No. of elements in a set $m(A)$

$m(A)$: size of set A

$m(I) = N$

$m(z) = 0$

set

10 Profissinal →
- A 3 assemblers
- M 5 Mechanicals
- C 2 clerks

$m(A) = 3$ $m(M) = 5$ $m(c) = 2$ $m(I) = 10$

ven diagram

A	M	C

A, M and C Muthally exclusive

Q: Both A and M

$M(Q) = AM = 0 = M(z)$

F: Mechanist oR assemblers (other than c

$F = A + M$, $m(F) = m(A) + m(M) = 3 + 5 = 8$

A and M are ME

$M(F) = m(I) - m(c) = 10 - 2 = 8$

→
- 8 full time enginer
- 3 full time superviser
- 2 Both supervises and engi
- 3 assemblers
- 5 Mechanicals
- 2 clerks

A	M	C	engineer 8	Both 2	super 3

$M(I) = 23$, $M(E) = 10$, $M(S) = 5$

$M(E+S) = 8 + 3 + 2 = 19$

since common elements will come twic and must be subtract

$M(b+E) \neq M(E) + M(S)$

$m(E+S) = m(E) + m(S) - m(B)$

متنافية → Mutualy exclussive events

Ex: Tossing a coin

$S = [H, T]$

$P(S) = 1$

H and T are disjoint (منفصلة)

$S = (H \cup T)$

$P(S) = P(H) + P(T)$

$P(T) = P(S) - P(H)$

$P(T) = 1 - P$

Ex: Batteres coming off an assembly line

$$\text{Battery} \xrightarrow{\text{تعمل}} S \qquad \text{Battery} \xrightarrow{\text{عاطلة}} F$$

Events are → $E_1 = (S)$

$E_2 = (FS)$

$E_3 = (FFS)$

$E_4 = (FFFS)$

Probability of sucess $= 0.99 = P(S)$

$P(E_1) = P(S) = 0.99$

$P(E_2) = P(F \cap S) = P(F) \cdot P(S) = 0.01 * 0.99$

$P(E_3) = P(F) \cdot P(F) \cdot P(S) = 0.01^2 * 0.99$

$P(E_4) = P(F) \cdot P(F) \cdot P(F) \cdot P(S) = 0.01^3 * 0.99$

مهم جدا!! $P(F) * P(S) \rightarrow (1-P)^{n-1} * P$

Ex: A: 60% subscribe to

B: 80% subscribe to local news paper

A and B: 50% subscribe to both

[illegible]

ⓐ what is Probability to <u>at least one of two newPaper</u> ? صحيفة أو أكثر

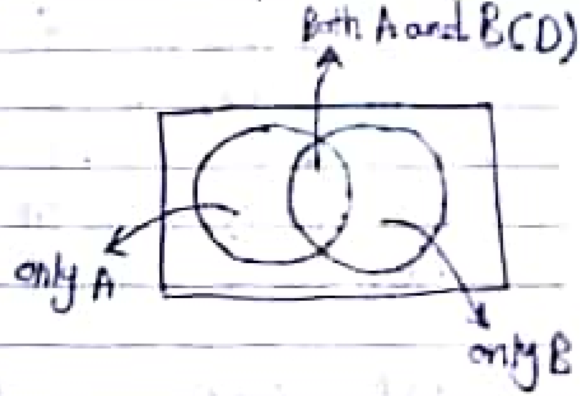
ⓑ <u>exactly one</u> of the news paper ? فقط A أو فقط B

$P(A) = 0.6$, $P(B) = 0.8$, $P(A \cap B) = 0.5$

ⓐ $P(A \text{ or and } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.8 - 0.5$

$= 0.9$

ⓑ exactly one and Both are usually HE



$(C \cup D) = (A \cup B)$

$P(C) + P(D) = P(A \cup B)$

$P(C) = P(A \cup B) - P(D)$

$= 0.9 - 0.5 = 0.4$

Permutation تباديل (الترتيب مهم)

Ex: ○○○ □□□□

$n = 3$ $m = 4$ $T = 7$

$$\frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5!}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} = \frac{7 \times 5 \times 4!}{4!} = 35$$

Combination (الترتيب غير مهم)

Ex: set A B C D E $n=5$
subset $r=3$

لـ الترتيب غير مهم

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{5!}{3!(5-3)!} = 10$$

$$P_{r\,in} = \frac{n!}{(n-r)!} = \frac{5!}{2!} = \frac{5\times4\times3\times2!}{2!} = 60$$

Ex:- 10 male (M)
15 female (F)
make groups of 6 at random
▲ Prob. that 3 are M and 3 are F ?
D_3 = exactly 3 of the 6 are F
Any set of the 6 are equally likely

$P(D_3) = \frac{N(D_3)}{N}$ → no. of ways choosing 3 F
N → total no. of choosing 6 from 25

$$N = \binom{25}{6} = \frac{25!}{6!(25-6)!} = \frac{25!}{6!\,19!}$$

$N(D_3)$
n_1 = No of ways of choosing 3 F = $\binom{15}{3} = \frac{15!}{3!\,12!}$
n_2 = " " " " " 3 M : $\binom{10}{3}$ $\frac{10!}{3!(10-3)!}$

$$N(D_3) = n_1 \times n_2 = \binom{15}{3}\binom{10}{3}$$

$$P(D_3) = \frac{\binom{15}{3}\binom{10}{3}}{\binom{25}{6}} = \frac{\left(\frac{15!}{3!\,12}\right)\left(\frac{10!}{\ }\right.}{\ } = 0.308$$

Scanned by CamScanner

Prob at least 3F ?

D_4 exactly 4F → 2M

D_5 " 5f → 1M

D_6 " 6F → 0M

$$P(\text{at least } 3F) = P(D_3 \cup D_4 \cup D_5 \cup D_6)$$
$$= P(D_3) + P(D_4) + P(D_5) + P(D_6)$$
$$= \frac{\binom{15}{3}\binom{10}{3}}{\binom{25}{6}} + \frac{\binom{15}{4}\binom{10}{2}}{\binom{25}{6}} + \frac{\binom{15}{5}\binom{10}{1}}{\binom{25}{6}} + \frac{\binom{15}{6}\binom{10}{0}}{\binom{25}{6}}$$

Ex :- Two assembly lines A and A'

A Produced a components 2 of which are defective (B) and 6 are not defective (B')

" " " 1 " " " " " " 9 " " "

	B	B'	
A	2	6	8
A'	1	9	10
	3		18

all 18 outcomes are equally likely

$$P(A) = \frac{N(A)}{N} = \frac{8}{18} = 0.44$$

on examination of A ____ dificted

(B) has acurve, the elements of space B

by ~~expression~~ inspection

$$P(A/B) = \frac{2}{3} = \frac{2/18}{3/18} = \frac{P(A \cap B)}{P(B)} \Rightarrow$$ ratio of unconditional Probability P(intersection): P(conditioning exist)

$\frac{1}{P(B)}$: constant

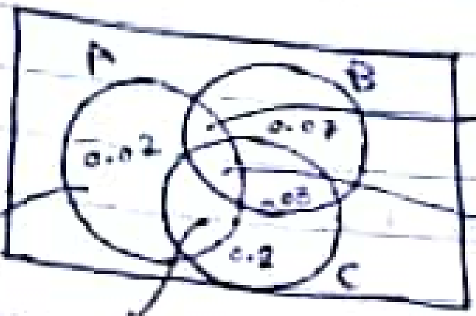
$$P(B/B) = \frac{P(B \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Ex :- Reading Habits

Colums A: Art B: Books C: cinema

Read Regulary	A	B	C	$A\cap B$	$A\cap C$	$B\cap C$	$A\cap B\cap C$
	0.14	0.23	0.37	0.08	0.09	0.13	0.05



$\rightarrow (B\cap C) - (A\cap B\cap C)$

$\rightarrow 0.05$

0.14
0.05
0.03
0.04

$(A\cap C) - (A\cap B\cap C) = 0.04$

$A - 0.05 - 0.04 - 0.03 = 0.02$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C)$
$+ P(A\cap B\cap C)$
$= 0.49$

$P(\overline{A\cup B\cup C}) = 1 - 0.49 = 0.51$

$P(A \mid B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.08}{0.23} = 0.348$

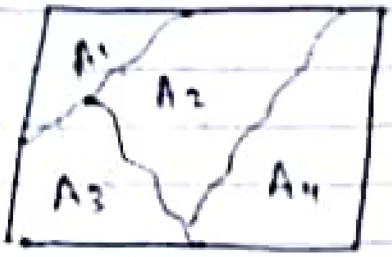
$\cancel{P(A \mid (A\cup B)) = \dfrac{P(A\cap(A\cup B))}{P(A\cup B)}}$ ~~(0.04 + 0.05 + 0.03~~

$P(A / (B\cup C)) = \dfrac{P(A\cap(B\cup C))}{P(B\cup C)} = \dfrac{0.04 + 0.05 + 0.03}{0.47}$

$P(A / \text{at least one}) = P(A / (A\cup B\cup C))$
$= \dfrac{P(A\cap(A\cup B\cup C))}{P(A\cup B\cup C)} = \dfrac{P(A)}{P(A\cup B\cup C)} = \dfrac{0.14}{0.49} = .286$

$P((A\cup B) / C) = \dfrac{P((A\cup B)\cap C)}{P(C)} = \dfrac{0.04 + 0.05 + 0.08}{0.37} = 0.459$

Ex:



$P(B/A_i)$

$P(B) = \sum_{i=1} P(B/A_i) * P(A_i)$

unconditional Pro — conditional — weeting

	B	B'		$P(A_i)$
A_1	2	6	8	8/18
A_2	1	4	10	10/18
	3	15	18	
$P(B_i)$	3/18	15/18		

$P(B) = P(B \cap A_1) + P(B \cap A_2)$

$= P(B/A_1) \cdot P(A_1) + P(B/A_2) P(A_2)$

$= \frac{2}{6} \cdot \frac{8}{18} + \frac{1}{10} \cdot \frac{10}{18} = \frac{3}{18}$

Ex: semiconductors Manufacturing

B (Failure)	A	% (expos
P(Failure	level of contamination	
0.1	High H	20%
0.01	Medium M	30%
0.001	Low L	50%

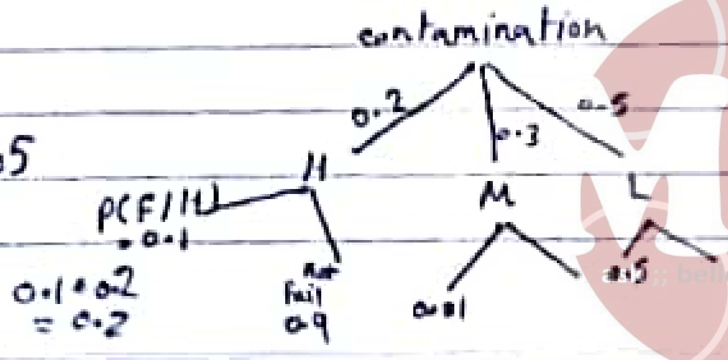
P(B): Prob of one comp fails ?

$= P(F/H) \cdot P(H) + P(F/M) \cdot P(M) + P(F/L) \cdot P(L)$

$= 0.1 * 0.2 + 0.01 * 0.3 + 0.001 * 0.5$

$= 0.0235$

$P(F) = 0.02 + 0.003 + 0.0005$



Ex: 84 air samples that contain rare molecules

A: all samples in which molecules 1 is Present

B: " " " " " 2 " "

Results:

		molecule 1		
		no	Yes	
molecule 2	No	32	24	56
	Yes	16	12	28 → contain molecule 2
		48	36	

↳ all contain molecule 1

$$P(B) = \frac{28}{84} = \frac{1}{3}$$

$$P(B \backslash A) = \frac{P(A \cap B)}{P(A)} = \frac{12/84}{36/84} = \frac{1}{3}$$

Knowlege of Present of ① didn't affect Present of ②

Also

$$P(A) = \frac{36}{84} = \frac{3}{7}$$

$$P(A/B) = \frac{P(A \cap B)}{P(A)} = \frac{12/84}{28/84} = \frac{3}{7}$$

Ex: Total Production in a day = 850 → 50 defective, → 800 not defective

بالتوالي

exp: selection of two Parts in succession

A: first Part selected is defective $\qquad$ A' (not defective)

B: second " " "

what is prob that 2^{nd} Part is defective: ① with replacement

② without replacement

① with replacement

$$P(B/A) = P(B) = \frac{50}{850} = P(B/A')$$

② without replacement

$$P(A) = \frac{50}{850}$$

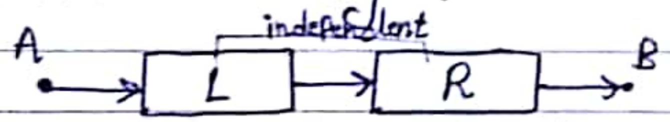
$$P(B/A) = 49/849$$

$$P(B) = P(B/A).P(A) + P(B/A').P(A')$$

$$P(B) = \frac{49}{849}\cdot\frac{50}{850} + \frac{50}{849}\cdot\frac{800}{850} = \frac{50}{850}$$

independent

Ex: circuit consisting of two units:

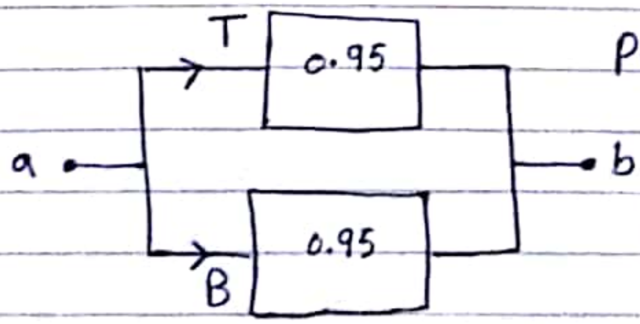


$P(L) = 0.8 \qquad P(R) = 0.9$

Prob circuit works?

$$P(L \cap R) = P(L).P(R) = 0.9 * 0.8 = 0.72$$

* Parallel circuits



$P(T \text{ or } B) = 1 - P(T \text{ or } B)'$

قانون

$= 1 - P(T' \text{ and } B')$

$= 1 - (0.05)^2 = 0.9975$

$P(T' \text{ and } B') = P(T' \cap B') =$

$P(T').P(B') =$

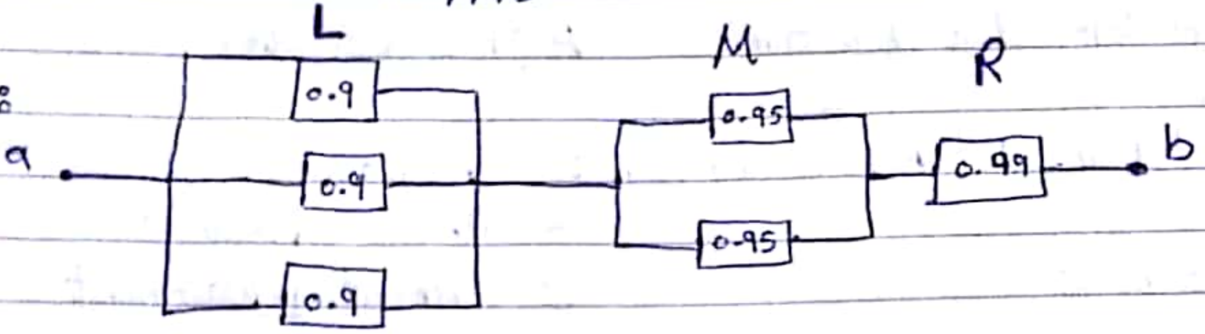
$(1-0.95)(1-0.95) = (0.05)^2$

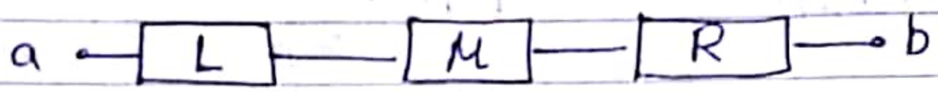
$P(T \text{ and/or } B) = P(T \cap B') + P(T' \cap B) + P(T \cap B)$

$= (0.95 * 0.05) + (0.95 * 0.05) + (0.95 * 0.95)$

$= 0.9975$

Ex:





$P(CW) = P(L) \cdot P(M) \cdot P(R)$

$= [1 - 0.1^3] \cdot [1 - 0.05^2] \cdot [0.99]$

$S = [P, aP, aaP, aaaP, \ldots]$

$P(X=1) = 0.01$

$P(X=2) = P(aP) = 0.99 \times 0.01 = 0.0099$

General formela

$P(X=x) = P(\underbrace{aa\cdots a}_{(x-1)\text{ a's}}P)$

$f(x) = P(X=x) = (1-P)^{x-1} P$

$\sum_{1}^{\infty} f(x) = 1$ Geometric distribution

$f(x_i) = P(X=x_i)$

experement has unbounded number of outcomes the R.V X is a discrete R.V with a countibly infinite range.

3.4 cumulative distribution function $F(x)$

For any discrete R.V with Possible values $X_1, X_2, X_3, \ldots, X_n$ the events $[X=x_1], [X=x_2], \ldots [X=x_n]$ are mutually exclusive $X \leq x$ union of events $(X=x_1), \ldots (X=x)$

$\because F(X) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$

x with Proparties :-

1) $0 \leq F(x) \leq 1$

2) if $x \leq y$, then $F(x) \leq F(y)$; $X \leq x$ is contain the event $X \leq$

3) $P(X > x_i) = 1 - P(X \leq x_i)$

Ex:- $X : [0, 1, 2, 3, 4]$

X	$f(x)$
0	0.6561
1	0.2916
2	0.0486
3	0.0036
4	0.0001
	1 mass function

$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= 0.9999$

$F(1.5) = P(X \leq 1.5) = P(X=0) + P(X=1) + P(X \overset{0}{=} 1.5)$
$= 0.9477$

* Mean and variance R.V

Expectation → long run average
Expectation → average value of statistic (based on sample)

Expected value of a R.V $E(X)$
by def $E(X) = \mu$

Discrete variable → Prob mass function (weight)

$\mu = E(X) = \sum_{x_i} f(x_i)\, x_i$

x_i : i^{th} value of X

$\equiv \sum$ mass * distance
$\equiv$ centroid of the Prob mass
$\equiv$ Weighted average of Possible value of X
$\equiv$ first non central moment (اهم تسمية)

Variance of X $\rightarrow$ $V(X)$, σ^2, $VAR(X)$

$\sigma^2 = E(X-\mu)^2 = \sum f(x)(x-\mu)^2$ = second central moment

$= \sum x^2 . f(x) - \mu^2$

* standard diviation of X is $\sigma = \sqrt{\sigma^2}$

For specific function of X in $h(x)$

$E[h(x)] = \sum h(x) . f(x)$

i.e $h(x) = (x-\mu)^2$

Ex:- $y = ax + b$

$E(y) = E(ax+b) = a\,E(x) + E(b) = a\mu_x + b$

$\mu_y = a\mu_x + b$

Ex:- $h(x) = x^2$

$E[h(x)] = \sum x^2 . f(x)$

Ex:- X: No of defectives In the next 4 selected Pices

$X : [0, 1, 2, 3, 4]$

X	0	1	2	3	4
f(x)	0.6561	0.2916	0.4810	0.0036	0.0001

$\mu = E(X) = \sum_{x=1}^{5} f(x_i) . x_i = 0.6561*0 + 0.2916*1 + 0.4810*2 \ldots$

$= 0.4$

$V(X) = \sigma^2 = \sum_{x=1}^{5} f(x_i) . (x_i - \mu)^2$

x_i	$x_i - \mu$	$(x_i-\mu)^2$	$f(x_i)$	$(x_i-\mu)^2 . f(x_i)$
0	-0.4	0.16	0.6561	
1	0.6	0.36	0.2916	
2	1.6	2.56	0.0481	
3	2.6	0.78	0.0036	
4	3.6	12.96	0.0001	
				$\sum = 0.36$

* Negative Binomial :

Generalisation of geometric distribution

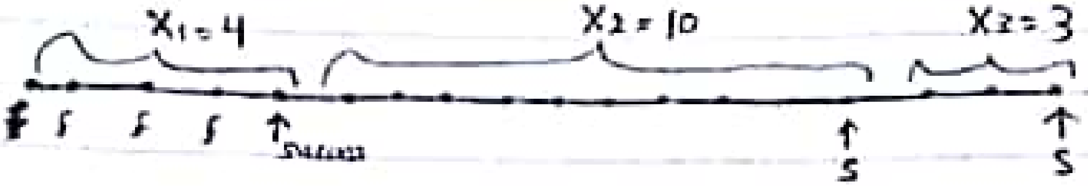
X: number of Bernolli trials required to obtain success

$f(x) = \binom{x-1}{r-1}(1-P)^{x-r}P^{r}$, $x = r, r+1, r+2, r+3$

Range of x : $r \rightarrow \infty$ since at least r trials are required to obtain r success

$\mu = \frac{r}{P}$, $\sigma^2 = \frac{r(1-P)}{P^2}$

-ve Binomial ≡ sum of geometric R.V



$X = X_1 + X_2 + X_3 = 17$

$r = 3$

* depending on Bernolli trials

Binomial : number of success in n trial

-ve Binomial : number of trial to obtain r successes

Geometric : Nu of trial to obtain 1 S.t successes

Ex :-

Data from 250 endothermic reactions involving Sodium Bichrbonate

X = Final Temp condition	No. of reaction (trials)	
266 K	70	$P(X=266) = 70/250 = 0.28$
271 K	80	$P(X=271) = 80/250 = 0.32$
274 K	100	$P(X=100) = 100/250 = 0.4$
	250	1

Prob mass function of final temp

X = final temp

$$f(x) = \begin{cases} 0.28 & x = 266 \\ 0.32 & x = 271 \\ 0.4 & x = 274 \end{cases}$$

a. prob that the first reaction to resulting in a final Temp less than 272 of the 10" reaction?

this is geometric R.V

$P(X < 272) = P(X = 266) + P(X = 271)$

$= 0.28 + 0.32 = 0.6$

$P(X = 10) = (1 - 0.6)^{x-1} \cdot 0.6$

$= 0.4^7 \cdot 0.6 = 0.000157$

b- what is the mean number of reaction until the first final Temp is less than 272?

$\mu = \frac{1}{p} = \frac{1}{0.6} = 1.6\dot{7}$

c- what is the Prob that the first reaction to result in a final temp less than 272 k occurs within 3 to fewer reactions.

$P(X \leq 3) = P(X = 3) + P(X = 2) + P(X = 1)$

$= 0.4^2 \times 0.6 + 0.4 \times 0.6 + 0.4^0 \times 0.6 = 0.936$

d. what is the mean number of reaction until two reaction result in a final temp less than 272?

$X \to$ is -ve binomial, $\mu = \frac{r}{p} = \frac{2}{0.6}$

* Poisson distribution

- Poisson Process : ~~~~ a random experiment with the following Properties.
- Consider an interval T of real numbers (time, length, area) partition into subintervals of small length Δt and assume that $\Delta t \to 0$

$$\left(n = \frac{T}{\Delta t}\right)$$

1) Prob of more than one event in a subinterval tends to zero
2) Prob of one event in a subinterval $= \frac{\lambda}{n} = \frac{\lambda \Delta t}{T}$

$\lambda \to$ average number of events in the interval

[Note: each subinterval can contain only one event]

3) event in each sub intervals is independent of other sub intervals

- these assumption apply that subintervals are approximatly bernoli trial with $P = \frac{\lambda}{n} = \lambda \frac{\Delta t}{T}$

$n = \frac{T}{\Delta t}$, $\lambda = P.n = \text{const}$ as $\Delta t \to 0$, $n \to \infty$

$\lambda = a.T$, a : is a rate of occurence Per unit interval

$$a = \frac{\lambda}{T} \longrightarrow \lambda = a.T$$

X: no. of events in a Poisson Process with Parameter λ

$$f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad , x = 0, 1, 2 \ldots$$

$\mu = E(x) = \lambda$

$\sigma^2 = \lambda$

تشوهات

X: No of flaws in a length of copper wire

λ: is the average no. of flaws in L mm

suppose X is Poisson R.V

$a = 2.3$ flaws/mm $\longrightarrow \lambda =$

~~Problem~~ Prob of exactly 2 flaws in 1 mm

X: no of flaws in 1mm $\longrightarrow$ L = 1 mm

$$\lambda = \frac{2.3}{1} = 2.3$$

- $P(X=2) = \frac{e^{-2.3} \cdot 2.3^2}{2!} = 0.265$

- Prob of 10 flaws in 5mm in this case

X: no of flaws in 5mm $\rightarrow L = 5mm$

$\lambda = a \cdot L$

$= 2.3 * 5 = 11.5$

$P(X=10) = \frac{e^{-11.5} \cdot 11.5^{10}}{10!} = 0.113$

- Prob of at least one flaw in 2mm of wire

$L = 2mm$

$\lambda = 2.3 * 2 = 4.6$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \frac{e^{-4.6} \cdot 4.6^{0}}{0!}$

$= 1 - e^{-4.6} = 0.9899$

Example:-

X : current measured in thin Cu wire mA

Range of X   0, 20 mA

is uniform $f(x) = \frac{1}{b-a} = \frac{1}{20-0} = 0.05$

$0 \leq X \leq 20$

$P(5<X<10)$

$f(5<X<10) = \int_{5}^{10} f(x).dx = 0.05 \times 5 = 0.25$

$F(X) = ?$

$F(x) = \frac{x-a}{b-a} = \frac{x-0}{20-0} = \frac{x}{20}$ $0 \leq X \leq 20$

Standard form :-

We defined normal variable

$Z = \frac{X-\mu}{\sigma}$, if x is normal then Z is also normal

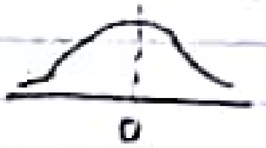
$dZ = \frac{1}{\sigma} dx$, $\frac{dx}{dz} = \sigma$

$Prob(X+\Delta X) = Prob(Z+\Delta Z)$

$\therefore f(x)\,dx = f(z)\,dz$

$f(z) = \frac{f(x)\,dx}{dz} = \sigma f(x)$

$f(z) = \frac{1}{\sqrt{2\pi}} e^{-1/2 z^2}$ ⇒ standard form



$E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = 0$

Variance $(x) = E\left[\left(\frac{X-\mu}{\sigma} - \mu\right)^2\right] = E \frac{(X-\mu)^2}{\sigma^2} = \frac{\sigma^2}{\sigma^2} = 1$

$$Z = \frac{X - \mu}{\sigma}$$

$\mu_Z = 0 \;,\; \sigma_Z = 1$

Ex: X: current in mA

$\mu = 10\,mA$

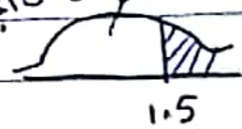
$\sigma^2 = 4\,(mA)^2$

المساواة وعدمها واحد $P(X \geq 13\,mA)$?

$$Z = \frac{X - 10}{2}$$

$P(X>13) = P\left(Z > \frac{\cancel{x}\,13 - 10}{2}\right) = P(Z > 1.5)$

0.9332 = نطلعها من الجدول



$= 1 - P(Z < 1.5)$

$= 1 - 0.9332 = 0.0668$

$P(9 < X < 11)$

$= P\left(\frac{9-10}{2} < Z < \frac{11-10}{2}\right)$

$= P(-0.5 < Z < 0.5)$

$= P(Z < 0.5) - P(Z < -0.5)$

$= 0.6915 - 0.3085$

$= 0.383$

Ex: Find x such that $P(X<x) = 0.98$

$P(X<x) = P\left(Z< \frac{x-10}{2}\right) = 0.98$

From the table the nearest value

$P(Z<2.05) = 0.9798$

$x = 10 + 2 \times 2.05 = 14.1 \text{ mA}$

Ex: X: no. of Bits received in error (Bionomial)

P: Prob. received in error

$n = 16000000$ bits

$p = 1 \times 10^{-5}$

$P(X \leq 150)$?

$P(X<150) = \sum_{x=0}^{150} \binom{16000000}{x} (10^{-5})^{x} (1-10^{-5})^{16000000-x}$

$\binom{16000000}{x} \rightarrow \frac{n!}{x!\,(n-x)!}$ very difficult

$P(X \leq 150) = P(X \leq 150+0.5) = P\left(Z \leq \frac{150.5 - 160}{\sqrt{160(1-10^{-5})}}\right)$

$= P(Z \leq -0.75)$

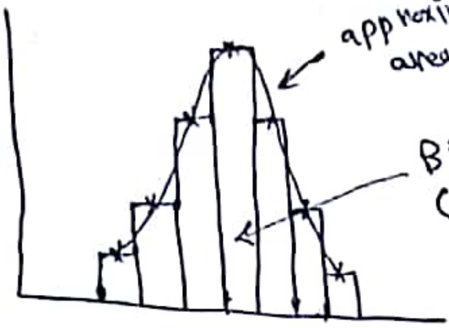
$= 0.2266$

Poisson $Z = \frac{x-\lambda}{\sqrt{\lambda}}$ بنوخدها كرقم

4.6 Normal Approximation to the Binomial Distribution:

Binomial Distribution $\simeq$ Normal Distribution for

$np > 5$ OR $n(1-p) > 5$; n must be larg



$$Z = \frac{X-\mu}{\sigma}$$

*for Bionomial
$\mu = np$
$\sigma = \sqrt{np(1-p)}$

if x is binomial RV with parameters n and P, the

$$Z = \frac{X-np}{\sqrt{np(1-p)}}$$

$E(x) = \mu = np$

$Var(X) = \sigma^2 = np(1-p)$

is approximately <u>a standard normal variable</u>

*<u>Continuity Correction:</u>

A continuity correction factor of 0.5 is added to OR subtracted from x to make the probability greater

$$P(X \le x) = P(X \le x+0.5) \simeq P\left(Z \le \frac{x+0.5-np}{\sqrt{np(1-p)}}\right)$$

$$P(X \ge x) = P(X \ge x-0.5) \simeq P\left(Z \ge \frac{x-0.5-np}{\sqrt{np(1-p)}}\right)$$

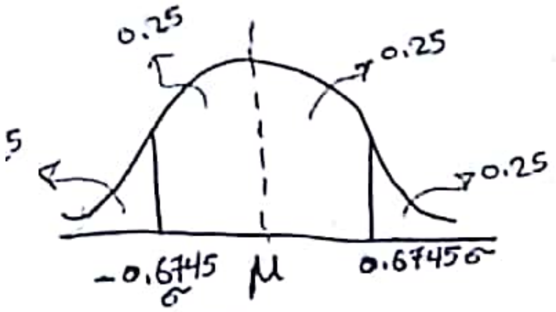
$P(X<X)$

$P(X \ge X)$

Probable Error

Deviation from the mean such that one half the area under the curve lies within the range of this deviation from either side.

This range lies within : $\pm 0.6745\sigma$



This means that an observation has an equal chance of falling within this range or outside it.

Standard Error

It is a measure of precision

It is a deviation of $\pm\sigma$ from the mean

$1\sigma = 68\%$

68.26 % of all observations fall within the range $\mu \pm \sigma$

Values exceeded by chance:

$5\% \Rightarrow \pm 1.96\sigma$

$1\% \Rightarrow \pm 2.576\sigma$

Ex:- X: denote T °c
Y: denote time in minutes for diesel engine to start

$$f_{xy}(x,y) = c(4x + 2y + 1) \qquad 0 \le x \le 40$$
$$0 \le y \le 2$$

$\boxed{1}$ Value of c

$$\int_0^2 \int_0^{40} f_{xy}(x, y)\, dx\, dy = 1$$

$$\int_0^2 \int_0^{40} c(4x + 2y + 1\, dx\, dy) = 1$$

$$c \int_0^2 \frac{4x^2}{2} + 2yx + x \Big|_0^{40} dy = 1$$

$$c \left[\frac{4yx^2}{2} + \frac{2xy^2}{2} + xy \Big|_0^{40} \Big|_0^{2} = 1 \right]$$

$$c \left[2 \times 40^2 \times 2 + 40 \times 2^2 + 40 \times 2 \right) = 1$$

$$c = \frac{1}{6640}$$

$\boxed{2}$ Find the Prob that in a random selected they, the air temp will exceeds 20 and at least 1 min to start

$X \geq 20$
$\hookrightarrow 20 \le X \le 40$

$P(X \geq 20 \text{ and } Y \geq 1)$

$1 \le Y \le 2$

$$\int_1^2 \int_0^{40} f_{xy}(x, y)\, dx\, dy$$

$$\int_{1}^{2}\int_{20}^{40} c\,[4x+2y+1]\,dx\,dy$$

$$= 2x^2y + xy^2 + xy \Big|_{20}^{40}\Big|_{1}^{2}$$

$$= c \cdot 2480$$

$$= \frac{2480}{6640} = 0.3735$$

$$f_X(x) = \int_0^2 f_{XY}(x,y)\,dy$$

$$= \int_0^2 c\,\underbrace{(4x+2y+1)}_{\text{Joint}}\,dy$$

$$= c\left[2xy + y^2 + y\right]_0^2$$

$$= c[8x+4+2] = c[8x+6]$$

$$f(x)^{(x)} = c[8x+c]$$

$$f_X(y) = \int_0^{40} f_{XY}(x,y)\,dx$$

$$f_X(y) = \int_0^{40} c\,[4x+2y+1]\,dx$$

$$= c\left[2x^2 + 2xy + x\right]_0^{40}$$

$$= c[3200 + 480y + 40]$$

$$= c[480y + 3240]$$

$P(x \geq 1) = \int_1^2 f_y(y)\,dy$

$= \int_1^2 c[3240 + 80y]\,dy$

$= c[3360] = \frac{3360}{6640} = 0.506$

من الجدول

$P(Y=1 \mid X=3) = \frac{P(X=3, Y=1)}{P(X=3)} = \frac{f_{x,y}(3,1)}{f_x(3)} = \frac{0.25}{0.55}$

conditional $f_{y/x}(y)$

	1	2	3
4	0.75	0.4	0.091
3	0.1	0.4	0.091
2	0.1	0.12	0.364
1	0.05	0.08	0.454
1	1	1	1

$f_{y/x}(y) = \frac{f_x(x,y)}{f_x(x)} = \frac{c[4x+2y+1]}{c[8x+6]} = \frac{4x+2y+1}{8x+6}$

find Prob $y>1$ given $x=20$

$P(y>1, x=20)$

$f_{xy}^{(x,y)} = \int_1^2 \frac{4x+2y+1}{8x+6}\,dy \quad x=20$

$= \int_1^2 \frac{80+2y+1}{160+6}\,dy \Rightarrow \frac{1}{166}\left[80y + y^2 + y\right]_1^2 = 0.5$

1

Case of Two Independent Normal populations: Parameters: μ_1, σ_1^2 and μ_2, σ_2^2

Sampling distributions of the means are: $\bar{X}_1$ and $\bar{X}_2$

Analysis of Differences: $\bar{X}_1 - \bar{X}_2$

In general, given random variables $X_1, X_2, \ldots, X_p$ and constants $c_1, c_2, \ldots, c_p$, then:

If $Y = c_1 X_1 \pm c_2 X_2 \pm \ldots \pm c_p X_p$ is a linear combination of $X_1, X_2, \ldots, X_p$ then:

$$E(Y) = c_1 E(X_1) \pm c_2 E(X_2) \pm \ldots \pm c_p E(X_p) \text{ and}$$

$$V(Y) = c_1^2 V(X1) + c_2^2 V(X2) + \cdots + c_p^2 V(Xp) + 2\sum_{i<j}\sum c_i c_j \mathrm{Cov}(X_i X_j)$$

the last term is equal to zero for independent variables.

Therefore:

- Expected Value:

$$E[\bar{X}_1 - \bar{X}_2] = \mu_{\bar{X}_1 - \bar{X}_2} = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2$$

- Variance:

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = V[(\bar{X}_1 - \bar{X}_2)]$$

$$= V(\bar{X}_1) + V(\bar{X}_2)$$
$$= \sigma^2_{\bar{X}_1} + \sigma^2_{\bar{X}_2}$$
$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

- Standard Normal Sampling Distribution of the Mean:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

• اذا كانت σ مجهولة بستخدم S

$$Z = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{25 - 50}{\sqrt{136}} = -2.14$$

$P[(\bar{x}_2 - \bar{x}_1) \geq 25] = P[Z \geq -2.14]$

$= 0.9838$

If the parent populations are normal, then Z is exactly normal, otherwise Z is approximately normal if the central limits theory can be applied.

Ex:- component with mean effective life = 5000 hr $\sigma = 40$ hr

P.P is normal $\mu_1 = 5000$
$\sigma_1 = 40$

Improvement component with mean effective life = 5050 hr $\sigma = 30$ hr

check on difference between means

$n_1 = 16$
$n_2 = 25$
$P[\bar{x}_2 - \bar{x}_1] \geq 25$?

continues

$$\sigma^2_{(\bar{x}_2 - \bar{x}_1)} = \sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_2} = \frac{40^2}{16} + \frac{30^2}{25} = 100 + 36 = 136$$

solution:-

$\bar{x}_1 = 5000$ $\sigma_{\bar{x}_1} = \frac{\sigma_1}{\sqrt{n_1}} = \frac{40}{\sqrt{16}}$

$\bar{x}_2 = 5050$ $\sigma_{\bar{x}_2} = \frac{\sigma_2}{\sqrt{n_2}} = \frac{30}{\sqrt{25}}$

$[\bar{x}_2 - \bar{x}_1]$ is normal with mean

$= \mu_2 - \mu_1 = 50$ hr

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Standard Error: (Standard Deviation of Sampling Distribution)

Introduction

The standard error of a statistic (or an estimator) is actually the standard deviation of the sampling distribution of that statistic (or $\sqrt{variance\ of\ estimator}$). Standard errors reflect how much sampling fluctuation a statistic will show. Increasing the sample size, the Standard Error decreases.

The size of standard error is affected by two values:

1. The Standard Deviation of the population which affects the standard error. Larger the population's standard deviation (σ), larger is standard error i.e. $\frac{\sigma}{\sqrt{n}}$
2. The standard error is affected by the number of observations in a sample. A large sample will result in a small standard error of an estimate.

Standard Error of Mean:

The standard error for the mean or standard deviation of the sampling distribution of the mean, measures the deviation/ variation in the sampling distribution of the sample mean ($\bar{X}$), denoted by ($\sigma_{\bar{x}}$) and calculated as a function of the standard deviation of the population and respective size of the sample. For a normal population with SD σ

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

And this is the standard error in the point estimator of the normal population mean μ.

When the standard deviation (σ) of the population is unknown, an estimate of the standard error is obtained using the standard deviation of the sample as an estimator (S) of (σ). In this case, an estimate of the standard error of the mean is:

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Ex- Thermal conductivity result are found at 100°F and 550

k in Btu/hr ft-°F

41.6 / 41.8 / 42.23 / 42.45 / 41.86 / 42.18 / 41.32 / 42.26 / 41.81 / 42.04

Points estimates → $\bar{X} = \Sigma x/n$ $\quad s = \Sigma(x-\bar{x})^2/n-1$

$\bar{X} = 41.924$

$s = 0.284$

$$\hat{\sigma}_{\bar{x}} = \frac{S}{\sqrt{n}} = \frac{0.284}{\sqrt{10}} = 0.0898$$

Y = coefficient of variation $= \frac{S.D}{mean} = \frac{0.0898}{41.924} = 0.21$

$2\hat{\sigma} = 0.1796 \rightarrow 41.744 \leq \mu \leq 42.104$
with high Prob

* this mean that the resulting Point estimate of μ is ~~reose~~ resaise of the sample mean

③

Standard Error for Difference between Means:

Standard error for difference between two independent quantities is:

$$\sigma_{\bar{x}_1 + \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where σ_1 and σ_2 are the respective variances of the two independent populations to be compared and $n_1 + n_2$ are the respective sizes of the two samples drawn from their respective populations.

For unknown variances: $\hat{\sigma}_{\bar{x}_1 + \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Methods of Point Estimation:

Method of moments:

Equate population moments, which are defined in terms of expected values, to the corresponding sample moments. The population moments are functions of unknown parameters (to be estimated). Then these equations are solved to yield estimates of the unknown parameters.

Moments:
Given random variables $X_1, X_2, \ldots, X_n$ taken from distribution f(x) (discrete or continuous). Then:

k^{th} population distribution moment is $E(X^{k})$, $k=1,2,\ldots$
k^{th} sample moment is $\frac{1}{n}\sum_{i=1}^{n} X_i^k$, $k=1,2,\ldots$ and n is sample size.

1st Population moment $= E(x) = \mu$
1st sample moment $= \frac{1}{n}\sum x = \bar{x}$
$\hat{\mu} = \bar{x}$ sample mean in the moment estimate of the population mean

Ex: Normal distribution moment estimators
1st Pop moment $E(x) = \mu$, $2^{nd} = E(x^2) = \sigma^2 + \mu^2$
sample 1st moment $= \frac{1}{n}\sum x_i = \bar{x}$, $2^{nd}: \frac{1}{n}\sum x_i^2$
$\hat{\mu} = \bar{x}$, $\hat{\mu}^2 + \sigma^2 = \frac{1}{n}\sum x_i^2$
solving equation $\hat{\mu} = \bar{x} = \frac{1}{n}\sum x_i$
$\hat{\sigma}^2 = \frac{1}{n}\sum x_i^2 - \left[\frac{1}{n}\sum x_i\right]^2$

$= \frac{\sum x_i^2 - n\bar{x}^2}{n}$

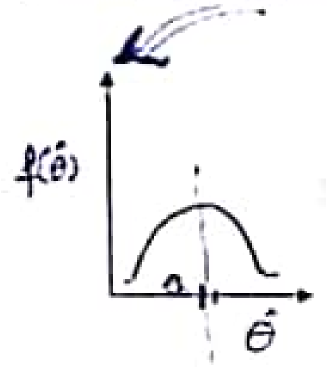
$\hat{\sigma}^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

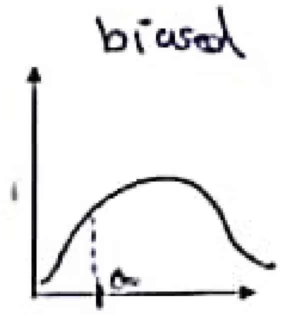
Properties of Estimators:

Quality of an estimator is judged by the distribution of the estimates (sampling distribution)

غير متحيز

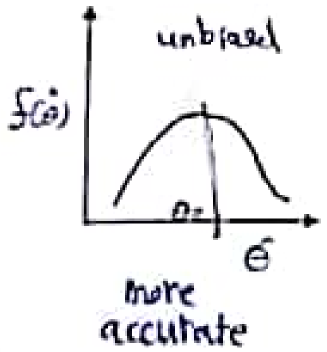
- Estimator should be <u>unbiased</u>:

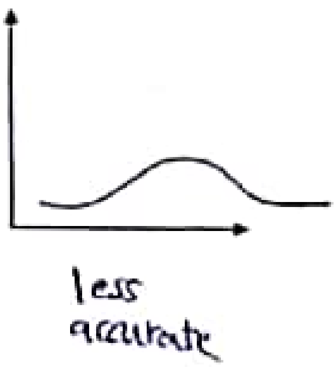




- Estimator should be consistent:

  Spread of distribution of estimate should be as small as possible. It is inversely proportional to the sample size.





Impact energy (J) on specimens of A 238 steel cut at 6°C

64.1 / 61.7 / 61.5 / 64.6 / 64.5

64.3 / 64.6 / 64.2 / 64.2 / 61.3

Assume impact energy in normally distributed

$\sigma = 1$ J find 95% CI for μ . mean

Impact energy

$$\bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} = \frac{\sum x}{n} = 64.46 \quad , \sigma = 1 \quad , \quad n = 10$$

$$(1-\alpha)100 = 95\% \rightarrow \alpha \, 100 = 5\%$$

$$\frac{\alpha}{2} = 2.5\%$$

$$Z_{\alpha/2} = Z_{0.025} = 1.96$$

$$95\% \, CI = 64.46 - 1.96 \frac{1}{\sqrt{10}} \leq \mu \leq 64.46 + 1.96 \frac{1}{\sqrt{10}}$$

* Confidence level and Precision of estimation

Length of CI

$$2 \cdot \left[Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right]$$

measure of Precision $\propto \frac{1}{\sqrt{n}}$

choise of sample size

error $E = |\bar{X} - \mu|$ should be less than $Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

with confidence $100(1-\alpha)\%$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2 \text{ , round up}$$

error $|\bar{X} - \mu|$ will not exceed E , this case

$2E$ is the resulting CI