Chapter 17 Instructor Notes

The objective of Chapter 17 is to introduce the foundations for the analysis of rotating electric machines. In Section 17.1, rotating electric machines are classified on the basis of their energy conversion characteristics and of the nature of the electric power they absorb (or generate). Section 17.2 reviews the physical structure of a DC machine and presents a simple general circuit model that is valid for both motors and generators, including dynamic equations. Section 17.3 contains a brief discussion of DC generators. Section 17.4 describes the characteristics of the various configurations of DC motors, both of the wound stator and permanent magnet types. The torque speed characteristics of the different configurations are compared, and the dynamic equations are given for each type of motor. The section ends with a brief qualitative discussion of speed control in DC motors.

The second half of the chapter is devoted to the analysis of AC machines. In Section 17.5, we introduce the concept of a rotating magnetic field. The next two sections describe synchronous generators and motors; the discussion is brief, but includes the analysis of circuit models of synchronous machines and a few examples. Circuit models for the induction motor, as well as general performance characteristics of this class of machines are discussed in Section 17.8, including a brief treatment of AC machine speed and torque control. Although the discussion of the AC machines is not particularly detailed, all of the important concepts that a non-electrical engineer would be interested in to evaluate the performance characteristics of these machines are introduced in the chapter, and reinforced in the homework problem set. The homework problems include a mix of traditional electric machinery problems based on circuit models and of more system-oriented problems. Problems 17.24-36 deal with the performance and dynamics of systems including DC motors. These problems are derived from the author's experience in teaching a Mechanical Engineering System Dynamics course with emphasis on electromechanics, and are somewhat unusual (although relevant and useful for non–electrical engineers) in this type of textbook. These problems are well suited to a more mature audience that has already been exposed to a first course in system dynamics. Problem 17.39 provides a link to the power electronics topics covered in Chapter 12. All other problems are based on the content of the chapter.

Learning Objectives

1. Understand the basic principles of operation of rotating electric machines, their classification, and basic efficiency and performance characteristics. Section 1.
2. Understand the operation and basic configurations of separately-excited, permanent-magnet, shunt and series DC machines. Section 2.
3. Analyze DC generators at steady-state. Section 3.
4. Analyze DC motors under steady-state and dynamic operation . Section 4.
5. Understand the operation and basic configuration of AC machines , including the synchronous motor and generator, and the induction machine. Sections 5, 6, 7, 8.

Section 17.1: Rotating Electric Machines

Problem 17.1

Solution:

Known quantities:
The relationship of the power rating and the ambient temperature is shown in the table. A motor with $P_e = 10\,kW$ is rated up to $85^\circ C$.

Find:
The actual power for the following conditions.
a) Ambient temperature is $50^\circ C$.
b) Ambient temperature is $25^\circ C$.

Assumptions:
None.

Analysis:
a)
The power at ambient temperature $50^\circ C$:

$$P_e^{'} = 10 - 10 \times 0.125 = 8.75\,kW$$

b)
The power at ambient temperature $30^\circ C$:

$$P_e^{'} = 10 + 10 \times 0.08 = 10.8\,kW$$

Problem 17.2

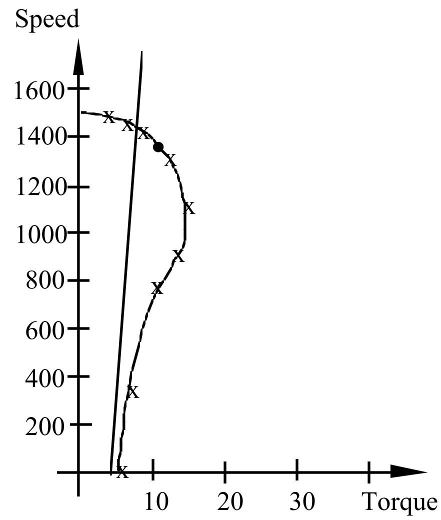
Solution:

Known quantities:
The speed-torque characteristic of an induction motor is shown in the table. The load requires a starting torque of $4\,N \cdot m$ and increase linearly with speed to $8\,N \cdot m$ at $1500\,rev/\text{min}$.

Find:
a) The steady state operating point of the motor.
b) The change in voltage if the load torque increases to $10\,N \cdot m$.

Assumptions:
None.

Analysis:
The characteristic is shown below:



a)
The operating point is:

$$n_m = 1425\, rev/\text{min}, \quad T = 7\, N \cdot m$$

b)
From the following equation:

$$\frac{T_{new}}{T_{old}} = \left|\frac{V_{s,new}}{V_{s,old}}\right|^2$$

$$\Rightarrow \frac{10}{7} = \left|\frac{KV_s}{V_S}\right|^2 = K^2$$

$$\Rightarrow K = 1.195$$

$$\therefore V_{s,new} = 1.195 V_{s,old}$$

Section 17.2: Direct Current Machines

Problem 17.3

Solution:

Known quantities:
Each conductor of the DC motor is $6\,in.$ long. The current is $90\,A$. The field density is $5.2\times10^{-4}\,Wb/in^2$.

Find:
The force exerted by each conductor on the armature.

Assumptions:
None.

Analysis:

$$F = BI\times l = 5.2\times10^{-4}\,\frac{Wb}{in^2}\times\frac{in^2}{(0.0254\,m)^2}\times90\times6\,in\times\frac{0.0254\,m}{in}$$

$$=11.06\,Nt$$

Problem 17.4

Solution:

Known quantities:
The air-gap flux density of the DC machine is $4\,Wb/m^2$. The area of the pole face is $2\,cm\times4\,cm$.

Find:
The flux per pole in the machine.

Assumptions:
None.

Analysis:
With $B=4\,kG=0.4\,T=0.4\,Wb/m^2$, we can compute the flux to be:

$$\phi = BA = 0.4\times0.02\times0.04 = 0.32\,mWb$$

Section 17.3: Direct Current Generators

Problem 17.5

Solution:

Known quantities:

A $120V, 10\,A$ shunt motor. The armature resistance is $0.6\,\Omega$. The shunt field current is $2\,A$.

Find:
The LVDT equations.

Assumptions:
None.

Analysis:
V_L at full load is $120V$ and

$$E_b = 120 + (2+10) \times 0.6 = 127.2V$$

$$R_f = \frac{120}{2} = 60\,\Omega$$

Assuming E_b to be constant, we have:

$$i_a = i_f = \frac{127.2}{0.6+60} = 2.1\,A$$

Therefore:

$$V_L = 127.2 - 2.1 \times 0.6 = 125.9V$$

$$Voltage\,reg. = \frac{125.9-120}{120} = 0.049 = 4.9\%$$

Problem 17.6

Solution:

Known quantities:

A $20\,kW, 230V$ separately excited generator. The armature resistance is $0.2\,\Omega$. The load current is $100\,A$.

Find:

a) The generated voltage when the terminal voltage is $230V$.

b) The output power.

Assumptions:
None.

Analysis:

If we assume rated output voltage, that is $V_L = 230V$, we have

a)

The generated voltage is $230V$.

b)

The output power is $23\,kW$.

If we assume rated output power, that is $P_{out} = 20\,kW$, we have

a)
The generated voltage is $200V$.
b)
The output power is $20\,kW$.

If we assume $E_b = 230V$, and compute the output voltage to be:

$$V_L = 230 - 100 \times 0.2 = 210V$$

We have:
a)
The generated voltage is $210V$.
b)
The output power is $21\,kW$.

Problem 17.7

Solution:

Known quantities:

A $10\,kW, 120\,VDC$ series generator. The armature resistance is $0.1\,\Omega$ and a series field resistance is $0.05\,\Omega$.

Find:

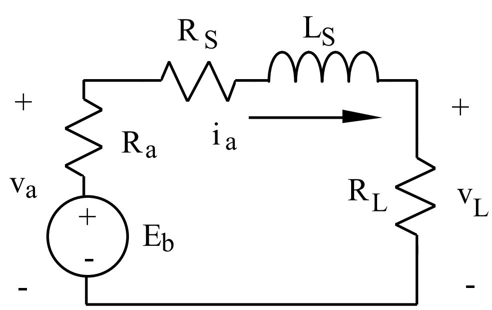
a) The armature current.
b) The generated voltage.

Assumptions:

The generator is delivering rated current at rated speed.

Analysis:

The circuit is shown below:



a)

$$i_a = \frac{P}{V_L} = \frac{10 \times 10^3}{120} = 83.33\,A$$

b)

$$V_a = 120 + i_a R_S = 124.17V$$

Problem 17.8

Solution:

Known quantities:

A $30\,kW, 440\,V$ shunt generator. The armature resistance is $0.1\,\Omega$ and a series field resistance is $200\,\Omega$.

Find:

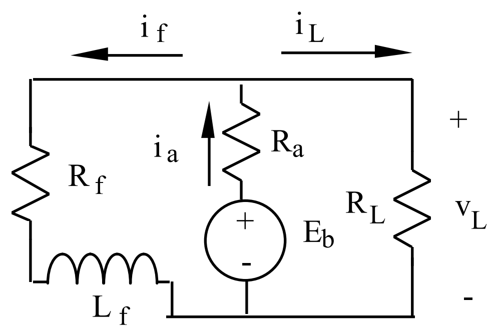
a) The power developed at rated load.
b) The load, field, and armature currents.
c) The electrical power loss.

Assumptions:

None.

Analysis:

The circuit is shown below:



$$i_L = \frac{30\times 10^3}{440} = 68.2\,A$$

$$i_f = \frac{440}{200} = 2.2\,A$$

$$i_a = 70.4\,A$$

a)

$$E_b = V_L + i_a R_a = 440 + 70.4\times 0.1 = 447.04\,V$$

$$P = E_b i_a = 31.471\,kW$$

b)

$$i_L = 62.8\,A$$

$$i_f = 2.2\,A$$

$$i_a = 70.4\,A$$

c)

$$P_{loss} = i_a^2 R_a + i_f^2 R_f = 1464\,W$$

Problem 17.9

Solution:

Known quantities:

A four-pole $450\,kW, 4.6\,kV$ shunt generator. The armature resistance is $2\,\Omega$ and a series field resistance is $333\,\Omega$. The generator is operating at the rated speed of $3600\,rev/\min$.

Find:

The no-load voltage of the generator and terminal voltage at half load.

Assumptions:
None.

Analysis:
For $n = 3600\,rev/\min, \omega_m = 377\,rad/\sec$:

$$i_L = \frac{450\times10^3}{4.6\times10^3} = 97.8\,A$$

$$i_f = \frac{4.6\times10^3}{333} = 13.8\,A$$

$$i_a = i_f + i_L = 111.6\,A$$

Using the relation:

$$E_b = V_L + i_a R_a = 4823.2V$$

At no-load,

$$V_L = E_b - i_a R_a = 4820.4V$$

At half load,

$$i_L = 48.9\,A$$

$$i_a = i_f + i_L = 62.7\,A$$

$$V_L = E_b - i_a R_a = 4810.7V$$

Problem 17.10

Solution:

Known quantities:

A $30\,kW, 240V$ generator is running at half load at $1800\,rev/\min$ with efficiency of 85 percent.

Find:

The total losses and input power.

Assumptions:
None.

Analysis:

$$P_{out} = \frac{1}{2}\,rated\ load = 15\,kW$$

At an efficiency of 0.85, the input power can be computed to be:

$$P_{in} = \frac{15\times10^3}{0.85} = 17.647\,kW$$

The total loss is:

$$P_{loss} = P_{in} - P_{out} = 2.647\,kW$$

Problem 17.11

Solution:

Known quantities:

A self excited DC shunt generator. At $200\,rev/\min$, it delivers $20\,A$ to a $100V$ line. The armature resistance is $1.0\,\Omega$ and a series field resistance is $100\,\Omega$. The magnetization characteristic is shown in Figure P17.11. When the generator is disconnected from the line, the drive motor speed up to $220\,rad/s$.

Find:
The terminal voltage.

Assumptions:
None.

Analysis:
From the figure, for $I_f > 0.5\,A, \omega = 200\,rad/\sec$

$$E_b = 40 I_f + 100$$

For $\omega = 220\,rad/\sec$, we have:

$$\frac{E_b^{'}}{220} = \frac{100 + 40 I_f}{200}$$

Therefore,

$$E_b^{'} = \frac{220}{200}(100 + 40 I_f) = 110 + 44 I_f$$

For no load, $I_a = I_f$. Therefore,

$$110 + 44 I_f = 101 I_f$$

$$\therefore I_f = 1.93\,A$$

The terminal voltage is:

$$V = I_f R_f = 193V$$

Section 17.4: Direct Current Motors

Problem 17.12

Solution:

Known quantities:

A $220V$ shunt motor. The armature resistance is $0.32\,\Omega$. A field resistance is $110\,\Omega$. At no load the armature current is $6\,A$ and the speed is $1800\,rpm$.

Find:

a) The speed of the motor when the line current is $62\,A$.

b) The speed regulation of the motor.

Assumptions:

The flux does not vary with load. Assume a $8\,N\cdot m$ brush drop.

Analysis:

a)

$$1800 = \frac{220 - 2 - 6(0.32)}{K_a\phi}$$

$$\Rightarrow K_a\phi = 0.12$$

$$\therefore n = \frac{220 - 2 - 6(0.32)}{K_a\phi} = 1657\,rpm$$

b)

$$\%reg = \frac{1800 - 1657}{1657} \times 100 = 8.65\%$$

Problem 17.13

Solution:

Known quantities:

A $50\,hp, 550\,volt$ shunt generator. The armature resistance including brushes is $0.36\,\Omega$. Operating at rated load and speed, the armature current is $75\,A$.

Find:

What resistance should be inserted in the armature circuit to get a 20 percent speed reduction when the motor is developing 70 percent of rated torque.

Assumptions:

There is no flux change.

Analysis:

$$T = K_a\phi I_a \Rightarrow I_a = 0.7(75) = 52.5\,A$$

$$n_R = \frac{550 - 75(0.36)}{K_a\phi} \Rightarrow K_a\phi\, n_R = 523$$

$$0.8 n_R = \frac{550 - 52.5 R_T}{K_a\phi} \Rightarrow 0.8 \times 523 = 550 - 52.5 R_T$$

$$\therefore R_T = 2.51\Omega$$

$$R_{add} = 2.51 - 0.36 = 2.15\Omega$$

Problem 17.14

Solution:

Known quantities:

A $100\,kW, 440\,V$ shunt DC motor. The armature resistance is $0.2\,\Omega$ and a series field resistance is $400\,\Omega$. The generator is operating at the rated speed of $1200\,rev/\min$. The full-load efficiency is 90 percent.

Find:

a) The motor line current.
b) The field and armature currents.
c) The counter emf at rated speed.
d) The output torque.

Assumptions:

None.

Analysis:

At $n = 1200\,rev/\min, \omega_m = 125.7\,rad/\sec$, the output power is $100\,hp = 74.6\,kW$.

From full-load efficiency of 0.9, we have:

$$P_{in} = \frac{74.6}{0.9} = 82.9\,kW$$

a)

From $P_{in} = i_S V_S = 82.9\,kW$, we have:

$$i_S = \frac{82.9 \times 10^3}{440} = 188.4\,A$$

b)

$$i_f = \frac{440}{400} = 1.1\,A$$

$$i_a = 187.3\,A$$

c)

$$E_b = V_L - i_a R_a = 402.5\,A$$

d)

$$T_{out} = \frac{P_{out}}{\omega_m} = 593.5\,N \cdot m$$

Problem 17.15

Solution:

Known quantities:

A $240\,V$ series motor. The armature resistance is $0.42\,\Omega$ and a series field resistance is $0.18\,\Omega$. The speed is $500\,rev/\min$ when the current is $36\,A$.

Find:

What is the motor speed when the load reduces the line current to $21\,A$.

Assumptions:
A $3\,volts$ brush drop and the flux is proportional to the current.

Analysis:

$$500 = \frac{240 - 3 - 36(0.6)}{K_a\phi} \Rightarrow K_a\phi = 0.431$$

$$n = \frac{240 - 3 - 21(0.6)}{(\frac{21}{36})(0.431)} = 893\,rpm$$

Problem 17.16

Solution:

Known quantities:
A $220VDC$ shunt motor. The armature resistance is $0.2\,\Omega$. The rated armature current is $50\,A$.

Find:
a) The voltage generated in the armature.
b) The power developed.

Assumptions:
None.

Analysis:
a)

$$E_b = V_L - i_a R_a = 220 - 50 \times 0.2 = 210V$$

b)

$$P = E_b i_a = 210 \times 50 = 10.5\,kW = 14.07\,hp$$

Problem 17.17

Solution:

Known quantities:
A $550V$ series motor. The armature resistance is $0.15\,\Omega$. The speed is $820\,rev/\min$ when the current is $112\,A$ and the load is $75\,hp$.

Find:
The horsepower output of the motor when the current drops to $84\,A$.

Assumptions:
The flux is reduced by 15 percent.

Analysis:

$$HP = \frac{2\pi \cdot n \cdot T}{33{,}000}$$

$$75 = \frac{2\pi(820)T}{33{,}000} \Rightarrow T = 480.4\, lb \cdot ft$$

$$T = K_a \phi I_a \Rightarrow 480.4 = K_a \phi (112) \Rightarrow K_a \phi = 4.29$$

$$T_n = 4.29(0.85)(84) = 306.2\, lb \cdot ft$$

$$n_n = \frac{550 - 84(0.15)}{0.85(0.65)} = 973\, rpm$$

$$HP_n = \frac{2\pi(973)(306.2)}{33{,}000} = 56.7\, hp$$

Problem 17.18

Solution:

Known quantities:

A $220\,VDC$ shunt motor. The armature resistance is $0.1\,\Omega$ and a series field resistance is $100\,\Omega$. The speed is $1100\, rev/\min$ when the current is $4\,A$ and there is no load.

Find:

E and the rotational losses at $1100\, rev/\min$.

Assumptions:
The stray-load losses can be neglected.

Analysis:

Since $n = 1100\, rev/\min$ corresponds to $\omega = 115.2\, rad/\sec$, we have:

$$i_S = 4\,A$$

$$i_f = \frac{200}{100} = 2\,A$$

$$i_a = i_S - i_f = 2\,A$$

Also,

$$E_b = 200 - 2 \times 0.1 = 199.8V$$

The power developed by the motor is:

$$\begin{aligned} P &= P_{in} - P_{copper\_loss} \\ &= 200 \times 4 - (2^2 \times 100 + 2^2 \times 0.1) \\ &= 399.6W \end{aligned}$$

Problem 17.19

Solution:

Known quantities:

A $230VDC$ shunt motor. The armature resistance is $0.5\,\Omega$ and a series field resistance is $75\,\Omega$. At $1100\,rev/\min$, $P_{rot} = 500W$. When loaded, the current is $46\,A$.

Find:

a) The speed P_{dev} and T_{sh}.

b) $i_a(t)$ and $\omega_m(t)$ if $L_f = 25\,H, L_a = 0.008\,H$ and the terminal voltage has a $115V$ change.

Assumptions:

None.

Analysis:

$$i_f = \frac{230}{75} = 3.07\,A$$

$$i_a = i_L - i_f = 42.93\,A$$

$$\omega_m = 117.3\,rad/\sec$$

At no load, $117.3 = \frac{230}{K_a\phi}$, therefore,

$$K_a\phi = 1.96$$

At full load,

$$\omega_m = \frac{230 - 0.5 \times 42.93}{K_a\phi}$$

The back emf is:

$$E_b = 230 - 0.5 \times 42.93 = 208.5V$$

The power developed is:

$$P_{dev} = E_b I_a = 8.952\,kW$$

The power available at the shaft is:

$$P_o = P_{dev} - P_{rot} = 8952 - 500 = 8452W$$

The torque available at the shaft is:

$$T_{sh} = \frac{P_o}{\omega_m} = 72.1\,N \cdot m$$

Problem 17.20

Solution:

Known quantities:

A $200 VDC$ shunt motor. The armature resistance is 0.1Ω and a series field resistance is 100Ω . At $955\, rev/\min$ with no load, $P_{rot} = 500 W$, the line current is $5 A$.

Find:

The motor speed, the motor efficiency, total losses, and the load torque when the motor draws $40 A$ from the line.

Assumptions:

Rotational power losses are proportional to the square of shaft speed.

Analysis:

$$i_S = 5 A$$

$$i_f = \frac{200}{100} = 2 A$$

$$i_a = i_S - i_f = 3 A$$

The copper loss is:

$$P_{copper} = i_f^2 R_f + i_a^2 R_a = 400.9 W$$

The input power is:

$$P_{in} = 5 \times 200 = 1 kW$$

Therefore,

$$P_{rot} + P_{SL} = 1000 - 409 = 599.1 W$$

at $\omega_m = 2\pi \frac{955}{60} = 100\, rad/\sec$.

Also, E_b at no load is:

$$E_b = 200 - 3 \times 0.1 = 199.7 V$$

$$K_a \phi = 1.997$$

When $i_S = 40 A$ with $i_f = 2 A$ and $i_a = 38 A$,

$$E_b = 200 - 38 \times 0.1 = 196.2 V$$

$$\omega_m = \frac{E_b}{K_a \phi} = 98.25\, rad/\sec = 938.2\, rev/\min$$

The power developed is:

$$P = E_b I_a = 196.2 \times 38 = 7456 W$$

The copper loss is:

$$P_{copper} = i_f^2 R_f + i_a^2 R_a = 544.4 W$$

The input power is:

$$P_{in} = 40 \times 200 = 8 kW$$

And

$$P_{SH} = 7456 - \frac{98.25}{100} \times 599.1 = 6867.4\,W$$

$$T_{SH} = \frac{6867.4}{98.25} = 69.9\,N \cdot m$$

Finally, the efficiency is:

$$eff = \frac{P_{SH}}{P_{in}} = 85.84\%$$

Problem 17.21

Solution:

Known quantities:

A $50\,hp, 230\,V$ shunt motor operates at full load when the line current is $181\,A$ at $1350\,rev/\min$. The field resistance is $17.7\,\Omega$. To increase the speed to $1600\,rev/\min$, a resistance of $5.3\,\Omega$ is cut in via the field rheostat. The line current is increased to $190\,A$.

Find:

a) The power loss in the field and its percentage of the total power input for the $1350\,rev/\min$ speed.

b) The power losses in the field and the field rheostat for the $1600\,rev/\min$ speed.

c) The percent losses in the field and in the field rheostat at $1600\,rev/\min$ speed.

Assumptions:

None.

Analysis:

a)

$$I_f = \frac{230}{17.7} = 13.0\,A$$

$$P_f = (230)(13.0) = 2988.7\,W$$

$$\frac{P_f}{P_m} = \frac{2988.7}{(230)(181)} = 0.072 = 7.2\%$$

b)

$$I_f = \frac{230}{(17.7 + 5.3)} = 10\,A$$

$$P_f = 10^2(17.7) = 1770\,W$$

$$P_R = 10^2(5.3) = 530\,W$$

c)

$$P_{in} = (230)(190) = 43{,}700\,W$$

$$\%P_f = \frac{1770}{43700} \times 100 = 4.05\%$$

$$\%P_R = \frac{530}{43700} \times 100 = 1.21\%$$

Problem 17.22

Solution:

Known quantities:

A $10\,hp, 230\,V$ shunt-wound motor. The armature resistance is $0.26\,\Omega$ and a series field resistance is $225\,\Omega$. The rated speed is $1000\,rev/\min$. The full-load efficiency is 86 percent.

Find:

The effect on counter emf, armature current and torque when the motor is operating under rated load and the field flux is very quickly reduced to 50 percent of its normal value. The effect on the operation of the motor and its speed when stable operating conditions have been regained.

Assumptions:

None.

Analysis:

$E_C = K\phi\, n$; counter emf will decrease.

$I_a = \frac{V - E_C}{r_a}$; armature current will increase.

$T = K\phi I_a$; effect on torque is indeterminate.

Operation of a dc motor under weakened field conditions is frequently done when speed control is an important factor and where decreased efficiency and less than rated torque output are lesser considerations.

$$n = \frac{V - I_a r_a}{K\phi} \Rightarrow 1000 = \frac{V - I_a r_a}{K\phi}$$

$$n_{new} = \frac{V - I_a r_a}{0.5K\phi}$$

Assume small change in the steady-state value of I_a. Then:

$$\frac{1000}{n_{new}} = \frac{0.5}{1} \Rightarrow n_{new} = 2000\,rpm$$

Problem 17.23

Solution:

Known quantities:

The machine is the same as that in Example 17.7. The circuit is shown in Figure P17.23. The armature resistance is $0.2\,\Omega$ and the field resistance is negligible. $n = 120\,rev/\min, I_a = 8A$. In the operating region, $\phi = kI_f, k = 200$.

Find:

a) The number of field winding turns necessary for full-load operation.
b) The torque output for the following speeds:

1. $n' = 2n$ 3. $n' = n/2$
2. $n' = 3n$ 4. $n' = n/4$

c) Plot the speed-torque characteristic for the conditions of part b.

Assumptions:

None.

Analysis:

in Example 17.7, $i_f = 0.6\,A$, the mmf F is:

$$F = 200 \times 0.6 = 120\,At$$

For a series field winding with:

a)

$i_{series} = i_a = 8\,A$, we have:

$$N_{series} = \frac{120}{8} = 15\,turns$$

b)

$$n_m = 120\,rev/\min$$

$$\omega_m = 12.57\,rad/\sec$$

$$E_b = V_S - i_a(R_a + R_S)$$

Neglecting R_S, we have

$$E_b = 7.2 - 8 \times 0.2 = 5.6V = k_a \phi \omega_m$$

$$k_a \phi = \frac{5.6}{12.57} = 0.446$$

From $T = k_T \phi\, i_a$ and $k_T = k_a$,

$$T = 0.446 \times 8 = 3.56\,N \cdot m$$

By using $\phi = k i_a$, we have:

$$E_b = k_a k i_a \omega_m = V_S - i_a R_a$$

$$T = k_T (k i_a) i_a = k_a k i_a^2$$

From $i_a = \dfrac{V_S}{R_a + K\omega_m}$, where $K = k_a k = \dfrac{5.6}{8 \times 12.57} = 0.056$

And $T \propto (\dfrac{1}{R_a + K\omega_m})^2$, we have:

$$\frac{T_X}{T} = (\frac{R_a + K\omega_m}{R_a + K\omega_X})^2 = (\frac{\frac{R_a}{K} + \omega_m}{\frac{R_a}{K} + \omega_X})^2$$

$$\because \frac{R_a}{K} = 3.59$$

$$\therefore T_X = 3.56(\frac{3.59 + \omega_m}{3.59 + \omega_X})^2$$

1. at $\omega_X = 2\omega_m = 25.12\,rad/\sec$,

$T_X = 1.13\,N \cdot m$

2. at $\omega_X = 3\omega_m = 37.71\,rad/\sec$,

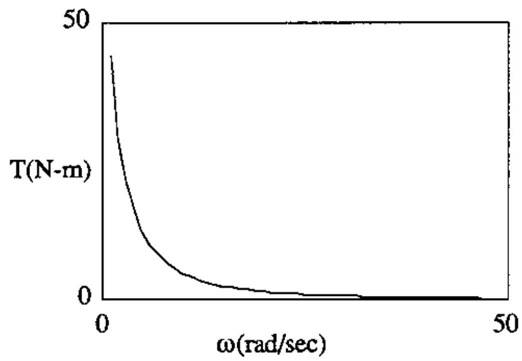
$T_X = 0.55\,N \cdot m$

3. at $\omega_X = 0.5\omega_m = 6.28\,rad/\sec$,

$T_X = 9.54\, N \cdot m$

4. at $\omega_X = 0.25\omega_m = 3.14\, rad/\sec$,
$T_X = 20.53\, N \cdot m$

c)
The diagram is shown below:



Problem 17.24

Solution:

Known quantities:

PM DC motor circuit model; mechanical load model. Example 17.9.

Find:
Voltage-step response of motor.

Assumptions:
None.

Analysis:
Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$V_L(t) \quad R_a I_a(t) \quad L_a \frac{dI_a(t)}{dt} \quad E_b(t) \quad 0$$

or

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad K_{aPM} \quad {}_m(t) \quad V_L(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d \quad (t)}{dt} \quad T(t) \quad T_{Load}(t) \quad b$$

or

$$K_{TPM} I_a(t) \quad J \frac{d \quad (t)}{dt} \quad b \quad (t) \quad 0$$

since the load torque is assumed to be zero. To derive the transfer function from voltage to speed, we use the result of Example 17.9 with $T_{load} = 0$:

$${}_m(s) \quad \frac{K_{TPM}}{sL_a \quad R_a \ (sJ \quad b) \quad K_{aPM}K_{TPM}} V_L(s)$$

The step response of the system can be computed by assuming a unit step input in voltage:

$${}_m(s) \quad \frac{K_{TPM}}{sL_a \quad R_a \ (sJ \quad b) \quad K_{aPM}K_{TPM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{(sL_a + R_a)(sJ + b) + K_{aPM} K_{TPM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{s[JL_a s^2 + (L_a b + R_a J)s + R_a b + K_a K_T]}$$

$$F(s) = \frac{K_{TPM}}{s\left[s^2 + \frac{(L_a b + R_a J)}{JL_a} s + \frac{(R_a b + K_a K_T)}{JL_a}\right]}$$

Set:

$$m = \frac{(L_a b + R_a J)}{JL_a} \qquad n = \frac{(R_a b + K_a K_T)}{JL_a}$$

$$F(s) = \frac{K_{TPM}}{s[s^2 + ms + n]} = \frac{K_{TPM}}{s\left[\left(s^2 + ms + \frac{m^2}{4}\right) + n - \frac{m^2}{4}\right]} = \frac{K_{TPM}}{s\left[\left(s + \frac{m}{2}\right)^2 + \left(n - \frac{m^2}{4}\right)\right]}$$

For

$$F(s) \quad \frac{1}{s[(s \quad a)^2 \quad b^2]} \qquad f(t) \quad \frac{1}{b_o^2} \quad \frac{1}{bb_o} e^{\quad at} \sin(bt \qquad)$$

where : $= \tan^{-1} \frac{b}{a}$ and $b_o \quad \sqrt{b^2 \quad a^2}$

Therefore:

$$a = \frac{m}{2} = \frac{(L_a b + R_a J)}{2JL_a} \qquad b = \sqrt{n - \frac{m^2}{4}} = \sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4}}$$

$$b_o^2 = \left[\frac{(L_a b + R_a J)}{2JL_a}\right]^2 + \frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4} = \frac{R_a b + K_a K_T}{JL_a}$$

$$\phi = \tan^{-1} \frac{\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4}}}{-\frac{(L_a b + R_a J)}{2JL_a}}$$

Thus giving the step response:

$$\Omega_m(t) = \frac{JL_a}{R_a b + K_a K_T} + \frac{1}{\left(\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4}}\right)\left(\sqrt{\frac{R_a b + K_a K_T}{JL_a}}\right)} e^{-\left[\frac{(L_a b + R_a J)}{2JL_a}\right]t} *$$

$$\sin\left[\left(\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4}}\right)t - \tan^{-1}\frac{\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a}\right)^2 \frac{1}{4}}}{-\frac{(L_a b + R_a J)}{2JL_a}}\right]$$

Expressions for the natural frequency and damping ratio of the second-order system may be derived by comparing the motor voltage-speed transfer function to a standard second-order system transfer function:

$$H(s) \quad \frac{K_S}{s^2 \quad 2\zeta\omega_n s \quad \omega_n^2}.$$

The motor transfer function is:

$$\frac{{}_m(s)}{V_L(s)} \quad \frac{K_{TPM}}{sL_a \quad R_a \ (sJ \quad b) \quad K_{aPM}K_{TPM}} \quad \frac{K_{TPM}}{JL_a s^2 \quad JR_a \quad bL_a \ s \quad R_a b \quad K_{aPM}K_{TPM}}$$

$$\frac{\frac{K_{TPM}}{JL_a}}{s^2 \quad \frac{JR_a \quad bL_a}{JL_a} s \quad \frac{R_a b \quad K_{aPM}K_{TPM}}{JL_a}}$$

Comparing terms, we determine that:

$$\omega_n^2 \quad \frac{R_a b \quad K_{aPM}K_{TPM}}{JL_a}$$

$$2\zeta\omega_n \quad \frac{JR_a \quad bL_a}{JL_a}$$

or

$$\omega_n \quad \sqrt{\frac{R_a b \quad K_{aPM}K_{TPM}}{JL_a}}$$

$$\zeta \quad \frac{1}{2}\frac{JR_a \quad bL_a}{JL_a}\sqrt{\frac{JL_a}{R_a b \quad K_{aPM}K_{TPM}}}$$

From these expressions, we can see that both natural frequency and damping ratio are affected by each of the parameters of the system, and that one cannot predict the nature of the damping without knowing numerical values of the parameters.

Problem 17.25

Solution:

Known quantities:

Torque-speed curves of motor and load: $T_m = a\omega + b$ (motor), $T_L = c\omega^2 + d$ (load),

Find:

Equilibrium speeds and their stability.

Assumptions:
All coefficients of torque-speed curve functions are positive constants

Analysis:
The first consideration is that the motor static torque, $T_{0,m} = b$ must exceed the load static torque, $T_{0,L} = d$; thus, the first condition is $b>d$.
The next step is to determine the steady state speed of the motor-load pair. If we set the motor torque equal to the load torque, the resulting angular velocities will be the desired solutions.

$$T_m \quad T_L \quad a\omega \quad b \quad c\omega^2 \quad d$$

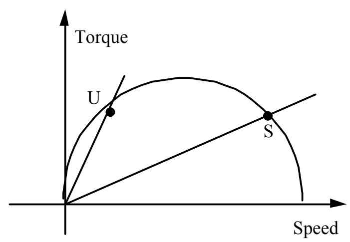
resulting in the quadratic equation

$$c\omega^2 \quad a\omega \quad d \quad b \quad 0$$

with solution

$$\omega \quad \frac{a \quad \sqrt{a^2 \quad 4c\, b \quad d}}{2c}.$$

Both solutions are positive, and therefore physically acceptable. The question of stability can be addressed by considering the following sketch.



In the figure, we see that the intersection of a line with a quadratic function when both solutions are positive leads to two possible situations: the line intersecting the parabola when the rate of change of both curves is positive, and the line intersecting the parabola when the rate of change of torque w.r. to speed of the latter is negative. The first case leads to an unstable operating point; the second case to a stable operating point (you can argue each case qualitatively by assuming a small increase in load torque and evaluating the consequences). We can state this condition mathematically by requiring that the following steady-state stability condition hold:

$$\frac{dT_L}{d} \quad \frac{dT_m}{d}$$

Evaluating this for our case, we see that

$$\frac{dT_L}{d} \quad \frac{dT_m}{d} \quad 2c \quad a.$$

From the expression we obtained earlier,

$$\frac{a \quad \sqrt{a^2 \quad 4c\, b \quad d}}{2c}$$

it is clear that $2c\omega>a$ always olds, since the term under the square root is a positive constant. Thus, this motor-load pair always leads to stable solutions. To verify this conclusion intuitively, you might wish to plot the motor and load torque-speed curves and confirm that the condition $\frac{dT_L}{d} \quad \frac{dT_m}{d}$ is always satisfied (note that the sketch above is **not** an accurate graphical representation of the two curves).

Problem 17.26

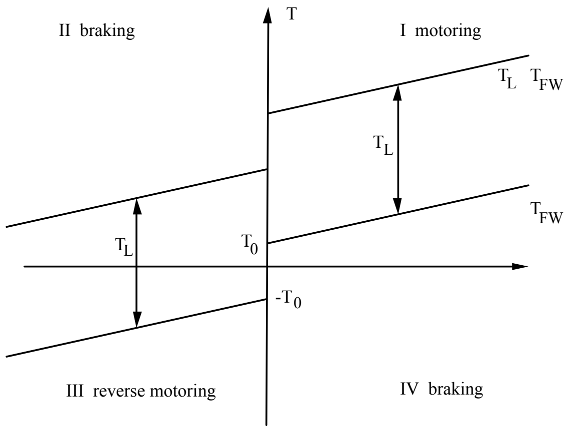
Solution:

Known quantities:
Expression for friction and windage torque, T_{FW}, functional form of motor torque, T, or load torque, T_L.

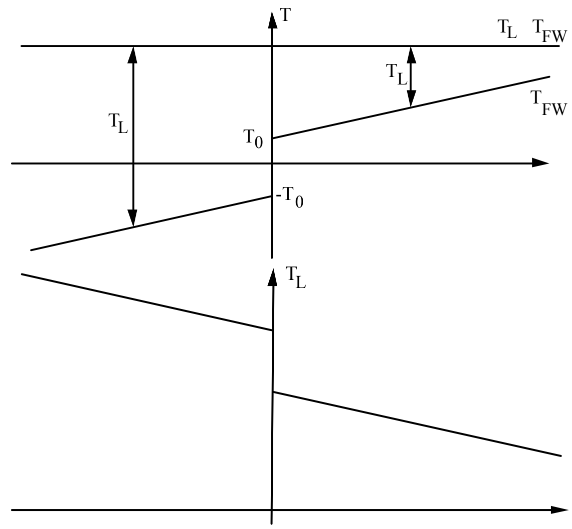
Find:
Sketch torque-speed curve

Assumptions:
None.

Analysis:
The sketches are shown below.



Torque-speed curve for constant load torque



Torque-speed curves for variable load torque

Problem 17.27

Solution:

Known Quantities:
A PM DC motor and parameters when 1) in steady-state, no load conditions, and 2) connected to a pump

Find:
a) A damping coefficient, sketch of the motor, the dynamic equations, the transfer function, and 3 dB bandwidth.
b) A sketch of the motor, dynamic equations, transfer function, and 3 dB bandwidth.

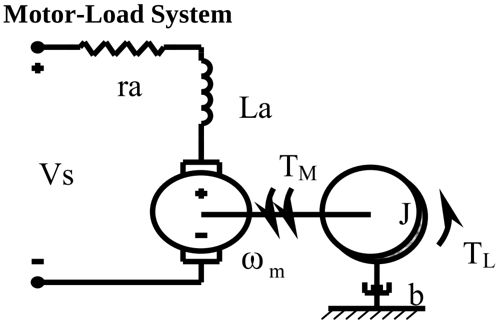
Assumptions:
$k_t \quad k_a \quad k_{PM}$

Analysis:

a)
The magnetic torque balanced the damping torque gives s:
$k_t * i_a \quad b * \quad _m$
or

$$b \quad \frac{k_{PM} * i_a}{\ _m} \quad \frac{7 * 10^{\ 3} \frac{\text{N} \quad \text{m}}{\text{A}} * .15 \text{ A}}{\frac{3350 \text{ rev}}{\text{min}} * \frac{2 \quad \text{rad}}{\text{rev}} * \frac{1 \text{ min}}{60 \text{ sec}}} \quad 2.993 * 10^{\ 6} \text{ N - m - sec}$$

Sketch: **PM DC Motor-Load System**



Dynamic equations:

$$V_s \quad L_a * \frac{di_a}{dt} \quad r_a * i_a \quad E_b$$
$$E_b \quad k_{PM} * \quad {}_m$$
$$V_s \quad L_a * \frac{di_a}{dt} \quad r_a * i_a \quad k_{PM} * \quad {}_m$$
$$J * \frac{d \quad {}_m}{dt} \quad T_m \quad T_L \quad b * \quad {}_m$$
$$T_m \quad k_{PM} * i_a$$
$$T_L \quad J * \frac{d \quad {}_m}{dt} \quad b * \quad {}_m \quad k_{PM} * i_a$$

To get the transfer function:

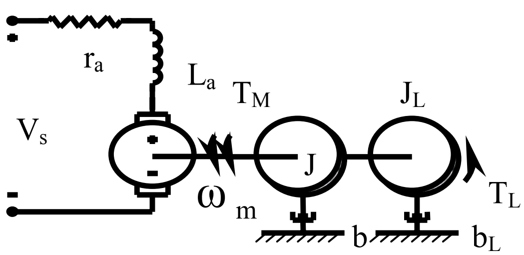
$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (Js + b) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (Js + b) \end{bmatrix}}$$

$$\left. \frac{\omega_m}{V_s}(s) \right|_{T_L = 0} = \frac{k_{PM}}{(J * L_a)s^2 + (r_a * J + L_a * b)s + r_a * b + k_{PM}^2}$$

b)

Sketch:



Dynamic Equations:

$$V_s \quad L_a * \frac{di_a}{dt} \quad r_a * i_a \quad E_b$$
$$E_b \quad k_{PM} * \quad {}_m$$
$$V_s \quad L_a * \frac{di_a}{dt} \quad r_a * i_a \quad k_{PM} * \quad {}_m$$
$$(J \quad J_L) * \frac{d \quad {}_m}{dt} \quad T_m \quad T_L \quad (b \quad b_L) * \quad {}_m$$
$$T_m \quad k_{PM} * i_a$$
$$T_L \quad (J \quad J_L) * \frac{d \quad {}_m}{dt} \quad (b \quad b_L) * \quad {}_m \quad k_{PM} * i_a$$

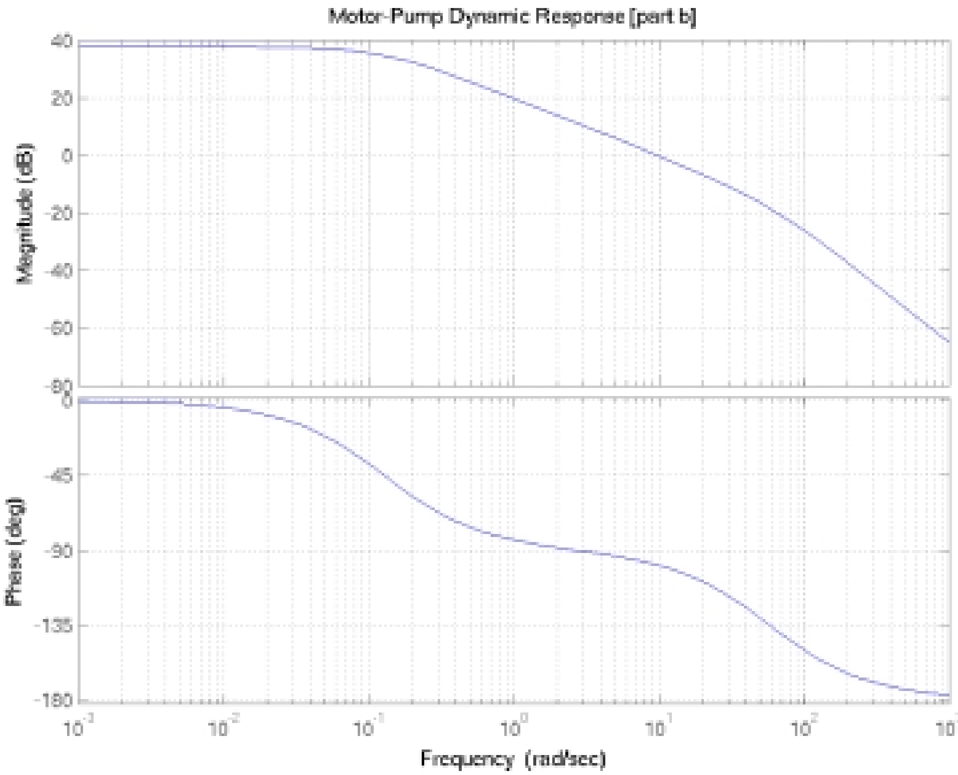
To get the transfer function:

$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J+J_L)s + (b+b_L)) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J+J_L)s + (b+b_L)) \end{bmatrix}}$$

$$\left.\frac{\omega_m}{V_s}(s)\right|_{T_L=0} = \frac{k_{PM}}{((J+J_L)*L_a)s^2 + (r_a*(J+J_L) + L_a*(b+b_L))s + r_a*(b+b_L) + k_{PM}^2}$$

Frequency Response:



Problem 17.28

Solution:

Known Quantities:

A PM DC motor is used to power a pump

Find:

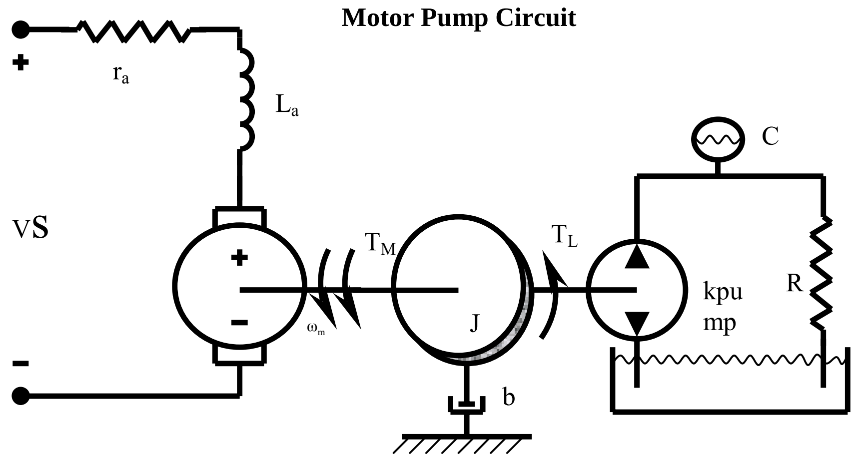
The dynamic equations of the system and the transfer function between the motor voltage and pressure.

Assumptions:

The inertia and damping of the motor and pump can be lumped together.

Analysis:

Sketch:



$$V_s \quad L_a \frac{di_a}{dt} \quad r_a i_a \quad E_b$$

$$V_s \quad L_a \frac{di_a}{dt} \quad r_a i_a \quad k_{PM} \quad {}_m$$

$$J \frac{d \quad {}_m}{dt} \quad T_m \quad T_L \quad b \quad {}_m$$

$$T_L \quad J \frac{d \quad {}_m}{dt} \quad b \quad {}_m \quad k_{PM} \quad {}_m$$

$$k_p \quad {}_m \quad \frac{p \quad 0}{R} \quad C_{acc} \frac{dp}{dt}$$

$$C_{acc} \frac{dp}{dt} \quad \frac{p}{R} \quad k_p \quad {}_m \quad 0$$

To get the transfer function:

$$\begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ p \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

$$p(s) = \frac{\det \begin{bmatrix} L_a s + r_a & k_{PM} & V_s \\ -k_{PM} & Js + b & 0 \\ 0 & -Rk_p & 0 \end{bmatrix}}{\det \begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix}}$$

$$\frac{p}{V_s}(s) = \frac{k_{PM} R k_p}{(L_a s + r_a)(Js + b)(RC_{acc}s + 1) + k_{PM}^2 (RC_{acc}s + 1) + Rk_p^2 (L_a s + r_a)}$$

Problem 17.29

Solution:

Known quantities:
Motor circuit shown in Figure P17.29 and magnetization parameters, load parameters. Operating point.
Note correction to the operating point*:* I_{a0} = 186.67 A;
Note correction to the parameter: k_f = 0.12 V-s/A-rad

Find:
a) System differential equations in symbolic form
b) Linearized equations

Assumptions:
The dynamics of the field circuit are negligible.

Analysis:
a) Differential equations
Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} \quad R_f I_f(t) \quad V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad k_f I_f(t) \; {}_m(t) \quad V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d \; {}_m(t)}{dt} \quad T_m(t) \quad T_L(t) \quad b \; {}_m$$

or

$$k_f I_f(t) I_a(t) \quad J \frac{d \; {}_m(t)}{dt} \quad b \; {}_m(t) \quad T_L(t)$$

Since the dynamics of the field circuit are much faster than those of the armature circuit (time constant $\frac{L_f}{R_f} \quad \frac{L_a}{R_a}$), we can write $I_f \quad \frac{V_S}{R_f}$ and the system of equations is now:

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad k_f \frac{V_S(t)}{R_f} \; {}_m(t) \quad V_S(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) \quad J \frac{d \; {}_m(t)}{dt} \quad b \; {}_m(t) \quad T_L(t)$$

b) Linearization
Define perturbation variables:

$$I_a(t) \quad \bar{I}_a \quad I_a(t)$$

$${}_m(t) \quad \bar{\ }_m \quad {}_m(t)$$

$$V_S(t) \quad \bar{V}_S \quad V_S(t)$$

Next, we write the steady-state equations (all derivatives equal to zero):

$$R_a \bar{I}_a \quad k_f \frac{\bar{V}_S}{R_f} \bar{\ }_m \quad \bar{V}_S$$

$$k_f \frac{\bar{V}_S}{R_f} \bar{I}_a \quad b \bar{\ }_m \quad \bar{T}_L$$

These equations must be satisfied at the operating point. We can verify this using numerical values:

$$0.75 \quad \bar{I}_a \quad 0.12\frac{150}{60}200 \quad 200$$

resulting in

$$\bar{I}_a \quad \frac{1}{0.75} \quad 200 \quad 0.12\frac{150}{60}200 \qquad 186.67$$

$$0.12\frac{150}{60}186.67 \quad 0.6 \quad 200 \quad \bar{T}_L$$

resulting in

$$\bar{T}_L \qquad 56 \quad 120 \quad 64 \text{ N - m}$$

Now, given that the system is operating at the stated operating point, we can linearize the differential equation for the perturbation variables around the operating point. To linearize the equation we recognize the nonlinear terms: $k_f \frac{V_S(t)}{R_f} \quad {}_m(t)$ and $k_f \frac{V_S(t)}{R_f} I_a(t)$.

To linearize these terms, we use the first-order term in the Taylor series expansion:

$$k_f \frac{V_S(t)}{R_f} \quad {}_m(t) \quad \frac{k_f}{R_f} \frac{V_S(t) \quad {}_m(t)}{V_S}\bigg|_{\bar{V}_S, \bar{}_m} \quad V_S(t) \quad \frac{k_f}{R_f} \frac{V_S(t) \quad {}_m(t)}{{}_m}\bigg|_{\bar{V}_S, \bar{}_m} \qquad {}_m(t)$$

$$\frac{k_f}{R_f} \bar{}_m \; V_S(t) \quad \bar{V}_S \quad {}_m(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) \quad \frac{k_f}{R_f} \frac{V_S(t) I_a(t)}{V_S}\bigg|_{\bar{V}_S, \bar{I}_a} \quad V_S(t) \quad \frac{k_f}{R_f} \frac{V_S(t) I_a(t)}{I_a}\bigg|_{\bar{V}_S, \bar{I}_a} \quad \bar{I}_a(t)$$

$$\frac{k_f}{R_f} \; \bar{I}_a \; V_S(t) \quad \bar{V}_S \; I_a(t)$$

Now we can write the linearized differential equations in the perturbation variables:

$$L_a \frac{d \; I_a(t)}{dt} \quad R_a \; I_a(t) \quad \frac{k_f}{R_f}\bar{V}_S \quad {}_m(t) \quad V_S(t) \quad \frac{k_f}{R_f} \bar{}_m \; V_S(t)$$

$$\frac{k_f}{R_f}\bar{V}_S \; I_a(t) \quad J\frac{d \quad {}_m(t)}{dt} \quad b \quad {}_m(t) \quad \bar{I}_a \; V_S(t) \quad T_L(t)$$

This set of equations is now linear, and numerical values can be substituted to obtain a numerical answer, valid in the neighborhood of the operating point, for given voltage and load torque inputs to the system.

Problem 17.30

Solution:

Known Quantities:

$$T_L = 5 + 0.05 \quad + 0.001 \quad ^2$$

$$k_{TPM} \quad k_{APM} \quad 2.42$$

$$R_a \quad 0.2$$

$$V_S \quad 50V$$

Find:

What will the speed of rotation be of the fan?

Assumptions:

The fan is operating at constant speed.

Analysis:

Applying KVL for a PM DC motor (note at constant current short inductor)

$V_S \quad i_a R_a \quad E_b$ (eqn. 17.48)

$i_a \quad \frac{T}{k_{TPM}}$

$E_b \quad k_{aPM} \quad {}_m$

$V_s \quad \frac{T}{k_{TPM}} R_a \quad k_{aPM} \quad {}_m$

$T \quad T_L \quad 5 \quad 0.05 \quad {}_m \quad 0.001 \quad {}_m^2$

$V_s \quad \frac{5 \quad 0.05 \quad {}_m \quad 0.001 \quad {}_m^2}{k_{TPM}} R_a \quad k_{aPM} \quad {}_m$

Plug in the known variables and solving for ${}_m$.

$50V \quad 5 \quad 0.05 \quad {}_m \quad 0.001 \quad {}_m^2 \quad \frac{0.02}{2.42 \text{ N m/Amp}} \quad 2.42 \, V \text{ sec/rad} \quad {}_m$

$8.26x10^{5} \quad {}_m^2 \quad 2.42 \quad 4.13x10^{3} \quad {}_m \quad (\ 50 \quad .413) \quad 0$

${}_m \quad 20.5, \text{ or } -29318 \text{ rad/sec}$

${}_m \quad 20.5 \text{ rad/sec}$

$N_m \quad 20.5 \text{ rad/sec} \quad \frac{60 \text{ sec}}{\text{min}} \quad \frac{\text{rev}}{2} \quad 196 RPM$

Problem 17.31

Solution:

Known Quantities:

A separately excited DC motor

$R_a = 0.1\Omega, R_f = 100\Omega, L_a = 0.2H, L_f = 0.02H, K_a = 0.8, K_f = 0.9$

$J = 0.5kg - m^2, b = 2N - m - rad / s$

Find:

a) A sketch of the system and its three differential equations
b) Sketch a simulation block diagram
c) Put the diagram into Simulink
d) Run the simulation with *Armature Control* with a constant field voltage $V_f = 100V$
   Plot the current and angular speed responses
   Run the simulation with *Field Control* with a constant armature voltage $V_f = 100V$
   Plot the current and angular speed responses

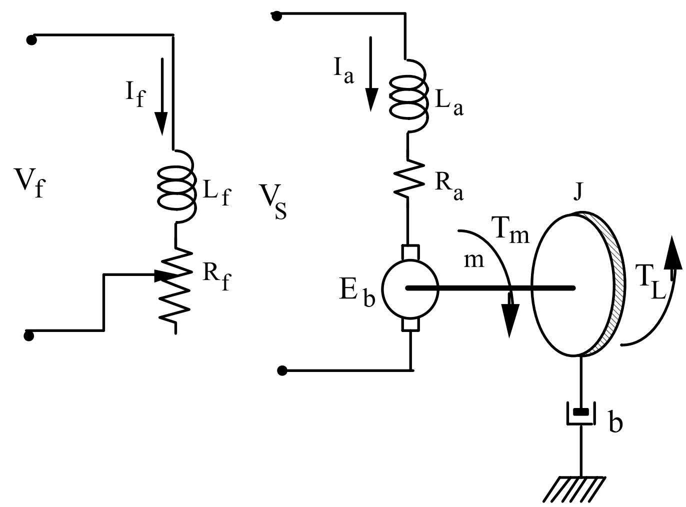
Assumptions:

No external load torque is applied

Analysis:

a) Sketch:

Separately Excited DC Motor



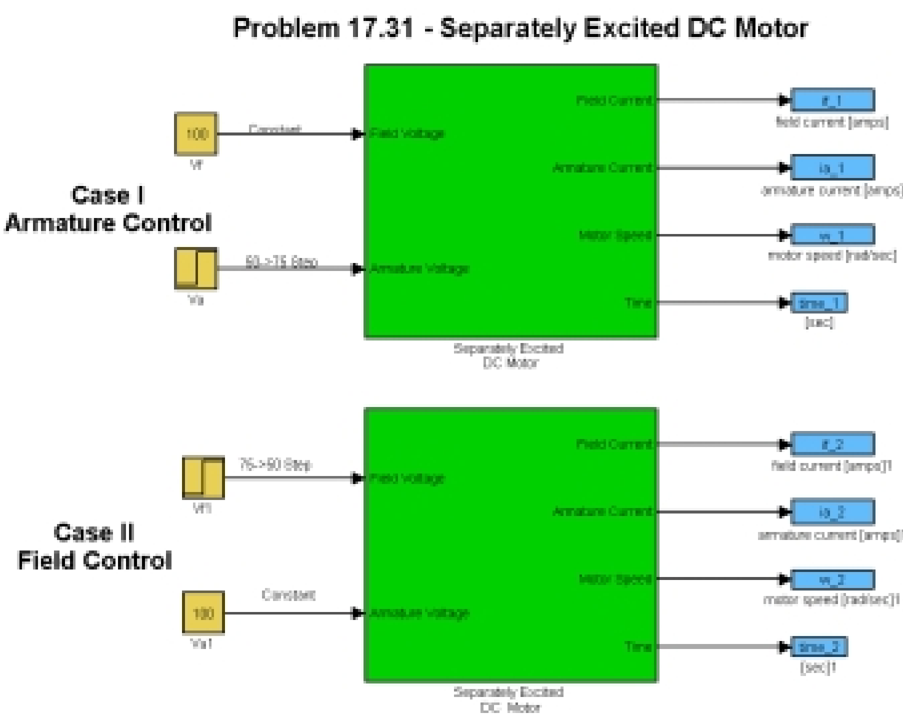
The three dynamic equations are:

$$V_f \quad L_f \frac{dI_f}{dt} \quad R_f I_f$$

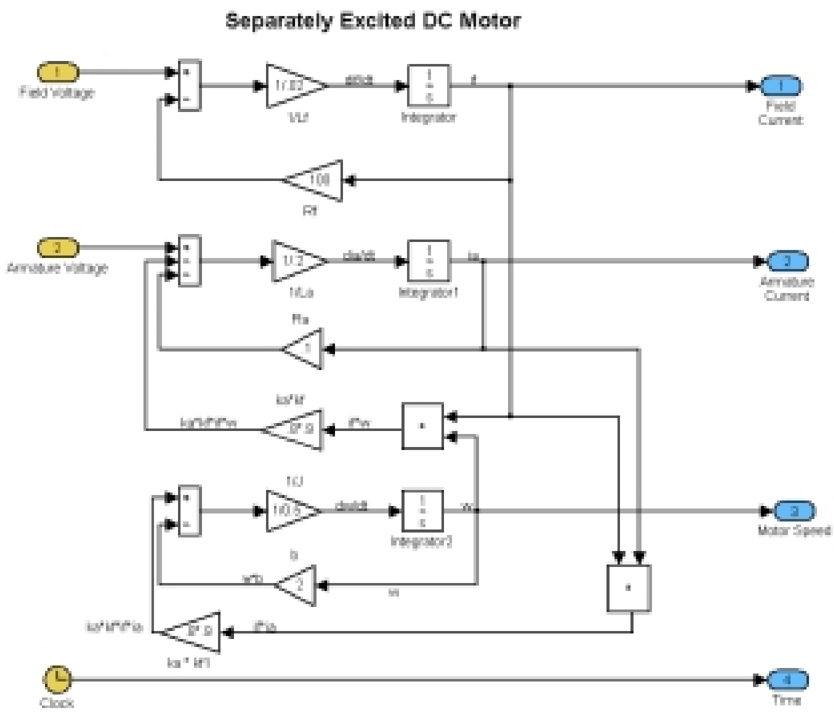
$$V_a \quad L_a \frac{dI_a}{dt} \quad R_a I_a \quad E_b$$

$$J\frac{d}{dt} \quad b \quad T_m \quad J\frac{d}{dt} \quad b \quad k_a i_a \quad J\frac{d}{dt} \quad b \quad k_a I_a k_f I_f$$

b) Simulink block diagram

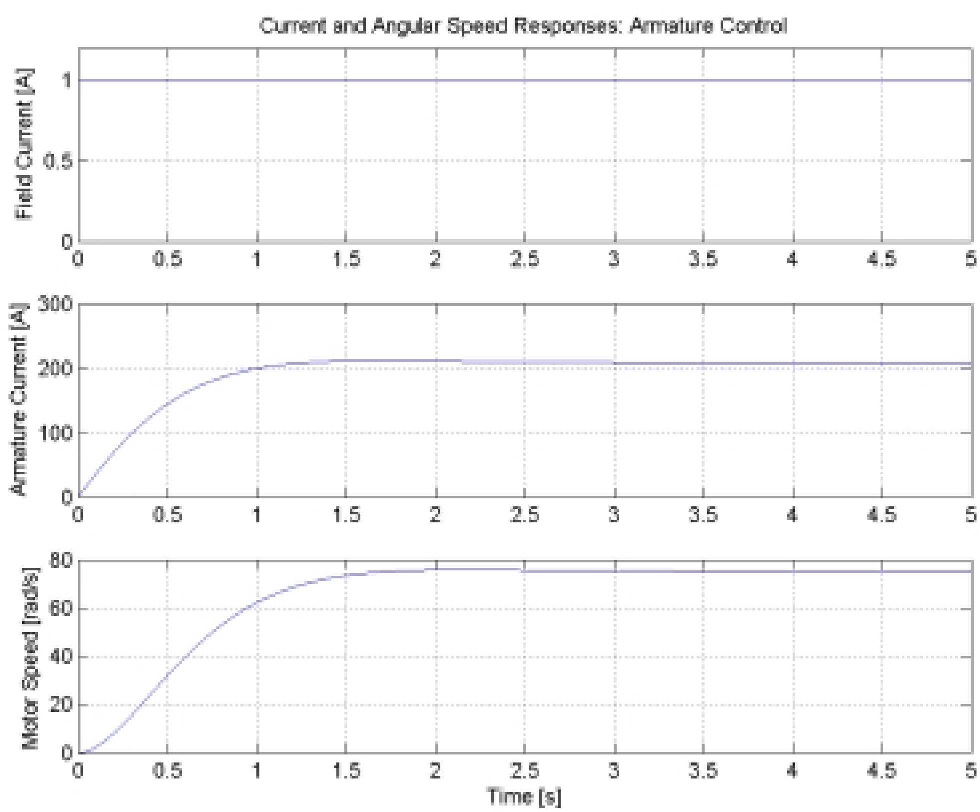


Simulink DC Motor Subsystem

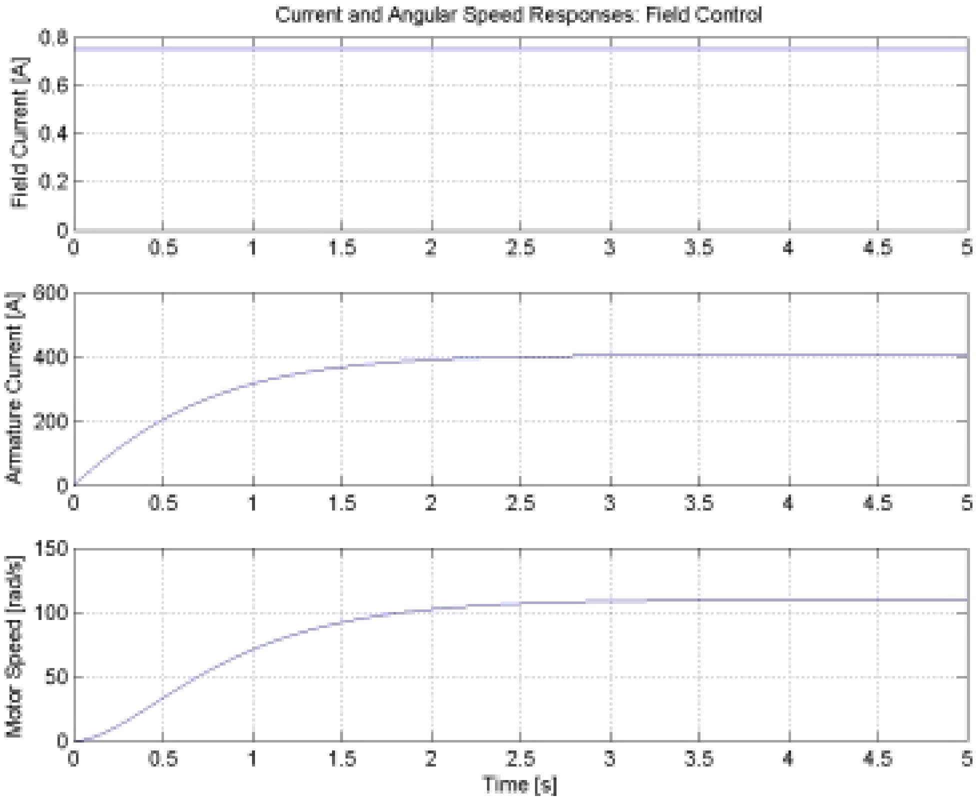


d) Simulink Responses:

Armature Control:



Field Control:



Problem 17.32

Solution:

Known quantities:

R_a, L_a, $k_a = k_T$, J_m, b_m, J, b, T_L.

Find:

Transfer functions from armature voltage to angular velocity and from load torque to angular velocity.

Schematics, diagrams, circuits and given data.

See equations 17.16-18 and Figure 17.20.

Assumptions:

Analysis:

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$V_a(t) \quad R_a I_a(t) \quad L_a \frac{dI_a(t)}{dt} \quad E_b(t) \quad 0$$

or

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad k_a \ \ _m(t) \quad V_a(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J_m \quad J \frac{d \quad (t)}{dt} \quad T_m(t) \quad T_L(t) \quad b_m \quad b$$

or

$$k_T I_a(t) \quad J_m \quad J \frac{d \quad (t)}{dt} \quad b_m \quad b \quad (t) \quad T_L(t)$$

To derive the transfer function, we Laplace transform the two equations to obtain:

$$sL_a \quad R_a \; I_a(s) \quad k_a \quad (s) \quad V_a(s)$$

$$k_a I_a(s) \quad s \; J_m \quad J \quad b_m \quad b \quad (s) \quad T_L(s)$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $\Omega_m(s)$ as a function of $V_a(s)$ and $T_L(s)$.

$$\begin{matrix} sL_a \quad R_a & k_a \\ k_a & s \; J_m \quad J \quad b_m \quad b \end{matrix} \quad \begin{matrix} I_a(s) \\ {}_m(s) \end{matrix} \quad \begin{matrix} V_a(s) \\ T_L(s) \end{matrix}$$

with solution

$$_m(s) \quad \frac{\det \begin{matrix} sL_a \quad R_a & V_a(s) \\ k_a & T_L(s) \end{matrix}}{\det \begin{matrix} sL_a \quad R_a & k_a \\ k_a & s \; J_m \quad J \quad b_m \quad b \end{matrix}}$$

or

$$_m(s) \quad \frac{sL_a \quad R_a}{sL_a \quad R_a \quad s \; J_m \quad J \quad b_m \quad b \quad k_a^2} T_L(s) \quad \frac{k_a}{sL_a \quad R_a \quad s \; J_m \quad J \quad b_m \quad b \quad k_a^2} V_a(s)$$

and finally

$$\left.\frac{_m(s)}{T_L(s)}\right|_{V_a(s) \quad 0} \quad \frac{sL_a \quad R_a}{sL_a \quad R_a \quad s \; J_m \quad J \quad b_m \quad b \quad k_a^2}$$

$$\left.\frac{_m(s)}{V_a(s)}\right|_{T_L(s) \quad 0} \quad \frac{k_a}{sL_a \quad R_a \quad s \; J_m \quad J \quad b_m \quad b \quad k_a^2}$$

Problem 17.33

Solution:

Known Quantities:
A PM DC motor that is coupled to a pump with a long shaft.

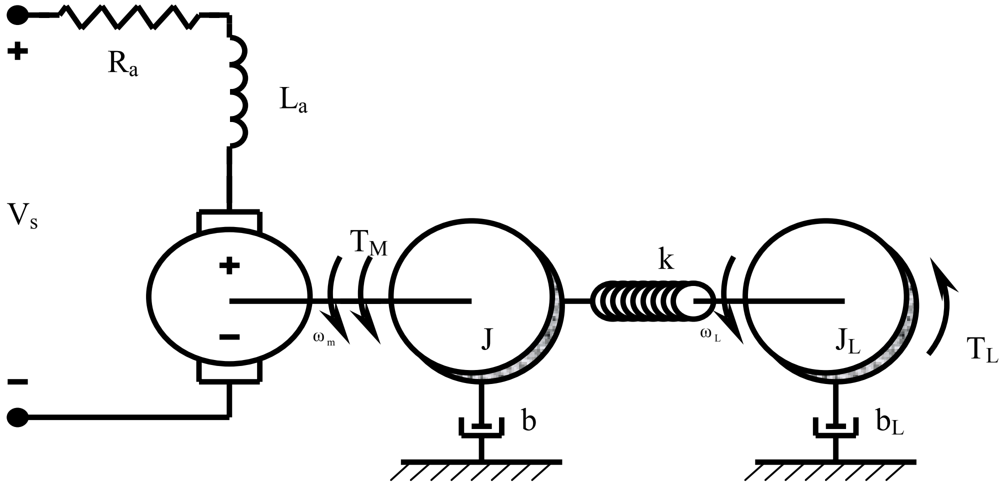
Find:
The dynamic equations of the system and the transfer function from input voltage to load inertia speed.

Assumptions:
The energy conversion is ideal.

Analysis:

Sketch:

PM DC Motor-Load Coupling



Knowing:

$$\frac{d}{dt} \qquad\qquad dt \quad \frac{1}{s}$$

Then the three dynamic equations for the system are:

$$V_a \quad L_a \frac{di_a}{dt} \quad R_a i_a \quad E_a \quad L_a \frac{di_a}{dt} \quad R_a i_a \quad k_a \quad {}_m$$

$$J_m \frac{d \quad {}_m}{dt} \quad b_m \quad {}_m \quad K \frac{{}_m}{s} \quad k_a i_a \quad K \frac{{}_L}{s} \quad 0$$

$$J_L \frac{d \quad {}_L}{dt} \quad b_L \quad {}_L \quad K \frac{{}_L}{s} \quad K \frac{{}_m}{s} \quad T_L$$

Putting them in matrix form:

$$\begin{bmatrix} L_a s + R_a & k_a & 0 \\ -k_a & J_m s^2 + b_m s + K/s & -K/s \\ 0 & -K/s & J_L s^2 + b_L s + K/s \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \omega_L \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -T_L \end{bmatrix}$$

$$\left.\frac{\omega_L}{V_a}(s)\right|_{T_L=0} = \frac{k_a K/s}{(L_a s + R_a)(J_m s + b_m + K/s)(J_L s + b_L + K/s) - (-K/s)^2 (L_a s + R_a) + (J_L s + b_L + K/s)(-k_a)(k_a)}$$

Problem 17.34

Solution:

Known quantities:

Field and armature circuit parameters; magnetization and armature constants; motor and load inertia and damping coefficients.

Find:

a) sketch system diagrams for shunt and series configuration

b) write expression for torque-speed curves for each configuration

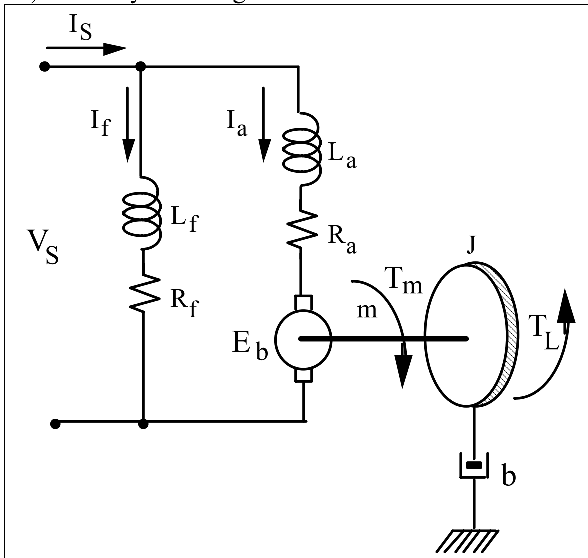
c) write the differential equations for each configuration

d) determine whether equations are linear or nonlinear and how they could be linearized

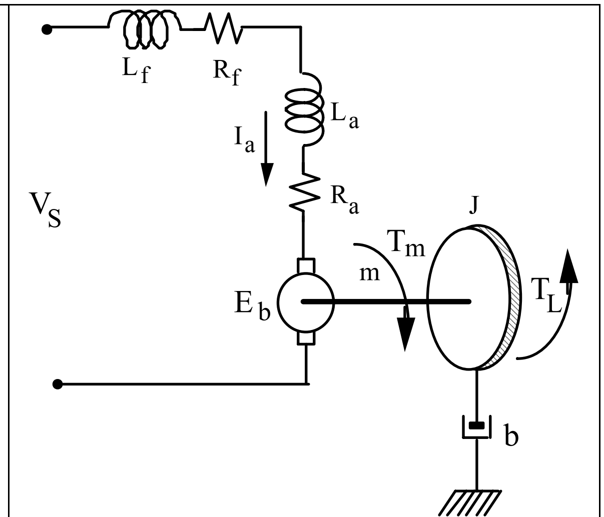
Assumptions:

Analysis:

a) System diagrams



Shunt motor-load system



Series motor-load system

b) Write expressions for the torque-speed curves

Shunt configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$V_S \quad R_f I_f$ field circuit

and

$V_S \quad R_a I_a \quad k_f I_f \quad {}_m$ armature circuit

or

$$V_S \quad R_a I_a \quad k_f \frac{V_S}{R_f} \quad {}_m$$

$$T_m \quad k_f I_f I_a \quad k_f \frac{V_S}{R_f} I_a \quad b \quad {}_m \quad T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$T_m \quad k_f I_f I_a$$

$$I_a \quad \frac{T_m}{k_f I_f} \quad \frac{T_m R_f}{k_f V_S}$$

and substitute the expression for I_a in the electrical circuit equation:

$$V_S \quad R_a I_a \quad k_f \frac{V_S}{R_f} \quad {}_m \quad \frac{R_a R_f}{k_f V_S} T_m \quad k_f \frac{V_S}{R_f} \quad {}_m$$

or

$$T_m \quad \frac{k_f V_S}{R_a R_f} V_S \quad k_f \frac{V_S}{R_f} \quad {}_m \quad \frac{k_f V_S^2}{R_a R_f} \quad \frac{k_f^2 V_S^2}{R_a R_f^2} \quad {}_m$$

Series configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$$V_S \quad R_a \quad R_f \; I_a \quad k_f I_a^2 \quad {}_m$$

$$T_m \quad k_f I_a^2 \quad b \quad {}_m \quad T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$I_a \quad \sqrt{\frac{T_m}{k_f}}$$

$$V_S \quad R_a \quad R_f \; I_a \quad k_f I_a^2 \quad {}_m = \; R_a \quad R_f \sqrt{\frac{T_m}{k_f}} \quad T_m \quad {}_m$$

which leads to a quadratic equation in T_m and ω_m.

c) Write the differential equations

Shunt configuration

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} \quad R_f I_f(t) \quad V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad k_f I_f(t) \; {}_m(t) \quad V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d \quad {}_m(t)}{dt} \quad T_m(t) \quad T_L(t) \quad b \quad {}_m$$

or

$$k_f I_f(t) I_a(t) \quad J \frac{d \quad {}_m(t)}{dt} \quad b \quad {}_m(t) \quad T_L(t)$$

Note that we have three differential equations that must be solved simultaneously. If the dynamics of the field circuit are much faster than those of the armature circuit (time constant $\frac{L_f}{R_f} \quad \frac{L_a}{R_a}$, as is often the case) one can assume that the field current varies instantaneously with the supply voltage, leading to $I_f \quad \frac{V_S}{R_f}$ and to the equations:

$$L_a \frac{dI_a(t)}{dt} \quad R_a I_a(t) \quad k_f \frac{V_S(t)}{R_f} \omega_m(t) \quad V_S(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) \quad J \frac{d\omega_m(t)}{dt} \quad b\omega_m(t) \quad T_L(t)$$

Series configuration

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_a \quad L_f \frac{dI_a(t)}{dt} \quad R_a \quad R_f \; I_a(t) \quad k_f I_a(t)\omega_m(t) \quad V_S(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} \quad T_m(t) \quad T_L(t) \quad b\omega_m$$

or

$$k_f I_a^2(t) \quad J \frac{d\omega_m(t)}{dt} \quad b\omega_m(t) \quad T_L(t)$$

d) Determine whether the equations are nonlinear

Both systems of equations are nonlinear. In the shunt case, we have product terms in I_f and ω_μ, and in I_f and I_a (or in V_S and ω, and in V_S and I_a if we use the simplified system of two equations). In the series case, we have a quadratic term in I_a^2 and a product term in I_f and ω_m. In either case, no simple assumption leads to a linear set of equations; thus either linearization or nonlinear solution methods (e.g.: numerical simulation) must be employed.

Problem 17.35

Solution:

Known quantities:
A shunt-connected DC motor shown in Figure P17.35
Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant

Find:
Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:
None

Analysis:
Electrical subsystem

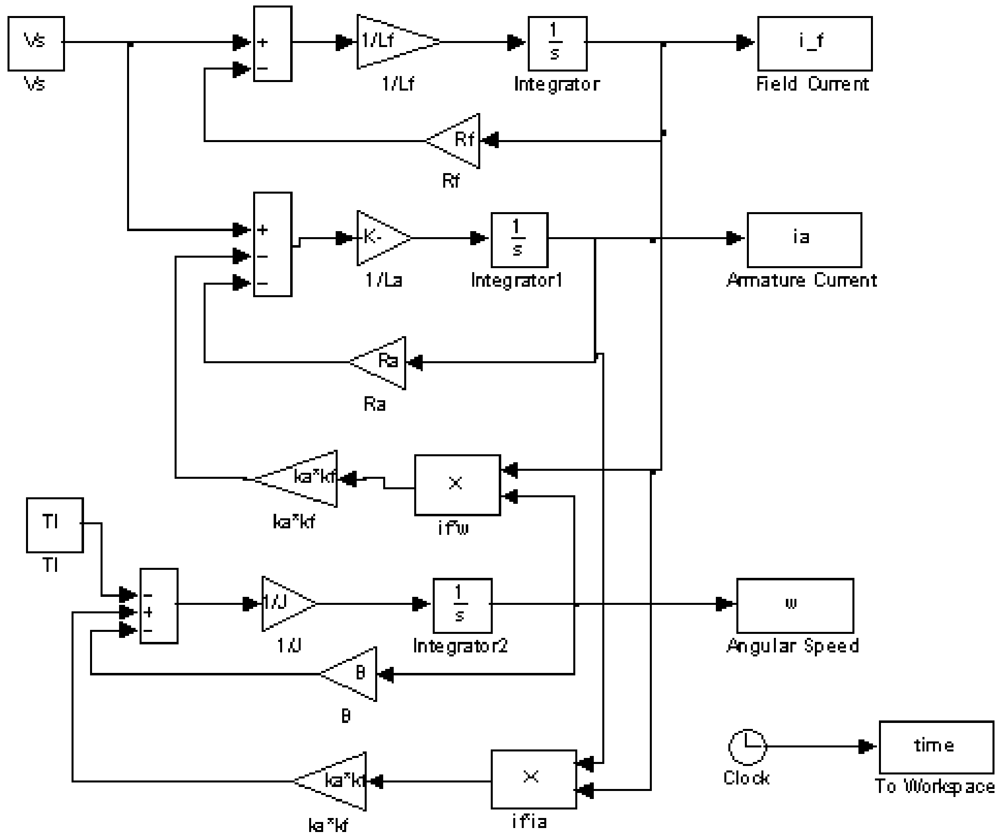
$$V_S(t) \quad L_f \frac{dI_f(t)}{dt} \quad R_f I_f(t) \quad \text{field}$$

$$V_S(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a \phi \omega_m(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a k_f I_f(t) \omega_m(t) \text{armature}$$

Mechanical subsystem

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m(t) = k_a \phi I_a(t) - T_L(t) - b\omega_m(t) = k_a k_f I_f(t) I_a(t) - T_L(t) - b\omega_m(t)$$

Simulation block diagram:



Problem 17.36

Solution:

Known quantities:
A series-connected DC motor shown in Figure P17.36
Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant

Find:
Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:
None

Analysis:
<u>Electrical subsystem</u>

$$V_S(t) \quad L_a \quad L_f \frac{dI_a(t)}{dt} \quad R_a \quad R_f \; I_a(t) \quad k_a \quad {}_m(t)$$

$$V_S(t) \quad L\frac{dI_a(t)}{dt} \quad RI_a(t) \quad k_a k_f I_a(t) \; {}_m(t)$$

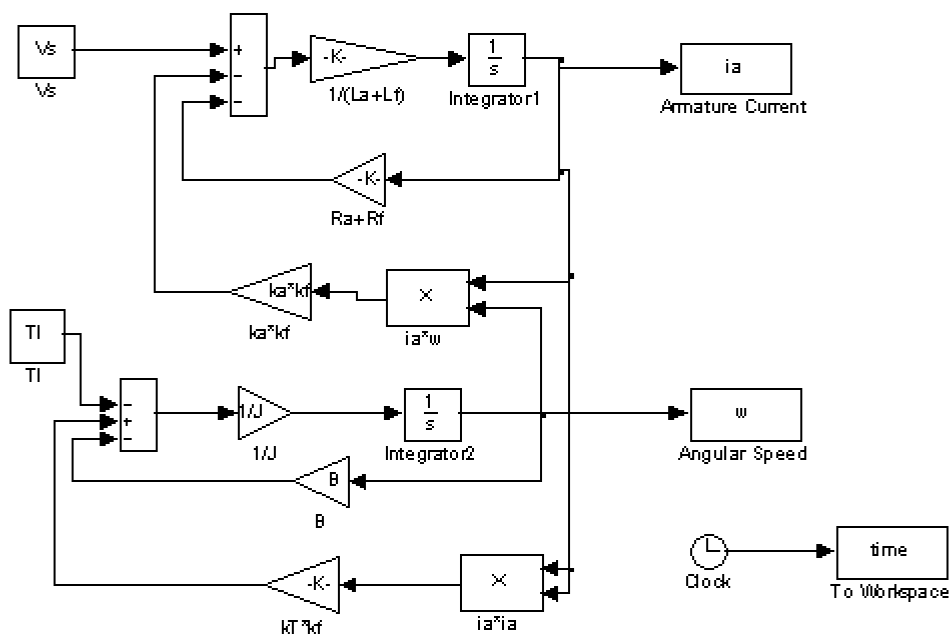
$$\frac{dI_a(t)}{dt} \quad \frac{1}{L}V_S(t) \quad \frac{R}{L}I_a(t) \quad \frac{k_a k_f}{L}I_a(t) \; {}_m(t)$$

<u>Mechanical subsystem</u>

$$J\frac{d \quad {}_m(t)}{dt} \quad T_m(t) \quad T_L(t) \quad b \quad {}_m(t) \quad k_T \; I_a(t) \quad T_L(t) \quad b \quad {}_m(t) \quad k_T k_f I_a^2(t) \quad T_L(t) \quad b \quad {}_m(t)$$

$$\frac{d \quad {}_m(t)}{dt} \quad \frac{k_T k_f}{J}I_a^2(t) \quad \frac{1}{J}T_L(t) \quad \frac{b}{J} \quad {}_m(t)$$

Simulation block diagram:



Section 17.6: The Alternator (Synchronous Generator)

Problem 17.37

Solution:

Known quantities:

A $550V \cdot A, 20V$ rated automotive alternator. At rated $V \cdot A$, the power factor is 0.85. The resistance per phase is 0.05Ω. The field takes $2\,A$ at $12V$. The friction and windage loss is $25W$ and core loss is $30W$.

Find:
The percent efficiency under rated conditions.

Assumptions:
None.

Analysis:

$$I_a = \frac{500}{20} = 25\,A$$

$$P_a = I_a^2 R_a = 31.25W$$

$$P_{out} = 500(0.85) = 425W$$

$$P_f = 2(12) = 24W$$

$$P_{in} = P_{out} + P_a + 25 + 30 + 24 = 535.25W$$

$$\% = \frac{425}{535.25} \times 100 = 79.4\%$$

Problem 17.38

Solution:

Known quantities:

A three-phase $2300V, 500\,kV \cdot A$ synchronous generator. $X_S = 8.0\,\Omega, r_a = 0.1\,\Omega$. The machine is operating at rated load and voltage at a power factor of 0.867 lagging.

Find:
The generated voltage per phase and the torque angle.

Assumptions:
None.

Analysis:

$$I = \frac{500k}{\sqrt{3}(2300)} = 125.5\,A$$

$$\begin{aligned} E &= \frac{2300}{\sqrt{3}}\angle 0^\circ + 125.5\angle -30^\circ(0.1 + j0.8) \\ &= 1327.9 + 101.2\angle 52.9^\circ \\ &= 1389 + j80.7 \\ &= 1391.3\angle 3.3^\circ\,V \end{aligned}$$

$$\therefore E = 1391.3V$$

$$\delta = 3.3^\circ$$

Problem 17.39

Solution:

Known quantities:

As shown in Figure P17.39.

Find:

Explain the function of Q, D, Z, and SCR.

Assumptions:

None.

Analysis:

Q : The setting of R_1 determines the biasing of Q. When Q conducts, the SCR will fire, energizing the alternator's field.

D: This diode serves as a "free-wheeling" element, allowing the field current to circulate without interfering with the commutation of the SCR.

Z: The Zener diode provides a fixed reference voltage at the emitter of transistor Q; i.e., determination of when Q conducts is controlled solely by the setting of R_1.

SCR: The SCR acts as a half-wave rectifier, providing field excitation for the alternator. Without the field, of course, the alternator cannot generate.

Section 17.7: The Synchronous Motor

Problem 17.40

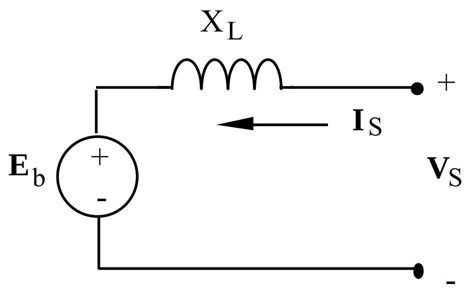
Solution:

Known quantities:
A non-salient pole, Y-connected, three phase, two-pole synchronous machine. The synchronous reactance is $7\,\Omega$ and the resistance and rotational losses are negligible. One point on the open-circuit characteristic is given by $V_0 = 400V$ (phase voltage) for a field current of $3.32\,A$. The machine operates as a motor, with a terminal voltage of $400V$ (phase voltage). The armature current is $50\,A$, with power factor 0.85 leading.

Find:
E_b, field current, torque developed, and power angle δ.

Assumptions:
None.

Analysis:
The per phase circuit is shown below:



Since the power factor is 0.85, we have:

$$\theta = 31.79^\circ$$

$$\omega_m = \frac{2\pi}{60}3600 = 377\,rad/\sec$$

From $V_{OC} = 400V$, we have

$$E_b = 400V\,(open\,circuit) = k\omega_m i_f$$

Therefore $k = \dfrac{400}{377 \times 3.32} = 0.3196$

$$E_b = 400\angle 0^\circ - 50\angle 31.79^\circ \times 7\angle 90^\circ$$
$$= 400 + 184.38 - j297.49$$
$$= 655.74\angle -26.98^\circ\,V$$

$$i_f = \frac{E_b}{120.48} = 5.44\,A$$

$$\theta_T = 31.79^\circ + 26.98^\circ = 58.77^\circ$$

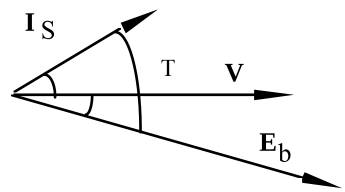
The torque developed is:

$$T = \frac{3}{377}|E_b||I_S|\cos\theta_T = 135.27\,N \cdot m$$

δ is the angle from V to E_b:

$$\delta = -26.98^\circ$$

The phase diagram is shown below:



Problem 17.41

Solution:

Known quantities:

A factory load of $900\,kW$ at 0.6 power factor lagging is increased by adding a $450\,kW$ synchronous motor.

Find:

The power factor this motor operates at and the KVA input if the overall power factor is 0.9 lagging.

Assumptions:

None.

Analysis:

$$P_{old} = 900\,kW \quad Q_{old} = 1200\,kVAR$$

$$P_m = 450\,kW$$

$$P_T = 1350\,kW \quad Q_T = 653.8\,kVAR$$

$$Q_m = 653.8 - 1200 = -546.2\,kVAR$$

$$pf_m = \cos(\tan^{-1}\frac{Q_m}{P_m}) = 0.636\,leading$$

$$S_m = \frac{P_m}{pf_m} = 708\,kVA$$

Problem 17.42

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, two-pole synchronous generator is connected to a $400\,V$ (line to line), $60\,Hz$, three-phase line. The stator impedance is $0.5 + j1.6$ (per phase). The generator is delivering rated current $36\,A$ at unity power factor to the line.

Find:

The power angle for this load and the value of E_b for this condition. Sketch the phasor diagram, showing E_b, I_S, and V_S.

Assumptions:
None.

Analysis:

$$V_L = \frac{400}{\sqrt{3}} \angle 0^\circ = 230.9 \angle 0^\circ \, V$$

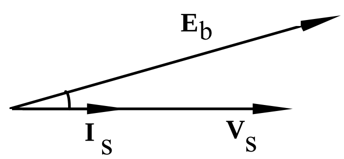
$$I_L = 36 \angle 0^\circ \, A$$

$$Z_S = 0.5 + j1.6 = 1.676 \angle 72.65^\circ \, \Omega$$

$$E_b = V_L + I_L Z_S = 248.9 + j57.6$$

$$= 255.5 \angle 13.03^\circ \, V$$

The power angle is 13.03°.



Problem 17.43

Solution:

Known quantities:
A non-salient pole, three phase, two-pole synchronous generator is connected in parallel with a three-phase, Y-connected load. The equivalent circuit is shown in Figure P17.43. The parallel combination is connected to a $220V$ (line to line), , three-phase line. The load current is $25\,A$ at a power factor of 0.866 inductive. $X_S = 2\,\Omega$. The motor is operating with $I_f = 1\,A, T = 50\,N \cdot m$ at a power angle of -30°.

Find:
I_S, P_{in} (to the motor), the overall power factor and the total power drawn from the line.

Assumptions:
Neglect all losses for the motor.

Analysis:
The phasor per-phase voltage is:

$$V_S = 127 \angle 0^\circ \, V$$

$$T_{dev} = 50\,N \cdot m = -\frac{3}{377} \frac{|E_b||V_S|}{X_S} \sin \delta$$

Therefore,

$$|E_b| = -\frac{50(377)2}{3(127)\sin(-30^\circ)} = 197.9V$$

$$E_b = 197.9 \angle -30^\circ \, V$$

For $i_f = 1\,A$,

$$I_S = 49.47 + j22.2 = 54.23 \angle 24.16^\circ \, A$$

The load current is:

$$I_L = 25\angle -\cos^{-1} 0.866 = 21.65 - j12.5$$

and

$$I_1 = I_L + I_S = 71.12 + j9.7 = 71.78\angle 7.77^\circ\ A$$

$$P_{in\_motor} = 3\times 54.23\times 127\times \cos 24.16^\circ = 18.85\,kW$$

$$P_{in\_total} = 3\times 71.78\times 127\times \cos 7.77^\circ = 27.10\,kW$$

The power factor is:

$$pf = \cos 7.77^\circ = 0.991\,leading$$

Problem 17.44

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, four-pole synchronous machine. The synchronous reactance is $10\,\Omega$. It is connected to a $230\sqrt{3}\,V$ (line to line), $60\,Hz$, three-phase line. The load requires a torque of $T_{shaft} = 30\,N\cdot m$. The line current is $15\,A$ leading the phase voltage.

Find:

The power angle δ and E for this condition. The line current when the load is removed. Is it leading or lagging the voltage.

Assumptions:

All losses can be neglected.

Analysis:

At $\omega_m = 188.5\,rad/\sec$, we can calculate

$$P_{out} = 30\times 188.5 = 5655W$$

Since $P_{in} = P_{out}$ and $P'_{in}(per\ phase) = 1885W = 230\times 15\cos\theta$, we calculate

$$\theta = \cos^{-1} 0.5464 = 56.88^\circ$$

Since $V_S = 230\angle 0^\circ\,V,\ \ I_S = 15\angle 56.88^\circ\ A$

$$E_b = 355.6 - j81.96 = 364.92\angle -12.98^\circ V$$

The power angle is:

$$\delta = -12.98_\circ$$

If the load is removed, the power angle is 0° and from

$$364.92\angle 0^\circ = 230\angle 0^\circ - 10\angle 90^\circ$$

$$\Rightarrow I = 13.495\angle 90^\circ\ A$$

The current is leading the voltage.

Problem 17.45

Solution:

Known quantities:

A $10\,hp, 230V, 60\,Hz$ Y-connected, three phase synchronous motor delivers full load at a power factor of 0.8 leading. The synchronous reactance is $6\,\Omega$. The rotational loss is $230W$, and the field loss is $50W$.

Find:
a) The armature current.
b) The motor efficiency.
c) The power angle.

Assumptions:
Neglect the stator winding resistance.

Analysis:

$$P_{out} = 10\,hp = 7460\,W$$

$$P_{in} = P_{out} + P_r + P_{copper} = 7740$$

$$\therefore P_{in}(\,per\ phase) = 2580 = V_S I_S 0.8$$

$$V_S = \frac{230}{\sqrt{3}} = 132.8V$$

$$\therefore I_S = \frac{2580}{132.8 \times 0.8} = 24.3\,A$$

That is:

$$V_S = 132.8\angle 0^\circ\,V,\ \ I_S = 24.3\angle 36.87^\circ\ A$$

$$E_b = V_S - I_S(6\angle 90^\circ) = 249.2\angle -27.9^\circ\,V$$

a)

$$I_S = 24.3\angle 36.87^\circ\ A$$

b)

$$efficiency = \frac{7460}{7740} = 0.964 = 96.4\%$$

c)

$$power\,angle = -27.9^\circ$$

Problem 17.46

Solution:

Known quantities:

A three-phase $2300V, 60\,Hz, 30\ poles, 2000\,hp$, unity power factor synchronous motor.
$X_S = 1.95\,\Omega$ per phase.

Find:
The maximum power and torque.

Assumptions:
Neglect all losses.

Analysis:

$$n_S = \frac{3600}{15} = 240\,rev/\min$$

$$\omega_S = 25.13\,rad/\sec$$

At full load,

$$P_{in} = 746 \times 2000 = 1.492\,MW$$

$$V_S = \frac{2300}{\sqrt{3}} = 1327.9\angle 0^\circ\,V$$

For unity power factor,

$$I_S = 374.5\angle 0^\circ\,A$$

$$E_b = V_S - I_S jX_S = 1327.9 - j730.3$$

$$= 1515.5\angle -28.2^\circ\,V$$

The maximum power and torque are:

$$P_{\max} = 3\frac{|E_b||V_S|}{X_S} = 3.096\,MW$$

$$T_{\max} = \frac{P_{\max}}{\omega_S} = 123.2\,kN \cdot m$$

Problem 17.47

Solution:

Known quantities:

A $1200V$ Y-connected, three phase synchronous motor takes $110\,kW$ when operated under a certain load at $1200\,rev/\min$. The back emf of the motor is $2000V$. The synchronous reactance is $10\,\Omega$ per phase.

Find:

The line current and the torque developed by the motor.

Assumptions:

Winding resistance is negligible.

Analysis:

$$V_S = \frac{1200}{\sqrt{3}} = 692.8\angle 0^\circ$$

The input power per phase is:

$$L = \frac{N^2}{\Re_T} = \frac{100^2}{12.51 \times 10^3} = 0.8\,H$$

The power developed is:

$$P = -3\frac{|E_b||V_S|}{X_S}\sin\delta$$

$$\therefore \sin\delta = -0.2646$$

$$\delta = -15.34^\circ$$

The torque developed is:

$$T = \frac{P}{\omega_S} = 875.1\, N \cdot m$$

Problem 17.48

Solution:

Known quantities:

A $600V$ Y-connected, three phase synchronous motor takes $24\,kW$ at a leading power factor of 0.707. The per-phase impedance is $5 + j50\,\Omega$.

Find:
The induced voltage and the power angle of the motor.

Assumptions:
None.

Analysis:

$$V_S = \frac{600}{\sqrt{3}} = 346.4\angle 0^\circ\, V$$

$$Z_S = 5 + j50 = 50.25\angle 84.29^\circ\, \Omega$$

From $pf = 0.707$, we have $\theta = 45^\circ$.

From $P_{in} = 3|V_S||I_S \cos\theta|$, we have

$$|I_S| = \frac{24 \times 10^3}{3 \times 346.4} \times 0.707 = 32.67\, A$$

$$I_S = 32.67\angle 45^\circ\, A$$

$$E_b = V_S - I_S Z_S = 1385 - j1270.6$$

$$= 1880.3\angle -42.51^\circ\, V$$

The power angle is:

$$\delta = -42.51^\circ$$

The power developed and the copper loss are:

$$P_{dev} = 3|E_b||I_S|\cos 87.51^\circ = 8.006\, kW$$

$$P_{loss} = 3|I_S|^2 R_S = 16.01\, kW$$

Section 17.8: The Induction Motor

Problem 17.49

Solution:

Known quantities:

A $74.6\,kW$ three-phase, $440V$ (line to line), four-pole, $60\,Hz$ induction motor. The equivalent circuit parameters are:

$$R_S = 0.06\,\Omega \quad R_R = 0.08\,\Omega$$

$$X_S = 0.3\,\Omega \quad X_R = 0.3\,\Omega \quad X_m = 5\,\Omega$$

The no-load power input is $3240\,W$ at a current of $45\,A$.

Find:

The line current, the input power, the developed torque, the shaft torque, and the efficiency at $s = 0.02$.

Assumptions:

None.

Analysis:

$$V_S = \frac{400}{\sqrt{3}} = 254\angle 0^\circ\,V$$

$$Z_{in} = 0.06 + j0.3 + \frac{j5(4 + j0.3)}{4 + j5.3}$$

$$= 2.328 + j2.294 = 3.268\angle 44.59^\circ\,\Omega$$

$$I_S = 77.7\angle -44.59\,A$$

$$P_{in} = 3 \times 254 \times 77.7\cos(-44.59^\circ) = 42.16\,kW$$

$$I_2 = \frac{j5}{4 + j5.3} I_S = 58.51\angle -7.55^\circ\,A$$

The total power transferred to the rotor is:

$$P_T = 3\frac{R_S}{S}\left|I_2\right|^2 = 41.1\,kW$$

$$P_m = P_T - P_{copper\_loss\_in\_rotor}$$

$$= 41.1 \times 10^3 (1 - s) = 40.25\,kW$$

$$\omega_m = (1 - s)\omega_S = 0.98 \times 188.5 = 184.7\,rad/\sec$$

Therefore, the torque developed is:

$$T_{dev} = \frac{P_m}{184.7} = 218\,N \cdot m$$

$$= 1880.3\angle -42.51^\circ\,V$$

The rotational power and torque losses are:

$$P_{rot} = 3240 - 3\times 45^2 \times 0.06 = 2875.5W$$
$$T_{rot} = 15.56\, N\cdot m$$

The shaft torque is:

$$T_{sh} = 218 - 15.56 = 202.4\, N\cdot m$$

Efficiency is:

$$T_{sh} = \frac{P_{out}}{P_{in}} = \frac{202.4\times 184.7}{42.16\times 10^3} = 0.887$$

Problem 17.50

Solution:

Known quantities:

A $60\,Hz$, four-pole, Y-connected induction motor is connected to a three-phase, $400V$ (line to line), $60\,Hz$ line. The equivalent circuit parameters are:

$$R_S = 0.2\,\Omega \qquad R_R = 0.1\,\Omega$$
$$X_S = 0.5\,\Omega \qquad X_R = 0.2\,\Omega \qquad X_m = 20\,\Omega$$

When the machine is running at $1755\, rev/\min$, the total rotational and stray-load losses are $800W$.

Find:

The slip, input current, total input power, mechanical power developed, shaft torque and efficiency.

Assumptions:

None.

Analysis:

From $n_S = 1800\, rev/\min$, we have

$$s = 0.025$$
$$\frac{R_R}{s} = 4$$
$$Z_{in} = 0.2 + j0.5 + \frac{j20(4 + j0.2)}{4 + j20.2}$$
$$= 3.972 + j1.444 = 4.226\angle 19.98^\circ\ \Omega$$

Therefore,

$$I_S = 54.6\angle -19.98^\circ\ A$$

$$P_{in} = 3(54.6)(\frac{400}{\sqrt{3}}\cos(-19.98^\circ)) = 35.6\,kW$$

$$P_t = P_{in} = 3\left|I_S\right|^2 R_S$$

$$= 35.6\times 10^3 - 3(54.6)^2 \times 0.2 = 33.81\,kW$$

$$P_m = (1-s)P_t = 32.97\,kW$$

$$P_{sh} = P_{out} = P_m - 800 = 32.17\,kW$$

$$\omega_m = 183.8\,rad/\sec$$

$$T_{sh} = 175\,N\cdot m$$

$$efficiency = \frac{32.17}{35.6} = 0.904$$

Problem 17.51

Solution:

Known quantities:

A three-phase, $60\,Hz$, eight-pole induction motor operates with a slip of 0.05 for a certain load.

Find:

a) The speed of the rotor with respect to the stator.
b) The speed of the rotor with respect to the stator magnetic field.
c) The speed of the rotor magnetic field with respect to the rotor.
d) The speed of the rotor magnetic field with respect to the stator magnetic field.

Assumptions:

None.

Analysis:

$$n_S = 900\,rev/\min, \qquad \omega_S = 94.25\,rad/\sec$$

a)

$$n_m = (1-s)n_S = 855\,rev/\min$$

b)

The speed of the stator field is $900\,rev/\min$, the rotor speed relative to the stator field is $-45\,rev/\min$.

c)

$$45\,rev/\min$$

d)

$$0\,rev/\min$$

Problem 17.52

Solution:

Known quantities:

A three-phase, $60\,Hz$, $400\,V$ (per phase), two-pole induction motor develops $P_m = 37\,kW$ at a certain speed.

The rotational loss at this speed is $800\,W$.

Find:

a) The slip and the output torque if the total power transferred to the rotor is $40\,kW$.

b) I_S and the power factor if $P_m = 45\,kW, R_S = 0.5\,\Omega$.

Assumptions:
Stray-load loss is negligible.

Analysis:
a)

$$P_m = 3(1-s)P_t = 37\,kW$$

$$1-s = 0.925 \Rightarrow s = 0.075$$

$$n_S = 3600\,rev/\text{min}, \quad \omega_S = 377\,rad/\text{sec}$$

$$\omega_m = (1-s)\omega_S = 348.7\,rad/\text{sec}$$

$$P_{sh} = P_{out} = 37 - 0.8 = 36.2\,kW$$

$$T_{sh} = \frac{P_{sh}}{348.7} = 103.8\,N \cdot m$$

b)

$$P_{in} = 3|I_S|^2 R_S + P_t$$

$$3|I_S|^2 R_S = 5\,kW$$

$$\therefore |I_S| = 57.7\,A$$

$$P_{in} = 3|V_S||I_S|\cos\theta = 45\,kW$$

The power factor is:

$$\cos\theta = 0.65\,lagging$$

Problem 17.53

Solution:

Known quantities:

The nameplate speed of a $25\,Hz$ induction motor is $720\,rev/\text{min}$. The speed at no load is $745\,rev/\text{min}$.

Find:
a) The slip.
b) The percent regulation.

Assumptions:
None.

Analysis:

a)

$$p \approx \frac{120(25)}{720} = 4.17 \Rightarrow p = 4$$

$$n_{sync} = \frac{120(25)}{4} = 750\, rpm$$

$$slip = \frac{750 - 720}{750} = 0.04 = 4\%$$

b)

$$reg = \frac{745 - 720}{720} = 0.035 = 3.5\%$$

Problem 17.54

Solution:

Known quantities:

The name plate of a squirrel-cage four-pole induction motor has $25\, hp, 220V, 60\, Hz, 830\, rev/\min, 64\, A$, three-phase line current. The motor draws $20{,}800W$ when operating at a full load.

Find:

a) slip.

b) Percent regulation if the no-load speed is $895\, rpm$.

c) Power factor.

d) Torque.

e) Efficiency.

Assumptions:

None.

Analysis:

a)

$$n_{sync} = 900\, rpm$$

$$slip = \frac{900 - 830}{900} = 0.078 = 7.8\%$$

b)

$$reg = \frac{895 - 830}{830} = 0.078 = 7.8\%$$

c)

$$pf = \frac{20{,}800}{\sqrt{3}(220)(64)} = 0.853\, lagging$$

d)

$$T = \frac{7.04(25 \times 746)}{830} = 158.2\, lb \cdot ft$$

e)

$$eff = \frac{25 \times 746}{20{,}800} = 0.897 = 89.7\%$$

Problem 17.55

Solution:

Known quantities:

A $60\,Hz$, four-pole, Y-connected induction motor is connected to a $200V$ (line to line), three-phase, $60\,Hz$ line. The equivalent circuit parameters are:

$$R_S = 0.48\,\Omega \qquad Rotational\ loss\ torque = 3.5\,N \cdot m$$
$$X_S = 0.8\,\Omega \qquad R_R = 0.42\,\Omega\,(referred\ to\ the\ stator)$$
$$X_m = 30\,\Omega \qquad X_R = 0.8\,\Omega\,(referred\ to\ the\ stator)$$

The motor is operating at slip $s = 0.04$.

Find:
The input current, input power, mechanical power, and shaft torque.

Assumptions:
Stray-load losses are negligible.

Analysis:

$$V_S = 115.5V$$
$$\omega_m = (1-s)188.5 = 181\,rad/\sec$$
$$Z_{in} = 0.48 + j0.8 + \frac{j30(10.5 + j0.8)}{10.5 + j30.8}$$
$$= 9.404 + j4.63 = 10.48\angle 26.2^\circ$$
$$\therefore I_S = 11.02\angle -26.2^\circ\ A$$
$$P_{in}(per\ phase) = 115.5 \times 11.02 \times \cos(-26.2^\circ)$$
$$P_{in}(total) = 3426W$$
$$P_t = P_{in}(total) - 3R_S\left|I_S\right|^2 = 3251W$$
$$\therefore P_m = (1-s)P_t = 3121W$$
$$T_{sh} = \frac{3121}{181} = 17.24\,N \cdot m$$

Problem 17.56

Solution:

Known quantities:

a) A three-phase, $220V$, $60\,Hz$ induction motor runs at $1140\,rev/\min$.

b) A three-phase squirrel-cage induction motor is started by reducing the line voltage to $V_S/2$ in order to reduce the starting current.

Find:

a) The number of poles (for minimum slip), the slip, and the frequency of the rotor currents.

b) The factor the starting torque and the starting current reduced.

Assumptions:
None.

Analysis:
a)

For minimum slip, the synchronous speed, $\frac{3600}{p/2}$, should be as close as possible to $1140\, rev/\min$, therefore,

$$n_S = 1200, \qquad p = 6\ poles$$

$$s = \frac{1200-1140}{1200} = 0.05$$

$$f_{rotor} = 3\,Hz$$

b)
If the line voltage is reduced to half, the starting current is reduced by a factor of 2. The developed torque is proportional to $|I_S|^2$. Therefore, the starting torque is reduced by a factor of 4.

Problem 17.57

Solution:

Known quantities:
A six-pole induction machine has a $50\,kW$ rating and is 85 percent efficient. If the supply is $220V$ at $60\,Hz$.

6 poles
60 Hz
50 kW
85% efficient
220 Volt
4% slip

Find:
The motor speed and torque at a slip $s = 0.04$·

Assumptions:
None.

Analysis:

a) $$n_s \quad \frac{120f}{p} \quad \frac{120 \quad 60}{6} \quad 1200\, rev/\min$$

@ slip of 4%

$$n \quad n_s\, 1 \quad s \qquad 1200 \text{ rev/ min } \ 1 \quad 0.04 \qquad 1152 \text{ rev/ min}$$

b)

$$P_{out} = P_{in} \times \text{efficiency} = (50kW)(0.85) = 42.5\text{ kW}$$

$$T_{out} = \frac{P_{out}}{\omega} = \frac{42500\text{ W}}{1152\text{ rev/min}} * \frac{1\text{ rev}}{2\pi\text{ rad}}\frac{60\text{ sec}}{\text{min}} = 352.3\text{ N - m}$$

Problem 17.58

Solution:

Known quantities: 6 poles

60 Hz
240 Volt rms
10% slip
Torque = 60 N-m

Find:

a) The speed and the slip of the induction machine if a load torque of 50 N-m opposes the motor.
b) The rms current when the induction machine is operating under the load conditions of part a.

Assumptions:

The speed torque curve is linear in the region of our interests.

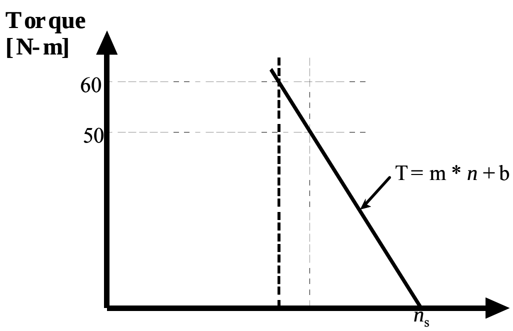
Analysis:

a)

$$n_s \quad \frac{120 f}{p} \quad \frac{120 \quad 60}{6} \quad 1200\, rev/\min$$

@ slip of 4%

$$n \quad n_s\ 1 \quad s \qquad 1200 \text{ rev/ min} \ \ 1 \quad 0.04 \qquad 1152 \text{ rev/ min}$$



torque

$$T \quad m\ n \quad b$$

$$m \quad \frac{60 \quad 0 \text{ N} \quad \text{m}}{1080 \quad 1200 \text{ rev/min}} \qquad 0.5\frac{\text{N} \quad \text{m}}{\text{rev/min}}$$

$$b \quad T \quad m\ n \qquad 60 \text{ N} \quad \text{m} \qquad 0.5\frac{\text{N} \quad \text{m}}{\text{rev/min}}\ 1080 \text{ rev/min} \qquad 600 \text{ N - m}$$

motor speed (@ 50 N-m)

$$n \quad \frac{T \quad b}{m} \quad \frac{\text{rev/min}}{.5 \text{ N} \quad \text{m}} \ \ 50 \text{ N - m} \qquad 600 \text{ N - m} \qquad 1000 \text{ rev/min}$$

n

slip (@ 50 N-m)

$$s \quad \frac{n_s \quad n}{n_s} \quad \frac{1200 \quad 1100}{1200} \qquad .0833 \qquad 8.33\%$$

b)

Output Power

$$P_{out} \quad T\omega \qquad 50 \text{ N - m} \ \ 1100 \ \frac{\text{rev}}{\text{min}}\frac{2\pi \text{ rad}}{\text{rev}}\frac{1\text{min}}{60\sec} \qquad 5760\ W$$

Input Power

$$P_{in} \quad \frac{P_{out}}{\text{efficiency}} \quad I_{rms} \ \ V_{rms}$$

Current

$$I_{rms} \quad \frac{P_{out}}{\text{efficiency}\ \ V_{rms}} \quad \frac{5760\ \text{W}}{0.92\ \ \ 240\ \text{volts}} \quad 26.1\ \text{amps}$$

Problem 17.59

Solution:

Known quantities:
A three-phase, $5\,hp$, $220V$, $60\,Hz$ induction motor. $V = 8V, I = 18\,A, P = 610W$.

Find:

a) The equivalent stator resistance per phase, R_S.

b) The equivalent rotor resistance per phase, R_R.

c) The equivalent blocked-rotor reactance per phase, X_R.

Assumptions:
None.

Analysis:
a)

$$R_S = \frac{1}{2}\frac{P_{BR}}{3I_{BR}^2} = 0.314\,\Omega$$

b)

$$R_R = 0.314\,\Omega$$

c)

$$Z_S = \frac{V_{BR}/\sqrt{3}}{I_{BR}} = \frac{48/\sqrt{3}}{18} = 1.54\,\Omega$$

$$X_R = \sqrt{Z_S^2 - R^2} = \sqrt{(1.54)^2 - (0.628)^2} = 1.4\,\Omega$$

Problem 17.60

Solution:

Known quantities:
The starting torque equation is:

$$T = \frac{m}{\omega_e} \cdot V_S^2 \cdot \frac{R_R}{(R_R + R_S)^2 + (X_R + X_S)^2}$$

Find:

a) The starting torque when it is started at $220V$.

b) The starting torque when it is started at $110V$.

Assumptions:
None.

Analysis:
a)

$$T = \frac{1}{\omega_S} \frac{q_1 V_1^2 (R_R/s)}{R^2 + X^2}; \quad s = 1$$

$$\therefore T = \frac{1}{377} \frac{3(127)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 17.1\, N \cdot m$$

b)

$$T = \frac{1}{377} \frac{3(63.5)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 4.28\, N \cdot m$$

Problem 17.61

Solution:

Known quantities:
A four-pole, three-phase induction motor drives a turbine load with torque-speed characteristic given by
$T_L = 20 + 0.006\varpi^2$
At a certain operating point, the machine has 4% slip and 87% efficiency.

Find:
Torque at the motor-turbine shaft
Total power delivered to the turbine
Total power consumed by the motor

Assumptions:
Motor run by 60-Hz power supply

Analysis:
Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120 f}{P} = \frac{60s/\min \times 60r/s}{4/2} = 1800 r/\min$$

$$\varpi_s = 1800 rev/\min \times \frac{2\pi rad/rev}{60 s/\min} = 188.5 rad/s$$

Rotor mechanical speed at 4% slip:

$$\varpi_m = (1-s)\varpi_s = (1-0.04)(188.5 rad/s) = 181.0 rad/s$$

Load torque at the shaft:

$$T_L = 20 + 0.006(181.0 rad/s)^2 = 216 N-m$$

Total power delivered to the turbine:

$$P = T_L \varpi_m = (216 N-m)(181.0 rad/s) = 39.1 kW$$

Total power consumed by the motor:

$$P_m = \frac{P}{\eta} = \frac{39.1 kw}{0.87} = 44.9 kW$$

Problem 17.62

Solution:

Known quantities:
A four-pole, three-phase induction motor rotates at 1700 r/min when the load is 100 N-m.
The motor is 88% efficient.

Find:
a) Slip
b) For a constant-power, 10-kW load, the operating speed of the machine
c) Total power consumed by the motor
d) Sketch the motor and load torque-speed curves on the same graph. Show numerical values.

Assumptions:
Motor run by 60-Hz power supply

Analysis:

a) Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120f}{P} = \frac{60s/\min \times 60r/s}{4/2} = 1800r/\min$$

Slip:

$$s = \frac{n_s - n}{n_s} = \frac{1800r/\min - 1700r/\min}{1800r/\min} = 0.056 = 5.6\%$$

b) Operating speed of machine for a constant-power load of 10-kW

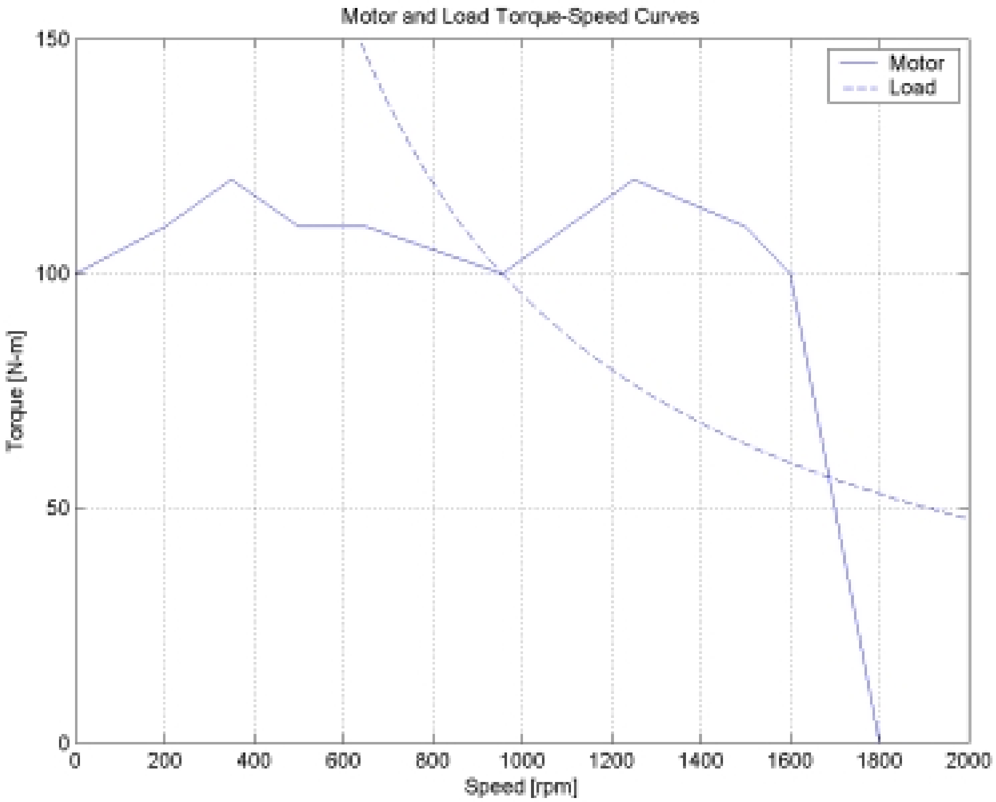
$$\varpi = \frac{P}{T_L} = \frac{10000W}{100N-m} = 100rad/\sec$$

$$n = \varpi \frac{60s/\min}{2\pi rad/rev} = 955r/\min$$

c) Total power consumed by the motor

$$P_m = \frac{P}{\eta} = \frac{10kW}{0.88} = 11.4kW$$

d) Sketch of motor and load torque-speed curves on the same graph, with the operating point at the first intersection:



Problem 17.63

Solution:

Known quantities:
A six-pole, three-phase motor.

Find:
The speed of the rotating field when the motor is connected to:
a) a $60\,Hz$ line.
b) a $50\,Hz$ line.

Assumptions:
None.

Analysis:
a)

For $60\,Hz$, $\omega_m = \frac{4\pi f}{P} = 125.7\,rad/\sec$, $n_m = 1200\,rev/\min$

b)

For $50\,Hz$, $\omega_m = 104.72\,rad/\sec$, $n_m = 1000\,rev/\min$

Problem 17.64

Solution:

Known quantities:
A six-pole, three-phase, $440 V, 60 Hz$ induction motor. The model impedances are:

$$R_S = 0.8\,\Omega \qquad X_S = 0.7\,\Omega$$
$$R_R = 0.3\,\Omega \qquad X_R = 0.7\,\Omega$$
$$X_m = 35\,\Omega$$

Find:
The input current and power factor of the motor for a speed of $1200\,rev/\min$.

Assumptions:
None.

Analysis:

$$V_S = \frac{440}{\sqrt{3}} = 254\angle 0^\circ\ V$$

For $n_m = n_S = 1200\,rev/\min$, $s = 0\,(no\,load)$.

$$Z_{in} = R_S + j(X_S + X_m) = 0.8 + j35.7$$
$$= 35.71\angle 88.7^\circ\ \Omega$$
$$I_S = 7.11\angle -88.7^\circ\ A$$

The power factor is:

$$\cos 88.7^\circ = 0.0224\,lagging$$
$$P_{in} = 3|I_S||V_S|\cos\theta = 121.4 W$$

Problem 17.65

Solution:

Known quantities:
A eight-pole, three-phase, $220 V, 60 Hz$ induction motor. The model impedances are:

$$R_S = 0.78\,\Omega \qquad X_S = 0.56\,\Omega$$
$$R_R = 0.28\,\Omega \qquad X_R = 0.84\,\Omega$$
$$X_m = 32\,\Omega$$

Find:
The input current and power factor of the motor for $s = 0.02$.

Assumptions:
None.

Analysis:
For 8 poles,

$$n_S = \frac{3600}{4} = 900\, rev/\min$$
$$\omega_S = 94.25\, rad/\sec$$
$$\omega_m = (1-s)\omega_S = 92.4\, rad/\sec$$

By using the equivalent circuit, we have:

$$Z_{in} = 0.78 + j0.56 + \frac{j32(\frac{0.28}{0.02} + j0.84)}{14 + j32.84}$$
$$= 12.03 + j6.17 = 13.52\angle 27.15^\circ\ \Omega$$
$$V_S = 127\angle 0^\circ\ V$$
$$I_S = 9.39\angle -27.15^\circ\ A$$
$$pf = \cos(-27.15^\circ) = 0.8898\, lagging$$

Problem 17.66

Solution:

Known quantities:
The nameplate is as given in Example 17.2.

Find:
The rated torque, rated volt amperes, and maximum continuous output power for this motor.

Assumptions:
None.

Analysis:
The speed is:

$$n_m = 3565\, rev/\min$$
$$\omega_m = \frac{2\pi \times 3565}{60} = 373.3\, rad/\sec$$

The rated $volt \cdot amperes$ is:

$$\sqrt{3} \times (230V) \times (106\,A) = 42.23\, kVA$$
$$or\ \sqrt{3} \times (460V) \times (53A) = 42.23\, kVA$$

The maximum continuous output power is:

$$P_O = 40 \times 746 = 29840\,W$$

The rated output torque is:

$$T = \frac{P_O}{\omega_m} = 79.93\, N \cdot m$$

Problem 17.67

Solution:

Known quantities:
At rated voltage and frequency, the 3-phase induction machine has a starting torque of 140 percent and a maximum torque of 210 percent of full-load torque.

Find:
a) The slip at full load.
b) The slip at maximum torque.
c) The rotor current at starting as a percent of a full-load rotor current.

Assumptions:
Neglect stator resistance and rotational losses. Assume constant rotor resistance.

Analysis:
a)

$$T_R = \frac{KV^2(R_2/s_R)}{(R_2/s_R)^2 + X^2}$$

$$T_{ST} = 1.4T_R \qquad s_{ST} = 1.0$$

$$T_{MT} = 2.1T_R \qquad s_{MT} = \frac{R_2}{X}$$

The above leads to 3 equations in 3 unknowns:

(1) $4.2XR_2 = 1.4R_2^2 + 1.4X^2$

(2) $\frac{R_2^2}{s_R} + s_R X^2 = 1.4R_2^2 + 1.4X^2$

(3) $4.2\frac{XR_2}{s_R} = (\frac{R_2}{s_R})^2 + X^2$

Solving the equations, we have:

$$\frac{R_2}{X} = 0.382$$

$$s_R = 0.097$$

b)

$$s_{MT} = \frac{R_T}{X} = 0.382$$

c)

$$I_R = \frac{KV}{4.06}, \qquad I_{ST} = \frac{KV}{1.07}$$

$$\frac{I_{ST}}{I_R}\times 100 = 379\%$$
