SOLVING SYSTEM OF LINEAR EQUATIONS & POLYNOMIALS

MATLAB Basics – Dr. Linda Al-Hmoud

Solving Systems of Linear Equations

- **To solve:**

$$5x = 3y - 2z + 10$$
$$8y + 4z = 3x + 20$$
$$2x + 4y - 9z = 9$$

- **First rearrange:**

$$5x - 3y + 2z = 10$$
$$-3x + 8y + 4z = 20$$
$$2x + 4y - 9z = 9$$

unknown quantities = known quantities

Solving Systems of Linear Equations

- This is now of the form AX = B,
  - A = matrix of the coefficients of the unknowns
  - X = vector of unknowns
  - B = vector containing the constants.

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

Solving Systems of Linear Equations

```
>> A = [5 -3 2; -3 8 4; 2 4 -9];
>> B = [10; 20; 9];
>> X=A\B
X =
    3.4442
    3.1982
    1.1868
```

- Backslash operator (\) is used to solve equations of the form AX = B, i.e. X = A\B.
- Now check your answer!

```
>> C = A*X
```

Exercise – work with a partner

- Solve the following systems of linear equations:

$$x_1 - 2x_2 - x_3 + 3x_4 = 10$$

$$2x_3 + 3x_2 + x_1 - 8 = 0$$

$$x_4 - 4x_3 - 2x_1 = 3$$

$$7 - x2 + 3x3 + x4 = 0$$

Polynomials

- Polynomial is an equation on the form:

  $$\mathbf{y\ (x) = a_0\, x^n + a_1\, x^{n-1} + \ldots. + a_{n-2}\, x^2 + a_{n-1}\, x + a_n}$$

  - $a_0, a_1, a_2 \ldots\ldots.a_n$ = **constants** called **x coefficient**
  - n = **positive integer** called the **order of the polynomial**
  - number of the polynomial terms = n+1

Polynomials

- Polynomials are described in MATLAB by row vectors with elements that are equal to the polynomial coefficients in **order of decreasing powers**.

like: $y(x) = \mathbf{x^3+3x^2+3x+1}$

```
>> y = [ 1 , 3 , 3 , 1 ];
```

Polynomials

- If there is a missing term in the polynomial, x coefficient is zero for this term, like:

g (x)= 5 x^5 - 4 x^3+ 11.5 x - 16

```
>> g = [5, 0, -4, 0, 11.5, -16]
```

polyval

- To substitute the polynomial with a value of x, command ***polyval*** is used as:
- let x=2, y(2) will be calculated as:

```
>>  polyval ( y , 2 )
ans =
        27
```

Exercise

- Create a row vector k that represents the following polynomial:

$$k(x) = (x + 1)(x^2 + 4x + 7)$$

```
>> k = [1, 5, 11, 7]
```

- Then evaluate the polynomial at x=-2:

```
>> polyval (k, -2)
ans = -3
```

Polynomials functions

roots

- The **roots** command is a convenient way to find the roots of a polynomial (roots are values of x that makes y=0):

```
>>  roots(k)
ans =
     -2.0000 + 1.7321i
     -2.0000 - 1.7321i
     -1.0000
```

This means that for: $k(x) = (x + 1) (x^2 + 4 x + 7)$,
k = 0.0 at x= -1, x= -2+1.7321i and x= -2-1.7321i

Exercise

- If $m=x^2-9$, find the roots of the equation.

```
>> m=[1 0 -9]
m =
   1  0  -9
>> roots(m)
ans=
     3
    -3
```

Polynomials functions

polyfit

- Suppose you have 2 vectors from the same size z & y where z is a polynomial in y from the 2nd order

$$z = a\,y2 + b\,y + c$$

- If you want to determine the values of the y coefficients, use the command **polyfit**:

```
>> y= 1:5
y =
     1      2      3      4      5
>> z= [2 4 8 16 32]
z =
     2      4      8     16     32
```

Polynomials functions

polyfit

```
>> w= polyfit (y,z,2)
w =  2.2857   -6.5143    6.8000
```

- Then: $z = 2.2857\ y^2 - 6.5143\ y + 6.8000$

Polynomials functions

polyfit

- If you want to get the value of each coefficient alone:

```
>> a=w(1),b=w(2),c=w(3)
a =
    2.2857
b =
   -6.5143
c =
    6.8000
```

Polynomials functions

poly

- Similarly, you can construct polynomials from its roots, like if x=3, x=4 & x=-5 are the roots of h(x), to get h use the command **poly**:

```
>> h= poly ( [ 3 , 4 , -5 ] )
h=
  1     -2    -23     60
```

Then

$$h(x) = x^3 - 2x^2 - 23x + 60$$

Polynomials functions

polyder

- The MATLAB function polyder returns the derivative of the polynomial whose coefficients are the elements of vector:

```
>> p = [1 5 6];
>> polyder (p)

ans =
     2     5
```

Exercise – work with a partner

- The following numbers are the roots of a polynomial: -1, 0, 1, and 2.
  - What is the order of this polynomial?
  - Find the coefficient of this polynomial.
  - Evaluate the polynomial at x = -5 and at x=10.
  - Derive the polynomial.
  - Find the roots of the derived polynomial.