

Engineering Economic and Analysis:

* Invest $\rightarrow$ Bank $\rightarrow$ profit

Value of money $\rightarrow$ (company project) $\rightarrow$ profit

* money (if invested) $\rightarrow$ generate money

* present value interest $\rightarrow$ New value

- present value of money

- life time (# of years)

- future value of money

- Interest

* The yearly interest rate = $\frac{F-P}{n}$

* Future value:

$$F = (1+ni)P$$

* types of interest:

1- simple: $F = P(1+ni)$

2- compound interest: $F = P(1+i)^n$ $\leftarrow$ standard

3- interest is changing w/ time: $F = P \prod_{i=1}^n (1+i)$

example:

How much you need to invest in saving account based on 6% interest rate to have \$5000 in 5 years??

sol:

$$F = 5000 \$$$

$$F = P(1+i)^n = P(1+0.06)^5 \Rightarrow P = 3736$$

Base for compound interest:

- yearly
- monthly
- daily
- continuous

$$r = \frac{i_m}{m}$$

number
of
periods

=>

example:

i = per year
needed per month

$$\Rightarrow r = \frac{i}{12}$$

Effective interest rate: $i_{\text{eff}} = (1 + \frac{i_m}{m})^n - 1$, $F = (1 + \frac{i_m}{m})^m$

example:-

what is the effective annual interest for nominal rate of 8% per annum when compounded monthly?

sol:

$$i_m = 8\% , m = 12$$

$$i_{\text{eff}} = (1 + \frac{8\%}{12})^{12} - 1 = 8.3\%$$

for a given p = 1000

$$F = p(1 + i_{\text{eff}})$$

$$= 1000(1 + \frac{0.08}{12})^{12} = 1083$$

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continuous compound interest:-

$$i_{\text{eff}} = e^{i_m} - 1$$

Example:

How much you would have to invest at 6% for 6 years to obtain \$1791, Assume Basic interest.

sol:

$$p = F(\frac{1}{(1+i)^n}) = 1791(\frac{1}{1+0.06})^6 = 12.63$$

Cash flow diagram:-



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x-axis $\rightarrow$ time

y-axis $\rightarrow$ cash flow

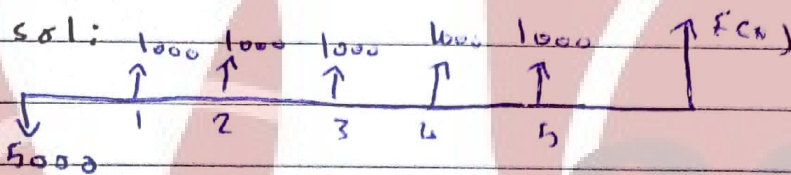
up word $\rightarrow$ earned money

down word $\rightarrow$ loss money

Example:-

you invest today with draw \$1000 at the end of each year
how much you have at the end of 6 years?

$i = 6\%$



$$P_1 = \frac{F_1}{(1+0.06)^1} = \frac{1000}{1+0.06} = 943.4$$

$$P_2 = \frac{F_2}{(1+0.06)^2} = \frac{1000}{(1+0.06)^2} = 890$$

$$P_3 = 839.6$$

$$P_4 = 792.1$$

$$P_5 = 747.3$$

$$P_x = \frac{x}{(1+0.06)^6} \Rightarrow 5000 = 943.4 + 890 + 839.6 + 792.1 + 747.3 + \frac{x}{(1+0.06)^6}$$

$$x = 1117.2 \$$$

Another sol: bring all money till the end

$$F_p = 5000(1+0.06)^6 = 7092.6$$

$$F_1 = 1000(1+0.06)^5 = 1338.2$$

$$F_2 = 1000(1+0.06)^4 = 1265.5$$

$$F_3 = 1000(1+0.06)^3 = 1191$$

$$F_4 = 1123.6$$

$$F_5 = 1060$$

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$$F_p = \sum_{i=1}^5 f_i = X = 1117.2$$

A uniform series of cash flow transaction :-

$$F_n = A [(1+i)^1 + (1+i)^2 + \dots]$$

$$F_n = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F_n = P(1+i)^n$$

$$F_n = A \left[\frac{(1+i)^n - 1}{i} \right]$$

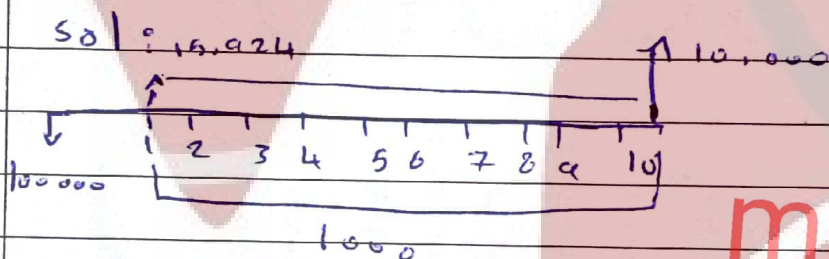
$$A = \frac{Fi}{(1+i)^n - 1}$$

$$A = \frac{P(1+i)^n \times i}{(1+i)^n - 1}$$

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

Example:-

A piece of equipment costs \$100,000. If an engineer is borrowed this amount from a bank with 9% interest and should this amount be paid within 10 years ~~at a profit~~ with a fixed payment yearly. The equipment needs \$1000/year for operation and yield a profit of \$15,924 yearly, she was expecting to sell this equip. ~~at~~ at the end with \$10,000. Do you think this engineer learn something from economy??



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$$A = 15,924 - 1,000 = 14,924$$

$$P = 100,000$$

$$F = 10,000$$

$$A = \frac{P(1+i)^n \cdot i}{(1+i)^n - 1} = \frac{100,000(1+0.09)^{10} \cdot 0.09}{(1+0.09)^{10} - 1} = 15,582$$

$$A = \frac{Fi}{(1+i)^n - 1} = \frac{10,000(0.09)}{(1+0.09)^{10} - 1} = 658.2$$

$$\sum Ai = 14,924 + 658.2 - 15,582 = \$0.2$$

Save based on $i = 0.08$

$$\sum Ai = 14,924 + 690.3 - 14,902 = \$712$$

Plant :-

2-11-2017

Depreciation :-

1- straight line :- $d_k^{sl} = \frac{FCI - SV}{n}$

2- Declining balance method :- $V_{at\ present} = V_{original} (1-f)^{n}$
no. of years
at year 2 as 22
at year 3 as 22.2

$$f = 1 - \left(\frac{S}{V} \right)^{\frac{1}{n}}$$

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Example :-

Original price of equipment \$22,000, used for 10 years, sell for \$2000. what is the price of this equipment after 5 years?

sol:

- straight line $d = \frac{FCI - SV}{n} = \frac{22,000 - 2,000}{10} = 2,000$

$$V_5 = FCI - nd = 22,000 - 5 \times 2,000 = 12,000$$

- Declining balance :

$$f = 1 - \left(\frac{S}{V} \right)^{\frac{1}{n}} = 1 - \left(\frac{2,000}{22,000} \right)^{\frac{1}{10}} = 0.2131$$

$$V_5 = V(1-f)^5 = 22,000(1-0.2131)^5 = \$6,650$$

3- Sum of year Digits method :-

$$d_k^{syd} = \frac{\left[\frac{n+1}{2} - k \right] (FCI - S)}{n/2 (n+1)}$$

total no. of years the year I want

why syd: $n/2 (n+1)$

year 1: 1	1
year 2: 2	3
year 3: 3	6
year 4	

4 - Double declining balance method :-

$$d_k^{DDB} = \frac{2}{n} \left[FCI - \sum_{i=0}^{k-1} d_i \right]$$

example:-

$$FCI = 150\,000\,000$$

$$S = 10\,000\,000$$

$$n = 7 \text{ years}$$

depreciation comparison ??

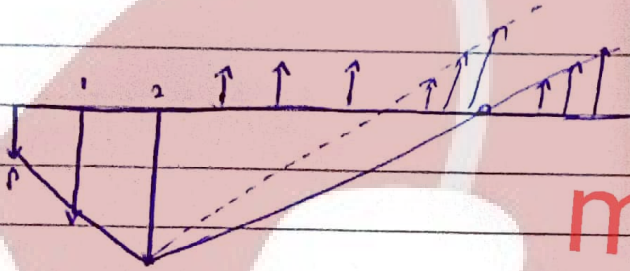
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year	d_k^{SL}	d_k^{SYN}	d_k^{DDB}	$BV = FCI - \sum d_i$ $= FCI - \sum d_i$ $\times 10^{-6}$
0	—	—	—	150
1	$\frac{150-10}{7} = 20$	$\frac{(7+1-1)(150-10)}{28} = 35$	$\frac{2}{7}(150-0) = 42.9$	$150 - 42.9 = 107.1$
2	20	= 30	$\frac{2}{4}(150 - 42.9) = 30.6$	$107.1 - 30.6 = 76.5$
3	20	= 25	$\frac{2}{3}(76.5) = 51.0$	$76.5 - 51.0 = 25.5$
4	20	= 20	15.6	39
5	20	= 15	11.1	27.9
6	20	= 10	8	19.9
7	20	= 5	9.9	10
			$FCI - \sum d_i - S$	
	140	140	140	10

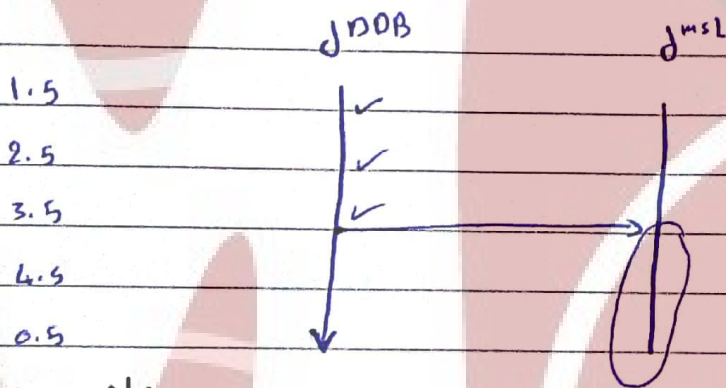
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Current depreciation method:-

Modified accelerated cost recovery system [MACRS]

 $\Rightarrow$ starting with DDB ending with MSL. $\Rightarrow$ work for 5 years.

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example:-

consider an equipment that has a FCI = 100000\$

calculate the modified accelerated cost recovery dep. system.

$$dk^{DDB} = 2/5 [FCI - \sum_{i=0}^{k-1} di] \quad , \quad dk^{SL} = \frac{FCI - \sum_{k=1}^{(n-k)} dk}{(n-k)}$$

gear

1/2

$dk^{DDB} = 2/5 [100000 - 0] \times 1/2$

$= 20K$

1.5

$dk^{DDB} = 2/5 [100 - 20]$

$= 32$

$(100 - 20) / 4.5 = 7.78$

2.5

$dk^{DDB} = 2/5 [100 - 52] = 19.2$

$(100 - 52) / 3.5 = 13.71$

3.5

$dk^{DDB} = 2/5 [100 - 71.2] = 11.52$

$(100 - 71.2) / 2.5 = 11.52$

4.5

$dk^{DDB} = 2/5 [100 - 82.7] = 6.91$

$(100 - 82.7) / 1.5 = 11.52$

0.5

dep for last 0.5 year.

$= 0.5 \left(\frac{100 - 82.7}{1.5} \right) = 5.76$

Depreciation

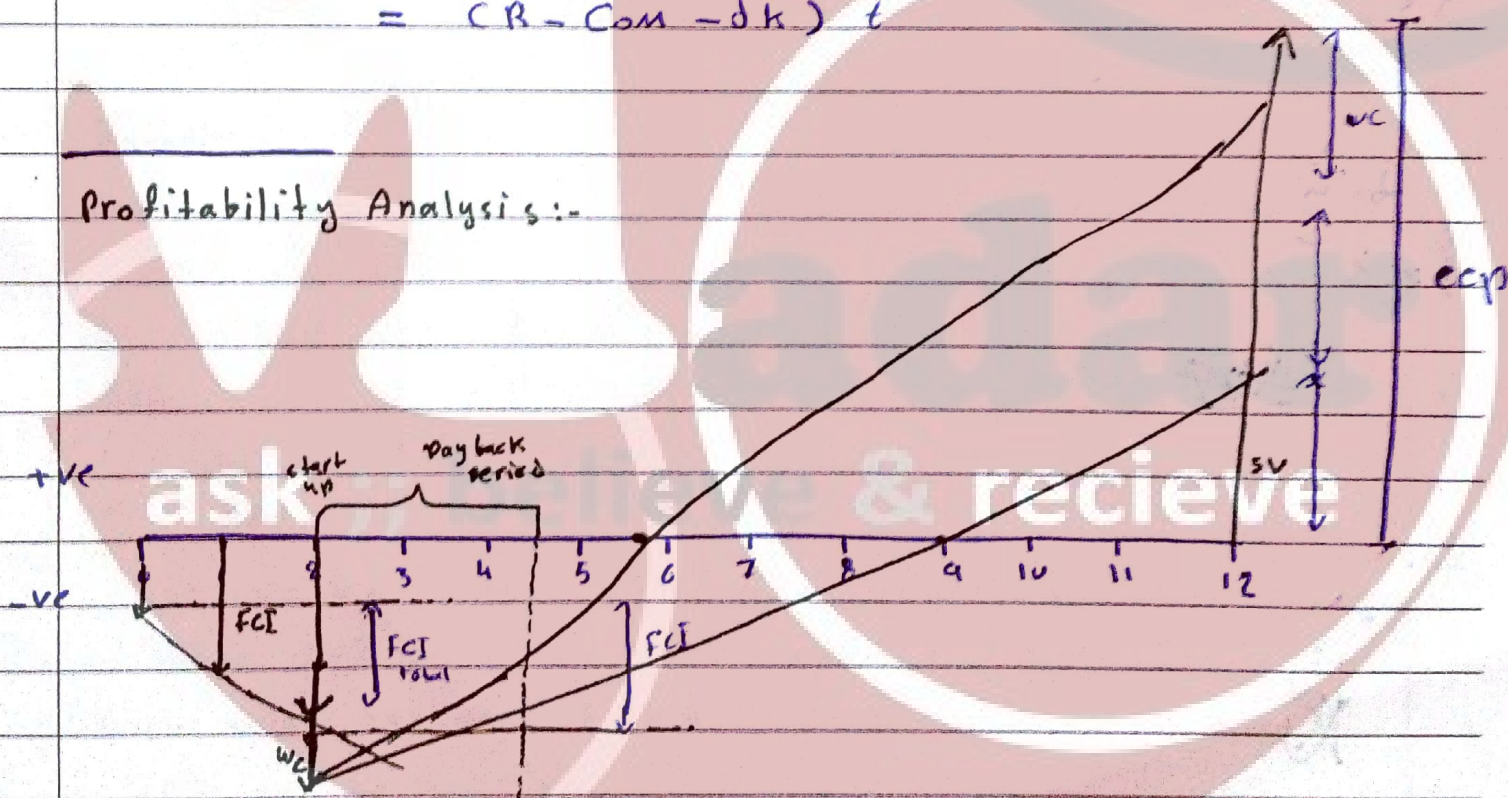
year	dk
0.5	20
1.5	32
2.5	19.5
3.5	11.52
4.5	11.52
5.5	5.76

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Taxation, cashflow, profit:-

$$\text{income tax} = (\text{Revenue} - \text{expenses}) \cdot \text{tax rate}$$

$$= (R - \text{Com} - dk) \cdot t$$

Profitability Analysis:-

- * Criteria:
- 1) time $\rightarrow$ payback period
 - 2) net positive value
 - 3) rate of return

1- time criteria:-

pay back period: time from start up, to recover the FCI

2- cash criteria:-

Cumulative cash position (CCP)

Cumulative Cash Recovery (CCR) , $\frac{\sum \text{of all positive values}}{\sum \text{of all negative cash}}$

$$CCR = 1 + \frac{CCP}{\text{land} + WC + FCI}$$

3- Interest Rate criteria:-

Rate of return on investment (RoRoI)

$$RoRoI = \frac{\text{Avg annual net profit}}{FCI} \Rightarrow \frac{\text{slope}}{FCI} \quad \text{① service life } [10 \text{ years}] \text{ from start up}$$

Example:-

A new chemical plant is going to be built with the required capital investment:

1)- Cost of land = \$10M

2)- FCI = \$150M

FCI₁ = \$90M

FCI₂ = \$60M

* plant starts up after 2 years from installation.

WC = 20% of FCI = $0.2 \times 150 = \$30M$

yearly sales revenue = \$75M

CoM = \$30M

Taxation = 45%

salvage value = \$10M

Depreciation based on 5 years MACRS

project life = 10 years.

* calculate , PBP, RoR, Net positive cash flow.

<u>End of year</u>	<u>investment</u>	<u>dk</u>	<u>FcI - E_{di}</u>	<u>B</u>	<u>com</u>	<u>net profit</u>
0	-10	-	0	-	-	-
1	-90	-	90	-	-	-
2	-30-60	-	150	-	-	-
3		36	120	75	30	38.25
4		48	72	75	30	46.35
5		28.8	43.2	75	30	37.71
6		17.28	25.92	75	30	32.53
7		17.28	8.64	75	30	32.53
8		8.64	0	75	30	28.64
9				75	30	24.74
10				75	30	
11				75	30	
12				75	30	24.75

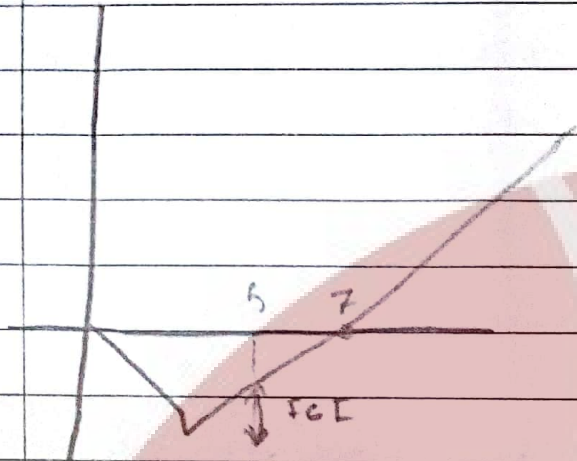
Cash flow

-10
-90
-90
38.25
46.35

net cash flow

-10
-100
-190
-151.75
-105.4
-67.69
-35.16
-2.64
+26
+50.75
+75.50
+100.25
+170.3

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$$\tan \theta = \frac{7-5}{67-69} = \frac{y}{67.69-40}$$

$$x = 0.818$$

$$\begin{aligned} \text{PBP} &= 0.188 + 3 \\ &= 3.1818 \text{ years} \end{aligned}$$

$$\text{CCP} = 170.5$$

$$\text{CCR} = 1 + \frac{\text{CCP}}{L + W_c + \text{FCL}} = 1 + \frac{170.5}{10 + 30 + 150} = 1.897$$

$$\text{RoBoI} = \frac{\text{average profit}}{\text{FCL}}$$

$$\text{net profit} = (\text{CR} - \text{COM} - d) (1 - \tan x)$$

time

0

1

2

8.25

3

-1.65

4

8.91

5

15.25

6

15.25

20

24.75

30.25

$$\begin{aligned} \text{average net} &= 17.05 \\ \text{Profit} \end{aligned}$$

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* Discounted Profitability criteria.

[DPPB] - Discounted payback period = time to Recover FCI

[DCCP] - cash criteria = Discounted cumulative cash position

= Net present value = cumulative cash at the end of project

present value ratio = $\frac{\text{Net +ve values}}{\text{Net -ve values}}$

* D R of return

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Example:- Repeat the same example for nondiscounted C.F but For discounted, rate = 10%?

year	non Discounted C.F	Discounted	cummulative
0	-10	-10	-10
1	-40	$\frac{-40}{(1+0.1)^1} = 81.82$	-91.82
2	-60-30	$\frac{-90}{(1+0.1)^2} = 74.39$	-166.2
3	38.25	-28.74	-137.46
4	46.38	-31.41	-105.8
5	37.71	-23.1	-82.39
6	32.52	-18.59	-64.03
7	32.52	-16.64	-47.34
8	28.64	-13.36	-33.98
9	24.75	-10.5	-23.48
10	24.75	-9.54	-13.94
11	24.75	-8.67	-5.26
12	70.25	-22.38	17.12



$$DPBP = 5.9 \text{ year}$$

$$NPV = 17.12$$

$$PVR = \frac{28.74 + 31.66 + \dots + 22.38}{10 + 81.82 + 74.38} = 1.1$$

(x)	(y)	} By interpolation NPV @ $i = 12.1\%$
i	NPV	
0 %	170.5	
10 %	17.12	
12 %	0.72	
13 %	-6.32	

$c =$ ١) که بازادت ال (discounted rate of return) من صالحي او لایق و ملاذای؟

- لو تغییر سعر الفلوس مع الزمن رج أخذ أربح
که بازادت ال زنا ربحان
من صالحي

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Comparing several large projects -

as $\uparrow$ DCROR $\rightarrow$ better choice for investment

DCROR is calculated at $NPV = 0$

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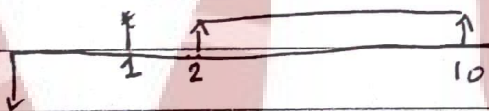
Example:-

Invest using three different project

Project A :	initial invest.	after tax cash flow	
	\$ 60 M	n=1	n=2 $\rightarrow$ 10
		\$ 10 M	\$ 12 M
Project B :	\$ 120 M	22	22
Project C :	\$ 100 M	12	20

The min acceptable ROR = 10%

soln:-



$$A- 0 = -60 \left(\frac{F}{P}, i, 10 \right) + 10 \left(\frac{F}{P}, i, 10 \right) + 12 \left(\frac{F}{A}, i, 9 \right) \Rightarrow i = 0.1435$$

$$B- 0 = -120 \left(\frac{F}{P}, i, 10 \right) + 22 \left(\frac{F}{P}, i, 10 \right) + 22 \left(\frac{F}{A}, i, 9 \right) \Rightarrow i = 0.1287$$

$$C- i = 0.1334$$

max $i = 14.35\% \Rightarrow$ so this is the best choice (A) ✓

Evaluation of equipment alternatives

used for equip. w/ same life time.

1) pump, CS

2) pump, SS

example:-

if we need to cool down a product using either Air cooler or exchanger based on water. if the following information are provided for each equip. what is your recommendation, based on life time of 12 years, $i = 8\%$.

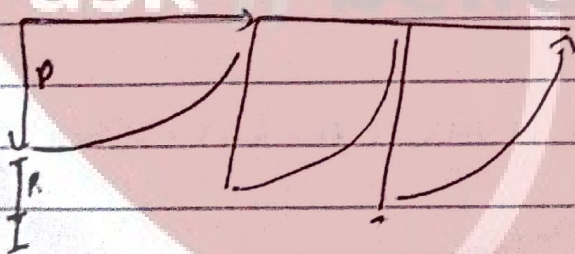
	<u>Initial invest</u>	<u>yearly op. cost.</u>
Air cooler	\$ 23,000	\$ 1700
exchanger	\$ 12,000	\$ 3,300

$$NPV = \text{initial invest.} + \text{op. cost} \left(\frac{P}{i}, i, n \right)$$

$$NPV = \$32,040 \Rightarrow \text{air cooler} \checkmark \Rightarrow \text{this is the best.}$$

$$NPV = \$36,870 \Rightarrow \text{exchanger}$$

Equipments with Different life time



$$R(1+i)^n - R = P$$

$$\text{capitalized cost} = P + R = P + \frac{P}{(1+i)^n - 1}$$

$$P = R(1+i)^n + P$$

equiv. capital cost :-

$$= \frac{[P(1+i)^n + YOC(F/P, i, n)]}{(1+i)^n - 1}$$

example:-

<u>pump</u>	<u>capital cost</u>	<u>op. cost</u>	<u>life time</u>
C S	8000	1800	4
S S	16,000	1600	7

Assume $i = 8\%$

sol :-

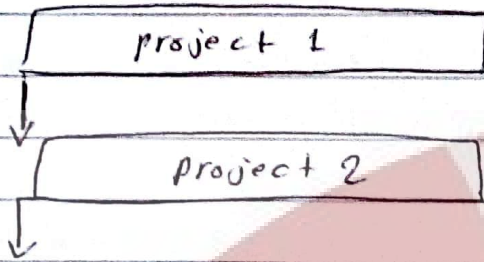
$$\text{capitalized cost (CS)} = (8000)(1.08)^4 + (1800) \frac{(1.08)^4 - 1}{0.08} = 52,700$$

$$\text{capitalized cost (SS)} = \frac{16000(1.08)^4 + (1600) \frac{(1.08)^4 - 1}{0.08}}{(1.08)^7 - 1} = 58,400$$

most feasible is carbon steel,

$$\text{capitalized cost} = 52,700$$

Equivalent Annual operating cost (EAC) :-



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$$EAC = (\text{Capital investment}) (A/P, i, n) + YOC$$

example:-

	capital cost	op. cost	n
CS (pump)	8000	1800	4
SS (pump)	16000	1600	7

i = 8%

$$EAC_{CS} = 8000 \left[\frac{0.08 (1.08)^4}{1.08^4 - 1} \right] + 1800 = 4220 \quad \checkmark$$

$$EAC_{SS} = 16000 \left[\frac{0.08 (1.08)^7}{1.08^7 - 1} \right] + 1600 = 4670$$

⇒

is not accurate if we have diff. life times.

Common denominator method:-

This method solve (EAC) by multiplying the life time of each eq. with the other.

$$\text{Total year} = nm = 4 \times 7 = 28 \text{ year}$$

$$NPV_{CS} = 8000 + 1800 (P/A, 0.08, 28) + 8000 (P/F, 0.08, 4) + 8000 (P/F, 0.08, 8) + \dots$$

$$NPV_{SS} = 16000 + 1600 (P/A, 0.08, 28) + 16000 (P/F, 0.08, 7) + 16000 (P/F, 0.08, 14) + \dots$$

Incremental Analysis for Retrofiling :-

- 1) Rate of return of incremental investment (RORII)
(should be max)

$$RORII = \frac{\text{investment yearly saving}}{\text{incremental invest.}}$$

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- 2) incremental Payback period (should be min)

$$IPBP = \frac{\text{incremental invest.}}{\text{invest. yearly saving}} = \frac{1}{RORII}$$

example:

you have a pipeline need to be insulated to save energy according to the following ~~options~~ options:-

option #:	project cost	saving saving	RORII	IPBP
1	0	0	—	—
2	3000	1400	$\frac{1400}{3000} = 0.47$	2.1
3	5000	1900	0.38	2.6
4	6000	2000	0.33	3
5	9700	2400	0.25	4

choose option (2)

change from (2) → (3)

$$\text{incremental invest} = 5000 - 3000 = 2000$$

$$\text{incremental saving} = 1900 - 1400 = 500$$

$$RORII = \frac{500}{2000} = 0.25 \quad \checkmark$$

Change from (3) $\rightarrow$ (4)

$$\text{Incr. invest} = 6000 - 5000 = 1000$$

$$\text{|| saving} = 2000 - 1900 = 100$$

$$\text{RoRII} = \frac{100}{1000} = 0.1 \quad \times$$

Change from (4) $\rightarrow$ (5)

$$\text{II} = 9700 - 6000 = 3700$$

$$\text{IS} = 2400 - 2000 = 400$$

$$\text{RoRII} = \frac{400}{3700} = 0.108 \quad \times$$

if the acceptable RoRII = 15% $\rightarrow$ then (3) $\rightarrow$ (4) and (4) $\rightarrow$ (5) is rejected.

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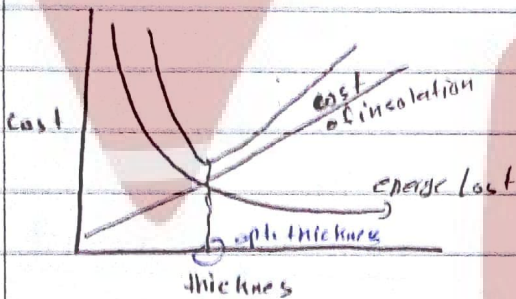
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Optimization Design and Design strategy :-

Factors need to be optimized :-

- 1- cost per unit production
- 2- profit
- 3- Amount of final product/unit time
- 4- conversion, yield, etc.

(Optimization per one unit variable):



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$$\text{capital cost (insulation)} = a + bx = f(x) \quad \text{the thickness of insulation}$$

$$\text{cost / energy loss} = c + \frac{d}{x}$$

$$\text{total cost} = a + bx + c + \frac{d}{x}$$

$$\frac{\partial C_T}{\partial x} = 0 = b - \frac{d}{x^2}$$

$$x = \left(\frac{d}{b}\right)^{-\frac{1}{2}}$$

example:-

	CC	CH
option	cost of insulation	cost related to heat loss
0	0	8000
1	2326	1000
2	4653	4500
3	6980	5666
4	9306	6250

$$CC = 2326.7 x$$

$$CH = 8000 - \frac{7000}{x}$$

$$CT = CC + CH = 2326.7 x + 8000 - \frac{7000}{x}$$

$$\frac{\partial CT}{\partial x} = 2326.7 + \frac{7000}{x^2} = 0$$

$$x^2 =$$

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(2) two variables :-

$$C_c = f(x) = a + bx + cx^2$$

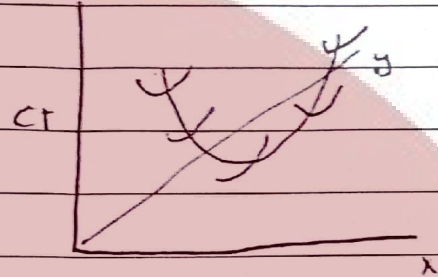
$$C_y = f(y) = \frac{d}{y^2} + fy$$

$$C_T = C_c + C_y$$

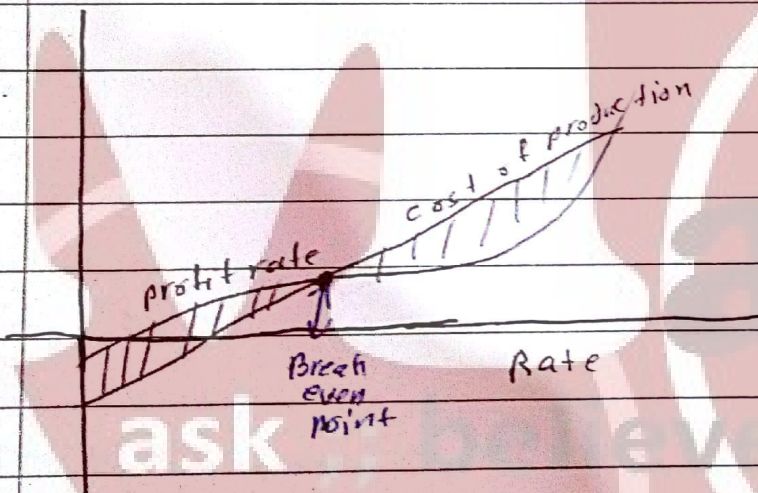
$$= a + bx + cx^2 + \frac{d}{y^2} + fy$$

$$\frac{\partial C_T}{\partial x} = 0 = b + 2cx \rightarrow x$$

$$\frac{\partial C_T}{\partial y} = -\frac{d}{y^3} + f = 0 \rightarrow y$$



Break even point :-



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