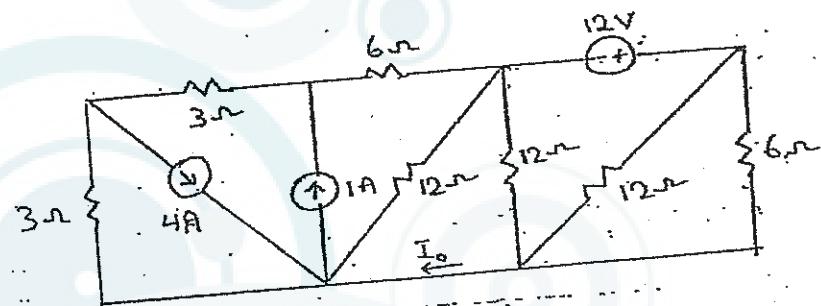


Problem (1)  
Using Source Transformation find the current  $I_o$  in the circuit  
Of Fig (1).



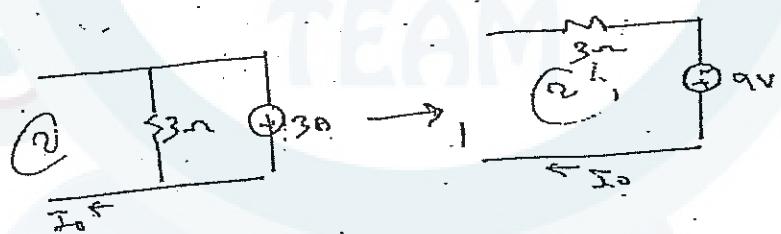
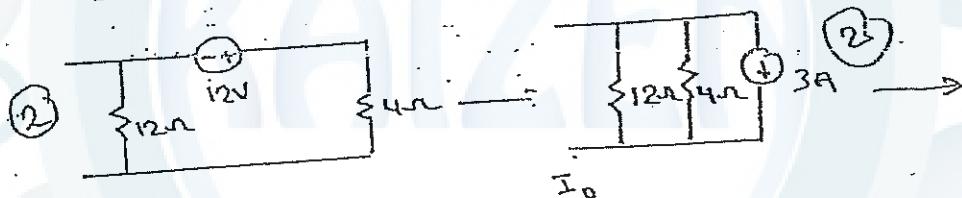
Solution:

Fig (1)

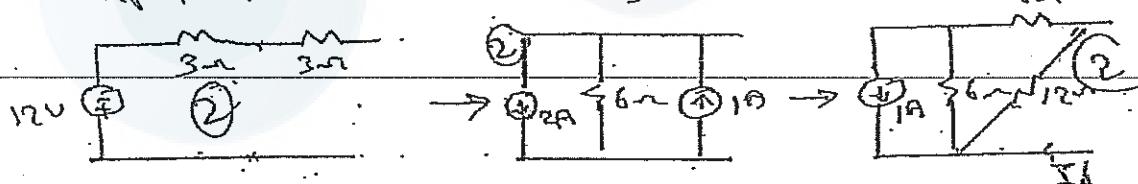
Starting from R.H.S. we have :

$$6/12 = \frac{6 \times 12}{18} = 4\Omega$$

By  $\approx$  source transformation we get:



Then we perform source transformation from L.H.S.  
up to the branch containing  $I_o$  as follows:



Continue next page ..

Problem (2)  
In the circuit of Fig (2), find the following:

- $R_L$  which will receive maximum power.
- The value of the maximum power transferred to  $R_L$ .
- Norton equivalent circuit between terminals of resistor  $R_L$ .

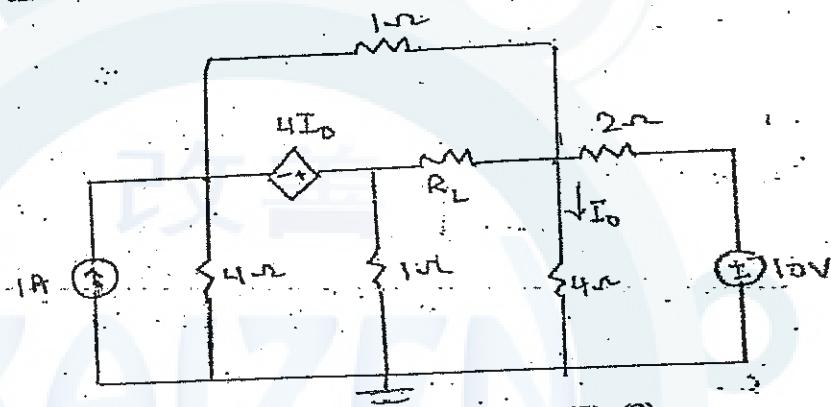
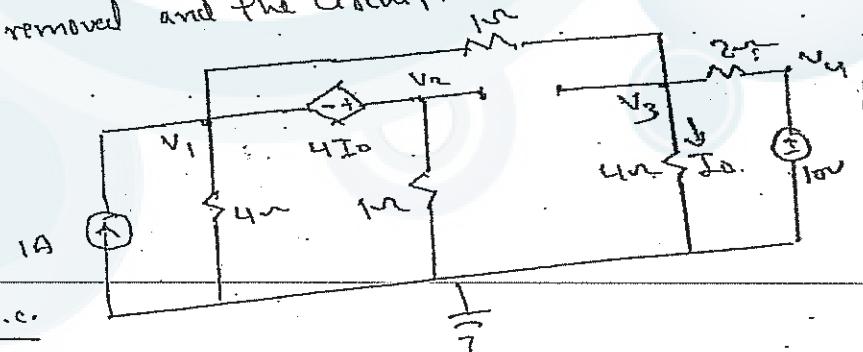


Fig (2)

Solution:

(a)  $R_{L_{max}} = R_{TH}$  to receive max. power  
since the circuit contains independent and dependent sources  
 $R_{TH} = \frac{V_{o.c.}}{I_{s.c.}}$  (2)

is removed and the circuit becomes as follows:



Calculation of V<sub>o.c.</sub>:

$$(1) V_{o.c.} = V_{TH} = V_2 - V_3$$

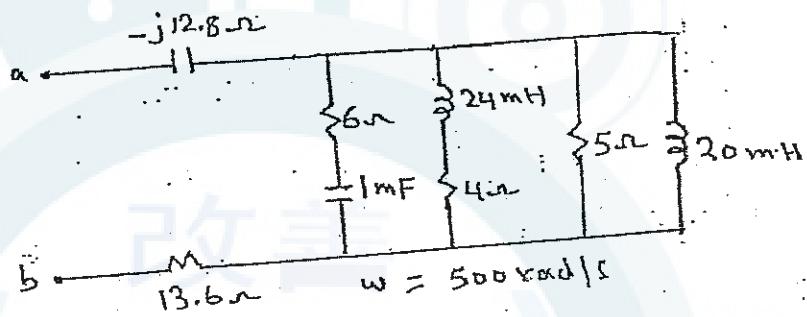
Applying nodal analysis we have:

at node 3 we have:

$$\therefore V_3 \left( \frac{1}{1} + \frac{1}{4} + \frac{1}{2} \right) - \frac{V_1}{1} - \frac{10}{2} \quad \left. \right\} (3)$$

$$\text{or } -4V_1 + 7V_3 = 20 \quad \dots (4)$$

Problem (3)  
In the circuit shown in Fig (3), find the admittance  $Y_{ab}$  in millimhos and in both Rectangular and Polar forms where  $\omega = 500 \text{ rad/sec}$



Solution:

Fig (3)

$$20 \text{ mH} \rightarrow j \times 500 \times 20 \times 10^{-3} = j12 \text{ mho} \quad (1)$$

$$24 \text{ mH} \rightarrow j \times 500 \times 24 \times 10^{-3} = j12.8 \text{ mho} \quad (1)$$

$$1 \text{ mF} \rightarrow -j \frac{1}{500 \times 1 \times 10^{-3}} = -j2 \text{ mho} \quad (1)$$

We find the admittance for the four branches in parallel in the R.H.S. of Fig.(3) and call it  $Y_1$ :

$$Y_1 = \frac{1}{6-j2} + \frac{1}{4+j12} + \frac{1}{5} + \frac{1}{j10} \quad (3)$$

$$= \frac{6+j2}{40} + \frac{4-j12}{160} + \frac{32}{160} - j0.1 \quad (2)$$

$$= \frac{60-j20}{160} = \frac{3-j1}{8} \quad (2)$$

$$Z_1 = \frac{1}{Y_1} = \frac{8}{3-j1} = 2+j+0.8 \quad (2)$$

$$\therefore Z_{ab} = 13.6 - j12.8 + 2+j+0.8 = 16-j12 \quad (2)$$

$$\therefore Y_{ab} = 20 \angle -36.87^\circ \quad (1)$$

$$\therefore Y_{ab} = 50 \angle 36.87^\circ \text{ mho} = 40+j30 \text{ mho} \quad (1)$$

**Problem (4)**

In the circuit shown in Fig (4) and using Mesh Analysis find:

(a) The currents  $I_a, I_b, I_c$ , and  $I_d$  in rectangular form and in time domain.

(b) Phase shift between each current in part (a) above and the relevant voltage across each element these currents are passing through.

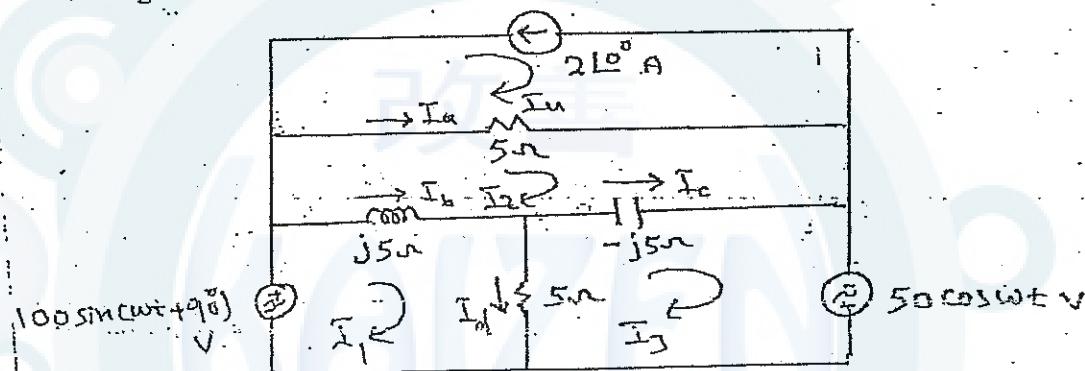


Fig (4)

Solution:

(a) In phasor domain voltage sources become:

$$① 100 \angle 0^\circ V \& 50 \angle 0^\circ V \quad ①$$

mesh #1:

$$100 = I_1(5+j5) - I_2j5 - I_3 \times 5 \quad \dots \quad ②$$

mesh #2:

$$\{ 0 = -I_1j5 + I_2(5+j5-j5) - I_3 \times 5 + I_4 j5 \\ \text{But } I_4 = -210^\circ A \}$$

∴ above eqn becomes:

$$-10 = -j5I_1 + 5I_2 + j5I_3 \quad \dots \quad ③$$

mesh #3:

$$50 \angle 0^\circ = -5I_1 + j5I_2 + (5-j5)I_3 \quad \dots \quad ④$$

Solving above 3 equations we get:

Q1. Write one equation only to determine  $V_a$  in the circuit shown in Fig.1.

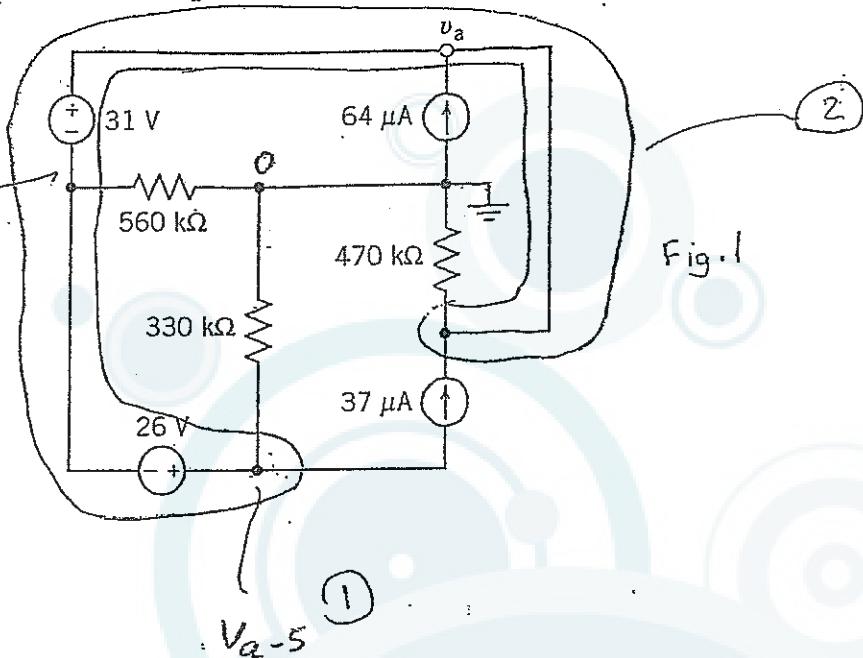


Fig. 1

$$\boxed{-64 \times 10^{-6} + \frac{V_a}{470 \times 10^3} + \frac{V_q - 31}{560 \times 10^3} + \frac{V_q - 5}{330} = 0}$$

$$-64 + 2.12766 V_a + 1.785 V_a - 55.357 + 3.03 V_a - 15.15 = 0$$

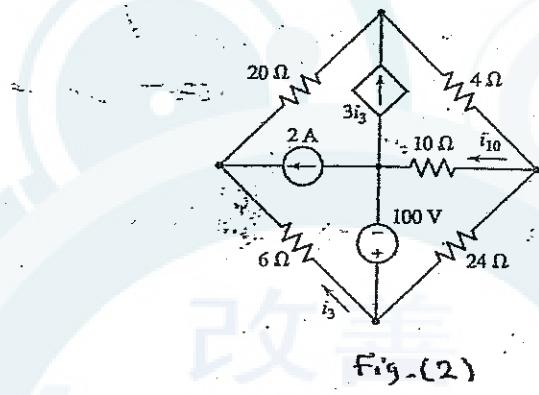
$$6.9436 V_a = 134.508$$

$$V_a = 19.37 \text{ V}$$

①

**Problem 2:**

Use mesh analysis method to find the two currents  $i_{10}$  and  $i_3$  in the circuit of Fig. (2).



**Solution**

\* On supermesh:

$$\begin{aligned} 0 &= 20(i_1) + 4(i_4) + 10(i_2 - i_4) - 10i_4 + 6i_3 \\ 20i_1 + 4i_4 + 10i_2 - 10i_4 + 6i_3 &= 0 \\ 100 &= 20i_1 + 4i_2 + 6i_3 - 10i_4 \end{aligned}$$

\* on the other mesh:

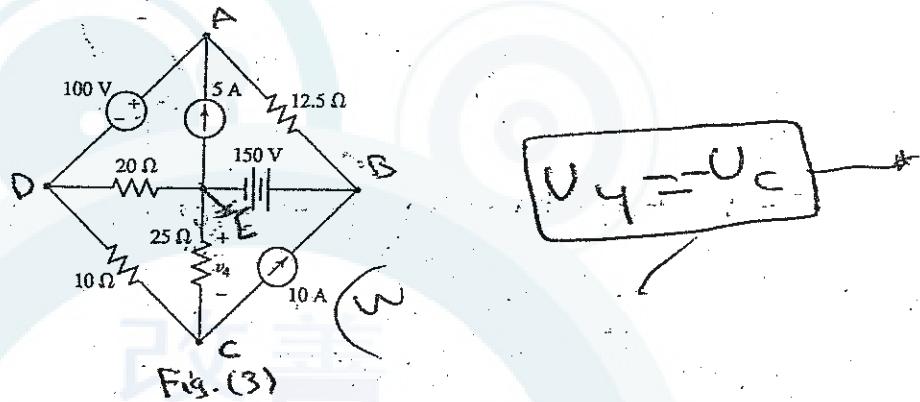
$$\begin{aligned} 100 + 10(i_4 - i_2) + 24(i_4) &= 0 \\ -100 &= 10i_4 - 10i_2 + 24i_4 \quad (1) \\ -100 &= 34i_4 - 10i_2 \quad (2) \end{aligned}$$

\* relations between currents:

$$3i_3 = i_2 - i_1 \quad (3) \quad i_2 = i_1 - i_3 \quad (4)$$

**Problem 3:**

Use nodal analysis method to find  $v_4$  in the circuit shown in Fig. (3).



**Solution**

$$* U_E = 0 \text{ V}, \quad U_B = 150 \text{ V} \quad (2)$$

\* Super node at B and A:

$$U_A - U_D = 100 \quad (3)$$

$$\frac{U_A - U_B}{2.5} + \frac{U_D - U_C}{10} + \frac{U_D}{20} = 5 \quad (4)$$

$$.08U_A - .08U_B + .1U_D - .1U_C + .05U_D = 5$$

$$.08U_A + .15U_D - .1U_C = 5 + 12$$

$$.08U_A + .15U_D - .1U_C = 17 \quad (5)$$

at node C:

$$\frac{U_C - U_D}{10} + \frac{U_C}{25} = -10 \quad (6)$$

$$-10U_C - 10U_D + .04U_C = -10 \Rightarrow .16U_C - .1U_D = 10 \quad (7)$$

**Problem (1)**

Use Nodal Analysis method to find  $i_{20}$  and  $v_{20}$  in the circuit of Fig (1).  
(Crossed wires not marked by a solid dot are not in physical contact).

$$V_E = 0 \quad V_B = V_A - 400$$

$$V_A - V_B = 400 \text{ V}$$

$$V_B - V_C = 4V_{20} \quad V_d$$

$$\Rightarrow V_d = 200 \text{ V} \quad \Rightarrow V_B - V_C = 4(V_C - 200)$$

$$V_B - V_C = 4V_C - 800$$

at A & B, Super Node

$$V_B - 5V_C = -800 \quad \text{Eqn 1}$$

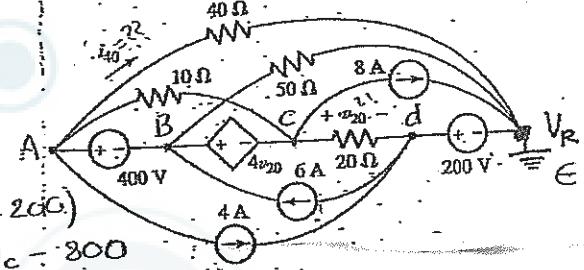


Fig (1)

$$+6 - 4 = V_A \left( \frac{1}{10} + \frac{1}{40} \right) - V_C \cdot \frac{1}{10} \quad \Rightarrow 2 = \frac{1}{8} V_A + \frac{1}{50} V_B - \frac{1}{10} V_C \quad \text{Eqn 2}$$

at B + E  $\Rightarrow$  Super Node

$$-8 + 6 = V_B \left( \frac{1}{50} \right) + V_C \left( \frac{1}{20} + \frac{1}{10} \right) - V_d \frac{1}{20} - V_A \frac{1}{10}$$

$$\Rightarrow -2 = -\frac{1}{10} V_A + \frac{1}{50} V_B + \frac{3}{20} V_C - \frac{200}{20} \cdot 10$$

$$8 = -\frac{1}{10} V_A + \frac{1}{50} V_B + \frac{3}{20} V_C$$

~~$$+4 - 6 = V_d \left( \frac{1}{20} \right) - V_C \left( \frac{1}{20} \right)$$~~
~~$$-2 = \frac{1}{20} (200) - \frac{1}{20} V_C$$~~

$V_A$

$$i_{20} = \frac{V_A}{40}$$

Problem (2)

In the circuit of Fig (2), use Mesh Analysis method to determine the power associated with the (2.2V) voltage source and state whether this source is delivering or absorbing power.

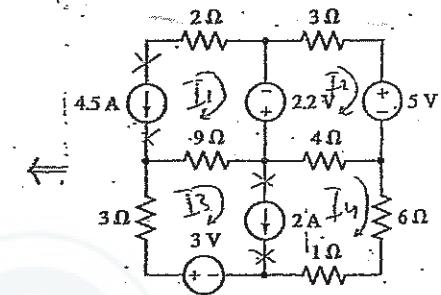
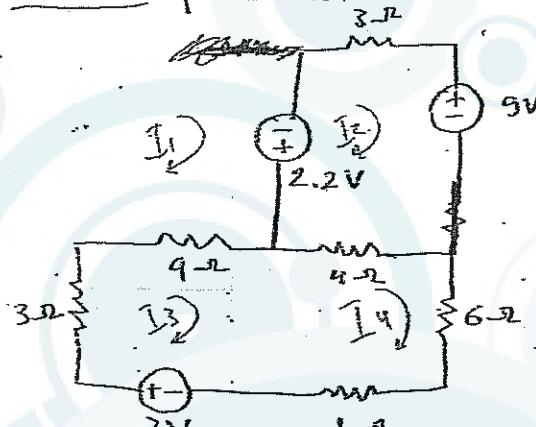


Fig (2)

$$I_1 = -4.5 \text{ A} \quad \text{---} \textcircled{*}$$

$$I_3 - I_4 = 2 \text{ A} \quad 8 \quad \text{---} \textcircled{1}$$

$$12I_3 - 4.5 = 11I_4 \quad \text{---} \textcircled{2}$$

$$3 = I_3(3+9) - 9I_1 + I_4(4+6+1) - 4I_2/6 \quad \text{---} \textcircled{2}$$

$$= -4.5 \quad = -1.5 \quad \text{---} \textcircled{2}$$

$$-5 - 2.2 = I_2(3+4) - I_4(4) \quad \text{---} \textcircled{3}$$

$$I_3 - I_4 = 2 \quad \text{---} \textcircled{1}$$

$$-4I_2 + 12I_3 + 4I_4 = -1.5 \quad \text{---} \textcircled{2}$$

$$7I_2 + 0 - 4I_4 = -7.2 \quad \text{---} \textcircled{3}$$

$$\begin{bmatrix} 0 & 1 & -1 \\ -4 & 12 & 11 \\ 7 & 0 & -4 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2 & 1 & -1 \\ 1.5 & 12 & 11 \\ -7.2 & 0 & 4 \end{vmatrix}}{14}$$

$$P = (I_2 - I_1) 2.2$$

$$+ \quad \text{Abs}$$

$$I_3 = \frac{\begin{vmatrix} 0 & 1 & -1 \\ -4 & 12 & 11 \\ -7.2 & 0 & 4 \end{vmatrix}}{14}$$

$$I_4 = \frac{\begin{vmatrix} 0 & 1 & 2 \\ -4 & 12 & -15 \\ 7 & 0 & -7.5 \end{vmatrix}}{14}$$

**Problem 2:**

The variable resistor in the circuit in Fig. (2) is adjusted for maximum power transfer to  $R_o$ .

What is the value of this max. power?

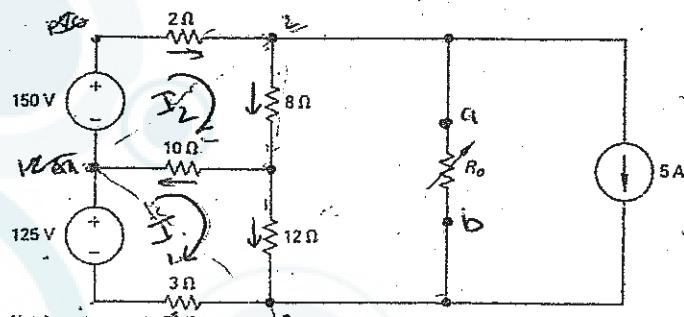


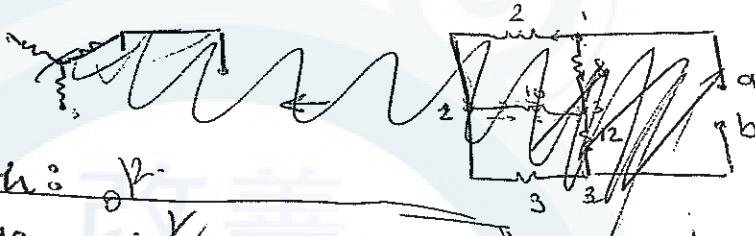
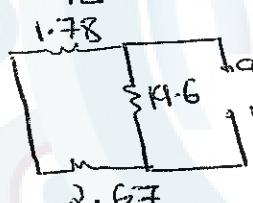
Fig. (2)

**Solution** $R_{Th}$ to calculate  $R_{Th}$ :

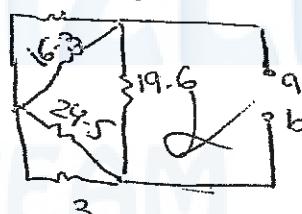
$$R_1 = \frac{10 \times 12 + 8 \times 12 + 8 \times 10}{10} = 19.6 \text{ } \cancel{\Omega}$$

$$R_2 = \frac{19.6}{8} = 24.5 \text{ } \cancel{\Omega}$$

$$R_3 = \frac{19.6}{12} = 16.3 \text{ } \cancel{\Omega}$$



$$\begin{aligned} R_1 &= 2 \times 8 = 0.8 \Omega \\ R_2 &= 2 \times 10 = 1 \Omega \\ R_3 &= 10 \times 8 = 4 \Omega \end{aligned}$$



$$R_{Th} = \frac{(1.78 + 2.67) + 19.6}{(1.78 + 2.67) + 19.6} = 3.63 \Omega$$

$$150 - 2I_2 - 8I_2 - 10(I_2 - I_1) = 0$$

$$I_1(10) + I_2(-2 - 8 - 10) = -150 \quad \text{--- (1)}$$

$$125 + 10(I_2 - I_1) - 12I_1 - 3I_1 = 0$$

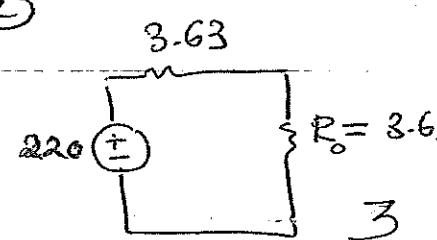
$$I_1(-10 - 12 - 3) + I_2(10) = -125 \quad \text{--- (2)}$$

$$I_1 = 10 \text{ A}, I_2 = 12.5 \text{ A}$$

$$V_{Th} = -3(10) + 125 + 150 - 2(12.5) = 220 \text{ V}$$

$$\max P_{max} = \left[ \frac{220}{(2 \times 3.63)} \right]^2 \times 3.63 = 252.97 \text{ W}$$

P 12



3

**Problem (3)**

In the circuit shown in Fig (3), use Superposition Principle to find the current  $i_A$  and the power developed in the dependent voltage source.

$v_D$  due to  $30V_{(1)}$  only

Mech  $-53i_2 + 53i_1 = 3I_1 - 3I_2 - 5I_3$   
 $i_2 - i_3 = -45I_1 + 50I_2 - 5I_3$   
 $-53i_A = I_1(3+5) - 3I_2 - 5I_3$

$$30 = I_3(20+2+5) - 5I_1 - 20I_2$$

$$0 = I_2(7+3)+20 - 20I_3 - 3I_1$$

$$\Rightarrow 0 = -45I_1 + 50I_2 - 5I_3$$

$$30 = -5I_1 - 20I_2 + 25I_3$$

$$0 = -3I_1 + 20I_2 - 20I_3$$

$$I_2 = \frac{\begin{bmatrix} -45 & 50 & -5 \\ -5 & -20 & 25 \\ -3 & 20 & -20 \end{bmatrix} \begin{bmatrix} 0 \\ 30 \\ 0 \end{bmatrix}}{\det \begin{bmatrix} -45 & 50 & -5 \\ -5 & -20 & 25 \\ -3 & 20 & -20 \end{bmatrix}}$$

$$I_3 = \frac{\begin{bmatrix} 45 & 50 & 0 \\ -5 & -20 & 30 \\ -3 & 20 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix}}{\det \begin{bmatrix} 45 & 50 & 0 \\ -5 & -20 & 30 \\ -3 & 20 & 0 \end{bmatrix}}$$

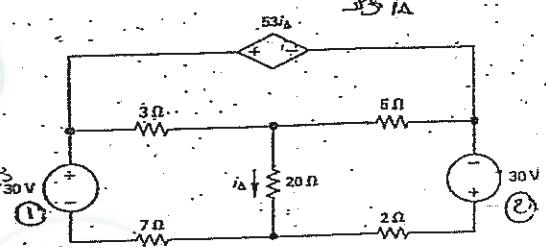
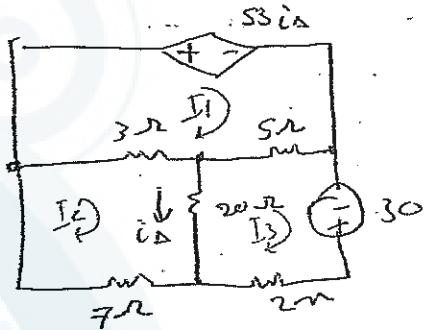


Fig (3)



$$I_A = I_2 - I_3$$

$$I_1 = \frac{-180}{13}$$

$$I_2 = \frac{-177}{13}$$

$$I_3 = \frac{-150}{13}$$

$$-\frac{177}{13} + \frac{150}{13} = -\frac{27}{13}$$

" due to  $30V_{(2)}$  only

$$0 = -45 + 50I_2 - 5I_3$$

$$53i_A$$

Mech

$$-53(I_2 - I_3) = I_1(2) - 3I_2 - 5I_3 \quad \textcircled{1}$$

$$30 = I_2(20) - 20I_3 - 3I_1 - \textcircled{2}$$

$$0 = I_3(27) - 5I_1 - 20I_2 \quad \textcircled{3}$$

$$I_2 = \frac{\begin{bmatrix} -45 & 50 & -5 \\ -3 & 20 & -20 \\ 0 & 0 & 27 \end{bmatrix} \begin{bmatrix} 0 \\ 30 \\ 0 \end{bmatrix}}{\det \begin{bmatrix} -45 & 50 & -5 \\ -3 & 20 & -20 \\ 0 & 0 & 27 \end{bmatrix}}$$

$$I_3 = \frac{\begin{bmatrix} 45 & 50 & 0 \\ -3 & 20 & 0 \\ 0 & 0 & 27 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix}}{\det \begin{bmatrix} 45 & 50 & 0 \\ -3 & 20 & 0 \\ 0 & 0 & 27 \end{bmatrix}}$$

$$I_2 - I_3 = -\frac{10}{13}$$

$$I_1 = -\frac{250}{13}$$

$$I_2 = -\frac{244}{13}$$

$$I_3 = -\frac{230}{13}$$

