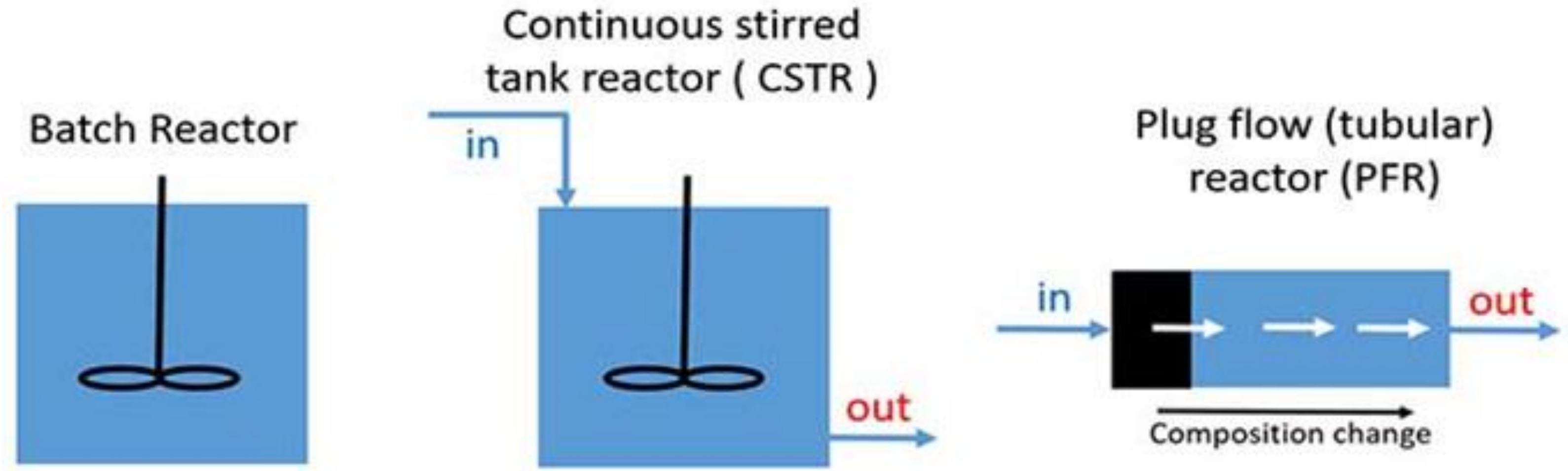
CHAPTER (3)

Reactors

for Wastewater Treatment



Reactors for Wastewater Treatment

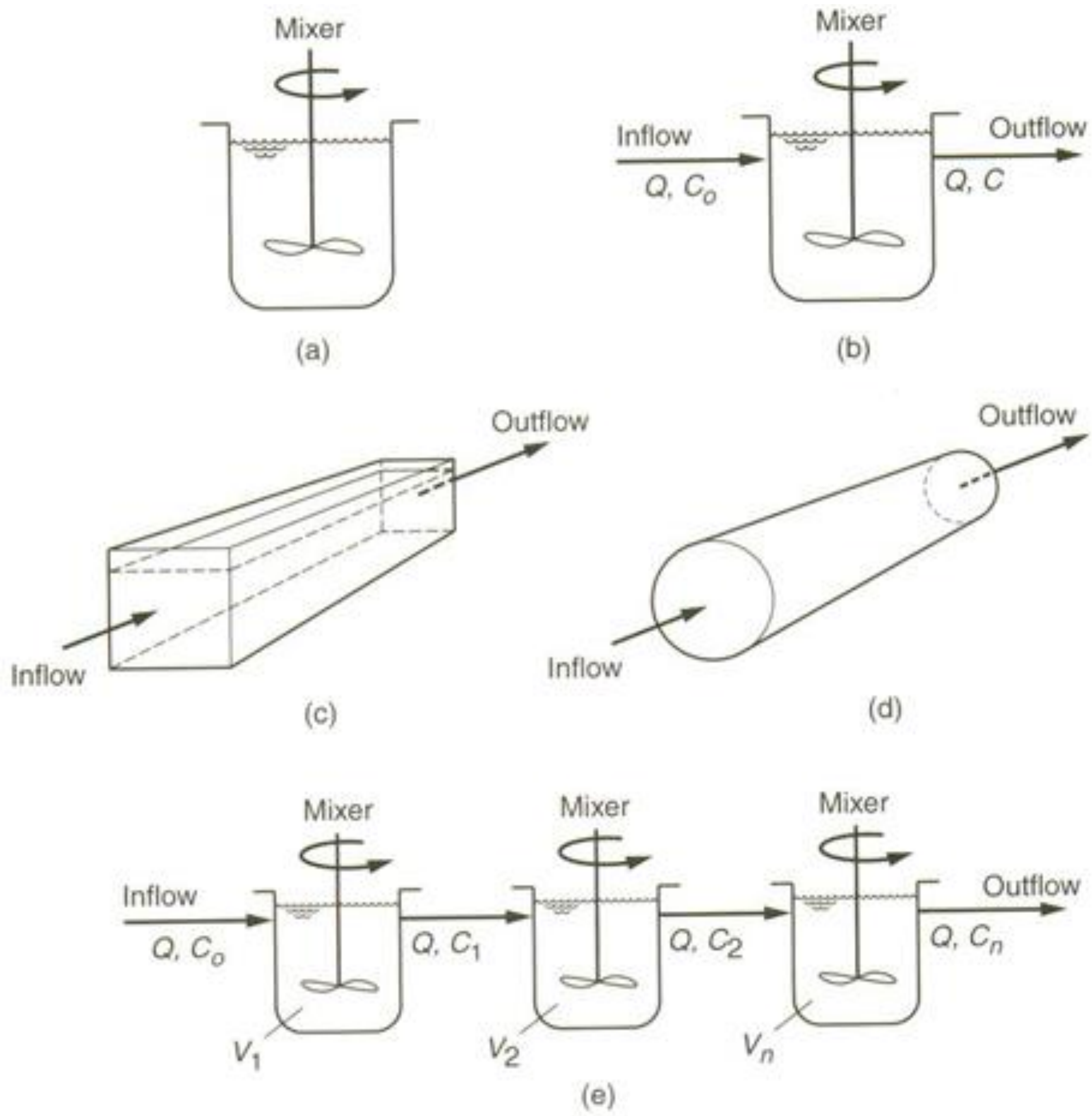
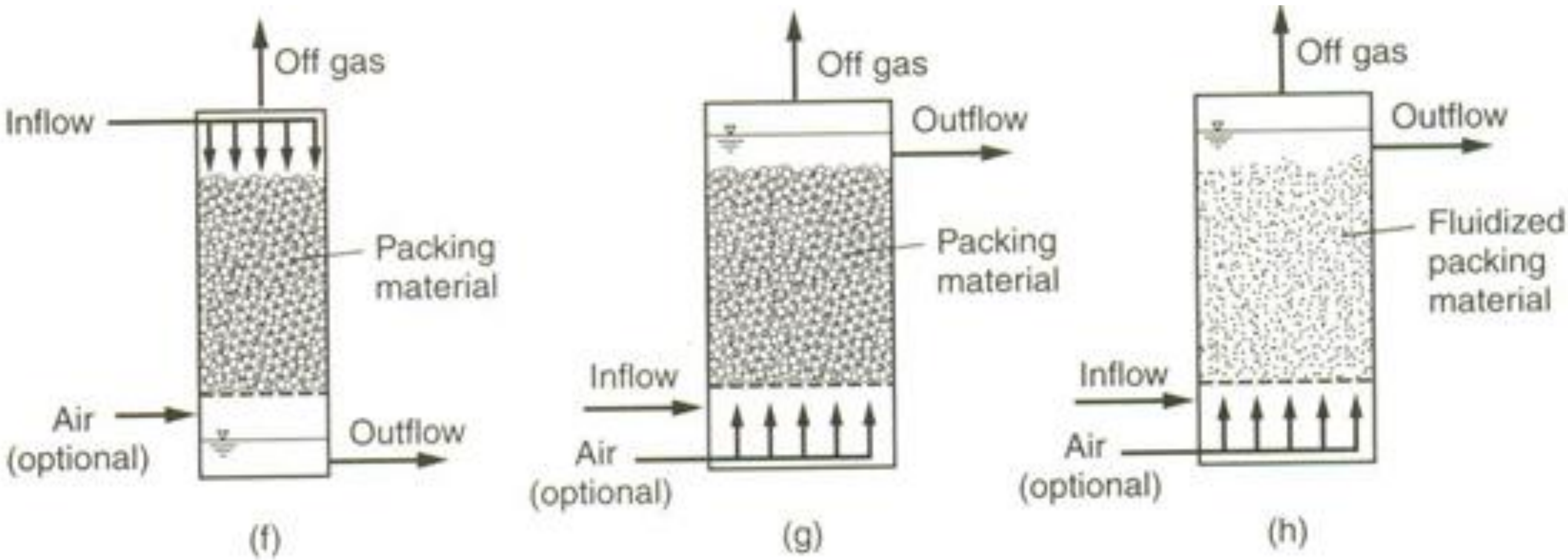


Figure 4–2

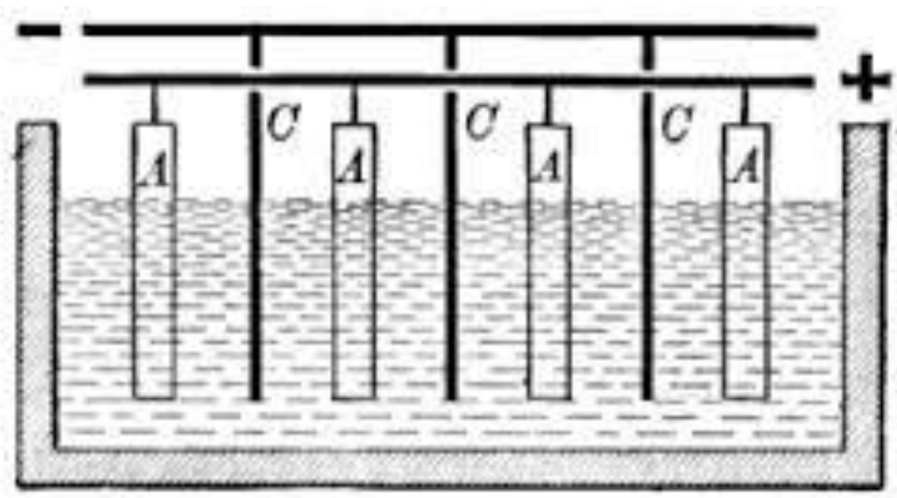
Definition sketch for the different types of reactors used for wastewater treatment: (*a*) batch reactor, (*b*) complete-mix reactor, (*c*) plug-flow open reactor, (*d*) plug-flow closed reactor also known as a tubular reactor, (*e*) complete-mix reactors in series,

Reactors for Wastewater Treatment



(*f*) packed-bed reactor,
(*g*) packed-bed upflow reactor, and
(*h*) expanded-bed upflow reactor.

Electrochemical Reactor



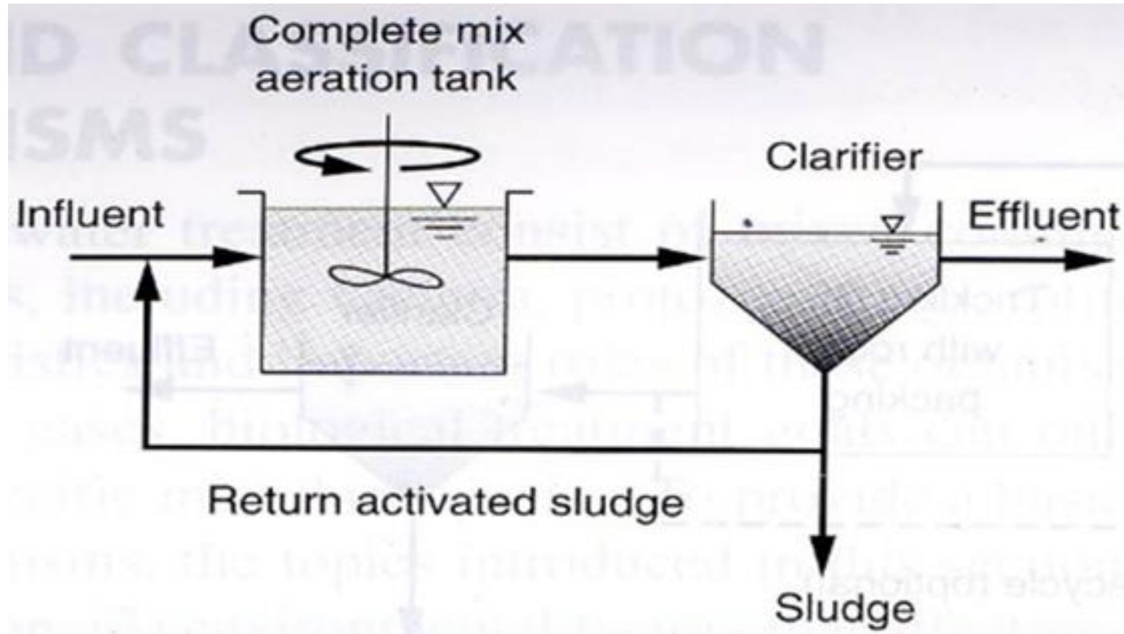
Reactors for Wastewater Treatment

Principal applications of reactor types used for water/wastewater treatment

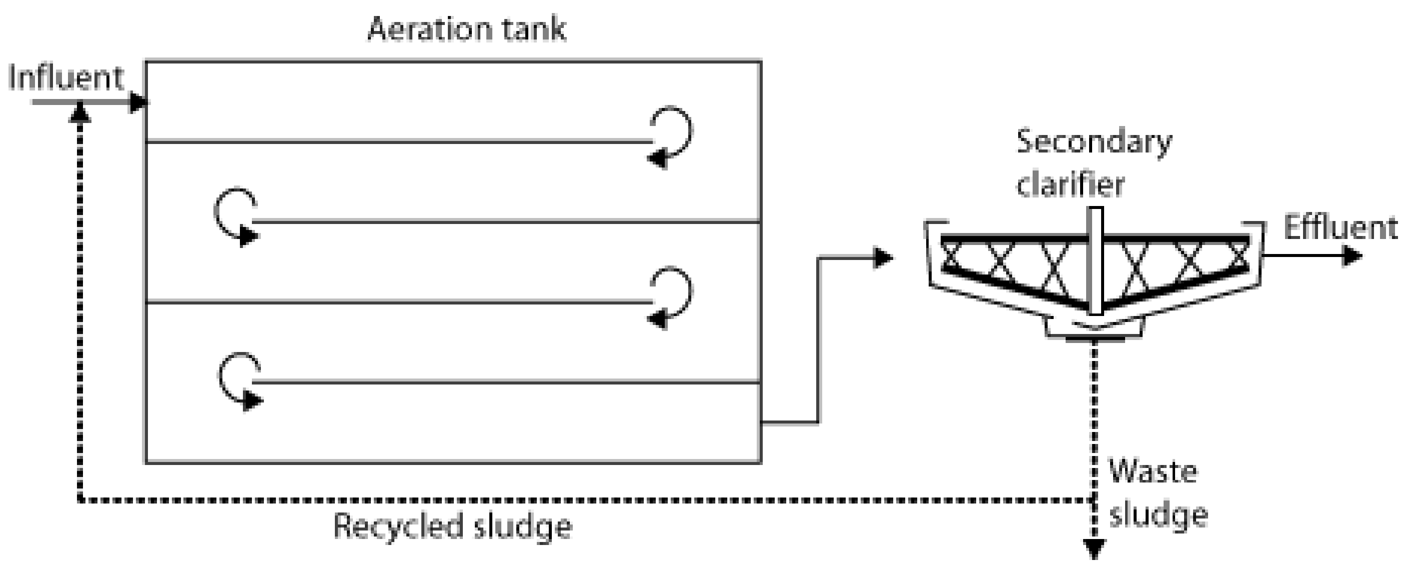
Type of reactor	Application in wastewater treatment
Batch	Activated-sludge biological treatment in a sequence batch reactor, mixing of concentrated solutions into working solutions
Complete-mix	Aerated lagoons, aerobic sludge digestion
Complete-mix with recycle	Activated-sludge biological treatment
Plug-flow	Chlorine contact basin, natural treatment systems
Plug-flow with recycle	Activated-sludge biological treatment, aquatic treatment systems
Complete-mix reactors in series	Lagoon treatment systems, used to simulate nonideal flow in plug-flow reactors
Packed-bed	Nonsubmerged and submerged trickling-filter biological treatment units, depth filtration, natural treatment systems, air stripping
Fluidized-bed	Fluidized-bed reactors for aerobic and anaerobic biological treatment, upflow sludge blanket reactors, air stripping

Activated Sludge Biological Treatment Process

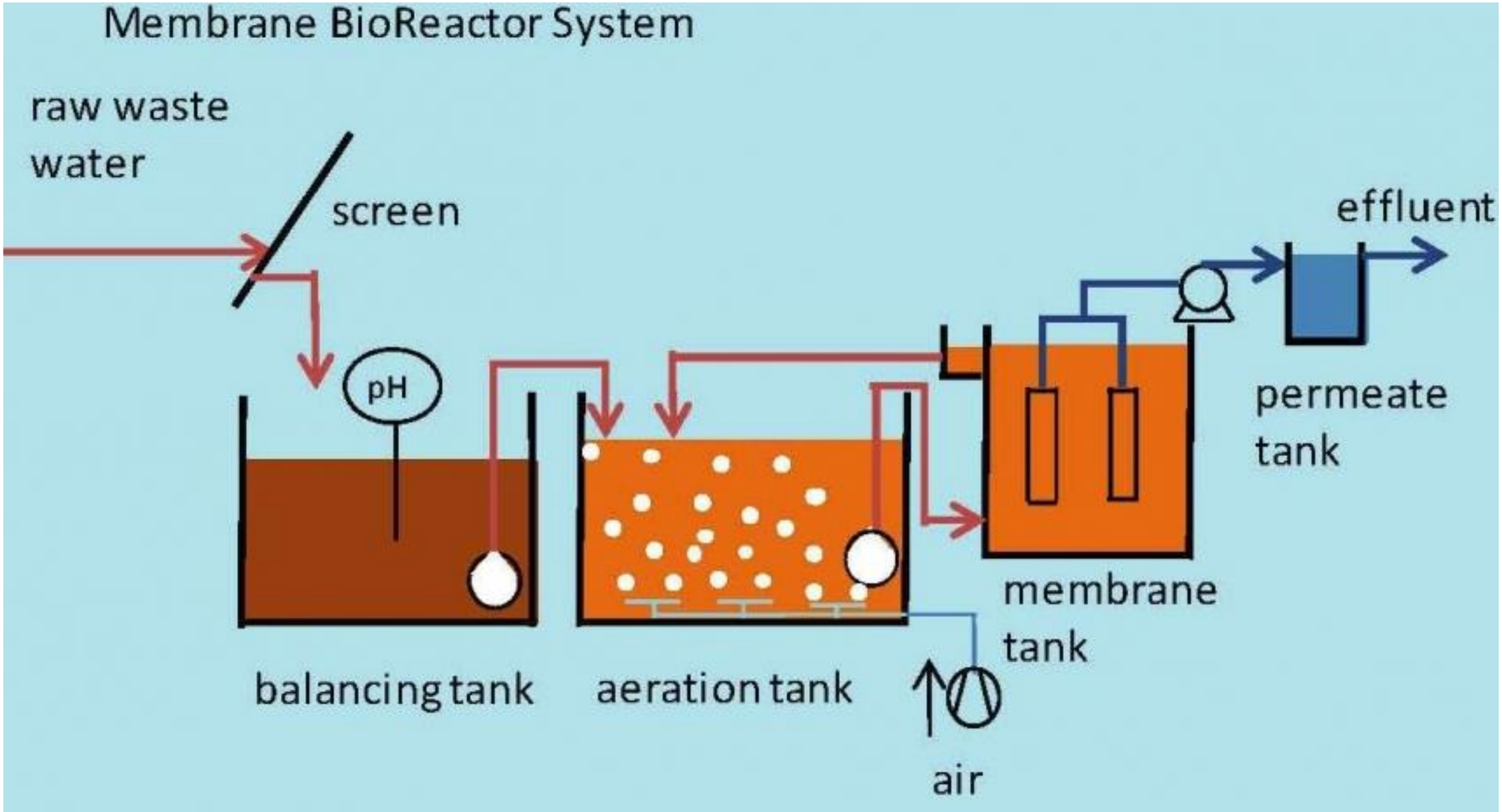
Completely-Mixed Reactor ASP



Plug-Flow Reactor ASP (oxidation ditch)

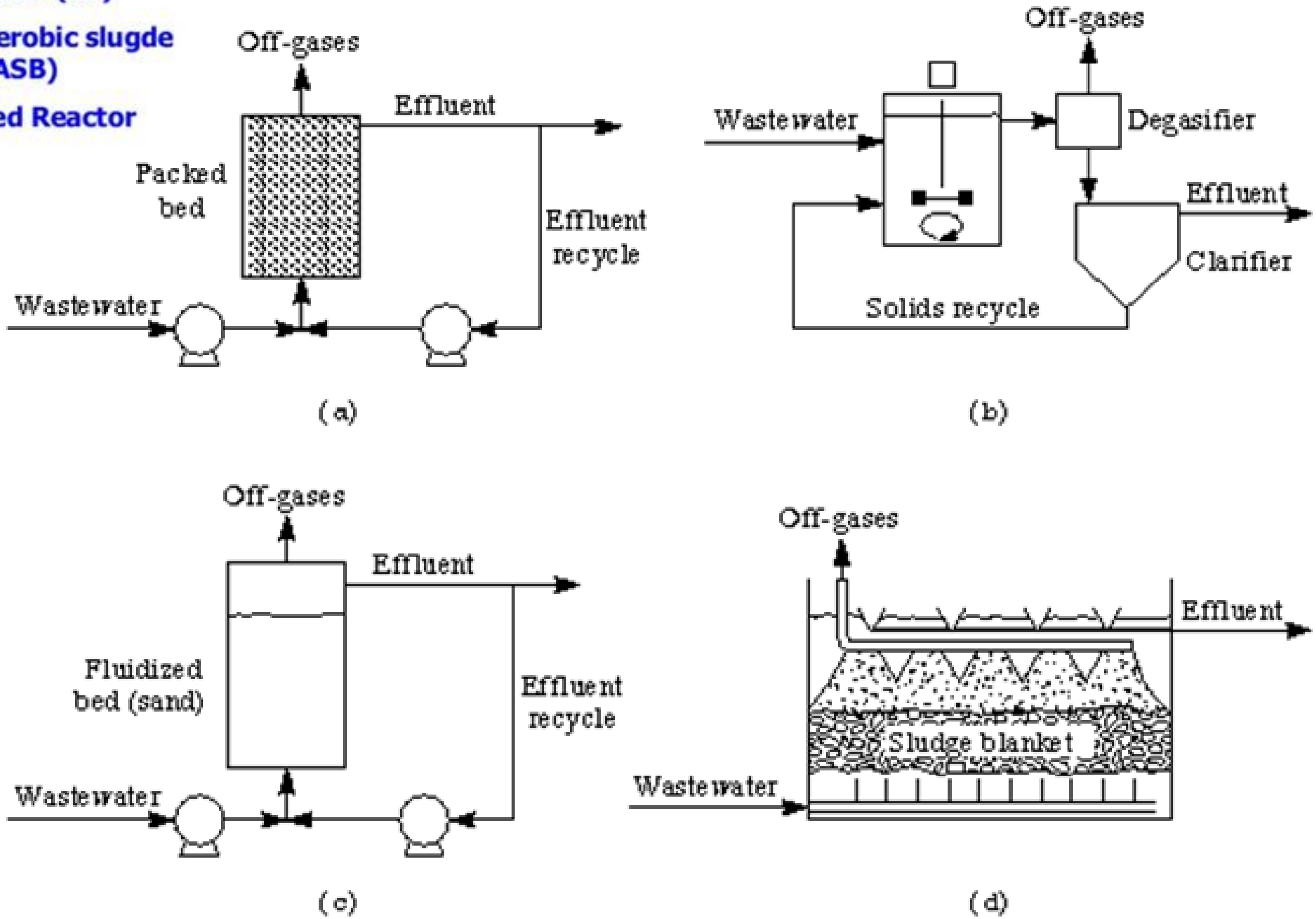


Membrane Bioreactor



Anaerobic Biological Treatment Processes

- Anaerobic contact process
- Anaerobic filter (AF)
- Upflow anaerobic slugde Blanket (UASB)
- Fluidized bed Reactor



Reaction Kinetics

- Consider the reaction in which A is being removed from wastewater (chemical or biochemical):

A → products

- The rate equation of reaction in which A is consumed:

$$r_A = -kC_A^n$$

- Reactions in environmental work are often treated, sometimes as only a rough approximation, as **first-order reactions** (i.e., n = 1) such that,

$$r_A = -kC_A$$

Reactor Kinetics and Design:

There are three commonly used “ideal” chemical reactor models:

1. **Continuous stirred tank reactor (CSTR)**
2. **Batch reactor**
3. **Plug flow reactor**

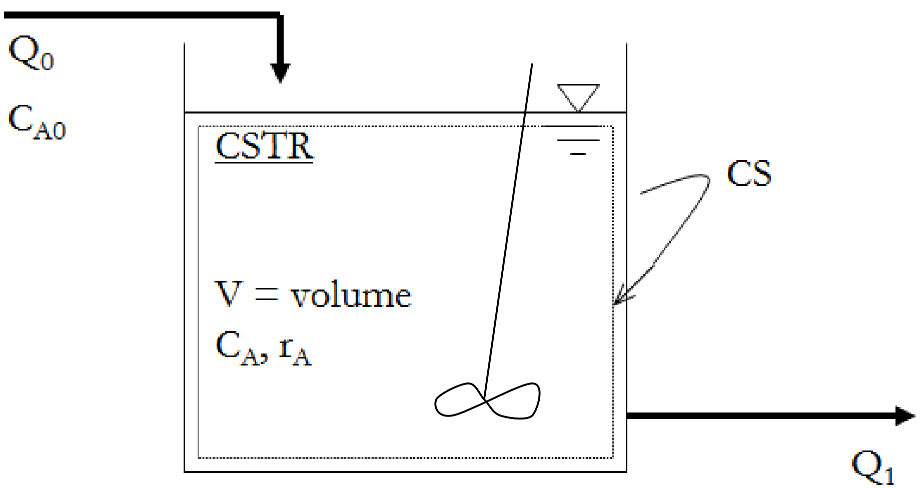
(i) Continuous Stirred Tank Reactor (CSTR) -

The contents of a CSTR exhibit no concentration gradients, i.e., concentrations of any given substance are equal at all points inside the tank (sometimes called, continuous flow completely mixed reactor, CFCMR).

Therefore, the concentration of any substance in the effluent is the same as the concentration of the substance inside the CSTR.

Continuous Stirred Tank Reactor (CSTR)

Design Equation for CSTR's - Steady-state water balance and mass analyses can be used to derive a design equation. Consider the following general process flow schematic for a single CSTR:



(1) Water balance around the CS,

$$\sum_{All} Q_{in} = \sum_{All} Q_{out}$$

$$Q_0 = Q_1$$

$$Q = Q_0 = Q_1 \qquad \text{Let}$$

(2) Mass balance on A around the CS,

$$\frac{dM_A}{dt} = \dot{m}_{A0} - \dot{m}_{A1} + r_A V$$

0, Steady-State

Continuous Stirred Tank Reactor (CSTR)

- Consider the two mass flux terms and apply

(i) $$\dot{m}_{A0} = QC_{A0}$$

$$0 = \dot{m}_{A0} - \dot{m}_{A1} + r_A V$$

(ii) $$\dot{m}_{A1} = QC_{A1}$$

- Substituting into the mass balance equation,

$$QC_{A0} - QC_{A1} + r_A V = 0$$

- Solving for V to yield a design equation for CSTR's,

$$V = \frac{Q\left(C_{A0} - C_{A1}\right)}{-r_A}$$

Steady-state, for any arbitrary $r_A = f(C_A)$

Continuous Stirred Tank Reactor (CSTR)

- Define a common design parameter, Hydraulic Detention Time, θ:

$$\theta \equiv \frac{V}{Q}$$

- Divide the design equation by Q and substitute the definition of θ to yield an alternate form of the CSTR design equation,

$$\theta = \frac{(C_{A0} - C_{A1})}{-r_A}$$

Steady-state, for any arbitrary $r_A = f(C_A)$

Continuous Stirred Tank Reactor (CSTR)

- For the special case of a first-order decay reaction,

$$r_A = - kC_A$$

- Remembering that for a CSTR, $\mathbf{C_{A1} = C_A}$, then,

$$r_A = -kC_{A1}$$

- Substituting into the CSTR design equation,

$$V = \frac{Q\left(C_{A0} - C_{A1}\right)}{-\left(-kC_{A1}\right)}$$

Continuous Stirred Tank Reactor (CSTR)

Simplifying:

$$V = \frac{Q}{k}\left(\frac{C_{A0} - C_{A1}}{C_{A1}}\right)$$

Steady-state, only for **first-order decay** kinetics, $r_A = -kC_{A1}$

Similarly, the alternate form of the design equation becomes,

$$\theta = \frac{1}{k}\left(\frac{C_{A0} - C_{A1}}{C_{A1}}\right)$$

Steady-state, only for **first-order decay** kinetics, $r_A = -kC_{A1}$

Continuous Stirred Tank Reactor (CSTR)

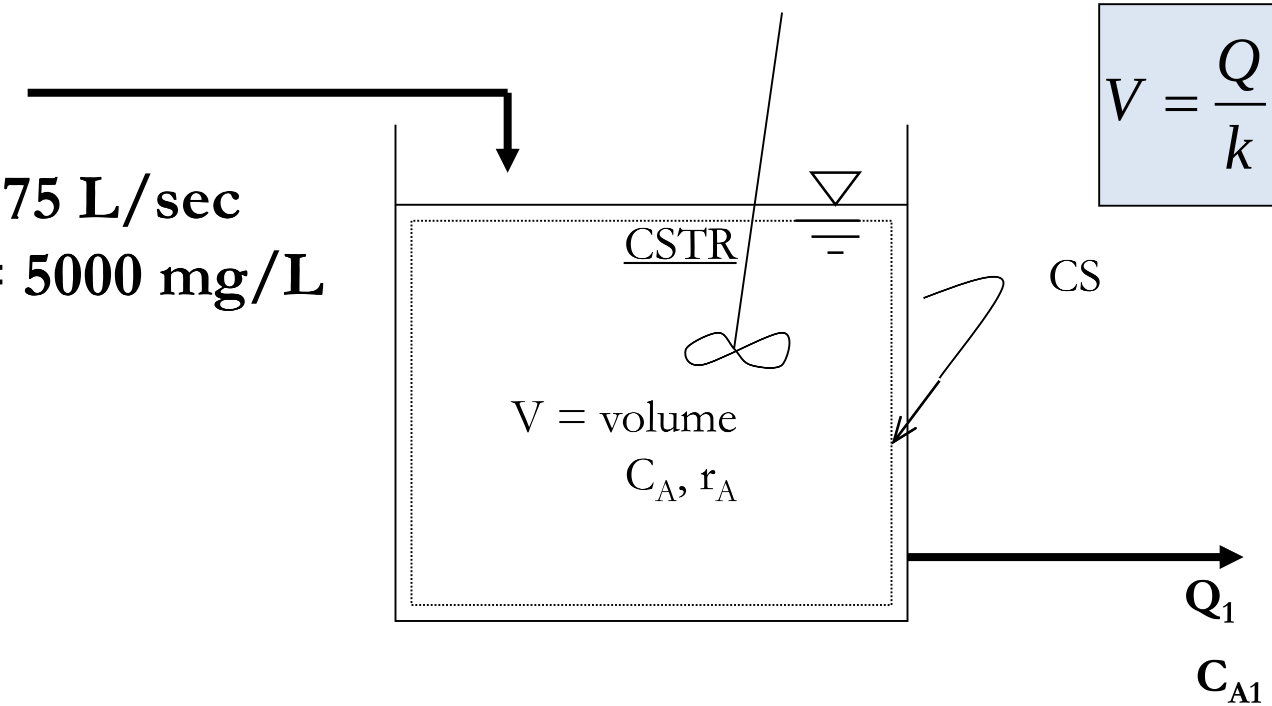
Exercise

Design a waste treatment unit for 98% destruction of a chemical pollutant, A. The reaction kinetics can be described as a first-order decay, i.e., $r_A = -kC_A$ with $k = 0.10\ sec^{-1}$

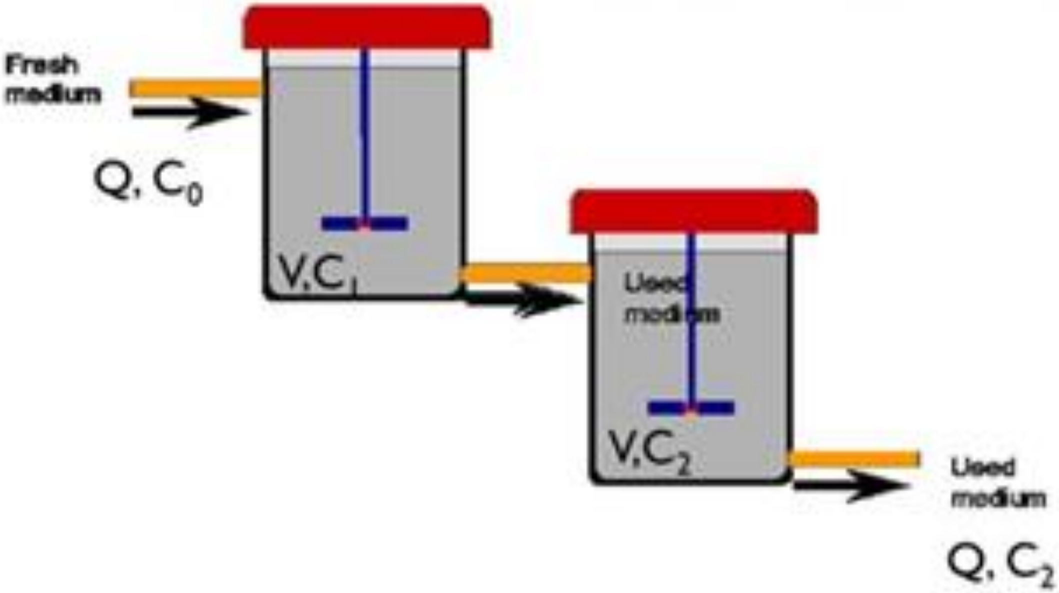
Design equation of CSTR:

$$V = \frac{Q}{k}\left(\frac{C_{A0} - C_{A1}}{C_{A1}}\right)$$

$Q_0 = 75$ L/sec
$C_{A0} = 5000$ mg/L



Continuous Stirred Tank Reactor (CSTR) In Series



$$\frac{C_1}{C_0} = \frac{1}{1 + kt_{CSTR}}$$

$$\frac{C_2}{C_1} = \frac{1}{1 + kt_{CSTR}}$$

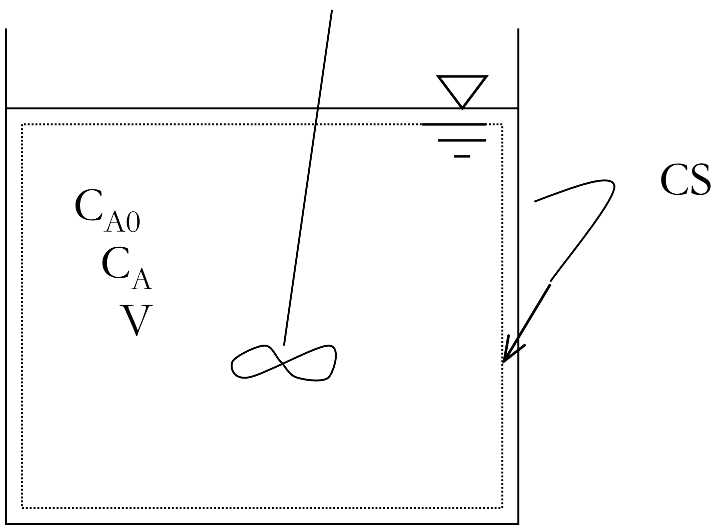
$$\frac{C_2}{C_0} = \frac{C_1}{C_0}\frac{C_2}{C_1} = \left(\frac{1}{1 + kt_{CSTR}}\right)^2$$

$$\frac{C_n}{C_0} = \left(\frac{1}{1 + kt_{CSTR}}\right)^n$$

$$nt_{CSTR} = \frac{n}{k}\left[\left(\frac{C_0}{C_e}\right)^n - 1\right]$$

(ii) Batch Reactor:

- In this case, the reactor is charged with the feed containing the reactants (e.g., an organic compound and a chemical oxidizing agent) and the chemical reaction is allowed to proceed for a period of time sufficient to achieve the desired conversion of the reactants.
- Schematically, this is represented as follows:



Batch Reactor

There are no inflow and no outflow streams in Batch Reactor.

But the mass of A in the reactor changes with time, i.e.,

$$\frac{dM_A}{dt} \neq 0$$

(i.e. there is accumulation of mass in reactor)

Where:

C_{A0} = concentration of A at time t = 0 [M/L^3]

C_A = concentration of A at any time t > 0 [M/L^3]

Batch Reactor

<u>Design Equation for Batch Reactors</u> - A mass balance on arbitrary chemical substance A around the designated CS can be used to derive a design equation as follows,

$$\frac{dM_A}{dt} = \dot{m}_{A0} - \dot{m}_{A1} + r_A V$$

No inflow and no outflow of substance A; Then,

$$\frac{dM_A}{dt} = r_A V$$

Batch Reactor

Substitute the basic equation $M_A = VC_A$ **,**

$$\frac{d(VC_A)}{dt} = r_A V$$

V is constant for a given batch of waste and can be factored from the differential term and divided out,

$$V\frac{dC_A}{dt} = r_A V$$

$$\boxed{\frac{dC_A}{dt} = r_A}$$

Batch Reactor

In order to determine the required time, θ, to proceed form C_{A0} to the desired final concentration of A, C_{A1}, solve this differential equation by separation of variables remembering that $r_A = f(C_A)$,

$$\int_{C_{A0}}^{C_{A1}} \frac{dC_A}{r_A} = \int_0^{\theta} dt$$

Then,

$$\theta = \int_{C_{A0}}^{C_{A1}} \frac{dC_A}{r_A}$$

For any arbitrary $r_A = f(C_A)$

Batch Reactor

For the special case of a first-order decay reaction,
$r_A = -kC_A$

$$\theta = \int_{C_{A0}}^{C_{A1}} \frac{dC_A}{-kC_A}$$

Factor out the constant (-1/k):

$$\theta = -\frac{1}{k}\int_{C_{A0}}^{C_{A1}} \frac{dC_A}{C_A}$$

Integrating,

$$\theta = -\frac{1}{k}\left[\ln C_A\right]_{C_{A0}}^{C_{A1}}$$

Evaluating the term in brackets:

$$\theta = -\frac{1}{k}\left[\ln C_{A1} - \ln C_{A0}\right]$$

Multiply through by (-1) on te right hand side:

$$\theta = \frac{1}{k}\left[\ln C_{A0} - \ln C_{A1}\right]$$

Batch Reactor

Apply the property of logarithms that: ln(x)-ln(y)=ln(x/y),

$$\theta = \frac{1}{k}\ln\left(\frac{C_{A0}}{C_{A1}}\right)$$

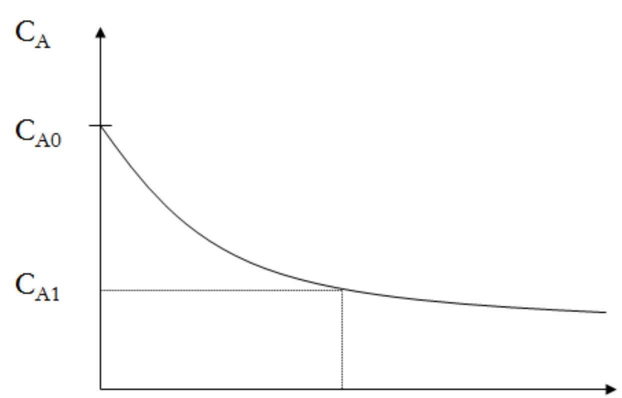
<u>Only</u> for first-order decay kinetics, $r_A = -kC_A$

Multiply through by (-k) and take the exponential of both sides of the equation,

$$C_{A1} = C_{A0}e^{-k\theta}$$

<u>Only</u> for first-order decay kinetics, $r_A = -kC_A$

The last equation is plotted as:



(iii) Plug Flow Reactor (PFR)

In this case, material enters the reactor at one end and flows through the reactor in a "slug" or "plug" of material in a linear or one-dimensional fashion until it exits at the opposite end.

A Steady-State mass balance analysis for an infinitesimal "plug" of the PFR yields,

$$V = Q \int_{C_{A0}}^{C_{A1}} \frac{dC_A}{r_A}$$

Steady-state, for any arbitrary $r_A = f(C_A)$

Plug Flow Reactor (PFR)

Divide through by Q and apply the definition of $\theta = V/Q$ to yield and alternate of the design equation,

$$\theta = \int_{C_{A0}}^{C_{A1}} \frac{dC_A}{r_A}$$

Steady-state, for any arbitrary $r_A = f(C_A)$

Notice that this is identical to the integral form of the design equation for a batch reactor.

Plug Flow Reactor (PFR)

For the special case of a first-order decay reaction,
$r_A = -kC_A$

$$V = \frac{Q}{k}\ln\left[\frac{C_{A0}}{C_{A1}}\right]$$

Steady-state, only for first-order decay kinetics, $r_A = -kC_{A1}$

or

$$\theta = \frac{1}{k}\ln\left[\frac{C_{A0}}{C_{A1}}\right]$$

Steady-state, only for first-order decay kinetics, $r_A = -kC_{A1}$

Further algebra yields,

$$C_{A1} = C_{A0}e^{-k\theta}$$

Exercise

Design a PFR for the same waste of the previous CSTR design *(98% destruction of a chemical pollutant, A).*

Q_0 = 75 l/sec C_{A0} = 5,000 mg/l

C_{A1} = 100 mg/l k = 0.10 sec^{-1}

Design equation of PFR:

$$V = \frac{Q}{k}\ln\left[\frac{C_{A0}}{C_{A1}}\right]$$

References

- *Unit Operations in Environ. Eng., Reynolds & Richards, 1996.*
- *Wastewater Eng., Chapter 4, 5th ed., Metcalf & Eddy, 2013.*

Reactor / Reaction Equations Summary

Table 5.2 Summary of Ideal Reactor Performance

Reaction Order	CMF Single Reactor	CMF n Reactors	Plug-Flow Reactor
Zero	$V=\frac{Q}{k}(C_0-C)$	$V=\frac{Q}{k}(C_0-C_n)$	$V=\frac{Q}{k}(C_0-C)$
First	$V=\frac{Q}{k}\left(\frac{C_0}{C}-1\right)$	$V=\frac{Qn}{k}\left[\left(\frac{C_0}{C}\right)^{1/n}-1\right]$	$V=\frac{Q}{k}\ln\frac{C_0}{C}$
Second	$V=\frac{Q}{k}\left(\frac{C_0}{C}-1\right)\frac{1}{C}$	complex	$V=\frac{Q}{k}\left(\frac{1}{C}-\frac{1}{C_0}\right)$

Note: Material destroyed; V = reactor volume; Q = flow rate; k = reaction constant; C_0 = influent concentration; C = effluent concentration; n = number of CMF reactors in series.