



# Chemical Reaction Engineering

## Rate Laws and Stoichiometry

Dr.-Eng. Zayed Al-Hamamre

Chemical Engineering Department | University of Jordan | Amman 11942, Jordan  
Tel. +962 6 535 5000 | 22888



## Content



- Reversible Reaction
- Reaction Rate Constant
- Activation Energy
- Stoichiometry



# Reversible Reactions



- All rate laws for reversible reactions *must* reduce to the thermodynamic relationship relating the reacting species concentrations at equilibrium.
- At equilibrium, the rate of reaction is identically zero for all species

for the general reaction



- The concentrations at equilibrium are related by the thermodynamic relationship for the equilibrium constant  $K_C$

$$K_C = \frac{C_{Ce}^c C_{De}^d}{C_{Ae}^a C_{Be}^b}$$

The units of the thermodynamic equilibrium constant,  $K_C$ , are  $(\text{mol/dm}^3)^{d+c-b-a}$

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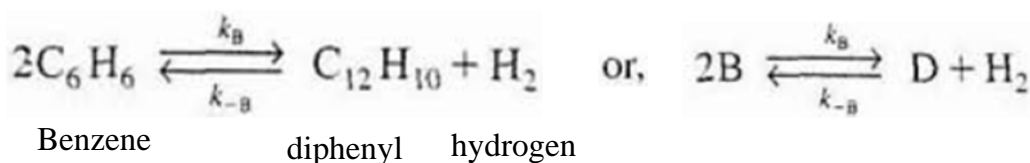


# Reversible Reactions



## Rate laws for reversible reactions

For the reaction



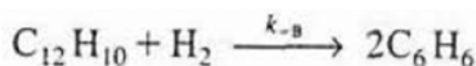
$k_B$  and  $k_{-B}$ , are the forward and reverse specific reaction rate constants respectively and defined with respect to benzene.

The rate of disappearance of benzene  $-r_{B, \text{forward}} = k_B C_B^2$

Or the rate of disappearance of benzene for the forward reaction

$$r_{B, \text{forward}} = -k_B C_B^2$$

- For the reverse reaction



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# Reversible Reactions



The rate of formation of benzene

$$r_{B, \text{reverse}} = k_{-B} C_D C_{H_2}$$

➤ The net rate of formation of benzene

$$r_B \equiv r_{B, \text{net}} = r_{B, \text{forward}} + r_{B, \text{reverse}}$$



$$r_B = -k_B C_B^2 + k_{-B} C_D C_{H_2}$$

the rate of disappearance of benzene,  $-r_B$ :

$$-r_B = k_B C_B^2 - k_{-B} C_D C_{H_2} = k_B \left( C_B^2 - \frac{k_{-B}}{k_B} C_D C_{H_2} \right)$$



$$-r_B = k_B \left( C_B^2 - \frac{C_D C_{H_2}}{K_C} \right)$$

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# Reversible Reactions



where

$$\frac{k_B}{k_{-B}} = K_C = \text{Concentration equilibrium constant}$$

➤ The equilibrium constant

- Decreases with increasing temperature for exothermic reactions and
- Increases **with** increasing temperature for endothermic reactions

➤ The rate of formation of diphenyl is:

$$r_D = k_D \left[ C_B^2 - \frac{C_D C_{H_2}}{K_C} \right]$$

➤ Also, from stoichiometry

$$\frac{r_D}{1} = \frac{r_B}{-2} = \frac{k_B}{2} \left[ C_B^2 - \frac{C_D C_{H_2}}{K_C} \right]$$

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# Reversible Reactions



Comparing

$$-r_B = k_B \left( C_B^2 - \frac{C_D C_{H_2}}{K_C} \right)$$

And

$$r_D = k_D \left[ C_B^2 - \frac{C_D C_{H_2}}{K_C} \right]$$

And utilizing stoichiometry

$$\frac{r_D}{1} = \frac{r_B}{-2} = \frac{k_B}{2} \left[ C_B^2 - \frac{C_D C_{H_2}}{K_C} \right]$$



$$k_D = \frac{k_B}{2}$$

At equilibrium,  $-r_B \equiv 0$ ,  $-r_B \equiv 0 = k_B \left[ C_{Be}^2 - \frac{C_{De} C_{H_2e}}{K_C} \right]$



$$K_C = \frac{C_{De} C_{H_2e}}{C_{Be}^2}$$

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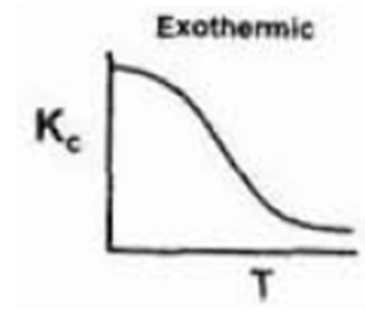
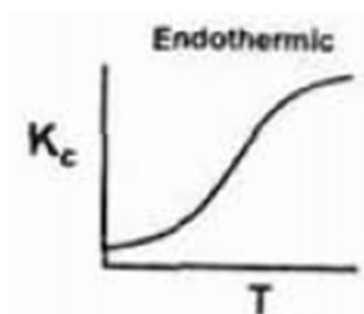


# Reversible Reactions



- When there is no change in the total number of moles and the heat capacity term,  $\Delta C_p = 0$

$$K_C(T) = K_C(T_1) \exp \left[ \frac{\Delta H_{Rx}}{R} \left( \frac{1}{T_1} - \frac{1}{T} \right) \right]$$



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# The Reaction Rate Constant



## The specific reaction rate or the rate constant

- Is merely independent of the concentrations of the species involved in the reaction.
- Is almost always strongly dependent on temperature.
- Depends on whether or not a catalyst is present,
- In gas-phase reactions, it may be a function of total pressure
- In liquid systems it can also be a function of other parameters, such as ionic strength and choice of solvent

➤ The temperature dependence of the specific reaction rate,  $k_A$  could be correlated by

$$k_A(T) = Ae^{-E/RT}$$

Arrhenius Equation

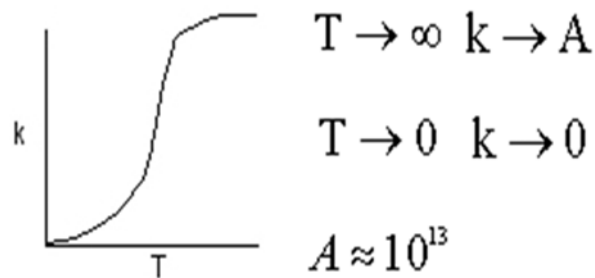
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# The Reaction Rate Constant



where  $A$  = preexponential factor or frequency factor  
 $E$  = activation energy, J/mol or cal/mol  
 $R$  = gas constant = 8.314 J/mol · K = 1.987 cal/mol · K  
 $T$  = absolute temperature, K

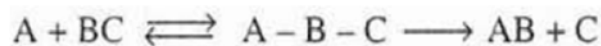


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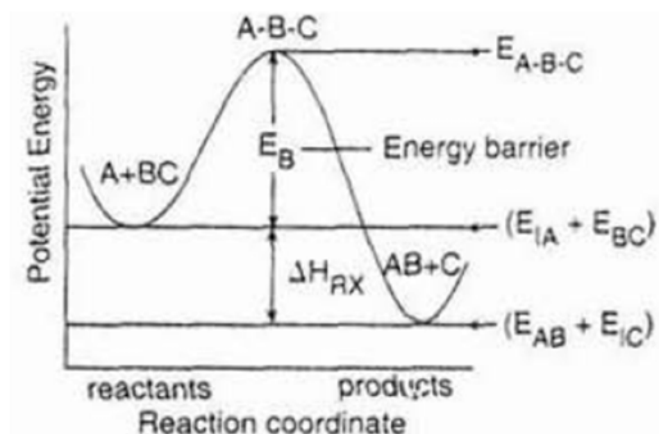


# The Activation Energy

- The activation energy can be thought of as a barrier to energy transfer (from **the** kinetic energy to the potential energy) between reacting molecule that must be overcome.



reactants                  intermediate                  products



the potential energy of the system as a function of the progress along the reaction path

- The distance between A and B decreases, and the AB bond begins to form.
- As we proceed further, the distance between AB and C increases and
- The energy of the reacting pair decreases to that of the AB bond energy.

$$E_B = E_{fA-B-C}^{\circ} - (E_{fA}^{\circ} + E_{fB-C}^{\circ})$$

$E_{f_i}^{\circ}$  : Energies of formation

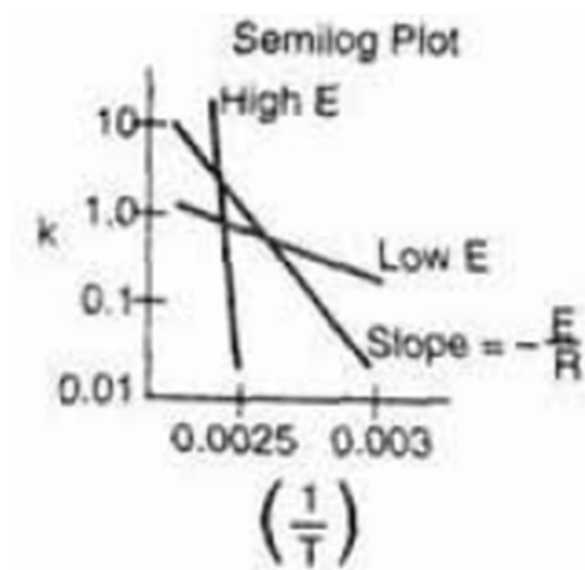
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# The Activation Energy

$$\ln k_A = \ln A - \frac{E}{R} \left( \frac{1}{T} \right)$$

- The larger the activation energy, the more temperature-sensitive is the rate of reaction.

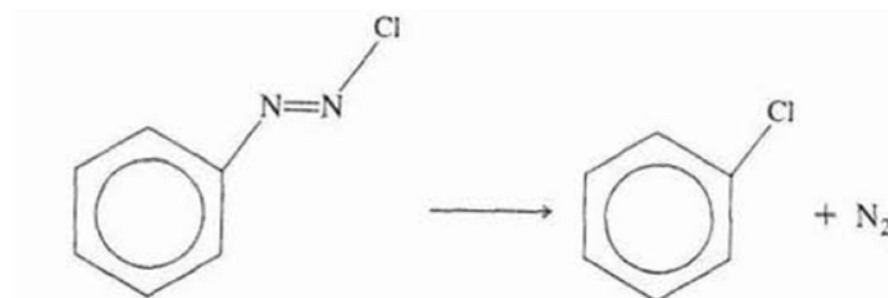


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## Example

Calculate the activation energy for the decomposition of benzene diazonium chloride to give chlorobenzene and nitrogen:



using the information

$k \text{ (s}^{-1}\text{)}$	0.00043	0.00103	0.00180	0.00355	0.00717
$T \text{ (K)}$	313.0	319.0	323.0	328.0	333.0

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## Example Cont.

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## Example Cont.



- If the specific reaction rate  $k_o(T_o)$  at a temperature,  $T_o$ , and we know the activation energy,  $E$ , we can find the specific reaction rate  $k(T)$  at any other temperature,  $T$ , for that reaction

$$k(T) = k(T_o)e^{\frac{E}{R}\left(\frac{1}{T_o} - \frac{1}{T}\right)} \quad \text{where} \quad k(T_o) = Ae^{-E/RT_o}$$

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## Review: Reactor Sizing and Design



- The rate of disappearance of A,  $-r_A$ , might be
  - Known as a function of conversion,  $X$

$$-r_A = g(X)$$

- Expressed in terms of the concentration of the reacting species

$$-r_A = [k_A(T)][\text{fn}(C_A, C_B, \dots)]$$

- The concentration of the reacting species may be written in terms of the conversion  $X$

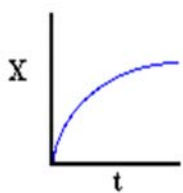
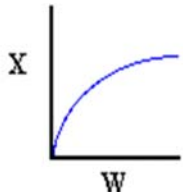
$$C_j = h_j(X)$$

- If the rate law depends on more than one species, it is necessary to relate the concentrations of the different species to each other

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# Review: Reactor Sizing and Design

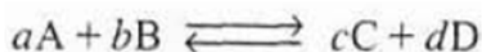
Reactor	Differential	Algebraic	Integral		
<u>Batch</u>	$N_{A0} \frac{dX}{dt} = -r_A V$		$t = N_{A0} \int_0^X \frac{dX}{-r_A V}$	<a href="#">Derive</a>	
<u>CSTR</u>		$V = \frac{F_{A0} X}{-r_A}$		<a href="#">Derive</a>	
<u>PFR</u>	$F_{A0} \frac{dX}{dV} = -r_A$		$V = F_{A0} \int_0^X \frac{dX}{-r_A}$	<a href="#">Derive</a>	
<u>PBR</u>	$F_{A0} \frac{dX}{dW} = -r'_A$		$W = F_{A0} \int_0^X \frac{dX}{-r'_A}$	<a href="#">Derive</a>	

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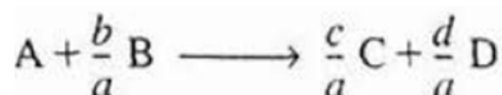
# Stoichiometry

- A stoichiometric table is a table that presents the stoichiometric relationships between reacting molecules for a single reaction.
- Such table make it easier to relate the concentrations of the different species to each other for the case when the rate law depends on more than one species.
- For the reaction



$$\frac{r_A}{-a} = \frac{r_B}{-b} = \frac{r_C}{c} = \frac{r_D}{d}$$

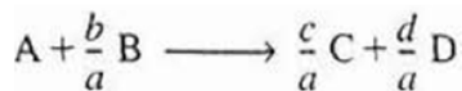
- In formulating our stoichiometric table, we shall take species A (limiting reactant) as a basis of calculation



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## Batch Systems



$$N_A = N_{A0} - N_{A0}X = N_{A0}(1 - X)$$

$N_{A0}$  the number of moles of A initially present in the reactor

$N_{A0}X$  moles of A are consumed

$N_A$  the number of moles of A remaining in the reactor

➤ How to express the number of moles of B, C, and D in terms of conversion of A?

$$\text{moles B reacted} = \frac{\text{moles B reacted}}{\text{moles A reacted}} \cdot \text{moles A reacted} = \frac{b}{a} (N_{A0}X)$$

$$\rightarrow N_B = N_{B0} - \frac{b}{a} N_{A0}X$$

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## Batch Systems

Species	Initially (mol)	Change (mol)	Remaining (mol)
A	$N_{A0}$	$-(N_{A0}X)$	$N_A = N_{A0} - N_{A0}X$
B	$N_{B0}$	$-\frac{b}{a} (N_{A0}X)$	$N_B = N_{B0} - \frac{b}{a} N_{A0}X$
C	$N_{C0}$	$\frac{c}{a} (N_{A0}X)$	$N_C = N_{C0} + \frac{c}{a} N_{A0}X$
D	$N_{D0}$	$\frac{d}{a} (N_{A0}X)$	$N_D = N_{D0} + \frac{d}{a} N_{A0}X$
I (inerts)	$N_{I0}$		$N_I = N_{I0}$
Totals	$N_{T0}$		$N_T = N_{T0} + \left( \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) N_{A0}X$

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# Batch Systems

Let 
$$\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$$

Then 
$$N_T = N_{T0} + \delta N_{A0}X$$

The concentration of A is the number of moles of A per unit volume:

$$C_A = \frac{N_A}{V} \quad \rightarrow \quad C_A = \frac{N_A}{V} = \frac{N_{A0}(1-X)}{V}$$

$$C_B = \frac{N_B}{V} = \frac{N_{B0} - (b/a)N_{A0}X}{V} \quad C_C = \frac{N_C}{V} = \frac{N_{C0} + (c/a)N_{A0}X}{V}$$

$$C_D = \frac{N_D}{V} = \frac{N_{D0} + (d/a)N_{A0}X}{V}$$

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# Batch Systems

defining the parameter  $\Theta_i$ ,

$$\Theta_i = \frac{N_{i0}}{N_{A0}} = \frac{C_{i0}}{C_{A0}} = \frac{y_{i0}}{y_{A0}}$$

Then

$$C_B = \frac{N_{A0}[N_{B0}/N_{A0} - (b/a)X]}{V} = \frac{N_{A0}[\Theta_B - (b/a)X]}{V}$$

$$\text{with } \Theta_B = \frac{N_{B0}}{N_{A0}}$$

$$C_C = \frac{N_{A0}[\Theta_C + (c/a)X]}{V}, \quad \text{with } \Theta_C = \frac{N_{C0}}{N_{A0}}$$

$$C_D = \frac{N_{A0}[\Theta_D + (d/a)X]}{V}, \quad \text{with } \Theta_D = \frac{N_{D0}}{N_{A0}}$$

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# Constant-Volume Batch Reaction Systems

- For the constant-volume systems

$$V = V_0$$

$$C_A = \frac{N_{A0}(1-X)}{V_0} = C_{A0}(1-X)$$

$$C_B = N_{A0} \frac{[(N_{B0}/N_{A0}) - (b/a)X]}{V_0} = \frac{N_{A0}[\Theta_B - (b/a)X]}{V_0} = C_{A0} \left( \Theta_B - \frac{b}{a}X \right)$$

$$C_C = N_{A0} \frac{[(N_{C0}/N_{A0}) + (c/a)X]}{V_0} = C_{A0} \left( \Theta_C + \frac{c}{a}X \right)$$

$$C_D = N_{A0} \frac{[(N_{D0}/N_{A0}) + (d/a)X]}{V_0} = C_{A0} \left( \Theta_D + \frac{d}{a}X \right)$$

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# Batch Systems

- For liquid-phase reactions (or as we will soon see for isothermal and isobaric gas-phase reactions with no change in the total number of moles,

For  $-r_A = k_A C_A C_B$

Then  $-r_A = k C_A C_B = k C_{A0}^2 (1-X) \left( \Theta_B - \frac{b}{a}X \right) = f(X)$

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## Example

For the reaction



Express the concentrations of the different species as function  $X$ .

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## Example Cont.

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## Example

If the initial mixture consists solely of sodium hydroxide at a concentration of  $10 \text{ mol/dm}^3$  (i.e.,  $10 \text{ mol/L}$  or  $10 \text{ kmol/m}^3$ ) and of glyceryl stearate at a of  $2 \text{ mol/dm}^3$ , what is the concentration of glycerine when the conversion of sodium hydroxide is (a) 20% and (b) 90%?

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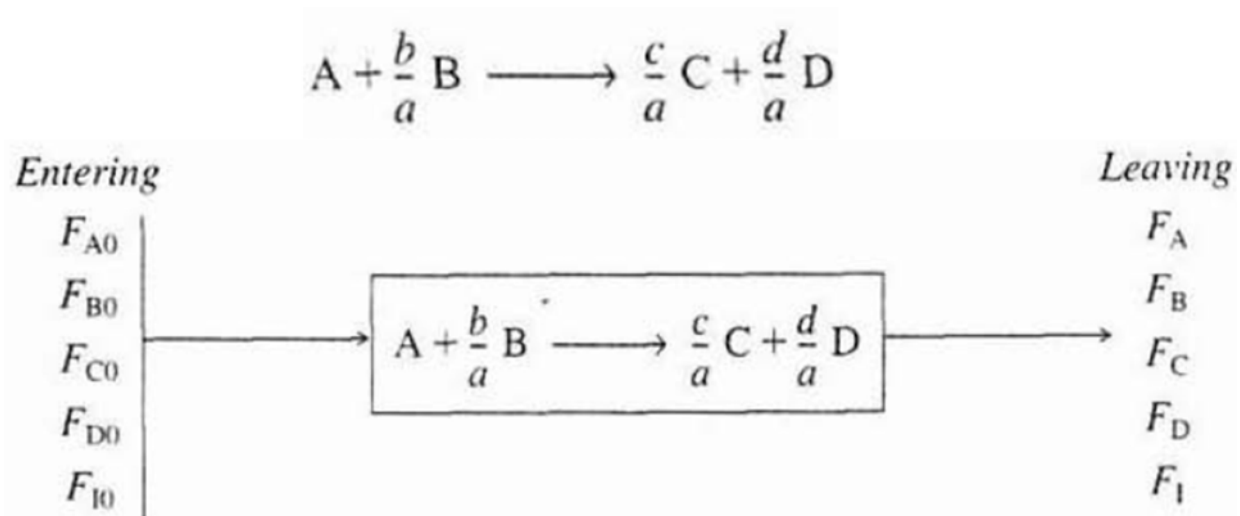


## Example Cont.

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## Flow Systems



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## Flow Systems

Species	Feed Rate to Reactor (mol/time)	Change within Reactor (mol/time)	Effluent Rate from Reactor (mol/time)
A	$F_{A0}$	$-F_{A0}X$	$F_A = F_{A0}(1 - X)$
B	$F_{B0} = \Theta_B F_{A0}$	$-\frac{b}{a} F_{A0}X$	$F_B = F_{A0} \left( \Theta_B - \frac{b}{a} X \right)$
C	$F_{C0} = \Theta_C F_{A0}$	$\frac{c}{a} F_{A0}X$	$F_C = F_{A0} \left( \Theta_C + \frac{c}{a} X \right)$
D	$F_{D0} = \Theta_D F_{A0}$	$\frac{d}{a} F_{A0}X$	$F_D = F_{A0} \left( \Theta_D + \frac{d}{a} X \right)$
I	$F_{I0} = \Theta_I F_{A0}$	—	$F_I = F_{A0} \Theta_I$
	$F_{T0}$		$F_T = F_{T0} + \left( \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) F_{A0}X$
			$F_T = F_{T0} + \delta F_{A0}X$

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## Concentrations in Flow Systems

where

$$\Theta_B = \frac{F_{B0}}{F_{A0}} = \frac{C_{B0} v_0}{C_{A0} v_0} = \frac{C_{B0}}{C_{A0}} = \frac{y_{B0}}{y_{A0}}$$

and  $\Theta_C$ ,  $\Theta_D$ , and  $\Theta_I$  are defined similarly.

### Equations for Concentrations in Flow Systems

For a flow system,

$$C_A = \frac{F_A}{v} = \frac{\text{moles/time}}{\text{liters/time}} = \frac{\text{moles}}{\text{liter}}$$

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## Concentrations in Flow Systems

$$C_A = \frac{F_A}{v} = \frac{F_{A0}}{v} (1 - X)$$

$$C_B = \frac{F_B}{v} = \frac{F_{B0} - (b/a) F_{A0} X}{v}$$

$$C_C = \frac{F_C}{v} = \frac{F_{C0} + (c/a) F_{A0} X}{v}$$

$$C_D = \frac{F_D}{v} = \frac{F_{D0} + (d/a) F_{A0} X}{v}$$

### Liquid-Phase Concentrations

- For liquids, volume change with reaction is negligible when no phase change are taking place

$$v = v_0$$

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# Liquid-Phase Concentrations

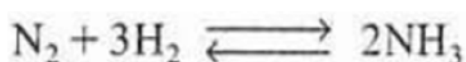
$$C_A = \frac{F_{A0}}{v_0} (1 - X) = C_{A0} (1 - X)$$

$$C_B = C_{A0} \left( \Theta_B - \frac{b}{a} X \right)$$

➔  $-r_A = f(X)$

## Change in the Total Number of Moles with Reaction in the Gas Phase

- Variation on the flow rate occurs quite frequently in gas-phase reactions that do not have an equal number of product and reactant moles



4 mol of reactants gives 2 mol of product.

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# Variable Volume

- Variable-volume situation could occur in batch reactors where volume changes with time as in internal-combustion engine

## Batch Reactors with Variable Volume

$$PV = ZN_T RT$$

where

$V$  = volume and  $N_T$  = total number of moles

$T$  = temperature, K

$P$  = total pressure, atm (kPa; 1 atm = 101.3 kPa)

$Z$  = compressibility factor

$R$  = gas constant =  $0.08206 \text{ dm}^3 \cdot \text{atm/mol} \cdot \text{K}$

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## Batch Reactors with Variable Volume

This equation is valid at any point in the system at any time  $t$ .

At  $t = 0$

$$P_0 V_0 = Z_0 N_{T0} R T_0$$

$$\rightarrow V = V_0 \left( \frac{P_0}{P} \right) \frac{T}{T_0} \left( \frac{Z}{Z_0} \right) \frac{N_T}{N_{T0}}$$

But  $N_T = N_{T0} + \delta N_{A0} X$

where  $\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$   $\delta = \frac{\text{Change in total number of moles}}{\text{Mole of A reacted}}$



## Batch Reactors with Variable Volume

divide by  $N_{T0}$ :

$$\frac{N_T}{N_{T0}} = 1 + \frac{N_{A0}}{N_{T0}} \delta X = 1 + \overbrace{\delta y_{A0}}^{\epsilon} X$$

where  $y_{A0}$  is the mole fraction of A initially present,

$$\rightarrow \frac{N_T}{N_{T0}} = 1 + \epsilon X$$

where

$$\epsilon = \left( \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) \frac{N_{A0}}{N_{T0}} = y_{A0} \delta$$

For both batch and flow systems



## Batch Reactors with Variable Volume

Also

$$\varepsilon = \frac{N_T - N_{T0}}{N_{T0}X}$$

at complete conversion, (i.e.,  $X = 1$  and  $N_T = N_{Tf}$ )

➔ 
$$\varepsilon = \frac{N_{Tf} - N_{T0}}{N_{T0}}$$

$$= \frac{\text{Change in total number of moles for complete conversion}}{\text{Total moles fed}}$$

If all species are in the gas phase,

➔ 
$$V = V_0 \left( \frac{P_0}{P} \right) \frac{T}{T_0} \left( \frac{Z}{Z_0} \right) (1 + \varepsilon X)$$

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## Batch Reactors with Variable Volume

- If the temperatures and pressures are changed such that the compressibility factor will not change significantly during the course of the reaction

$$Z_0 \cong Z$$

Then, for a variable volume batch system, the volume of gas at any time  $t$  is

$$V = V_0 \left( \frac{P_0}{P} \right) (1 + \varepsilon X) \frac{T}{T_0}$$

### Flow Reactors with Variable Volumetric Flow Rate

- The total concentration can be defined as

$$C_T = \frac{F_T}{v} = \frac{P}{ZRT}$$

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# Flow Reactors with Variable Volumetric Flow Rate



At the entrance to the reactor,

Taking the ratio 
$$C_{T0} = \frac{F_{T0}}{v_0} = \frac{P_0}{Z_0 R T_0}$$

→ 
$$v = v_0 \left( \frac{F_T}{F_{T0}} \right) \frac{P_0}{P} \left( \frac{T}{T_0} \right)$$

→ the concentration of species  $j$  for a flow system

$$C_j = \frac{F_j}{v} = \frac{F_j}{v_0 \left( \frac{F_T}{F_{T0}} \frac{P_0}{P} \frac{T}{T_0} \right)} = \left( \frac{F_{T0}}{v_0} \right) \left( \frac{F_j}{F_T} \right) \left( \frac{P}{P_0} \right) \left( \frac{T_0}{T} \right)$$

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→ 
$$C_j = C_{T0} \left( \frac{F_j}{F_T} \right) \left( \frac{P}{P_0} \right) \left( \frac{T_0}{T} \right)$$

And 
$$F_T = F_A + F_B + F_C + F_D + F_I + \dots = \sum_{j=1}^n F_j$$

But 
$$F_T = F_{T0} + F_{A0} \delta X$$

Then 
$$v = v_0 \frac{F_{T0} + F_{A0} \delta X}{F_{T0}} \left( \frac{P_0}{P} \right) \frac{T}{T_0}$$

$$= v_0 \left( 1 + \frac{F_{A0}}{F_{T0}} \delta X \right) \frac{P_0}{P} \left( \frac{T}{T_0} \right) = v_0 (1 + y_{A0} \delta X) \frac{P_0}{P} \left( \frac{T}{T_0} \right)$$

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# Flow Reactors with Variable Volumetric Flow Rate



$$\rightarrow v = v_0(1 + \varepsilon X) \frac{P_0}{P} \left( \frac{T}{T_0} \right)$$

The concentration of species  $j$  is  $C_j = \frac{F_j}{v}$

The molar flow rate of species  $j$  is  $F_j = F_{j0} + v_j(F_{A0}X) = F_{A0}(\Theta_j + v_jX)$

where  $v_i$  is the stoichiometric coefficient, negative for reactants  
positive for products.

➤ For the reaction  $A + \frac{b}{a} B \longrightarrow \frac{c}{a} C + \frac{d}{a} D$

$v_A = -1$ ,  $v_B = -b/a$ ,  $v_C = c/a$ ,  $v_D = d/a$ , and  $\Theta_j = F_{j0}/F_{A0}$ .

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# Flow Reactors with Variable Volumetric Flow Rate



$$\rightarrow C_j = \frac{F_{A0}(\Theta_j + v_jX)}{v_0 \left( (1 + \varepsilon X) \frac{P_0}{P} \frac{T}{T_0} \right)}$$

Rearranging

$$C_j = \frac{C_{A0}(\Theta_j + v_jX)}{1 + \varepsilon X} \left( \frac{P}{P_0} \right) \frac{T_0}{T}$$

Recall that

$$y_{A0} = F_{A0}/F_{T0}, \quad C_{A0} = y_{A0}C_{T0}, \quad \varepsilon = y_{A0}\delta$$

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The stoichiometric table for the gas-phase reaction

$$\begin{aligned}
 C_A &= \frac{F_A}{v} = \frac{F_{A0}(1-X)}{v} = \frac{F_{A0}(1-X)}{v_0(1+\epsilon X)} \left( \frac{T_0}{T} \right) \frac{P}{P_0} = C_{A0} \left( \frac{1-X}{1+\epsilon X} \right) \frac{T_0}{T} \left( \frac{P}{P_0} \right) \\
 C_B &= \frac{F_B}{v} = \frac{F_{A0}[\Theta_B - (b/a)X]}{v} = \frac{F_{A0}[\Theta_B - (b/a)X]}{v_0(1+\epsilon X)} \left( \frac{T_0}{T} \right) \frac{P}{P_0} = C_{A0} \left( \frac{\Theta_B - (b/a)X}{1+\epsilon X} \right) \frac{T_0}{T} \left( \frac{P}{P_0} \right) \\
 C_C &= \frac{F_C}{v} = \frac{F_{A0}[\Theta_C + (c/a)X]}{v} = \frac{F_{A0}[\Theta_C + (c/a)X]}{v_0(1+\epsilon X)} \left( \frac{T_0}{T} \right) \frac{P}{P_0} = C_{A0} \left( \frac{\Theta_C + (c/a)X}{1+\epsilon X} \right) \frac{T_0}{T} \left( \frac{P}{P_0} \right) \\
 C_D &= \frac{F_D}{v} = \frac{F_{A0}[\Theta_D + (d/a)X]}{v} = \frac{F_{A0}[\Theta_D + (d/a)X]}{v_0(1+\epsilon X)} \left( \frac{T_0}{T} \right) \frac{P}{P_0} = C_{A0} \left( \frac{\Theta_D + (d/a)X}{1+\epsilon X} \right) \frac{T_0}{T} \left( \frac{P}{P_0} \right) \\
 C_I &= \frac{F_I}{v} = \frac{F_{A0}\Theta_I}{v} = \frac{F_{A0}\Theta_I}{v_0(1+\epsilon X)} \left( \frac{T_0}{T} \right) \frac{P}{P_0} = \frac{C_{A0}\Theta_I}{1+\epsilon X} \left( \frac{T_0}{T} \right) \frac{P}{P_0}
 \end{aligned}$$

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## Example



Show under what conditions and manipulation the expression for  $C_B$  for a gas flow system reduces to that given in Table 3-5.

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## Example Cont.



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## Example



A mixture of 28%  $\text{SO}_2$  and 72% air is charged to a flow reactor in which  $\text{SO}_2$  is oxidized.



First, set up a stoichiometric table using only the symbols (i.e.,  $\Theta_i$ ,  $F_i$ ) and then prepare a second stoichiometric table evaluating numerically as many symbols as possible for the case when the total pressure is 1485 kPa (14.7 atm) and the temperature is constant at 227°C.

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## Example Cont.



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## Example Cont.



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## Example Cont.



### Example

The reversible gas-phase decomposition of nitrogen tetroxide,  $\text{N}_2\text{O}_4$ , to nitrogen dioxide,  $\text{NO}_2$ ,



is to be carried out at constant temperature. The feed consists of pure  $\text{N}_2\text{O}_4$  at 340 K and 202.6 kPa (2 atm). The concentration equilibrium constant,  $K_C$ , at 340 K is  $0.1 \text{ mol/dm}^3$ .

- Calculate the equilibrium conversion of  $\text{N}_2\text{O}_4$  in a constant-volume batch reactor.
- Calculate the equilibrium conversion of  $\text{N}_2\text{O}_4$  in a flow reactor.
- Assuming the reaction is elementary, express the rate of reaction solely as a function of conversion for a flow system and for a batch system.
- Determine the CSTR volume necessary to achieve 80% of the equilibrium conversion.

## Example Cont.



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## Example Cont.



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