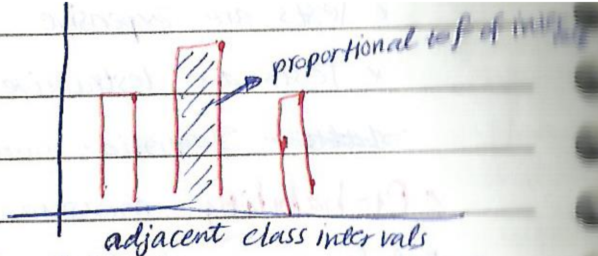
Histograms:

1- the Area of rectangles:

~~is~~ proportional to f of intervals



if width equal for intervals $\Rightarrow$ 2- the highest proportional to f or f/n

Density scale: by calculating rectangular hight $= \dfrac{f/n}{\text{class width}}$

Area of rectangle $= \dfrac{f}{n}$ & $\sum \text{Area} = 1$

2 Numerical Methods: sensitive to outliers

* Mean ⇒ ① Sample mean (Arithmetic) ⇒ $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

② Geometric mean ⇒ $\bar{x}_g = \left[\prod_{i=1}^{n} x_i\right]^{\frac{1}{n}} = \sqrt[n]{x_1 * x_2 * x_3 * \dots * x_n}$

③ Harmonic mean ⇒ $\bar{x}_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$

④ Population mean ⇒ $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$

⑤ log Mean (av.) ⇒ $T_{lm} = \frac{\log T_2 - \log T_1}{\log \frac{T_2}{T_1}}$

⑥ Sauter mean = $\frac{\text{Volume}}{\text{Surface Area}}$
(drop size in liqu. extraction)

* Median ⇒ 50% above (Median) 50% below

n odd ⇒ $x_{\left(\frac{n+1}{2}\right)}$

n even ⇒ $\frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$

* Mode : most frequently encountered observation (max. distribution)

* Measure of variability (dispersion) : how elements spread around the mean.

1- Range = $x_{max} - x_{min}$

2- Sample variance $(s^2) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$ population $(\sigma^2) = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$

3- Sample standard deviation $(s) = \sqrt{s^2}$ population $(\sigma) = \sqrt{\sigma^2}$

↓ s & Range → observations are closely clustered around the mean

↑ s & Range → " " spread out around the mean.

68% ⇒ $\bar{x}\ (\pm s)$ 99.7% ⇒ $\bar{x}\ (\pm 3s)$

95% ⇒ $\bar{x}\ (\pm 2s)$

* grounded data : presents in the form of a (f) distribution

↳ $\bar{x} = \frac{\sum xf}{n}$ ⇒ $n = \sum f$

$s^2 = \frac{\sum x^2 f - \frac{(\sum xf)^2}{n}}{n-1}$ $s = \sqrt{s^2}$

* coefficient of variation = $\frac{\text{standard deviation}}{\text{Arithmetic mean}} \times 100\%$ ⇒

sample ⇒ $CV = \frac{s}{\bar{x}} \times 100\%$ population ⇒ $CV = \frac{\sigma}{\mu} \times 100\%$

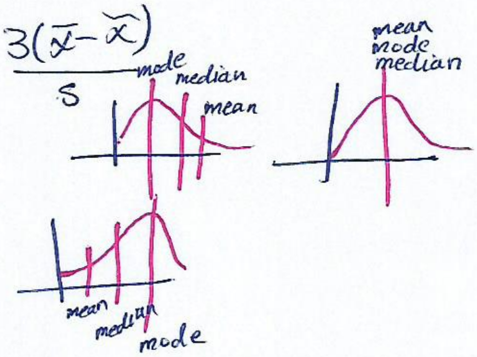
* Skewness (Measures of symmetry) * don't provide any information about the shape of distribution around mean or median

* coefficient of skewness (SK) = $\frac{3(\text{Mean} - \text{Median})}{\text{standard deviation}} = \frac{3(\bar{x} - \tilde{x})}{s}$

mode < median < mean ⟷ +ve → right skew

(1 mode) mean = median = mode ⟶ 0 → Symmetrical

mode > Median > mean ⟷ −ve → left skew



Probability:

* Randomness & uncertainty *
* Interpretation of probability:

(1-) Classical : event can occures in N equally likely and different ways

A = an attribute n = ways have A

$P_r(A) = \frac{n}{N}$

(2-) frequency (empirical) int: N = times of doing experiment A = particular attribute ⇒

n = times of doing A

$P_r(A) = \frac{n}{N}$ $N \geq \infty$

(3-) Subjected int. = measure the degree of belief one holds in a specified proposition

* Sets: finite and infinite collection of distinct elements with some common distinguishing charact.

Subset: partition of the set some further charact. that differentiates the elements in the subset from the set

Identity or reference set : all elements under ~~con~~ consideration (I)

Zeroset / nullset / empty set : set with no elements (2)

* Sample space (S) : all possible outcomes of experiment

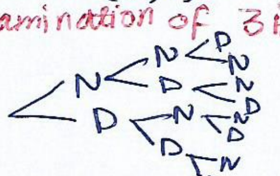
1- discrete : finite set of outcomes

2- Continuouse: an interval (finite / infinite) of real numbers

experiment: action / process that generates observations.

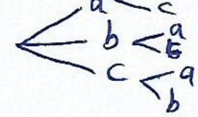
~~Ex~~: coin : (H, T) = S childborn S = (M, F)

Examination of 3 items for being D → Defective N → Not defective.



{(N,N,D), (N,N,N), (N,D,D), (N,D,N), (D,D,D), (D,N,D), (D,D,N)}

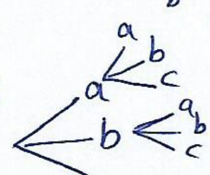
Sampling without replacement : exp. S = {a, b, c} 2 items



Swithout = {(a,b), (ac), (ba), (bc), (ca), (cb)}

Sampling with replacement :

Swith = {(aa), (ab), (ac), (ba), (bb), (bc), (ca), (cb), (cc)}



Events:

is any collection (subset) of outcome contained in the S

↳ ① simple: contain exactly one outcome.

↳ ② compound: contain more than one outcome

$E = \{g\}$ → E has occured → event that the child is a girl

impossible event = zero set

Ex: 3 cars R → right L → left

{LLL, RLL, LRL, LLR, LRR, RLR, RRL, RRR}

1- A exactly one car of the 3 turn right ⇒ A = {RLL, LRL, LLR}. بس وحدة فقط

2- B at most one car of the 3 turn right ⇒ B = {LLL, RLL, LRL, LLR} ① اكثر شي → 1/0

3- C 3 cars turn in the same direction ⇒ C = {LLL, RRR}

Basic Operation:

1. (OR) union: $\cup$

Mutually exclusive ⇒ A or B = A+b = $A \cup B$

Not Mutually exclusive ⇒ A or/and B = A+b = $A \cup B$ → A or B or both.

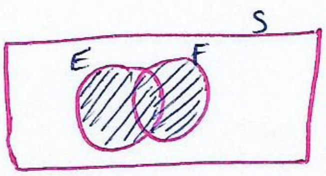
mutually exclusive/disjoint events = A & B have no common outcomes. لايوجد نتائج مشتركة

2. (AND) intersection: $\cap$

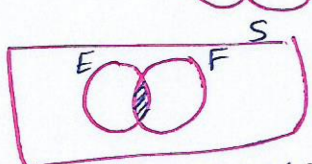
$A * B = A \cap B$

3. (Not) complement: $\bar{A}$ تتمة

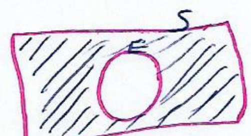
Note: $\bar{C} + C = 1$



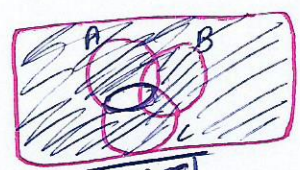
$E \cup F$
$= P(E) + P(F) - P(E \cap F)$



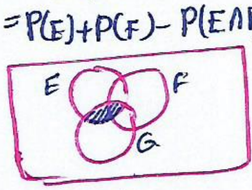
$E \cap F = P(E) * P(F)$



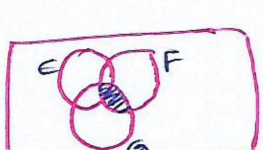
$\bar{E}$ $P(\bar{E}) = 1 - P(E)$



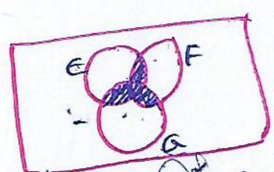
$\overline{(A \cap B)}$



$E \cap G$

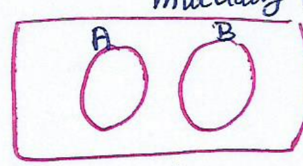


$F \cap G$

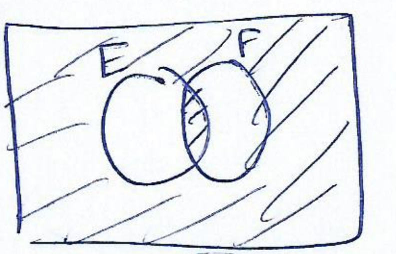


$(E \cap G) \cap F \cap E)$

mutually exclusive



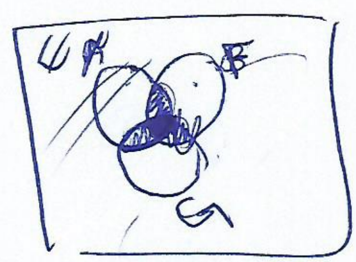
$A \cap B = \phi$



$1 - P(E \cap F)$

$1 - (P(E) * P(F))$

$\frac{P(N)}{P(N)} = 1$



$1 - P(A \cap C)$

$= P(\overline{A \cap C})$

Propability Laws:

- Commutative law: $E \cup F = F \cup E$ $\quad E \cap F = F \cap E$
- Associative law: $(E \cup F) \cup G = E \cup (F \cup G)$ $\quad (E \cap F) \cap G = E \cap (F \cap G)$
- Distributive law: $(E \cup F) \cap G = (E \cap G) \cup (F \cap G)$ $\quad (E \cap F) \cup G = (E \cup G) \cap (F \cup G)$
- De Morgan's law: $(E \cup F)^c = E^c \cap F^c$ $\quad (E \cap F)^c = E^c \cup F^c$

Laws: 1. $P(A) \geq 0 \Rightarrow P(S) = 1$

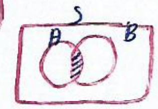
$P(\bar{A}) = 1 - P(A)$

2. Mutually exclusive events: $A \& B$ $P(A_1 \cup A_2 \cup \cdots) = \sum_{i=1}^{n} P(A_i)$ {finite events} (infinit: ∞)

3. Mutually exclusive events $A \& B$: $P(A \cap B) = 0$

4. any 2 events : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5. Joint probability: $P(A \cap B) = P(AB) = P(A) \cdot P(B)$ {and}



6. " " of n independent events: $P(A_1 \cap A_2 \cap \cdots) = P(A_1) \cdot P(A_2) \cdots$

7. equally likely events: N = outcomes $\quad P(\text{each outcome}) = \frac{1}{N}$ $\quad P(A) = \frac{N(A)}{N}$ → # of outcomes contained in A

Counting Techniques:

- Ordered Pairs (Product) n_1 = way to select the first elements, n_2 = second element $\Rightarrow N = n_1 n_2$ (??) 2nd elements are dependent or independent of the 1st element

Ex: $A_1 \; A_2 \; A_3$ ($n_1 = 3$) $\quad B_1 \; B_2 \; B_3 \; B_4$ ($n_2 = 4$) $\quad N = 4 * 3 = 12$

$A_1B_1, A_1B_2, A_1B_3, A_1B_4, A_2B_1, A_2B_2, A_2B_3, A_2B_4, A_3B_1, A_3B_2, A_3B_3, A_3B_4$.

Multiplication rule:

$N = n_1 \cdot n_2 \cdot n_3 \cdots n_k$

Permutations:

number of ordered sequences of objects that can be formed by successively selecting the objects without replacement from a fixed set consisting of n distinct elements

total num. of Perm. ← $P_r^n = P_{r,n} = \frac{n!}{(n-r)!}$ $\quad r \in [1, n]$ $\quad P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$ $\boxed{r=n}$

$\boxed{r=1}$ $P_1^n = \frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

P_r^n → n total elements, r → elements selected from n

*similar objects:

$n = n_1 + n_2 + \cdots + n_r$

$P_r^n = \frac{n!}{n_1! \, n_2! \, n_3! \cdots n_r!}$

Combinations:

Number of unordered subsets of (r) objects that can be selected from a set of n distinct objects

$C_r^n = C_{r,n} = \binom{n}{r}$ ← n: total set elements, r: unordered subset of size r

$\binom{n}{r} = \frac{P_{r,n}}{r!}$

Conditional Probability:

Concerns how the information (an event B has occurred) affects the probability assigned to A

$P(A)$: Original Pr. event

$P(A|B)$ = conditional Pr. of A given that event B has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$P(A \cap B) \neq P(A) \cdot P(B) \Rightarrow$ A & B are dependent

$P(A|B) = 0 \Rightarrow$ A & B Mutually exclusive

$P(A|B) = P(A)$ & $P(B|A) = P(B) \Rightarrow$ A & B are independent

$$\left.\begin{aligned} B &= (B \cap A_1) + (B \cap A_2) \\ P(B) &= P(B \cap A_1) + P(B \cap A_2) \\ P(B) &= P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) \end{aligned}\right\}$$

$A_i \Rightarrow$ are mutually exclusive & exhustive