Engineering Economy

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Engineering Economy
(0901420)

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Slides 1

This week topics:

- "Engineering Economy", Sullivan, Wicks, & Koelling, 17th Edition, 2018.
- Introduction to Engineering Economy.
- Solutions to engineering problems.
- Fundamental principles of engineering economy
- Engineering economic analysis procedure
- Development of prospective outcomes.
- Electronic spreadsheets
- Examples

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Engineering Economy", Sullivan, Wicks, & Koelling, 17th Edition, 2018

- Chapter 5, will discuss the methods commonly used to analyze the economic consequences and profitability of an alternative.
- Chapter 6, covers these methods and their proper use in the comparison of alternatives.
- Chapter 7, the additional details required to accomplish engineering economy studies on an after-tax basis.

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Chapter 1

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

راح نحكي بشكل عام عن الاقتصاد الهندسي وشو الشغلات يلي راح نفعلها
وكيف الـ procdure يلي راح ابتعها

Introduction to Engineering Economy

The purpose of this class is to develop and illustrate the principles and methodology required to answer the basic economic question of any design.

Engineering Economy involves the systematic evaluation of the economic merits of proposed solutions to engineering problems.

كما يكون عندي أكثر من حل لاقتراح حل مشكلة كلهم بحقوي الـ criterion يلي بدي إياها، بفضل عندي أنا بدي آخذ الناحية الاقتصادية (to maximize my profit or minimze my loses)

* لنفرض أنا بشتغل بمصنع وبدي أشتري ماكينة كام ما أختار 3 ماكينات بأنواع مختلفة هيك أنا بكون وصلت لـ 3 حلول بعرف إني لو اخترت أي وحدة راح تقوم بالغرض يلي بدي إياه منها بس بفضل اني بدي اختار الماكينة يلي بتحملي أدنى إيرادات أو يلي ما بدها تكاليف صيانة عالية أو operating costs عالية.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

الحلول المقترحة لحل المشكلات الهندسية

Solutions to engineering problems must:

القواريي بدي آخذه لازم يكون ماخذ بعين الاعتبار انو الشركة بدها تعمل ربح وتكون قادرة تو في روابت آخر الشهر.

1. Promote the well-being and survival of an organization,

لازم تشتغل خارج الصندوق وتكون قادرين تواكبا التغيرات التكنولوجية السريعة يلي بتقيي بالهندسة يلي بشتغل فيها و أتقلم معها.

2. Embody creative and innovative technology and ideas,

3. Permit identification and scrutiny of their estimated outcomes, and

طالما يكون عندي حل لازم يكون عندي فكرة وأعرف شو النتائج يلي راح تنتج نتيجة هاد الحل

4. Translate profitability to the "bottom line" through a valid and acceptable measure of merit.

لازم أزيد الإيرادات بالتالي بزيد الربح أو أقلل التكاليف يلي بتدفعها الشركة وبرضو بزيد الربح.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

مع أعمل تصميم لبناء بدي أشوف ② أنو أحسن slab وأي واحد لا يؤثر على متانة البناء يلي عندي مثلا في كمية حديد أقل أو كمية باطون أقل عشان أزيد الأرباح أو أقلل الخسائر.

بعض المشكلات يلي بتواجهنا وبنحتاج نتخذ فيها اقتراحات ونحط حلول

Engineering economic analysis can play a role in many types of situations.

لفرن بشتغل بصنع وبحتاج كمهندس ميكانيك بدك تكون عارف شو درجة الحرارة يلي بدي إياها وأديش الفترة الزمنية يلي راح يعمل فيها شغال شو الـ gas furnace يلي بختاره متعلق الـ situation يلي موجودة بالصنع عشان أزيد النوابح وعلى هذا الأساس بدي أتخذ القرار.

gas furnace

1. Choosing the best design for a high-efficiency gas furnace.

2. Choosing the best design for the type of slab for a certain building.

3. Selecting the most suitable machine for a welding operation on an automotive assembly line.

مثلا بماكينة اللحام عندي أكثر من نوع أنو واحد بحتاج عمال أقل للتشغيل وتكون تكاليف الصيانة أقل.

4. Making a recommendation about whether jet airplanes for an overnight delivery service should be purchased or leased.

أنو أحسن يسترد طيارة ولا يستأجروها لنقل الـ delivers تلازم يكون محدد الفترة الزمنية يلي بحكي عنها (سنة، 5، ...)

5. Determining the optimal staffing plan for a computer help desk.

بدي اختار الشغلات يلي راح يشتغلو عندي على الـ help desk في الشركة يلي بشتغل منها بدي حسب مدة الشغل أشوف هل انو احسن أوظف عامل واحد شغلت مسائي وواحد صباحي ولا عاملين صباحي وعامل مسائي ولا العكس وهكذا...

عندي مشكلة هندسية بدي أخذ لها قرار شو لازم آخذ بعين الاعتبار.

There are seven fundamental principles of engineering economy:

- Develop the alternatives → أشوف شو الحلول أو الاختيارات الموجودة عندي وبي راح تحل المشكلة
- Focus on the differences → لازم يكون في اختلاف في الحلول
- Use a consistent viewpoint → على أي أساس بدي أخذ القرار هل بدي أدفع فلوس وكا أزيد الربح، بدي أوظف عمال زيادة وأدفع فلوس زيادة لهيك لما بدي أقارن بين viewpoint أكثر من حل لازم آخذ بعين الاعتبار وحدة وأثبتها وأقارن بين الحلول
- Use a common unit of measure → على أساس هاي الـ viewpoint بدي أحدد وحدة قياس
- Consider all relevant criteria
- Make uncertainty explicit → حاول قدر الإمكان اعرف شو راح يصير نتيجة هذا الكل شوال بدي outcome عندي بدي أشوف كل الأشياء المتشابهة الإيجابية.
- Revisit your decisions

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

شو الخطوات لي راح أتبعها عشان أتأكد انو عندي decision صحيحة

Engineering economic analysis procedure

1) Problem definition. → شو المشكلة وشو الحلول لي بدي إياها.
2) Development of alternatives. → عندي خيارات وبدي أحدد شو أفضل خيار.
3) Development of prospective outcomes. → بدي أشوف كل خيار شو راح ينتج منو
4) Selection of a decision criterion. → على أي أساس بدي أخذ القرار، هل عشان أزيد الربح وكا عشان أسهل حياتي وما عندي مشكلة لو دفعت أكثر فبدي أحدد الـ criterion وعلى أساسها آخذ القرار.
5) Analysis and comparison of alternatives. → بدي أقارن بين الحلول
6) Selection of the preferred alternative. → بدي اختار أحسن حل بحل مشاكلي.
7) Performance monitoring and post-evaluation of results. → كم بشوف النتائج، فبصير أعرف بالمستقبل لو الحطيت بنفس المشكلة شو القرار لي لازم اخذه لأنو بصير عندي خبرة لبعض المشاكل لي بتواجهني.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

عندي شركة بتصنع أثاث بدها تعمل على زيادة الربح عشان تحصل على قرض جديد وتشتري ماكينة مين. * اقترحت هاي الشركة عشان تزيد الربح انو التجارة الزيادة يلي كانت تستعملها لتدفئة المصنع بتبيعها لمصنع فحم.

Example:

A small furniture-manufacturing company wants to increase its profits to get a loan from the bank to purchase a more modern pattern-cutting machine. One proposed solution is to sell waste wood chips and shavings to a local charcoal manufacturer instead of using them to fuel space heaters for the company's office and factory areas.

مشكلة الشركة انها ما بتحصل على ربح مبنح عشان تحصل على القرض يلي بدها إياه.

➢ Define the company's problem.

The company's problem appears to be that revenues are not sufficiently covering costs.

اذا زادت الإيرادات وقلت التكاليف بالتالي بزيد الربح وباخذ القرض.

Several reformulations can be conducted:

1. The problem is there is a need to increase revenues while reducing costs.
2. The problem is to maintain revenues while reducing costs.
3. The problem is an accounting system that provides distorted cost information.
4. The problem could be that the new machine is not needed (and hence there is no need for a bank loan).

أو أحيانًا تكون معلومات غلط والمشكلة من المحاسب.

أحيانًا ما في داعي للماكينة الجديدة بالتالي ما في داعي لكل هاي الخيارات

* لهلق أنا بس أفكر بطريقة مادية كد :-

إذا بعت الخشب كم تكلفة إني أجيب آلة جديد لبدفي المصنع ، طلو التدفئة راح تكلفني أكثر معناها هاد ما بكون حل مبنح لأنو ما راح يحقق الهدف يلي بدي إياه.

Example (Cont.):

* الربح بزيد من خلال زيادة الإيرادات (الفلوس يلي بتدخل علي) وتقليل التكاليف (الفلوس يلي بعي أدفعها)

- An alternative is to sell wood chips and shavings as long as increased revenue exceeds the extra expenses that may be required to heat the buildings.
- Another alternative is to discontinue the manufacture of specialty items and concentrate on standardized, high-volume products.
- The alternative is to pool purchasing, accounting, engineering, and other white-collar support services with other small firms in the area by contracting with a local company involved in providing these services.

عندي شخصين بدهم يغيرو سياراتهم الـ criterion من هذا التغيير هو انهم يقللو كمية الغاز يلي راح يصرفوها خلال سنة راح يمشو 12000 ميل <=

Jerry → 10 mpg → 15 mpg

Linda → 25 mpg → 50 mpg (hybrid)

Example:

Linda and Jerry are faced with a car replacement opportunity where an interest rate can be ignored. Jerry's old clunker that averages 10 miles per gallon (mpg) of gasoline can be traded in for a vehicle that gets 15 mpg. Or, as an alternative, Linda's 25 mpg car can be traded in for a new hybrid vehicle that averages 50 mpg. If they drive both cars 12,000 miles per year and their goal is to minimize annual gas consumption, which car should be replaced—Jerry's or Linda's? They can only afford to upgrade one car at this time.

* بدنا نحسب الـ saving أكبر saving هو يلي راح نغير سيارته بالتالي راح نغير سيارة Jerry

Solution

Jerry's trade-in will save = (12,000 miles/year)/10 mpg − (12,000 miles/year)/15 mpg
= 1,200 gallons/year − 800 gallons/year = 400 gallons/year.

Linda's trade-in will save = (12,000 miles/year)/25 mpg − (12,000 miles/year)/50 mpg
= 480 gallons/year − 240 gallons/year = 240 gallons/year.

← القرار هون تم اتخاذه على أساس مين راح يقلل استهلاك الغاز لهيك أنا بدور على saving أكبر وهو يلي راح أختاره.

Therefore, Jerry should trade in his vehicle to save more gasoline.

الـ outcomes بتلعب دور كبير بكيف بدي أختار الـ decision الصح (وهذا يعتمد على أولويات الشركة)

Development of prospective outcomes:

In addition to the economic aspects of decision-making, *some factors often play* a significant role in the final recommendation.

راح نحكي أهداف ثانية للشركة غير زيادة الربح وتقليل الخسائر.

Examples of objectives other than profit maximization or cost minimization that can be important to an organization include the following:

ماعندهم مشكلة لو زادو المصاريف لهيك ضروري أحدد الـ objective يلي على أساسه راح أتخذ القرار

1. Meeting or exceeding customer expectations;
2. Safety for employees and the public;
3. Improving employee satisfaction;
4. Maintaining production flexibility to meet changing demands;
5. Meeting or exceeding all environmental requirements;
6. Achieving good public relations or being an exemplary member of the Community.

بدي آخذ بعين الاعتبار سلامة العمال والناس لهيك راح يكون في تكاليف زيادة عشان راح أعمل أشياء للحماية سواء حماية الناس أو العمال فأنا هيك بحمي حالي على المدى الطويل

اذا أربح تكاليف زيادة.

* بدنا نوخذ كل الأشياء يلي ممكن تؤثر في اتخاذ القرار بعين الاعتبار.

عنا شخص اشترى مبنى فيه شقق تكلفته 100 000 دولار هو دفع 10 000 دولار من البنك عشان يغطي الـ 90000 يلي حلو، كل سنة هو بدفع للبنك 10500 دولار (التكاليف يلي بدفعها) وهو مقدر انو تقويتا راح يحط كل سنة 15000 دولار على صيانة المبنى، عندو شقق كل شقة بقدر يؤجرها بـ 360 دولار كل شهر (الإيرادات)

Economic analysis example

Someone bought a small apartment building for $100,000. He spent $10,000 of his own money on the building and obtained a mortgage from a local bank for the remaining $90,000.

The annual mortgage payment to the bank is $10,500. He also estimates that annual maintenance on the building and grounds will be $15,000.

There are four apartments in the building that can each be rented for $360 per month.

- Problem? *Yes*
  - التكاليف ← *money spent ($10,500 + $15,000 = $25,500) every year* ← كل سنة قاعد بدفع 25500 دولار
  - الإيرادات ← *revenue ($360×4×12 = $17,280) every year* ← بدخل عليه 17280
  - *→ $8,220 loss per year.* ← عندو 8220 دولار بضيعو كل سنة وهاي هي المشكلة

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

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* حلول مقترحة على المشكلة :-

① ممكن يرفع الإيجار بالتالي بتزيد الإيرادات وبقدر يغطي التكاليف يلي عندو.
② بيع المبنى (لأنو بجيب وجع راس كتير)
③ يقلل تكلفة الصيانة - لأنو ما بقدر يقلل من المبلغ يلي راح يدفعه للبنك لأنو مغصوب عليه
④ احكي للبنك بدبش أدفع حلو علي كتير.

Economic analysis example (Cont.)

- Development of alternatives: حلول مقترحة
  1. ➢ *Raise the rent.*
  2. ➢ *Decrease maintenance cost.*
  3. ➢ *Sell the building.*
  4. ➢ *Abandon the building.*

* كل حل من هدول الو جانب سلبي لكمان.

money spent ($10,500 + $15,000 = $25,500)
revenue ($360×4×12 = $17,280)
→ $8,220 loss per year.

- Development of prospective outcomes:

Option 1: raise the rent so the net balance is zero. ⇝ أديش المبلغ يلي لازم أزيده على الإيجار

$8,220/4 Apts./ 12 months = $171.25 increase per apartment per month (48% increase).

Option 2: lower monthly expenses.

$10,500 + X = $17,280 → X = $6,780 per year (maintenance) → $565 per month.

Option 3: Selling the building.

Option 4: Abandoning the building.

* كل هدول الـ option منأنا بحدد لحالي على criterion بيي أخذ القرار

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

* الهدف من هاي المحاضرة أعرف كيف أنفذ بطريقة مادية عشان أحل المشكلات الهندسية

Economic analysis example (Cont.)

- Selection of a decision criterion?

  Minimization of losses. ← بدي أقلل الخسائر
  يعني بدي أقلل التكاليف

- Analysis and comparison of alternatives?

  Based on the selected criterion.

  Option 1: raise rent ($171.25 increase per apartment per month (48% increase)). ← بزيد الإيرادات أو أقلل التكاليف زي تكاليف الصيانة.

  Option 2: lower monthly expenses ($565 per month).

- Selection of the preferred alternative?

  Select the best achievable option.

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Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Electronic spreadsheets

ليش بنفضل نستعمل الـ spreadsheets بالحل

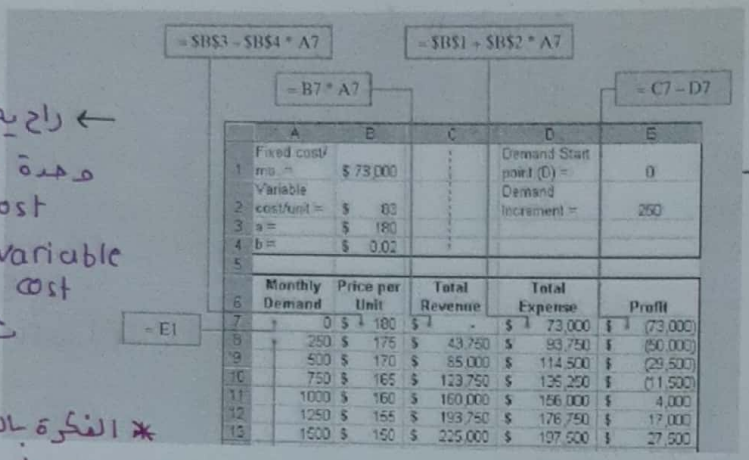
Most engineering economy problems are amenable to spreadsheet solutions for the following reasons:

1. They consist of structured, repetitive calculations that can be expressed as formulas that rely on a few functional relationships. ← ما بنلاقش حتى معادلات وقوانين بدل ما أحسبها الآلة الحاسبة الإكسل بوفر عندي الـ functions اللي بحتاجها.
2. The parameters of the problem are subject to change. ← 2- عندي كثير parameters راح تتغير ونتيجة هذا التغير أنا بقدر يكون عندي 2 cells محتوية على كل هاي المتغيرات
3. The results and the underlying calculations must be documented. ← 3- بعد ما خلص الحل، لازم يكون حدا القدرة على تفصيل النتائج كاملة (بالإكسل بيكون عملية save وكل اشي موجود)
4. Graphical output is often required, as well as control over the format of the graphs. ← الإكسل بطلع عندي charts بدل ما أحاول أرسم.

Which can make it a powerful addition to the analysis arsenal.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Spreadsheets



	A	B	C	D	E
1	Fixed cost/ mo. =	$ 73,000		Demand Start point (D) =	0
2	Variable cost/unit =	$ 80		Demand Increment =	250
3	a =	$ 180			
4	b =	$ 0.02			
5					
6	Monthly Demand	Price per Unit	Total Revenue	Total Expense	Profit
7	0	$ 180	$ -	$ 73,000	$ (73,000)
8	250	$ 175	$ 43,750	$ 93,750	$ (50,000)
9	500	$ 170	$ 85,000	$ 114,500	$ (29,500)
10	750	$ 165	$ 123,750	$ 135,250	$ (11,500)
11	1000	$ 160	$ 160,000	$ 156,000	$ 4,000
12	1250	$ 155	$ 193,750	$ 176,750	$ 17,000
13	1500	$ 150	$ 225,000	$ 197,500	$ 27,500

← راح يكون عنا معادلة وحدة فيها الـ Fixed cost ثابتة والـ variable cost ثابتة وعشان B و A ثوابت

→ price demand relation ship (learning curve)

* الفكرة بالـ spreadsheets انو بقدر أغير 2cells فبتتغير كل الـ output الموجودة عشان مين alternative أو مشروع

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

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In summary

techniques نهاية هاي المادة رح نتعلم شوية بساعدنا لحل المشاكل الهندسية اللي ممكن نواجهها in order to make decision بطريقة شوي اقتصادية

An engineering economy is a collection of problem-solving tools and techniques that are applied to engineering, business, and environmental issues.

Common, yet often complex, problems involving money are easier to understand and solve when you have a good grasp on the engineering economy approach to problem-solving and decision-making.

The problem-solving in this class will enable you to master the theoretical and applied principles of engineering economy.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

ما أبلش أحكي عن الفلوس لازم أتطلع عليها From engineering economy prospective in order to make a correct decision

Engineering Economy
(0901420)

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Slides 2

Chapter 2: Cost Concepts and Design Economics

بهاد الشابتر راح نحكي عن الـ concepts
لـ cost ، وبعدين راح نحكي شوي صغيرة
عن الـ price
demand relationship

Objective:

Analyze short term-alternatives when the time value of money is not a factor.

Topics to be covered this week:

1. Cost categories
2. Cost terminologies
3. General price-demand relationship
4. Present economy studies

بدنا نحكي عن التكاليف وأنواع التكاليف
مسميات للتكاليف

* راح نتطلع على الأشكال الهندسية الموجودة عندنا من منظور اقتصادي .

① fixed cost ← عبارة عن تكاليف راح أدفعها بكل الأحوال سواء أنا شغال أو لا راح أدفعها ، مثلاً <= إذا بدي أعمل مصلحة جديدة رح بدي أستأجر المحل الأجار يلي راح أدفعو هو fixed cost سواء المحل باع أو لا سواء ربحت أو خسرت عندي هذا الأجار بدي أدفعه آخر الشهر (ثابتة مالها علاقة بال operating conditions.

Cost Categories

أنواع التكاليف يلي بدفعها بأي مشروع

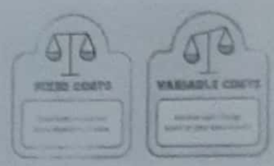
① • **Fixed costs:** costs unaffected by changes in a specific range of operating conditions.

Examples: insurance on facilities, taxes on facilities, general management and administrative salaries, license fees, and interest costs on borrowed capital. → مثال ثاني

لو بدي أأمن على المصنع يلي بشتغل فيه راح أدفع ضريبة برضو هاي fixed cost

② • **Variable costs:** costs that vary with the quantity of output.

Examples: materials and labor costs.



Retrieved from: https://difftz.com/fixed-cost-vs-variable-cost/

③ • **Incremental cost:** additional cost resulting from increasing the output of a system by one or more units.

Examples: production of 100 vs. 120 units.

③ التكاليف الإضافية يلي بدفعها عشان أنتج قطع زيادة ، يعني لو بدي أرجع لمصنع الأثاث وبدي استغل 100 كرسي دبعرف انو تكلفتهم حوالي 800 دينار داجاني زبون بدو كمان كرسيين مع انو الـ production capacity يلي بقدر أعملها خلال الشهر بتغطي 100 كرسي ، وبدي أعمل للزبون الكرسي يعني الـ production صار 102 ، التكلفة لي تكلفتها عشان أعمل الكرسيين الزيادة هاي incremental cost

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

② variable cost ← بتعتمد على الكمية يلي بدي أشغلها ، مثلاً <= عندي مصنع بعمل أثاث الـ variable cost يلي هي كمية الخشب يلي بدو يشتريها ديدخلها على المصنع بتعتمد بشكل كبير على الـ Final product يلي راح يطلعها ، بدي مثلاً اطلع 100 كرسي راح أدخل كمية خشب غير عن لو بدي أطلع 1000 كرسي وهكذا .

Cost Categories (Cont.)

* الفرق بين الـ salary والـ wages :-

salary ← راتب متفق فيه مع الموظف دراح أدفعه بنهاية الشهر في جميع الحالات (fixed cost)

wages ← بنحكيها للعمالة شو ما اشتغل راح أعطيه ، اشتغل ساعة بعطيه للساعة اشتغل 10 بعطيه لـ 10 وهكذا ، بتغير بتغير الساعات يلي راح يشتغلها العامل (variable cost)

* Classify each of the following cost items into fixed or variable costs:

- Raw materials → المواد يلي بدي أجيبها دراح تختلف باختلاف الإنتاج)
- Direct labor wages
- Supplies → الاحتياجات يلي بجيبها للمشروع عشان أقدر أقوم بالمشاريع
- Property taxes — ضريبة الأرض يلي بشتغل عليها
- Utilities (i.e. electricity bill)
- Administrative salaries
- Sales commission
- Rent — أجار — العمالات يلي بعطيها تبعتي ، مثلاً اذا بدي اشتري 3 أو 4
- Shipping charges — نتيجة شغلهم

الخدمات — ماكينات كام حسب الـ production line & ماكينة بدي أوردها للمصنع .

*Utilities → زي فاتورة الكهرباء ممكن أعتبرها fixed وممكن variable بتعتمد على حسب السيناريو لي عنا

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

shipping charges → بدي شحن شي للمصنع بدي أبعث منتجات معينة product

* مثال ثاني ع :-

لنفرض بشتغل بمصنع فيه ماكينة لحام within the production line السبب لي بشتغل على تشغيل الماكينة التكلفة بتعتبر راح تكون direct على الـ production line الموجود عنا متكاليف التشغيل لي بدفعها للعامل بتكون direct cost ، بالمصنع كامل عنا فريق للصيانة بعمل على صيانة الماكينات المختلفة في المصنع لهيك ما أعطيه فلوسه ما بقدر أحطها على production line وحدة لأنو معلياً بشتغل عليهم كلهم ، فالتكلفة لي بدفعها الو جزء منها راح يكون indirect على الـ production line للماكينة لي حكينا عنها

Cost Categories (Cont.)

Classify each of the following cost items into fixed or variable costs:

Raw materials	Variable
Direct labor wages	Variable
Supplies	Variable
Property taxes	Fixed
Utilities (electricity bill) ← أي خدمة من الحكومة بدفع عليها فاتورة .	Fixed and Variable
Administrative salaries	Fixed
Sales commission	Variable
Rent	Fixed
Shipping charges	Variable

* مثال :-

لنفرض إني بشتغل بشركة استشارات اهكابت بتعتها بعمان وفي عندهم 4 مشاريع موزعات في عمان ، أنا كمهندس إذا كنت شغال على اول مشروع تكلفتي بتكون direct cost على مشروع 1 ، وزميلي لي اشتغل على مشروع 3 تكلفته ورواتبه بتكون direct cost لمشروع 3 ، راح يكون عنا مدير المشاريع التكاليف بتعتبر راح تكون موزعة على كل المشاريع لأنو هو لي أدارها فمابقدر أحكي انو تكلفة على مشروع 1 أو 2 لهيك جزء من تكاليفه بقدر أحطها على مشروع (1) كـ indirect cost ،

Other Categories of Cost

تكاليف مباشرة (تكلفة بقدر أحطها على activity معينة .

④ • Direct costs (directly measured and allocated to a specific outcome or work activity).

Examples: labor and material costs associated with certain construction activities are direct costs for that activity.

⑤ • Indirect costs (overhead or burden): difficult to allocate to a specific output or work activity. A specific formula can be used (proportions).

Examples: plant operating costs, common tools, general supplies, and general maintenance.

⑥ • Standard costs (established ahead of production or service delivery): anticipated labor and material costs + overhead cost per unit.

Useful for bidding, cost estimation, comparison, and evaluation.

عبارة عن التكاليف المباشرة وغير المباشرة لي أنا لازم آخذها بعين الاعتبار لـ activity معينة وبالعادة بستعملها

مثل الـ activity عشان أقدر التكاليف أدقيق راح تكون .

*مثال :-
لو أنا بشركة مقاولات وبدي أسعّر مشروع بحسب التكاليف يلي أنا راح أدفعها على هذا المشروع، بما انو قادر احسبها اذا هاي التكاليف عبارة عن direct cost وبنفس الوقت انا عندي المكتب بتع الشركة راح أدفع منو ورق المشروع فواتير أفتار مهندسين بساعدوني في عملية التصميم عبارة عن indirect cost باخذها بعين الاعتبار.

Other Categories of Cost

بقدر استعملها عشان ساعدني في اتخاذ القرار.

- Standard costs (established ahead of production or service delivery): anticipated labor and material costs + overhead cost per unit).

اذا بدي أهدم عظام أو بدي أحفر تقويبًا كم التكاليف راح تكون باخذ نسبة لـ indirect وبقدر أطلع التكاليف.

➢ Some typical uses of standard costs are:

1. Estimating future manufacturing costs;
2. Measuring operating performance by comparing actual cost per unit with the standard unit cost;
3. Preparing bids on products or services requested by customers;
4. Establishing the value of work in process and finished inventories.

قبل ما نشغل أي مشروع بكون هنا بقدر كم التكاليف راح تكون، بعد ما أعمل واشتغل على المشروع بقدر أحسب التكاليف يلي بدفعها وأقارن مابين التكاليف الـ actual يلي بدفعها وبين الـ standard يلي كنت متوقعها أو حاسبها.

*مثال على الـ (sunk cost) :-
مثلًا بدي اشتري سيارة، شغلت سيارة شغالة على الغاز وبنزين فاز وبنزين حكيت لصاحب السيارة اناراح أشتريها وأعملك عطاء (بقولو) 100 دينار، وأنا مروح شغلت سيارة hybred بتشتغل على الغاز بدها شي غيار زيت وصيانة فلما آخذ القرار وأختار اذا راح اشتري الـ hybred

*مثال :-
أنا اشتريت سيارة للمشروع بتكلفة السيارة اشتريت السيارة هاد عبارة عن cash cost بين ما استهلك السيارة وأستعملها خلال سنة من بداية المشروع أنا فعليًا صار عندي تكاليف استهلاك للسيارة، سواء السيارة بعد الاستهلاك بسعيها Book cost لأنو فعليًا هاي موجودة بس لو بدي ابيعها راح تكون تكلفتها أقل من ما اشتريتها

Cost Terminology

ما باخذ بعين الاعتبار الـ 100 يلي دفعتها لأول سيارة، بقارن بين السيارتين وعلى أساسها بقرر، فأنا لو اشتريت الـ hybred الـ 100 دينار بكون خسرتها opportunity cost s

- Cash cost: involves payment of cash and results in cash flow. → الفلوس يلي بتطلع من جيبتي وبودعها بالبنك أو بعطيها لـ supplier اذا في cash cost بسميها cash involvo

Example: estimates for the cost of travel, labor, material, etc.

- Book cost: does not involve a cash transaction and is normally reflected as a noncash cost. → تكاليف انا بحسبها بالـ accounting record الموجودة عندي عشان أحملها بعدين على المشروع

Example: depreciation due to the use of assets such as equipment (not a cash flow).

- Sunk cost: payment occurred in the past with no relevance to the future cost and revenue estimates (not a part of future cash flows and is typically disregarded in engineering economy problems).

Example: non-refundable down payment on a car.

تكاليف مفكك أدفعها في الماضي وما بتأثر على اختياراتي للقرار.

* مثال =>
أنا لما كنت توجهي اجيتا بدي أبلش جامعة كان عندي خيارين اما أبلش جامعة وأدرس وأدفع تكاليف الجامعة أو اشتغل سنة وأوفر شوية فلوس اذا اخترت اول خيار، فالفلوس اللي كنت ممكن أحصلها من الخيار الثاني هاي عبارة عن opportunity cost أنا خسرتها لأني ما اخذت هذا القرار.

Cost Terminology (Cont.)

opportunity cost ممكن يكون عندي أكثر من خيار، ال هو أحسن خيار أنا ما اخذته.

- **Opportunity costs:** monetary advantage foregone due to limited resources or the cost of the best-rejected opportunity.

  Example: working and getting paid for one year or going to college and paying tuition.

- **Life-cycle cost:** summation of all costs related to a product, a system, a structure, or a service during its lifespan.
  - Acquisition phase (need, design, alternatives) ... greatest potential for savings occurs here.
  - Operation phase (production or delivery until product/service is retired or disposed of).

  Example: buying a modern hybrid car vs. an old SUV.

عدد التكاليف اللي بتكون موجودة لحياة ال alternative أو ال project وأنا بسعقلها لأخذ قرار

طالبي اخذ قرار أخا سيارة بدي اشتري لازم أحط مثلاً كل التكاليف اللي ممكن أدفعها خلال 10 سنين لقدام هاي التكاليف بسميها life cycle cost

أنا بشتغل بشركة وشريت ملكينة بـ 50 000 دولار حالياً باد company records لو بدي أبيعها سعرها راح يكون 20000 (هاي تمثل Book cost) بس لما سألت بالسوق عن سعرها ال market value كانت 5000 ، اذا الشركة قررت تبيع الماكينة أديش ال opportunity cost ؟! راح تكون 5000 لأني ما بعت

Cost Terminology (Cont.)

- **Opportunity costs example:** your firm considered the replacement of an existing piece of equipment that originally cost $50,000, and is presently shown on the company records with a value of $20,000 but has a present market value of only $5,000. For purposes of an engineering economic analysis of whether to replace the equipment or keep it, what is the opportunity cost if the firm decided to keep the equipment?

The $5,000 immediate selling price is the investment cost of not replacing the equipment and is based on the opportunity cost concept. Where the firm is giving up the opportunity to obtain $5,000 from its disposal.

* لنفرض انو اخترت أبيع الماكينة كم راح تكون ال opportunity cost :-

15000 دولار

Cost Terminology (Cont.)

- Life-cycle cost example:

كل التكاليف يلي عنا من بداية المشروع لنهاية الـ operation phase

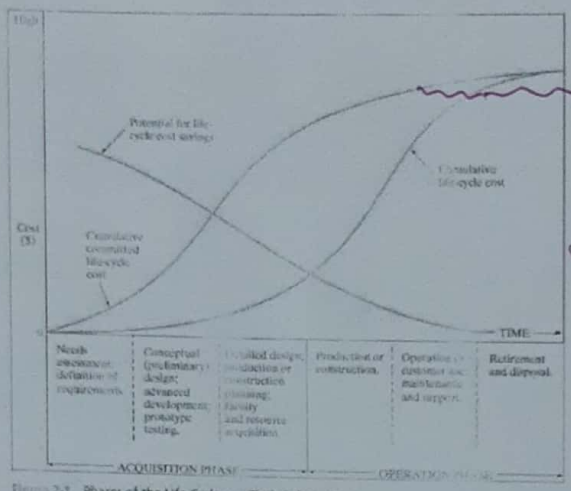


Figure 2-1 Phases of the Life Cycle and Their Relative Cost

Base or s curve

لأنو من بداية المشروع ما بيكون معي كثر معاري أصرفها كما بتبلش الانتاج بتبلش التكاليف تزيد وبنهاية المشروع التكاليف بترجع stable.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

* عندي مقاول بدو يعمل زفتة لطريق سريع ، المقاول عندو اختيارين بحط فيهم الماكينة يلي بدو يخلط فيها الزفتة .
* الخيارات إما بدو يختار موقع A أو موقع B .
* بدي أتخذ القرار بناءً على الموقع يلي راح يكون أقل.

Example: highway paving

A new highway is to be paved and the contractor has two locations to set up their asphalt mixing equipment. The job requires 50,000 yd^3 of asphaltic material and the project duration is estimated to be 4 months (17 weeks of 5 working days). Which option is better?

بدي أشوف الاختلافات بين الموقعين

Cost Factor	Site A	Site B
Average hauling distance	4 miles	3 miles
Monthly rental of site	$2,000	$7,000
Cost to set up and remove equipment	$15,000	$50,000
Hauling expense	$2.75/$yd^3$-mile	$2.75/$yd^3$-mile
Flagperson	Not required	$150/day

البعد
الإيجار الشهري
كلفة التركيب وإزالة
كلفة النقل (نقل الأسفلت من الكسارة للشارع).

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Example: highway paving (Cont.)

حسبنا الـ Fixed والـ variable cost بالمدة يلي راح بيشتغل فيها (4 mounth)

Cost	Fixed	Variable	Site A	Site B
Rent	✓		= $8,000	= $28,000
Setup/removal	✓		= 15,000	= 50,000
Flagperson	✓		= 0	5(17)($150) = 12,750
Hauling		✓	4(50,000)($2.75) = 550,000	3(50,000)($2.75) = 412,500
			Total: $573,000	$503,250

=> Site B is better.

B هو الموقع الأفضل عشان تكاليفه أقل

- Assume the contractor is paid $12/yd³ for asphalt delivered to the site and assume the cost of material is $1.5/yd³. At what point does he break even and begin to make a profit?

لـ بعد ما المقاول يبيع كم متر مكعب بنى الـ 50 000 يكون فعلي التكاليف يلي دفعها وبلش يعمل ربح .

break even → يتحقق لما تكون التكاليف يلي أنا دافعها تساوي الإيرادات والفلوس يلي دخلت علي .

Example: highway paving (Cont.)

- Assume the contractor is paid $12/yd³ for asphalt delivered to the site and
- assume the cost of material is $1.5/yd³. At what point does he break even and begin to make a profit?

Break-even means: Total expenses= Total revenue

(Rent + setup/removal + Flag person)+ Hauling + Material = Asphalt revenue

$$\$90{,}750 + [\$2.75 \times 3 \times X] + [\$1.5 \times X] = \$12 \times X$$

$$90{,}750 + 8.25X + 1.5X = 12X$$

$$\$90{,}750 + 9.75x = 12X$$

$X = 40{,}333\ yd^3$ ← بعد ما المقاول يورد للموقع

هيول التكاليف

كم متر مكعب أنا وردت للموقع

الإيرادات

من أصل 50 000 يكون 40333

حقق التكاليف بتعته وبعد هيك بعير بيلش يعمل ربح

هل في علاقة تربط بين ال price
(سعر ال product) يلي بقدمه مع ال
product يلي هو الطلب على ال demand
أو الخدمة يلي بقدمها.

General price-demand relationship

$$p = a - b \times D$$

↳ linear relation ship

Where:

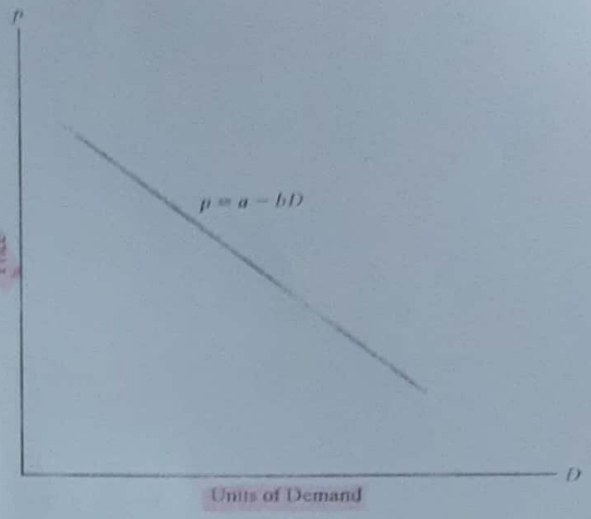
p = price

D = demand

a and b = constants that depend on the product or service.

$0 < D < a/b \quad a > 0; b > 0$

price كلما قل ال D كلما زاد ال
الاعتماد على ال production
rate يلي أنا بشتغل فيها



* مثال على سيناريو 2 :-
بتغير من بشتغل بصنع بغض النظر عن كمية البيع والزبائن يلي موجودين فنا أنا
ال product بتبقى راح يكون له Fixed price وهذا راح يفضل ثابت على
ال product طالما عملية الانتاج .

Total revenue and breakeven point

عند القطع يلي بيعها راح يعمل
تأثير على ال over all price of the product

Total revenue (TR) from selling a product or a service is:

1. Scenario 1: Demand is a function of price.
2. Scenario 2: Demand and price are independent of each other.

$TR = price \times demand = p \times D$ or

↳ الإيرادات

$$TR = (a - b \times D) \times D = aD - bD^2$$

Total costs (C_T) = Fixed costs (C_F) + Variable costs (C_V)

↳ التكاليف
يلي راح أدفعها

Assuming a linear relationship between variable costs and demand,

$C_V = cv \times D$, where c_v is the variable cost per unit demand.

$$C_T = C_F + c_V \times D$$

صار عندي معادلة لل TC وال TR بقدر منها أحسب الربح
↳ total cost

* حكينا ببداية حياة المشروع عندي Fixed cost بدي أدفعها بغض النظر أنا كم بيع وال variable cost راح تقل تزيد بالتالي ال total cost راح تزيد وال total revenue راح يمثلوني هذا ال curve

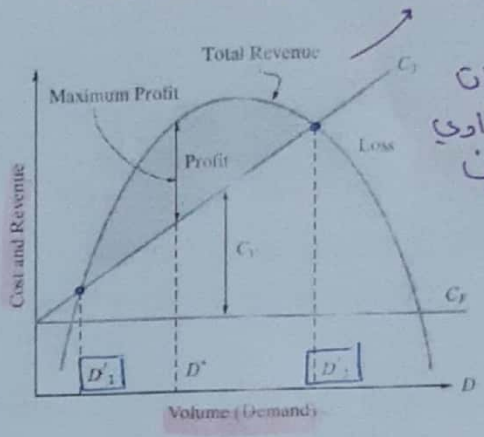
Profit – scenario 1

راح يكون عندي نقطتين أحقق فيهم ال (Breakeven) يعني الإيرادات راح تساوي التكاليف

Profit = Total revenue – Total costs (الإيرادات – التكاليف)

$$Profit = (aD - bD^2) - (C_F + c_V D)$$

$$Profit = -bD^2 + (a - c_V)D - C_F$$



To maximize profit,

$$\frac{d(Profit)}{dD} = a - c_v - 2bD = 0$$ ← اشتقيت المعادلة وساويتها بالصفر

Optimal $D \rightarrow D^* = \frac{a - c_v}{2b}$

Two breakeven points (profit = 0) are found by using the Quadratic Formula : Total revenue = Total cost

$$D' = \frac{-(a - c_v) \pm \sqrt{(a - c_v)^2 - 4(-b)(-C_F)}}{2(-b)}$$

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

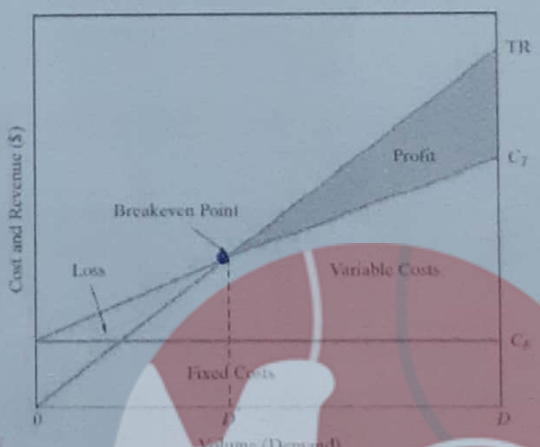
* range of profability ال من D_1 لـ D_2 ، يعني أنا بين ال D_1 وال D_2 بكون فعلياً بعمل profit حتى هيك اتأكد لأنو التكاليف عالية وبعد هيك برضو التكاليف أعلى من الإيرادات .

* عشان أحقق المنفعة القصوى بدي أشتق ال profit عشان أحصل على (optimal D) يلي هو النقطة يلي راح أحقق فيها ال max profit

* أنا كمهندس بشتغل بمصنع لازم أكون عارف شو ال Range يلي بحقق منيه الربح ، دائماً بفضل ال production line تحتي ال product تساوي ال optimal D عشان أحقق أعلى ربح .

Profit – scenario 2

Price per unit (p) and demand (D) are independent of each other. → ما في علاقة بين ال price وال demond ال price راح يكون ثابت كلما بعد كلما زادت الإيرادات



Profit = Total revenue – Total costs

$$Profit = p\,D - (C_F + c_V D)$$

- ☐ Only one breakeven point.
- ☐ No optimal demand.

* لهيك بالأسئلة اذا ال price ثابت بكون بسيناريو 2

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Example

A company produces an electronic timing switch that is used in consumer and commercial products. The fixed cost (C_F) is \$73,000 per month, and the variable cost (c_v) is \$83 per unit. The selling price per unit is $p = \$180 - 0.02(D)$

1) Determine the optimal volume for this product and confirm that a profit occurs (instead of a loss) at this demand.

2) Find the volumes at which breakeven occurs; that is, what is the range of profitable demand?

(1)

$$D^* = \frac{a - c_v}{2b} = \frac{\$180 - \$83}{2 * 0.02} = 2{,}425 \text{ units per month (maximum profit).}$$

Or write down the equation of profit, derive, and equate to zero.

$$P = 180\,D - 0.02\,D^2 - (73{,}000 + 83\,D) = -0.02\,D^2 + 97\,D - 73{,}000$$

For a profit to occur, the 2nd derivative should be negative (-0.04).

Also, substitute the optimal demand (D^*) in the profit equation:

Profit= [\$180 × 2,425 – 0.02 × 2,425^2] – [\$73,000 + \$83 × 2,425] = \$44,612 (+ve >>> profit).

(2) $a = 180$ $b = 0.02$ $c_v = 83$ $C_F = 73000$

$$D' = \frac{-(180 - 83) \pm \sqrt{(180 - 83)^2 - 4(-0.02)(-73{,}000)}}{2(-0.02)}$$

$D'1 = 932$ units and $D'2 = 3{,}918$ units.

*بما انو بالسؤال ما أعطاني علاقة بين D و P بالتالي أنا سيناريو 2 (اذ P ثابت)

عنا شركة مقاولات (استشارات مثلاً) بيحسبو الربح بتقلهم حسب كم ساعة خدمة بقدمو

Example

An engineering consulting firm measures its output in a standard service hour unit. The variable cost (c_v) is \$62 per standard service hour and the charge-out rate [i.e., selling price (p)] is \$85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost (C_F) is \$2,024,000 per year. What is the breakeven point in standard service hours and in the percentage of total capacity?

اعطلوب بعدد كم ساعة خدماتية الشركة راح تزبح الفلوس يلي دفعتها مابين CV واذ CP وغيرها وهاي الساعات مش بنسبتها من اذ 160000 ساعة يلي بتقدرو يقدموها بالسنة.

أكبر عدد ساعات ممكن تشتغلها

$$\text{Total revenue} = \text{Total costs}$$

$$D' \times \$85.56/h = \$2,024,000 + \$62 \times D'$$

$D' = 85,908\ h.$ ← هيك الشركة بتكون غطت التكاليف وبعدها راح بتلش تعمل ربح

$$\%\ of\ total\ capacity = \frac{85,908}{160,000} \times 100\% = 54\%$$

هاي الشركة تقريبًا تشتغل نص اذ تبكون حققت اذ break even وبعد هيك راح تبلش تعمل profit

*مثلاً => لوبدي أقارن بين ماكينتين لحام وبدي أشوف تكاليف الماكينة الأولى والثانية القرار راح يكون على الماكينة يلي راح تكلفني أقل، أما إذا بعرف اذ production كم وبعرف سعر البيع بقدر أحسب اذ profit فالقرار بكون للماكينة يلي بتحقق أكبر ربح.

*يعني الخلاصة القرار دايمًا حسب مين بعطيني max profit or min loses

Present Economy Studies

Duration Less than one year: time influence on money is ignored (present economy).

زي السؤالين يلي قبل أنا بعرف الإيرادات يلي بتفوت على وبقدر أحدد الربح فالخيار راح يكون حسب مين بعطيني أكبر ربح

Comparing multiple alternatives:

(1) For variable known revenue and benefits, select the alternative with maximum profit.

(2) For constant or unknown revenue and benefits, select the alternative with a minimum total cost per defect-free product or service.

راح أختار أقل يلي بتقلل الخسائر والتكاليف

مثلاً لو بعمل كراسي وواحد الو 3 رجلين هاد مابكون صفر الإيرادات لأني ماراح أبيعه بالتالي بشغله صفر التكاليف

10000 قطعة

يعني لو اكصنع اشتغل
راح يبيعهم كلهم

Example

The demand for a certain part is 100,000 units. The part is produced on a high-speed turret lathe, using screw-machine steel costing $0.30 per pound. A study was conducted to determine whether it might be cheaper to use brass screw stock, costing $1.40 per pound. Because the weight of steel required per piece was 0.0353 pounds and that of brass was 0.0384 pounds, the material cost per piece was $0.0106 for steel and $0.0538 for brass. However, when the manufacturing engineering department was consulted, it was found that, although 57.1 defect-free parts per hour were being produced by using steel, the output would be 102.9 defect-free parts per hour if brass were used. Assuming the machine attendant is paid **$15.00 per hour**, and the variable (i.e., traceable) overhead costs for the turret lathe are estimated to be $**10.00 per hour**. Which material should be used for this part?

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Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

بـ كل معلومات هاد
السؤال عبارة عن التكاليف
فأنا بدي أشوف تكلفة
القطعة الواحدة بالساعة كم
راح تكون للعاديتن

✳Unknown or constant revenue (demand is constant) → compare the cost per defect-free unit

	Steel	Brass
Material	$0.30 × 0.0353 = $0.0106	$1.40 × 0.0384 = $0.0538
Labor	$15.00/57.1 = 0.2627	$15.00/102.9 = 0.1458
Variable overhead	$10.00/57.1 = 0.1751	$10.00/102.9 = 0.0972
Total cost per piece	$0.4484	$0.2968

Saving per piece by use of brass = $0.4484 − $0.2968 = $0.1516

→ Select brass

£ راح نختار الـ
لأنها كإنتاجية
steel الـ brass
كانت أفضل من
وقللت التكاليف

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

راح أخذ القرار بناءً على الـ

Example

كم قطعة بعمل بالساعة production capacity
راح أشوف عدد القطع اللي بقدر أبيعها وأشغل منها اللي ما راح تتباع.

Two machines with approximately the same capital investment are being considered to produce a part. The important differences between the machines are their production capacities (production rate × available production hours) and their reject rates (percentage of parts produced that cannot be sold). Consider the following table:

كم ساعة راح اشتغل
100 كل ساعة بنتج منها
A لـ
قطعة 3 منهم ما راح أشتغلهم.

	Machine A	Machine B
Production rate	100 parts/hour	130 parts/hour
Hours available for production	7 hours/day	6 hours/day
Percent parts rejected	3%	10%

كل قطعة ما منها عيوب
بقدر أبيعها بـ 12 دولار

تكلفة القطعة الوحدة

The material cost is $6.00 per part, and all defect-free parts produced can be sold for **$12 each** (rejected parts have negligible scrap value). For either machine, the operating cost is $15.00 per hour and the variable overhead rate for traceable costs is $5.00 per hour. → indirect cost

تكلفة التشغيل

Assume that the daily demand for this part is large enough that all defect-free parts can be sold. Which machine should be selected?

لـ بدنا نعرف الدخل
القطع اللي راح أنتجها وتكون سليمة راح أبيعها بـ 12 دولار

عشان إيرادات جندي أشوف اد
machine لكل profit واختار اللي بتحقق لي
max profit

✳ Variable total revenue → Rule #1 → Profit maximization

Profit per day = Total revenue per day – Total costs per day

$$= \left[\text{production rate} \times \text{production hours} \times \frac{\$12}{\text{part}} \times \left(1 - \frac{\text{rejected\%}}{100}\right)\right] - \left[\text{production rate} \times \text{production hours} \times \frac{\$6}{\text{part}}\right] - \left[\text{production hours} \times \left(\frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}}\right)\right]$$

↳ revenue

↳ cost
اللي راح اتكلفها

For machine A:

$$\text{Profit per day} = \left[\frac{100\text{ parts}}{\text{hour}} \times \frac{7\text{ hours}}{\text{day}} \times \frac{\$12}{\text{part}} \times (1 - 0.03)\right] - \left[\frac{100\text{ parts}}{\text{hour}} \times \frac{7\text{ hours}}{\text{day}} \times \frac{\$6}{\text{part}}\right] - \left[\frac{7\text{ hours}}{\text{day}} \times \left(\frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}}\right)\right] = \$3{,}808$$

material cost

operating & overhead cost

For machine B:

$$\text{Profit per day} = \left[\frac{130\text{ parts}}{\text{hour}} \times \frac{6\text{ hours}}{\text{day}} \times \frac{\$12}{\text{part}} \times (1 - 0.10)\right] - \left[\frac{130\text{ parts}}{\text{hour}} \times \frac{6\text{ hours}}{\text{day}} \times \frac{\$6}{\text{part}}\right] - \left[\frac{6\text{ hours}}{\text{day}} \times \left(\frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}}\right)\right] = \$3{,}624$$

→ Select machine A

Example

Two pumps delivering 100hp (the consumption rate: 1hp= 0.746kW) will be operated for one year (4,000h) for agricultural purposes. Assuming the electricity costs $0.1 per kWh. Which pump would you select?

كل kw انا
بسحبه بكلفني
٠.١ دولار

	Pump A	Pump B
Purchase price ← السعر	$2,900	$6,200
Maintenance cost ← تكاليف الصيانة	$170	$510
Efficiency ← الكفاءة	80%	90%

السؤال ما عندي داتا أد
إيرادات السنوي راح أقارن بين
التكاليف واختار الـ pump يلي
eff راح تكلفني أقل بس آخذ الـ
بعين الاعتبار لأنها راح تلعب دور
باستهلاك الكهرباء

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

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Example (Cont.)

[Elect consumption($)= Power delivered efficiency × # hours × price]

- For pump A:
  - Consumption= (100hp /0.8 × 0.746kW/hp) × 4,000h × $0.10kWh = $37,300
  - Total owning and operating cost = $37,300 + $2,900 + $170 = $40,370

- For pump B:
  - Consumption= (100hp/0.9 × 0.746kW/hp) × 4,000h ×$0.10kWh= $33,156
  - Total owning and operating cost = $33,156 + $6,200 + $510 = $39,866

Select pump B

التكاليف يلي دفعتها

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Engineering Economy
(0901420)

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Slides 4

* احنا كمهندسين بنا نشتغل بمشاريع ومصانع فمتعية الفلوس بجيب الها اعتبارات بالحسابات.

راح نتكلم فحكي عن الـ time value of money، كيف الفلوس بختلف سعرها أو equivalence بالها خلال فترة من الزمن

Chapter 4: Time value of money

Topics to be covered this week:

- Objective (الهدف) ⟶ How we can a deal with money when ever the time value of money is a complecated factor.
- Interest:
  - Simple Interest
  - Compound Interest

  ⟶ راح نحكي عن أنواع الـ interest اللي ممكن تؤثر على الـ value of money
- The concept of economic equivalence ⟶ بيش الفلوس اليوم لو بدي أعرف قيمتها الشرائية بعد 5 سنوات كيف ممكن أحسب هاي الـ value.
- Cash-flow diagram
- Relating present and future equivalent values
- Interest and Annuity Tables for Discrete Compounding

⟶ alternatives تمثيل أوالـ project اللي بدنا نتخذ فيها قرار.

Chapter 4: Time value of money

راح آخذ بعين الاعتبار القيمة الزمانية للفلوس
وأعرف كيف أطلع الـ equivelance لها
خلال فترات مختلفة من الزمن .

Objective:

- The objective of Chapter 4 is to explain time value of money calculations and to illustrate economic equivalence.

Time value of money is important because money can earn more money overtime (interest on capital).

دينار اليوم مش زي دينار قبل ١٥ سنين لهيك راح آخذ الـ time value of money والـ interest rate
طالبي أخذ قرار .

capital يعني رأس المال ، الفلوس اللي بدي استثمرها مثل
أمثل فلوس ، دائمًا عشان نعمل project بتحتاج فلوس ، سواء كانت
فلوس أو أرض أو مبنى بدك تؤجره لعمتك كلها بدي capital بدي استثمرها
وأعمل فلوس خلال فترة من الزمن .

Time value of money

- *Capital* refers to wealth in the form of money or property that can be used to produce more wealth.

- Engineering economy studies involve the commitment of capital for extended periods of time.

- A dollar today is worth more than a dollar one or more years from now (for several reasons).

Interest

الفائدة

بسيطة ، باخذ فائدة بقدر أحسبها بسهولة مباشرة وعلى هذا الأساس القيمة ما راح تتغير بتضل ثابتة مع مرور السنوات ، أنا باخذ فائدة على الـ capital أو على principle ، في كثير من البنوك الإسلامية بتاخذ simple وبتزيد عليها شوي ليكونوا مكافئين لـ compound interest .

① ➤ Simple interest

- Not commonly used.
- Total interest is linearly proportional to the initial loan amount (principal).

فائدة مركبة

② ➤ Compound interest

- More common in personal and professional financing.
- Interest is based on the remaining principal + any accumulated interest.

لـ مثلاً اذا بتروح على supplier تشتري material تكيلوا أنا بدي أحاسبك شيكات بياخذ compound interest وحسبهالك خلال الفترة اللي بدك تسد فيها الـ material اللي أخذتها من عنده

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

معظم الحسابات والبنوك بستخدموها ، الفائدة المركبة ، عم تاخذ فائدة على الأصل مع مرور الوقت بعين آخذ فائدة على الفوائد اللي بتتراكم وبتركب على الـ capital اللي بلشت فيه المشروع .

بقدر من خلالها أحسب الفوائد اللي راح أدفعها في نهاية الفترة اللي بدي أخذ القرار على أساسها .

Simple Interest

كل الأرقام اللي عندي بتربطها مع بعض بنسبة الفائدة بنفس الـ simple interest

When the total interest earned or charged is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods, the total amount of interest are said to be *simple*.

- The total simple interest, I, earned or paid may be computed using the formula below:

الـ principal اللي بدي آخذها

بدي امدده بعد 5 سنوات

$$I = P \times N \times i$$

قيمة الفائدة 5٪

لنفترض أخذت قرض من بنك مثلاً 10000 دينار

- I: Total simple interest paid or earned.
- P: Principal amount lent or borrowed.
- N: Number of interest periods (e.g., years).
- i: Interest rate per interest period.

هيول مع بعض بمعطوني الـ simple interest اللي راح ترجعها وتدفعها للبنك

The total amount repaid at the end of N interest periods is $P + I$.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Simple Interest

الـ n و الـ i لازم يكونوا نفس
الباقي اذا year أو month اذا مش نفس
الباقي يحول (لازم نفس الوحدة)

Example:

A $1,000 loan for 3 years at a simple interest rate of 10% per year.

↳ $\frac{10}{100} = 0.1$

P= Principal = $1,000.

N= Number of interest periods = 3 years.

i: Interest rate per interest period = 10% per year.

➢ The total interest paid = I= $1000 × 10% × 3 years = $300.

The total amount repaid at the end of the loan period = principal (P) + interest (I)

= $1000 + $300 = $1300.

إذا بدي أسد القرض بعد 3 سنوات بفائدة 10٪ راح يكون بقيمة 1300 $

Simple Interest

Example: You borrowed $5,000 at a simple interest rate = 0.5% per month to be repaid after 4 years.

How much will you pay back? Or

what is the future equivalent of the borrowed $5,000?

السؤالين نفس الجواب (F)

simple هاي الحركة بنعملها بـ لأنها بسيطة مباشرة.

P= Principal = $5,000.

N= Number of interest periods = 4 years.

i: Interest rate per interest period = 0.5% per month × 12 months/year = 6% per year.

➢ The total interest paid = I= $5,000 × 6% × 4 years = $1,200.

The total amount repaid (or future equivalent) = $5,000 + $1,200 = $6,200.

Compound Interest

- Interest is based on the remaining principal + accumulated interest.

Example: $1,000 loan for 3 years at a compound interest rate of 10% per year.

Period	(1) Amount owed at beginning of period	(2)=(1)x10% Interest amount for period	(3)=(1)+(2) Amount owed at end of period
1	$1,000	$100	$1,100
2	$1,100	$110	$1,210
3	$1,210	$121	$1,331

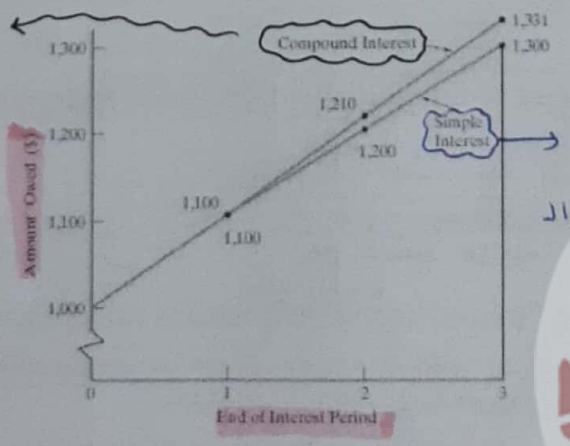
← الفائدة مركبة يعني بدي احسبها لكل سنة ، الفائدة لكل سنة راح تزاكم على السنة يلي بعدها وهكذا لآخر سنة بحسب فائدة على الفوائد يلي تراكمت على مدار الـ 3 سنوات .

المبلغ يلي بدي أسدده للبنك بعد 3 سنوات

الفرق هون بين الـ simple والـ compound (31) مش كتير كبير لأن الـ 1000$ مبلغ مو كتير كبير يعني

Simple Versus Compound Interest

$1,000 loan for 3 years at a simple versus compound interest rate of 10% per year.



الـ compound راح يبلش زي بعض بأول سنة من الـ 1000 ولكن من السنة الثانية يبلش بـ 1100 وهكذا ، الـ compound بتزيد كل ما زادت الفترة الزمنية كلما زادت حتمية الفوائد يلي بدي أدفع عليها فائدة .

الـ simple أنا بحسب فائدة على الأصل يلي هو الـ 1000 $

Simple Versus Compound Interest

Repayment of $17,000 in Four Months with Interest at 1% per Month:

Simple Interest VS Compound Interest

Month	(2) Amount Owed at Beginning of Month	(3) Interest Accrued for Month	(4) Total Month-End Owed at End of Month	(5) Principal Payment	(6) = (3) + (5) Total End-of-Month Payment (Cash Flow)
	Pay interest due at end of each month and principal at end of fourth month.				
1	$17,000	$170	$17,170	$0	$170
2	17,000	170	17,170	0	170
3	17,000	170	17,170	0	170
4	17,000	170	17,170	17,000	17,170
	68,000 \$-mo.	[680] (total interest)			

Month	(2) Amount Owed at Beginning of Month	(3) = 1% × (2) Interest Accrued for Month	(4) = (2) + (3) Total Month-End Owed at End of Month	(5) Principal Payment	(6) = (3) + (5) Total End-of-Month Payment (Cash Flow)
	Pay principal and interest in one payment at end of fourth month				
1	$17,000	$170	$17,170	$0	$0
2	17,170	171.70	17,341.70	0	0
3	17,341.70	173.42	17,515.12	0	0
4	17,515.12	175.15	17,690.27	17,000	17,690.27
	69,026.83 \$-mo.	[690.27] (total interest)			

← حيقة الفائدة لي بتي ادفعها باد comp بتكون أكبر .

← كلما زادت الفترة ، كلما زادت الفائدة لي بدفعها باد comp وكلما زادت الحيقة لي بتي أرجع أسدها ؟ .

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

الفائدة بحي, الها equivalence نتيجة فائدة معينة لازم أدفعها في حال اقترضت فلوس من بنك أو supplier أو مصرف .

The concept of economic equivalence

Used for comparing alternatives when time value of money is a factor (compound interest is involved).

- Each alternative can be reduced to an *equivalent basis* dependent on
  - Interest rate, نوعها وحيقتها
  - Amount of money involved, and → كم حيقة الـ principal لي بلشت فيها
  - Timing of monetary receipts or expenses. → متى راح يحي التسليم هل بعد انتهاء الفترة ولا على دفعات خلال هاي الفترة .
- Using these elements, we can "move" cash flows so that we can compare them at "particular" points in time.
  - Cash-flow diagram is an essential tool in economic equivalence.

في جميع الحالات بتي أطلع equiv. basis حتى أقدر أقارن بين الـ alternative

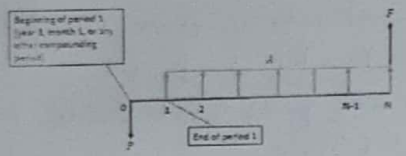
← دايمًا لما بتي أقارن بين الفلوس لازم أقارنهم بنفس حيقة الـ equivalence لأنهم بنفس الفترة الزمنية . ما بحير أقارن ١٥ دنانير اليوم مع ١٥ دنانير بعد ١٥ سنوات ، يا إما بطلع الـ equiv لـ ١٥ دنانير اليوم بعد ١٥ سنوات كم راح تكون وبقارنها يا إما برجع الـ ١٥ لي بالمستقبل لليوم وبقارنهم بنفس الفترة .

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

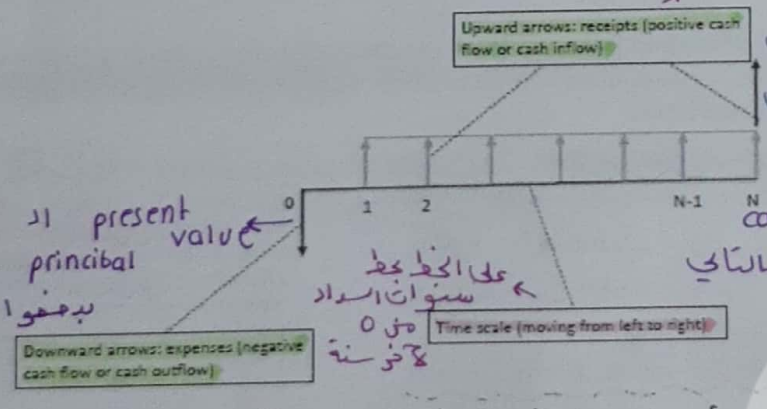
The concept of economic equivalence (Cont.)

Notation used in formulas for compound interest calculations, and used for drawing the cash-flow diagram

- i: effective interest rate per interest period.
- N: number of compounding interest periods (e.g., years)
- P: present sum of money (or the equivalent sum of one or more cash flows at present time).
- F: future sum of money (or the equivalent sum of one or more cash flows at future time).
- A: end-of-period cash flow (or the equivalent end-of-period value) in a uniform series starting at the end of first period and continuing through the last period.



Cash-flow diagram



Direction of the arrows depends on the point of view (lender vs. borrower)

* مشروع بدي استثمر فيه 10000 ، هاد المشروع كل سنة بدخل عليه 5000 لمدة 5 سنوات بعد 5 سنوات راح بيع المحل بـ 2000 ، تكاليف الصيانة كل سنة 3000 .

Cash-flow diagram

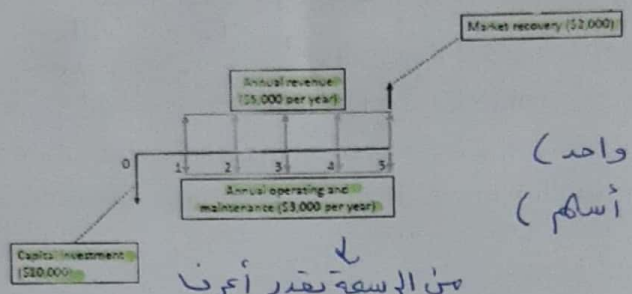
Example: An investment of $10,000 will produce a uniform annual revenue of $5,000 for 5 years and have a market (recovery) value of $2,000 at the end of year (EOY) five. Annual operating and maintenance expenses are estimated at $3,000 at the end of each year. Draw a cash-flow diagram from the corporation's viewpoint.

ايرادات

* الأموال اللي بتدخل على (سهم أو أسهم) للأعلى :-

(5 أسهم) كل سنة 5000 ①

آخر سنة (سهم واحد) 2000 ②



من الرسمة بقدر أعرف انو كل سنة أنا بدخل على 2000 دولار (بدفع 3000 وبدخل على 5000) .

* الأموال اللي ← تكاليف دفعتها (سهم أو أسهم) للأسفل :-

10 000 عند الزمن 0 (سهم واحد) ①

3000 صيانة كل سنة (5 أسهم) ②

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Cash-flow diagram

In a company's renovation of a small office building, two feasible alternatives for upgrading the heating, ventilation, and air conditioning (HVAC) system have been identified. Either Alternative *A* or Alternative *B* must be implemented. The costs are as follows:

➢ Alternative *A* *Rebuild (overhaul) the existing HVAC system*

- Equipment, labor, and materials to rebuild : $18,000 → تكاليف للعمال
- Annual cost of electricity : $32,000 → كهرباء
- Annual maintenance expenses : $2,400 → للصيانة

➢ Alternative *B* *Install a new HVAC system that utilizes existing ductwork*

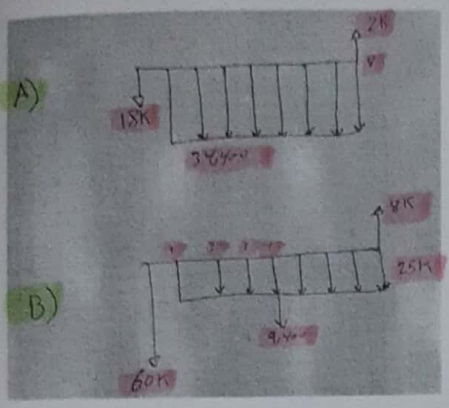
- Equipment, labor, and materials to install : $60,000 → تكاليف التركيب
- Annual cost of electricity : $9,000 → الكهرباء
- Annual maintenance expenses : $16,000 → الصيانة
- Replacement of a major component four years : $9,400

لو بدي أبيع بـ

At the end of eight years, the estimated market value for Alternative *A* is $2,000 and for Alternative *B* it is $8,000. Assume that both alternatives will provide comparable service (comfort) over an eight-year period and assume that the major component replaced in Alternative *B* will have no market value at EOY eight.

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Cash-flow diagram



= -25000 - 9400 | = C3 - B3 | = SUM(D$3:D3)

	A	B	C	D	E
1		Alternative A	Alternative B	Difference	Cumulative
2	End of Year	Net Cash Flow	Net Cash Flow	(B-A)	Difference
3	0 (now)	$ (18,000)	$ (60,000)	$ (42,000)	$ (42,000)
4	1	$ (34,400)	$ (25,000)	$ 9,400	$ (32,600)
5	2	$ (34,400)	$ (25,000)	$ 9,400	$ (23,200)
6	3	$ (34,400)	$ (25,000)	$ 9,400	$ (13,800)
7	4	$ (34,400)	$ (34,400)	$ -	$ (13,800)
8	5	$ (34,400)	$ (25,000)	$ 9,400	$ (4,400)
9	6	$ (34,400)	$ (25,000)	$ 9,400	$ 5,000
10	7	$ (34,400)	$ (25,000)	$ 9,400	$ 14,400
11	8	$ (32,400)	$ (17,000)	$ 15,400	$ 29,800
12	Total	$ (291,200)	$ (261,400)		

= -34400 + 2000

= SUM(B3:B11)

= -25000 + 8000

لنفرض بعد ما أتخرج بدي أشتري سيارة بتكلف 20 000 وحالياً معي ثروة مهاري مبدي أفرض عم بدي أحط بالبنك اليوم بفائدة معينة عشان أقدر أحصل الـ 20 000 بعد ما أخلص جامعة.

Relating present and future equivalent values

We can apply compound interest formulas to "move" cash flows along the cash flow diagram.

For a single cash flow and using the compound interest rate formula using the standard notation, we can find that a present amount, P, can grow into a future amount, F, in N time periods at interest rate i according to

اذا عندي الـ present وبدي أحسب الـ future

$$F = P(1+i)^N$$

or

هيك بقلعهم من الجداول

$$F = P(F/P, i\%, N) \text{ from tables}$$

In a similar way we can find P given F by

$$P = F(1+i)^{-N}$$

هون العكس.

or

$$P = F(P/F, i\%, N) \text{ from tables}$$

Interest and Annuity Tables for Discrete Compounding

APPENDIX C

Interest and Annuity Tables for Discrete Compounding

- For various values of *i* from 1/4% to 25%
  - *i* = effective interest rate per period (usually one year)
  - *N* = number of compounding periods

نحدد الـ interest rate ونروح على الجدول المطلوب ونشوف على N اللي بدي اياها ونوجد المطلوب .

Relating present and future equivalent values

Example:

P (present)

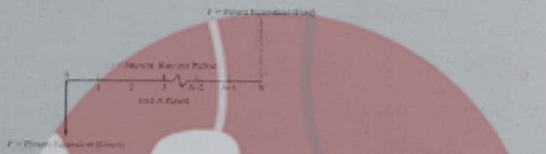
Suppose that you borrow $8,000 now, promising to repay the loan principal plus accumulated interest in four years at *i* = 10% per year. How much would you repay at the end of four years?

N

TABLE C-13 Discrete Compounding, $i = 10\%$

	Single Payment			Uniform Series			Uniform Gradient	
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
	To Find F Given P	To Find P Given F	To Find F Given A	To Find P Given A	To Find A Given F	To Find A Given P	To Find P Given G	To Find A Given G
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G	N
1	1.1000	0.9091	1.0000	0.9091	1.0000	1.1000	0.000	0.0000	1
2	1.2100	0.8264	2.1000	1.7355	0.4762	0.5762	0.826	0.4762	2
3	1.3310	0.7513	3.3100	2.4869	0.3021	0.4021	2.329	0.9366	3
4	1.4641	0.6830	4.6410	3.1699	0.2155	0.3155	4.378	1.3812	4
5	1.6105	0.6209	6.1051	3.7908	0.1638	0.2638	6.862	1.8101	5

OR



$F = \$8{,}000\ (F/P, 10\%, 4)$
$= \$8{,}000\ (1.4641)$
$= \$11{,}713.$

$F = P\ (1 + i)^4 = \$8{,}000\ (1+0.1)^4$
$= \$11{,}713$

Relating present and future equivalent values

You need $10,000 after five years so you decided to save money now. How much do you need to deposit now in the bank given that the interest rate is 5% per year?

F= $10,000, N= 5 years, i= 5%, Find P= ?

P= $10,000(1+0.05)^-5= $7,835.26

or

TABLE C-8 Discrete Compounding; i = 5%

	Single Payment		Uniform Series				Uniform Gradient	
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Present Worth Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor	
	To Find F Given P	To Find P Given F	To Find F Given A	To Find P Given A	To Find A Given P	To Find P Given G	To Find A Given G	
N	F/P	P/F	F/A	P/A	A/P	P/G	A/G	N
1	1.0500	0.9524	1.0000	0.9524	1.0500	0.000	0.000	1
2	1.1025	0.9070	2.0500	1.8594	0.5378	0.907	0.4878	2
3	1.1576	0.8638	3.1525	2.7232	0.3672	2.635	0.9675	3
4	1.2155	0.8227	4.3101	3.5460	0.2820	5.103	1.4391	4
5	1.2763	0.7835	5.5256	4.3295	0.2310	8.237	1.9025	5

From the Tables, i= 5% page to find $(P/F,5\%,5)$

P= $10,000 $(P/F,5\%,5)$ = $10,000 (0.7835) = $7835

Relating present and future equivalent values

Example Problems (All Using an Interest Rate of i = 10% per Year—See Table C-13 of Appendix C)

To Find:	Given:	(a) In Borrowing-Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram*	Solution
For single cash flows:					
F	P	A firm borrows $1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year?	What is the future equivalent at the end of eight years of $1,000 at the beginning of those eight years?	$P = \$1{,}000$; $N = 8$; 0; $F = ?$	$F = P(F/P, 10\%, 8)$ $= \$1{,}000(2.1436)$ $= \$2{,}143.60$
P	F	A firm wishes to have $2,143.60 eight years from now. What amount should be deposited now to provide for it?	What is the present equivalent of $2,143.60 received eight years from now?	$F = \$2{,}143.60$; 0; $N = 8$; $P = ?$	$P = F(P/F, 10\%, 8)$ $= \$2{,}143.60(0.4665)$ $= \$1{,}000.00$

Finding the Interest Rate (i) Given P, F, and N

There are situations in which we know two sums of money (P and F) and how much time separates them (N), but we don't know the interest rate (i) that makes them equivalent.

اذا عندي كل المعلومات
بقدر أحسب i

$$i = \sqrt[N]{F/P} - 1$$

Finding the Interest Rate (i) Given P, F, and N

Example: What is the interest rate that will double an investment of $50,000 in 10 years?

P= $50,000, F= $100,000, N= 10 years, i= ?

- $i = \sqrt[10]{(100{,}000/50{,}000)} - 1 = 0.0718$
- i= 7.18%

$$i = \sqrt[N]{F/P} - 1$$

To use Appendix C tables, you need interpolation. P = F (P/F, i, N) >> (P/F, i, 10)= 0.5

لح بيطوفا من الجداول وبين
قيمة P/F قريبة من الـ 0.5

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Finding N when Given P, F, and i

Sometimes we are interested in finding the amount of time needed for a present sum to grow into a future sum at a specified interest rate.

$$N = \frac{\log(F/P)}{\log(1+i)}.$$

لما بدي أعرف N

Finding N when Given P, F, and i

Example: How many years does it take to double my money at an interest rate of 5% per year?

↳ $F/P = 2$

- $F/P = 2$, $i = 5\%$, $N = ?$
- $N = \log(2)/\log(1+0.05) = 14.2$ years

N=15 years

$$N = \frac{\log(F/P)}{\log(1+i)}.$$

To use Appendix C tables, you need interpolation. (صح)

لما بروح على الجدول عند

$2 = F/P$ و $i = 5\%$

إما بحكي 15 أو بفعل interpolation بين الـ 14 والـ 15

Engineering Economy
(0901420)

Muhammad T. Hatamleh, PhD
Department of Civil Engineering
Slides 5

Chapter 4: Time value of money

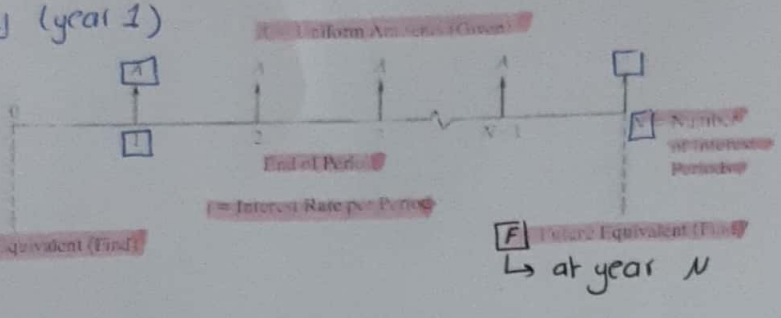
Topics to be covered this week:

- The concept of economic equivalence
  - Uniform series (annuity) to present and future
  - Deferred Annuities
  - Uniform (arithmetic) gradient of cash flows

* لو أخذت قرض 10000 راح أسدده على مدة 5 سنوات
كل سنة بدفع 17000 بشكل ثابت من أول سنة لغاية ما انهي
وأسدد القرض بكي على ٥ سنوات.

The Concept of Economic Equivalence

A: series of uniform (equal) payments occurring at the end of each period for *N* periods (also called annuity).



بتلش من the first intrest period (year 1)
و بتنتهي مع نهاية (year N) المشروع
(قيمة ثابتة)
at year zero
↳ at year N

Example: repaying a loan in uniform monthly payments.

* Start at the end of year 1 and end by the end of the last year of a project.

Uniform Series (Annuity)

Uniform series (annuity) to present and future:

➢ Finding *F* given *A*

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

or

$$F = A\,(F/A, i\%, N)\ \text{from tables in Appendix C}$$

➢ Finding *P* given *A*

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

or

$$P = A\,(P/A, i\%, N)\ \text{from tables in Appendix C}$$

Uniform Series (Annuity)

- Find **A** given **F**

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] \quad \text{or} \quad A = F\,(A/F, i\%, N)\ \textit{from tables in Appendix C}$$

- Find **A** given **P**

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] \quad \text{or} \quad A = P\,(A/P, i\%, N)\ \textit{from tables in Appendix C}$$

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Uniform Series (Annuity)

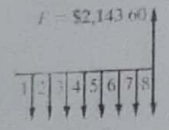
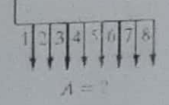
TABLE 4.2 Engineering Economy Examples Illustrating Equivalence

Example Problems (All Using an Interest Rate of i = 10% per Year—See Table C-13 of Appendix C)

To Find	Given:	(a) In Economy Study Problem Statement Terminology	(b) In Equivalent Terminology	Cash Flow Diagram	Solution
For uniform series:					
F	A	If eight annual deposits of $187.45 each are placed in an account, how much money has accumulated immediately after the last deposit?	What amount at the end of the eighth year is equivalent to eight EOY payments of $187.45 each?	$F = ?$; 1 2 3 4 5 6 7 8; $A = \$187.45$	$F = A(F/A, 10\%, 8)$ $= \$187.45(11.4359)$ $= \$2,143.60$
P	A	How much should be deposited in a fund now to provide for eight EOY withdrawals of $187.45 each?	What is the present equivalent of eight EOY payments of $187.45 each?	$A = \$187.45$; 1 2 3 4 5 6 7 8; $P = ?$	$P = A(P/A, 10\%, 8)$ $= \$187.45(5.3349)$ $= \$1,000.00$

Uniform Series (Annuity)

Example Problems (All using an interest Rate of i = 10% per Year—See Table C-13 of Appendix C)

Given: Find:	(a) Answering Engineering Terminology?	(b) Equivalent or Terminology?	Cash Flow Diagram	Solution
Given: F Find: A	What uniform annual amount should be deposited each year in order to accumulate $2,143.60 at the time of the eighth annual deposit?	What uniform payment at the end of eight successive years is equivalent to $2,143.60 at the end of the eighth year?	$F = \$2,143.60$  $A = ?$	$A = F(A/F, 10\%, 8)$ $= \$2,143.60(0.0874)$ $= \$187.45$
Given: A Find: P	What is the size of eight equal annual payments to repay a loan of $1,000? The first payment is due one year after receiving the loan.	What uniform payment at the end of eight successive years is equivalent to $1,000 at the beginning of the first year?	$P = \$1,000$  $A = ?$	$A = P(A/P, 10\%, 8)$ $= \$1,000(0.18745)$ $= \$187.45$

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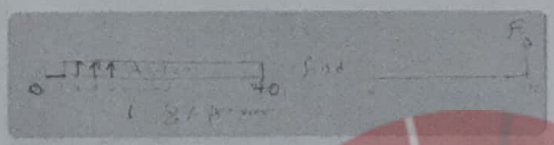
Uniform Series (Annuity)

Example: How much will you have in 40 years if you invest $3,000 of your income each year in a project that earns 8% per year?

A= $3000, i= 8% per year, N= 40 years

→ F = ?



$$F = 3{,}000\left[\frac{(1+0.08)^{40}-1}{0.08}\right] = \$777{,}169.6$$

→OR

From Appendix C tables, $(F/A, 8\%, 40) = 259.0565$ → F= \$3,000 × 259.0565 = \$777,169.5

* لنفرض بدي آخذ قرض من أشتري سيارة وبابي 600 دينار بالشهر بقدر آخذ منهم 200 دينار ، بسأل في البنوك كم بعطوني فوائد ، 200 دينار بالشهر البنك العربي i=0.5% ، الاسكان i=0.75% وهكذا ، فأنا عارف الفوائد وحديش بقدر أدفع بدي أسددهم على فترة 4 سنوات ، بقدر أحسب بالزبط شو الرقم اللازم آخذه من البنك وكم حيقه الفائدة وعلى أي أساس بدي أدفعها .

Uniform Series (Annuity)

Example: You took a loan which is to be repaid in uniform payments over 4 years. Assuming the interest rate is 1% per month, and your monthly payment is $300. What is the principal amount (the amount of money borrowed)?

↳ A → راح أدفع 300 كل شهر مدة 4 سنوات

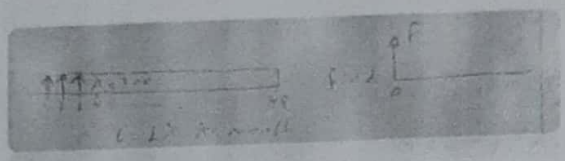
↳ P = ?!

A = $300, i = 1% per month, N = 4 years × 12 months/year = 48 months

→ The period N should be consistent with the interest rate (interest per month, then, period in months)

→ P = ?

$$P = 300\left[\frac{(1+0.01)^{48}-1}{0.01(1+0.01)^{48}}\right] = \$11{,}392.2$$



→ OR

From Appendix C tables, $(P/A,1\%,48) = 37.9740$ → $P = \$300 \times 37.9740 = \$11{,}392.2$

* الـ Final بكون بعكس مكان آخذ value Future

* الـ present بكون جنب بوحدة ، يعني بالسؤال لو طلبوا الـ present (P) بتكون at year (-1)

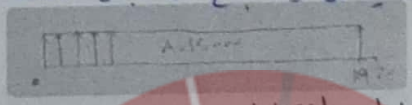
Uniform Series (Annuity)

بدي أحط 5000 كل سنة بحساب توفير

Example: Calculate the compounded future value at EOY 20 of 20 annual payments of $5,000 each into a savings account that earns 6% per year. All 20 payments are made at the beginning of each year.

A → الدفعات بتنفع بداية كل سنة يعني كل مرة بدفع من بداية السنة لحد أنا منها لهيك بنبلش من الـ 0

↳ i



- Definition of annuity occurs at the end of each compounding period.
- In the example, payments are made at the beginning of each period.

0 → 19 آخر interest period

Payments start at the beginning of each year, so the first annuity is at time 0. Hence, the present equivalent is at year -1 and ***the future equivalent*** is at ***year 19***. We first use the (F/A) relationship to determine the future equivalent at EOY 19 and then we determine the future equivalent at EOY 20 using the (F/P, 6%, 1).

F هيك أنا أوجدت F عند نهاية (19) ، المطلوب عند نهاية 20 لهيك

عند نهاية سنة 20 ، راح اتركه سنة مبتحكي القيمة على عند الـ 19 بالنسبة لـ P

$$F = \$5{,}000\,(F/A,6\%,20)\,(F/P,6\%,1)$$

$$= \$5{,}000 \times 36.7856 \times 1.06 = \$194{,}963.68$$

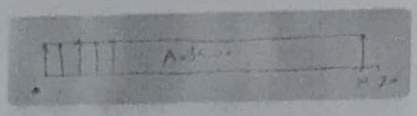
عند الـ 19 هي بداية سنة 20 والمطلوب نهايتها

Uniform Series (Annuity)

Example: Calculate the compounded future value at EOY 20 of 20 annual payments of $5,000 each into a savings account that earns 6% per year. All 20 payments are made at the beginning of each year.

(1 → 19) من

لعشرين



Another way to solve:

❖ $F = \$5{,}000\,(F/P,6\%,20) + \$5000\,(F/A,6\%,19)\,(F/P,6\%,1)$

$F = \$5{,}000 \times 3.2071 + \$5000 \times 33.7600 \times 1.06 = \$194{,}963.68$

ممكن أحكي من ← A هدول 1 → 19 وال 5000 الأولى تكون P

* المبلغ ال F النهائي بعمل ال equi د P باد التكاليف + Future بنتجت من ال A

أخذت قرض 10 000 بدي أسدده على 4 دفعات متساوية كل سنة، كل دفعة أنا بدفعها جزء منها بروح للقرض يلي باخذه وجزء منها راح يغطي الفوائد

Uniform Series (Annuity)

3155 ← interest → principal

Example: A loan of $10,000 is to be repaid in 4 equal payments (over 4 years) and the interest rate is 10% per year. Determine the interest paid and principal repayment every year.

First → Find the annual payment (annuity) $A = P\,(A/P,10\%,4) = \$10{,}000 \times 0.3155 = \$3{,}155$ per year.

يلي راجع من أصل الدين

يلي بدفعه بداية كل سنة ← Loan amount — الفوائد

Second →

Fill out a table to determined the principal repayment amount

Year	Amount owed at beginning of period	Interest	Annual payment	Principal repayment
1	$10,000	$1,000	$3,155	$2,155
2	$7,845	$785	$3,155	[$2,371]
3	$5,475	[$547]	$3,155	$2,608
4	$2,867	$287	$3,155	$2,868

= amount owed at beginning of previous year − principal repayment in previous year

= amount owed at beginning of period × interest rate

= annual payment − interest

لـ الفوائد قلت بالتالي قيمة سداد الدين زادت

Uniform Series (Annuity)

Solving for N

Example: You borrowed $100,000 at an interest rate of 7% per year. If the annual payment is $8,000, how many years does it take to repay the loan?

$$12.5 = \frac{1.07^N - 1}{0.07\,(1.07)^N} \Rightarrow 0.125\,(1.07)^N = 1$$

- $100,000=$8,000 $(P/A,7\%,N)$

N= 30.73 years or use the tables to find $(P/A,7\%,N)=12.5$

Example: You invested $20,000 in a project and you are expected to gain $4,000 annually. At a 10% interest rate, when will you recover your investment?

- $20,000=$4,000 $(P/A,10\%,N)$

$$5 = \frac{1.1^N - 1}{0.1\,(1.1)^N} \Rightarrow 0.5\,(1.1)^N = 1$$

N= 7.27 years

Uniform Series (Annuity)

Example: Your company has a $100,000 loan for a new security system it just bought. The annual payment is $8,880 and the interest rate is 8% per year for 30 years. Your company decides that it can afford to pay $10,000 per year. After how many payments (years) will the loan be paid off?

- The original loan payment was found as following:

A = $100,000 $(A/P, 8\%, 30)$ = $100,000 (0.0888) = $8,880 per year, However

We can calculate using: $100,000 = $10,000 $(P/A, 8\%, N)$ → $(P/A, 8\%, N) = 10$.

➢ We can now use the interest tables provided in Appendix C to find N. Looking down the Present Worth Factor column (P/A), we see that

$(P/A, 8\%, 20) = 9.8181$ and $(P/A, 8\%, 21) = 10.0168$

Uniform Series (Annuity)

Solving for i

Example: You wanted to start saving so that you will have $60,000 (F) in your bank account eight years (N) from now. Each year, you deposit $6,000 (A) in your bank account. What should be the interest rate so you can achieve your goal?

A: $6,000,   F=$60,000,   N: 8years,   i=?

To solve:

- Trial and error. (تجريب)
- Interpolation. (تحليل؟)
- Calculators with solver.
- Spreadsheets (Excel function: Rate).

$$\$60{,}000 = \$6{,}000\left[\frac{(1+i)^8 - 1}{i}\right]$$

→ Using the tables (F/A, i, 8)= 10

[i= 6.29 %]

لو رحنا على الجداول راح نلاحظ انو ال F/A=10 عند N=8 محصورة بين ال 6% وال 7% بالتالي بعمل interp

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

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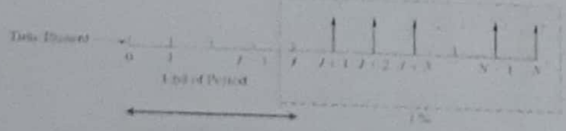
لنفرض أنا بشتغل وشريت محل أواعي، التكاليف أول سنة سنتين كانت شوي عالية على السنة الثالثة بلش يدخل مبيعات قلت التكاليف، السنة الرابعة بلشت تعمل ربح والسنة الخامسة والسادسة عملت نفس الربح صار في عندك قيمة ثابتة بتتكرر هاي بتختلف عن ال amuity العادية لأن ال amuity بتبلش من بداية المشروع لنهايته.

Deferred Annuities

- Ordinary annuity (uniform series) appears at the end of the first period.
- Deferred annuity (also uniform series) begins at later time.

ordinary A ← هي بس مش من بداية حياة المشروع



Annuity is deferred for J periods

Finding the value at time 0 of a deferred annuity is a two-step process.

$$P = A(P/A, i\%, N-J)\,(P/F, i\%, J)$$

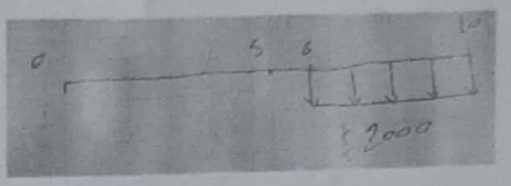
لهاد للفترة لي كل قيمها ثابتة A → هون بجيبها لـ 0

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

من سنة 6 لسنة 10 يعني عندي 5 دفعات
مابجيب احكي (6-10) بدي عندي 5 دفعات

Deferred Annuities

Example: You just purchased a new sports car and wants to also set aside cash for future maintenance expenses. The car has a bumper-to-bumper warranty for the first five years. It was estimates that the car will need approximately $2,000 per year in maintenance expenses for years 6-10, at which you will sell the vehicle. How much money should you deposit into an account today, at 8% per year, so that you will have sufficient funds in that account to cover the projected maintenance expenses?



- Find the present

$$P = \$2{,}000\,(P/A,8\%,5)(P/F,8\%,5)$$

هاي هيقة تكافئ الـ 5 دفعات هاي راح تطلع لنهاية السنة الخامسة
0 → 5

الـ 5 دفعات يلي فلت

عم هيقة الفلوس يلي لازم أحطها بالبنك لمدة 12 سنة عشان أكون قادر أسحب 309 دينار كل سنة لمدة 5 سنوات من سنة 14

Deferred Annuities

Example: How much money should be deposited each year for 12 years if you wish to withdraw $309 each year for five years, beginning at the end of the 14th year? Assume the interest rate is 8% per year.

* بنقدر نحل بطريقة ثانية بنحسب 309 (P/A, 8%, 5) بحسبها الـ Future equiv. لإنها راح بتطلع على سنة 13 وبنقلوا لليمين لـ 13 منضرب بـ F/P

- Find the present worth of both annuities and equate:

$$A(P/A,8\%,12) = \$309\,(P/A,8\%,5)(P/F,8\%,13)$$

بدي الـ 5 دفعات بدي أنقلها لـ year 0

$$A \times 7.5361 = \$309 \times 3.9927 \times 0.3677$$

$$A = \$60.2$$

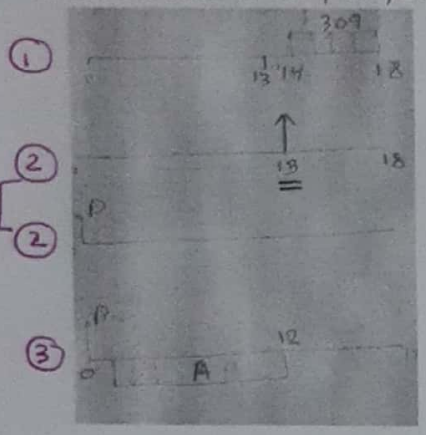
لازم أحط بالبنك كل سنة لمدة 12 سنة 60 دولار عشان أقدر أسحب 309 لمدة 5 سنوات من نهاية سنة 14.

* نقدر أعمل نفس الكلام بس بوجد الـ F عند الـ 18 وبعدين وجد الـ P عند الـ 0 وبوجد A العادية

① 309 (F/A 8%, 5) * (P/F 8%, 18) * (A/P 8%, 12)

Deferred Annuities

Example: How much money should be deposited each year for 12 years if you wish to withdraw $309 each year for five years, beginning at the end of the 14th year? Assume the interest rate is 8% per year.



الـ 5 دفعات من الـ 14 لـ 18 بدي أحولهم present equi لـ

$= \$309\ (P/A,8\%,5) = \$309 \times 3.9927 = [\$1233.7443]$ ← الرقم P لسنة 13 (قبل بسنة)

$= \$1233.7443\ (P/F,8\%,13) = \$1233.7443 \times 0.3677 = [453.6478]$ ← القيمة اللي طلعت خوت بدي أرجعها لـ zero عشان F وبدي P single

لهاي القيمة مكافئة لـ 5 دفعات عند الـ zero

$A = 453.6478 \times (A/P, 8\%, 12) = 453.6478 \times 0.1327 = [\$60.2]$

هيك بكون طلعت القيمة عند الـ zero لـ P وبدي A من (1-12) بحسب من (A/P)

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Compounding-Interest Factors

TABLE 4.3 Discrete Compounding-Interest Factors and Symbols

To Find:	Given:	Factor by which to Multiply (Given)	Factor Name	Symbol (Functional Symbol)
*For single cash flows:				
F	P	$(1+i)^N$	Single payment compound amount	(F/P, i%, N)
P	F	$\frac{1}{(1+i)^N}$	Single payment present worth	(P/F, i%, N)
*For uniform series (annuities):				
F	A	$\frac{(1+i)^N-1}{i}$	Uniform series compound amount	(F/A, i%, N)
P	A	$\frac{(1+i)^N-1}{i(1+i)^N}$	Uniform series present worth	(P/A, i%, N)
A	F	$\frac{i}{(1+i)^N-1}$	Sinking fund	(A/F, i%, N)
A	P	$\frac{i(1+i)^N}{(1+i)^N-1}$	Capital recovery	(A/P, i%, N)

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

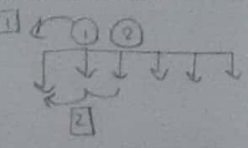
* لو كان دفعة واحدة اما بحسب F من P أو العكس

* لو عندي من 14 → 18 وبدي (F/A) الرقم اللي بطلع بكون عند الـ 18

* لو بدي P/A الرقم اللي بطلع بكون قبل أول سنة يعني عند الـ 13

نقدر أعامل الـ A انها single كل سنة بس بزيد حسابات هيك هيول أركع

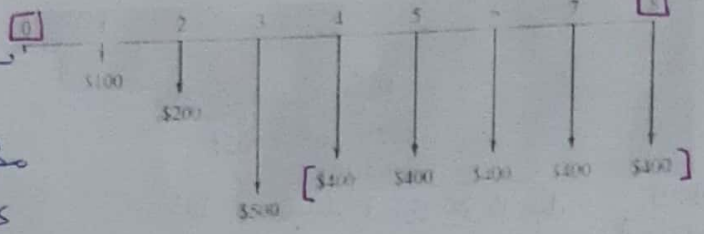
* لو عندي 5 سنوات وبدي P كل مرة برجع سنوات لـ zero



Compounding-Interest Factors

- EXAMPLE 4-16: The cash flow below have a problem with a series of year-end cash flow extending over eight years. The amounts are $100 for the first year, $200 for the second year, $500 for the third year, and $400 for each year from the fourth through the eighth. These could represent something like the expected maintenance expenditures for a certain piece of equipment or payments into a fund.

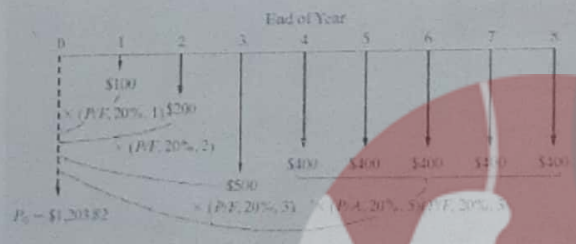
* Find the present equivalent expenditure if the annual interest rate is 20%?



Compounding-Interest Factors

Find the present equivalent expenditure?

$$
\begin{aligned}
P_0 &= F_1(P/F,20\%,1) && = \$100(0.8333) && = \$83.33\\
&+F_2(P/F,20\%,2) && +\$200(0.6944) && +138.88\\
&+F_3(P/F,20\%,3) && +\$500(0.5787) && +289.35\\
&+A(P/A,20\%,5)\times(P/F,20\%,3) && +\$400(2.9906)\times(0.5787) && +692.26\\
& && && \$1,203.82
\end{aligned}
$$



* مثلاً لو بشتغل بشركة والشركة بتعطيك زيادة سنوية مبلغها 50 دينار، فإنت بتزيد عملية ثابتة مهما هذا الأساس بتقدر تستعمل القوانين الموجودة لـ uniform gradient

حيث تزيد وتنقص with a const amount each period

Uniform (arithmetic) gradient of cash flows

Cash flow that changes by a constant amount (G) each period.

End of Period	Cash Flow
1	(0)G
2	(1)G
3	(2)G
.	.
.	.
N − 1	(N − 2)G
N	(N − 1)G

① Present equivalent

$$P = G \times \left\{ \frac{1}{i} \left[\frac{(1+i)^N}{i(1+i)^N} - \frac{N}{(1+i)^N} \right] \right\}$$

Or $P = G \times (P/G, i\%, N)$ ... tables in Appendix C

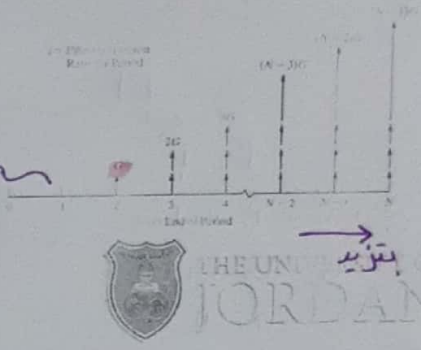
② Annuity equivalent

$$A = G \times \left[\frac{1}{i} - \frac{N}{(1+i)^N - 1} \right]$$

$$A = G \times (A/G, i\%, N)$$

③ Future equivalent

$$F = \frac{G}{i} \times (F/A, i\%, N) - \frac{N \times G}{i}$$



أول سنة فيها zero of const amount

خلها أحسب الـ P راح يكون قبل 2 interest period من الـ G

* الـ P عند الـ 0 تمثل الـ uniform gradient

Uniform (arithmetic) gradient of cash flows

Example: suppose that we have cash flows as follows:

End of Year	Cash Flows ($)
1	5,000
2	6,000
3	7,000
4	8,000

في زيادة بقيمة 1000 → const amount بتزيد بالتالي uniform grad

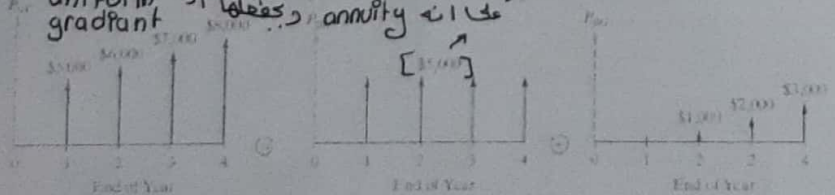
Calculate their present equivalent at $i = 15\%$ per year

- $PQT = P0A + P0G$

$= A(P/A, 15\%, 4) + G(P/G, 15\%, 4)$ ← حولت الـ A present equiv.

$= \$5{,}000(2.8550) + \$1{,}000(3.79)$

$= \$14{,}275 + 3{,}790 = \$18{,}065.$



أخذت أصغر رقم وجعلها الـ 5000 على انه annuity

Uniform (arithmetic) gradient of cash flows

Example: suppose that we have cash flows as follows:

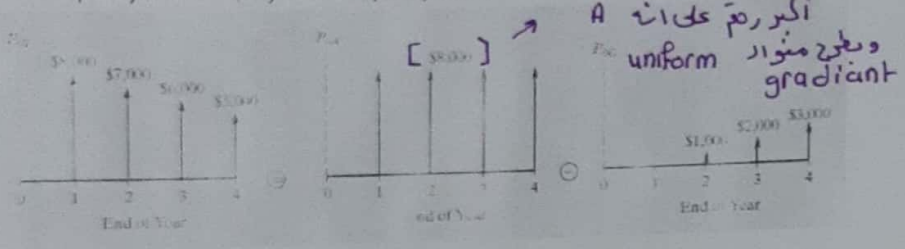
Calculate their present equivalent at $i = 15\%$ per year

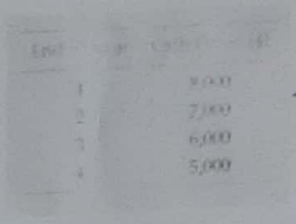
- $P0T = P0A - P0G$

$= A(P/A, 15\%, 4) - G(P/G, 15\%, 4)$

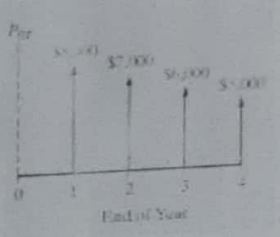
$= \$8{,}000(2.8550) - \$1{,}000(3.79)$

$= \$22{,}840 - \$3{,}790 = \$19{,}050.$





تتناقص بمقدار 1000



تتناقص متناظرة A أكبر رقم على أنه uniform ويطرح منوار gradiant

Engineering Economy
(0901420)

Muhammad T. Hatamleh, PhD
Department of Civil Engineering
Slides 6

Chapter 4: Time value of money

Topics to be covered this week:

- Geometric sequence of cash flows
- Interest Rates that Vary with Time
- Nominal and effective interest rates
- Continuous compounding

Geometric sequence of cash flows

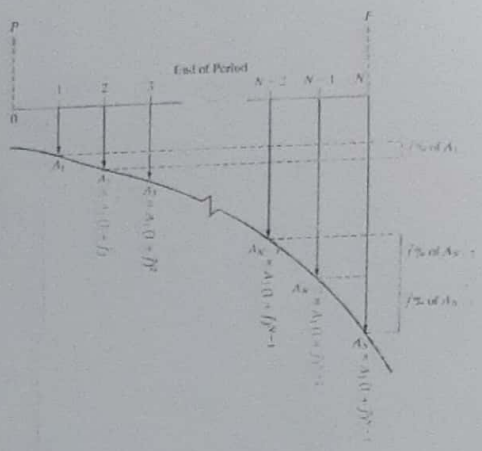
Cash flow that changes by a constant rate ($\bar{f}$) each period.

First payment at EOY 1.

بنسبة معينة وإما مثلاً زيادة بمقدار ١٥٪ وهكذا

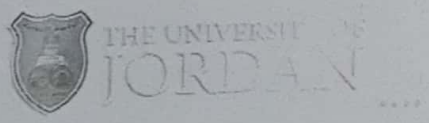
ما حكي قانون لـ F بس اذا حكى P بقدر أوجد F من F/P

$$P = \begin{cases} \dfrac{A_1[1-(1+i)^{-N}(1+\bar{f})^N]}{i-\bar{f}} & \bar{f} \neq i \\ A_1 N(1+i)^{-1} & \bar{f} = i. \end{cases}$$



$$P = \begin{cases} \dfrac{A_1[1-(P/F,i\%,N)(F/P,\bar{f}\%,N)]}{i-\bar{f}} & \bar{f} \neq i \\ A_1 N(P/F,i\%,1) & \bar{f} = i. \end{cases}$$

لـ P من الجداول



* $\bar{P}$ ← اذا بتزيد بتكون (+) اذا بتقل (-)
اذا سالبة ما بقدر أستعمل الجدول بدي أحل من المعادلة .

Geometric sequence of cash flows

القيمة الكبيرة يلي بدي ابلش منها ← A_1 ، $\bar{P}$

Example: Assume that a payment of $1,000 is made at EOY 1 and decreases by 20% per year after the first year for 4 years. At a 25% interest rate, Determine the present equivalent, A, and F.

↳ N ، ↳ i

The value of $\bar{f}$ is −20% in this case. The desired quantities are as follows:

$$P = \frac{\$1,000[1-(P/F,25\%,4)(F/P,-20\%,4)]}{0.25-(-0.20)}$$

$$= \frac{\$1,000}{0.45}\left[1-(0.4096)(1-0.20)^4\right]$$

$$= \$2,222.22(0.83222)$$

$$= \$1,849.38;$$

$$A = \$1,849.38(A/P,25\%,4) = \$783.03;$$

$$F = \$1,849.38(F/P,25\%,4) = \$4,515.08.$$

$(F/P, i\%, N) = (1+i)^N$

Geometric sequence of cash flows

• **Example:** On your 23rd birthday you decide to invest $4,500 (10% of your annual salary) in a mutual fund earning 7% per year. You will continue to make annual deposits equal to 10% of your annual salary until you retire at age 62 (40 years after you started your job). You expect your salary to increase by an average of 4% each year during this time. How much money will you have accumulated in your mutual fund when you retire?

$$P = \frac{\$4{,}500[1 - (P/F, 7\%, 40)(F/P, 4\%, 40)]}{0.07 - 0.04}$$

$$P = \frac{4{,}500[1 - (0.0668)(4.8010)]}{0.03}$$

$$P = \$101{,}894.$$

$$F = \$101{,}894(F/P, 7\%, 40)$$

$$F = \$101{,}894(14.9745)$$

$$F = \$1{,}525{,}812.$$

Interest Rates that Vary with Time

Interest rates often change with time (e.g., a variable rate mortgage).

We often must resort to moving cash flows one period at a time, reflecting the interest rate for that single period.

The present equivalent of a cash flow occurring at the end of period N can be computed with the equation below, where i_k is the interest rate for the k^{th} period.

$$P = \frac{F_N}{\prod_{k=1}^{N}(1 + i_k)}$$

If $F_3 = \$2{,}500$ and $i_1 = 8\%$, $i_2 = 10\%$, and $i_3 = 11\%$, then

$$P = \$2{,}500(P/F, 8\%, 1)(P/F, 10\%, 1)(P/F, 11\%, 1)$$

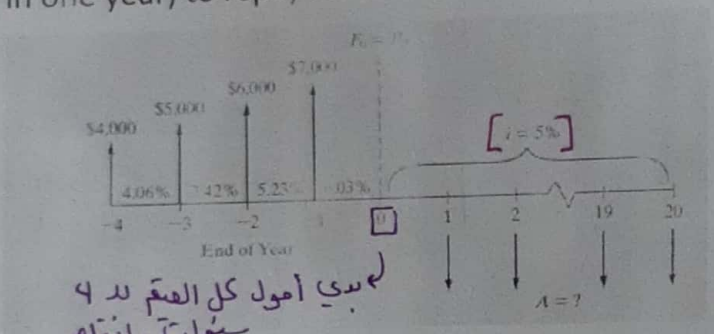
$$P = \$2{,}500(0.9259)(0.9091)(0.9009) = \$1{,}896$$

Interest Rates that Vary with Time

EXAMPLE 4-27: Smith is a 22-year-old senior who used the Stafford loan program to borrow $4,000 four years ago when the interest rate was 4.06% per year. $5,000 was borrowed three years ago at 3.42%. Two years ago he borrowed $6,000 at 5.23%, and last year $7,000 was borrowed at 6.03% per year. Now he would like to consolidate his debt into a single 20-year loan with a 5% fixed annual interest rate. If Smith makes annual payments (starting in one year) to repay his total debt, what is the amount of each payment?

A

مطلوب أحسب قيمة A يلي بدو smith يدفعها لمدة 20 سنة حتى يسد القروض يلي أخذهم هالكل دراسته



لبدي أعمل كل القيم لد 0 سنوات وانقلهم لد 0 يعني للمستقبل وهيك بكون قلعت P الكائنة ومنها حسب A لد 20 سنة

Muhammad T. Hatamleh. Extracted from Sullivan et al. (2018)

$(F/P, i\%, N) = (1 + i)^N$

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Interest Rates that Vary with Time

من القوانين لـ (1+i)^N بأثر ماعنا مواصل باكمال

قيمة الـ 4000 عند سنة 3

المصاري يلي أخذتهم بنفس السنة

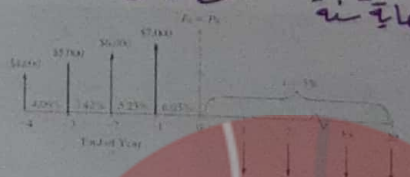
- $F-3 = \$4,000(F/P, 4.06\%, 1) + \$5,000 = \$4,000(1.0406) + \$5,000 = \$9,162.40$
- $F-2 = \$9,162.40(F/P, 3.42\%, 1) + \$6,000 = \$15,475.75$
- $F-1 = \$15,475.75(F/P, 5.23\%, 1) + \$7,000 = \$23,285.13$
- $F0 = \$23,285.13(F/P, 6.03\%, 1) = \$24,689.22$

المبلغ يلي حتى نهاية سنة 3

هيك بكون سد القرض كامل مع الفوائد بس هو بدو يدفعهم بدفعات سنوية

$(F/P, i\%, N) = (1 + i)^N$

$A = \$24,689.22(A/P, 5\%, 20) = \$24,689.22(0.0802) = \$1,980.08$ per year



Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

في حال كانت الفائدة شهرية والدفعات سنوية بسمي الـ effective interest rate يلي بتوحد الفترة يلي بحتسب منها الفائدة .

Nominal and Effective interest rates

* If the compounding period is less than a year: بعطيك إياها البنك وهي أقل من الحقيقة

- Annual rate is called nominal interest rate or annual percentage rate (APR). الأصلية يلي بين أدفعها أو أدفع عليها فوائد .
- Actual or exact rate is called effective interest rate.

Example: if annual interest rate is 10% compounded annually? ← الفائدة سنوية

then the Effective rate = Nominal rate = 10%.

حتى لو ١٥٪ ، الفائدة بتراكم بشكل سنوي وراد ١٥٪ سنوية

Nominal = eff متساويات بالباي راد

Nominal and Effective interest rates

راد nominal متأثرة بقدر أقول من سنة لشهر والعكس ، أما راد effective هي يلي بحسبلها من الفائدة

$$i = \left(1 + \frac{r}{M}\right)^{M} - 1$$

nominal

actual or effective

Where:

i: effective interest rate per year.

r: nominal interest rate per year.

M: number of compounding periods per year. ← عدد المرات يلي راح احتسب منها الفائدة بالفترة راد eff يلي أنا بحسبها (كم مرة بحتسب الفائدة مثلاً خلال السنة)

Compounding Frequency	Number of Compounding Periods per Year, M	Effective Rate (%) for Nominal Rate of 6%	8%	10%	12%	15%	24%
Annually	1	6.00	8.00	10.00	12.00	15.00	24.00
Semiannually	2	6.09	8.16	10.25	12.36	15.56	25.44
Quarterly	4	6.14	8.24	10.38	12.55	15.87	26.25
Bimonthly	6	6.15	8.27	10.43	12.62	15.97	26.53
Monthly	12	6.17	8.30	10.47	12.68	16.08	26.82
Daily	365	6.18	8.33	10.52	12.75	16.18	27.11

كل سنة بحتسب فائدة

كل الفائدة راح احتسبها مرتين .

$$i = \left(1 + \frac{6\%}{2}\right)^{2} - 1$$

Nominal and Effective interest rates

Suppose that a $100 lump-sum amount is invested for 10 years at a nominal interest rate of 6% compounded quarterly. How much is it worth at the end of the 10th year?

There are four compounding periods per year, or a total of 4 × 10 = 40 interest periods.

The interest rate per interest period is 6%/4 = 1.5%.

F = P(F/P, 1.5%, 40) = $100.00(1.015)^40 = $100.00(1.814) = $181.40.

OR

Alternatively, the effective interest rate from $i = \left(1 + \frac{r}{M}\right)^M - 1$

i = $(1 + \frac{0.06}{4})^4 - 1$ = 6.14% Therefore,

F = P(F/P, 6.14%, 10) = $100.00(1.0614)^10 = $181.40.

Nominal and Effective interest rates

Example: A credit card company charges 1.375% per month on the unpaid balance. They claim that the annual interest rate is (12 × 1.375% = 16.5%). Is that true?

➢ What is the effective interest rate per month?

Since compounding is monthly, effective monthly rate = nominal monthly rate = 1.375%.

➢ What is the effective interest rate per year?

r (nominal interest rate per year.) = 16.5%

M = 12 compounding periods per year

$$i = \left(1 + \frac{0.165}{12}\right)^{12} - 1 = 17.81\%$$

➢ Does this card provide a better deal than another card which charges 16.8% annual rate compounded monthly?

M = 6 compounding periods per year ⇒ i for the other card = $\left(1 + \frac{0.168}{6}\right)^6 - 1 = 18.02\%$

17.81% < 18.02% ⇒ the first card (16.5% per year compounded monthly) is better.

أخذت قرض لمدة 8 سنوات عليه فائدة 10٪ يتم احتسابها كل سنة
هاد القرض بدي أسدده على دفعتين ، نهاية السنة الرابعة ونهاية السنة الخامسة
(دفعتين متساويتين)

Nominal and Effective interest rates

Example: A loan of $2,000 at 10% annual interest rate for 8 years is to be repaid in [two equal payments] @ EOY 4 and EOY 8. What is the value of the payments?

لو بدي أحسب كل 4 سنوات عبارة عن interest per. period
وحدة بالباقي عندي 2 interest و A ثابتة بحسب لـ 4 سنوات

[Consider every 4 years as one payment]

$\Rightarrow r = 40\%$ per 4 years compounded annually.

$$i = \left(1 + \frac{0.4}{4}\right)^4 - 1 = 46.41\% \text{ per 4 years}$$

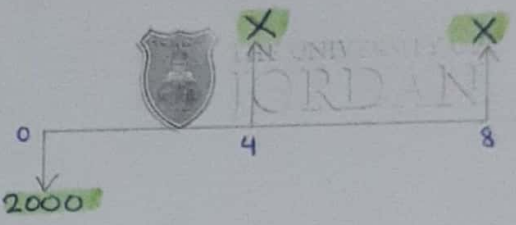
$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

Using A/P relationship:

$$A = \$2{,}000 \times \frac{(0.4641 \times 1.4641^2)}{1.4641^2 - 1} = \$1{,}739.9 \text{ every 4 years}$$

* طريقة حل ثانية نرسم cash flow وخط
X عند 4 وعند 8 ونرجعهم لـ <= zero

لـ كل 4 سنوات راح أدفع هذا
المبلغ عشان أسدد الـ 2000 اللي أخذتهم
خلال 8 سنوات .



$2000 = X(P/F, 10\%, 4) + X(P/F, 10\%, 8)$

$2000 = 0.6830X + 0.4665X$

$2000 = 1.1495X$

$X = 1739.9\ \$$

Nominal and Effective interest rates

If the monthly interest rate is 1%, what is the effective [semi-annual] rate?

لـ الـ eff
بكل 6 أشهر

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

$\Rightarrow$ Monthly rate = 1% = effective monthly = nominal monthly (no additional info on compounding is provided).

Nominal semi-annual rate = [6 × 1% = 6%]

Effective semi-annual rate $i = \left(1 + \frac{0.06}{6}\right)^6 - 1 = 6.15\%$

Nominal and Effective interest rates

راح أدفع دفعات
بشكل شهري
بقيمة 477 ↗

EXAMPLE 4-32: A loan of $15,000 requires monthly payments of $477 over a 36-month period of time. These payments include both principal and interest.

مدة 36 شهر

(a) What is the nominal interest rate (annual percentage rate (APR)) for this loan?

(b) What is the effective interest rate per year?

(c) Determine the amount of unpaid loan principal after 20 months.

↳ بعد ما أدفع 20 دفعة اد 16 دفعة
يلي منلو ع اد equiv. بالهم

Nominal and Effective interest rates

(a) What is the nominal interest rate (APR) for this loan?

P = $15,000, A = $477, and N = 36 months.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$\$477 = \$15{,}000(A/P, i\text{mo}, 36) \rightarrow (A/P, i\text{mo}, 36) = 0.0318$$

We can now look through Appendix C to find values of i that have an $(A/P, i, 36)$ value close to 0.0318. From Table C-3 (i = 3/4%), we find $(A/P, 3/4\%, 36) = 0.0318$. Therefore,

$i\text{mo} = 0.75\%$ per month And $r = 12 \times 0.75\% = 9\%$ per year, compounded monthly.

(b) What is the effective interest rate per year?

$$i_{eff} = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.0938 \text{ or } 9.38\% \text{ per year.}$$

(c) Determine the amount of unpaid loan principal after 20 months?

$P20 = \$477(P/A, 3/4\%, 16) = \$477(15.0243) = \$7{,}166.59$

- Notice that we used the monthly interest rate of 3/4% in our calculation since the cash flows are occurring monthly

Continuous compounding

Allowing interest to compound continuously throughout the period ⇒ M approaches ∞.

$$i = e^r - 1$$

Where i is the effective rate and r is the nominal rate.

* عندي هلا يقين الحل، اما بحول لـ i
وبستخدم القوانين بي جنبك، أو بخلي
r وبستخدم القوانين بي هون.

Continuous compounding factors

$$(F/P, r\%, N) = e^{rN}$$

$$(P/F, r\%, N) = e^{-rN} = \frac{1}{e^{rN}}$$

$$(F/A, r\%, N) = \frac{e^{rN} - 1}{e^r - 1}$$

$$(P/A, r\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^r - 1)}.$$

Notice: r is substituted (not i)
So we can use these formulas or we can substitute the effective interest rate (i) in P/F, F/A, and P/A equations presented earlier in the chapter

Continuous compounding

Example: A bank offers loans at an annual interest rate of 12% compounded continuously,

- What is the effective annual interest rate?

$r = 0.12$ (nominal annual)

$i = e^{0.12} - 1 = 0.1275 = 12.75\%$

- What is the effective monthly interest rate?

$r = \frac{0.12}{12} = 0.01$ (nominal monthly)

$i = e^{0.01} - 1 = 0.01005 = 1.005\%$

- If you borrowed $10,000 (P) on these terms, what is the future equivalent (F) of this loan after 5 years (N)?

$(F/P, r\%, N) = e^{rN} = e^{0.12 \cdot 5} = 1.8221 \Rightarrow F = \$10{,}000 \times 1.8221 = \$18{,}221$

Or $F = P \times (1+i)^N = \$10{,}000 \times (1 + 0.1275)^5 = \$18{,}221$

Or using the monthly interest: $F = \$10{,}000 \times (1 + 0.01005)^{60} = \$18{,}221$

* قوانين الـ cont. ← القوانين القديمة
* اذا بدي بالشهر
5 years × 12 = 60
برضو بطلع نفس الجواب N = 60

Continuous compounding

Example: A nominal interest rate of 8% is compounded continuously.

- What is the uniform EOY amount for 10 years that is equivalent to $8,000 at EOY 10?

$F = \$8{,}000$ $A = ?$

$$A = \$8{,}000\,(A/F, 8\%\ nominal, 10) = \$8{,}000 \times \frac{e^{0.08}-1}{e^{0.08\times 10}-1} = \$543.68$$

- What is the present equivalent value of $1,000 per year for 12 years?

$A = \$1{,}000$ $P = ?$

$$P = \$1{,}000\,(P/A, 8\%\ nominal, 12) = \$1{,}000 \times \frac{e^{0.08\times 12}-1}{e^{0.08\times 12}(e^{0.08}-1)} = \$7{,}409.3$$

Or $P = \$1{,}000\ (P/A, i_{eff=0.08329}, 12) = \$1{,}000 \times 7.4094 = \$7{,}409.4$

- What is the future equivalent at the end of the 6th year of $243 payments made **every** 6 months during the 6 years (first payment occurs 6 months from the present and the last occurs at EOY 6)?

$243 ← مدة 6 سنوات يدفع كل 6 أشهر

$$F = \$243\,(F/A, 4\%, 12) = \$243 \times \frac{e^{0.04\times 12}-1}{e^{0.04}-1} = \$3{,}668.3$$

The end

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Engineering Economy
(0901420)

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Slides 8

Chapter 5: Evaluating a Single Project

Topics to be covered this week:

- Evaluating a single project
- Equivalent worth methods
  - Present worth (PW). ⇝ at year 0.
  - Future worth (FW). ⇝ at future intrest period.
  - Annual worth (AW). ⇝ come from a uniform payment.
- Applications PW: Bonds

2- اذا بشتغل ب org. والمشروع موجود عندي ، هل بتحمل أخسر شوي صغيرة أو بتحمل ال profit يكون margenaly مقارنة بال capital inv. ، فبقدر أحدد شو قيمة أقل نسبة بدي أربحها بمشروع معين .

3- هل أنا كشركة ما بدي آخذ risks عن أي مشروع وبدي دائما MARR تكون عالية وإذا مو عالية ما بدي المشروع .

Methods for evaluating a single project

اذا كان عندي inv. وعملت الحسبة باستعمال ال MARR وكانت ال revenue تساوي ال expenses أو أكثر معناها إنت حققت MARR

Minimum Attractive Rate of Return **(MARR):** The lowest internal **rate of return** that the organization would consider a good investment.

- The Minimum Attractive Rate of Return (MARR) is usually a policy issue resolved by an organization's top management given numerous considerations.

قديش الفلوس الموجودة في ال org بتفرق وعلى هذا الأساس بقدر أشوف شو ال profet rate يلي بدي تدخل علي من ال inves

Among these considerations are the following:

1. Amount, source, and cost of money available
2. Number and purpose of good projects available
3. Perceived risk of investment opportunities
4. Type of organization (i.e., government, public utility, or private industry)

وال source of money ، مثلاً لو أخذت قرض عشان أبلش بال inve يلي أنا بدي أشتغله والقرض ال %6 = i ال MARR بدها تكون أكثر من ال i ؟ حتى أتأكد انو راح يقوت علي inv. يلي بدي أشتغل والفائدة يلي راح تتراكم عليك .

بيعتمد على الشركة وعلى ال culture وال structure لل org يلي بشتغل فيها .

الشركات الحكومية MARR بتكون محددة من وزارة الأشغال أو رئاسة الوزراء .

AW → بحول كل اشي ل uniform payment وبجمع كل ال revene يلي تمثل anuety وبطرح من ال expenses

عندي فكرة مشروع وبدي أشوف هل راح يعطيني ربح وأفوت فيه أو راح يخليني أخسر وما بدي اياه

Methods for evaluating a single project

- Present worth (PW) (the most common method). → بدي أرسم ال cash flow وأحول كل اشي ل year 0
- Future worth (FW). → نفس ال PW بس بجمع كل اشي بال future
- Annual worth (AW).
- Internal rate of return (IRR). — it will be by the end of year (inves. t)
- External rate of return (ERR). — لو المشروع 5 سنوات بوجدلها FW بنهاية السنة الخامسة
- Payback period: least common method. → الفترة الزمنية يلي بحتاجها لأرجع الفلوس يلي استثمرتها في المشروع .

PW = revenue − expenses

الفلوس يلي بتفوت علي حقيقتها at year 0

الفلوس يلي أنا بدفعها وال equi الها at year 0

PW > 0 → revenue أكثر بالتالي good inv إذا أقل bad inv وما بدي اياه .

A project must provide a return that is equal to or greater than the Minimum Attractive Rate of Return (MARR).

لازم دائما آخذ ال MARR للشركة يلي بشتغل فيها بعين الاعتبار .

Methods for evaluating a single project

ملاحظة سؤال بشكل عام بدي أعتقد انه ال interest rate تكون at the end of year period وبنفس الوقت الفلوس بجلها simple مش compounded مثلاً، لو بالسؤال السادس إجاني 5000 أنا بجلهم بنهاية السنة الأولى، في حال ماحكو الدفعات متى بتجي بفرض إنها راح تتحمل نهاية حياة المشروع.

Unless otherwise specified, the end-of-period cash-flow convention and discrete compounding of interest are used throughout this and subsequent chapters. A planning horizon, or study (analysis) period, of *N* compounding periods (usually years), is used to evaluate prospective investments throughout the remainder of this course.

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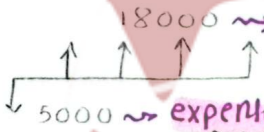
Present worth (PW)

* بالسؤال ما بجد ال method يلي بستخدمها بس بالعادة بنروح PW أو FW

- All cash inflows and outflows are discounted to the present time at an interest rate (generally MARR).
  ↳ at year 0

- PW $(i = \text{MARR}) \geq 0 \Rightarrow$ acceptable project (profit required by investors is satisfied or exceeded).
  ↳ منيح راح يحقق ربح

Example: A project has a capital investment of $50,000 (P) and returns $18,000 (A) per year for 4 (N) years. At a 12% MARR (i), is this a good investment?



$PW = -50{,}000 + 18{,}000\,(P/A, 12\%, 4)$
لانها تكاليف

$PW = -50{,}000 + 18{,}000 \times 3.0373 = \$4{,}671.40$ → It is a good investment.

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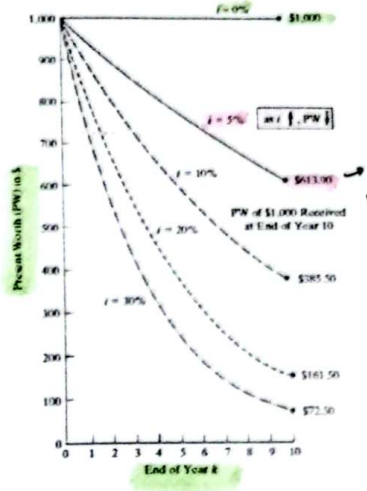
PW Example

كلما زادت الـ i ، كلما قلت الـ PW (علاقة عكسية)

- It is important to observe that the higher the interest rate and the farther into the future a cash flow occurs, the lower its PW is.

➢ PW of $1,000 Received at the End of Year *k* at an Interest Rate of *i*% per Year

* كلما زادت الفترة الزمنية كلما قلت الـ PW الها .



كلما قلت i ، زادت الـ PW

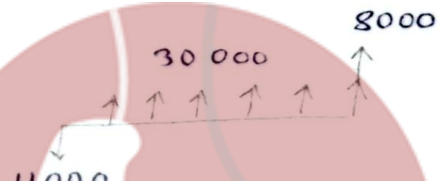
PW Example

بما أنها saving بقدر أموالها لفلوس وأخرنا حدش وفرت

A new heating system is to be purchased and installed for $110,000. This system will save approximately 300,000 kWh of electric power each year for a 6-year period with no additional O&M costs. Assume the cost of electricity is $0.10 per kWh, and company's MARR is 15% per year, and the market value of the system will be $8,000 at EOY 6. Using the PW method, is this a good idea?

P

= F



$$\text{Estimated annual savings} = \frac{300{,}000\ \text{kWh}}{\text{year}} \times \frac{\$0.10}{\text{kWh}} = \$30{,}000 \text{ per year.}$$

$$PW\ (i = 15\%) = -\$110{,}000 + \$30{,}000\,(P/A, 15\%, 6) + \$8{,}000\,(P/F, 15\%, 6)$$
$$= -\$110{,}000 + \$30{,}000 \times 3.7845 + \$8{,}000 \times 0.4323 = \$\,6{,}993.4$$

➔ Good investment

Present worth (PW)

There are two assumptions that we make when using PW:

- First, it is assumed that we know the future with certainty (For example, we presume to know with certainty future interest rates and other factors).
- Second, it is assumed we can borrow and lend money at the same interest rate (i.e., capital markets are perfect).

Future Worth (FW)

The future Worth (FW) method is an alternative to the PW method

- Looking at FW is appropriate since the primary objective is to maximize the future wealth of owners of the firm.
- FW is based on the equivalent worth of all cash inflows and outflows at the end of the study period at an interest rate that is generally the MARR.
- Decisions made using FW and PW will be the same.

Future Worth (FW)

في حال كان المطلوب FW
واحنا حسبنا على الـ PW
بس كل اللي علينا
نحول الجواب اللي
طلع لـ Future
(F/P)

- FW is equivalent to PW $[FW = PW\,(F/P, i\%, N)]$.
- FW ≥ 0, project is economically justified.

Example: A $45,000 investment in a new conveyer system is projected to improve throughout and increase revenue by $14,000 per year for five years. The estimated market value of the conveyer at the end of five years is $4,000. Using the FW method at a MARR of 12%, is this a good investment?

$FW = -\$45{,}000\,(F/P, 12\%, 5) + \$14{,}000\,(F/A, 12\%, 5) + \$4{,}000 = \$13{,}635.7$

$\Rightarrow$ It is a good investment.

اشتريت conveyer بـ 45000 بتجة اني اشتريته راح
لحسن الأداء ويزيد الإيرادات بـ 14000 كل سنة لمدة
5 سنوات ، بعد ما تخلص الـ 5 سنوات بدنا نبيعه كخردة
بما يقارب 4000 د.ك

Future Worth (FW)

Example: A $110,000 retrofitted space-heating system was projected to save $30,000 per year in electrical power and be worth $8,000 at the end of the six-year study period. Use the FW method to determine whether the project is still economically justified if the system has zero market value after six years. The MARR is 15% per year.

هل المشروع
راح يفضل مربح إذا أنا ما أخذت
الـ 8000 اللي راح نبيعها نهاية
السنة السادسة ، بس لو مش
حاكي هاي المعلومة ، بدي أجمع
8000

- FW(15%) = $-\$110{,}000\,(F/P, 15\%, 6) + \$30{,}000\,(F/A, 15\%, 6)$
  $= -\$110{,}000\,(2.3131) + \$30{,}000\,(8.7537)$
  $= \$8{,}170.$
  ↳ positive

بالتالي هنا بحقق ربح

Annual Worth (AW)

Annual Worth (AW) is another way to assess projects.

- Equal annual series equivalent to the cash inflows and outflows at a specific interest rate (normally MARR).
- AW is equivalent to PW and FW.
- AW ≥ 0, the project is economically justified.

برضو هون نفس الكبدأ
لو حسب PW بقدر أحسب AW (A/P)

Annual Worth (AW)

The AW of a project is annual equivalent revenue or savings minus annual equivalent expenses, less its annual capital recovery (*CR*) amount.

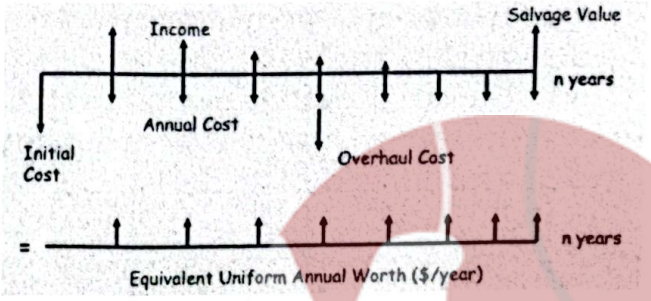
$$AW(i\%) = R - E - CR(i\%)$$

annuity الـ
الناتجة من الـ P
تاخفي الـ A الناتجة من
F ، أو بجول كل اشي لـ
A وعلى أساسها بتخذ
القرار.

Where:

R: annual equivalent revenue.

E: annual equivalent expenses.



CR: annual capital recovery which covers the loss in value of the asset and interest (at MARR) on invested capital.

$$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$$

* اذا A (+) انا بربح
كل سنة. اذا (-) أنا خسرت كل سنة.

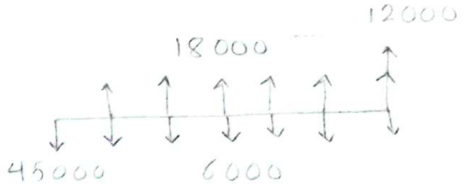
Where I is the initial cost and S is the salvage value.

* P ← صار بسنة .
* F ← بنفس السنة .

Annual Worth (AW)

Example: A project requires an initial investment of $45,000, has a salvage value of $12,000 after six years, incurs annual expenses of $6,000, and provides annual revenue of $18,000. Using a MARR of 10%, determine the AW of this project.

$$AW(10\%) = \underline{R} - \underline{E} - CR(10\%)$$



R = $18,000

E = $6,000

$$CR(10\%) = \$45{,}000\,(A/P, 10\%, 6) - \$12{,}000\,(A/F, 10\%, 6) = \$8{,}777$$

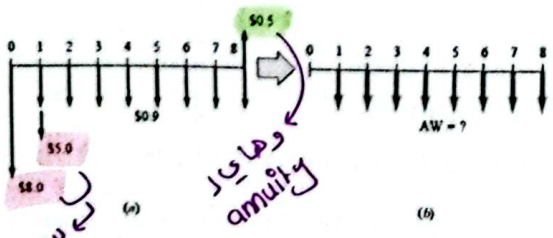
$$AW(10\%) = 18{,}000 - 6{,}000 - 8{,}777 = \$3{,}223$$

Since the *AW* is positive, it's a good investment.

رأس المال يلي بدي استعمله راح يكون
13 مليون
8 مليين at year 0
5 مليين at year 1

Annual Worth (AW)

Example: Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in opening up new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of $13 million, with $8 million committed now and the remaining $5 million expended at the end of year 1 of the project. Annual operating costs for the system are expected to start the first year and continue at $0.9 million per year. The useful life of the tracker is 8 years with a salvage value of $0.5 million. Calculate the CR and AW values for the system, if the corporate MARR is 12% per year.



$$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$$

$$AW(i\%) = R - E - CR(i\%)$$

وطلع annuity

بدي أحول هيك annuity

Annual Worth (AW)

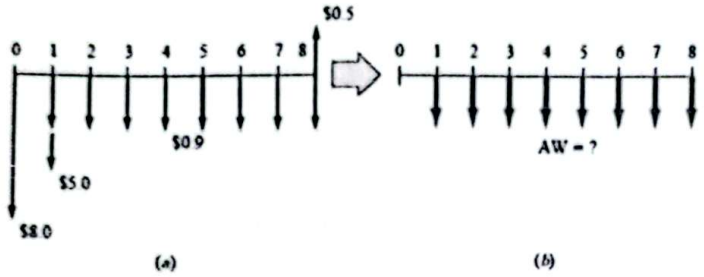
$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$

I = 8 + 5(P/F, 12%, 1) = $12.46M; S = 0.5 M

CR = 12.46M(A/P, 12%, 8) - 0.5M(A/F, 12%, 8) = 12.46M(0.20130) - 0.5M(0.08130) = $2.47M

$AW(i\%) = R - E - CR(i\%)$

AW = - 0.9 -2.47 = $-3.37 million per year



انت بتشتري سند من البنك ، بتدفع مبلغ عشان
كمثل هاد الـ Bond

Applications PW: Bonds

السندات

& by default you are start getting an annual monthly or quarterly depend on the intrest period throuout the lifespand by the bond ... وهيك

Bond value is a good example of present worth.

$V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

Where:

بدها تكون بالمستقبل

V_N: value (price) of the bond N interest periods prior to redemption (or present worth).

Z: face, or par value of the bond.

C: redemption or disposal price (usually equal to Z).

r: bond rate (nominal interest) per interest period.

N: number of periods before redemption.

i: bond yield rate per period.

Applications PW: Bonds

Example: A bond with a face value of \$5,000 pays interest of 8% per year. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due one year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for \$4,600, what annual bond yield would the buyer receive?

(a) $V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

- The value of *VN* can be determined:

C = Z = \$5,000, r = 8%, i = 10%, N = 20

V_N = \$5,000(*P/F*, 10%, 20) + \$5,000(0.08)(*P/A*, 10%, 20)

= \$743.00 + \$3,405.44 = \$4,148.44.

لم أبلغ بحيا مفروض
Bond تدفعه للحقل ال

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Applications PW: Bonds

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لو دفعنا ← V_N = 4600 حتى نبش ال interest rate

(b) we are given *VN* = \$4,600, and we must find the value of *i*%??

$V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

C = Z = \$5,000, r = 8%, i = ??, N = 20

\$4,600 = \$5,000(*P/F*, *i*'%, 20) + \$5,000(0.08)(*P/A*, *i*'%, 20).

$(P/F, i\%, N) = \frac{1}{(1+i)^N}$; $(P/A, i\%, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$

- To solve for *i*'%, we can resort to an iterative trial-and-error procedure (e.g., try 8.5%, 9.0%) to determine that

i'% = 8.9% per year.

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ask ;; believe & recieve

Applications PW: Bonds

Example: A bond has a face value of $10,000 and matures in 8 years. The bond stipulates a fixed nominal interest of 8% per year, but interest payments are made to the bondholder every 3 months. The bondholder wishes to earn 10% nominal annual interest (compounded quarterly). Assuming the redemption value is equal to the face value, how much should be paid for the bond now?

Z = face value = $10,000.

C = redemption value = $10,000.

i = 10% nominal per year = 2.5% per quarter. 10%/4

N = 8 years = 32 quarters. 8*4

r = 8% per year = 2% per quarter. 8%/4

$V_N = \$10{,}000\,(P/F, 2.5\%, 32) + 0.02 \times \$10{,}000\,(P/A, 2.5\%, 32) = \$8{,}907.55$

⇒ The bondholder should pay no more than $8,907.55 for the purchase of the bond.

Applications PW: Bonds

Example: What is the value of a 6%, 10-year bond with a par (and redemption) value of $20,000 that pays dividends semi-annually, if the purchaser wishes to earn an 8% return?

6%/2 10*2 8%/2

$V_N = \$20{,}000\,(P/F, 4\%, 20) + (0.03)\$20{,}000\,(P/A, 4\%, 20)$

$V_N = \$20{,}000\,(0.4564) + (0.03)\$20{,}000\,(13.5903)$

$V_N = \$17{,}282.18$

Engineering Economy
(0901420)

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Slides 9

Chapter 5: Evaluating a Single Project

Topics to be covered this week:

- Applications PW: Capitalized worth
- Internal Rate of Return (IRR)
- External Rate of Return (ERR)

Engineering Economy
(0901420)

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Slides 8

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Chapter 5: Evaluating a Single Project

عشي مشروع بدي أحلل أعطله ↩ وأشوف إذا فتت على هاد evaluation المشروع بالدنا والمعلومات يلي عندي ، هل راح أحقق ربح ولا أخسر ، على هذا الأساس كمهندسين بدنا نتخذ القرار .

Topics to be covered this week:

- Evaluating a single project
- Equivalent worth methods
  - Present worth (PW). ⇝ at year 0.
  - Future worth (FW). ⇝ at future intrest period.
  - Annual worth (AW). ⇝ come from a uniform payment.
- Applications PW: Bonds

2- اذا بشتغل ب org. والمشروع موجود عندي ، هل بتحمل أخسر شوي صغيرة أو بتحمل ال profit يكون margenaly مقارنة بال capital inv. فبقدر أحدد شو قيمة أقل نسبة بدي أربحها بمشروع معين . 3- هل أنا كشركة مابدي آخذ risks عن أي مشروع وبدي دائماً MARR تكون عالية وإذا مو عالية مابدي المشروع.

Methods for evaluating a single project

Minimum Attractive Rate of Return (MARR): The lowest internal **rate of return** that the organization would consider a good investment.

- The Minimum Attractive Rate of Return (MARR) is usually a policy issue resolved by an organization's top management given numerous considerations.

Among these considerations are the following:

1. Amount, source, and cost of money available
2. Number and purpose of good projects available
3. Perceived risk of investment opportunities
4. Type of organization (i.e., government, public utility, or private industry)

إذا كان عندي inves. وعملت الحسبة باستعمال ال MARR وكانت ال revenue تساوي ال expenses أو أكثر معناها انت حققت MARR

قديش المصاري الموجودة في ال org وعلى هذا الأساس بقدر أشوف شو ال profet rate يلي بدي تدخل علي من ال inves وال source of money ، مثلاً لو أخذت قرض عشان أبلش بال inve يلي أنا بدي أشتغله والقرض ال 6% ؟ ال MARR بدها تكون أكثر من ال ؟ حتى أتأكد انو راح يفوت علي invest

بيعتمد على الشركة وعلى ال culture وال structure لل org يلي بشتغل فيها .

للشركات الحكومية MARR بتكون محددة من وزارة الأشغال أو رئاسة الوزراء .

بقدر تقيس الفائدة يلي راح تتراكم عليك .

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

AW → بحول كل اشي ل uniform payment وبجمع كل ال revene يلي تمثل anuety وبطرح من ال expenses

عندي فكرة مشروع وبدي أشوف هل راح يعطيني ربح وأفوت فيه أو راح يخليني أخسر ومابدي إياه

Methods for evaluating a single project

- Present worth (PW) (the most common method). → بدي أرسم ال cash flow وأحول كل اشي ل year 0
- Future worth (FW). → نفس ال PW بس بجمع كل اشي بال Future it will be by the end of year (inves. t) لو المشروع 5 سنوات بوجد ال FW بنهاية السنة الخامسة
- Annual worth (AW).
- Internal rate of return (IRR).
- External rate of return (ERR).
- Payback period: least common method. → الفترة الزمنية يلي بحتاجها لأرجع الفلوس يلي استثمرتها في المشروع .

PW = revenue − expenses

الفلوس يلي بتفوت علي حقيقتها at year 0

الفلوس يلي أنا بدفعها وال equi لها at year 0

PW > 0 → revenue أكثر بالتالي good inv إذا أقل bad inv ومابدي إياه .

A project must provide a return that is equal to or greater than the Minimum Attractive Rate of Return (MARR).

لازم دائماً آخذ ال MARR للشركة يلي بشتغل فيها بعين الاعتبار .

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Methods for evaluating a single project

ملاحظة سؤال بشكل عام برجي أعتقد انه الـ interest rate تكون at the end of year period وبنفس الوقت الفلوس بجلها simple مش compounded مثلاً، لو بالشهر السادس إجاني 5000 أنا بجلهم بنهاية السنة الأولى، في حال ماتكو الدفعات متى بتجيبو بفرض إنها راح تحصل نهاية حياة المشروع.

Unless otherwise specified, the end-of-period cash-flow convention and discrete compounding of interest are used throughout this and subsequent chapters. A planning horizon, or study (analysis) period, of *N* compounding periods (usually years), is used to evaluate prospective investments throughout the remainder of this course.

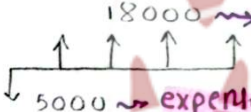
Present worth (PW)

* بالسؤال ما بجد الـ method برجي بستخدمها بس بالعادة بنزح PW أو FW

- All cash inflows and outflows are discounted to the present time at an interest rate (generally MARR).
  ↳ at year 0

- PW (i = MARR) ≥ 0 ⇒ acceptable project (profit required by investors is satisfied or exceeded).
  ↳ منيح راح يحقق ربح

Example: A project has a capital investment of $50,000 (P) and returns $18,000 (A) per year for 4 (N) years. At a 12% MARR (i), is this a good investment?



$PW = -50{,}000 + 18{,}000\,(P/A, 12\%, 4)$ لإنها تكاليف

$PW = -50{,}000 + 18{,}000 \times 3.0373 = \$4{,}671.40$ → It is a good investment.

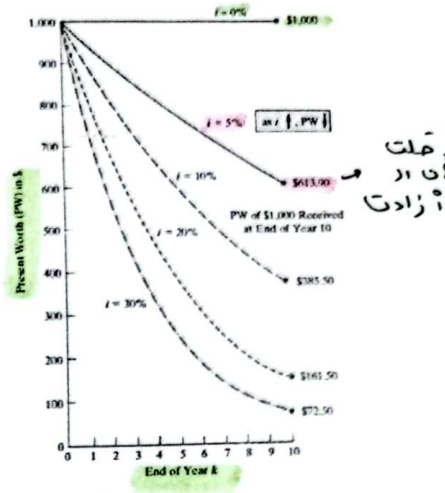
PW Example

كلما زادت الـ i كلما قلت الـ PW (علاقة عكسية)

- It is important to observe that the higher the interest rate and the farther into the future a cash flow occurs, the lower its PW is.

➢ PW of $1,000 Received at the End of Year *k* at an Interest Rate of *i*% per Year

* كلما زادت الفترة الزمنية كلما قلت الـ PW الها.

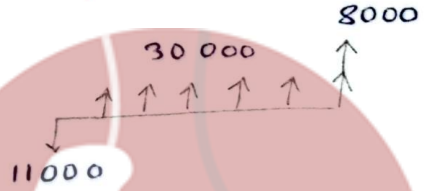


قلت لأن الـ i زادت

PW Example

بما إنها saving بقدر أحولها لفلوس وأحرنا قديش وفرت

A new heating system is to be purchased and installed for $110,000 (P). This system will save approximately 300,000 kWh of electric power each year for a 6-year period with no additional O&M costs. Assume the cost of electricity is $0.10 per kWh, and company's MARR is 15% per year, and the market value of the system will be $8,000 (F) at EOY 6. Using the PW method, is this a good idea?



$$\text{Estimated annual savings} = \frac{300{,}000\ \text{kWh}}{\text{year}} \times \frac{\$0.10}{\text{kWh}} = \$30{,}000 \text{ per year.}$$

$$\text{PW}\ (i = 15\%) = -\$110{,}000 + \$30{,}000\,(P/A, 15\%, 6) + \$8{,}000\,(P/F, 15\%, 6)$$
$$= -\$110{,}000 + \$30{,}000 \times 3.7845 + \$8{,}000 \times 0.4323 = \$\,6{,}993.4$$

➔ Good investment

Present worth (PW)

There are two assumptions that we make when using PW:

- First, it is assumed that we know the future with certainty (For example, we presume to know with certainty future interest rates and other factors).
- Second, it is assumed we can borrow and lend money at the same interest rate (i.e., capital markets are perfect).

Future Worth (FW)

The future Worth (FW) method is an alternative to the PW method

- Looking at FW is appropriate since the primary objective is to maximize the future wealth of owners of the firm.
- FW is based on the equivalent worth of all cash inflows and outflows at the end of the study period at an interest rate that is generally the MARR.
- Decisions made using FW and PW will be the same.

Future Worth (FW)

في حال كان اكطلوب FW
وإحنا حسبنا على اد PW
بس كل يلي علينا
نحول الجواب يلي
طلع د Future
(F/P)

- FW is equivalent to PW $[FW = PW\,(F/P, i\%, N)]$.
- FW ≥ 0, project is economically justified.

Example: A $45,000 investment in a new conveyer system is projected to improve throughout and increase revenue by $14,000 per year for five years. The estimated market value of the conveyer at the end of five years is $4,000. Using the FW method at a MARR of 12%, is this a good investment?

FW = $-\$45{,}000\,(F/P, 12\%, 5) + \$14{,}000\,(F/A, 12\%, 5) + \$4{,}000 = \$13{,}635.7$

⇒ It is a good investment.

اشتريت conveyer بـ 45000 نتيجة اني اشتريته راح
يحسن الأداء ويزيد الإيرادات بـ 14000 كل سنة لمدة
5 سنوات ، بعد ما تخلص الـ 5 سنوات بدنا نبيعه كخردة
بما يقارب 4000 دينار.

Future Worth (FW)

Example: A $110,000 retrofitted space-heating system was projected to save $30,000 per year in electrical power and be worth $8,000 at the end of the six-year study period. Use the FW method to determine whether the project is still economically justified if the system has zero market value after six years. The MARR is 15% per year.

هل المشروع ↳
راح يضل مربح إذا أنا ما أخذت
الـ 8000 يلي راح نبيعها نهاية
السنة السادسة ، بس لو مش
حاكي هاي المعلومة ، بيي أجمع
8000

FW(15%) = $-\$110{,}000\,(F/P, 15\%, 6) + \$30{,}000\,(F/A, 15\%, 6)$

$= -\$110{,}000\,(2.3131) + \$30{,}000\,(8.7537)$

$= \$8{,}170.$

↳ positive

بالتالي هنا يحقق ربح

Annual Worth (AW)

Annual Worth (AW) is another way to assess projects.

- Equal annual series equivalent to the cash inflows and outflows at a specific interest rate (normally MARR).
- AW is equivalent to PW and FW.
- AW ≥ 0, the project is economically justified.

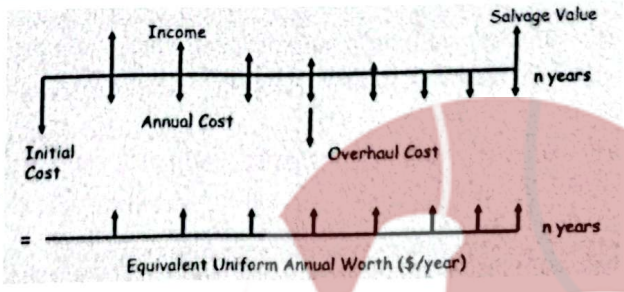
برضو هون نفس الكبدأ
لو حسب PW بجدر أحسب AW (A/P)

Annual Worth (AW)

The AW of a project is annual equivalent revenue or savings minus annual equivalent expenses, less its annual capital recovery (*CR*) amount.

$$AW(i\%) = R - E - CR(i\%)$$

annuity اد الناتجة من اد P
ناقص اد A الناتجة من F، أو بجول كل اشي لـ A وبكل أسلها بتجذ القوار.



Where:

R: annual equivalent revenue.

E: annual equivalent expenses.

CR: annual capital recovery which covers the loss in value of the asset and interest (at MARR) on invested capital.

$$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$$

Where I is the initial cost and S is the salvage value.

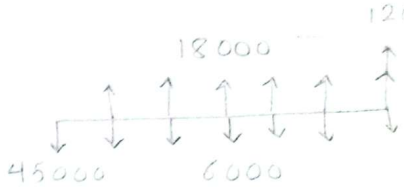
* اذا A (+) انا بربح
كل سنة إذا (-) أنا خسران كل سنة.

* P ← صار بسنة .
* F ← بنفس السنة .

Annual Worth (AW)

Example: A project requires an initial investment of $45,000, has a salvage value of $12,000 after six years, incurs annual expenses of $6,000, and provides annual revenue of $18,000. Using a MARR of 10%, determine the AW of this project.

$$AW(10\%) = \underline{R} - \underline{E} - CR(10\%)$$



R = $18,000

E = $6,000

$$CR(10\%) = \$45{,}000\,(A/P, 10\%, 6) - \$12{,}000\,(A/F, 10\%, 6) = \$8{,}777$$

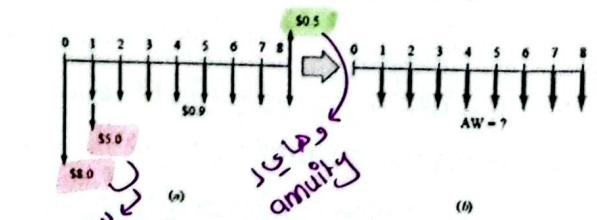
$$AW(10\%) = 18{,}000 - 6{,}000 - 8{,}777 = \$3{,}223$$

Since the *AW* is positive, it's a good investment.

رأس المال يلي بدي استعمله راح يكون
13 مليون
8 ملايين at year 0
5 ملايين at year 1

Annual Worth (AW)

Example: Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in opening up new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of $13 million, with $8 million committed now and the remaining $5 million expended at the end of year 1 of the project. Annual operating costs for the system are expected to start the first year and continue at $0.9 million per year. The useful life of the tracker is 8 years with a salvage value of $0.5 million. Calculate the CR and AW values for the system, if the corporate MARR is 12% per year.



هدول أصول بدي ياهم annuity

وهاي annuity

$$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$$

$$AW(i\%) = R - E - CR(i\%)$$

Annual Worth (AW)

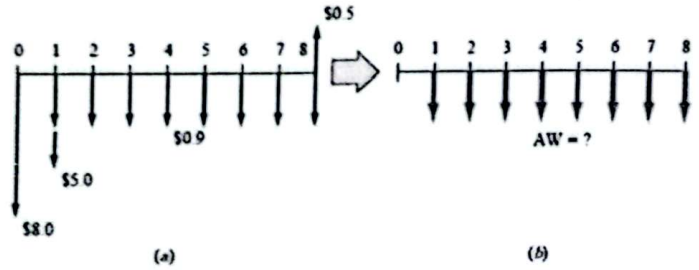
$CR(i\%) = I\,(A/P, i\%, N) - S\,(A/F, i\%, N)$

I = 8 + 5(P/F, 12%, 1) = $12.46M; S = 0.5 M

CR = 12.46M(A/P, 12%, 8) - 0.5M(A/F, 12%, 8) = 12.46M(0.20130) - 0.5M(0.08130) = $2.47M

$AW(i\%) = R - E - CR(i\%)$

AW = - 0.9 -2.47 = $-3.37 million per year



انت بتشتري سند من البنك ، بتدفع مبلغ عشان
كمثل هاد الـ Bond

Applications PW: Bonds السندات

& by default you are start getting an annual monthly or quarterly depend on the intrest period throout the lifespand by the bond ...

Bond value is a good example of present worth.

$V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

Where: بدها تكون بالمستقبل

V_N: value (price) of the bond N interest periods prior to redemption (or present worth).

Z: face, or par value of the bond.

C: redemption or disposal price (usually equal to Z).

r: bond rate (nominal interest) per interest period.

N: number of periods before redemption.

i: bond yield rate per period.

Applications PW: Bonds

Example: A bond with a face value of $5,000 pays interest of 8% per year. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due one year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for $4,600, what annual bond yield would the buyer receive?

(a) $V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

- The value of *VN* can be determined:

C = Z = $5,000, r = 8%, i = 10%, N = 20

V_N = $5,000(*P/F*, 10%, 20) + $5,000(0.08)(*P/A*, 10%, 20)

= $743.00 + $3,405.44 = $4,148.44.

لم المبلغ يجي مفروض
تدفعه للحقل ال Bond

Applications PW: Bonds

Example: A bond with a face value of $5,000 pays interest of 8% per year. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due one year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for $4,600, what annual bond yield would the buyer receive?

لو دفعت ↳ V_N ~ 4600 حد بش ال interest rate

(b) we are given *VN* = $4,600, and we must find the value of *i*%??

$V_N = C\,(P/F, i\%, N) + r\,Z\,(P/A, i\%, N)$

C = Z = $5,000, r = 8%, i = ??, N = 20

$4,600 = $5,000(*P/F*, *i*'%, 20) + $5,000(0.08)(*P/A*, *i*'%, 20).

$(P/F, i\%, N) = \frac{1}{(1+i)^N}$; $(P/A, i\%, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$

- To solve for *i*'%, we can resort to an iterative trial-and-error procedure (e.g., try 8.5%, 9.0%) to determine that

i'% = 8.9% per year.

Applications PW: Bonds

Example: A bond has a face value of $10,000 and matures in 8 years. The bond stipulates a fixed nominal interest of 8% per year, but interest payments are made to the bondholder every 3 months. The bondholder wishes to earn 10% nominal annual interest (compounded quarterly). Assuming the redemption value is equal to the face value, how much should be paid for the bond now?

Z = face value = $10,000.

C = redemption value = $10,000.

i = 10% nominal per year = 2.5% per quarter. 10%/4

N = 8 years = 32 quarters. 8*4

r = 8% per year = 2% per quarter. 8%/4

$V_N = \$10{,}000\,(P/F, 2.5\%, 32) + 0.02 \times \$10{,}000\,(P/A, 2.5\%, 32) = \$8{,}907.55$

⇒ The bondholder should pay no more than $8,907.55 for the purchase of the bond.

Applications PW: Bonds

6%/2 10*2

Example: What is the value of a 6%, 10-year bond with a par (and redemption) value of $20,000 that pays dividends semi-annually, if the purchaser wishes to earn an 8% return? ↳ 8%/2

$V_N = \$20{,}000\,(P/F, 4\%, 20) + (0.03)\$20{,}000\,(P/A, 4\%, 20)$

$V_N = \$20{,}000\,(0.4564) + (0.03)\$20{,}000\,(13.5903)$

$V_N = \$17{,}282.18$

Engineering Economy
(0901420)

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Chapter 5: Evaluating a Single Project

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Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Applications PW: Capitalized worth

Capitalized worth is a special variation of present worth

- Capitalized worth (CW): special case of PW; where revenues or expenses occur over an infinite length of time.
- If only expenses are considered, then it is called capitalized cost.

The capitalized worth method is especially useful in problems involving grants and public projects with indefinite lives.

The CW of a series of end-of-period uniform payments A, with interest at *i%* per period, is *A(P/A, i%, N).*

Applications PW: Capitalized worth

The capitalized worth of a project with an interest rate of i% per year: is the annual equivalent of the project over its useful life divided by i.

The CW of a series of end-of-period uniform payments A, with interest at *i%* per period, is *A(P/A, i%, N).*

- As *N* becomes very large (if the *A* are perpetual payments), the (P/A) term approaches *[1/i]*. So, *CW = A(1/i).*

$$CW(i\%) = PW_{N\to\infty} = A(P/A, i\%, \infty) = A\left[\lim_{N\to\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\left(\frac{1}{i}\right)$$

Applications PW: Capitalized worth

As *N* becomes very large (if the *A* are perpetual payments), the (P/A) term approaches [1/i]. So, CW = A(1/i).

تـ القيمة الثابتة يلي بتكرر خلال ∞ period of time

TABLE C-7 Discrete Compounding: i = 4%

	Single Payment		Uniform Series				Uniform Gradient		
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor	
N	To Find F Given P F/P	To Find P Given F P/F	To Find F Given A F/A	To Find P Given A P/A	To Find A Given F A/F	To Find A Given P A/P	To Find P Given G P/G	To Find A Given G A/G	N
1	1.0400	0.9615	1.0000	0.9615	1.0000	1.0400	0.000	0.0000	1
2	1.0816	0.9246	2.0400	1.8861	0.4902	0.5302	0.925	0.4902	2
3	1.1249	0.8890	3.1216	2.7751	0.3203	0.3603	2.703	0.9739	3
4	1.1699	0.8548	4.2465	3.6299	0.2355	0.2755	5.267	1.4510	4
5	1.2167	0.8219	5.4163	4.4518	0.1846	0.2246	8.555	1.9216	5
6	1.2653	0.7903	6.6330	5.2421	0.1508	0.1908	12.506	2.3857	6
7	1.3159	0.7599	7.8983	6.0021	0.1266	0.1666	17.066	2.8433	7
8	1.3686	0.7307	9.2142	6.7327	0.1085	0.1485	22.181	3.2944	8
9	1.4233	0.7026	10.5828	7.4353	0.0945	0.1345	27.801	3.7391	9
10	1.4802	0.6756	12.0061	8.1109	0.0833	0.1233	33.881	4.1773	10
11	1.5395	0.6496	13.4864	8.7605	0.0741	0.1141	40.377	4.6090	11
12	1.6010	0.6246	15.0258	9.3851	0.0666	0.1066	47.248	5.0343	12
13	1.6651	0.6006	16.6268	9.9856	0.0601	0.1001	54.455	5.4533	13
14	1.7317	0.5775	18.2919	10.5631	0.0547	0.0947	61.962	5.8659	14
15	1.8009	0.5553	20.0236	11.1184	0.0499	0.0899	69.736	6.2721	15
16	1.8730	0.5339	21.8245	11.6523	0.0458	0.0858	77.744	6.6720	16
17	1.9479	0.5134	23.6975	12.1657	0.0422	0.0822	85.958	7.0656	17
18	2.0258	0.4936	25.6454	12.6593	0.0390	0.0790	94.350	7.4530	18
19	2.1068	0.4746	27.6712	13.1339	0.0361	0.0761	102.893	7.8342	19
20	2.1911	0.4564	29.7781	13.5903	0.0336	0.0736	111.565	8.2091	20
21	2.2788	0.4388	31.9692	14.0292	0.0313	0.0713	120.341	8.5779	21
22	2.3699	0.4220	34.2480	14.4511	0.0292	0.0692	129.202	8.9407	22
23	2.4647	0.4057	36.6179	14.8568	0.0273	0.0673	138.128	9.2973	23
24	2.5633	0.3901	39.0826	15.2470	0.0256	0.0656	147.101	9.6479	24
25	2.6658	0.3751	41.6459	15.6221	0.0240	0.0640	156.104	9.9925	25
30	3.2434	0.3083	56.0849	17.2920	0.0178	0.0578	201.062	11.6274	30
35	3.9461	0.2534	73.6522	18.6646	0.0136	0.0536	244.877	13.1198	35
40	4.8010	0.2083	95.0255	19.7928	0.0105	0.0505	286.530	14.4765	40
45	5.8412	0.1712	121.0294	20.7200	0.0083	0.0483	325.403	15.7047	45
50	7.1067	0.1407	152.6671	21.4822	0.0066	0.0466	361.164	16.8122	50
60	10.5196	0.0951	237.9907	22.6235	0.0042	0.0442	422.997	18.6972	60
80	23.0498	0.0434	551.2450	23.9154	0.0018	0.0418	511.116	21.3718	80
100	50.5049	0.0198	1237.6237	24.5050	0.0008	0.0408	563.125	22.9800	100
∞				25.0000		0.0400			∞

Applications PW: Capitalized worth

For example, a bridge was constructed for $1,900,000 and the annual upkeep cost is $25,000. It is also estimated that maintenance will be required for $350,000 every 8 years. What is the capitalized worth of the bridge over its life assuming MARR = 8%?

$$CW(8\%) = -\$1,900,000 - \$350,000\frac{A/F, 8\%, 8}{0.08} - \frac{\$25,000}{0.08}$$

$$= -\$2,623,815$$

P ← F ↓ بـ حولهاد A وراز A بيتسهلها لك

أ ← تتــا A (مشيتها لك) i

- If the bridge has an expected life of 50 years, what is the capitalized worth (CW) of the bridge over a 100-year study period?

$$CW(8\%) = -\$1,900,000 - \$1,900,000\ (P/F, 8\%, 50) - [\$350,000\ (A/F, 8\%, 8)]/0.08 - \$25,000/0.08.$$

Applications PW: Capitalized worth

Betty has decided to donate some funds to her local community college. Betty would like to fund an endowment that will provide a scholarship of $25,000 each year in perpetuity and a special award, "Student of the Decade," every ten years (again, in perpetuity) in the amount of $50,000. How much money does Betty need to donate today, in one lump sum, to fund the endowment? Assume the fund will earn a return of 8% per year.

* كم الفلوس يلي لازم أحطها اليوم حشان أحدر كل سنة اسحب 25000 وكل ١٠ سنين أسحب 50000 .

$A = \$50,000\ (A/F, 8\%, 10) = \$3,451.47$

$$CW = \frac{\$25,000 + \$3,451.47}{0.08} = \$355,643.43$$

* بمسك الـ revenue وبجولهم بدي أحسب شو الـ interest rate يلي بتخليني الـ revenue يساوي expenses متساويات والـ interest rate لـ year 0 وبنفس الاشي لـ expenses ، بعدين بدي أحسب هيقة أ يلي بتخلي الـ rev. يساوي الـ exp at the same period of time, we will do that at year zero.

Internal Rate of Return (IRR)

- The most widely used rate of return method in engineering economic analysis.
- Also called the investor's method, the discounted cash flow method, and the profitability index.
- The IRR is the interest rate that equates the equivalent worth of an alternative's cash inflows (revenue, R) to the equivalent worth of cash outflows (expenses, E).
- The IRR is sometimes referred to as the breakeven interest rate.

بدي أحسب أ يلي أسعاد تانية الها ؟ ∴ PW = 0

* IRR Decision Rule: If IRR ≥ MARR, the project is economically justified.

ernal Rate of Return (IRR)

R is the interest $i'\%$ at which

$$equivalent\ worth\ of\ cash\ inflows = equivalent\ worth\ of\ cash\ outflows$$

$$\sum_{k=0}^{N} R_k\,(P/F, i'\%, k) = \sum_{k=0}^{N} E_k\,(P/F, i'\%, k)$$

* بدي أشوف E at k و R at k و شو اذا في بدي بتخلي اذ cash inflow = cash outflow و i هو unknown

Where:

R_k: net revenue or savings for the k^{th} year.

E_k: net expenditures including any investment costs for the k^{th} year.

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الفائدة اللي بدي استرجع منها الفلوس اللي دفعتها .

nal Rate of Return (IRR)

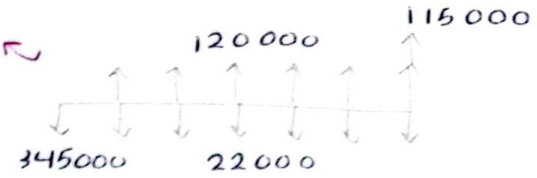
or the IRR is a bit more complicated than PW, FW, or AW.

تحديات ممكن نواجهها

es in applying the IRR method:

- omputationally difficult without proper tools. ممكن نحتاج نفعل interpolation أو trial & error و نشتغل إكسل
- e instances, multiple rates of return can be found. ممكن يكون عنا أكثر من حل ، بحقق الكل .
- RR method must be carefully applied and interpreted when comparing two more ally exclusive alternatives (e.g., do not directly compare internal rates of return).

Internal Rate of Return (IRR)



Example: A company is considering the purchase of a digital camera for the maintenance of design specifications by feeding digital pictures directly into an engineering workstation. The capital investment requirement is $345,000 and the estimated market value of the system after a six-year study period is $115,000. Annual revenues attributable to the new camera system will be $120,000, whereas additional annual expenses will be $22,000. You have been asked by management to determine the IRR of this project and to make a recommendation. The corporation's MARR is 20% per year.

$$PW = 0 = -\$345{,}000 + (\$120{,}000 - \$22{,}000)(P/A, i'\%, 6) + \$115{,}000(P/F, i'\%, 6)$$

$i'\% = ?$

To find i' or the IRR:
- Interpolation.
- Trial and error.
- Calculators with solver.
- Spreadsheets.

➢ At $i' = 20\%$: $PW = -\$345{,}000 + \$98{,}000(3.3255) + \$115{,}000(0.3349) = +\$19{,}413$

Since the PW is positive at 20%, we know that $i' > 20\%$.

➢ At $i' = 25\%$: $PW = -\$345{,}000 + \$98{,}000(2.9514) + \$115{,}000(0.2621) = -\$25{,}621$

✳ $i' \approx 22.16\%$.

✳ Spreadsheets IRR = 22.03% > MARR → Acceptable project

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Internal Rate of Return (IRR)

Example: A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is $25,000, and the equipment will have a market (salvage) value of $5,000 at the end of its expected life of five years. Increased productivity attributable to the equipment will amount to $8,000 per year after extra operating costs have been subtracted from the value of the additional production. Use a spreadsheet to evaluate the IRR of the proposed equipment. Is the investment a good one? Recall that the MARR is 20% per year.

- The IRR for the proposed piece of equipment is 21.58%
- PW(MARR= 20%) = $934.29;
- FW(MARR= 20%) = $2,324.80;
- AW(MARR= 20%) = $312.40.

$PW = 0$

$-25000 + 8000\,(P/A\ i'\%, 5) + 5000\,(P/F\ i'\%, 5)$

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

بدي أحسب كل الـ E الـ equiv بالهم
at year 0 بسعر (ε%) اللي بالسؤال، وبدي
أجمع كل الـ revenue وأوديهم لـ
(end of study period) period N
وأشوف شو الـ ERR اللي
بتساويهم في بعض.

External Rate of Return (ERR)

three steps are used in the calculating procedure:

1. all net cash *outflows* are discounted to time zero (the present) at ∈% per compounding period.
2. all net cash *inflows* are compounded to period *N* at ∈%.
3. the ERR, which is the interest rate that establishes equivalence between the two quantities, is determined.

$$\sum_{k=0}^{N} R_k\,(F/P, \varepsilon\%, N-k) = \sum_{k=0}^{N} E_k\,(P/F, \varepsilon\%, k)(F/P, i'\%, N)$$

Where:

R_k: excess of receipts over expenses in period k.

E_k: excess of expenses over receipts in period k.

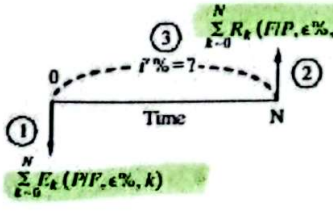
ε: external reinvestment rate per compounding period.

ERR ≥ MARR; project is economically justified.

External Rate of Return (ERR)

هون بدهم نساوي بينها الموجودة في
السؤال هاي الطلوبة، اللي بالسؤال
حولنا بينها الـ E لـ 0 والـ R لـ N

Graphically, we have the following (the numbers relate to the three steps)



→

$R = E\,(F, P, \epsilon\%, N-K)$

$E = R * (P, F\ \epsilon\%, N-K)$

↓

unknown variable

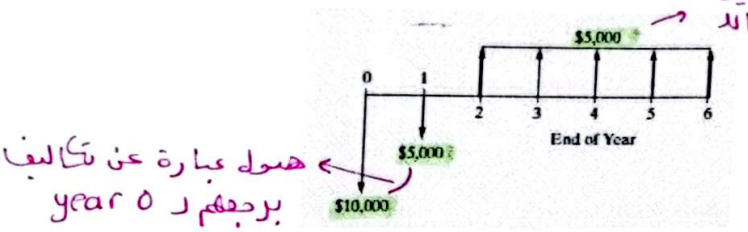
اللي راح أعمل الحسابات
عشانها.

The ERR method has two basic advantages over the IRR method:

1. It can usually be solved for directly, without needing to resort to trial and error. → ما بحتاج trail & error (جد)
2. It is not subject to the possibility of multiple rates of return.

External Rate of Return (ERR)

Example: When ϵ=15% and MARR = 20% per year, determine whether the project (whose net cash-flow diagram appears next) is acceptable.



$$[\$10{,}000 + \$5{,}000(P/F, 15\%, 1)](F/P, i'\%, 6) = \$5{,}000(F/A, 15\%, 5);$$

$i'\% = 15.3\%$

The i'% is less than the MARR = 20%; therefore, this project would be unacceptable according to the ERR method.

External Rate of Return (ERR)

Example: For the cash flows given below, find the ERR when the external reinvestment rate (ε) = MARR = 12%.

Year	0	1	2	3	4
Cashflow	($15,000)	($7,000)	$10,000	$10,000	$10,000

Expenses: $\$15{,}000 + \$7{,}000\ (P/F, 12\%, 1) = \$21{,}250$

Revenue: $\$10{,}000\ (F/A, 12\%, 3) = \$33{,}744$

Solving for ERR:

$\$21{,}250\ (F/P, i'\%, 4) = \$33{,}744$ $\Rightarrow i' = 12.26\% > 12\%$ (acceptable project)

Engineering Economy
(0901420)

Muhammad T. Hatamleh, PhD
Department of Civil Engineering
Slides 10

Evaluating a Single Project

Topics to be covered this week:

- Payback (payout period)

متى راح رجع
أصل الفلوس يلي
أنا دفعتها .

ɔack (payout period)

es of payback period:

ds presented thus far reflect the *profitability* of a proposed alternative for a study period of *N*. The ıethod, which is often called the *simple payout method*, mainly indicates a project's *liquidity* n its profitability.

ıidity deals with how fast an investment can be recovered.

time value of money بتوخذ ا بعين الاعتبار

back period: ignores the time value of money

$-I \geq 0$, where θ is the simple payback period ($\theta \leq N$)

② - Discounted payback period: time value of money is considered و بترجع ا R و ا E ل year 0

$$\sum_{k=1}^{\theta'} (R_k - E_k)(P/F, i\%, k) - I \geq 0$$, where θ′ is the discounted payback period ($\theta' \leq N$)

nple payback period, θ, ignores the time value of money and all cash flows that after θ.

مشروعي كلفني ٥ كل سنة بطوت عل بالتالي بعد 5 سنوات أرجع ا 5000 يلي دفع time value of mong ا

ɔack (payout period)

e: An investment of $5,000,000 (P) yields net annual revenue of $1,500,000 (A). What is *ple* payback period?

$$\text{Payback period} = \frac{\$5,000,000}{\$1,500,000} = 3.33 \text{ years}$$

⇒ Simple payback period (θ) is 4 years. بعد 4 سنين ← برجع ا 5000 000 (simple حل مباشر دنها)

payback period can produce misleading results, and it is recommended as ɔlemental information only in conjunction with one or more of the five methods ·iously discussed.

Payback (payout period)

- Discounted payback period: time value of money is considered

$$\sum_{k=1}^{\theta'} (R_k - E_k)\,(P/F, i\%, k) - I \geq 0, \text{where } \theta' \text{ is the discounted payback period } (\theta' \leq N)$$

➢ $i\%$ is the MARR, I is the capital investment usually made at the present time ($k = 0$).

Payback (payout period)

The payback calculation for Example 5-13 in the book.

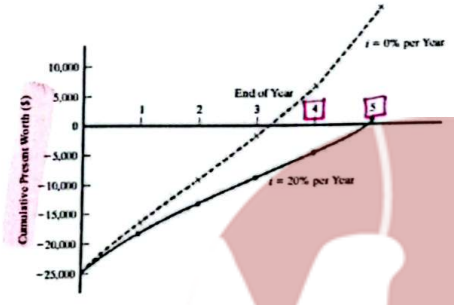
TABLE 5-2 Calculation of the Simple Payback Period (θ) and the Discounted Payback Period (θ') at MARR = 20% for Example 5-13[a]

Column 1 End of Year k	Column 2 Net Cash Flow	Column 3 Cumulative PW at $i = 0\%$/yr through Year k	Column 4 PW of Cash Flow at $i = 20\%$/yr	Column 5 Cumulative PW at $i = 20\%$/yr through Year k
0	−$25,000	−$25,000	−$25,000	−$25,000
1	8,000	−17,000	6,667	−18,333
2	8,000	−9,000	5,556	−12,777
3	8,000	−1,000	4,630	−8,147
4	8,000	+7,000	3,858	−4,289
5	13,000		5,223	+934

$\theta = 4$ years because the cumulative balance turns positive at EOY 4.

$\theta' = 5$ years because the cumulative discounted balance turns positive at EOY 5.

[a] Notice that $\theta' \geq \theta$ for MARR $\geq 0\%$.



Payback (payout period)

Example: For the following cash flows, what is the simple and discounted payback periods at i = 6%?

EOY	0	1	2	3	4	5
Net cash flow	-$42,000	$12,000	$11,000	$10,000	$10,000	$9,000

EOY	Net cash flow	Cumulative PW at 0%	Cumulative PW at 6%
0	-$42,000	-$42,000	-$42,000
1	$12,000	-$30,000	-$30,679
2	$11,000	-$19,000	-$20,889
3	$10,000	-$9,000	-$12,493
4	$10,000	$1,000	-$4,572
5	$9,000		$2,153

simple

بدي أشيل
من ال 42000
12000 × (P/F, 6, 1)

Keep calculating cumulative PW until reaching a positive value.

From the table:

$\theta = 4$ years and $\theta' = 5$ years

* بدي الطلع اا لإلها equi
at year 0
وأطرحه من ال
capital investment

The end

Engineering Economy
(0901420)

Muhammad T. Hatamleh, PhD
Department of Civil Engineering
Slides 11

Chapter 6: Comparison and Selection Among Alternatives

↳ بدنا نستخدم الـ tech. يلي أخذناهم في شابتر 5 حتى نقارن بين الـ alternative.

Topics to be covered this week:

- Comparison and Selection Among Alternatives.
- Evaluating multiple projects
  - Comparing Alternatives with equal lives:
    1. Equivalent-Worth Methods (PW, FW, AW)
    2. Rate-of-Return Methods (IRR, ERR)

* أنا بشتغل في مصنع وبدنا نشتري conveyer built راح يكون عنا أكثر من خيار إذا اخترت واحد واشتريته وركبته في المصنع ما راح اشتري غيرها.

* إذا بدك تشتري سيارة وكان عندك 4 أو 5 خيارات انت معك فلوس لتشتري سيارة واحدة فإذا شريتها ما راح تتفوج على الباقي.

Comparison and Selection Among Alternatives

Making decisions means comparing alternatives.

- In this chapter, we examine *feasible design alternatives*.
- The decisions considered are those selected from among a set of ***mutually exclusive*** alternatives (when selecting one excludes the choice of any of the others).

بدو يكون عنا أكثر من alternatives وبدنا نختار بس واحد

Mutually exclusive alternatives (MEAs) characteristics:

- We examine them on the basis of economic considerations alone.
- The alternatives may have different initial investments and their annual revenues and costs may vary.
- The alternatives must provide comparable "usefulness" criteria: performance, quality, etc.
- The basic methods from chapter 5 provide the basis for the economic comparison of the alternatives.

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Comparison and Selection Among Alternatives

ما يكون معك فلوس تشتري 3 أو 4 سيارات بالتالي حارد

Alternatives:

- ***Mutually exclusive:*** The selection of one alternative *excludes* the others.
- ***Independent:*** Selection of one alternative does not exclude other alternatives.

indep. alter اختيارك لوحدة ما راح يلغي باقي الاختيارات.

An acceptable alternative with the least capital investment is called the base alternative.

ما يكون عندي أكثر من خيار، الخيار يلي فيه راس المال أو الفلوس يلي بحتاجها بسعيه base alternative

If the incremental investment (over the base alternative) is justified by extra benefits:

- Investment should be made.

يطلع واحد من الـ alternative من الباقي، إذا (+) الخيار الأول أفضل، إذا (-) الخيار الثاني أفضل.

Two basic types of alternatives:

راح أحط رأس مال
وعلى أساسها بدي أتوقع
يجيني ربح (بختار الكل يلي بحقق ربح)

- ***Investment Alternatives***

Those with initial (or front-end) capital investment that produces positive cash flows from increased revenue, savings through reduced costs, or both.

هون بختار الكل يلي راح يقلل التكاليف.

- ***Cost Alternatives***

Those with all negative cash flows, except for a possible positive cash flow from the disposal of assets at the end of the project's useful life. The decision involves the most economical way of conducting an activity/project.

Two basic types of alternatives:

بدي اشوف أي واحد بعطي
ربح أكثر

Ex) Use a MARR of 10% and useful life of 5 years to select between the investment alternatives below:

Alternative	Capital investment	Annual revenues less expenses
A	-$100,000	$34,000
B	-$125,000	$41,000

* base alternative هو (A) لأنه فيه رأس مال أقل.

$$PW_A = -100,000 + 34,000(P/A, 10\%, 5) = 28,887$$

$$PW_B = -125,000 + 41,000(P/A, 10\%, 5) = 30,423$$

Both alternatives are attractive, but Alternative B provides a greater present worth, so is better economically.

B أحسن لأنو ← max my profit

Two basic types of alternatives:

بدي أشوف بحي التكاليف أقل

Ex) Use a MARR of 12% and useful life of 4 years to select between the cost alternatives below:

Alternative	Capital investment	Annual expenses
C	-$80,000	-$25,000
D	-$60,000	-$30,000

$$PW_C = -80,000 - 25,000(P/A, 12\%, 4) = -155,933$$

$$PW_D = -60,000 - 30,000(P/A, 12\%, 4) = -151,119$$

Alternative D costs less than Alternative C, it has a greater PW, so is better economically.

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Study period

Study period (*or planning horizon*): selected time period over which mutually exclusive alternatives are compared.

المقارنة بحي بنعملها of the same period

Study period cases:

- Useful lives of all mutually exclusive alternatives (MEAs) are the same and equal to the study period.
  - ⇒ No cash flow adjustment.
  - ⇒ MEAs are compared using equivalent worth methods (PW, FW, or AW) or rate of return methods (IRR or ERR).

إذا تخي راح استعمل وحدة من هدول ي

- Useful lives are unequal and at least one does not match the study period.
  - ⇒ Repeatability assumption.
  - ⇒ Co-terminated assumption.

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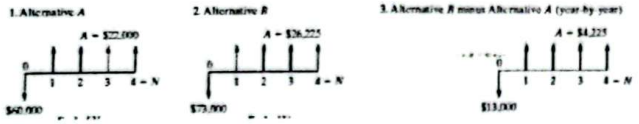
Comparing Alternatives with equal lives.

A → base alternative

- If PW (B-A) is positive, the additional capital invested in *B* is justified. @ MARR = 10%

	Alternative		
	A	*B*	Δ(*B* − *A*)
Capital investment	−$60,000	−$73,000	−$13,000
Annual revenues less expenses	22,000	26,225	4,225



* إما بطلع PW لـ A و B
وبشوف مين أعطى ربح أكثر
أو بجيد الـ base
PW
وبجعل □ - base
وبشوف اذا (+) الأول
واذا (−) الثاني أفضل

PW (10%) $_A$ = -$60,000 + $22,000 (P/A, 10%, 4) = $9,738.

PW (10%) $_B$ = -$73,000 + $26,225 (P/A, 10%, 4) = $10,131.

PW (10%) $_{B-A}$ = -$13,000 + $4,225 (P/A, 10%, 4) = $393. ⇝ (+) → B بالتالي أحسن

Incremental analysis

⇒ Both alternatives are acceptable (PW @ MARR ≥ 0) ... investment alternatives.

⇒ Alternative A is the *base alternative* (acceptable + lowest capital).

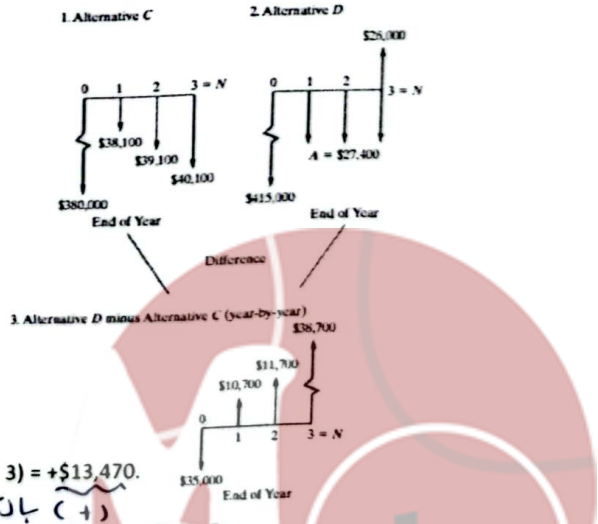
Select alternative B (higher PW @ MARR) ⇒ additional capital investment in B is justified.
Also, PW $_{B-A}$ is positive ⇒ additional investment is justified.

Comparing Alternatives with equal lives.

- If PW (D-C) is positive, the additional capital invested in *D* is justified. @ MARR = 10%.
- Alternative C is the *base alternative* (lowest capital).

	Alternative		
End of Year	*C*	*D*	Δ(*D* − *C*)
0	−$380,000	−$415,000	−$35,000
1	−38,100	−27,400	10,700
2	−39,100	−27,400	11,700
3	−40,100	−27,400	12,700
3[a]	0	26,000	26,000

[a] Market value.



PW (10%) $_C$ = -$380,000 − $38,100 (P/A, 10%, 3) − $1,000 (P/G, 10%, 3) = -$477,077.

PW (10%) $_D$ = -$415,000 − $27,400 (P/A, 10%, 3) + $26,000 (P/F, 10%, 3) = -$463,607.

* PW (10%) $_{D-C}$ = -$35,000 + $10,700 (P/A, 10%, 3) + $1,000 (P/G, 10%, 3) + $ 26,000 (P/F, 10%, 3) = +$13,470.

(+) بالتالي D أحسن

PW @ MARR < 0 for both alternatives ... cost alternatives.

Select alternative D (higher PW @ MARR) ➔ Additional capital investment in D is justified.

Comparing Alternatives with equal lives.

	Alternatives			
	A	B	C	D
Capital investment	-$150,000	-$85,000	-$75,000	-$120,000
Annual revenues	$28,000	$16,000	$15,000	$22,000
Annual expenses	-$1,000	-$550	-$500	-$700
Market Value (EOL)	$20,000	$10,000	$6,000	$11,000
Life (years)	10	10	10	10

Use a MARR of 12% to select one alternative among the four alternatives:

PW_A = - $150,000 + $27,000 (P/A, 12%, 10) + $20,000 (P/F, 12%, 10) = $8,995.

PW_B = - $85,000 + $15,450 (P/A, 12%, 10) + $10,000 (P/F, 12%, 10) = $5,516.

PW_C = - $75,000 + $14,500 (P/A, 12%, 10) + $6,000 (P/F, 12%, 10) = $8,860.

PW_D = - $120,000 + $21,300 (P/A, 12%, 10) + $11,000 (P/F, 12%, 10) = $3,891.

→ نختار ال alternative يلي بحقق أكبر ربح

All alternatives are acceptable (PW > 0)

➔ C is the base alternative (lowest capital).

➔ Select alternative A (highest PW)

Repeat using AW and FW methods
➔ Should get the same conclusion

لو استغلت هدول برجعو لازم A يكون الأفضل.

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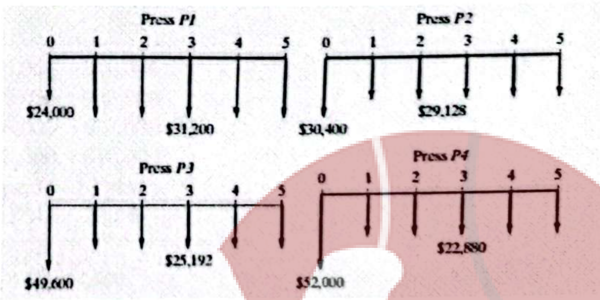
Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Comparing Alternatives with equal lives: Equivalent-Worth Methods

Example 6.2: A company is planning to install a new automated plastic-molding press. Four different presses are available. The initial capital investments and annual expenses for these four mutually exclusive alternatives are as follows @ MARR = 10%:

	Press			
	P1	P2	P3	P4
Capital investment	$24,000	$30,400	$49,600	$52,000
Useful life (years)	5	5	5	5
Annual expenses				
Power	2,720	2,720	4,800	5,040
Labor	26,400	24,000	16,800	14,800
Maintenance	1,600	1,800	2,600	2,000
Property taxes and insurance	480	608	992	1,040
Total annual expenses	$31,200	$29,128	$25,192	$22,880



لو بما انو هدول التكاليف بتأثر كمهم وبستغل زمهم بدل ما اربح كل واحد منهم

The preferred alternative will minimize the equivalent worth of total costs over the five-year analysis period

Same output → same revenue → minimize cost

Calculate the PW of all costs and select the one with the highest value.

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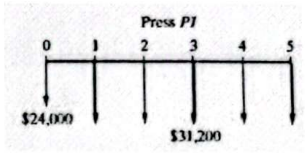
Comparing Alternatives with equal lives: Equivalent-Worth Methods

Example 6.2:



PW(10%)*P1* = –$24,000 – $31,200(*P/A*, 10%, 5) = –$142,273,

AW(10%)*P1* = –$24,000(*A/P*, 10%, 5) – $31,200 = –$37,531,

FW(10%)*P1* = –$24,000(*F/P*, 10%, 5) – $31,200(*F/A*, 10%, 5) = –$229,131.

	Press (Equivalent-Worth Values)			
Method	*P1*	*P2*	*P3*	*P4*
Present worth	–$142,273	–$140,818	–$145,098	–$138,734
Annual worth	–37,531	–37,148	–38,276	–36,598
Future worth	–229,131	–226,788	–233,689	–223,431

الأفضل لأن فيها أقل تكاليف

❑ The preference ranking (*P4* > *P2* > *P1* > *P3*)

The results are the same from the analysis for all three methods.

Comparing Alternatives with equal lives: Equivalent-Worth Methods

Example: Three mutually exclusive design alternatives are being considered. The estimated cash flows for each alternative are given in the following table. At a MARR of 20% per year, which one will you select?

	A	*B*	*C*
Investment cost	$28,000	$55,000	$40,000
Annual expenses	$15,000	$13,000	$22,000
Annual revenues	$23,000	$28,000	$32,000
Market value	$6,000	$8,000	$10,000
Useful life	10 years	10 years	10 years
IRR	26.4%	24.7%	22.4%

كلهم أكثر من الـ 20٪ بالتالي كلهم راح يعطوا ربح

PW (20%) $_A$ = -$28,000 + ($23,000 – $15,000) (P/A, 20%, 10) + $6,000 (P/F, 20%, 10) = $6,509.

PW (20%) $_B$ = -$55,000 + ($28,000 – $13,000) (P/A, 20%, 10) + $8,000 (P/F, 20%, 10) = $9,180.

PW (20%) $_C$ = -$40,000 + ($32,000 – $22,000) (P/A, 20%, 10) + $10,000 (P/F, 20%, 10) = $3,540.

→ Select B

*** selection based on maximum IRR is wrong ***

لما بجيب أحكي انو A اكبر بالتالي A أفضل ، لازم أحسب الـ PW وعلى أساسها أختار.

Comparing Alternatives with equal lives: Rate of Return Method

The return on investment (rate of return) is a popular measure of investment performance.

- Selecting the alternative with the largest rate of return can lead to incorrect decisions (do not compare the IRR of one alternative to the IRR of another alternative).
  - *The only legitimate comparison is the IRR to the MARR.* → بس عشان نشوف اذا بحقق لح أو لا ، إما إذا بدي أختر القرار بناءًا عليها لازم اعمل incremental analysis
- Remember, the *base alternative* must be attractive (rate of return greater than the MARR), and the *additional* investment in other alternatives must itself make a satisfactory rate of return on that increment.

Comparing Alternatives with equal lives: Rate of Return Method

@ MARR = 10%. Calculate PW and IRR for both of the following alternatives:

	Alternative		
	A	B	Δ(B – A)
Capital investment	–$60,000	–$73,000	–$13,000
Annual revenues less expenses	22,000	26,225	4,225

بدي احسب الـ IRR و أشوف → اذا أكبر من ١٥ بختار B و اذا أقل من ١٥ بختار A

- Both have positive PW & IRR > MARR: Both are acceptable alternatives.
  - Based on PW ⇒ select B (the PW method is **always correct**).
  - Based on IRR ⇒ select A (misleading).

Alternative	IRR	PW (10%)
A	17.3%	$9,738
B	16.3%	$10,131

- To solve using IRR, the *base alternative is A:* find the IRR of the incremental cash flow (B – A)
  - IRR_{B-A} = 11.4% > MARR ... The incremental investment is justified.
  - PW_{B-A} = $393 > 0 ... same conclusion.

Select alternative B.

Comparing Alternatives with equal lives: Rate of Return Method

Use the incremental investment analysis procedure:

1. Arrange (rank order) the feasible alternatives based on increasing capital investment.
2. Establish a base alternative.
   - Cost alternatives—the first alternative is the base.
   - Investment alternatives—the first *acceptable* alternative (*IRR*>*MARR*) is the base.
3. Iteratively evaluate differences (incremental cash flows) between alternatives until all have been considered.
4. Work up the order of ranked alternatives from smallest to largest.
5. Subtract cash flows of the lower-ranked alternative from the higher ranked

Comparing Alternatives with equal lives: Rate of Return Method

Use the incremental investment analysis procedure:

6. Determine if the incremental initial investment in the higher-ranked alternative is *attractive* (e.g., *IRR*>*MARR*, *PW*, *FW*, *AW* all >*0*). If it is attractive, it is the "winner." If not, the lower-ranked alternative is the "winner." The "loser" from this comparison is removed from consideration. Continue until all alternatives have been considered.
7. This works for both *cost* and *investment* alternatives.

Comparing Alternatives with equal lives: Rate of Return Method

Six mutually exclusive alternatives with equal useful lives (10 years) are analyzed and compared using the IRR method. Assuming MARR = 10%, which alternative will you select?

	A	B	C	D	E	F
Capital investment	$900	$1,500	$2,500	$4,000	$5,000	$7,000
Net annual income	$150	$276	$400	$925	$1,125	$1,425
IRR	10.6%	13.0%	9.6%	19.1%	18.3%	15.6%

- IRR is computed for each alternative … alternatives are ranked from lowest to highest capital.
- IRR < MARR for alternative C →→→ "C is eliminated" ⇝ C ← IRR أقل من الـ 10 بالتالي ماني ربح (بشطبها).
- The rest of alternatives (A, B, D, E, and F) are all acceptable alternatives.

Alternative A has the lowest capital among all acceptable alternatives

"A is the base alternative".

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Comparing Alternatives with equal lives: Rate of Return Method

A, B, D, E, and F from the lowest capital

	A	B – A	D – B	E – D	E – E
Δ Capital	$900	$600	$2,500	$1,000	$2,000
Δ Annual income	$150	$126	$649	$200	$300
IRR Δ	10.6%	16.4%	22.6%	15.1%	[8.1%] → أقل من 10٪ بالتالي نختار E
Is increment justified?	-	yes	yes	yes	no

لـ مادام حقق الشرط باجاي B أحسن منالكفوة بعد بقارن D بـ B وهكذا ...

B becomes the new base (A is eliminated)

Select E

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Comparing Alternatives with equal lives: Rate of Return Method

Check Example 6-4

This example is a *cost-type situation with four mutually exclusive cost alternatives*. The solution demonstrates the use of the incremental analysis procedure to compare cost alternatives

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Comparing Alternatives with equal lives: Rate of Return Method

Four alternatives are compared at MARR = 9%.

Alternative	Capital	IRR	ΔIRR		
			A	B	C
A	$15,000	12%	-	-	-
B	$20,000	15%	15%	-	-
C	$25,000	10%	9.3%	10%	-
D	$30,000	20%	9.5%	12%	7%

D – C

يعني اختيار واحد
مابلغي الاختيارات
الثانية

أقل من ٩٪ يالتالي نختار الـ [C]

- If all alternatives are independent, which one will you select?
  All (all have IRR > MARR and the selection of one does not exclude others).
- If all alternatives are mutually exclusive, which one will you select?
  Alternative C.

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Engineering Economy
(0901420)

Muhammad T. Hatamleh, PhD
Department of Civil Engineering
Slides 12

Chapter 6: Comparison and Selection Among Alternatives

alternative دائمًا بنختار الـ
يلي

max my profit & min my loses.

Topics to be covered this week:

- Evaluating multiple projects
  - Comparing Alternatives with Unequal useful lives:
    1. Repeatability
    2. Co-terminated
  - Exercises and practical examples

أو بستعمل الـ IRR (Incermantal analysis)

مثلا بدي اطبخ معجنات وأشغله مدة 10 سنوات، رح أشتري equip الـ useful life عبارة عن 5 سنوات بهاي الحالة الـ 5 سنوات ما غطت الـ study period.

Comparison and Selection Among Alternatives: Unequal useful lives

1. The useful life of an alternative is less than the study period:

- Cost alternatives → min loses
  - Contracting or leasing for remaining years may be appropriate
  - Repeat part of the useful life and use an estimated market value to truncate
- Investment alternatives → max profit
  - Cash flows reinvested at the MARR at the end of the study period
  - Replace with another asset, with possibly different cash flows, after the study period

بعد ما يخلص الـ 5 سنوات بشتري equip ثاني ويرجع أشغله عالـ 5 سنوات الباقي فوضح إني راح استعمل نفس الـ equip لأغطي الـ study period كاملة، أو بدي أوقف الـ production line وأجيب واحد ثاني من شركة ثانية.

revenue أخذت الـ وحطيتها في البنك وأخذت عليها فوائد بقيمة الـ MARR للنهاية الـ study period.

مثلا عندي معجنات مدة 10 سنوات، شريت ماكينة لحام الـ useful لها 15 سنة في هاي الحالة بدي أعمل الـ decision الاعتماد على my study period.

Comparison and Selection Among Alternatives: Unequal useful lives

2. The useful life of an alternative is greater than the study period:

- Truncate the alternative at the end of the study period, using an estimated market value.
- The underlying principle in all such analysis is to compare the MEAs in a decision situation over the same study (analysis) period.

The repeatability assumption, when applicable, simplified comparison of alternatives.

↳ بدي أوحد بينهم.

I am repeating the same process to cover my study period.

Comparison and Selection Among Alternatives: Unequal useful lives, repeatability

بدي أوحد الـ alternative مع الـ study period المشروع بدي.

The least common multiple of useful lives should be found to facilitate repeatability:

العامل المشترك والأصغر

- **Example:** Alternative A with 2 years useful life, B with 3 years. what is the least common multiple of useful lives?
  - A can be repeated 3 times → 6 years
  - B can be repeated 2 times → 6 years

لو شغلت A 3 سنين و B سنتين الـ LCM راح يكون 6 سنوات

* معمول دايما بيكون حاصل ضرب لو A ← 6 و B ← 3 الـ LCM بيكون 6 بشرط الـ 6 يكون الـ 3، 3 مرات.

- **Example:** Alternative A with 4 years useful life, B with 3 years. what is the least common multiple of useful lives?
  - A can be repeated 3 times → 12 years
  - B can be repeated 4 times → 12 years

Comparison and Selection Among Alternatives: Unequal useful lives, repeatability

عشان A و B وبدي اختار 1

Example: Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and using the repeatability assumption, which alternative would you pick?

	A	B
Capital investment	$3,500	$5,000
Annual net cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

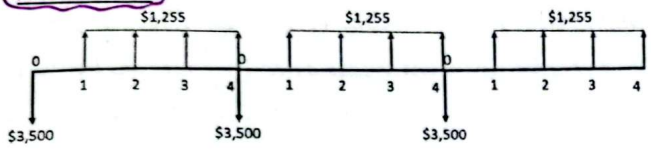
→ الـ LCM راح 12، 3 كورسات A مرات و B مرتين.

Comparison and Selection Among Alternatives: Unequal useful lives, repeatability

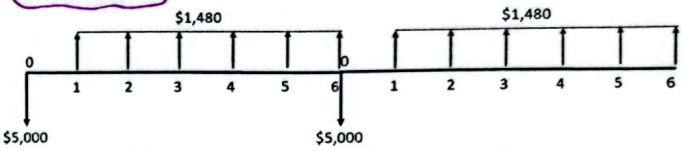
The least common multiple of useful lives = 12 years.

- A is repeated 3 times.
- B is repeated 2 times.

Alternative A



Alternative B



$PW(10\%)_A = -\$3,500 - \$3,500\,[(P/F, 10\%, 4) + (P/F, 10\%, 8)] + \$1,255\,(P/A, 10\%, 12) = \$1,028.$

$PW(10\%)_B = -\$5,000 - \$5,000\,(P/F, 10\%, 6) + \$1,480\,(P/A, 10\%, 12) = \$2,262.$

Select B

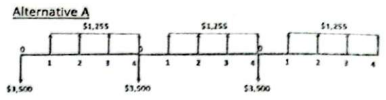
لأن ال profit أكبر

Comparison and Selection Among Alternatives: Unequal useful lives, repeatability

- If repeatability can be assumed, the MEAs are most easily compared by finding the annual worth (AW) of each alternative over its own useful life, and recommending the one having the most economical value.

لو حليت على أساس ال AW راح تكون كاتبة بشكل تكرار كانها كاملة لعمل decision

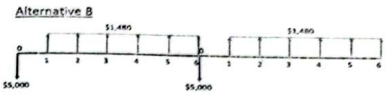
Comparison and Selection Among Alternatives: Unequal useful lives, repeatability



Solve using the annual worth method

$AW(10\%)_A = -\$3{,}500\,(A/P, 10\%, 12) - \$3{,}500\,[(P/F, 10\%, 4) + (P/F, 10\%, 8)]\,(A/P, 10\%, 12) + \$1{,}255 = \$151.$

Or $AW(10\%)_A = PW(10\%)_A \times (A/P, 10\%, 12) = \$151.$



$AW(10\%)_B = -\$5{,}000\,(A/P, 10\%, 12) - \$5{,}000\,(P/F, 10\%, 6)\,(A/P, 10\%, 12) + \$1{,}480 = \$332.$

Or $AW(10\%)_B = PW(10\%)_B \times (A/P, 10\%, 12) = \$332.$

Select B

Calculate the annual worth of each alternative over one useful life cycle:

$AW(10\%)_A = -\$3{,}500\,(A/P, 10\%, 4) + \$1{,}255 = \$151.$

$AW(10\%)_B = -\$5{,}000\,(A/P, 10\%, 6) + \$1{,}480 = \$332.$

ينقدر نستخدم الـ AW مباشرة بما انه طالب نستخدم الـ repeat. ass.

Select B

⇒ Same result

⇒ For repeatability assumption, we can calculate the AW for each alternative over its own useful (single) life and compare directly.

Comparison and Selection Among Alternatives: Unequal useful lives, repeatability

EXAMPLE 6-11: Modeling Estimated Expenses as Arithmetic Gradients

(Check it out)

Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

- The repeatability assumption, when applicable, simplified comparison of alternatives.

إذا كان مطلوب ← بنستعمل هاي الطريقة بتقدر مباشرة
حسب الـ AW لكل alternative وبنختار الأقل.

- If repeatability cannot be used, an appropriate study period must be selected (the co-terminated assumption).

لـ طريقة ثانية.

مثلاً ← شريت ماكينة راح تخدمني 5 سنوات الـ study period
8 سنوات، يعني لو بدي أكررها مرتين صارت ١٥ سنوات زادت عن الـ study period
بتبقى بالتالي بستعملها أول مرة وبشتريها ثاني مرة وبستعملها مدة 3 سنوات وبيعها at the end of my study period

Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

1) Useful life < study period → assumption لازم أعمل

بس repeat. إما بعمل study period مع نهاية الـ market value وبعوض عنها بـ

a) Cost alternatives:

- Contracting or leasing equipment/service for the remaining years.
- Repeat part of the useful life and truncate at the end of the study period with an estimated market value.

b) Investment alternatives:

- Cash flows reinvested at MARR to the end of the study period.
- Replace with another asset with possibly different cash flows over the remaining life.

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Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

2) Useful life > study period: truncate at the end of the study period with an estimated market value.

ما كتاج أكرر
بس بالنهاية لازم
يكون عندي estimated market

For Co-termination, use any equivalent worth methods using the cash flows available for the study period.

Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and the study period is 6 years, which alternative would you pick?

ما بقدر
أستعمل
repeat.
assumption

	A	B
Capital investment	$3,500	$5,000
Annual cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

- 6 years isn't a multiple of both lives ⇒ [*repeatability isn't applicable for the study period.*]

↳ Co-terminated assumption: Assume that the money at EOY 4 for alternative A will be reinvested at MARR till the end of the study period

لو راح نستثمره بفائدة
10٪ لمدة السنتين يلي
فلو لنفس الـ study period

Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

✳ Co-terminated assumption: Study period of 6 years

	A	B
Capital investment	$3,500	$5,000
Annual cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

For A, useful life < study period ... needs cash flow adjustment.
Assume money will be reinvested at MARR until the end of the study period.
$FW(10\%)_A = [-\$3,500\ (F/P, 10\%, 4) + \$1,225\ (F/A, 10\%, 4)] \times (F/P, 10\%, 2) = \$847.$

عشان السنتين لي خلو (؟)

For B, useful life = study period ... no cash flow adjustment is needed.
$FW(10\%)_B = -\$5,000\ (F/P, 10\%, 6) + \$1,480\ (F/A, 10\%, 6) = \$2,561.$

لـ بختار B = to max my profit

✳ الخلاصة :-
اذا بدي استعمل الـ rep. ass. بقدر مباشرة أحسب الـ AW لكل مشروع
على الـ useful life تبعته ماشي داي أكرر لأن الـ AW راح تكون موحدة .
اذا الـ rep. ass ما بنلح نستخدمها بدي استخدم الـ co.. بس لازم أحط assumption .

Comparison and Selection Among Alternatives: Unequal useful lives, Co-terminated

- In summary, utilizing the repeatability assumption for unequal lives among alternatives reduces to the simple rule of "comparing alternatives over their useful lives using the AW method, at i = MARR."
- This simplification, however, May not apply when a study period, selected to be shorter or longer than the common multiple of lives
  - ➢ (Co-terminated assumption), is more appropriate for the decision situation.
- When utilizing the co-terminated assumption, cash flows of alternatives need to be adjusted to be terminated at the end of the study period.
- Adjusting these cash flows usually requires estimating the market value of assets at the end of the study period or extending service to the end of the study period through leasing or some other assumption.

Comparison and Selection Among Alternatives: Unequal useful lives.

Example: Two mutually exclusive alternatives with different useful lives. At 5% per year MARR:

	A	B
Capital investment	$6,000	$14,000
Annual expenses	$2,500	$2,400
Useful lives (years)	12	18
Market value at end of useful life	0	$2,800

→ LCM → 36
A ← بكررها 3 مرات
B → بكررها مرتين

Determine which alternative to select assuming repeatability applies.

↳ بحسب مباشرة AW

$AW\ (5\%)_A = -\$6,000\ (A/P, 5\%, 12) - \$2,500 = -\$3,176.8.$
$AW\ (5\%)_B = -\$14,000\ (A/P, 5\%, 18) - \$2,400 + \$2,800\ (A/F, 5\%, 18) = -\$3,497.6.$
⇒ select A (lower cost).

Comparison and Selection Among Alternatives: Unequal useful lives.

Example: Two mutually exclusive alternatives with different useful lives. At 5% per year MARR:

	A	B
Capital investment	$6,000	$14,000
Annual expenses	$2,500	$2,400
Useful lives (years)	12	18
Market value at end of useful life	0	$2,800

- Determine which alternative to select if the repeatability does not apply, study period is 18 years, and a new system can be leased for $8,000 per year after the useful life of alternative A is over.

من سنة 13 لسنة 18 عشان 8000

$PW\ (5\%)_A = -\$6,000 - \$2,500\ (P/A, 5\%, 12) - \$8,000\ (P/A, 5\%, 6)\ (P/F, 5\%, 12) = -\$50,767.45.$
$PW\ (5\%)_B = -\$14,000 - \$2,400\ (P/A, 5\%, 18) + \$2,800\ (P/F, 5\%, 18) = -\$40,885.54.$
⇒ select B (lower cost).

Comparison and Selection Among Alternatives: Unequal useful lives.

Example: Which alternative should be selected assuming MARR = 20%? Use the IRR method.

	A	B
Capital investment	$3,500	$5,000
Annual cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful lives	0	0

- Study period isn't determined ....

➢ Repeatability means finding AW. Which means you should use repeatability method

$AW(i^*)_B - AW(i^*)_A = 0$

$-\$3{,}500\,(A/P, i^*, 4) + \$1{,}255 = -\$5{,}000\,(A/P, i^*, 6) + \$1480$

$i^* = 26\%$ ... increment is justified ... Select B

The End

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Engineering Economy
(0901420)

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Slides 14

Chapter 7: Depreciation and Income Taxes

Topics to be covered this week:

- Taxes

Taxes

Types of taxes

that you will pay by the end of the financial year based in your income as an individual or an org. (راح تحكي عنها)

مثلا عنا ضريبة مسقفات بندفعها على الشباك أو الأراضي يلي بنمتلكها سواء اكسبنا فيها على الأرض ربح أو لا ما إلها علاقة راح أدفع الضريبة بكل الحالات.

- **Income taxes:** function of gross revenue minus allowable deductions.
- **Property taxes:** function of the property (e.g., land, building, equipment, etc.) value (**independent of income or profit**).
- **Sales taxes:** function of the value of purchased goods or services (**independent of income or profit**). (للمنتجات والخدمات)
- **Excise taxes:** taxes imposed on the purchase of non-necessities (**independent of income or profit**). (يعني عاليات انت بتشتريها)

Taxes

After-tax analysis

- After-tax MARR ≅ Before-tax MARR × (1 – effective income tax rate) (product لـ product تختلف من وحسب صناعة لصناعة ومن منطقة لمنطقة .. وبشكل كل التكاليف يلي بتكلفيها خلال هاي السنة)
- Taxable income = Gross income – All expenses (except capital investment) – Depreciation deductions

Example: A company generates $1,500,000 of gross income during its tax year and incurs operating expenses of $800,000. Property taxes on business assets amount to $48,000. The total depreciation deductions for the tax year equal $114,000. What is the taxable income of this firm?

(ايرادات) (تكاليف) (ضريبة أبنية)

Taxable income = $1,500,000 – $800,000 – $48,000 – $114,000 = $538,000.

(لـ بالآخير راح أدفع tax على ال taxable income)

Taxes

After-tax cash flow (ATCF)

➢ After-tax economic analysis is the same as before-tax analysis except:

الضرايب يلي بدفعها ← $T_k = -t\,(R_k - E_k - d_k)$ ← سالب لأن أنا يلي بدفع الضريبة

$(R_k - E_k - d_k)$: Taxable income

Where:

T_k: income tax consequence during year k.
R_k: revenue (and savings) or cash inflow during year k.
E_k: cash outflows during year k.
d_k: sum of book costs during year k or accumulated depreciation.
t: effective income tax rate.

*بدفع ضريبة على صافي الربح بعد ما أدفع هاي الضريبة صافي الربح راح ينقص فأنا لما بدي أخذ القرارات لازم تكون مأخوذه على اد $ATCF_k$

Taxes

After-tax cash flow (ATCF)

Before-tax cash flow ⋯▸ $\text{BTCF}_k = R_k - E_k$

$T_k = -t\,(R_k - E_k - d_k)$

$(R_k - E_k - d_k)$: Taxable income

$$\text{ATCF}_k = \text{BTCF}_k + T_k$$
$$= (R_k - E_k) - t\,(R_k - E_k - d_k)$$
$$= (1 - t)(R_k - E_k) + t\,d_k$$

Taxes

Example: A new equipment is estimated to cost $180,000 and is expected to reduce net annual expenses by $36,000 for 10 years and to have a $30,000 market value at the end of the 10th year. Using the SL depreciation method, and assuming a 40% effective income tax rate, develop the ATCF and BTCF. ($-t \times$ taxable income)

EOY k	Capital	R_k	E_k	BTCF	d_k	Taxable income	Income tax	ATCF
0	-$180,000	-	-	-$180,000	-	-	-	-$180,000
1	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
2	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
3	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
4	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
5	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
6	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
7	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
8	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
9	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
10	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
10*	-	$30,000	-	$30,000	-	0	0	$30,000

(BTCF + income tax)

$d = \frac{\$180,000 - \$30,000}{10}$ — عنا const

$BTCF - d_k$ — ما بحسبلها taxes

سالب عشان أنا بدفعه

Taxes

Example: A company wants to purchase a machine with an initial cost of $100,000 with additional $10,000 installation and transportation costs and a salvage value after 10 years of $10,000. If the annual revenue is $20,000 and the annual expenses are $5,000, and using the SL depreciation method and a 30% income tax rate:

- What is the BTCF for the 3rd year?

$BTCF_3 = \$20,000 - \$5,000 = \$15,000.$ → راح يكون نفسه لأي وحدة من السنوات

R_k E_k

- What is the ATCF for the 2nd year?

$BTCF_2 = \$20,000 - \$5,000 = \$15,000.$

$d_k = (\$100,000 + \$10,000 - \$10,000)/10 = \$10,000$ per year.

Taxable income for year 2 = BTCF - d_k = $15,000 – $10,000 = $5,000.

$T_k = -0.30 \times \$5,000 = -\$1,500.$

$ATCF_2 = BTCF_2 + T_k = \$15,000 + -\$1,500 = \$13,500.$

Taxes

Example: Assume the cost basis is $35,000, annual revenue is $30,000, annual expenses are $13,000 in the first year and increasing by $1,000 per year. The useful life is 4 years and the SOYD is the applicable depreciation method, develop BTCFs and ATCFs using a 15% income tax rate.

EOY k	Capital	R_k	E_k	BTCF	d_k	Taxable income	Income tax	ATCF
0	-$35,000	-	-	-$35,000	-	-	-	-$35,000
1		$30,000	-$13,000	$17,000	$14,000	$3,000	-$450	$16,550
2		$30,000	-$14,000	$16,000	$10,500	$5,500	-$825	$15,175
3		$30,000	-$15,000	$15,000	$7,000	$8,000	-$1,200	$13,800
4		$30,000	-$16,000	$14,000	$3,500	$10,500	-$1,575	$12,425

d_k
-
= 4/(1+2+3+4) × ($35,000 – $0)
= 3/(1+2+3+4) × ($35,000 – $0)
= 2/(1+2+3+4) × ($35,000 – $0)
= 1/(1+2+3+4) × ($35,000 – $0)

The end

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Engineering Economy
(0901420)

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Chapter 7: Depreciation and Income Taxes

equiv value لا *
in money لاستهلاك
depreciation بنسميه

Depreciation

- Accounting concept (noncash or book cost).
- Measures the decrease in the value of physical properties with time and use.
- Begins once the property is placed in service for business.

Depreciable properties

- Tangible properties (machinery, vehicles, furniture, buildings).
- Intangible properties (patents, copyrights, franchises).

أي نوع من الاشكال منها
معادي وأنا استهلكتها
بالتالي بحسب إلها اد

* أي اشي أنا بستهلكه وبتنزل من قيمته
ميكون الها Dep. وراح نحسبها بناء
على مجموعة من ال methods .

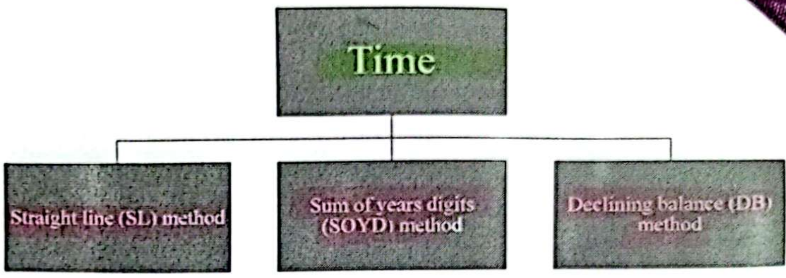
Chapter 7: Depreciation and Income Taxes

Depreciation Methods:

① ➤ Time

الها 3 طرق

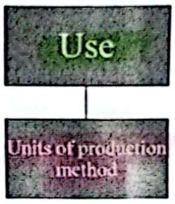
- Straight line (SL) method
- Sum of years digits (SOYD) method
- Declining balance (DB) method



② ➤ Use

- Units of production method

الي بيحصون استعمالنا
لل prop. لغرض بشتغل
مصنع وعنا conveyr built
شريته ب 50000 JD بعد ٤ سنة
بسبب الاستعمال والصيانة قلت
تكلفته هاد النقصان هو ال
depreciation



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Dep = cash cost − book cost

← الاستهلاك

← رأس المال أو الفلوس الي شريت فيها ال equip.

السعر الي شريته اذا شلت منه الاستهلاك بعطينا ال book cost

Straight line (SL) method

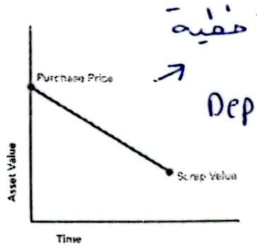
*The constant amount is depreciated each year over the depreciable (useful) l

* مثلاً سيارة شريتها وبدي
استعملها 5 سنوات بقدر من خلال
هاي ال method اعرف قديش
ال Def كل سنة .

$$d_k = \frac{B - SV_N}{N}$$

$$d_k^* = k \times d_k, \text{for } 1 \le k \le N$$

$$BV_k = B - d_k^*$$



علاقة خطية
بين ال Dep وال time

Where:

N: depreciable life of the asset in years.

B: cost basis, which is the initial cost of acquiring an asset + other associated expenses (sales tax, transportation, setup, etc.)

d_k: annual depreciation in year k.

BV_k: book value at end of year k (worth of a depreciable property on accounting records = cost basis – all allowable depreciation).

SV_k: estimated salvage value at end of year N.

d_k^*: cumulative depreciation through year k.

* لو شريت سيارة ب 20000
وبعد 5 سنوات بعتها ب 10000
يعني أنا خسرت 10000 خلال ال 5 سنوات
كل سنة كنت استهلك 2000 بالتالي حسبت
الاستهلاك .

$$\to d_k = \frac{20000 - 10000}{5} = 2000$$

Muhammad T. Hatamleh, Extracted from Sullivan et al. (2018)

Straight line (SL) method

Example: A tool has a cost basis of $200,000 and a five-year depreciable life. The estimated salvage value is $20,000 at the end of five years. Determine the annual depreciation using SL method and tabulate the annual depreciation amounts and book values at the end of each year.

B = $200,000 SV_N = $20,000 N = 5 years

$$d_1 = d_2 = d_3 = d_4 = d_5 = \frac{200{,}000 - 20{,}000}{5} = \$36{,}000$$

EOY, K	d_K	BV_K
0	-	$200,000
1	$36,000	$164,000
2	$36,000	$128,000
3	$36,000	$92,000
4	$36,000	$56,000
5	$36,000	$20,000

$d_1^* = 1 \times \$36{,}000 = \$36{,}000$ $BV_1 = \$200{,}000 - \$36{,}000 = \$164{,}000$

$d_2^* = 2 \times \$36{,}000 = \$72{,}000$ $BV_2 = \$200{,}000 - \$72{,}000 = \$128{,}000$

$d_3^* = 3 \times \$36{,}000 = \$108{,}000$ $BV_3 = \$200{,}000 - \$108{,}000 = \$92{,}000$

$d_4^* = 4 \times \$36{,}000 = \$144{,}000$ $BV_4 = \$200{,}000 - \$144{,}000 = \$56{,}000$

$d_5^* = 5 \times \$36{,}000 = \$180{,}000$ $BV_5 = \$200{,}000 - \$180{,}000 = \$20{,}000$

Sum of years digits (SOYD) method

This method accelerates the recognition of depreciation (most depreciation is recognized in the first few years of the asset's life).

$$\text{Depreciation expense} = \frac{\text{Remaining useful life of the asset}}{\text{Sum of the years' digits}} \times \text{Depreciable cost}$$

$$SOYD = \frac{[N(N+1)]}{2}$$

$$d_K = \frac{[N-K+1]}{SOYD} * (B - SVN)$$

$$d_k^* = \sum_{n=1}^{k} d_k, \text{for } 1 \le k \le N$$

$$BV_k = B - d_k^*$$

Sum of years digits (SOYD) method

Example: A property has a cost basis of $33,000 and a salvage value of $3,000 with a 5-year useful life. Use the SOYD method to determine the annual depreciation and book values at end of each year.

$$SOYD = \frac{[5(5+1)]}{2} = 15$$

$$d_1 = \frac{[5-1+1]}{15} * (\$33,000 - \$3,000) = \frac{5}{15} * (\$30,000) = \$10,000$$

$$d_2 = \frac{4}{15} \times \$30,000 = \$8,000$$

$$d_3 = \frac{3}{15} \times \$30,000 = \$6,000$$

$$d_4 = \frac{2}{15} \times \$30,000 = \$4,000$$

$$d_5 = \frac{1}{15} \times \$30,000 = \$2,000$$

BV = 33000 − (10000 + 8000 + 6000) = 9000

EOY, k	d_k	BV_k
0	-	$33,000
1	$10,000	$23,000
2	$8,000	$15,000
3	$6,000	$9,000
4	$4,000	$5,000
5	$2,000	$3,000

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Declining Balance (DB) method

Also called the constant-percentage method.

- Annual depreciation is a fixed percentage of the BV at the beginning of the year.

$$d_k = \mathrm{B}\,(1-R)^{k-1}\,R$$

$$d_k^* = \mathrm{B}[1-(1-\mathrm{R})^k]$$

$$\mathrm{BV}_k = \mathrm{B}(1-\mathrm{R})^k$$

Where:

R: constant percentage ratio [2/N] when 200% DB is being used (double declining balance – DDB) and [1.5/N] when 150% DB is used].

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Declining Balance (DB) method

Example: A new cutting machine has a cost basis of $4,000 and a 10-year depreciable life. The machine has no market value at the end of its life. Use the DB method to calculate the annual depreciation when:

(a) $R = 2/N$ or 200% DB or DDB.

(b) $R = 1.5/N$ or (150% DB).

$$R = \frac{2}{10} = 0.2$$

$d_1 = \$4{,}000\,(1-0.2)^{1-1} \times 0.2 = \$800$, $BV_1 = \$4{,}000(1-0.2)^1 = \$3{,}200$.

$d_2 = \$4{,}000\,(1-0.2)^{2-1} \times 0.2 = \$640$, $BV_2 = \$4{,}000(1-0.2)^2 = \$2{,}560$.

... and so on

$$R = \frac{1.5}{10} = 0.15$$

$d_1 = \$4{,}000\,(1-0.15)^{1-1} \times 0.15 = \$600$, $BV_1 = \$4{,}000(1-0.15)^1 = \$3{,}400$.

$d_2 = \$4{,}000\,(1-0.15)^{2-1} \times 0.15 = \$510$, $BV_2 = \$4{,}000(1-0.15)^2 = \$2{,}890$.

... and so on

200% DB Method Only

EOY, k	d_k	BV_k
0	—	$4,000
1	$800	3,200
2	640	2,560
3	512	2,048
4	409.60	1,638.40
5	327.68	1,310.72
6	262.14	1,048.58
7	209.72	838.86
8	167.77	671.09
9	134.22	536.87
10	107.37	429.50

Declining Balance (DB) method

- Sample calculations for year six are as follows:

a)

$R = 2/10 = 0.2$,

$d_6 = \$4{,}000(1-0.2)^5(0.2) = \$262.14$,

$d^*_6 = \$4{,}000[1-(1-0.2)^6] = \$2{,}951.42$,

$BV_6 = \$4{,}000(1-0.2)^6 = \$1{,}048.58$.

b)

$R = 1.5/10 = 0.15$,

$d_6 = \$4{,}000(1-0.15)^5(0.15) = \$266.22$,

$d^*_6 = \$4{,}000[1-(1-0.15)6] = \$2{,}491.40$,

$BV_6 = \$4{,}000(1-0.15)^6 = \$1{,}508.60$.

200% DB Method Only

EOY, k	d_k	BV_k
0	—	$4,000
1	$800	3,200
2	640	2,560
3	512	2,048
4	409.60	1,638.40
5	327.68	1,310.72
6	262.14	1,048.58
7	209.72	838.86
8	167.77	671.09
9	134.22	536.87
10	107.37	429.50

Declining Balance (DB) method

200% DB method only		
EOY k	d_k	BV_k
0	-	$4,000
1	$800	$3,200
2	$640	$2,560
3	$512	$2,048
4	$409.6	$1,638.4
5	$327.68	$1,310.72
6	$262.14	$1,048.58
7	$209.72	$838.86
8	$167.77	$671.09
9	$134.22	$536.87
10	$107.37	$429.50

عشان نحل بـ مشكلة انو ال $BV \neq SV$

الأصل تكون صفر

Never reaches SV

Switchover to SL method

- Switchover occurs in the year in which the SL depreciation is greater than or equal to the DB depreciation.

switch يعمل بنهاية الحالة

Declining Balance (DB) method

200% DB method only				
Year, k	BV @ beginning of year	d_k DDB method	d_k SL method	d_k selected
1	$4,000	$800	$400	$800
2	$3,200	$640	$355.56	$640
3	$2,560	$512	$320	$512
4	$2,048	$409.6	$292.57	$409.6
5	$1,638.4	$327.68	$273.07	$327.68
6	$1,310.72	$262.14	Switch $262.14	$262.14
7	$1,048.58	$209.72	$262.14	$262.14
8	$786.44	$167.77	$262.14	$262.14
9	$524.30	$134.22	$262.14	$262.14
10	$262.14	$107.37	$262.14	$262.14
11	0			

- d_k SL method is calculated based on the BV @ beginning of each year, SV, and the remaining years.

For year 1, d_k SL = ($4,000 – 0)/10 = $400.

For year 2, d_k SL = ($3,200 – 0)/9 = $355.56.

كنت أحسب على اكهيتن لغاية ماوصلنا لقيمة لا بأ تساوي ال DDB بعدها عملنا switch وكملنا على ال d_k SL

Units of production method

A decrease in value is a function of use.

$$\text{Depreciation per unit of production} = \frac{B - SV_N}{\text{estimated lifetime production units}}$$

Example: An equipment has a basis of \$50,000 and is expected to have a \$10,000 SV when replaced after 30,000 hours of use. Find the depreciation rate per hour of use and find its book value after 10,000 hours of operation.

$$\text{Depreciation per unit of production} = \frac{\$50{,}000 - \$10{,}000}{30{,}000 \text{ hours}} = \$1.33 \text{ per hour}$$

$$\text{After } 10{,}000 \text{ hours, } BV = \$50{,}000 - \frac{\$1.33}{\text{hour}} \times 10{,}000 \text{ hours} = \$36{,}700.$$

The end

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