, A_b, can be defined as:

$$A_b = \frac{L}{mG}$$



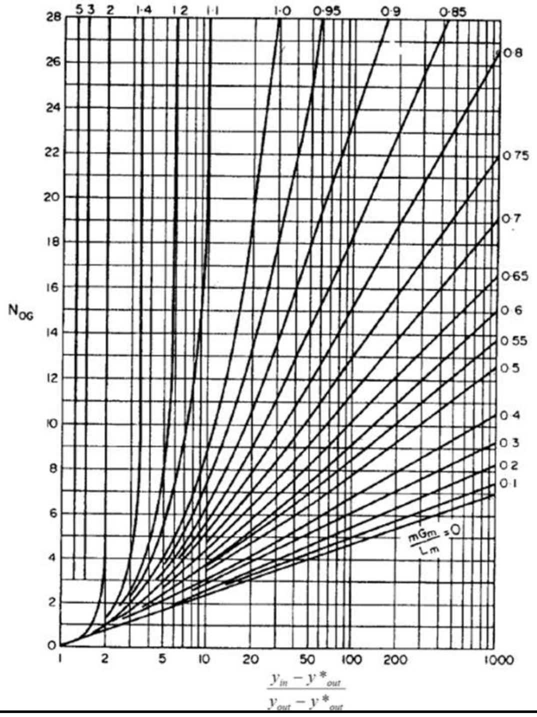
Analytical Solution: Dilute Solution Case

or

$$n_{\text{OG}} = \frac{1}{1 - \frac{mG}{L}} \ln\left[\frac{\left(1 - \frac{mG}{L}\right)y_{in} + \frac{mG}{L}y_{out} - mx_{in}}{y_{out} - mx_{in}}\right]$$

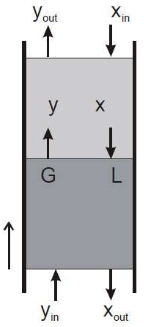
Since $y^*_{\text{out}} = mx_{\text{in}}$, we have

$$\Rightarrow \quad n_{\text{OG}} = \frac{1}{1 - \frac{mG}{L}} \ln\left[\left(1 - \frac{mG}{L}\right)\frac{y_{in} - y^*_{out}}{y_{out} - y^*_{out}} + \frac{mG}{L}\right]$$



Analytical Solution: Dilute Solution Case

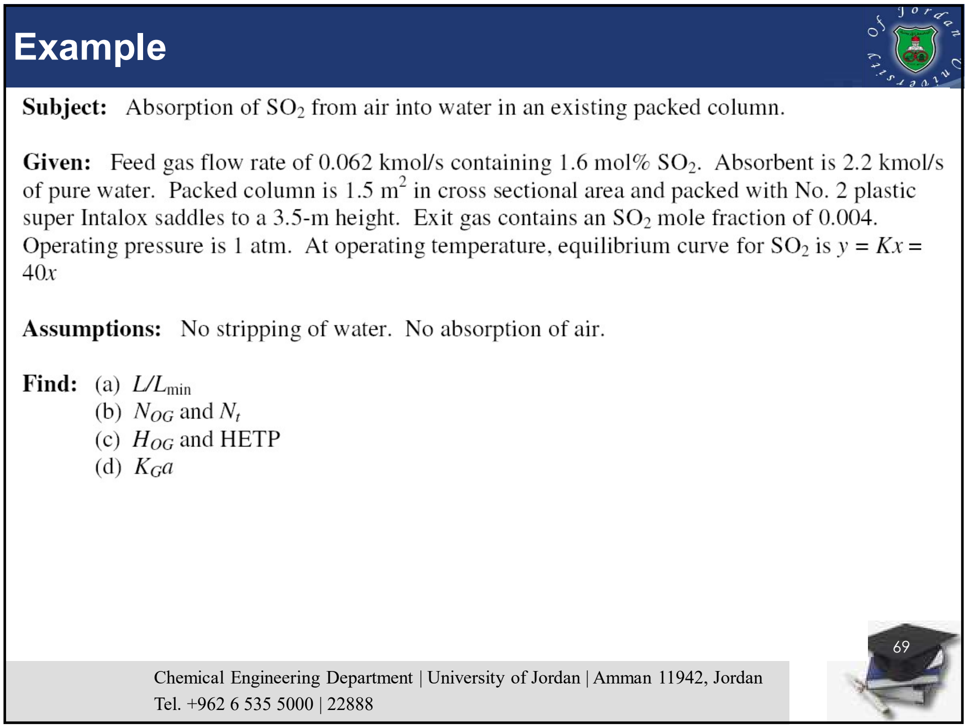
We can follow a similar procedure to obtain the number of overall liquid transfer unit



$$n_{OL} = \int_{x_{in}}^{x_{out}} \frac{dx_A}{(x - x^*)}$$

$$n_{OL} = \frac{1}{1 - \frac{L}{mG}} \ln\left[\left(1 - \frac{L}{mG}\right)\frac{x_{in} - x^*_{out}}{x_{out} - x^*_{out}} + \frac{L}{mG}\right]$$

where $x^*_{out} = y_{in}/m.$



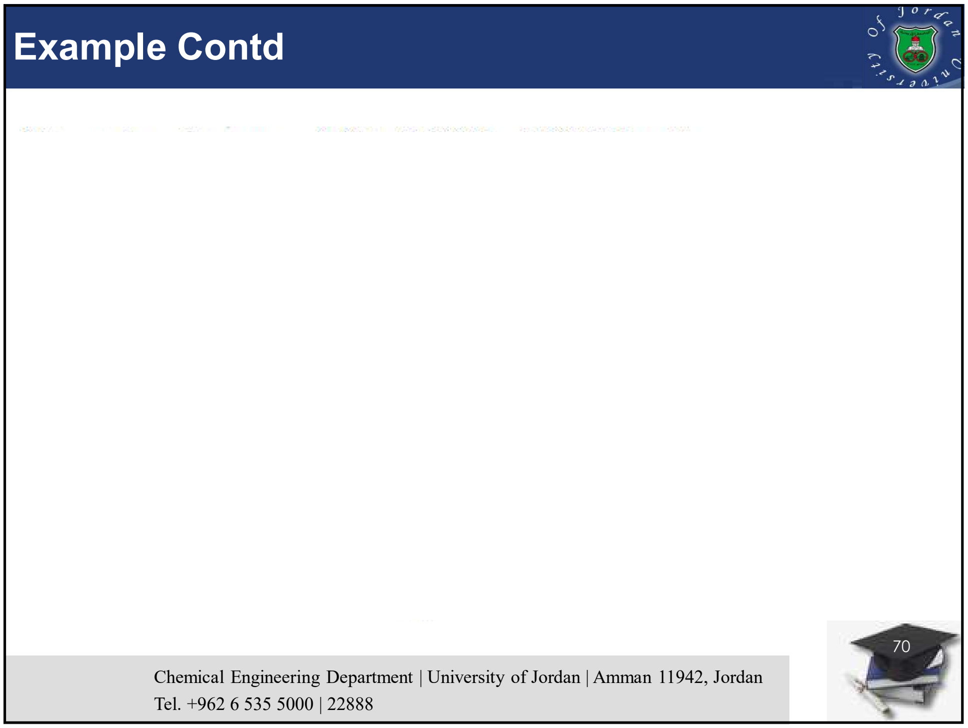
Example

Subject: Absorption of SO_2 from air into water in an existing packed column.

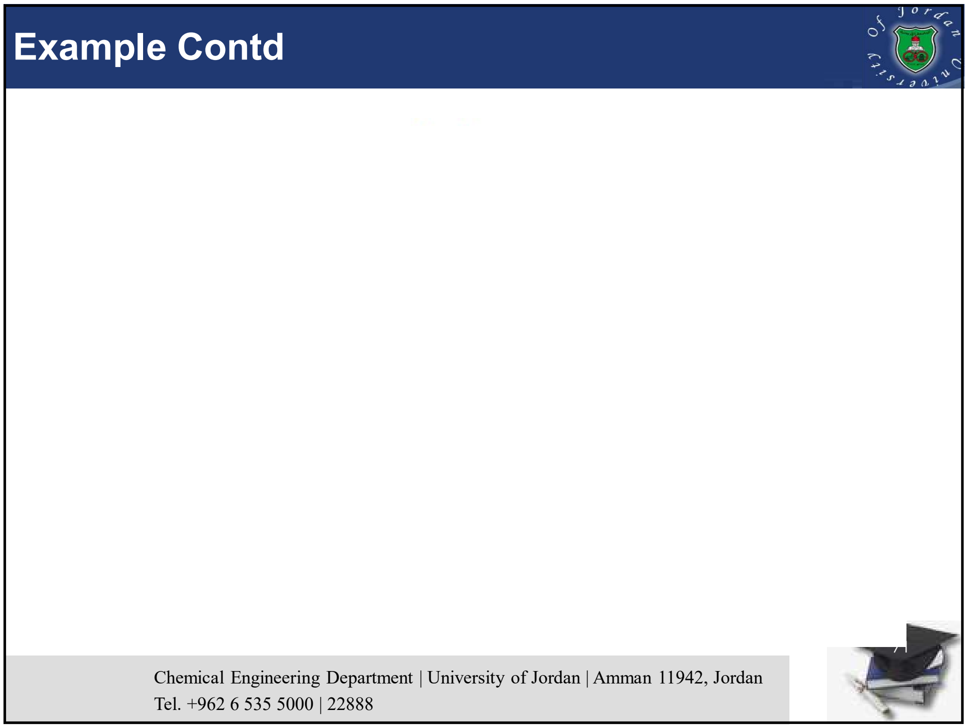
Given: Feed gas flow rate of 0.062 kmol/s containing 1.6 mol% SO_2. Absorbent is 2.2 kmol/s of pure water. Packed column is 1.5 m^2 in cross sectional area and packed with No. 2 plastic super Intalox saddles to a 3.5-m height. Exit gas contains an SO_2 mole fraction of 0.004. Operating pressure is 1 atm. At operating temperature, equilibrium curve for SO_2 is $y = Kx = 40x$

Assumptions: No stripping of water. No absorption of air.

Find: (a) $L/L_{\min}$
(b) N_{OG} and N_t
(c) H_{OG} and HETP
(d) K_Ga



Example Contd



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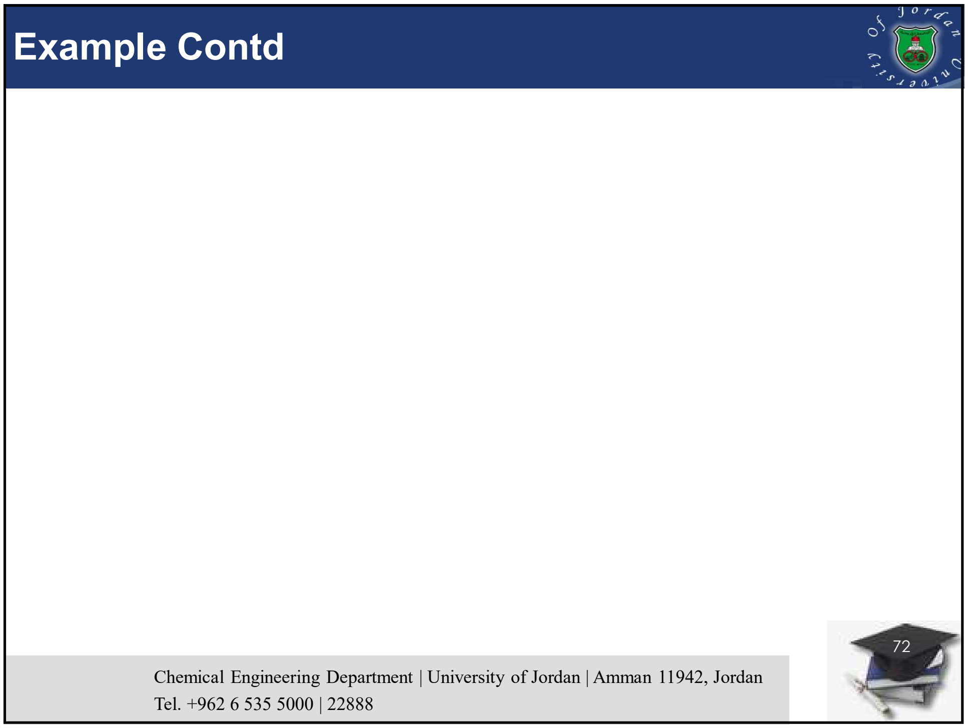

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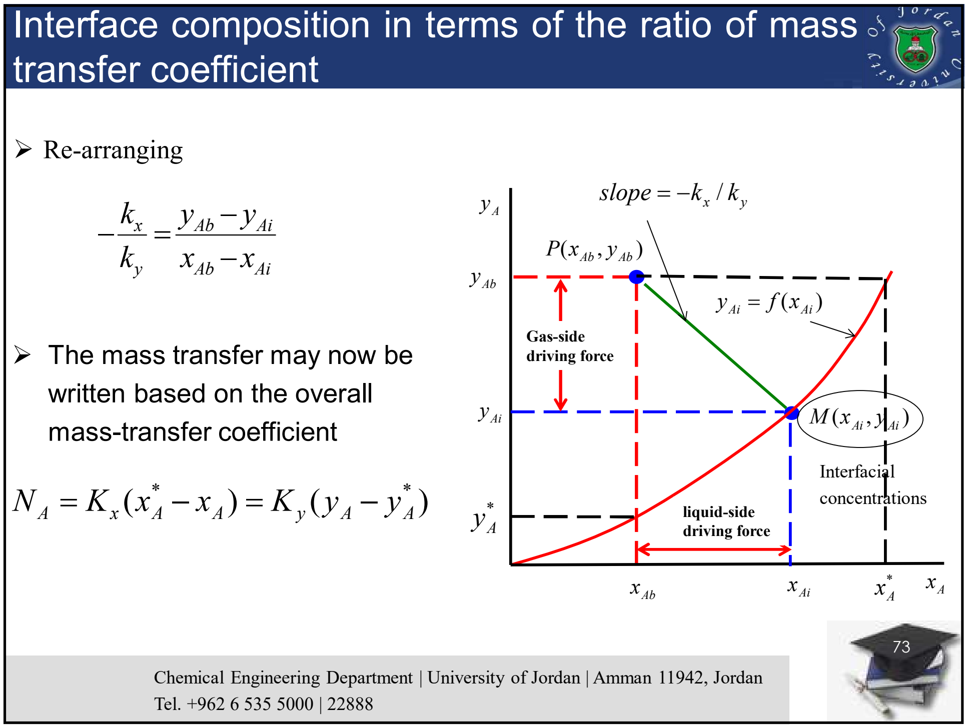


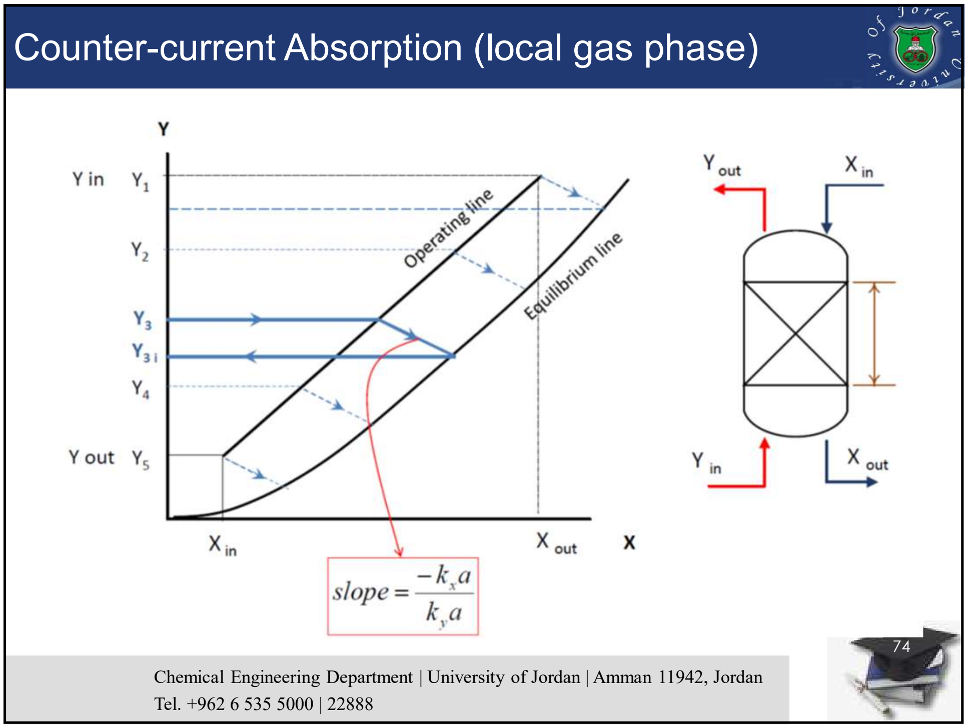
Interface composition in terms of the ratio of mass transfer coefficient

- Re-arranging

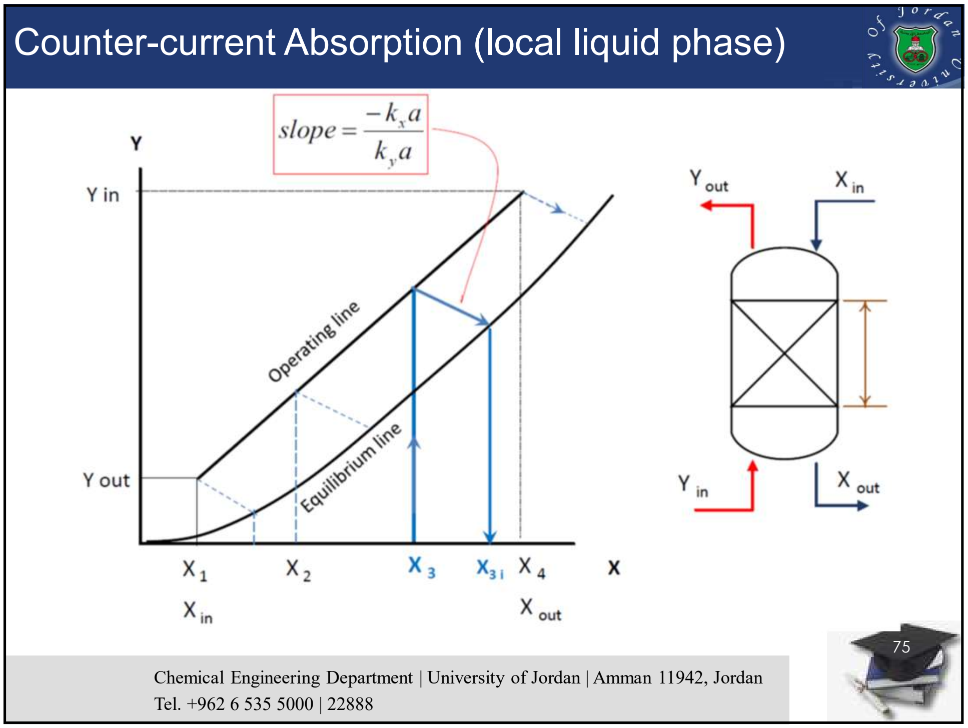
$$-\frac{k_x}{k_y}=\frac{y_{Ab}-y_{Ai}}{x_{Ab}-x_{Ai}}$$

- The mass transfer may now be written based on the overall mass-transfer coefficient

$$N_A=K_x(x_A^*-x_A)=K_y(y_A-y_A^*)$$

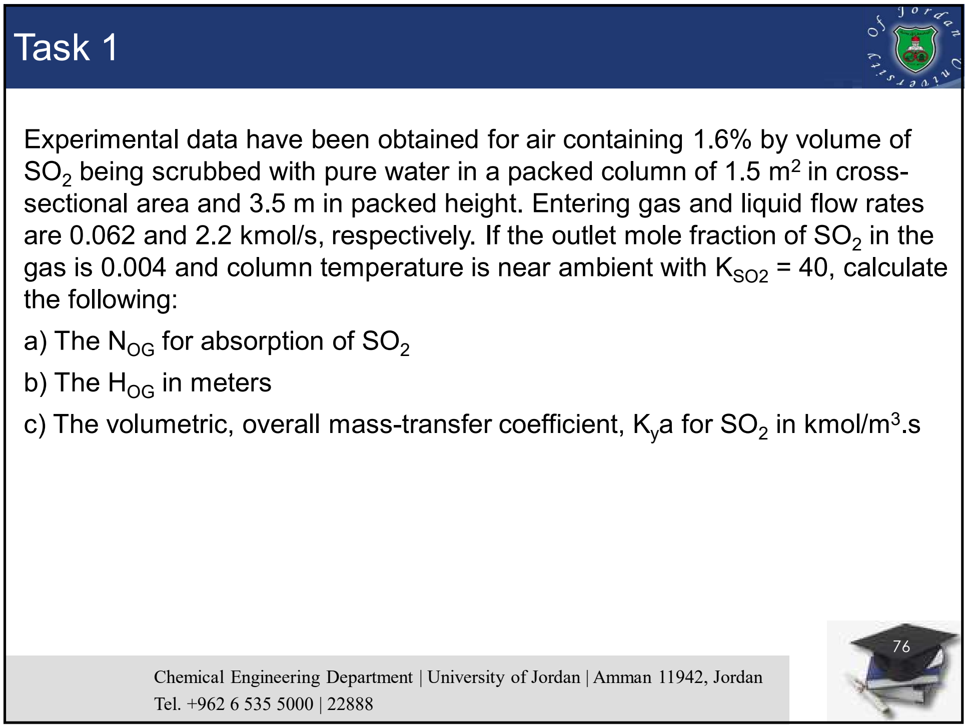


Counter-current Absorption (local gas phase)



Counter-current Absorption (local liquid phase)

$$slope = \frac{-k_x a}{k_y a}$$



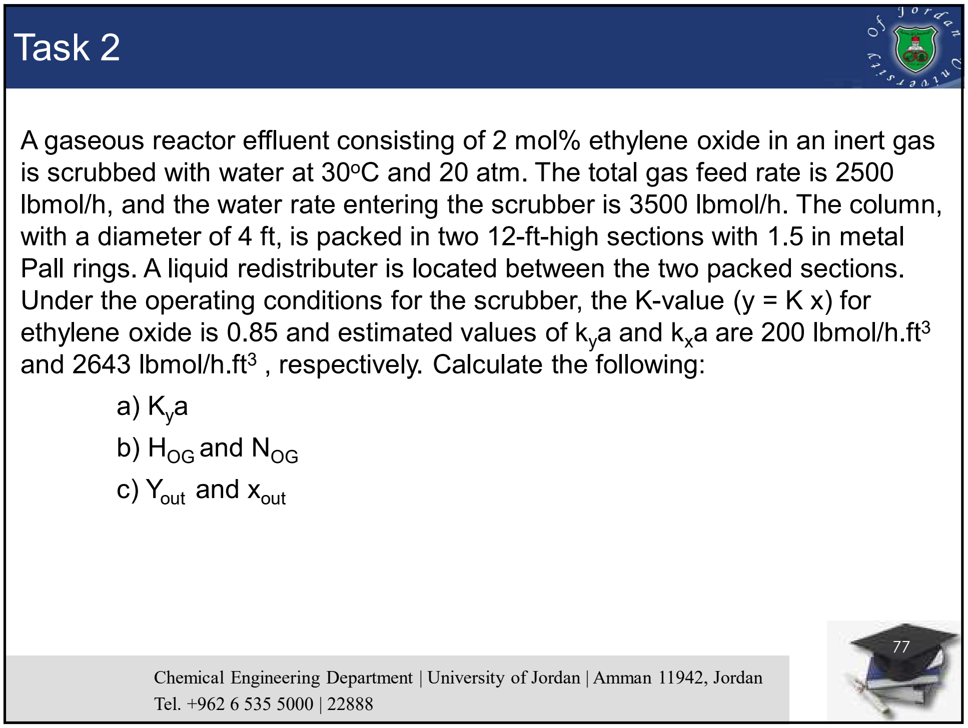
Task 1

Experimental data have been obtained for air containing 1.6% by volume of SO_2 being scrubbed with pure water in a packed column of 1.5 m^2 in cross-sectional area and 3.5 m in packed height. Entering gas and liquid flow rates are 0.062 and 2.2 kmol/s, respectively. If the outlet mole fraction of SO_2 in the gas is 0.004 and column temperature is near ambient with $K_{SO2} = 40$, calculate the following:

a) The N_{OG} for absorption of SO_2

b) The H_{OG} in meters

c) The volumetric, overall mass-transfer coefficient, $K_y a$ for SO_2 in kmol/m^3.s



Task 2

A gaseous reactor effluent consisting of 2 mol% ethylene oxide in an inert gas is scrubbed with water at 30^{o}C and 20 atm. The total gas feed rate is 2500 lbmol/h, and the water rate entering the scrubber is 3500 lbmol/h. The column, with a diameter of 4 ft, is packed in two 12-ft-high sections with 1.5 in metal Pall rings. A liquid redistributer is located between the two packed sections. Under the operating conditions for the scrubber, the K-value (y = K x) for ethylene oxide is 0.85 and estimated values of k_ya and k_xa are 200 lbmol/h.ft^3 and 2643 lbmol/h.ft^3 , respectively. Calculate the following:

a) K_ya

b) H_{OG} and N_{OG}

c) Y_{out} and x_{out}