Modeling of ASS (CSTR): Substrate and Biomass Mass Balance

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Activated Sludge System Design

- One of the characteristics of the ideal complete-mix reactor (CSTR) is that the effluent leaves with the same concentration as in the liquid in any part of the reactor. This implies that the value of S and X are the same in the reactor, as well as in the effluent.

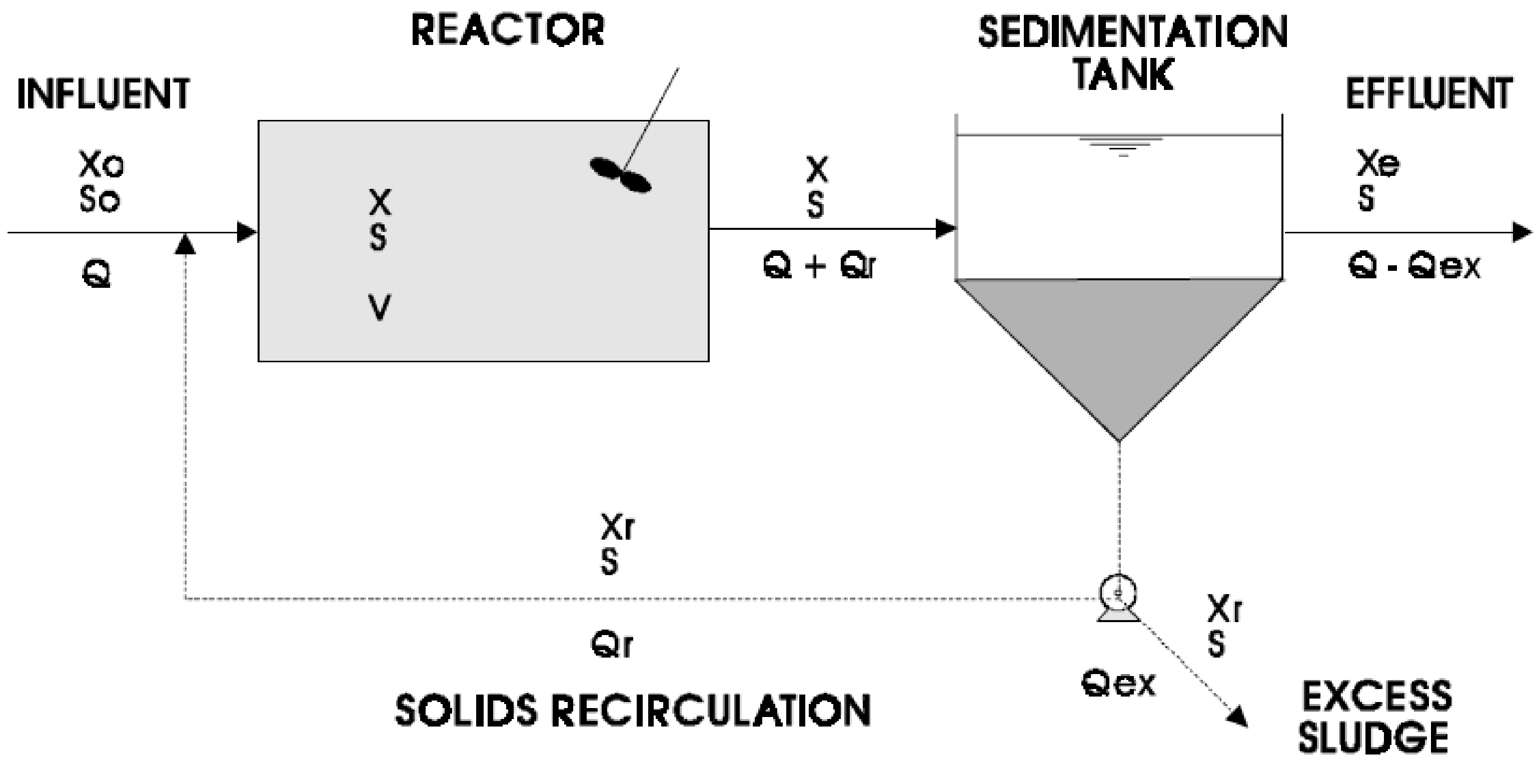
- X is the concentration of the solids. In the reactor, these solids are mainly biological solids, represented by the biomass (microorganisms) produced in the reactor at the expense of the available substrate.

- In contrast, in the influent to the reactor, the solids are those present in the wastewater, and the presence of biological solids is frequently neglected in the general mass balance. For simplicity, it is usually considered that X_0 = 0 mg/L (although this assumption does not apply in all situations).

- Two mass balances can be done, one for the substrate and the other for the biomass. These mass balances are essential for design and operational control of the biological reactor, and are detailed in this section.

AS Systems with solids recirculation

Reactor with a final sedimentation unit and with solids recirculation

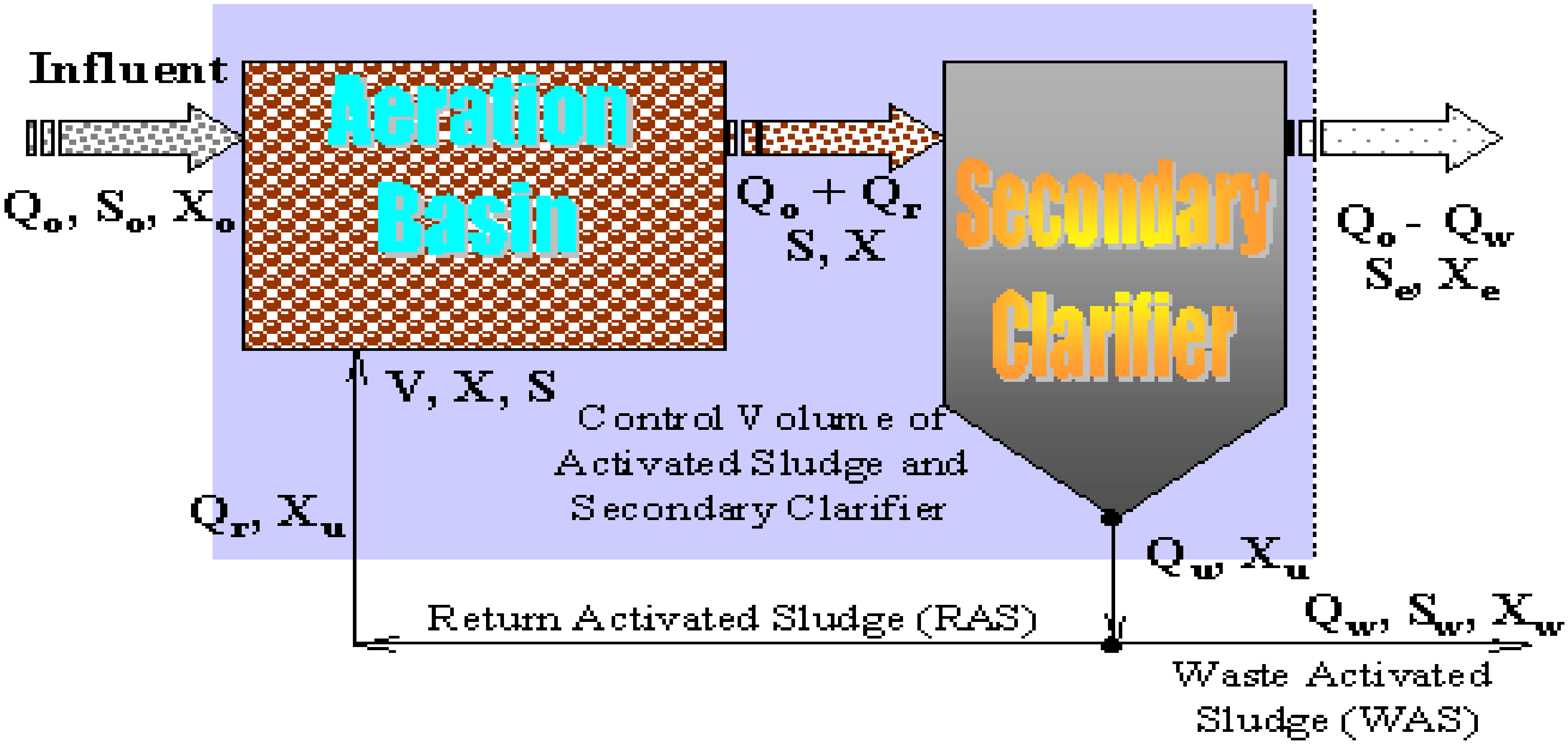


Q_r	=	recycle or return sludge flow (m^3/d)
Q_{ex}	=	excess (surplus or waste) sludge flow (m^3/d)
X_r	=	concentration of suspended solids in the return sludge (mg/L or g/m^3)

AAS with solids recirculation

- The sludge accumulated up to a certain period at the bottom of the settling unit consists mainly of bacteria that are still active in terms of their capacity to assimilate organic matter.
- The greater the biomass concentration, the greater the substrate utilization or, in other words, the greater the BOD removal.
- Therefore, if the settled sludge is returned, with a concentration higher than in the reactor, the system will be able to assimilate a much higher BOD load.
- This recirculation has also the important role of increasing the average time in which the microorganisms remain in the system. The recirculation of biomass is the basic principle of systems, such as activated sludge, which is accomplished by a recirculation pumping station.
- The value of X_r is higher than X, that is, the return sludge has a greater suspended solids concentration, what allows the increase of SS concentration in the reactor.
- there is another flow line, which corresponds to the excess sludge. This is based on the concept that the biomass production (bacterial growth) must be compensated for by the wastage of an equivalent quantity, for the system to be maintained in equilibrium. If there were no such a wastage, the mass of suspended solids in the reactor would progressively increase, and these solids would then be transferred to the settling tank, until a point when the settler would become overloaded.

AS Design Equations



Mass balance of biomass production (CSTR)

Influent biomass + biomass production = effluent biomass + sludge wasted

$$Q_o X_o + V\frac{dX}{dt} = (Q_o - Q_w)X_e + Q_w X_w$$

Substitute biomass production equation

$$Q_o X_o + V\left(\frac{\mu_m S}{K_s + S} X - k_d X\right) = (Q_o - Q_w)X_e + Q_w X_w$$

Assume that influent and effluent biomass concentrations are negligible and solve

$$\frac{\mu_m S}{K_s + S} = \frac{Q_w X_w}{VX} + k_d$$

Mass balance of food substrate

Influent substrate + substrate consumed = effluent substrate + sludge wasted substrate

$$Q_o S_o + V\frac{dS}{dt} = (Q_o - Q_w)S_e + Q_w S_w$$

Substitute substrate removal equation

$$Q_o S_o + \frac{V}{Y}\left(\frac{\mu_m XS}{K_s + S}\right) = (Q_o - Q_w)S_e + Q_w S_w$$

Assume that no biochemical action takes place in clarifier. Therefore the substrate concentration in the aeration basin is equal to the substrate concentrations in the effluent and the waste activated sludge. Solve:

$$\frac{\mu_m S}{K_s + S} = \frac{Q_o Y}{VX}(S_o - S)$$

Hydraulic detention time and solids retention time

- In a system with solids recycling, the solids are separated and concentrated in the final settling unit and subsequently returned to the reactor.

- The liquid, on the other hand, in spite of the recirculation (which is internal in the system), does not vary quantitatively, apart from the withdrawal of the excess sludge flow, which is negligible in the overall calculation ($Q_{ex} \approx 0$).

- Therefore, only the solids are retained in the system, owing to the separation, thickening and recycling. Thus, the solids remain longer in the system than the liquid. It is thus necessary to distinguish the concepts of solids retention time and hydraulic detention time.

The hydraulic detention time t (or hydraulic retention time – HRT) given by:

Since the volume of liquid that enters is the same as the one that leaves, the following generalization can be made:

$$\text{hydraulic detention time} = \frac{\text{volume of liquid in the system}}{\text{volume of liquid removed per unit time}}$$

$$t = \frac{V}{Q}$$

Similarly, the **solids retention time SRT** (or mean cell residence time – MCRT or **sludge age** - θ_c) is given by:

$$\text{sludge age} = \frac{\text{mass of solids in the system}}{\text{mass of solids produced per unit time}}$$

In the steady state, the quantity of solids removed from the system is equal to the quantity of sludge produced. Hence, the sludge age can also be expressed as:

$$\text{sludge age} = \frac{\text{mass of solids in the system}}{\text{mass of solids removed per unit time}}$$

$$\boxed{\theta_c = \frac{1}{\mu - K_d}}$$

Depending on inclusion or not of sludge recycle, the following two conditions are obtained:

- Systems without solids retention: $t = \theta_c$
- Systems with solids retention: $t > \theta_c$

Overall equations

- Combine the mass balance equations for food and biomass:

$$\frac{Q_w V_w}{VX} + k_d = \frac{Q_o Y}{VX}(S_o - S)$$

The cell residence time is:

$$\theta_c = \frac{VX}{Q_w X_w}$$

and the hydraulic retention time is, $\theta = V/Q_o$

Substitute and rearrange:

$$X = \frac{\theta_c (Y)(S_o - S)}{\theta(1 + k_d \theta_c)}$$

- The fact that the biomass stays longer than the liquid in the system justifies the greater efficiency of systems with solids recirculation, compared with systems without solids recirculation.

- It can also be said that, for the same removal efficiency, systems with solids recirculation require much smaller reactor volumes than the systems without recirculation.

- The biochemical reactions occur only in the reactor. The reactions of the conversion of organic matter and of cellular growth in the settling unit can be neglected.

- The biomass is assumed to be present only in the reactor. In the calculation of the sludge age, the solids present in the final settling unit and in the recirculation line have not been considered.

- The mechanisms take place according to the steady state. In the dynamic state, the mass of solids produced is not equal to the mass wasted, which alters the interpretation of the sludge age concept.

Example

Calculate the hydraulic detention time and the sludge age in the sewage treatment system ! (without a settling tank and solids recirculation). The main relevant data :

Reactor volume: $V = 9{,}000\ m^3$

Input and output variables:

- Influent flow: $Q = 3{,}000\ m^3/d$
- Influent substrate (BOD_5 total): $S_o = 350$ mg/L
- Effluent substrate (BOD_5 soluble): $S = 9.1$ mg/L

Model coefficients:

- Maximum specific growth rate: $\mu_{max} = 3.0\ d^{-1}$
- Half-saturation coefficient: $K_s = 60$ mg/L
- Endogenous respiration coefficient: $K_d = 0.06\ d^{-1}$

Solution:

a) Hydraulic detention time

$$t = \frac{V}{Q} = \frac{9{,}000\ m^3}{3{,}000\ m^3/d} = 3.0\ d$$

b) Sludge age

The value of μ is

$$\mu = \mu_{max} \cdot \frac{S}{K_s + S} = 3.0 . \frac{9.1}{60 + 9.1} = 0.395\ d^{-1}$$

The sludge age is

$$\theta_c = \frac{1}{\mu - K_d} = \frac{1}{0.395 - 0.06} = 3.0\ d$$

As expected, in the present example $t = \theta_c$, since the system has no solids recirculation.

Loading rates on biological reactors

Sludge load (food-to-microorganism ratio)

- A relationship widely used by designers and operators of wastewater treatment plants is the **sludge load** or **F/M (food-to-microorganism) ratio**.

- It is based on the concept that the quantity of food or substrate available per unit mass of microorganisms is related to the efficiency of the system.

- Hence, it can be understood that, the higher the BOD load supplied per unit value of the biomass (high F/M ratio), the lower is the substrate assimilation efficiency, but, on the other hand, the lower is the required reactor volume.

- Conversely, when less BOD is supplied to the bacteria (low F/M ratio), the demand for food is higher, which implies a greater BOD removal efficiency and a larger reactor volume requirement.

- In a situation in which the quantity of food supplied is very low, the mechanism of endogenous respiration becomes prevalent.

The food load supplied is given by: $F = Q.S_0$

The microorganism mass is calculated as: $M = V.X_v$

where:

Q = influent flow (m^3/d)

S_0 = influent BOD_5 concentration (g/m^3)

V = reactor volume (m^3)

X_v = volatile suspended solids concentration (g/m^3)

Thus, the F/M ratio is expressed as:

$$\frac{F}{M} = \frac{Q.S_0}{V.X_v}$$

where:

F/M = sludge load ($gBOD_5$ supplied per day/g VSS)

Example

Calculate the values of F/M in a wastewater treatment plant with sludge recirculation, as described in Data:

$S_o = 300\ gBOD_5/m^3$
$S = 15\ gBOD_5/m^3$
$t = 0.25\ d$
$X_v = 2{,}540\ gVSS/m^3$

Solution:

a) Calculation of F/M

From Equation

$$\frac{F}{M} = \frac{S_o}{t.X_v} = \frac{300\ gBOD_5/m^3}{0.25\ d\ .\ 2{,}540\ gVSS/m^3} = 0.47 d^{-1}$$

$F/M = 0.47\ kgBOD_5/kgVSS.d$

Ref. Book: Vesilind

Example: 11.1
Example: 11.2
Example: 11.3
Example: 11.4
Example: 11.5