



UNIVERSITY OF JORDAN
CHEMICAL ENGINEERING DEPARTMENT
0905331 –PROCESS MODELING BY STATISTICAL METHODS

1. **(30 points)** A batch contains 36 bacteria cells and 12 of the cells are not capable of cellular replication. Suppose you examine three bacteria cells selected at random, without replacement.
 - a. What is the probability mass function of the number of cells in the sample that can replicate?
 - b. What are the mean and the variance of the number of cells in sample that can replicate?
 - c. What is the probability that at least one of the selected cells cannot replicate?
2. **(30 points)** Natural red hair consists of two genes. People with red hair have two dominant genes, two regressive genes, or one dominant and one regressive gene. A group of 1000 people was categorized as follows:

Gene 1	Gene 2		
	Dominant	Regressive	Other
Dominant	5	25	30
Regressive	7	63	35
Other	20	15	800

Let A denote the event that a person has a dominant red hair gene and let B denote the event that a person has a regressive red hair gene. If a person is selected at random from this group, compute the following.

- a. $P(A)$
 - b. $P(A \cap B)$
 - c. $P(A \cup B)$
 - d. $P(A' \cap B)$
 - e. $P(A|B)$.
3. **(30 points)** The time it takes a cell to divide (called mitosis) is normally distributed with an average time of one hour and a standard deviation of 5 minutes.
 - a. What is the probability that a cell divides in less than 45 minutes?
 - b. What is the probability that it takes a cell more than 65 minutes to divide?
 - c. By what time have approximately 99% of all cells completed mitosis?
4. **(30 points)** In a healthy individual age 20 to 29 years, the calcium level in the blood, X, is usually between 8.5 and 10.5 mg/dL and the cholesterol level, Y, and is usually between 120 and 240 mg/dL. Assume that for a healthy individual in this age group the random variable (X, Y) is uniformly distributed such that

$$f_{XY} = c \quad 8.5 \leq x \leq 10.5$$

$$120 \leq y \leq 240$$

- a. What is the value of the constant c that will make the function a mass density function?
 - b. Determine the probability that an individual's calcium level will lie between 9 and 10 mg/dL, whereas the cholesterol level is between 125 and 140 mg/dL.
 - c. Determine $\text{Cov}(X, Y)$.
 - d. Determine if X and Y are independent.
 - e. Determine ρ_{XY} .