



ELECTRICAL

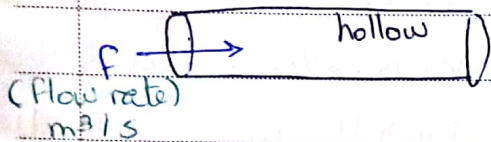
Dr. Mohammad Khasawneh



NO. 23/1/2019

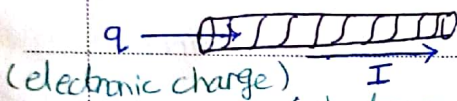
Introduction to electrical engineering

* f is analogous to I
Pipe



• $\uparrow$ diameter $\uparrow$ Flow if pressure is increased

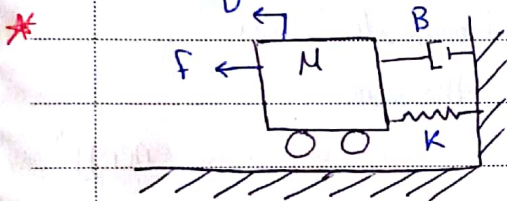
electrical wire (not hollow) • conductivity $\left\{ \begin{array}{l} \text{thermal} \\ \text{electrical} \end{array} \right.$ due to electrons



• copper and gold and iron are conductive

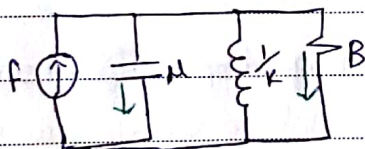
$$I = \frac{dq}{dt} = \frac{\text{Coulomb}}{\text{sec}} = \text{Ampere}$$

in future by fibers not metals



shock absorber of the car (chassis)
second order system

Analogous to



Force representing the current flowing through M

• We conclude that engineering types are opened on each other, it can be converted to circuits.

* e - charge of single electron (Coulomb) = 1.6×10^{-19} C (Coulomb)

$$1 \text{ e.v} = 1.6 \times 10^{-19} \text{ J} \quad 1 \text{ J} = 6.24 \text{ e.v}$$

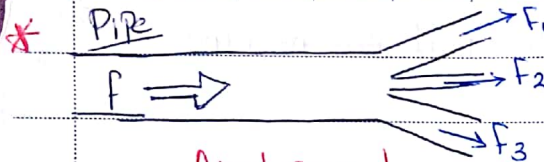
$$\text{الطاقة الكهربائية} = 200 \times 10^6 \text{ e.v}$$

$$1 \text{ GW electric Power} = 3 \text{ GW thermal Power}$$

intuitively

NO.

↑

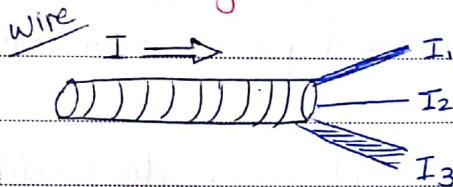


↓ diameter ↓ resistance ↓ F

$F = F_1 + F_2 + F_3$ "mass conservation law"

You can't make $F_2 = F_3$

Analogous to



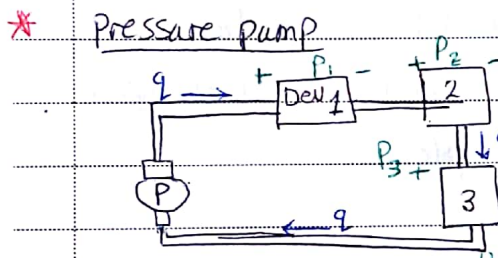
cross sectional area is called gauges

$I = I_1 + I_2 + I_3$ "Kirchoff's current law (KCL)"

↑ diameter ↑ I

switch fuser to prevent high current

Same Laws but different names !



same flow rate through all devices

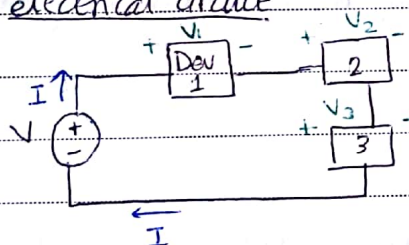
pressure drop is not the same

$P = P_1 + P_2 + P_3$ "law of conservation of energy"

Pressure per area → force

Info. on the pump: max (pressure, flow rate, power, current)

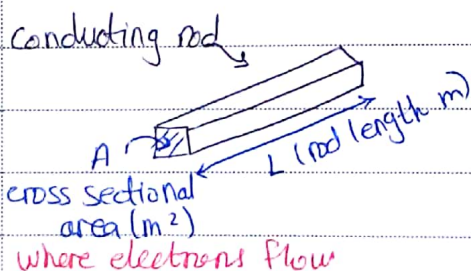
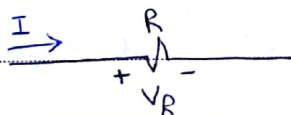
electrical circuit



same current, voltage drop is different

$V = V_1 + V_2 + V_3$ "Kirchoff's voltage law (KVL)"

* electrical resistance, usually made from carbon, or piezoelectric material
Unit of resistance is Ohms (Ω)



$R = \frac{\rho L}{A}$

ρ : material resistivity ($\Omega \cdot m$)
[R] = Ω

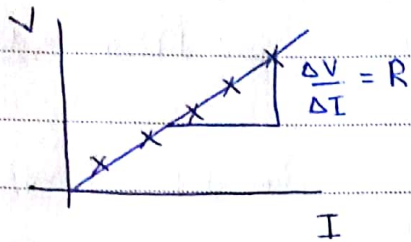
resistivity is material property

$\sigma = \frac{1}{\rho}$ σ : conductivity

From the law $\uparrow A \downarrow R$, does not apply in optical fiber

ask FIVE APPLE

NO.



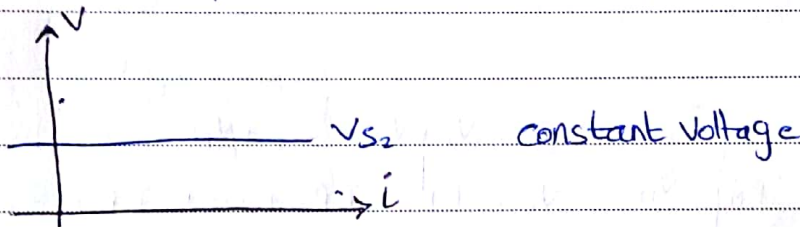
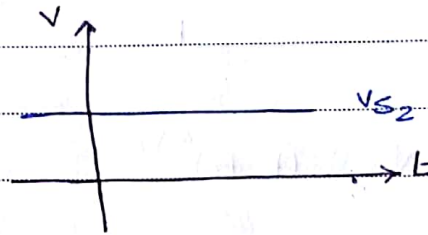
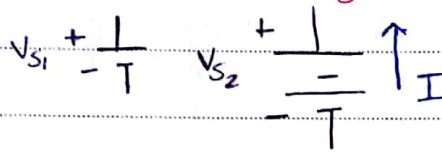
Ohm's

$$V_R = IR \quad \text{Ohm's law}$$

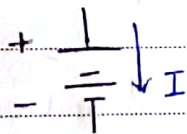
For microproducts, it does not apply on nanoscale

28/11/2019 Monday

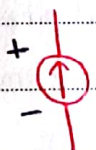
⇒ Electrical battery



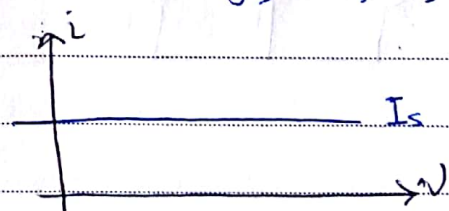
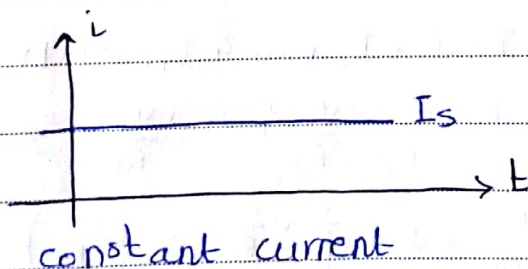
• But, I can make the current in the opposite direction



but here it will consume energy, not a source
when recharging the battery Rare

⇒ Current Source  I_S current direction the same as arrow direction

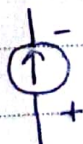
الفولتية قابلة للتغير



التيار الكهربائي ليس required voltage

• Here, I can change the current direction

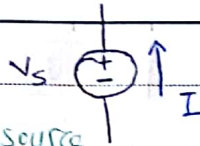
current direction in the opposite direction of the arrow



ask & receive

NO.

⇒ Voltage Source

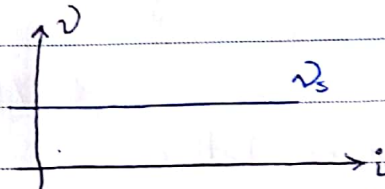
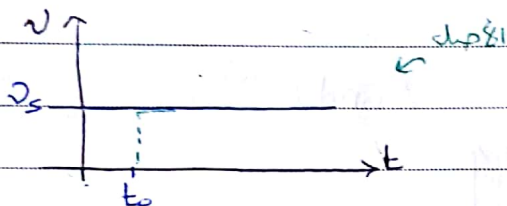


Conventionally

التيار الكهربائي قابل للتغير

battery is voltage source but with a difference

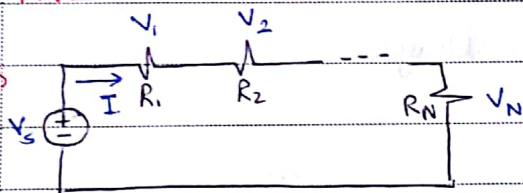
لكنها مختلفة بنفس الطريقة



Step function $V(t) = V_s U_s(t - t_0)$ ^{خطوة}

⇒ KVL

Series Resistances



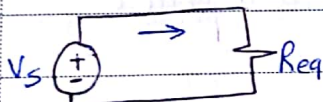
$$V_s = V_1 + V_2 + \dots + V_N$$

$$V_s = IR_1 + IR_2 + \dots + IR_N$$

$$IR_{eq} = I(R_1 + R_2 + \dots + R_N)$$

↓ equivalent to

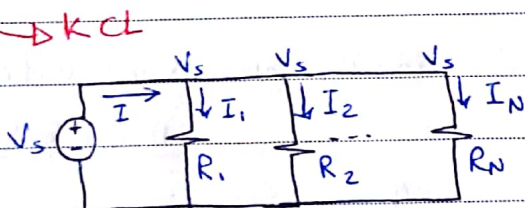
$$R_{eq} = R_1 + R_2 + \dots + R_N \quad \text{In Series}$$



Node definition :

in the electrical sense, it is a point where current merge together or forke out.

Parallel Resistances



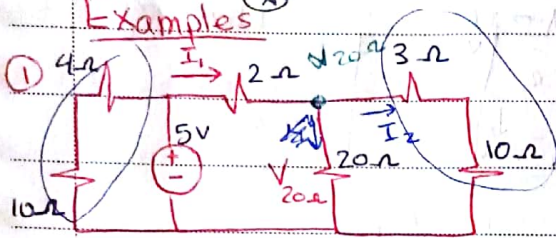
$$I = I_1 + I_2 + \dots + I_N \quad (\text{KCL})$$

$$I = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \dots + \frac{V_s}{R_N}$$

$$\frac{V_s}{R_{eq}} = V_s \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \quad \text{In Parallel}$$

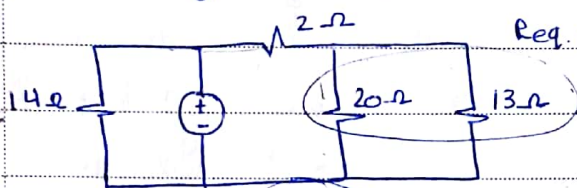
Examples



1. Req. as seen by voltage source?

2. I_1 , $V_{20\Omega}$, I_2 , $V_{4\Omega}$?

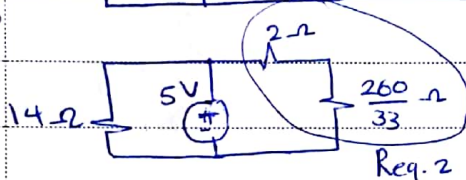
Voltage source is in the middle



Req.1

$$\frac{1}{Req.1} = \frac{1}{20} + \frac{1}{13}$$

$$Req.1 = \frac{260}{33} \Omega$$



Req.2

$$I_1 = \frac{V}{R_1} = \frac{5}{\frac{260}{33}} = \frac{165}{326} A$$

$$\frac{1}{Req.2} = \frac{1}{2} + \frac{1}{\frac{260}{33}}$$

$$Req.2 = \frac{326}{33} \Omega$$

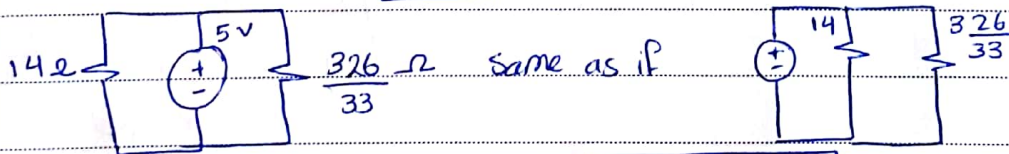
 $I_2 =$

$$V_{20\Omega} = \frac{260}{33} \left(\frac{165}{326} \right) = \frac{650}{163} V = V_{20\Omega}$$

Look

$$I_2 = \frac{V_1}{R_1} = \frac{650}{163} \times \frac{1}{13} = \frac{50}{163} A$$

$$V_{4\Omega} = \frac{5}{14} \times 4 = \frac{20}{14} V$$



same as if

$$\frac{1}{Req.} = \frac{1}{14} + \frac{33}{326} = \frac{197}{1141}$$

$$Req. = \frac{1141}{197} \Omega$$

$$I_{5V} = \frac{5}{14} + \frac{165}{326} = \frac{985}{1141} A$$

NO.

②

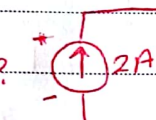
$R_{eq}?$

$V_{2A}?$

$V_{2A}?$

$V_{2\Omega}?$

$I_{20\Omega}?$



$I_{20\Omega}$

$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{3}{50}$$

$$R_{eq} = 2 + \frac{44}{3}$$

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{55}$$

$$R_{eq} = \frac{44}{3} \Omega$$

$$R_{eq} = \frac{50}{13}$$

$$= \frac{50}{3}$$

$$V_{2A} = 2 \times \frac{50}{13} = \frac{100}{13} \text{ V}$$

$$I_E = \frac{100/13}{50/3} = \frac{6}{13} \text{ A}$$

$$V_{2\Omega} = 2 I_E = \frac{2 \times 6}{13} = \frac{12}{13} \text{ V}$$

$$V_a = \frac{100}{13} - \frac{12}{13} = \frac{88}{13} \text{ V}$$

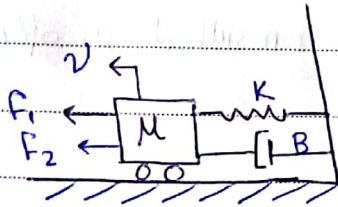
$$I_{20\Omega} = \frac{\frac{88}{13}}{20} = \frac{22}{65} \text{ A}$$



30/11/2019

Wednesday

⇒ Superposition Principle :-



You can add $F_1 + F_2$ together, or you can see the effect of each force and its velocity then add both velocities which is the superposition principle.

Linear systems

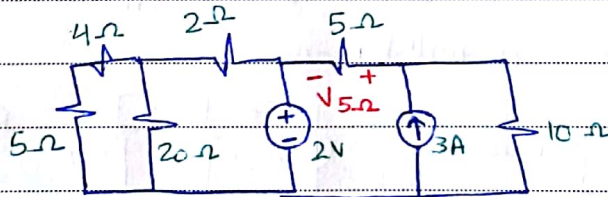
Linear function... recall from math, $F(\cdot)$ is said to be linear (iff) $\iff$

$$F(\alpha x + \beta y) = \alpha F(x) + \beta F(y) \text{ For } \alpha \text{ \& } \beta \text{ being scalars.}$$

For linearity (1) $F(x+y) = F(x) + F(y)$ ← additive

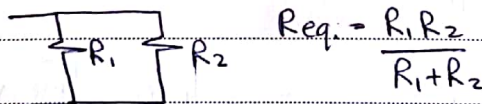
(2) $F(\alpha x) = \alpha F(x)$ ← homogeneity property

Consider



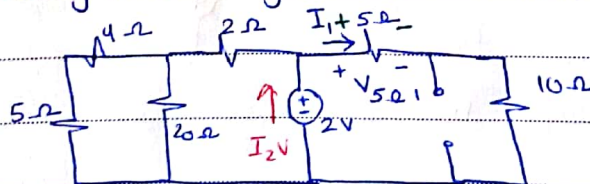
Req as seen current source voltage source

Very basic bricks :

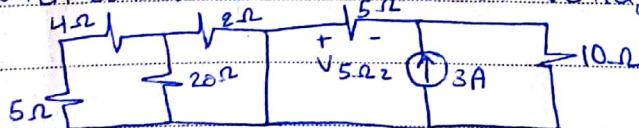


current source $\rightarrow$ voltage source
(superposition) Answer

(1) dealing with voltage source without current source



(2) with current source without voltage source



current prefer to flow in the plain wire

→ Terminologies

• Open circuit

is caused in a branch of large circuit as a result of an open path along current flow.

• Short circuit

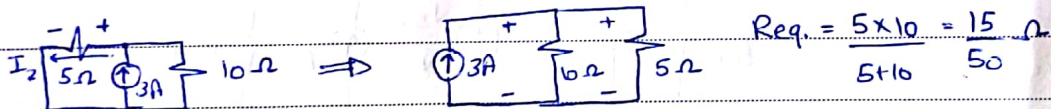
is caused along a cross two points along a branch when we connect the two points with a plain wire.

For circuit ① $I_1 = \frac{25V}{5\Omega + 10\Omega} = \frac{2}{15} A$

$$V_{5\Omega_1} = -I_1(5\Omega) = -\frac{2}{15} \times 5 = \boxed{-\frac{2}{3} V}$$

-ve because it is the opposite direction of potential.

For circuit ② it will become

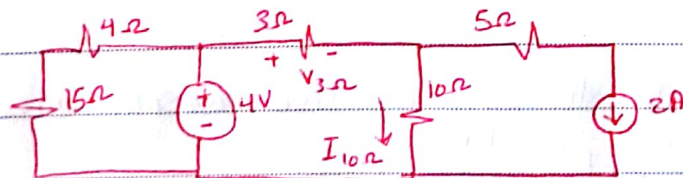


$$V = 3 \times \frac{50}{15} = 10V = V_{5\Omega_2} \quad \checkmark \text{ the same direction}$$

Now, $V_{5\Omega} = V_{5\Omega_1} + V_{5\Omega_2} = -\frac{2}{3} + 10 = \boxed{\frac{28}{3} V}$

This is called superposition

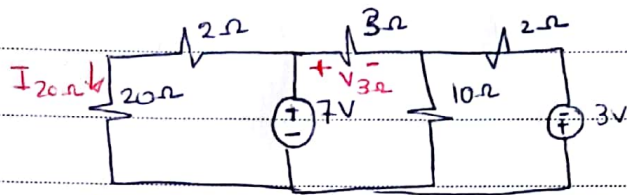
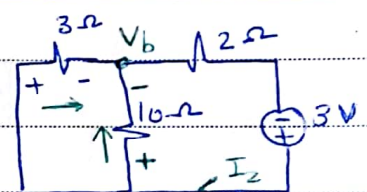
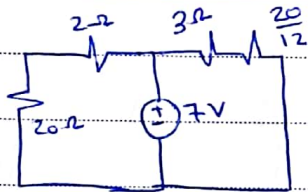
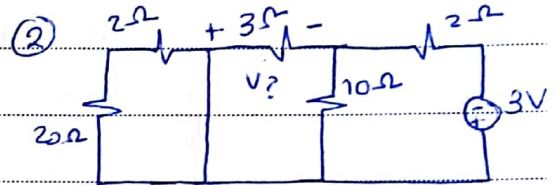
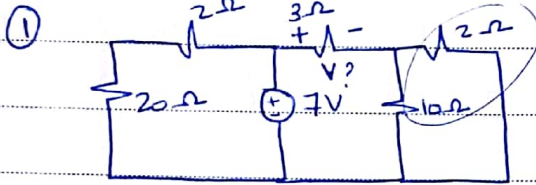
H.W
Assignment



Find $V_{3\Omega}$ & $I_{10\Omega}$

2, in the consider example, Find I_{2V} .

• Until now, if there are more than 1 source, you must use the superposition principle.

• ExampleFind, $I_{20\Omega}$ & $V_{3\Omega}$
H.W.K

$$V_{3\Omega} = \left(\frac{7}{3 + \frac{20}{12}} \right) (3\Omega) = \boxed{\frac{63}{4}} \frac{9}{2}$$

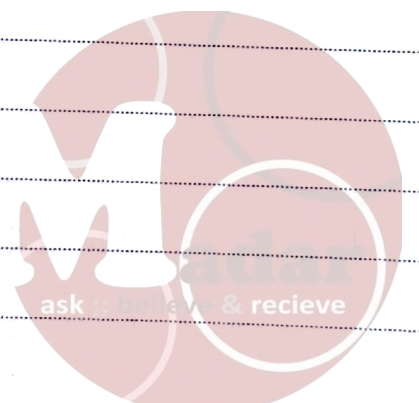
$$R_{eq} = \frac{3 \times 10}{3 + 10} = \frac{30}{13} \Omega \quad R_{eq, all} = 2 + \frac{30}{13} = \frac{56}{13} \Omega$$

$$I_2 = \frac{3}{R_{eq}} = \frac{3 \times 13}{56} = \frac{39}{56} A$$

$$V_{3\Omega} = V_{3\Omega_1} + V_{3\Omega_2} = \frac{63}{41} + \frac{45}{28} = \boxed{\frac{3609}{1148}} \quad \boxed{\frac{171}{28} V}$$

$$V_b = 3 - 2I_2 = \boxed{\frac{45}{28} V = V_{3\Omega_2}}$$

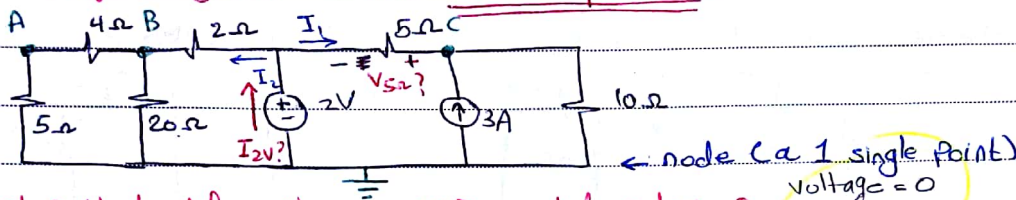
Equivalent for the whole circuit does not exist until studying here, even Req.



NO.

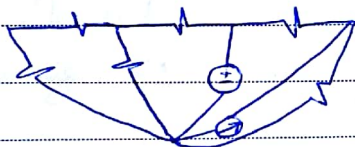
4/11/2019 Monday

⇒ Answering Assignments By Nodal Equations



→ steps that define the process for nodal analysis :-

- ① pick a common/reference node that is (preferably) connected to most/all energy sources in the circuit. $V_{\text{node}} = 0$



- ② identify all possible unknown nodes.

explained on the above circuit

- ③ at each identified node from step ②, write KCL equations but in terms of the unknown voltages.

assume them positive nodes (positive voltages)

• writing KCL at node A $\frac{V_a - V_b}{4} + \frac{V_a}{5} = 0$

#DoF = 0

$$\frac{9}{20} V_a - \frac{1}{4} V_b = 0 \rightarrow ①$$

• writing KCL at node B $\frac{V_b - V_a}{4} + \frac{V_b}{20} + \frac{V_b - 2}{2} = 0$

$$\frac{1}{4} + \frac{1}{20} + \frac{1}{2} = \frac{5+1+10}{20} = \frac{16}{20}$$

$$-\frac{1}{4} V_a + \frac{16}{20} V_b = 1 \rightarrow ②$$

• writing KCL at node C $\frac{V_c - 2}{5} + \frac{V_c}{10} - 3 = 0$

$$\frac{3}{10} V_c = \frac{17}{5} \rightarrow ③$$

$$V_c = \frac{17 \times 10}{5 \times 3} = \frac{34}{3} \text{ V}$$

FIVE APPLE

ask : believe & recieve

Taking (1) & (2)

$$\frac{9}{20} V_a - \frac{1}{4} V_b = 0 \rightarrow (1)$$

$$\boxed{V_a = \frac{100}{119} V} \quad \boxed{V_b = \frac{180}{119} V}$$

$$-\frac{1}{4} V_a + \frac{16}{20} V_b = 1 \rightarrow (2)$$

$$\rightarrow V_a = \frac{100}{119} V \quad V_b = \frac{180}{119} V \quad V_c = \frac{34}{3} V$$

$$\text{Now } V_{5\Omega} = V_c - 2 = \frac{34}{3} - 2 = \boxed{\frac{28}{3} V}$$

$$I_{2V} = I_1 + I_2$$

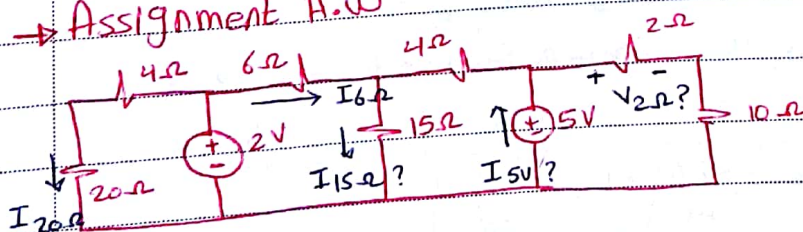
$$\rightarrow I_1 = \frac{-V_{5\Omega}}{5} = \frac{-28}{3 \times 5} = \frac{-28}{15} A$$

$$\rightarrow I_2 = \frac{2 - V_b}{2} = \frac{2 - 180/119}{2} = \frac{29}{119} A$$

$$I_{2V} = \frac{29}{119} - \frac{28}{15} = \boxed{-1.622969 A}$$

- This means that the voltage source dissipate energy, it is a load instead of being a source.

→ Assignment H.W



Find the unknowns using nodal analysis AND compare your results of when you would use superposition.

Loop Analysis of electrical networks

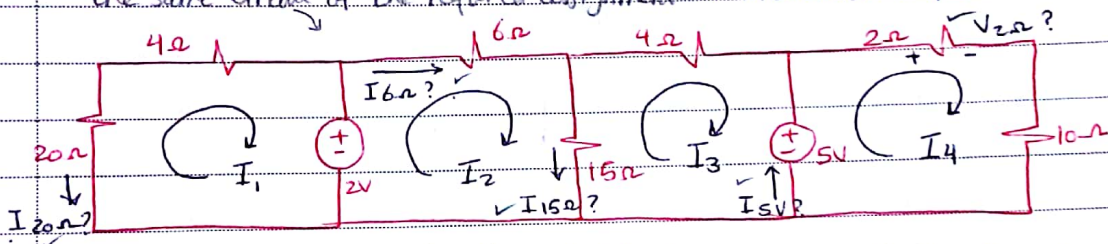
(mesh)

* The loop can be

* ~~Answering the previous assignment~~

basic loops = 4, the other loops are called meshes

the same circuit of the required assignment it intersects loops.



- You can take any direction in the loop. مع ضروري كلهم نفس الاتجاه
لكن يجب أن نلزم بالاتجاهات المفروضة

Procedure:

we take each of the identified loops & write KVL around each loop.

• writing KVL around loop 1 $20I_1 + 4I_1 + 2 = 0 \rightarrow ①$

$$24I_1 = -2 \rightarrow \boxed{I_1 = -1/12 \text{ A}}$$

• writing KVL around loop 2 $-2 + 6I_2 + 15(I_2 - I_3) = 0 \rightarrow ②$

$$21I_2 - 15I_3 = 2$$

• writing KVL around loop 3 $15(I_3 - I_2) + 4I_3 + 5 = 0 \rightarrow ③$

$$19I_3 - 15I_2 = -5$$

• writing KVL around loop 4 $-5 + 2I_4 + 10I_4 = 0 \rightarrow ④$

$$12I_4 = 5 \rightarrow \boxed{I_4 = 5/12 \text{ A}}$$

taking ② & ③

$$\boxed{I_2 = -\frac{37}{174} \text{ A}}$$

$$\boxed{I_3 = -\frac{25}{68} \text{ A}}$$

Now $I_{5V} = I_4 - I_3 = \boxed{\frac{295}{348} \text{ A}}$

$$I_{6\Omega} = I_2 = \boxed{-\frac{37}{174} \text{ A}}$$

$$I_{15\Omega} = I_2 - I_3 = \boxed{\frac{19}{87} \text{ A}}$$

$$I_{20\Omega} = -I_1 = \boxed{\frac{1}{12} \text{ A}}$$

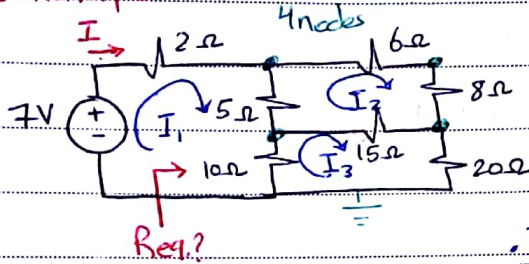
$$V_{2\Omega} = 2I_4 = \boxed{\frac{5}{6} \text{ V}}$$

Wednesday

Supermesh analysis & Y-Δ Transformation

6/2/2019

Example



it is difficult to find Req. by the original way.

if # node equation < # loop equations then choose the less # equations (here nodal)

⇒ writing loop (KVL) for loop 1 & 2 & 3

loop 1 $-7 + 2I_1 + 5(I_1 - I_2) + 10(I_1 - I_3) = 0$

$17I_1 - 5I_2 - 10I_3 = 7$ ①

loop 2 $5(I_2 - I_1) + 6I_2 + 8I_2 + 15(I_2 - I_3) = 0$

$-5I_1 + 34I_2 - 15I_3 = 0$ ②

loop 3 $10(I_3 - I_1) + 15(I_3 - I_2) + 20I_3 = 0$

$-10I_1 - 15I_2 + 45I_3 = 0$ ③

⇒ $I_1 = \frac{1827}{3232} = 0.56528A$ $I_2 = \frac{525}{3232} = 0.162438A$ $I_3 = \frac{581}{3232} = 0.17976A$

Now, solve for the unknowns

$I = I_1 = \frac{1827}{3232} A$

$Req. = \frac{7V}{I} = \frac{7 \times 3232}{1827} = \frac{22624}{1827} = 12.38547 \Omega$

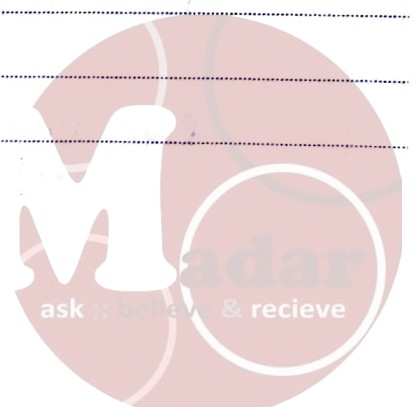
checked

It can be ~~solved~~ by another way

$17\left(\frac{1827}{3232}\right) - 5\left(\frac{525}{3232}\right) - 10\left(\frac{581}{3232}\right) = 7$ # eq. #1 is checked

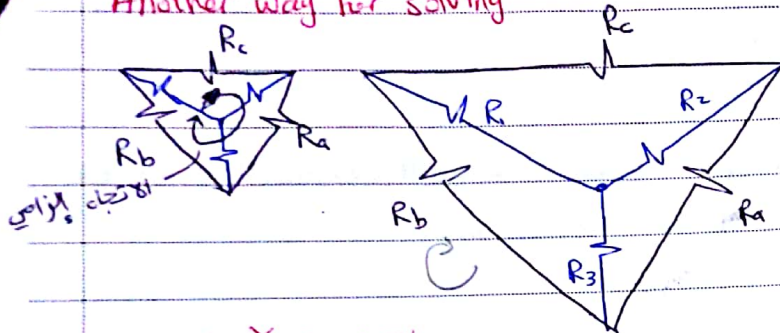
so the answer is true

You can use another equations to check (eq. #2 + eq. #3)



NO.

Another way for solving



Δ -connected resistors

Y-connected resistors

التحويل من المثلث إلى النجمة

Δ -Y conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

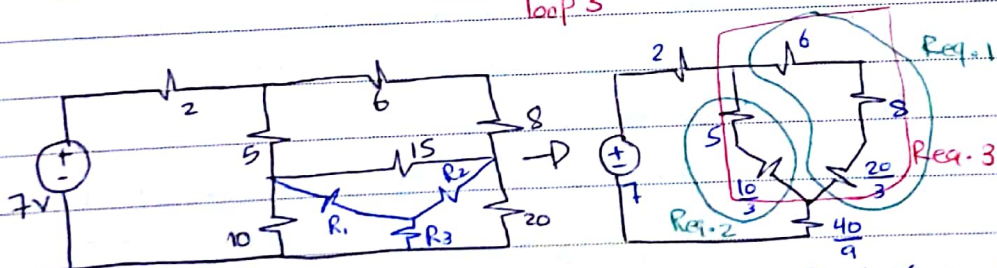
Y- Δ conversion

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

- Solving the previous example Imp & You can replace only loop 2 or only loop 3



$$R_1 = \frac{(15)(10)}{15+10+20} = \frac{10}{3} \Omega$$

$$R_2 = \frac{(15)(20)}{15+10+20} = \frac{20}{3} \Omega$$

$$R_3 = \frac{(10)(20)}{15+10+20} = \frac{40}{9} \Omega$$

$$Req.1 = \frac{20}{3} + 6 + 8 = \frac{62}{3} \Omega$$

$$Req.2 = 5 + \frac{10}{3} = \frac{25}{3} \Omega$$

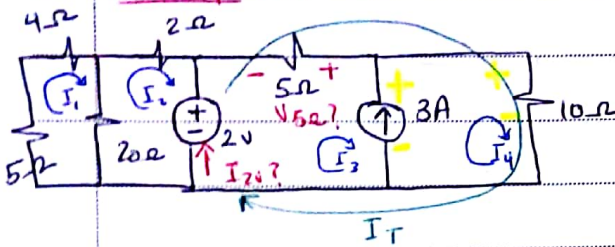
$$Req.3 = Req.1 \parallel Req.2 = \frac{\frac{62}{3} \times \frac{25}{3}}{\frac{62}{3} + \frac{25}{3}} = \frac{1550}{261} \Omega$$

$$Req. = \frac{1550}{261} + \frac{40}{9} + 2 = \frac{3232}{261} \Omega$$

The same answer #

FIVE APPLE

ask & believe & recieve

Example 8

• if we use loop 3 & 4 there will be problems
we don't know the voltage of the current source
To solve this problems, take meshes instead of
these two loops, but don't name them.

→ 2 loops are crossed with a current source → "super mesh"

loop 1 $5I_1 + 4I_1 + 20(I_1 - I_2) = 0$ $29I_1 - 20I_2 = 0$ (1)

loop 2 $20(I_2 - I_1) + 2I_2 + 2 = 0$ $-20I_1 + 22I_2 = -2$ (2)

loop in green (supermesh) $-2 + 5I_3 + 10I_4 = 0$ $5I_3 + 10I_4 = 2$ (3)

I need also an additional equation to solve the 4 unknowns

$I_4 - I_3 = 3$ (4)

⇒ Rewrite equations

$29I_1 - 20I_2 + 0I_3 + 0I_4 = 0$

$-20I_1 + 22I_2 + 0I_3 + 0I_4 = -2$

$0I_1 + 0I_2 + 5I_3 + 10I_4 = 2$

$0I_1 + 0I_2 + -1I_3 + I_4 = 3$

$I_1 = \frac{-20}{119} A$ $I_2 = \frac{-29}{119} A$

$I_3 = \frac{-28}{15} A$ $I_4 = \frac{17}{15} A$

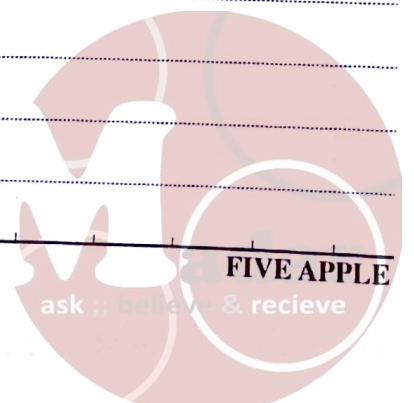
source current into 3A 10Ω 182, 384 182

Now, solve for the unknowns

• $I_{2V} = I_3 - I_2 = \frac{-28}{15} + \frac{29}{119} = -1.622969 A$

• $V_{5\Omega} = -5I_3 = -5 \times \frac{-28}{15} = \frac{28}{3} V$

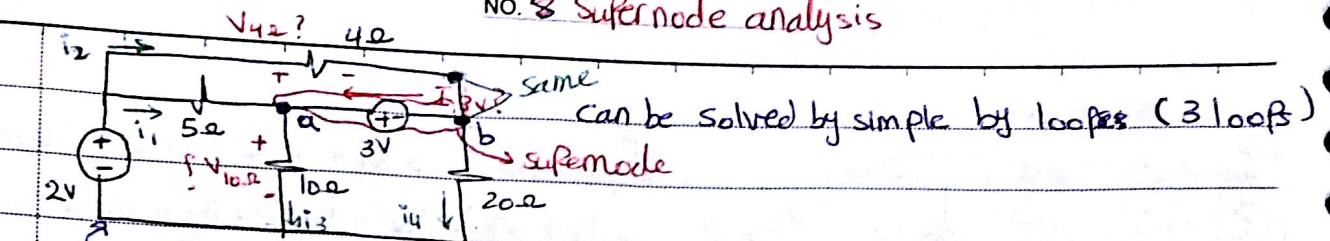
• $V_{3A} = 10 \times 3 - 30V$ $I_4 \times 10$



Monday

11/2/2019

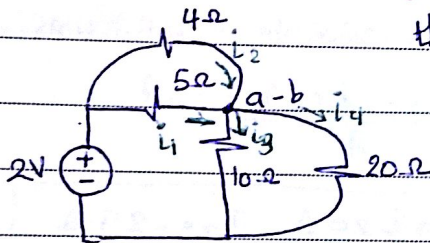
NO. 8 Supernode analysis



if we want to solve it by nodal equations, but I have 2 nodes: a and b, but there is a problem; I don't know the current which pass through the voltage source.

How Assignment: solve it by loops.

→ By supernode analysis & since the voltage source is not connected to the ground. they are floating.



∴ writing KCL @ node a-b.

$$\rightarrow \frac{2-V_a}{5} + \frac{2-V_b}{4} = \frac{V_a}{10} + \frac{V_b}{20} \quad \frac{-3V_a}{10} - \frac{3V_b}{10} = -\frac{9}{10} \quad (1)$$

$$\rightarrow V_a - V_b = 3 \quad (2)$$

$$V_a = 3V \quad V_b = 0V$$

∴ Now, solve for the unknowns

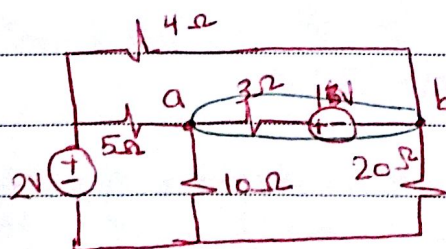
$$I_{3V} = \frac{V_a - V_b}{3}$$

$$I_{3V} = -i_1 + i_3 = \frac{3-2}{5} + \frac{3}{10} = \frac{1}{2} A$$

$$V_{4\Omega} = 2 - V_b = 2V$$

$$V_{10\Omega} = V_a = 3V$$

How Assignment: by supernode and loops



all supernode (resistance + voltage source)

$$V_a - V_b = 18V - 3I_{3\Omega}$$

FIVE APPLE

NO Gaussian elimination

→ To solve system of linear equations

* Example

$$17I_1 - 5I_2 - 10I_3 = 7$$

$$-5I_1 + 34I_2 - 15I_3 = 0$$

$$-10I_1 - 15I_2 + 45I_3 = 0$$

$$\begin{bmatrix} 17 & -5 & -10 \\ -5 & 34 & -15 \\ -10 & -15 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}$$

vector of right hand side coefficients

square matrix vector of unknowns

→ Consider the system of linear equations

* Example

$$2x - 3y + 7z = -1$$

$$\frac{3}{2}x + 2y - z = 2$$

$$-5x + y + 11z = 3$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3/2 & 2 & -1 \\ -5 & 1 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

each equation is plane actually

2 planes intersect into line (if they are not parallel)

intersect with the 3rd plane → point (unless line in plane)

* Gaussian elimination steps:-

same steps in calculator and matlab

① Form an augmented matrix

$$\begin{bmatrix} 2 & -3 & 7 & -1 \\ 3/2 & 2 & -1 & 2 \\ -5 & 1 & 11 & 3 \end{bmatrix}$$

1st pivot element

② Pick your 1st pivotal element out of the matrix Pivot: $\frac{2}{1}$

$$R_2^* = R_2 - \frac{3}{4}R_1 \rightarrow \begin{bmatrix} 2 & -3 & 7 & -1 \\ 0 & \frac{17}{4} & \frac{35}{4} & \frac{11}{4} \\ 0 & -\frac{13}{2} & \frac{57}{2} & \frac{1}{2} \end{bmatrix}$$

$$R_3^* = R_3 + 5R_1 \rightarrow \begin{bmatrix} 2 & -3 & 7 & -1 \\ 0 & \frac{17}{4} & \frac{35}{4} & \frac{11}{4} \\ 0 & -\frac{13}{2} & \frac{57}{2} & \frac{1}{2} \end{bmatrix}$$

opposite sign

These equations are called elementary law operations

③ Pick a 2nd Pivotal element (must not be on the same column or row of Pivot #1)

$$\begin{bmatrix} 2 & -3 & 7 & -1 \\ 0 & \frac{17}{4} & -\frac{25}{4} & \frac{11}{4} \\ 0 & \frac{-13}{2} & \frac{57}{2} & \frac{1}{2} \end{bmatrix} \xrightarrow{R_3^* = R_3 + \frac{26}{17} R_2} \begin{bmatrix} 2 & -3 & 7 & -1 \\ 0 & \frac{17}{4} & -\frac{25}{4} & \frac{11}{4} \\ 0 & 0 & \frac{322}{17} & \frac{80}{17} \end{bmatrix}$$

• Gaussian elimination :

1. Forward elimination (the previous steps)

$$\text{Row \# 3} \quad \frac{322}{17} z = \frac{80}{17} \quad \boxed{z = \frac{80}{322}}$$

$$\text{Row \# 2} \quad \frac{17}{4} y = \frac{25}{4} \times \frac{80}{322} + \frac{11}{4} = \frac{2771}{644} \quad \boxed{y = \frac{163}{161}}$$

$$\text{Row \# 3} \quad 2x - 3 \times \frac{163}{161} + 7 \times \frac{80}{322} = -1 \quad \boxed{x = \frac{24}{161}}$$

2. (the previous steps) Back substitution

→ To check, substitute in any equation.

$$\text{eqn. \# 2} \quad \frac{2 \times 24}{3 \times 161} + \frac{2 \times 163}{161} + \frac{-1 \times 80}{322} = 2 \quad \#$$

• Gaussian elimination can be applied on any matrix !

→ Tricks

- if pivot #1 is zero, switch with the biggest number but multiply the determinate by zero.
- if condition number is very large, you can multiply, but it will not be the same.

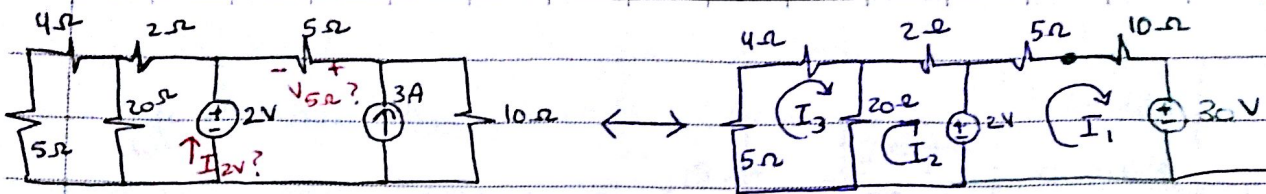
Calculator may solve in wrong way.



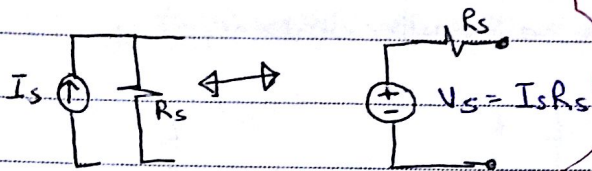
Wednesday

13/2/2019

NO. Source Transformation



→ Source Transformation:-



- it can be solved by loops as simple, instead of supermesh or nodes

• see example on slides.

You can also convert to current source after calculations

• loop 1 $-2 + 15I_1 + 30 = 0$ $15I_1 = -28$ $I_1 = \frac{-28}{15} A$ ①

• loop 2 $20(I_2 - I_3) + 2I_2 + 2 = 0$ $22I_2 - 20I_3 = -2$ ②

• loop 3 $5I_3 + 4I_3 + 20(I_3 - I_2) = 0$ $-20I_2 + 29I_3 = 0$ ③

$$I_2 = \frac{-29}{119} A$$

$$I_3 = \frac{-20}{119} A$$

OR by Gaussian elimination

$$\begin{bmatrix} 22 & -20 & -2 \\ -20 & 29 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 22 & -20 & -2 \\ 0 & \frac{119}{11} & \frac{-20}{11} \end{bmatrix}$$

$$I_3 = \frac{-20}{11} \times \frac{11}{119} = \frac{-20}{119} A$$

$$I_2 = \left(-2 + 20 \times \frac{-20}{119} \right) \div 22 = \frac{-29}{119} A$$

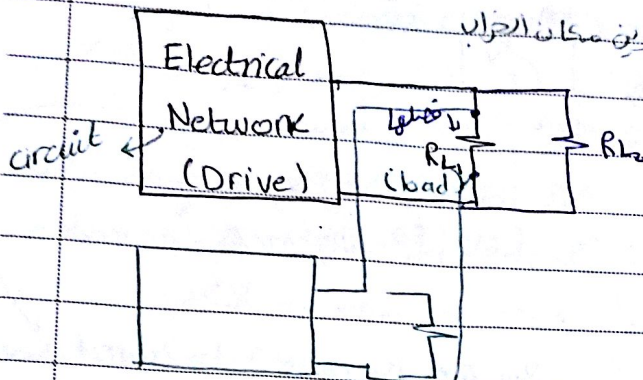
→ Now, substitute for the unknowns

• $V_{5\Omega} = -I_1 \cdot 5 = \frac{28}{3} V$

• $I_{2V} = I_1 - I_2 = -1.622969 A$

• $I_{5V} = \frac{28}{3} \times \frac{1}{5} = \frac{28}{15} A$

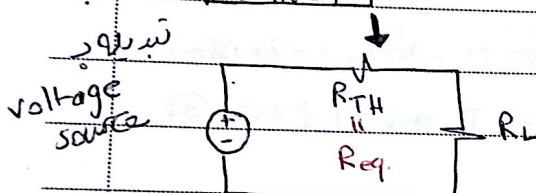
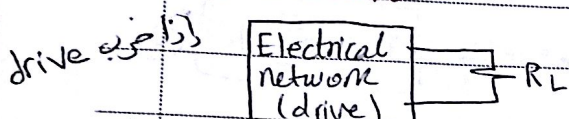
NO. Thevenin Etet equivalent circuit



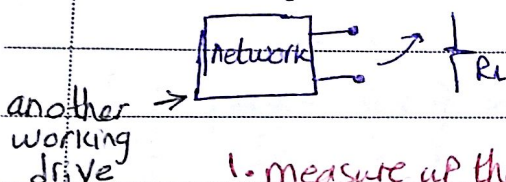
إذا ضرب خط الإنتاج مثلاً، بدي أي فرد من مكان الجواب
إذا لم نستطع R_L يعني drive ضايع

OR
إذا اشغل R_L يعني drive ضايع
add another drive

→ Thevenin



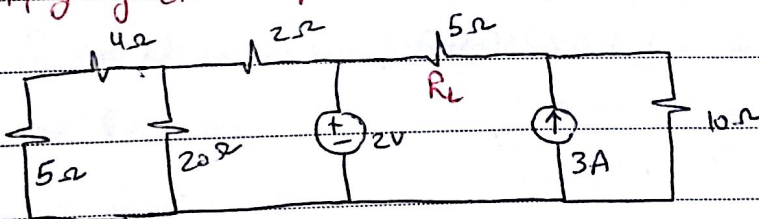
removing R_L



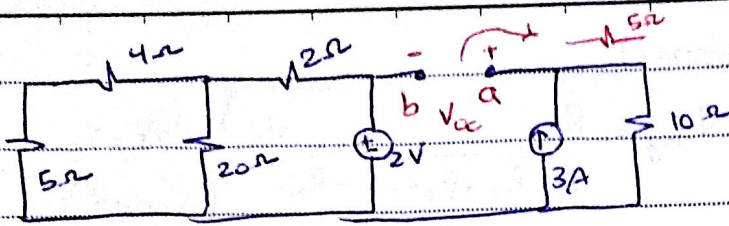
1. measure up the ckt voltage, V_{cc} , while the drive is on.

2. measure the value of R_{eq} looking backward into the drive network
this is done while the drive is turned off.

• Applying on the previous example



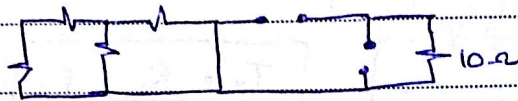
NO.



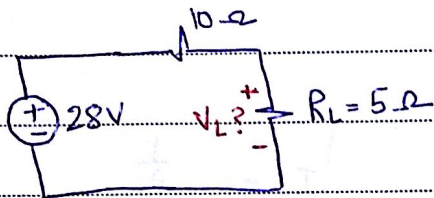
$V_a = 3 \times 10 = 30V$ $V_b = 2V$ $V_{OC} = V_a - V_b = 30 - 2 = \boxed{28V}$
 $\downarrow$
 $= V_{TH}$

→ Req. → when sources are all

$Req. = R_{TH} = R_{10\Omega}$



→ Our circuit will be



$V_L = \left(\frac{28}{10+5} \right) 5 = \boxed{\frac{28V}{3}}$

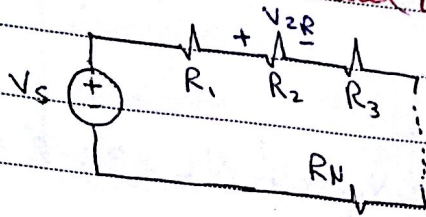
$I_{5\Omega} = I_L = 28 / (10+5) = \boxed{28/15A}$

The volt is experiencing the same voltage as it was in load.



NO. Some Tricks (Shortcuts)

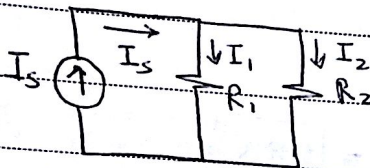
* Voltage division rule (in series)



$$V_{R_j} = \frac{R_j}{R_1 + R_2 + R_3 + \dots + R_N} V_s$$

($j = 1, \dots, N$)
For all j

* Only For 2 resistances

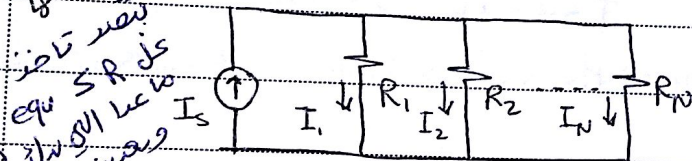


$$\frac{R_1 R_2}{R_1 + R_2} I_s = R_1$$

$$I_1 = \frac{R_2}{R_1 + R_2} I_s$$

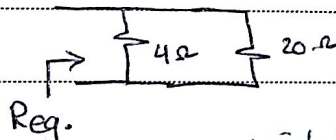
$$I_2 = \frac{R_1}{R_1 + R_2} I_s$$

For more than 2 resistances



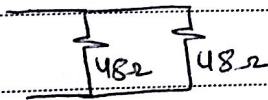
$$I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}} I_s$$

Example



$$Req. = 4 \Omega \parallel 20 \Omega = \frac{4 \times 20}{4 + 20} = \frac{10}{3}$$

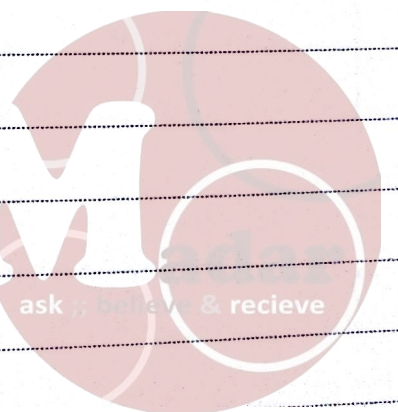
IF



$$Req. = 24 \Omega$$

$$Req. = \frac{48 \times 48}{2 \times 48} = \frac{48}{2} = 24 \Omega$$

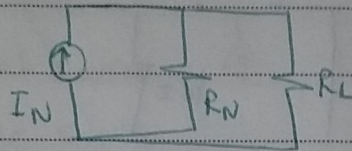
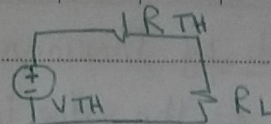
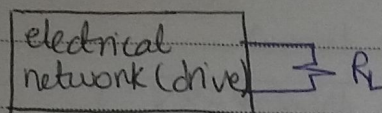
مقاومة الجوزي * المقاومة المكافئة أقل من أقل من المقاومة الموجودة في السلسلة



Monday

18/2/2019

NO. Norton circuit equivalent



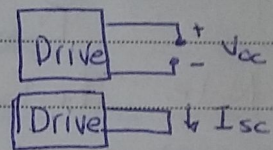
In reality, it is found by measurement.

Steps :

1. measure the open circuit voltage, V_{oc}^{open}

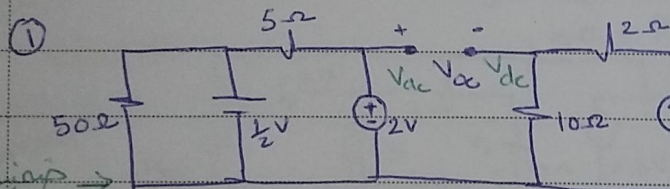
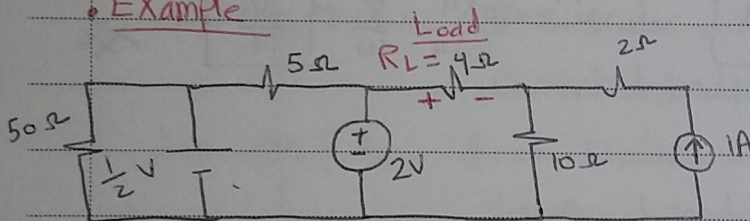
2. measure the short circuit current, I_{sc}

3. Find R_N $R_N = \frac{V_{oc}}{I_{sc}}$



Thevenin & Norton are best

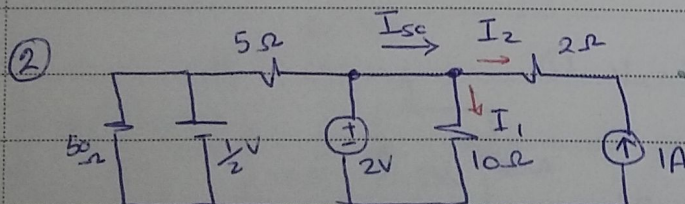
Example



$$V_{oc} = V_a - V_b = 2 - 10 = -8V$$

$$V_a = 2V$$

$$V_b = 10 \times 1 = 10V$$



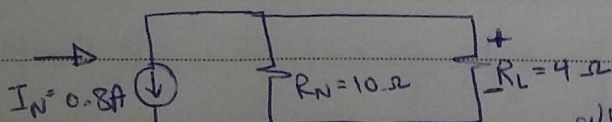
using loops is not the easiest way.

$$I_{sc} = I_1 + I_2 = 0.2 - 1 = -0.8A$$

$$I_1 = 2/10 = 0.2A$$

$$I_2 = -1A$$

$$R_N = \frac{-8}{-0.8} = 10\Omega$$



$$V_L = +0.8 \times 4$$

التيار في الحمل

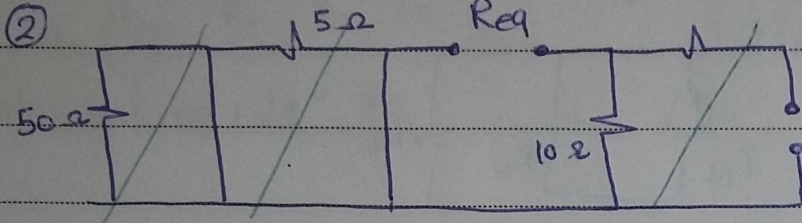
ask

to receive FIVE APPLE

• The previous example by Thevenin

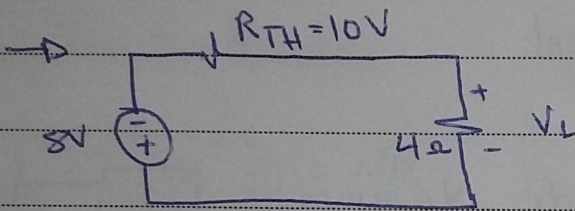
① V_{oc} is the same as the value in Norton

$$V_{oc} = -8V$$



$$R_{eq} = 10\Omega = R_{TH}$$

Look



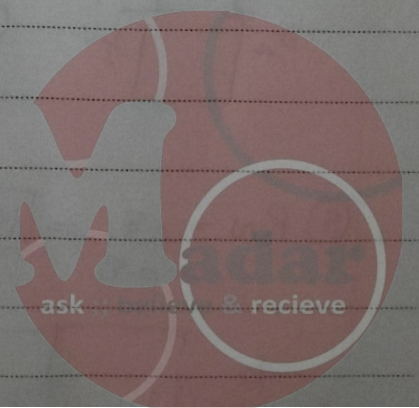
Note The load experiences the same conditions

Thevenin $V_L = \frac{4}{10+4} (-8) = \boxed{\frac{-16}{7} V}$

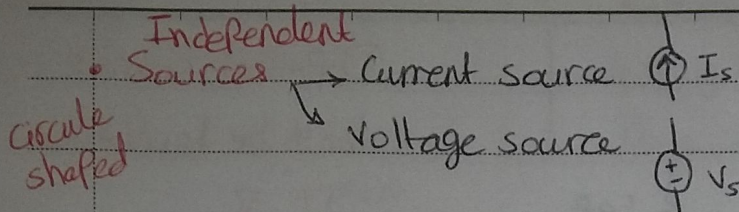
$$I_{RL} = \frac{-8}{14} = \boxed{\frac{-4}{7} A}$$

Norton $I_{RL} = \frac{10}{10+4} \left(\frac{8}{10} \right) = \boxed{\frac{-4}{7} A}$

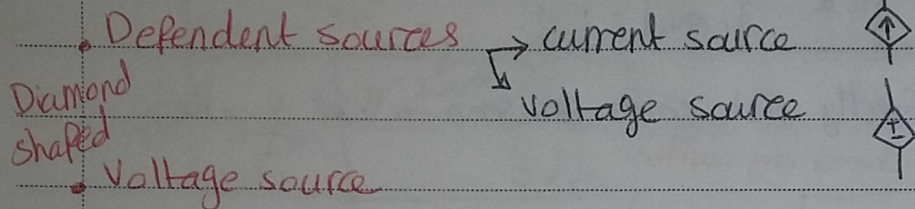
$$V_L = -\frac{10 \times 4}{10+4} \cdot \frac{8}{10} = \boxed{\frac{-16}{7} V} \quad \times$$



NO. Dependent sources



المصدر المستقل / التيار المستمر I_s
 المصدر المستقل / الجهد المستمر V_s



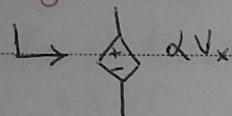
Dependent sources

مصادر معتمدة

مصادر معتمدة

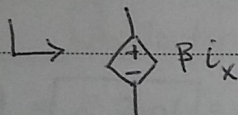
أكثر ما يعتمد على المتغير

للتيار الكهربائي



α is scalar, usually less than 1

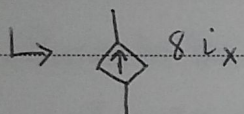
"Voltage-dependent voltage source"



β is scalar, usually less than 1

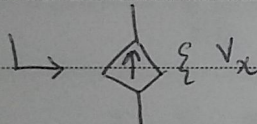
"Current-dependent voltage source"

Current source



γ is scalar, usually less than 1

"Current-dependent current source"



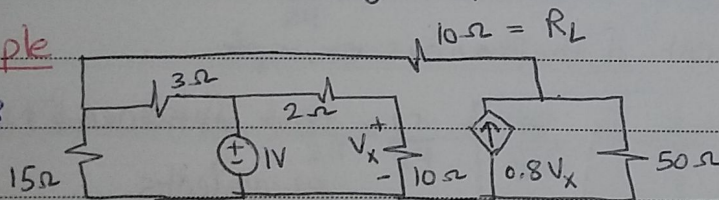
ξ is scalar, usually less than 1

"Voltage-dependent current source"

Look

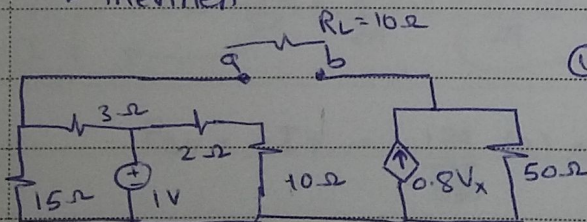
Example

المثال



Find Thevenin's & Norton's equivalent at R_L

→ Thevenin



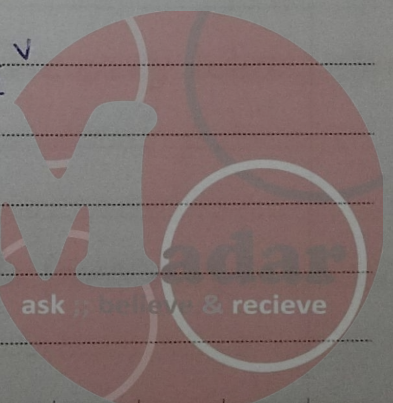
$$\textcircled{1} V_a = V_{15\Omega} = \left(\frac{15}{3+15} \right) 1 = \frac{15}{18} \text{ V}$$

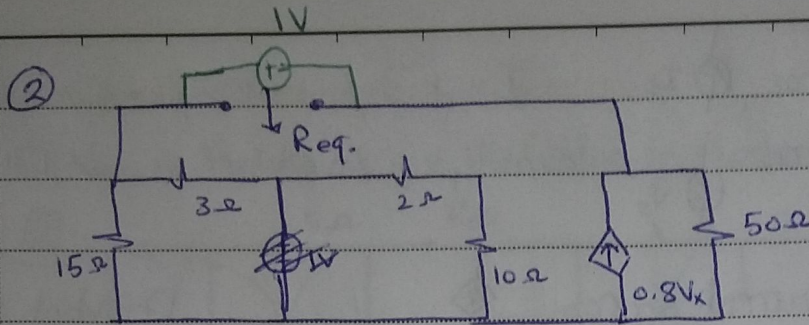
$$V_b = 0.8V_x (50\Omega)$$

$$V_x = \frac{10}{2+10} (1) = \frac{10}{12} \text{ V}$$

$$V_b = \frac{100}{3} \text{ V}$$

$$V_{TH} = V_{oc} = V_a - V_b = \frac{15}{18} - \frac{100}{3} = \frac{-65}{2} \text{ V}$$

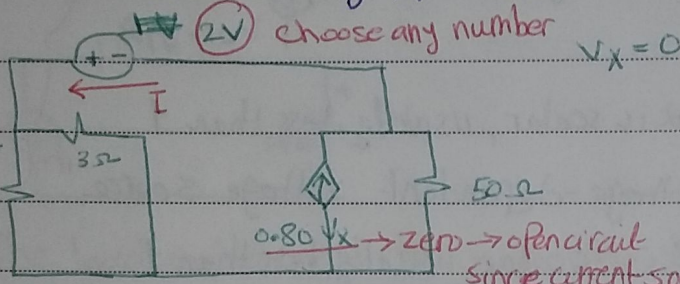




Dependent sources
can't be turned
off!

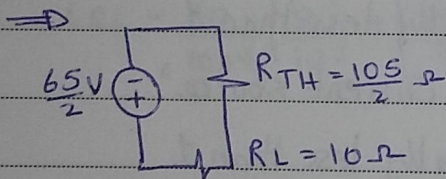
can't be solved directly, put source as seen $\rightarrow$ Then $\frac{V}{I} = R_{eq}$

If R_{eq} is
hard to be found
Put current or voltage
source to find it.



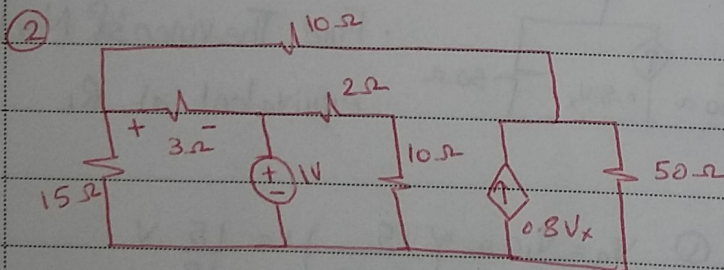
$0.8V_x \rightarrow \text{zero} \rightarrow \text{open circuit}$
since current source / not always

$$I = \frac{2}{\frac{3 \times 15}{3 + 15} + 50} = \frac{4}{105} \text{ A} \quad \rightarrow \quad R_{eq} = \frac{2}{4/105} = \boxed{\frac{105}{2} \Omega} = R_{TH}$$

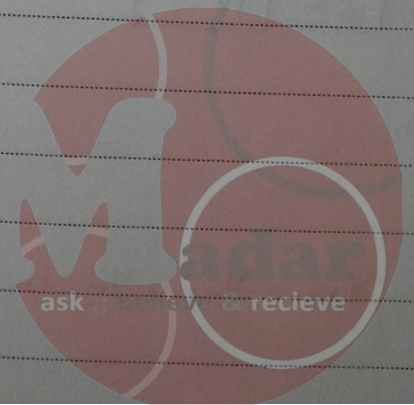


④ How Assignment

① Find Norton equivalent of the previous example



Find Thevenin & Norton
equivalents



$$1 \text{ hp} = 746 \text{ W}$$

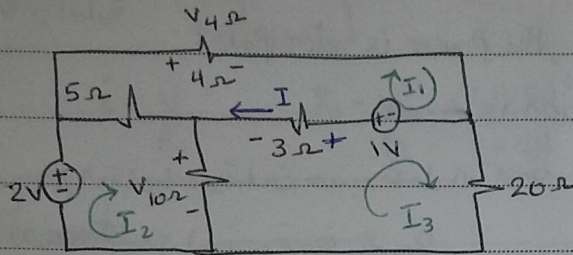
Wednesday

20/2/2019

NO. Power calculations
Rated Power & Power rating
Wheatstone Bridge
Maximum Power Transfer

20/2/19
20/2/19
20/2/19

Power Calculations



Power = Across variable x Thru variable

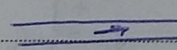
mechanically Power = (W)(r)
Angular velocity (F)(velocity)

In electrical engineering
Voltage electric current
 $P_R = VI$

Heat exchanger

(ΔT) (amount of heat Flow)

Pipe



(P)(Q)
Through

Across variable

"Power dissipated across resistor"
↑ consumed
wasted

Back to the example

$$I_1 = \frac{23}{116} \text{ A} \quad I_2 = \frac{139}{580} \text{ A} \quad I_3 = \frac{7}{116} \text{ A}$$

$$I = I_1 - I_3 = \frac{4}{29} \text{ A}$$

$$V_{3\Omega} = 3I = \frac{12}{29} \text{ V}$$

$$P_{3\Omega} = IV_{3\Omega} = \frac{4}{29} \times \frac{12}{29} = \frac{48}{841} \text{ W}$$

The same for dependent source

$$P_{1V} = -IV = -\frac{4}{29} (1) = -\frac{4}{29} \text{ W}$$

negative if I from -ve to +ve
This is Power produced/generated by the voltage source.

$$P_{2V} = -IV = -\frac{139}{580} (2) = -\frac{139}{290} \text{ W}$$

لو التيار بالاكس يصبغ بـ سالب

Assignment 8 Find all the Powers of all resistances & Prove energy balance equation

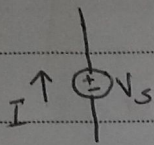
Imp. Energy balance equation

$$\sum P_{\text{generated power}} = \sum P_{\text{dissipated}} \quad \sum P_i = 0$$

usually dependent sources will dissipate not generate power, try on previous examples and prove on more than 1 example.

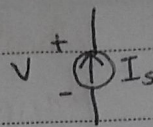
ask & receive

FIVE APPLE

Generally

$$P = -I V_s$$

generated



$$P = -I_s V$$

generated

التيار I_s و الجهد V
Voltage for
current source
هو الجهد في التيار

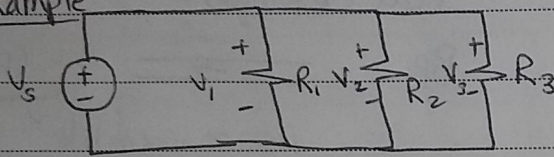
But any other resistance, the power is dissipated
what ever the direction is

→ Power rating & rated power

You put it Putted on product

الحد الأقصى للقدرة التي يمكن أن يتحملها العنصر

القدرة التي يتم وضعها على المنتج

Example

$$V_s = 3V$$

$$i_1 = 0.2 \text{ mA}$$

$$i_3 = 1.2 \text{ mA}$$

$$i_2 = 0.4 \text{ mA}$$

Find the power produced by the voltage source

$$P_s = -V_s I_s = -3 \times 1.8 \text{ mA} = -5.4 \text{ mW}$$

$$I_s = i_1 + i_2 + i_3 = 1.8 \text{ mA}$$

Find voltages and resistance R_1, R_2, R_3

$$V_1 = V_2 = V_3 = 3V$$

$$R_1 = \frac{V_1}{i_1} = \frac{3}{0.2 \times 10^{-3}} = 15 \times 10^3 \Omega = 15 \text{ k}\Omega$$

$$R_2 = \frac{V_2}{i_2} = \frac{3}{0.4 \times 10^{-3}} = 7.5 \times 10^3 \Omega = 7.5 \text{ k}\Omega$$

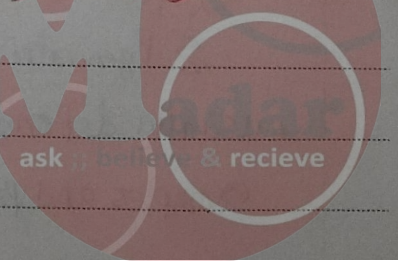
$$R_3 = \frac{V_3}{i_3} = \frac{3}{1.2 \times 10^{-3}} = 2.5 \text{ k}\Omega$$

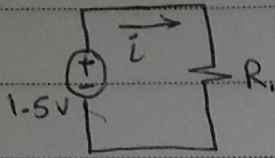
$$I = \frac{dq}{dt}$$

التيار هو معدل التغير في الشحنة
differentiation
integration أو إذا كانت نسبة

$$\text{Power} = IV = I^2 R = \frac{V^2}{R}$$

القدرة هي حاصل ضرب الجهد في التيار



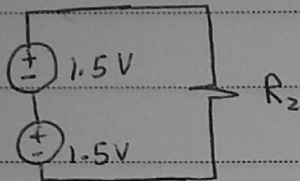
Example

with a $\frac{1}{4}$ W rating, what is the minimum resistor size that can be solved?

$$i = \frac{V}{R_1}$$

$$R_1 = \frac{V^2}{P_R} = \frac{(1.5)^2}{(0.25)}$$

$$R_1 = 9 \Omega$$

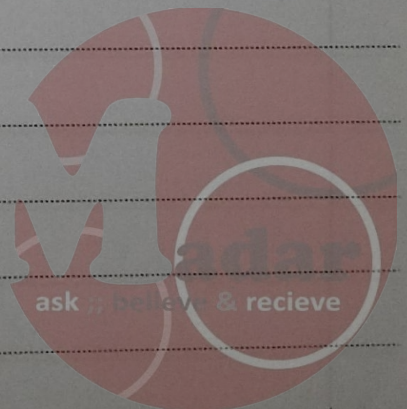
Example

$$P_{R_2} = V \cdot i = V \left(\frac{V}{R} \right)$$

$$R_2 = \frac{V^2}{P_{R_2}} = \frac{3^2}{0.25} = 36 \Omega$$

↑ V ↑ R

def rating 0.25 W 9 Ω 36 Ω, 0.25 W

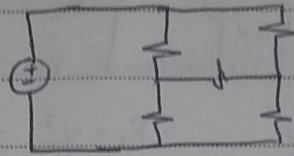
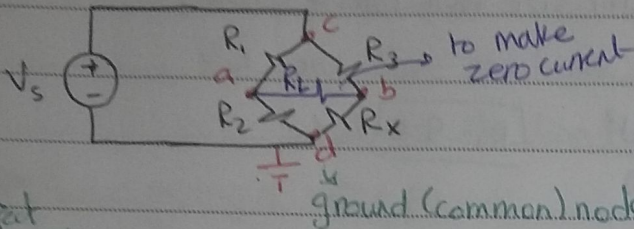


Monday

25/2/20189

NO.

⇒ Wheatstone bridge → To find an unknown resistance value



with respect to d node →

$$V_{ab} = V_a - V_b$$

$$V_a = \frac{R_2}{R_1 + R_2} V_s$$

$$V_{ab} = \left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right) V_s$$

we want to make $V_{ab} = 0$

$$V_b = \frac{R_x}{R_3 + R_x} V_s$$

$$= \left(\frac{R_2 R_3 + R_2 R_x - R_1 R_x - R_2 R_x}{(R_1 + R_2)(R_3 + R_x)} \right) V_s$$

Way #1 To make $V_{ab} = 0 \Rightarrow$ when $R_2 R_3 - R_1 R_x = 0$

$$R_2 R_3 = R_1 R_x$$

$$\boxed{\frac{R_2}{R_1} = \frac{R_x}{R_3}}$$

OR

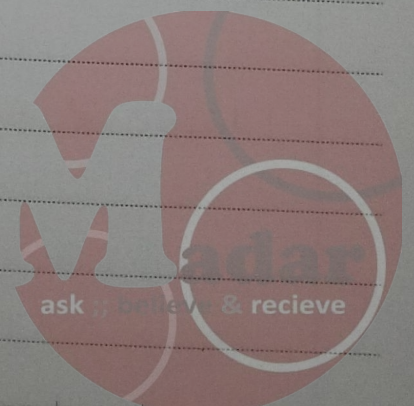
$$\boxed{\frac{R_x}{R_3} = \frac{R_2}{R_1}}$$

Way #2 Not necessary to make $V_{ab} = 0$

$$V_{ab} = V_s \left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right) \text{ by making } R_x \text{ in terms of other variables}$$

$$\boxed{R_x = \frac{R_3 \left(\frac{R_2}{R_1 + R_2} - \frac{V_{ab}}{V_s} \right)}{\left(1 - \frac{R_2}{R_1 + R_2} + \frac{V_{ab}}{V_s} \right)}}$$

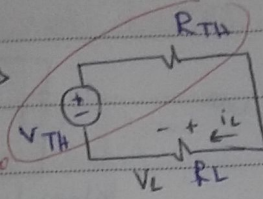
Solve examples on the slides. Imp



Maximum Power Transfer

Electrical Network

control variable
Power is to be



when would I be delivering maximum power to R_L ?

Answer

$$P_{RL} = V_L \cdot I_L$$

$$P_{RL} = \left[\left(\frac{R_L}{R_L + R_{TH}} \right) V_{TH} \right] \left(\frac{V_{TH}}{R_L + R_{TH}} \right) = \frac{R_L V_{TH}^2}{(R_L + R_{TH})^2} \quad \text{"power delivered to the load"}$$

To know the max. power $\rightarrow$ derive with respect to R_L because it is the controlled variable. (R_{TH} is from network properties).

By Quotient Rule

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 (R_L + R_{TH})^2 - 2 R_L V_{TH}^2 (R_L + R_{TH})}{(R_L + R_{TH})^4}$$

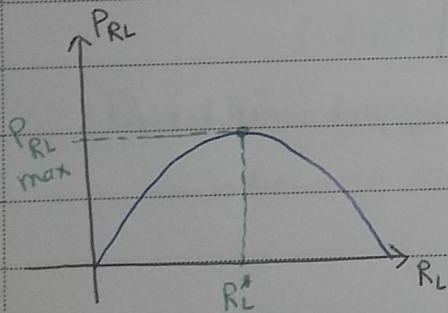
Best R_L value

$$\frac{\partial P_L}{\partial R_L} \bigg|_{R_L = R_L^*} = 0$$

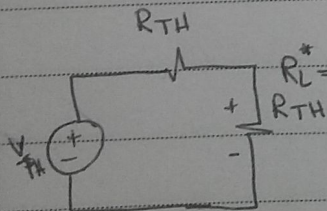
$$V_{TH}^2 (R_L + R_{TH})^2 - 2 R_L V_{TH}^2 (R_L + R_{TH}) = 0$$

$$V_{TH}^2 (R_L^2 + 2 R_L^* R_{TH} + R_{TH}^2) - 2 V_{TH}^2 (R_L^2 + R_L^* R_{TH}) = 0$$

$$-R_L^{*2} + R_{TH}^2 = 0 \quad R_L^* = R_{TH} \quad \boxed{R_L^* = R_{TH}} \quad R_L = R_L^*$$



if $R_L \neq R_{TH}$ it doesn't occur at max. power



$$V_{RL} = \frac{1}{2} V_S$$

$$P_{RL} = \frac{R_L V_{TH}^2}{(R_L + R_{TH})^2} \quad \text{But here } R_L = R_{TH}$$

$$P_{out, max} = \frac{R_L V_{TH}^2}{(2 R_L)^2} = \boxed{\frac{V_{TH}^2}{4 R_L}}$$

check efficiency η (unitless)

is defined as

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{V_{TH}^2}{4 R_L} \cdot \frac{2 R_L}{V_{TH}^2} \times 100\% = \boxed{50\%} \quad \text{Big value!}$$

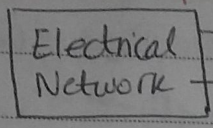
note: +ve value

$$P_{in, max} = V_{TH} I_S = V_{TH} \cdot \frac{V_{TH}}{2 R_L} = \boxed{\frac{V_{TH}^2}{2 R_L}}$$

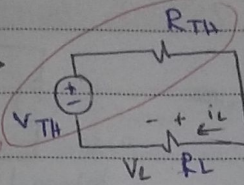
* most engineering systems good if $\eta = 33\%$

ask FIVE APPLE gas cooled reactors = higher η

⇒ Maximum Power Transfer



Power available



when would I be delivering maximum power to R_L ?

Answer

$$P_{RL} = V_L \cdot I_L$$

$$P_{RL} = \left[\left(\frac{R_L}{R_L + R_{TH}} \right) V_{TH} \right] \left(\frac{V_{TH}}{R_L + R_{TH}} \right) = \frac{R_L V_{TH}^2}{(R_L + R_{TH})^2} \quad \text{"power delivered to the load"}$$

To know the max. power → derive with respect to R_L because it is the controlled variable. (R_{TH} is from network properties).

By Quotient Rule

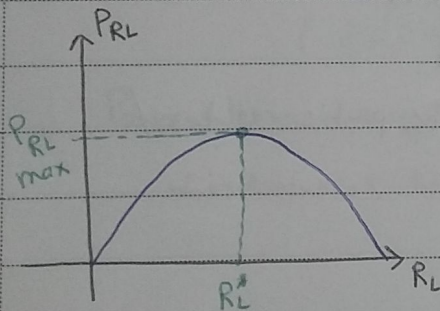
$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 (R_L + R_{TH})^2 - 2R_L V_{TH}^2 (R_L + R_{TH})}{(R_L + R_{TH})^4}$$

Best R_L value

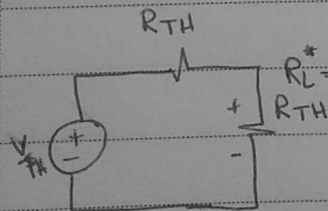
$$\left. \frac{\partial P_L}{\partial R_L} \right|_{R_L = R_L^*} = 0 \quad V_{TH}^2 (R_L + R_{TH})^2 - 2R_L V_{TH}^2 (R_L + R_{TH}) = 0$$

$$V_{TH}^2 (R_L^2 + 2R_L R_{TH} + R_{TH}^2) - 2V_{TH}^2 (R_L^2 + R_L R_{TH}) = 0$$

$$-R_L^2 + R_{TH}^2 = 0 \quad R_L^2 = R_{TH}^2 \quad \boxed{R_L^* = R_{TH}} \quad R_L = R_L^*$$



if $R_L \neq R_{TH}$ it doesn't occur at max. power



$$V_{RL} = \frac{1}{2} V_S$$

$$P_{RL} = \frac{R_L V_{TH}^2}{(R_L + R_{TH})^2} \quad \text{But here } R_L = R_{TH}$$

$$P_{out, max} = \frac{R_L V_{TH}^2}{(2R_L)^2} = \boxed{\frac{V_{TH}^2}{4R_L}}$$

→ check efficiency η (unitless)

is defined as

$$\eta \left(\frac{\Delta}{\Delta} \right) = \frac{P_{out}}{P_{in}} \times 100\% = \frac{V_{TH}^2}{4R_L} \cdot \frac{2R_L}{V_{TH}^2} \times 100\% = \boxed{50\%} \quad \text{Big value!}$$

the value

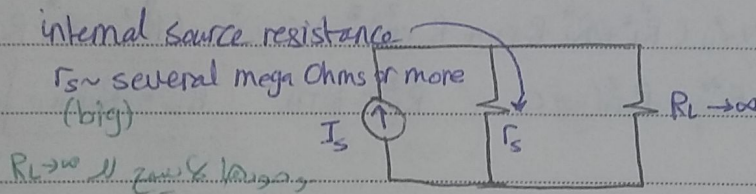
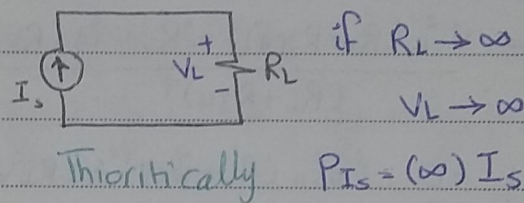
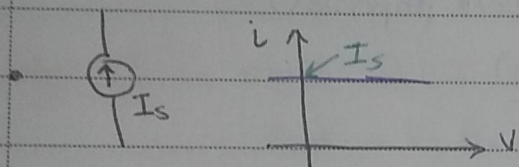
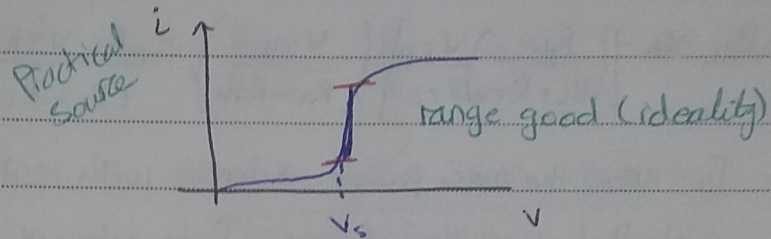
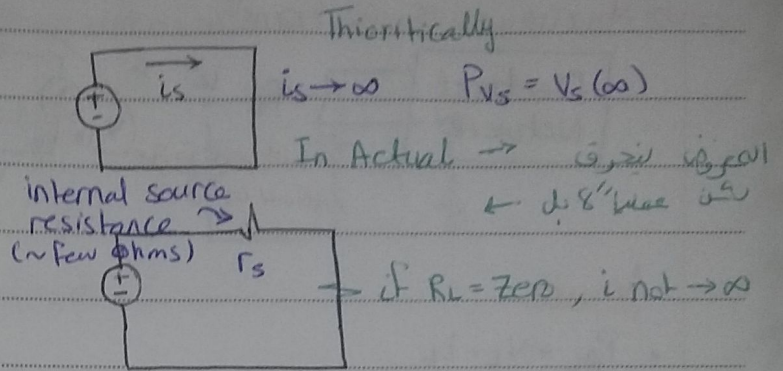
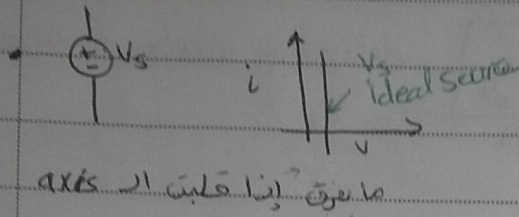
$$P_{in, max} = V_{TH} I_S = V_{TH} \cdot \frac{V_{TH}}{2R_L} = \boxed{\frac{V_{TH}^2}{2R_L}}$$

* most engineering systems good if $\eta = 33\%$

gas cooled reactors is higher η

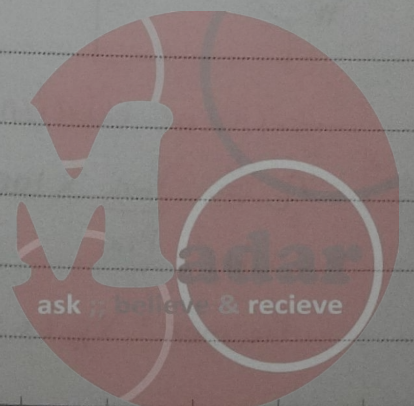
FIVE APPLE

⇒ Practical sources



See the plot in slides

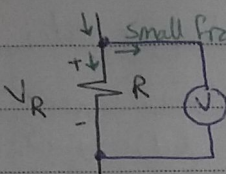
سوف نرى العنصر يتصرف كأنه له مقاومة داخلية كبيرة



measuring.
(R, I, V, P)

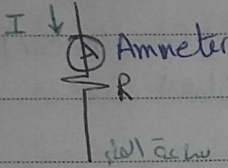
⇒ Electrical Quantities Measurement devices

Voltmeter



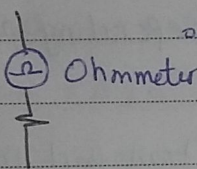
small fraction of current
voltage
while the circuit is ON
use Ohm's law

Ammeter



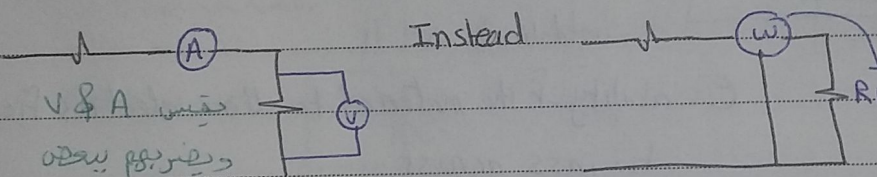
with small resistance
while the circuit is ON

Ohmmeter



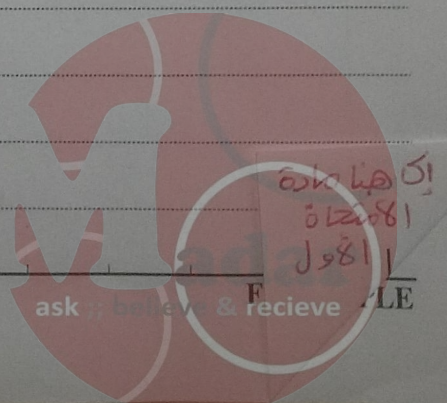
current
while the circuit is OFF
Req.

Wattmeter



Instead
3 legs
what
Wattmeter
Power * time = energy

$$\text{Power} \times \text{time} = \text{energy}$$



Tank - open → less energy storage
closed

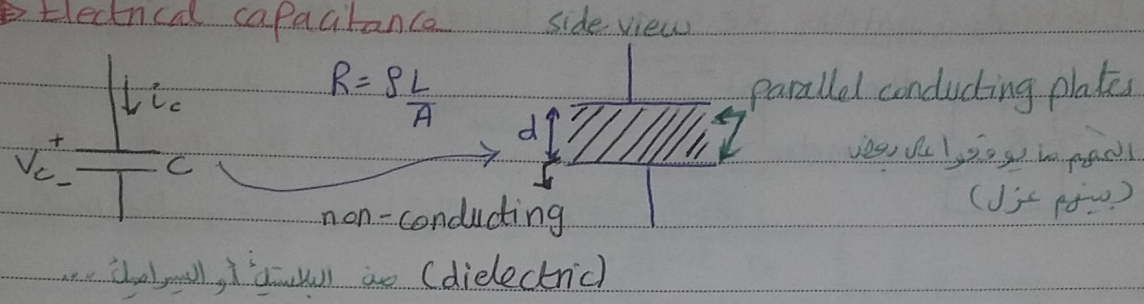
Capital letter → Fixed quantity
small letter →

Wednesday

27/2/2019

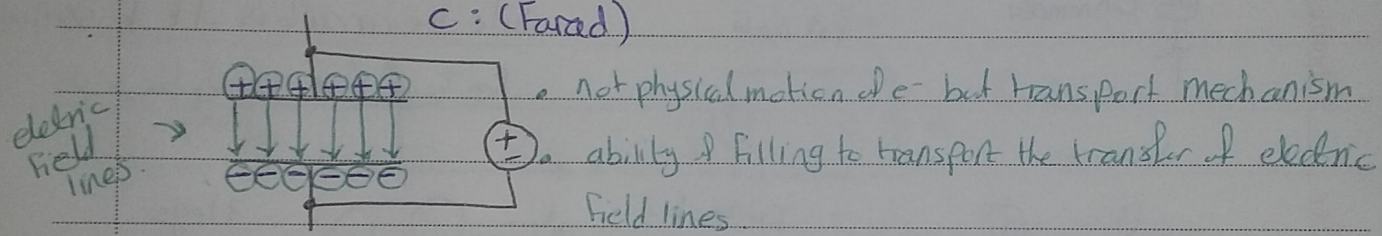
NO. Storage elements

→ Electrical capacitance

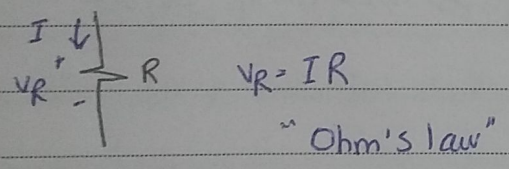


$$C = \frac{\epsilon A}{d}$$

Permittivity of dielectric material (Farad/meter)
 ϵ : material property (permittivity)
 A : surface area of plate (m^2)
 d : distance between two plates (separating plates) (m)
 C : (Farad)

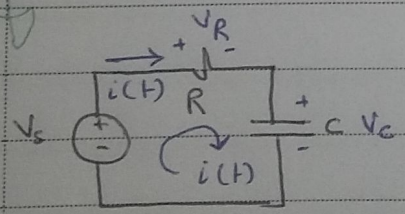


ϵ : ability of the material to allow electric field lines to pass across



In small letters $V_C(t) = \frac{1}{C} \int_{-\infty}^t i_C(\tau) d\tau$

Dynamics of resistor operation



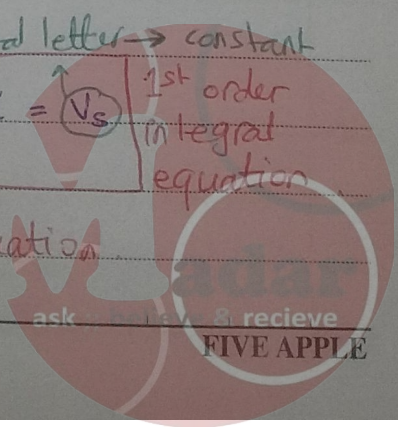
By loops

$$-V_s + V_R + V_C = 0$$

$$-V_s + i(t)R + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = 0$$

$$Ri(t) + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = V_s$$

solved by integral differential equation



-∞ doesn't p/s *
until to the current value
(point) which I'm interested
in

NO.

- we want to convert this equation to differential equation → Take derivative w.r.t t

$$R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

$$R di(t') = \frac{1}{C} i(t') dt'$$

$$R \int_{i(t'=-\infty)}^{i(t'=t)} di(t') = \frac{1}{C} \int_{t=-\infty}^t i(t') dt' \quad \text{we will not use this procedure in solving}$$

Guess a solution

$$\text{Assume } i(t) = A e^{st} \quad e^{st} \neq 0$$

$$\rightarrow RA s e^{st} + \frac{1}{C} A e^{st} = 0 \quad (RCs + 1) e^{st} = 0$$

$$(RCs + 1) e^{st} = 0 \quad \text{"characteristic equation"}$$

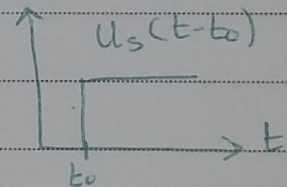
it has all the properties of the system

$$RCs + 1 = 0 \quad \boxed{s = -\frac{1}{RC}} \quad \text{Then go to the assumption.}$$

$$i(t) = A e^{-\frac{1}{RC} t}, \quad t \geq 0$$

$$i(t) = A e^{-\frac{1}{RC} t} u_s(t) \rightarrow$$

step function



In order to find A → From initial conditions

$$\text{Assume } i(t=0) = 2A$$

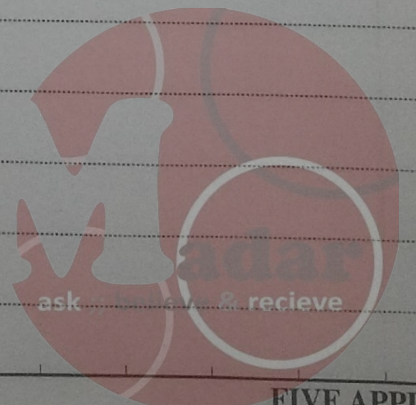
$$i(0) = A e^0 = 2 \quad A = 2$$

$$i(t) = 2 e^{-\frac{1}{RC} t}, \quad t \geq 0$$

$$(\text{at } t = \infty \Rightarrow i(t) = 0)$$

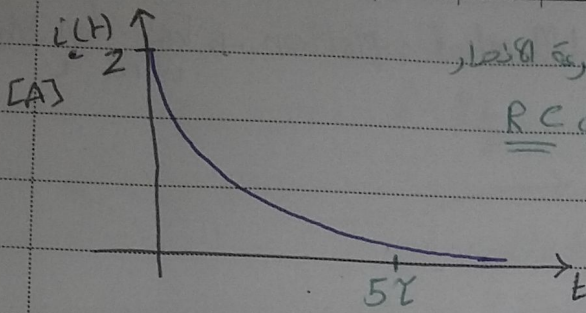
$$V_c(t) = \frac{1}{C} \int_{t'=0}^t 2 e^{-\frac{1}{RC} t'} dt' = -2 \frac{RC}{C} e^{-\frac{1}{RC} t'} \Big|_0^t = -2R (e^{-\frac{1}{RC} t} - 1)$$

$$V_c(t) = \underbrace{2R}_{A \cdot R} (1 - e^{-\frac{1}{RC} t}) \underbrace{[V]}_V \quad t \geq 0$$



FIVE APPL E

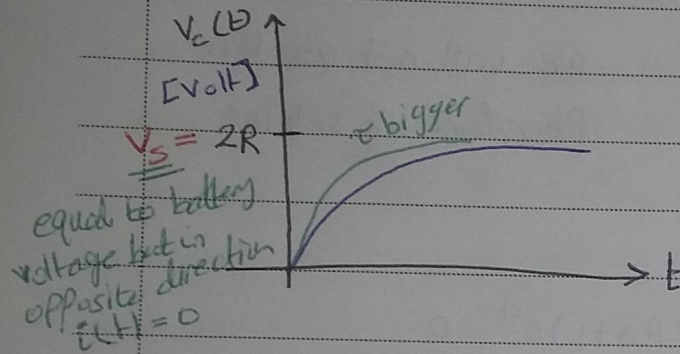
NO.



also says $RC = \tau$ (time constant) (small quantity)

$$RC \frac{di(t)}{dt} + i(t) = 0$$

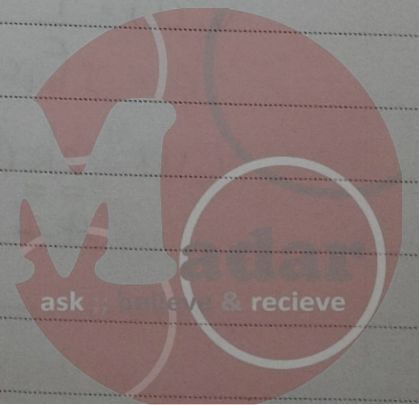
$\downarrow \tau$ $\uparrow$ response
depends on capacitor



"Deflection"

$\downarrow \tau$ $\uparrow$ response
charging the capacitor

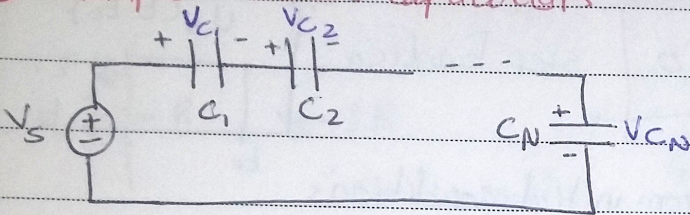
- This analysis is called "Transient analysis"
- differential equation
 - Folks math $\rightarrow$ homogeneous, Particular
 - engineering folks $\rightarrow$ transient, steady state



differential equation $\xrightarrow{\text{math}}$ homogeneous, particular
 engineering $\xrightarrow{\text{folks}}$ transient $\downarrow$ steady state

4/3/2019

Example Series capacitors



writing KVL

$$-V_s + V_{C1} + V_{C2} + \dots + V_{CN} = 0$$

$$\rightarrow V_s = V_{C1} + V_{C2} + \dots + V_{CN} \quad \text{But } V_C(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

$$\frac{1}{C_{eq}} \int_{-\infty}^t i(\tau) d\tau = V_s = \frac{1}{C_1} \int_{-\infty}^t i(\tau) d\tau + \frac{1}{C_2} \int_{-\infty}^t i(\tau) d\tau + \dots + \frac{1}{C_N} \int_{-\infty}^t i(\tau) d\tau$$

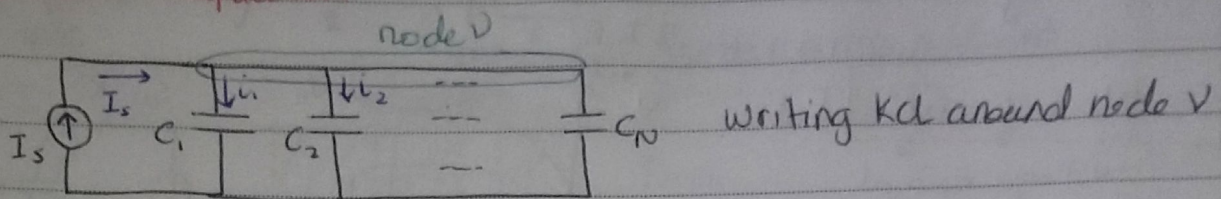
$$V_C(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}} \quad \text{In series}$$

FIVE APPLE



Parallel Capacitors

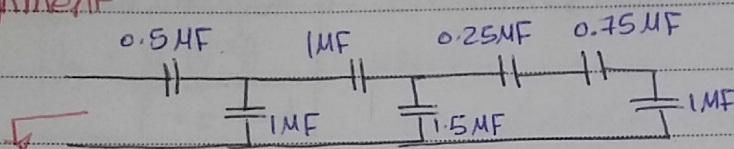


$$I_s = i_1 + i_2 + \dots + i_N$$

$$C_{eq} \frac{dv}{dt} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$

⑥ Assignment



Find C_{eq} ?

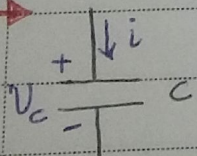
$$v_c(t) = \frac{1}{c} \int_{-\infty}^t i(\tau) d\tau \quad \int i(t) = \frac{dQ(t)}{dt} \text{ take integration}$$

$$\int_{-\infty}^t i(t') dt' = \int_{-\infty}^t \frac{dQ(t')}{dt'} dt' \Rightarrow \int_{-\infty}^t i(\tau) d\tau = \int_{Q(t'=-\infty)}^{Q(t)} dQ(t') = Q(t') \Big|_{-\infty}^t$$

$$= Q(t) - Q(-\infty) = Q(t)$$

it wasn't charged

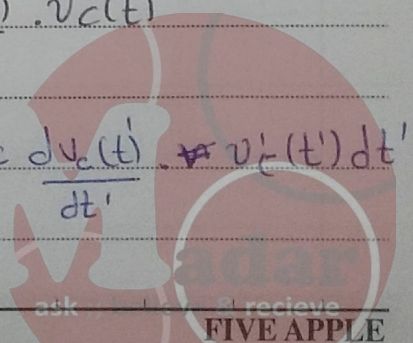
$$v_c(t) = \frac{1}{c} \int_{-\infty}^t i(\tau) d\tau = \frac{1}{c} Q(t)$$



$$P_c(t) = i_c(t) * v_c(t) = c \frac{dv_c(t)}{dt} \cdot v_c(t)$$

$$W_c = \int_{-\infty}^t P_c(t') dt' = \int_{-\infty}^t i_c(t') \cdot v_c(t') dt' = \int_{-\infty}^t c \frac{dv_c(t')}{dt'} \cdot v_c(t') dt'$$

work done to store energy
(energy stored in capacitor)



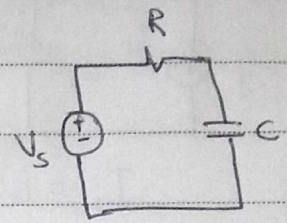
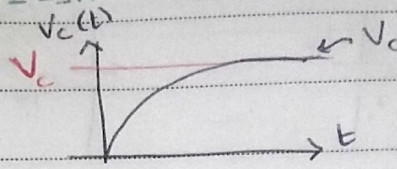
NO.

$$= C \int_{-\infty}^t v_C(\tau) dv_C(\tau) = C \left. \frac{v_C^2(\tau)}{2} \right|_{-\infty}^t = \frac{1}{2} C (v_C^2(t) - \underbrace{v_C^2(-\infty)}_{\text{Zero}})$$

$$E_C = W_C = \frac{1}{2} C V_C^2$$

voltage $\bar{q} \bar{q} \bar{q} \bar{q} \bar{q}$

Joules



look to examples in slides

CRT elements resistance

charge $\bar{q} \bar{q} \bar{q} \bar{q} \bar{q}$ $\rightarrow$ capacitor $\rightarrow$ $\bar{q} \bar{q} \bar{q} \bar{q} \bar{q}$

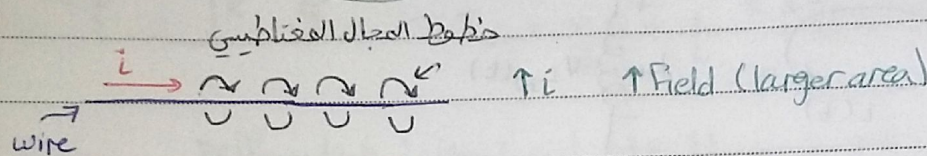
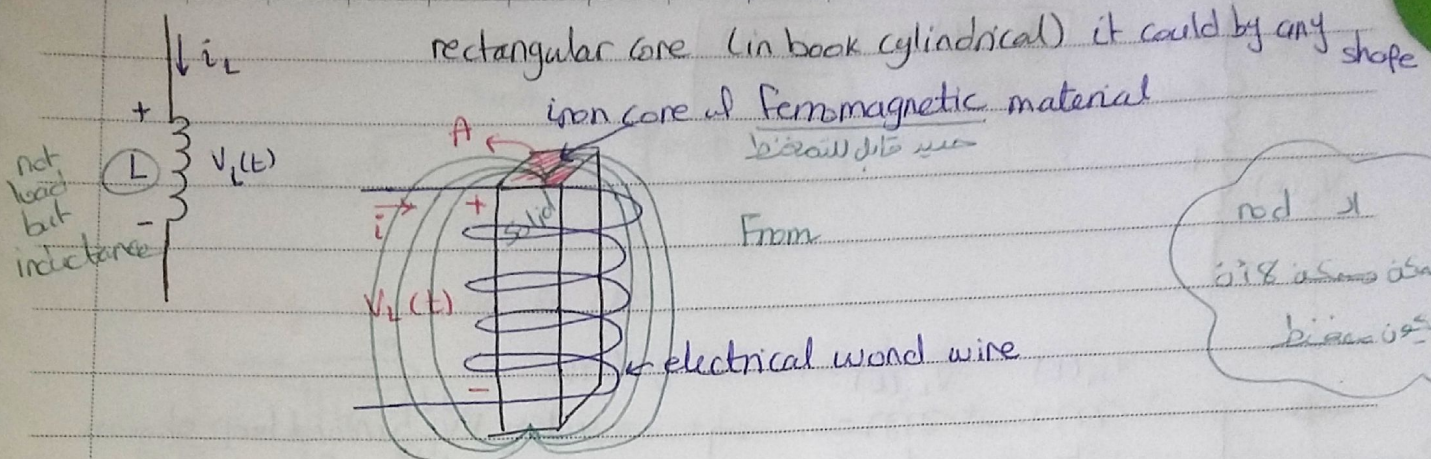
sources

electrical inductance $\rightarrow$ magnetic field $\bar{q} \bar{q} \bar{q} \bar{q} \bar{q}$

$$C = \frac{\epsilon A}{d} \quad R = \frac{\rho L}{A}$$



NO. Electrical Inductance



$$L = \frac{\mu N^2 A}{l + 0.45 d}$$

where

L : inductance (Henries)

l : length of magnetic rod (m)

d : diameter of rod cross section (m)

A : cross sectional area of rod (m^2) (unitless)

N : # of turns of wire that's wound around the rod

property for magnetic rod μ : Permeability of magnetic rod (Henry/m)

$\Rightarrow \mu$

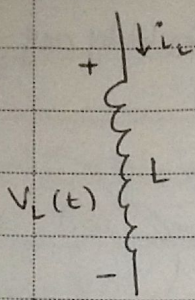
$$\mu = \mu_0 \mu_r \quad [\mu] = \text{Henry/m}$$

μ_0 : permeability of air or free space $\mu_0 = 1.255663 \times 10^{-6} \text{ Henry/m}$

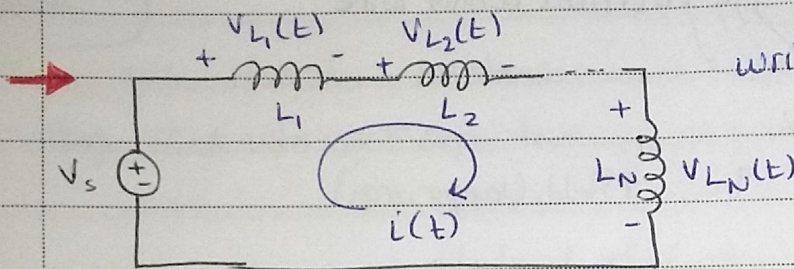
memorise

μ_r : relative permeability (from tables) [unitless]

This magnetic phenomenon prevent earth's aurora (Northern lights) (Aurora Borealis)



$$V_L(t) = L \frac{di_L(t)}{dt}$$



writing KVL around loop shown

$$-V_s + V_{L1} + V_{L2} + \dots + V_{LN} = 0$$

$$V_s = V_{L1} + V_{L2} + \dots + V_{LN}$$

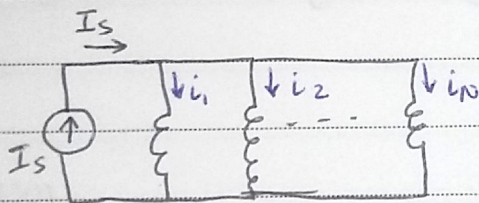
$$L \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_N \frac{di}{dt}$$

measures
overall
effect

$$L_{eq} = L_1 + L_2 + \dots + L_N$$

In series

Wednesday
6/13



writing KCL

$$I_s = i_1 + i_2 + \dots + i_N$$

$$\text{But } V_L = L \frac{di}{dt} \iff i(t) = \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau$$

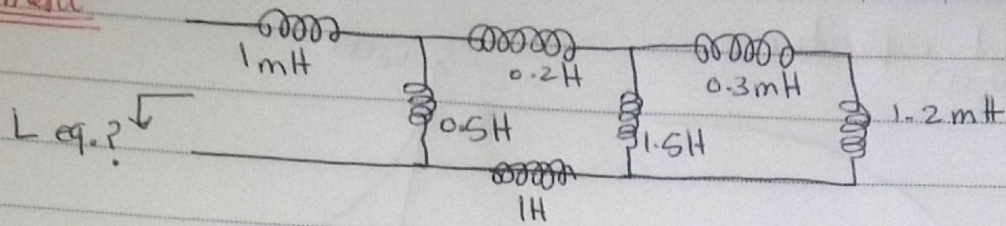
$$\frac{1}{L} \int_{-\infty}^t v(\tau) d\tau = I_s = \frac{1}{L_1} \int_{-\infty}^t v(\tau) d\tau + \frac{1}{L_2} \int_{-\infty}^t v(\tau) d\tau + \dots + \frac{1}{L_N} \int_{-\infty}^t v(\tau) d\tau$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

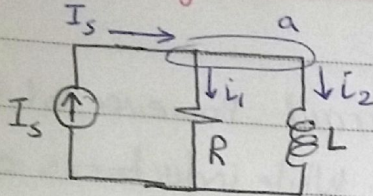
In series



7 Assignment



* Including inductance in ckt



writing KCL at node a

$$I_s = i_1 + i_2$$

$$I_s = \frac{V_a}{R} + \frac{1}{L} \int_{-\infty}^t V_a(\tau) d\tau \quad \text{"First order integral equation"}$$

Differentiate instead $\frac{1}{R} \frac{dv(t)}{dt} + \frac{1}{L} v(t) = 0$ "First order differential equation"

Time constant $\frac{L}{R} \frac{dv(t)}{dt} + v(t) = 0$ $\Rightarrow v(t) = Ae^{st}$ $s = -\frac{R}{L}$
 whenever it is a first order equation

$$v(t) = Ae^{-\frac{t}{\tau}} \quad t \geq 0 \text{ V}$$

$$\Rightarrow v(t=0) = V_0$$

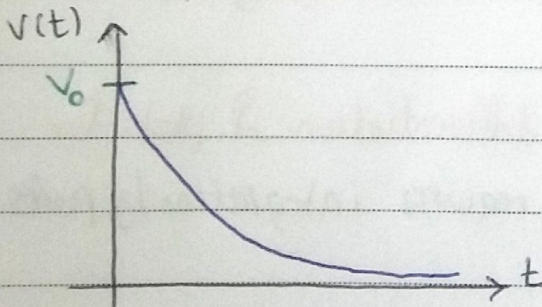
$$v(t) = V_0 e^{-\frac{t}{\tau}} \quad t \geq 0 \text{ Volts}$$

Now, $i(t) = \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau = \frac{V_0}{L} \int_0^t e^{-\tau/(L/R)} d\tau$

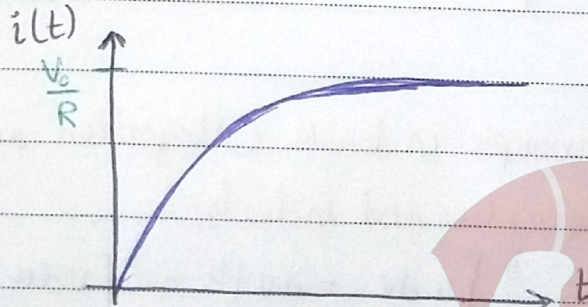
$$= -\frac{V_0}{L} \left(\frac{L}{R} \right) \left[e^{-\tau/(L/R)} \right]_0^t = -\frac{V_0}{R} \left[e^{-\frac{t}{\tau}} - 1 \right]$$

$$i(t) = \frac{V_0}{R} \left[1 - e^{-t/(L/R)} \right]; t \geq 0 \text{ A}$$

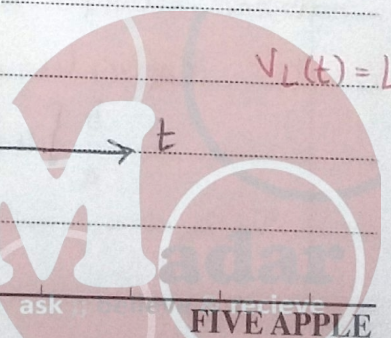
→ PLOT, inductor

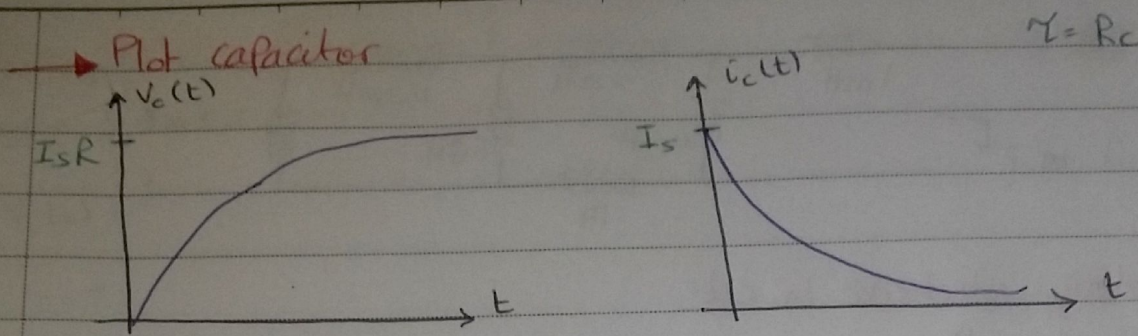


$$\tau = \frac{L}{R} \quad \left[\tau \right] = \text{Seconds}$$



$$v_L(t) = L \frac{di(t)}{dt}$$





- Initially, capacitor behaves as a short circuit. However, at the end time, it behaves as an open circuit. While inductor is opposite.

Device name	$t = 0$	$t \rightarrow \infty$
Inductor	open circuit	short circuit
Capacitor	short circuit	open circuit

Energy

Inductor circuit diagram: A voltage source V is connected to an inductor L . Current i flows through the inductor. The voltage across the inductor is V_L and the current is i_L .

$$P_L(t) = i_L(t) \cdot V_L(t)$$

$$E_L = \int_{-\infty}^t P_L(t') dt' = \int_{-\infty}^t i_L(t') V_L(t') dt'$$

$$= \int_{-\infty}^t L \frac{di_L(t')}{dt'} i_L(t') dt' = \int_{-\infty}^t L i_L(t') di_L(t')$$

$$= \frac{L}{2} i_L^2(t) \Big|_{-\infty}^t = \frac{L}{2} [i_L^2(t) - i_L^2(-\infty)]$$

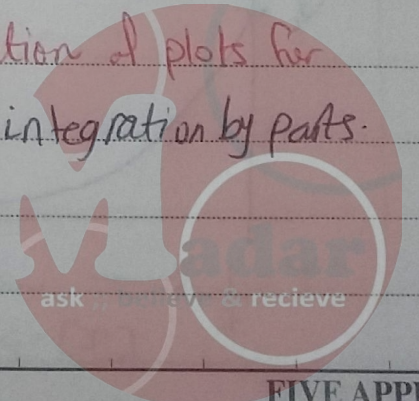
assuming uncharged $i_L(-\infty) = 0$

$W_L = E_L = \frac{1}{2} L I_L^2$ in Joules.

maximum current through inductor $= \frac{V_0}{R}$

See example in book (integration and differentiation of plots for capacitor and inductor). requires integration by parts.

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$



$$V_L(t) = L \frac{di}{dt}$$

$$i_C(t) = C \frac{dV_C(t)}{dt}$$

they are not d.c elements by A.C

NO. _____



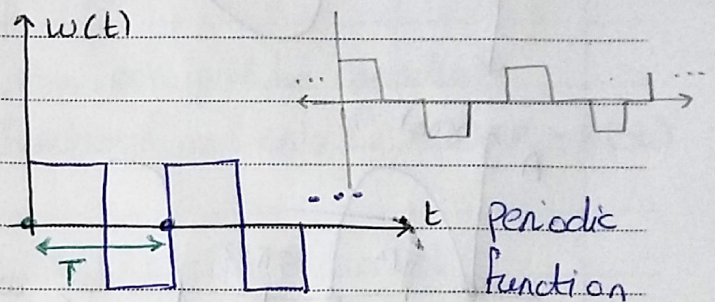
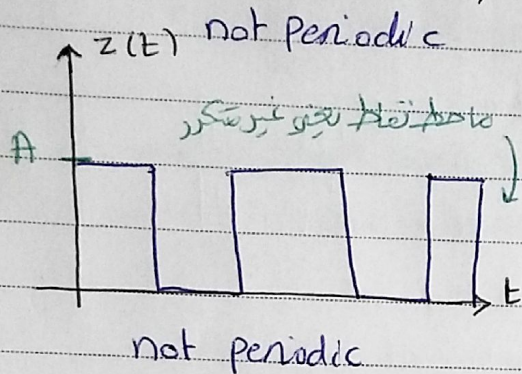
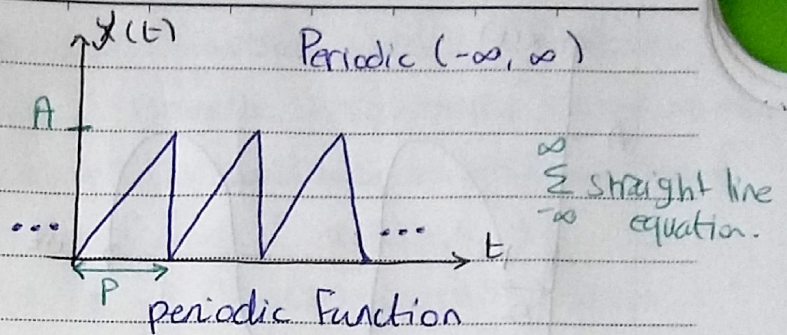
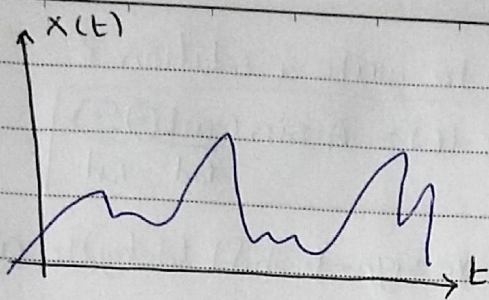
$$V_L(t) = L \frac{di}{dt}$$

$$i_c(t) = C \frac{dv_c(t)}{dt}$$

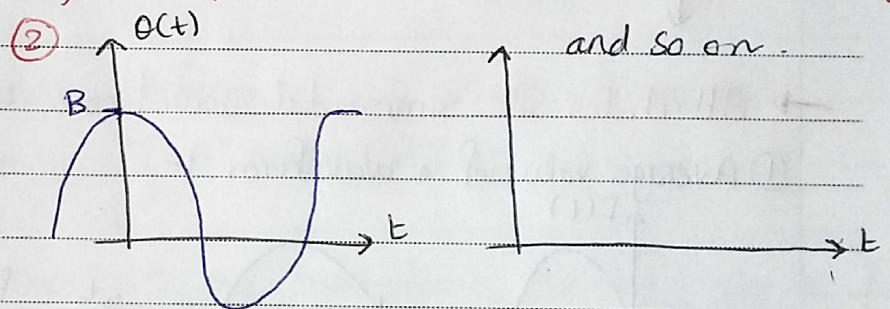
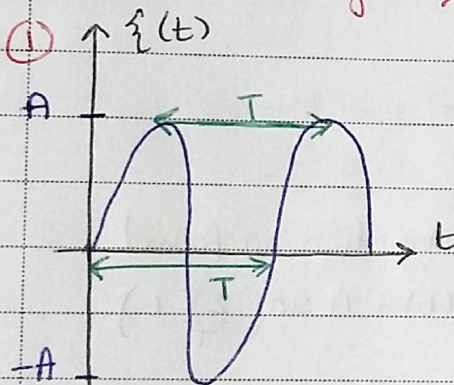
They are not d-c elements by A-C

Monday
11/3/2019

NO. Periodic Functions



• we will focus only on ^{specific} types of periodic functions (Sinosoidal Form) $\left\{ \begin{matrix} \sin \\ \cos \end{matrix} \right.$



and so on.

① $i(t) = A \sin(\omega t)$
 ω : Frequency (rad/s)
 A : amplitude

T : Period of the wave form.

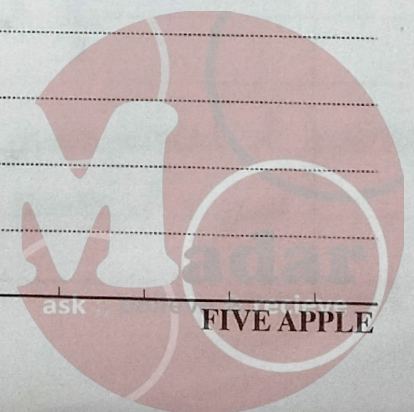
$$\omega = \frac{2\pi}{T} \quad [\omega] = \text{rad/s}$$

$$F = \frac{1}{T} \quad [F] = \text{Hz (or per second)}$$

$$\omega = 2\pi F$$

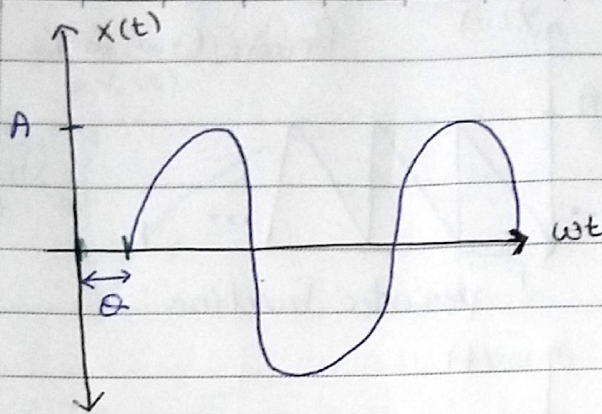
② $\theta(t) = B \cos(\omega t)$ same above
 B : amplitude
 ω : angular (radian) Frequency.

• angular named because of the scientist imagine



• Go to chapter in book and solve examples (min 6-7 examples)

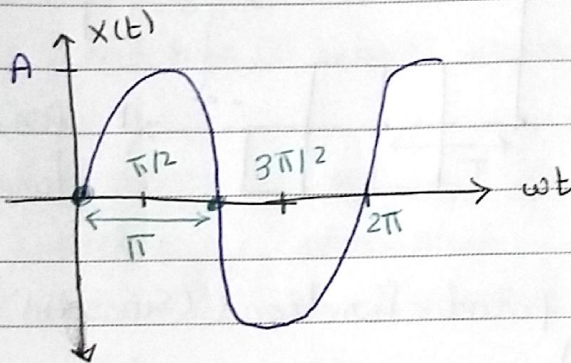
NO.



how to write a relation?

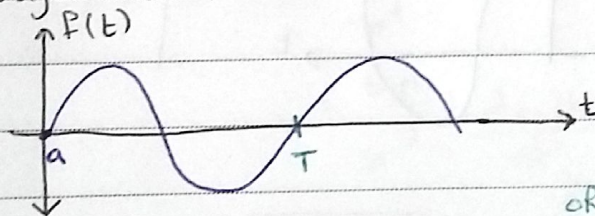
$$x(t) = A \sin\left(\frac{\omega t}{\text{rad}} - \frac{\theta}{\text{rad}}\right)$$

-ve sign $\rightarrow$ shift (delay) to right



$\rightarrow$ Attributes of Sinusoidal waveforms 8-

(i) Average value of a waveform :-



integration on period

$$f(t) = A \sin\left(\frac{2\pi}{T} t\right)$$

or $0 \rightarrow \pi$
(same)

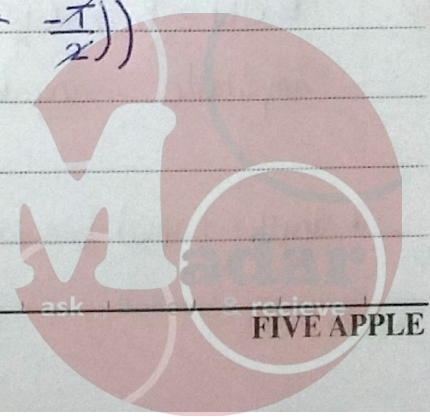
Average Value

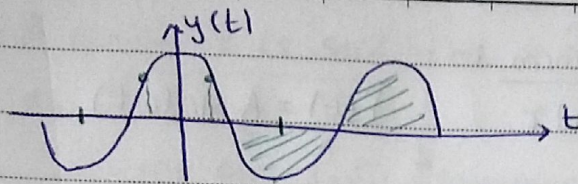
$$\bar{f}(t) = \overline{f(t)} = \langle f(t) \rangle \triangleq \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(\tau) d\tau$$

$$\bar{f}(t) = \frac{1}{T} \int_{-T}^T A \sin \frac{2\pi}{T} t' dt' = -\frac{A}{T} \cos \frac{2\pi}{T} t' \bigg|_{-\frac{T}{2}}^{\frac{T}{2}} \frac{T}{2\pi}$$

$$= -\frac{A}{2\pi} \left(\cos\left(\frac{2\pi}{T} \cdot \frac{T}{2}\right) - \cos\left(\frac{2\pi}{T} \cdot -\frac{T}{2}\right) \right)$$

$$= -\frac{A}{2\pi} (\cos \pi - \cos(-\pi))$$





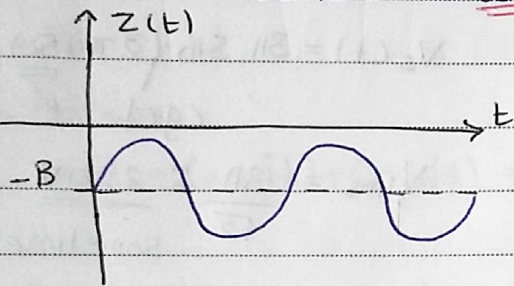
• cos function is an even function
(^{not} symmetric since symmetric around an axis)

* $x(t)$ is said to be an even function of t if & only if $x(t) = x(-t)$

$$\text{So } \frac{-A}{2\pi} (\cos(\pi) - \cos(-\pi)) = \frac{-A}{2\pi} (\cos(\pi) - \cos(\pi)) = \text{zero}$$

→ This make sense since the two areas will be cancelled.

* $x(t)$ is said to be an odd function if and only if $x(t) = -x(-t)$



$$z(t) = -B + A \sin \omega t$$

$$\bar{z}(t) = -B$$

② Mean square value for a waveform :-

$$f(t) = A \sin(\omega t)$$

$$\bar{f}^2 = \langle f^2(t) \rangle \triangleq \frac{1}{T} \int_{-T/2}^{T/2} A^2 \sin^2 \omega \tau d\tau$$

$$= \frac{A^2}{T} \int_{-T/2}^{T/2} \sin^2(\omega \tau) d\tau$$

From identity $\sin^2 x = \frac{1 - \cos 2x}{2}$

You can prove it

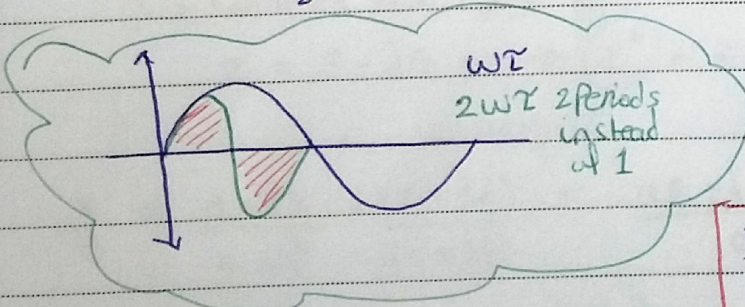
since

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= \frac{A^2}{T} \int_{-T/2}^{T/2} \frac{1 - \cos 2\omega \tau}{2} d\tau = \frac{A^2}{2T} \left[\int_{-T/2}^{T/2} 1 d\tau - \int_{-T/2}^{T/2} \cos 2\omega \tau d\tau \right]$$

(same as sin function)



$$= \frac{A^2}{2T} T = \frac{A^2}{2}$$

$$\bar{f}^2 = \frac{A^2}{2}$$

measurement of average power of signal

NO.

if index $\rightarrow$ small letter

(RMS) (RMS) $\rightarrow$ if index $\rightarrow$ small letter
 (3) Root mean square value of a waveform :-

$$F_{rms} = \sqrt{\overline{F^2}} \triangleq \sqrt{\frac{1}{T} \int_{-T/2}^{T/2} F^2(\tau) d\tau} \quad F(t) = A \sin(\omega t)$$

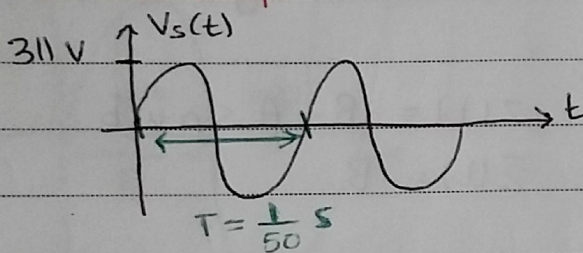
$$F_{rms} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

we take only +ve value because rms as you are measuring quantities or lengths.

$\rightarrow$ effective value of the waveform = F_{rms}

part of signal that do work.

$\rightarrow$ Oscilloscope



$$V_s(t) = 311 \sin(2\pi(50)t)$$

cycle $\downarrow$

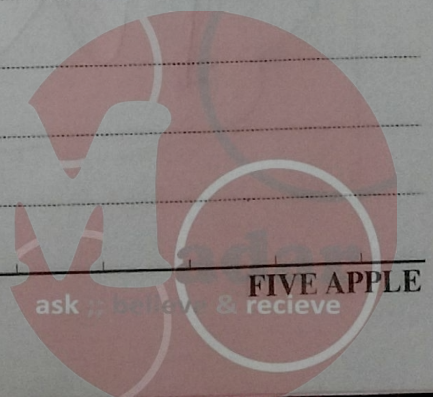
$$V_{rms} = \frac{311}{\sqrt{2}} \approx 220V$$

sometimes 240

$$240\sqrt{2} = 339.41 \text{ Volts if } 240V$$

it is in a range of 311 to 339.41 Volt.

Voltmeter in electricity $\rightarrow$ measures rms not voltage



Wednesday

NO. Complex numbers → used in AC systems and arithmetics

Complex numbers *

A number x is said to be a complex number if it can be represented as

$$x = a + jb$$

↑ ↑
real part $j = \sqrt{-1}$ imaginary part

Historically arised from

$$as^2 + bs + c = 0 \rightarrow s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{b^2 - 4ac}$ ← discriminant was negative
↑
reason for fluctuation

* Complex arithmetics

let $x = a + jb$

$y = c - jd$

→ Addition

$$x + y = (a + jb) + (c - jd) = \underbrace{(a + c)}_{\text{real part}} + j \underbrace{(b - d)}_{\text{imaginary part}}$$

→ Substraction

$$y - x = (c - jd) - (a + jb) = \underbrace{(c - a)}_{\text{real part}} + j \underbrace{(-d - b)}_{\text{imaginary part}}$$

→ Multiplication

$$x * y = (a + jb)(c - jd) = ac - jad + jbc + bd = \underbrace{(ac + bd)}_{\text{real part}} + j \underbrace{(bc - ad)}_{\text{imaginary part}}$$

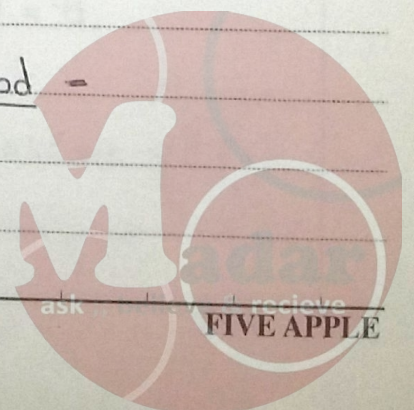
→ Division

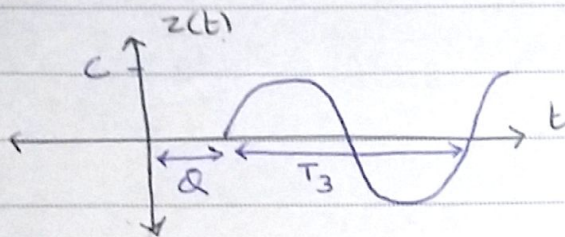
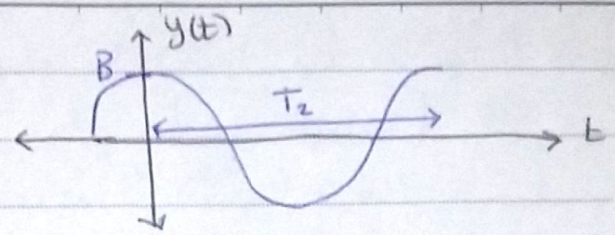
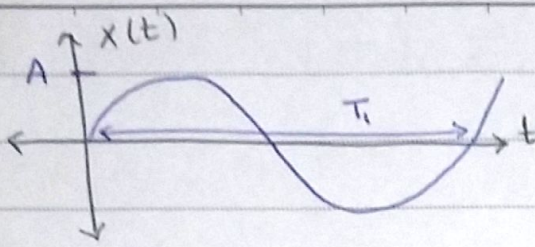
$$\frac{x}{y} = \frac{a + jb}{c - jd} \quad \text{multiply by dominator conjugate}$$

↳ Complex conjugate of a complex number

$$z = 3 - j5 \rightarrow \underbrace{(z^*)}_{\text{conjugate}} = (3 - j5)^* = 3 + j5$$

$$\frac{x}{y} = \frac{a + jb}{c - jd} \cdot \frac{(c + jd)}{(c + jd)} = \frac{ac + jad + jbc - bd}{c^2 + d^2} =$$
$$\frac{\underbrace{(ac - bd)}_{\text{real part}}}{c^2 + d^2} + j \frac{\underbrace{(ad + bc)}_{\text{imaginary part}}}{c^2 + d^2}$$

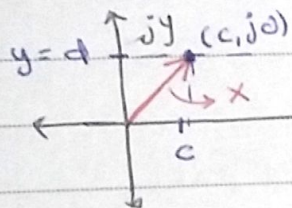




$$x(t) = A \sin\left(\frac{2\pi}{T_1} t\right) \quad y(t) = B \cos\left(\frac{2\pi}{T_2} t\right) \quad z(t) = C \sin\left(\frac{2\pi}{T_3} t - \phi\right)$$

⇒ Representation of complex numbers

① $X = c + jd$

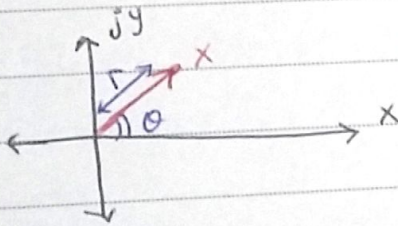


empty plot → "complex plane"

there is an imaginary axis (jy)

• This representation is "Cartesian graphical representation of x "

②



• This representation is "polar representation of a complex number in the complex plane"

⇒ Transformation of coordinates

• Cartesian → polar

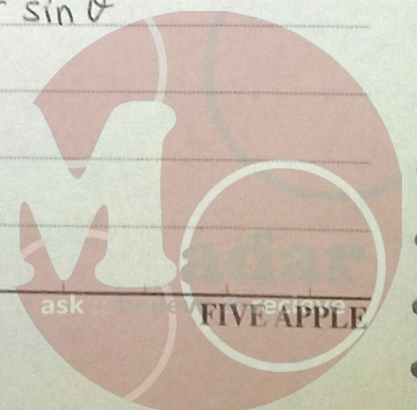
$$r = \sqrt{c^2 + d^2}$$

$$\theta = \tan^{-1}\left(\frac{d}{c}\right) \begin{matrix} \rightarrow \text{imaginary} \\ \rightarrow \text{real} \end{matrix}$$

• Polar → Cartesian

$$c = r \cos \theta$$

$$d = r \sin \theta$$



⇒ Euler's identity

$$e^{jx} = \cos x + j \sin x$$

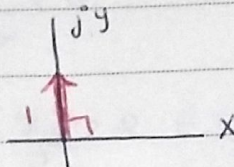
$$e^{-jx} = \cos x - j \sin x$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$



$$j = ? = e^{j\frac{\pi}{2}}$$

$$j = \sqrt{-1}$$



OR

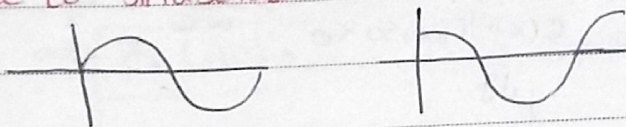
$$e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2}$$

zero

$$= j = \angle \frac{\pi}{2} \quad (\theta = \frac{\pi}{2} \rightarrow j)$$

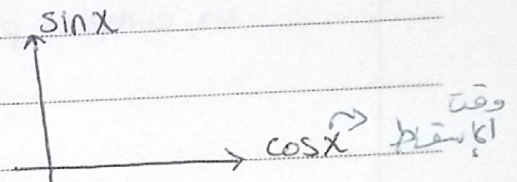
زاوية <

→ Back to sinusoidal



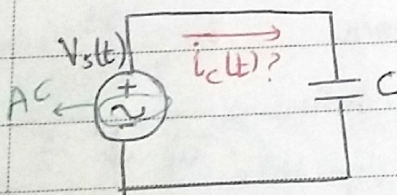
$$x = A \sin(\omega t) = A \cos(\omega t - \frac{\pi}{2})$$

$$y = B \cos(\omega t) = B \sin(\omega t + \frac{\pi}{2})$$



I want all functions to be in cos function (to consider it starting from zero axis)

⇒ Example



$$V_s(t) = 100 \sin(2\pi(50)t) \text{ Volts}$$

$$V_s(t) = 100 \cos(2\pi(50)t - \frac{\pi}{2})$$

$$i_c(t) = C \frac{dV_c}{dt}$$

$$\frac{dV_s(t)}{dt} = \frac{dV_c}{dt} = 100(2\pi(50)) \cos(2\pi(50)t)$$

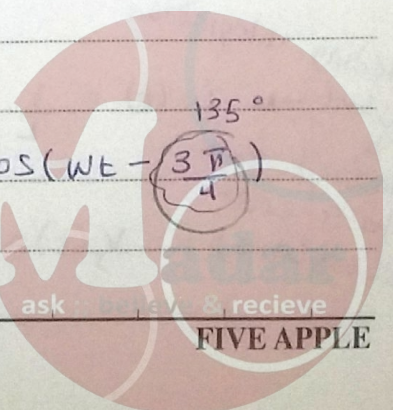
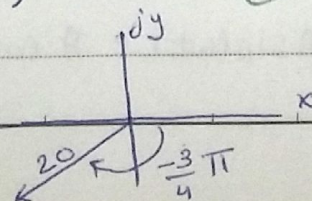
$$i_c(t) = C \cdot 100 \times 10^4 \pi \cos(2\pi(50)t)$$

Transfer to Frequency (Polar Coordinates)

$$V_s(j\omega) = 100 \angle -\frac{\pi}{2} \text{ volts} \rightarrow \text{explanation}$$

$$i_F Z_{\frac{1}{j\omega}}(t) = 20 \sin(\omega t - \frac{\pi}{4}) \rightarrow i_F Z_{\frac{1}{j\omega}}(t) = 20 \cos(\omega t - \frac{3\pi}{4})$$

$$\text{capital} \leftarrow Z_{\frac{1}{j\omega}}(j\omega) = 20 \angle -\frac{3\pi}{4}$$



addition & subtraction of polar coordinates is hard.

NO.

$$I_c(j\omega) = 10 \times 10^4 C < 0 \text{ Ampere}$$

* Multiplication of polar coordinates ^{add angles} multiply numbers

$$X = 10 < \pi$$

$$X \cdot Y = (10)(5) < \pi - \frac{\pi}{5} = 50 < \frac{4\pi}{5}$$

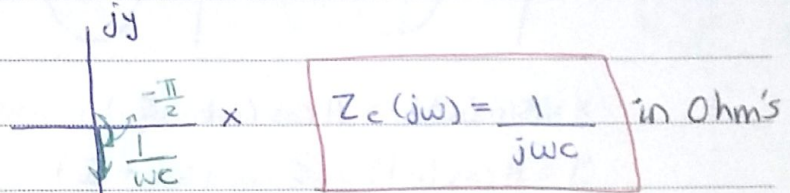
$$Y = 5 < -\frac{\pi}{5}$$

* Division ^{subtract angles} divide numbers

$$\frac{X}{Y} = \frac{10 < \pi}{5 < -\frac{\pi}{5}} = \left(\frac{10}{5}\right) < \pi + \frac{\pi}{5} = 2 < \frac{6\pi}{5}$$

$$\text{Now } \frac{V_c(j\omega)}{I_c(j\omega)} = \frac{100 < -\frac{\pi}{2}}{10^4 \pi C < 0} = \frac{100 < -\frac{\pi}{2}}{C(2\pi)(100)50 < 0} = \frac{1}{\omega C} < -\frac{\pi}{2} = \text{Resistance}$$

in complex plain



in AC: instead of saying resistance ← Impedance of the capacitor
in DC: resistance ← ^{مقاومة التيار}

Frequency of DC is zero → $Z \rightarrow \infty$ since $\omega = 0$

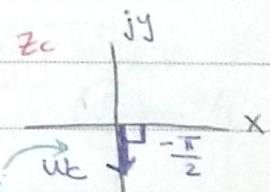
Monday

$$C \frac{1}{T}$$

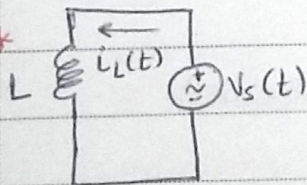
Voltage and current in Frequency domain

$$\frac{V(j\omega)}{I(j\omega)} \triangleq Z_c(j\omega) = \frac{1}{j\omega C} \text{ in ohms}$$

what does this means?



2 Inductor



$$V_s(t) = A \sin(\omega t)$$

$$V_L(t) = L \frac{di_L(t)}{dt} \rightarrow i_L(t) = \frac{1}{L} \int_{-\infty}^t V_L(\tau) d\tau$$

integration

But I don't know the angle at ∞

by adding constant → neglected because we have zero initial conditions

$$i_L(t) = \frac{1}{L} \int A \sin(\omega t') dt' = -\frac{A}{L\omega} \cos(\omega t) = \frac{A}{L\omega} (\omega t + \pi) \text{ or } -\pi$$

$$V_s(t) = A \sin(\omega t) = A \cos\left(\omega t - \frac{\pi}{2}\right)$$

(doesn't matter)
removing -ve sign
ask by retrieve
FIVE APPLE

NO.

- Convert to phaser (polar representation)

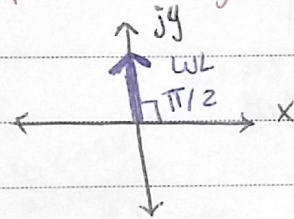
$$V_s(j\omega) = V_L(j\omega) = A \angle -\frac{\pi}{2}$$

$$I_L(j\omega) = \frac{A}{L\omega} \angle \pi$$

$$Z_L(j\omega) = \frac{V_L(j\omega)}{I_L(j\omega)} = \frac{A \angle -\frac{\pi}{2}}{\left(\frac{A}{L\omega}\right) \angle \pi} = L\omega \angle -\frac{\pi}{2} - \pi = L\omega \angle -\frac{3\pi}{2} = L\omega \angle \frac{\pi}{2} = j\omega L$$

$$\Rightarrow Z_L(j\omega) = j\omega L \text{ in ohms}$$

- Impedance diagram for Z_L



→ What does this mean?

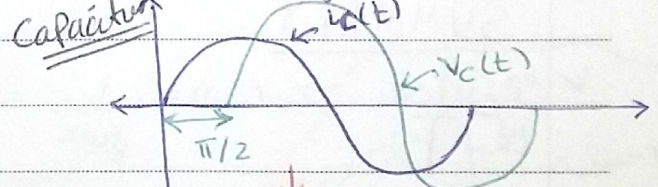
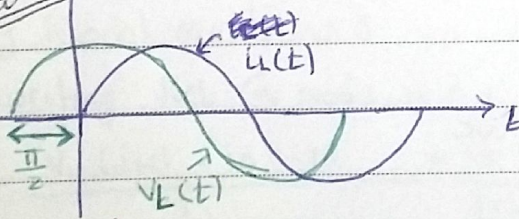
For $Z_C \rightarrow -\frac{\pi}{2}$

$Z_L \rightarrow \frac{\pi}{2}$ late

Inductor
Capacitor

voltage is late $\frac{\pi}{2}$ from current

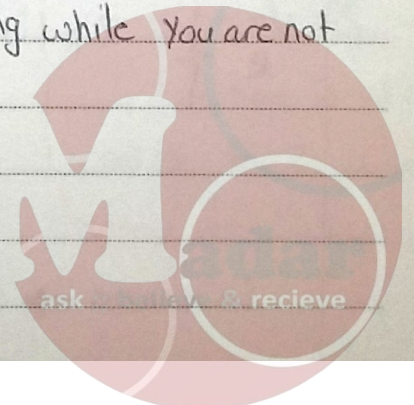
voltage faster than current $\frac{\pi}{2}$



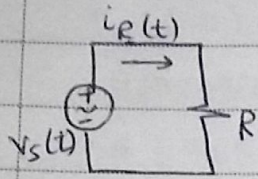
There is pre flow rate but no pressure

Problem for government

loss of voltage accounted, devices is working while you are not paying money for this electricity



③ Resistance impedance



• $V_R(t) = V_s(t)$ since in parallel

$$V_R(t) = A \cos(\omega t)$$

$$V_R(j\omega) = A \angle 0 \text{ Volts}$$

$$V_s(t) \text{ given} = A \cos(\omega t)$$

$$i_R(t) = \frac{V_R(t)}{R} = \frac{A \cos(\omega t)}{R}$$

$$I_R(j\omega) = \frac{A}{R} \angle 0 \text{ Ampere}$$

$$Z_R(j\omega) = \frac{V_R(j\omega)}{I_R(j\omega)} = \frac{A \angle 0}{\frac{A}{R} \angle 0} = R \angle 0 = R$$

$$Z_R = R$$

• what does this means?

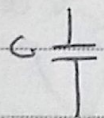
voltage and current together

Two notes

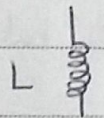
① Z_R is not a function of ω

② $i_R(t)$ and $v_R(t)$ are in-phase
تقريباً الزاوية

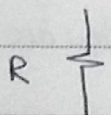
SUMMARY



$$Z_C(j\omega) = \frac{1}{j\omega C} = -j \frac{1}{\omega C}$$

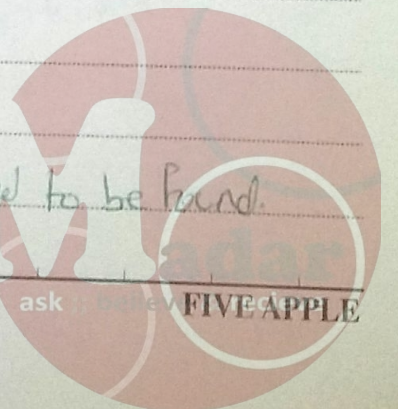


$$Z_L(j\omega) = j\omega L$$



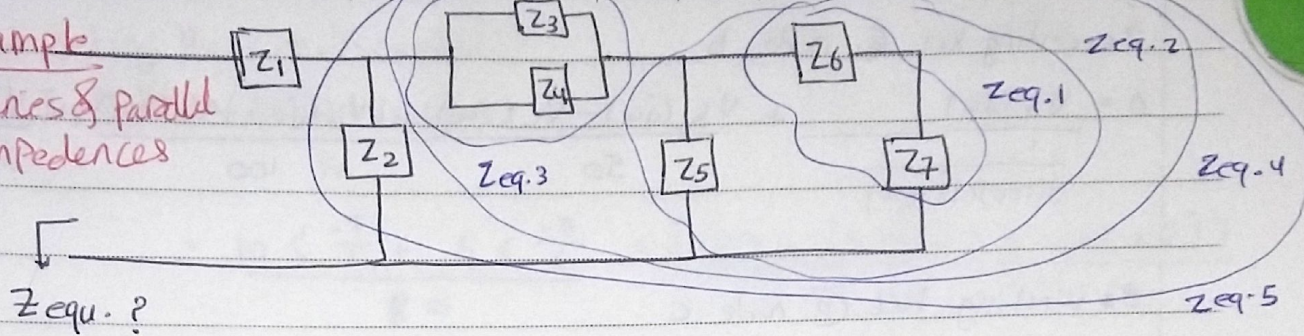
$$Z_R(j\omega) = Z_R = R$$

In exam $\rightarrow$ sin function and period are required to be found.
Solve in complex numbers.



NO.

Example
Series & Parallel
Impedances



$$Z_{eq.1} = Z_6 + Z_7$$

$$Z_{eq.3} = \frac{Z_3 Z_4}{Z_3 + Z_4}$$

$$Z_{eq.2} = \frac{Z_5 Z_{eq.1}}{Z_5 + Z_{eq.1}}$$

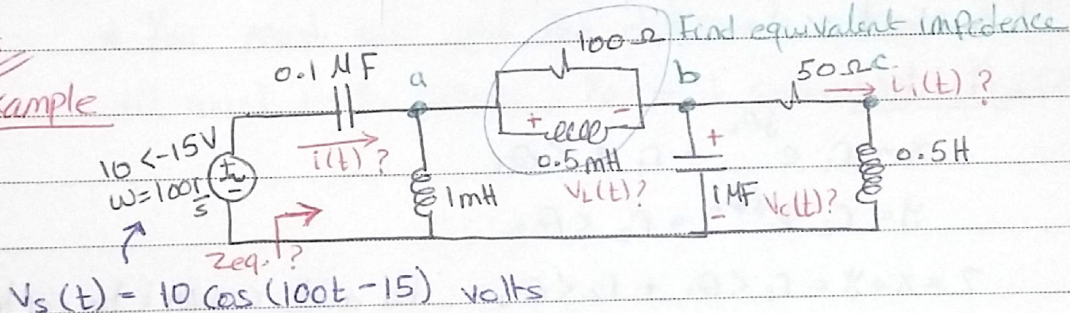
$$Z_{eq.4} = Z_{eq.2} + Z_{eq.3}$$

$$Z_{eq.5} = Z_2 \parallel Z_{eq.4} = \frac{Z_2 Z_{eq.4}}{Z_2 + Z_{eq.4}}$$

$$Z_{eq.} = Z_1 + Z_{eq.5}$$

Deal as impedances as in resistances.

Assignment
Example



$$V_s(t) = 10 \cos(100t - 15) \text{ volts}$$

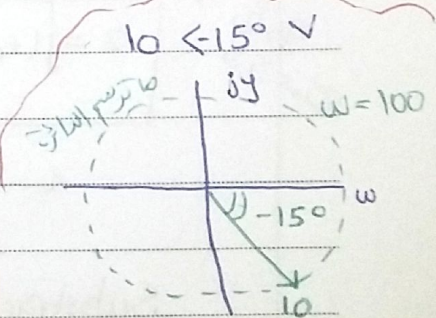
* Repeat by solving it by loop equations? 3 loops in Frequency domain

* By Nodal equations:

1 * writing KCL @ node a:

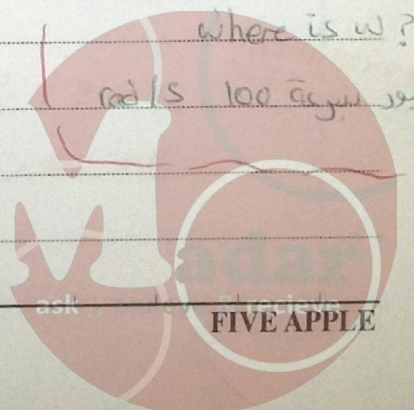
$$\frac{V_a(j\omega) - 10 \angle -15^\circ}{j \cdot 100 = 0.1 \times 10^{-6}} + \frac{V_a(j\omega)}{j(100)(0.001)} + \dots$$

$$\frac{V_a(j\omega) - V_b(j\omega)}{100} + \frac{V_a(j\omega) - V_b(j\omega)}{j(100)(0.5 \times 10^{-3})} = 0$$



where is ω ?

rad/s 100 راديان في الثانية



2* writing KCL @ node b

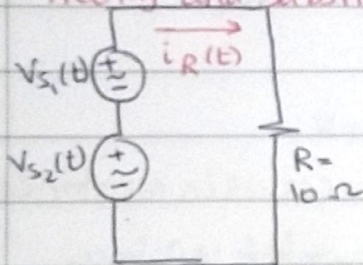
$$0 = \frac{V_b(j\omega)}{j(100)(1 \times 10^{-6})} + \frac{V_b(j\omega) - V_c(j\omega)}{50} + \frac{V_b(j\omega) - V_a(j\omega)}{100} + \frac{V_b(j\omega) - V_a(j\omega)}{j(100)(0.5 \times 10^{-6})}$$

3* Writing KCL @ node c

$$0 = \frac{V_c(j\omega) - V_b(j\omega)}{50} + \frac{V_c(j\omega)}{j(100)(0.5)}$$

3 equations with 3 variables ✓
after finding, return to time domain

→ Adding and subtracting of polar domain (Polar arithmetics)



$$V_{s1}(t) = 10 \sin(20t) \rightarrow 10 \cos(20t - \frac{\pi}{2}) \rightarrow 10 \angle -\frac{\pi}{2} \text{ V}$$

$$V_{s2}(t) = 5 \cos(30t - \frac{\pi}{6}) \rightarrow 5 \angle -\frac{\pi}{6} \text{ Volt}$$

Addition

$$x = r_1 e^{j\theta_1} = r_1 \angle \theta_1$$

$$y = r_2 e^{j\theta_2} = r_2 \angle \theta_2$$

$$z = x + y = r_1 \angle \theta_1 + r_2 \angle \theta_2 \quad \text{to add, convert to cartesian}$$

$$= r_1 \cos \theta_1 + r_2 \cos \theta_2 + j r_1 \sin \theta_1 + j r_2 \sin \theta_2$$

add each
part alone

$$= (r_1 \cos \theta_1 + r_2 \cos \theta_2) + j (r_1 \sin \theta_1 + r_2 \sin \theta_2)$$

نظرة
فنية

$$z = \sqrt{(r_1 \cos \theta_1 + r_2 \cos \theta_2)^2 + (r_1 \sin \theta_1 + r_2 \sin \theta_2)^2} \tan^{-1} \left[\frac{r_1 \sin \theta_1 + r_2 \sin \theta_2}{r_1 \cos \theta_1 + r_2 \cos \theta_2} \right]$$

← magnitude $\equiv$ length

→ angle

Subtraction ↑ sign change

Assignment

$$x = 5 \angle \frac{\pi}{12}$$

Add & subtract

$$y = -2 \angle -\frac{\pi}{6}$$



ask for level & price level

FIVE APPLE

Remember to change in radian

NO.

120°

completing the case discussed

$$I_R(j\omega) = \frac{V_{S1}(j\omega) + V_{S2}(j\omega)}{R}$$

$$= \frac{10 \angle -\frac{\pi}{2} + 5 \angle -\frac{\pi}{6}}{10} = \frac{j(10) + (0.866 \times 5 + -0.5 \times 5j)}{10}$$

$$= \frac{j10 + [4.33012 - 2.5j]}{10} = \frac{4.33012}{10} + j \frac{7.5}{10} = I_R(j\omega)$$

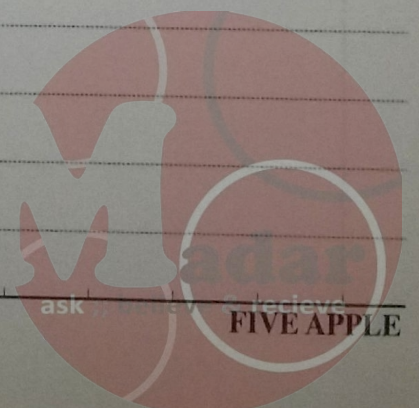
$$I_R(j\omega) = \sqrt{\left(\frac{4.33012}{10}\right)^2 + \left(\frac{7.5}{10}\right)^2} \angle \tan^{-1}\left(\frac{7.5}{4.33012}\right)$$

$$= 0.866 \angle 60^\circ \text{ Ampere}$$

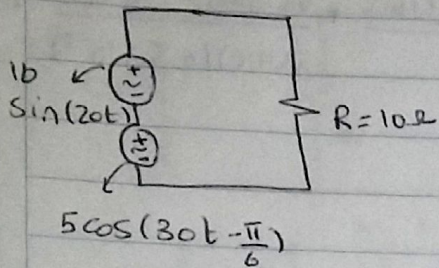
$$i_R(t) = 0.866025 \cos(t + 60^\circ)$$

* You must not add the different numbers of ω .

ω must be the same, You must solve this ذکر کرد



* Solving the previous example in the correct way



$$10 \sin(20t) = 10 \cos(20t - \frac{\pi}{2})$$

$$V_{s1}(j\omega) = 10 \angle -\frac{\pi}{2} \text{ Volt}$$

$$V_{s2}(j\omega) = 5 \angle -\frac{\pi}{6} \text{ Volt}$$

* You can solve in any phase since there is no impedance in ckt.

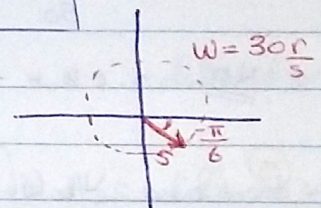
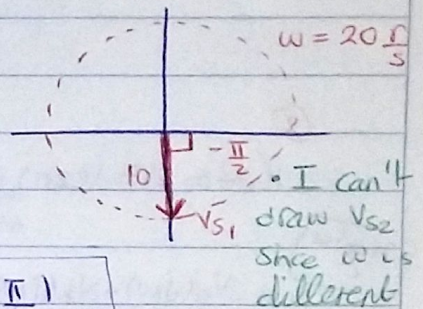
Due to $V_{s1} \rightarrow i_{R1}(t) = \frac{10}{10} \cos(20t - \frac{\pi}{2})$ Ampere

$$i_{R2}(t) = \frac{5}{10} \cos(30t - \frac{\pi}{6}) \text{ Ampere}$$

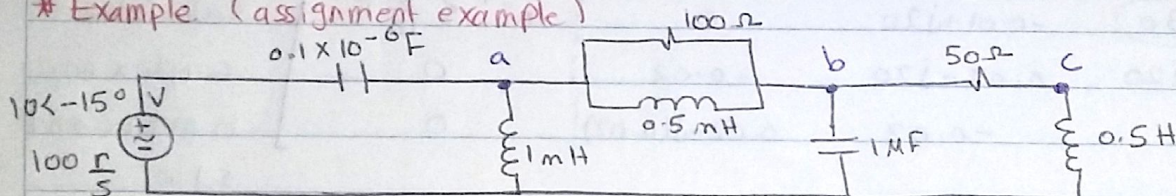
Superposition principle = $\cos(20t - \frac{\pi}{2}) + \frac{1}{2} \cos(30t - \frac{\pi}{6})$

* when you add two phases together, the

resulting phase is different



* Example (assignment example)



KCL (a)

$$\frac{V_a(j\omega) - 10 \angle -15^\circ}{j(100)(0.1 \times 10^{-6})} + \frac{V_a(j\omega)}{j(100)(1 \times 10^{-3})} + \frac{V_a(j\omega) - V_b(j\omega)}{100} + \frac{V_a(j\omega) - V_b(j\omega)}{j(100)(0.5 \times 10^{-3})} = 0$$

$$\underline{V_a} \left[\frac{1}{j(100)(0.1 \times 10^{-6})} - j \frac{1}{(100)(1 \times 10^{-3})} + 0.01 + \frac{1}{j(100)(0.5 \times 10^{-3})} \right] = 0.01 - j(30)$$

$$\underline{V_b} \left[-0.01 + j \frac{1}{100(0.5 \times 10^{-3})} \right] = -0.01 + j(20)$$

$$V_a(j\omega)(0.01 - j(30)) + (-0.01 + j(20))V_b(j\omega) = [10 \angle -15^\circ] j(100)(0.1 \times 10^{-6}) = j1 \times 10^{-4} \angle -15^\circ$$

$$= j1 \times 10^{-4} \angle -\frac{\pi}{12} \quad \text{But } j = \frac{\pi}{2}$$

$$1 \times 10^{-4} \angle -\frac{\pi}{12} + \frac{\pi}{32} = 1 \times 10^{-4} \angle \frac{5\pi}{12}$$

$$\textcircled{1} V_a(j\omega)(0.01 - j(30)) + (-0.01 + j(20))V_b(j\omega) = 1 \times 10^{-4} \angle \frac{5\pi}{12}$$

$$\frac{1}{j} = -j$$

ملاحظة السلك ذو اتجاه واحد

node b

$$\frac{V_b(j\omega)}{j(100)(1 \times 10^{-6})} + \frac{V_b(j\omega) - V_c(j\omega)}{50} + \frac{V_b(j\omega) - V_a(j\omega)}{100} + \frac{V_b(j\omega) - V_a(j\omega)}{j(100)(0.5 \times 10^{-3})} = 0$$

$$\underline{V_a} = -0.01 + j \frac{1}{100(0.5 \times 10^{-3})} = -0.01 + j(20)$$

$$\underline{V_b} = \frac{j(100)(1 \times 10^{-6}) + 0.02 + 0.01 + -j1}{j(100)(0.5 \times 10^{-3})} = 0.03 + -j(20)$$

②

$$(-0.01 + j(20))V_a(j\omega) + (0.03 - j(20))V_b(j\omega) - 0.02V_c(j\omega) = 0$$

node c

$$\frac{V_c(j\omega) - V_b(j\omega)}{50} + \frac{V_c(j\omega)}{j(100)(0.5)} = 0$$

$$\underline{V_c} = \frac{0.02 + -j \frac{1}{100 \times 0.5}}{100 \times 0.5} = 0.02 - j(0.02)$$

③

$$-0.02V_b(j\omega) + (0.02 - j(0.02))V_c(j\omega) = 0$$

Put Now in a matrix

$$\begin{bmatrix} 0.01 - j30 & -0.01 + j20 & 0 \\ -0.01 + j20 & 0.03 - j20 & -0.02 \\ 0 & -0.02 & 0.02 - j(0.02) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 10^{-4} < \frac{5}{12}\pi \\ 0 \\ 0 \end{bmatrix}$$

Calculates of complex no. in matrix can't solve.

$$\begin{bmatrix} 0.01 - j30 & -0.01 + j20 & 0 \\ 0 & 8.888889 \times 10^{-3} + j13.3 & -0.02 \\ 0.03 - j(20) - \left(\frac{-0.01 + j20}{0.01 - j30} \right) (-0.01 + j20) & = 6.0001 - j0.001 - j0.2 - j0.2 - 400 \\ & = -399.9999 - j(0.4)(0.01 + j30) \\ & = \frac{-3.999999 - j11999.997 - j(0.004) + 12}{(0.01)^2 + (30)^2} = \frac{8.000001 - j(12000.001)}{900.0001} \\ & = 8.888889012 \times 10^{-3} + j13.33333296 \end{bmatrix}$$

$$0.03 - j(20) - \left(\frac{-0.01 + j20}{0.01 - j30} \right) (-0.01 + j20) = 6.0001 - j0.001 - j0.2 - j0.2 - 400$$

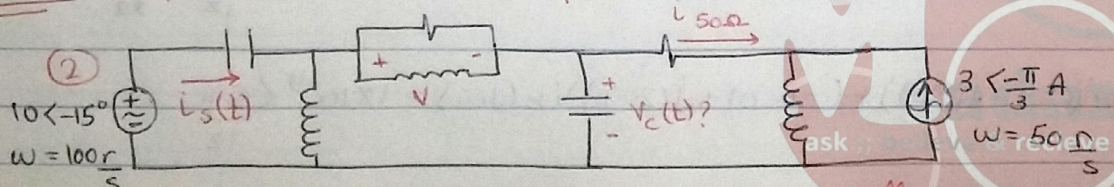
$$= -399.9999 - j(0.4)(0.01 + j30)$$

$$= \frac{-3.999999 - j11999.997 - j(0.004) + 12}{(0.01)^2 + (30)^2} = \frac{8.000001 - j(12000.001)}{900.0001}$$

$$= \frac{-3.999999 - j11999.997 - j(0.004) + 12}{(0.01)^2 + (30)^2} = \frac{8.000001 - j(12000.001)}{900.0001}$$

$$= 8.888889012 \times 10^{-3} + j13.33333296$$

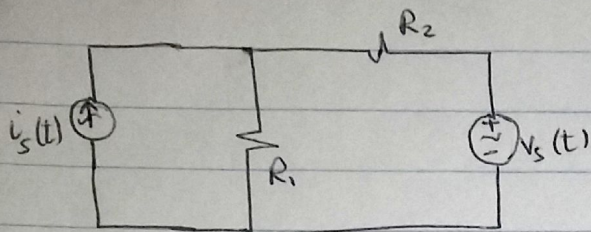
Assignment ① complete the above example go to time domain and find unknown



By superposition it can be solved because different ω .

in complex # use decimal # min. 4 decimal points

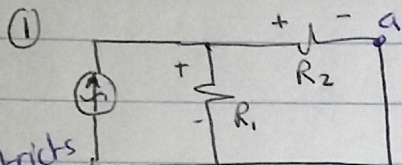
Example on slides



Superposition

$$i_s(t) =$$

$$V_s(t) =$$

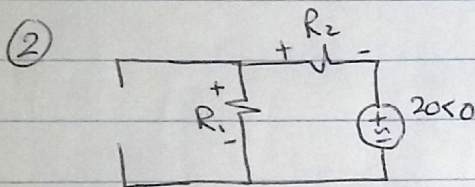


$$i_s(t) = 0.5 \angle 0$$

$$R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{(150)(50)}{200} = 37.5 \Omega$$

$$V_a^{(1)} = V_{R_1}^{(1)} = V_{R_2}^{(1)} = (0.5 \angle 0)(37.5) = 18.75 \text{ V}$$

Subscripts are capital



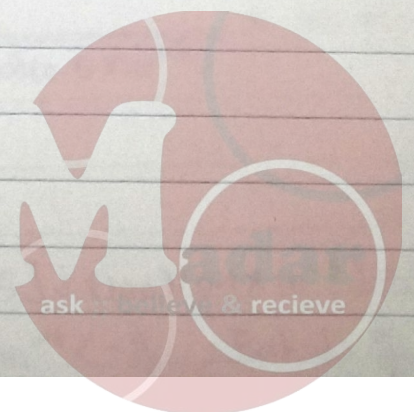
$$V_{R_1}^{(2)} = \left(\frac{R_1}{R_1 + R_2} \right) 20 \angle 0 = \frac{15}{200} (20 \angle 0) = 1.5 \angle 0 \text{ V}$$

$$V_{R_2}^{(2)} = - \left(\frac{R_2}{R_1 + R_2} \right) 20 \angle 0 = - \frac{50}{200} (20 \angle 0) = -5 \angle 0 \text{ V}$$

small letter

$$\rightarrow V_{R_1}(t) = 18.75 \cos(2\pi(100)t) + 1.5 \cos(2\pi(1000)t) \text{ Volts different } \omega$$

$$\rightarrow V_{R_2}(t) = 18.75 \cos(2\pi(100)t) + -5 \cos(2\pi(1000)t) \text{ Volts different } \omega$$



Remember to change in radian

NO.

120°

completing the case discussed

$$I_R(j\omega) = \frac{V_{S1}(j\omega) + V_{S2}(j\omega)}{R}$$

$$= \frac{10 \angle -\frac{\pi}{2} + 5 \angle -\frac{\pi}{6}}{10} = \frac{j(10) + (0.866 \times 5 + -0.5 \times 5j)}{10}$$

$$= \frac{j10 + [4.33012 - 2.5j]}{10} = \frac{4.33012}{10} + j \frac{7.5}{10} = I_R(j\omega)$$

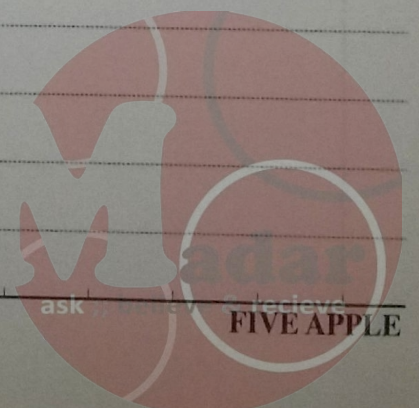
$$I_R(j\omega) = \sqrt{\left(\frac{4.33012}{10}\right)^2 + \left(\frac{7.5}{10}\right)^2} \angle \tan^{-1}\left(\frac{7.5}{4.33012}\right)$$

$$= 0.866 \angle 60^\circ \text{ Ampere}$$

$$i_R(t) = 0.866025 \cos(t + 60^\circ)$$

* You must not add the different numbers of ω .

ω must be the same, You must solve this

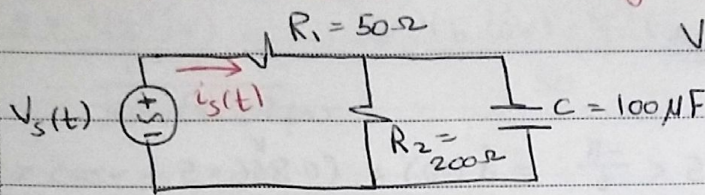


Monday

25/3/2019

NO.

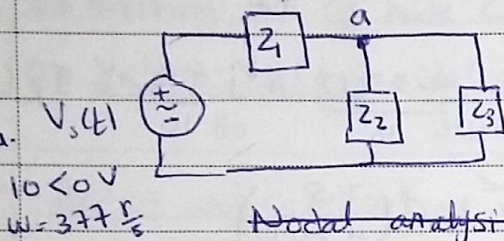
Example from slides (correcting mistakes in slides)



$$V_s(t) = 10 \sin(\omega t + \frac{\pi}{2}) \text{ Volts}$$

$$10 \cos(\omega t)$$

$$f = 60 \text{ Hz} \rightarrow \omega = 2\pi f = 377 \text{ rad/s}$$



$$Z_1 = R_1 = 50 \Omega$$

$$Z_2 = R_2 = 200 \Omega$$

$$Z_3 = \frac{1}{j\omega C} = \frac{1}{j(377)(10^{-4})}$$

Nodal analysis

$$i_s(t) = \frac{V_s(j\omega) - V_a(j\omega)}{Z_1} = \frac{V_a(j\omega)}{Z_2 \parallel Z_3}$$

$$\frac{V_s(j\omega)}{Z_1} = V_a(j\omega) \left[\frac{1}{Z_2 \parallel Z_3} + \frac{1}{Z_1} \right] \text{ Solve for } V_a(j\omega)$$

$$\frac{V_s(j\omega)}{Z_1} = V_a(j\omega) \left[\frac{Z_3 + Z_2}{Z_2 Z_3} + \frac{1}{Z_1} \right]$$

$$\frac{V_s(j\omega)}{R_1} = V_a(j\omega) \left[\frac{200 R_2 + \frac{1}{j\omega C} + 1}{R_2 \cdot \frac{1}{j\omega C}} + \frac{1}{R_1} \right] \rightarrow \text{after making common}$$

$$\frac{V_s(j\omega)}{R_1} = V_a(j\omega) \left[\frac{j\omega C R_2 + 1 + \frac{1}{R_1}}{R_2} \right] = V_a(j\omega) \left[\frac{j\omega C R_2 R_1 + R_1 + R_2}{R_1 R_2} \right]$$

$$V_a(j\omega) = \frac{V_s(j\omega) R_1 R_2}{R_1 (j\omega C R_2 R_1 + R_1 + R_2)}$$

*Polar division is easier than complex division

$$= 10 \angle 0^\circ \times \frac{200}{j(377)(10^{-4})(200)(50) + 50 + 200}$$

$$V_a(j\omega) = 4.421 \angle -0.9852 \text{ in rad. (no small degree)}$$

$$= 4.421 \angle 56.45^\circ \text{ volt}$$

$$\rightarrow \text{Find } I_s(j\omega) = \frac{V_s(j\omega) - V_a(j\omega)}{R_1} = \frac{10 \angle 0^\circ - 4.421 \angle -0.9852}{50}$$

Polar addition is can't be done

$$= 0.1681 \angle 0.4537 \text{ Ampere.}$$

$$\rightarrow i_s(t) = 0.1681 \cos(377t + 0.4537) \text{ in rad.}$$

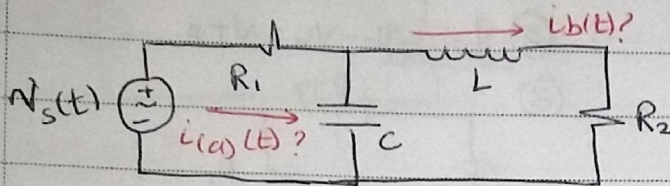
But I want in the time domain

You must be consistent in angles

You can find $i_s(1s)$ for example, just substitute in the above equation.

FIVE APPLE

Example from slides



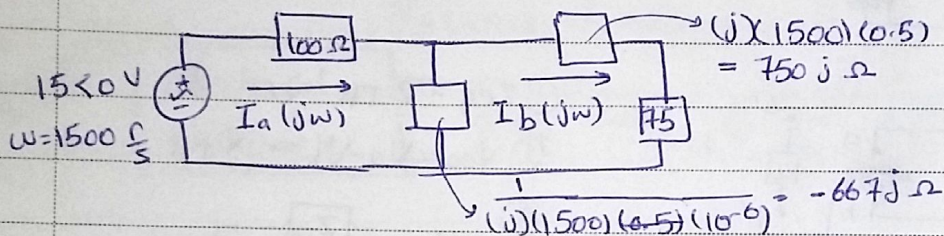
$$v_s(t) = 15 \sin(1500t + \frac{\pi}{2}) \text{ V}$$

$$R_1 = 100 \Omega \quad R_2 = 75 \Omega$$

$$L = 0.5 \text{ H} \quad C = 1 \mu\text{F}$$

Transfer the circuit to the ω domain and impedances (phasor notation)

* Don't put the sources in the cartesian coordinates, ONLY impedances can be written in the cartesian.



Can be solved by nodal and loop analysis.

Since I want current $\rightarrow$ loop analysis

$$(I_a) \quad -v_s(j\omega) + V_{100\Omega}(j\omega) + V_C(j\omega) = 0$$

$$-15\angle 0 + 100 I_a(j\omega) + \frac{1}{(j)(1500)(10^{-6})} (I_a(j\omega) - I_b(j\omega)) = 0$$

$$\rightarrow I_a(j\omega) [100 - j66.7] + I_b(j\omega) [66.7j] = 15\angle 0$$

$$(I_b) \quad -66.7j [I_b(j\omega) - I_a(j\omega)] + j(1500)(0.5) I_b(j\omega) + 75 I_b(j\omega) = 0$$

$$\rightarrow I_a(j\omega) [66.7j] + I_b(j\omega) [75 + 83j] = 0$$

- By gaussian elimination

$$\begin{bmatrix} 100 - j66.7 & 66.7j & 15\angle 0 \\ 66.7j & 75 + 83j & 0 \end{bmatrix}$$

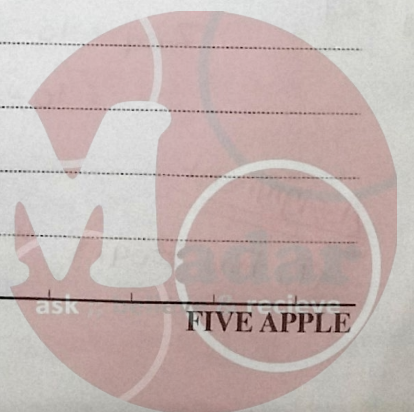
$$I_a(j\omega) = 0.0032 \angle 0.917 \text{ Ampere}$$

$$I_b(j\omega) = 0.019 \angle -1.49 \text{ Ampere}$$

$\Rightarrow$ In Time Domain (Converting From Frequency domain)

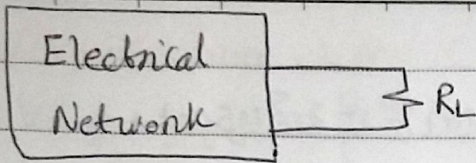
$$i_a(t) = 3.2 \cos(1500t + 0.917) \text{ mA}$$

$$i_b(t) = 19 \cos(1500t - 1.49) \text{ mA}$$



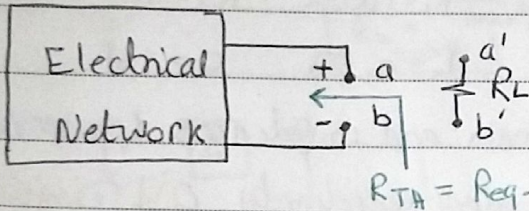
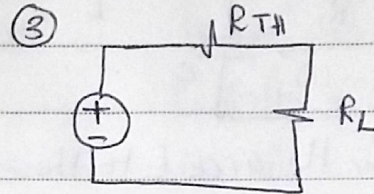
NO. Thevenin equivalent

DC

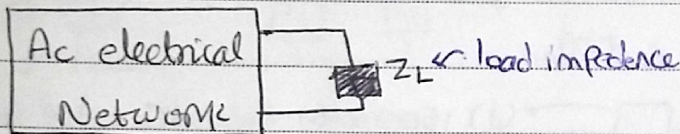


① R_{TH} ckt off

② $V_{oc} = V_a - V_b = V_{TH}$

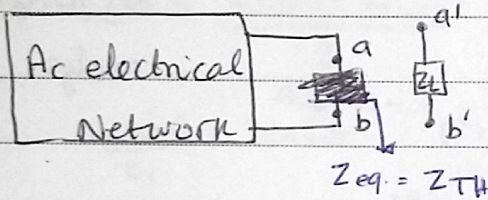
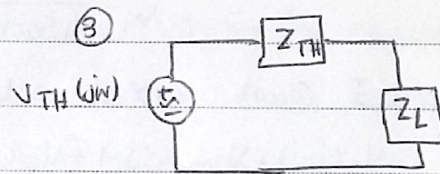


AC



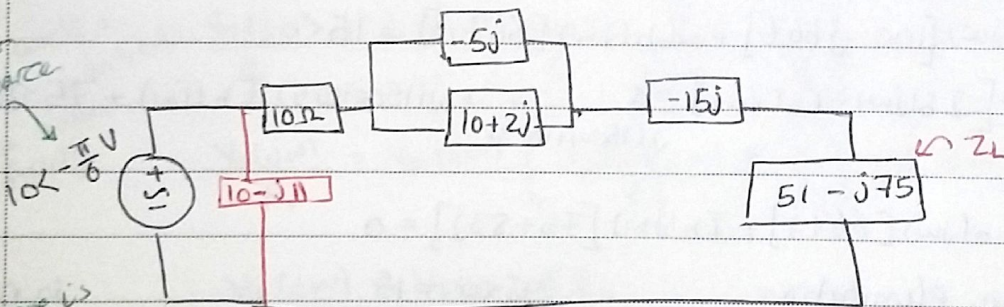
① $Z_{eq} = Z_{TH}$ ckt off

② $V_{oc} = V_a - V_b = V_{TH}$

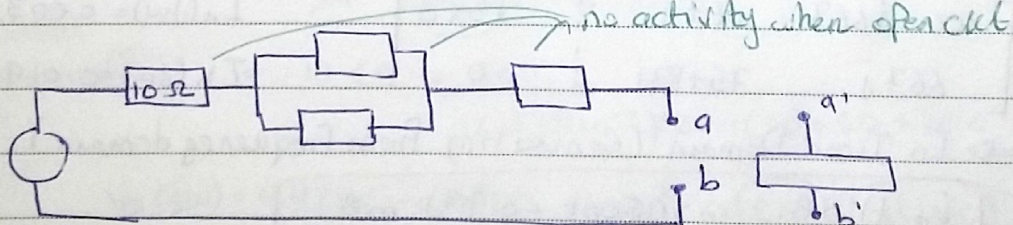


look to book example
maybe current source or voltage source

Example



maybe there is dependent source
Put source
Find current
Find Z_{eq}



no activity when open ckt

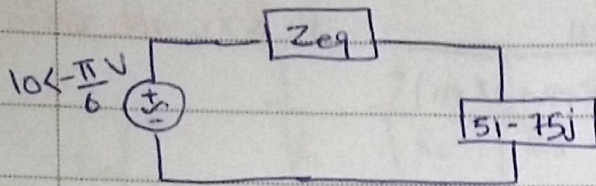
$$Z_{eq} = 10 - j15 + (-j5 \parallel (10 + j2))$$

$$= 10 - j15 + \frac{(-j5)(10 + j2)}{-(j5 + 10 + j2)}$$

$$V_{oc} = \text{Source} = 10 \angle -\frac{\pi}{6} \text{ V}$$

Assignment

- ① Find Z_{eq} .? ω is required to initiate impedance but here we have them
 ω is required for time domain



Assignment

② ckt Thevenin with additional impedance (in red)

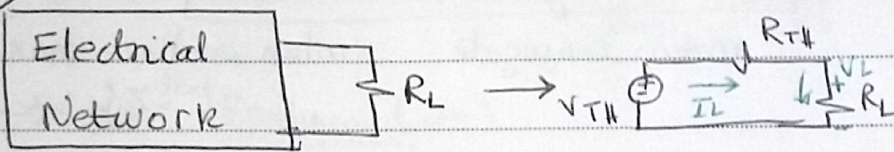
Find

Wednesday

27/3/2019

Maximum Power Transfer

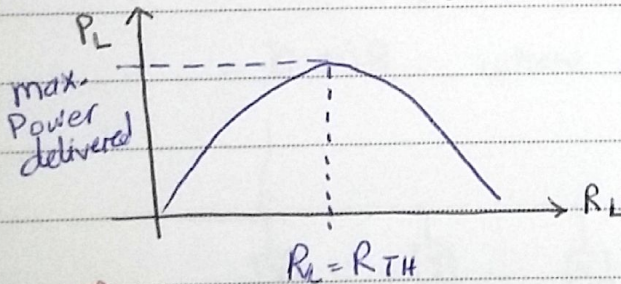
Previously



$$P_L = I_L V_L = \frac{V_{TH}}{R_L + R_{TH}} \cdot V_L \quad V_L = \frac{R_L}{R_L + R_{TH}} V_{TH}$$

$$= \frac{R_L V_{TH}^2}{(R_L + R_{TH})^2} \quad V_L = \frac{R_L}{R_L + R_{TH}} V_{TH}$$

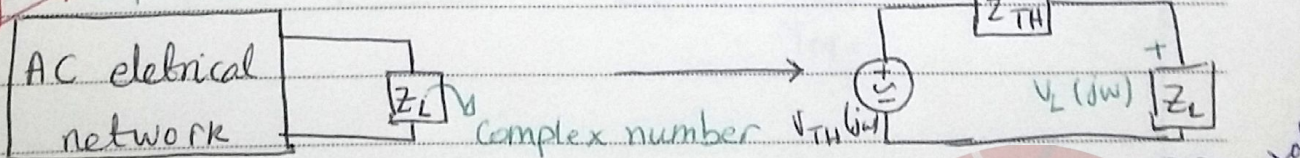
max. power $\frac{dP_L}{dR_L} \bigg|_{R_L=R_L^*} = 0 \rightarrow \boxed{R_L^* = R_{TH}}$



AC maximum Power delivery

* impedances and sources are AC, maybe have DC *

in exam



$$P_{ZL} = I_L(jw) V_L(jw)$$

$$I_L(jw) = \frac{V_{TH}(jw)}{Z_L(jw) + Z_{TH}(jw)}$$

$$V_L(jw) = \frac{Z_L}{Z_{TH} + Z_L} V_{TH}(jw)$$

$$V_L(jw) = \frac{Z_L}{Z_{TH} + Z_L} V_{TH}(jw)$$

$$Z_L = R_L + jX_L \quad \text{Reactance}$$

$$Z_{TH} = R_{TH} + jX_{TH} \quad \text{Reactance}$$

real number (not resistance)

ask

FIVE APPLE

NO.

$$P_{ZL} = \frac{Z_L V_{TH}^2}{(Z_{TH} + Z_L)^2} = \frac{(R_L + jX_L)^2 V_{TH}}{(R_L + jX_L + R_{TH} + jX_{TH})^2}$$

R_L & X_L are variables

Partial derivative

$$\frac{\partial P_{ZL}}{\partial R_L} \bigg|_{R_L = R_L^{best}} = 0 \rightarrow ①$$

$$\frac{\partial P_{ZL}}{\partial X_L} \bigg|_{X_L = X_L^{best}} = 0 \rightarrow ②$$

Assignment

Derive this partial derivatives

The final answer $R_L^{best} = R_{TH}$ $X_L^{best} = -X_{TH}$

$$Z_L^{best} = Z_{TH}^* = R_{TH} - jX_{TH}$$

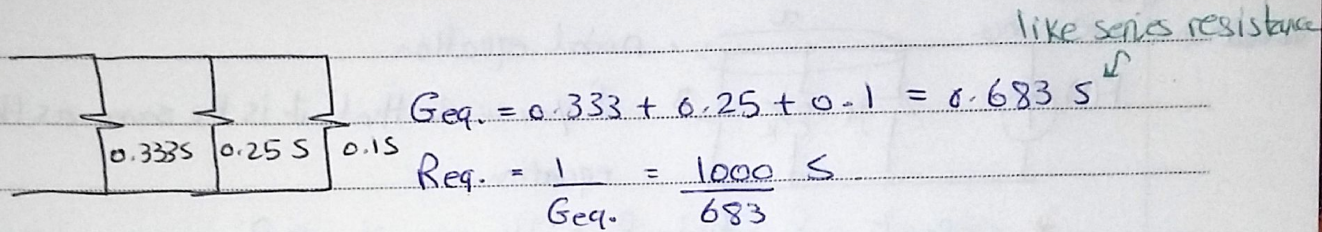
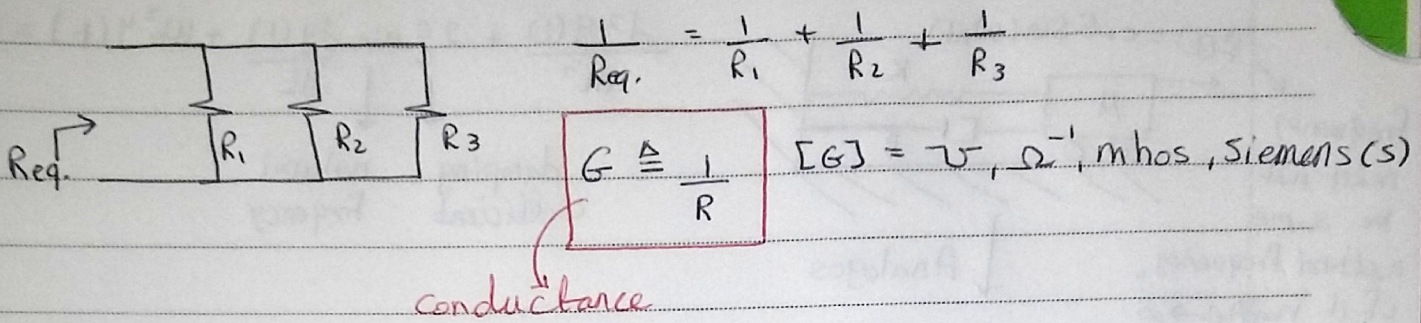
means conjugate

when we have max. power?

$$Z_L^{best} = Z_{TH}^* \text{ we can see}$$



NO. Admittance



$Z(j\omega)$ impedance (Ω)

$Y(j\omega) \triangleq 1/Z(j\omega)$ Admittance (it admits current) units of conductance

$Z(j\omega) = R + jX$

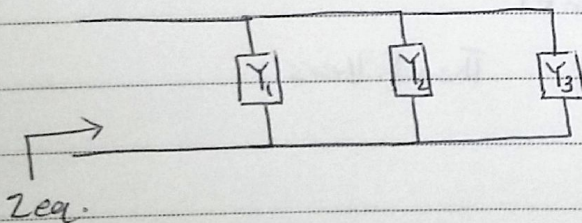
resistive component reactive components (reactance)

ωL $\frac{1}{\omega C}$

$Y(j\omega) = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2}$

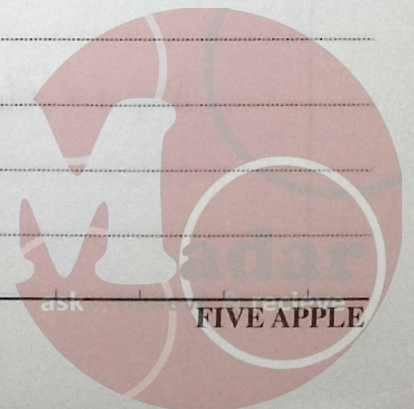
(G) conductance (B) susceptance

$= G + jB$ where $G = \frac{R}{R^2 + X^2}$ $B = \frac{-X}{R^2 + X^2}$



$Y_{eq} = Y_1 + Y_2 + Y_3$

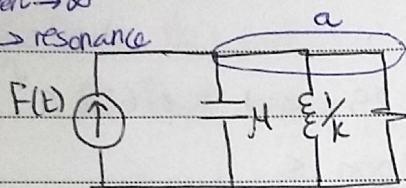
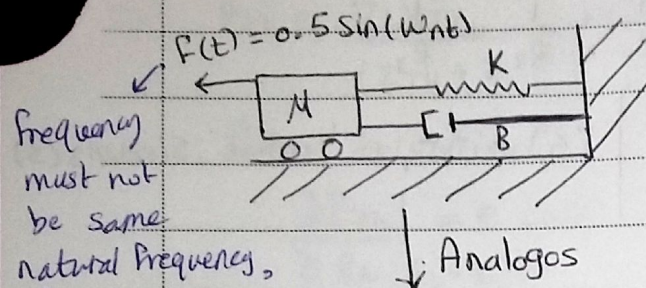
$Z_{eq} = \frac{1}{Y_{eq}}$



NO. RLC Resonance it is vibration preferred
not preferred but in electrical

$$\frac{d^2 y(t)}{dt^2} + 2\zeta \omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = F(t)$$

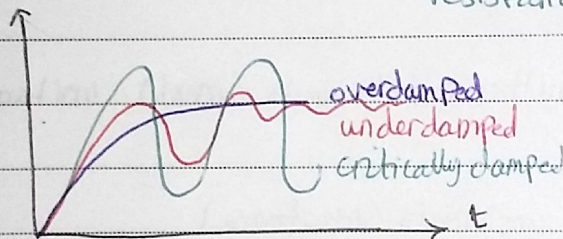
damping coefficient natural frequency



• nodal equation

if you write that, it is the same as the above equation.

* Capacitor & inductor in the circuit $\Rightarrow$ Resonance exist.
resistance



e.g. water hammering

Pressure is high and pipes are broken / San Francisco bridge requires "pipe line distillation"

TV $y(t) = 10 \sin(30000t)$ [changing frequency]

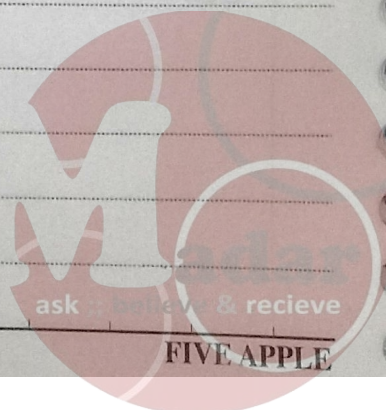
$x(t) = A \sin(30000t)$

$x(t)y(t) = 10A \sin^2(30000t)$

$\sin^2(\omega t) = \frac{1 - \cos 2\omega t}{2}$ Then Filters

Radio
[changing capacitor]

Popular electronics 1975



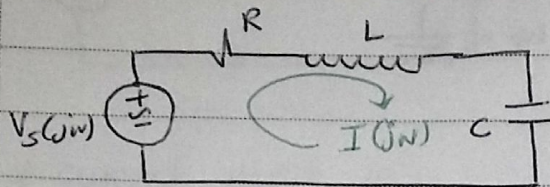
- radios, radar, phones, networks & computer → resonance
- actually no $-ve$ w , but mathematically we use $-ve$ w .

Sunday 1/4/2019

NO. RLC Resonance

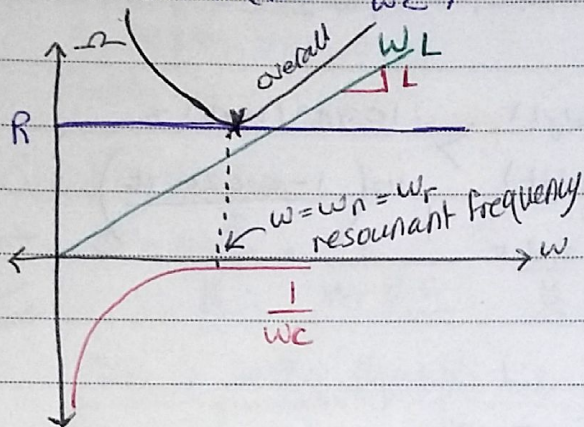
*2 Types :-

1. Series RLC Resonance



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$



$$|Z_{eq}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

• min. at $Z_{eq} = R \Rightarrow$ max. current

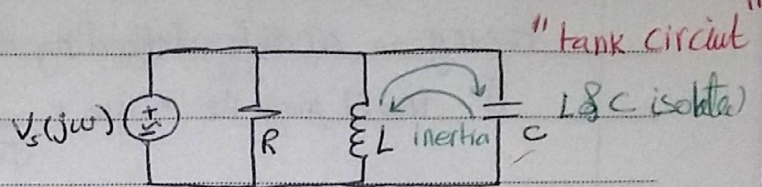
• At resonance:

$$\omega_r L - \frac{1}{\omega_r C} = 0 \quad \omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

2. Parallel RLC Resonance



$$Y_{eq}(j\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{j\omega L + R - \omega^2 LCR}{j\omega L R}$$

$$= (R - \omega^2 LCR) + j\omega L / j\omega L R$$

$$Z_{eq}(j\omega) = j\omega L R / (R - \omega^2 LCR + j\omega L)$$

multiply by complex conjugate

$$= \frac{j\omega L R}{R - \omega^2 LCR + j\omega L} \times \frac{(R - \omega^2 LCR) - j\omega L}{(R - \omega^2 LCR) - j\omega L}$$

$$Z_{eq} = \frac{j\omega L R (R - \omega^2 LCR) + \omega^2 L^2 R}{(R - \omega^2 LCR)^2 + (\omega L)^2}$$

@ resonance, imaginary part disappears

$$j\omega L R (R - \omega_r^2 LCR) = 0 \quad (\text{when } \omega = \omega_r)$$

$$\omega_r L R^2 (1 - \omega_r^2 LC) = 0$$

$$1 = \omega_r^2 LC \quad \omega_r^2 = \frac{1}{LC} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

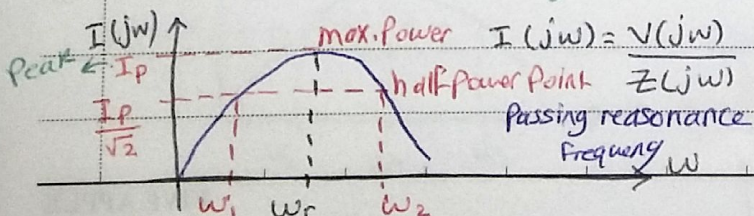
equal either parallel or series

• like inertia flowing to tank and then emptying tank, like isolated

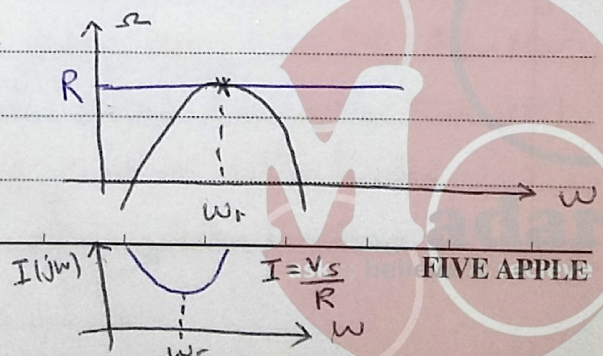
- in natural, there is no $-ve$ w

- capacitor & inductor cancel each other

at resonance. only R appears



$$\text{halfPower} = I^2 R = \frac{I_p^2 R}{2}$$



FIVE APPLE

* Resonance circuit is also called tuned circuit.

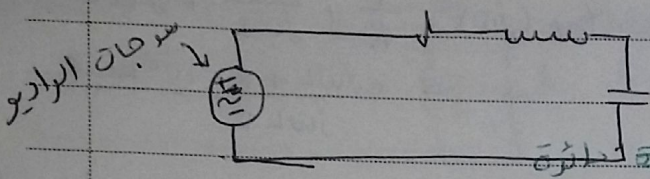
NO.

* In radio, we use series RLC, since I want to catch max.

Power, maximum current - OR in TV, radar, --

tuning on circuit depending on $\omega_r = \frac{1}{\sqrt{LC}}$

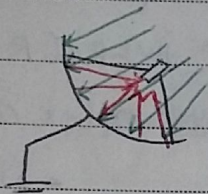
if $103.7 \text{ MHz} = \omega_r = \frac{1}{\sqrt{LC}}$



when tuning, the capacitor plates vary until having a value.

parallel plates

* For TV



$$\frac{5 \sin(\omega_r t)}{2 \sin(\omega_r t)}$$

$$\frac{10 \sin^2(\omega_r t)}{10 \left(\frac{1 - \cos 2\omega_r t}{2} \right)}$$

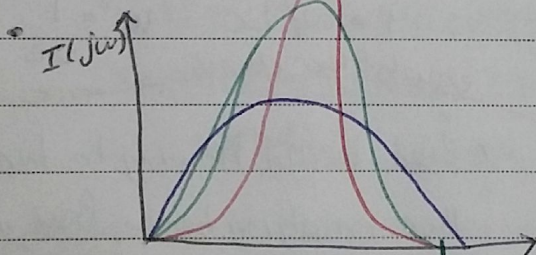
→ Back to Figures :

• ω_1, ω_2 are called half power frequencies

station disappears station disappears

• "Band width" = $B = \omega_2 - \omega_1$

• station clear at ω_r (approximately at $\frac{1}{2} B$)

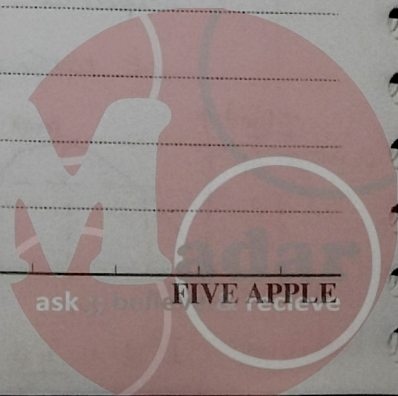


∴ best filter to filter the tune

problem when another station is

in the band width.

↓ band width ↑ height

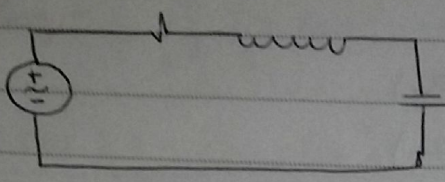


• solve examples on series resonance in slides

Wednesday 3/4

NO. Review & others

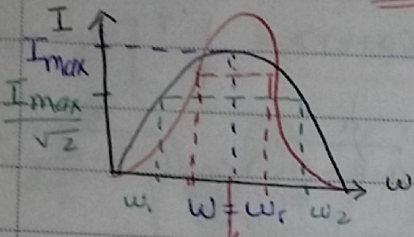
Series



$$\omega_r = \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{units (r/s)} \quad \omega = 2\pi f$$

$$F_r = \frac{1}{2\pi \sqrt{LC}}$$

* current w.r.t ω



ω_1, ω_2 : half power frequencies

Since power = $I^2 R$ (I & C are cancelled @ resoi.)

$$\text{Power} = \left(\frac{I_{\max}}{\sqrt{2}} \right)^2 R = \frac{I_{\max}^2 R}{2} \quad \text{half}$$

Find it in phaser domain

You can get ω_1 & ω_2

better filter band width $\downarrow$ (less tuning)

* Quality Factor

$$[R] = \frac{\text{time}}{\text{time}} = \frac{1}{\text{time}}$$

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r R C} = \frac{\omega_r}{B}$$

B : band width $[Q] = \text{unitless}$

$\uparrow Q$ is better By L/R big / RC small / B small

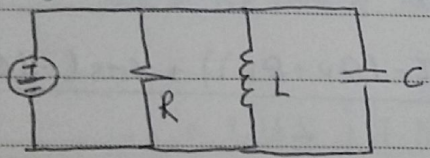
$$\omega_r = \sqrt{\omega_1 \omega_2} \quad [\omega_r] = \text{r/s}$$

$$B = \text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_r}{Q} \quad [B] = \text{r/s}$$

For ~~both~~ ONLY Parallel and series circuits

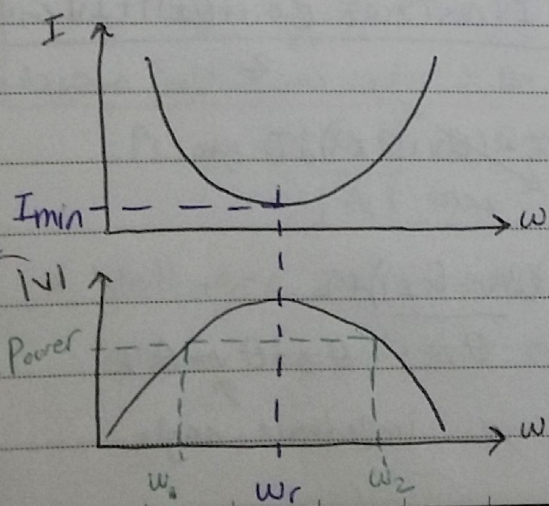
Parallel
"Tank ext"

high impedance at resonance $\rightarrow$ small current.



$$Q = \frac{R}{\omega_r L} = \frac{\omega_r R}{B} = \omega_r R C \quad \omega_r = \sqrt{\omega_1 \omega_2}$$

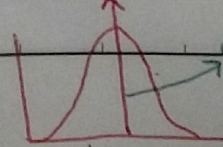
$$B = \frac{1}{R C} = \frac{\omega_r}{Q} = \omega_2 - \omega_1$$



IF $Q \gg 10$: $\omega_1, \omega_2 = \omega_r \pm \frac{B}{2}$

* Q must not be < 1 , order of magnitude is 10, 100, 1000, as $\uparrow Q$ $\uparrow$ better common preferred super (better than 100)

* if Q is $\infty \rightarrow$ delta function

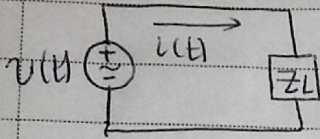


delta function

ask FIVEAPPLE

NO. Instantaneous & average power

$$p(t) = i(t)v(t)$$



$$i(t) = I \cos(\omega t - \theta_I)$$

$$v(t) = V \cos(\omega t - \theta_V)$$

- $p(t) = IV \cos(\omega t - \theta_V) \cos(\omega t - \theta_I)$ "instantaneous power"

- "average power" $P_{avg} = \bar{p}(t) = \langle p(t) \rangle = \frac{1}{T} \int_0^T p(\tau) d\tau$

$$= \frac{1}{T} \int_0^T \underbrace{IV \cos(\omega \tau - \theta_V)}_a \underbrace{\cos(\omega \tau - \theta_I)}_b d\tau$$

Different angles

From calculus

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$$

$$\text{Now, } \cos(a+b) + \cos(a-b) = \cos a \cos b - \sin a \sin b + \cos a \cos b + \sin a \sin b$$

$$\rightarrow \boxed{\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}} \text{ substitute above}$$

$$a+b = 2\omega\tau - \theta_V - \theta_I = 2\omega\tau - (\theta_V + \theta_I)$$

$$a-b = -\theta_V + \theta_I = -(\theta_V - \theta_I)$$

$$\Rightarrow \cos(\omega\tau - \theta_V) \cos(\omega\tau - \theta_I) = \frac{\cos(2\omega\tau - (\theta_V + \theta_I)) + \cos(-(\theta_V - \theta_I))}{2}$$

$$IV (\cos(\omega\tau - \theta_V) \cos(\omega\tau - \theta_I)) = \frac{IV \cos(2\omega\tau - (\theta_V + \theta_I)) + IV \cos(-(\theta_V - \theta_I))}{2}$$

$$\Rightarrow \bar{p}(t) = P_{avg} = \frac{1}{T} \int_0^T \frac{IV}{2} \cos(2\omega\tau - (\theta_V + \theta_I)) d\tau +$$

2 periods cancel each other

$$\frac{1}{T} \int_0^T \frac{IV}{2} \cos(-(\theta_V - \theta_I)) d\tau$$

$\theta_2 = \theta_V - \theta_I$
impedance angle

FIVE APPLE

ask, believe & receive

Complex Power is not included in the exam

choose the 1ans

NO.

$$P_{avg} = \frac{1}{T} \int_0^T \frac{IV}{2} \cos(-(\theta_2 - \theta_1)) d\tau = \frac{VI}{2} \cos \theta_2$$

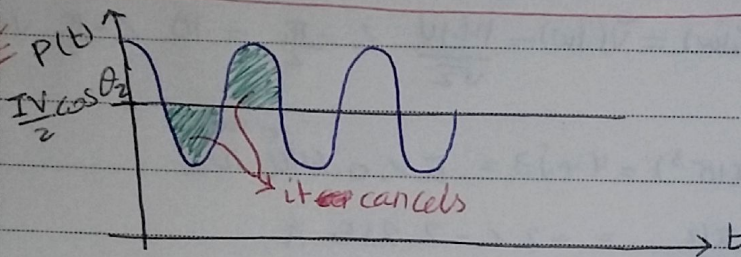
DC Power is

$$(\cos \theta_2 = \cos -\theta_2)$$

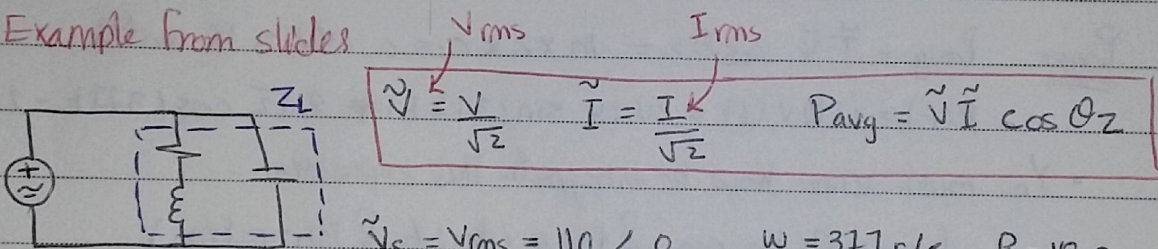
$$P(t) = \underbrace{\frac{IV}{2} \cos \theta_2}_{P_{avg} \rightarrow \text{DC Power}} + \underbrace{\frac{1}{T} \int_0^T \frac{IV}{2} \cos(2\omega t - (\theta_2 - \theta_1)) d\tau}_{\text{Pulsating Power}}$$

In example

Find the



Example from slides



$$\tilde{V}_s = V_{rms} = 110 \angle 0 \quad \omega = 377 \text{ rad/s} \quad R = 10 \Omega \quad L = 0.05 \text{ H}$$

$C = 470 \text{ nF}$ Find the average power dissipated by the load?

$$Z_L = R + j\omega L \parallel j\omega C = R + j\omega L \times \frac{1}{j\omega C} = \frac{Rj\omega L}{R + j\omega L + \frac{1}{j\omega C}} = \frac{Rj\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$= 1.16 - j7.18 \Omega = 7.27 \angle -1.41 \text{ rad}$$

$$P_{avg} = \frac{VI}{2} \cos \theta_2 = \frac{1}{2} \frac{V^2}{|Z|} \cos \theta_2 = \frac{1}{2} I^2 |Z| \cos \theta_2$$

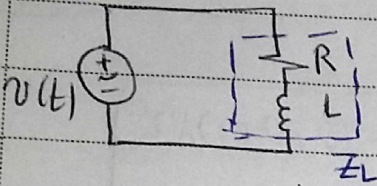
degrees angle is between vector of V & I

angle between $\tilde{I}$ & $\tilde{V}$ is existing, so don't take it

$$P_{L, avg} = \frac{|\tilde{V}_L|^2}{|Z_L|} \cos \theta_2 = \frac{(110)^2}{7.27} \cos -1.41 = 266 \text{ Watts}$$

* Voltages and currents are taken as magnitude since the angle is existing.

- Example from slides



$$v(t) = 14.14 \sin \omega t \quad \omega = 377 \text{ rad/s}$$

$$R = 4 \Omega \quad L = 8 \text{ mH}$$

Find instantaneous and average power dissipated across the load.

$$v(t) = 14.14 \sin 377t = 14.14 \left(\cos 377t - \frac{\pi}{2} \right)$$

$$V_s(j\omega) = 14.14 \angle -\frac{\pi}{2} \text{ V.olt.}$$

$$\tilde{V}_{\text{rms}}(j\omega) = \tilde{V}(j\omega) = \frac{14.14}{\sqrt{2}} \angle -\frac{\pi}{2} = 10 \angle -\frac{\pi}{2} \text{ Volts.}$$

$$Z_L = 4 + j(377)(8 \times 10^{-3}) = 4 + j3 = 5 \angle 0.644 \text{ rad}$$

$$I_{\text{rms}} = \tilde{I} = \frac{\tilde{V}}{Z} = \frac{10 \angle -\pi/2}{5 \angle 0.644} = 2 \angle -2.215 \text{ A}$$

Power $P_{\text{avg}} = \tilde{I} \tilde{V} \cos \theta_Z = 10 \times 2 \times \cos 0.644 = 16 \text{ watts}$

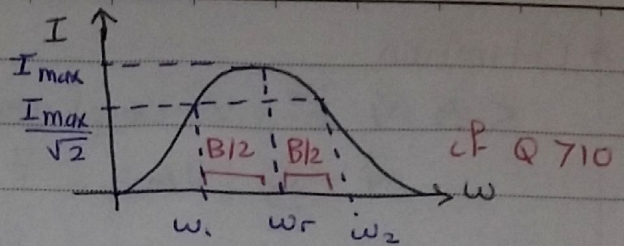
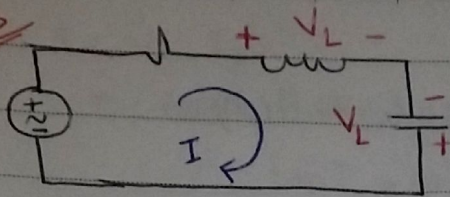
$$P(t) = i(t)v(t) = 14.14 \sin(377t) * 2\sqrt{2} \cos(377t - 2.215) \text{ W}$$

• You must know how to integrate this expression.

(convert to cos. & trig identity & integration)



Series



$Q > 10$ $\omega_1 = \omega_r - \frac{B}{2}$ $\omega_2 = \omega_r + \frac{B}{2}$
if not maybe the curve will be not symmetric

where B : band width $B = \omega_2 - \omega_1$

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r R C} = \frac{\omega_r}{B} \quad \omega_r = \sqrt{\omega_1 \omega_2} = \frac{1}{\sqrt{LC}}$$

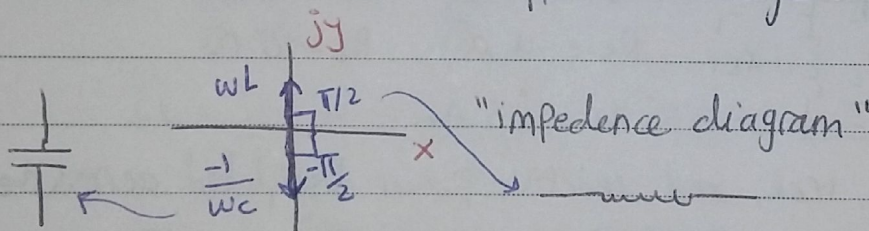
$$B = \frac{R}{L} = \frac{\omega_r}{Q}$$

* when $\omega = \omega_r = \frac{1}{\sqrt{LC}}$ $\rightarrow$ resonance

$\rightarrow V_L$ +ve because the impedance is +ve
 $\rightarrow V_C$ -ve because the impedance is -ve

Parallel

capacitor & inductor cancel because of opposite current, while in series case because of opposite voltage.



more than 1 impedance
can be drawn ONLY
of the same ω .

$$Z_C = \frac{V_C(j\omega)}{I_C(j\omega)}$$

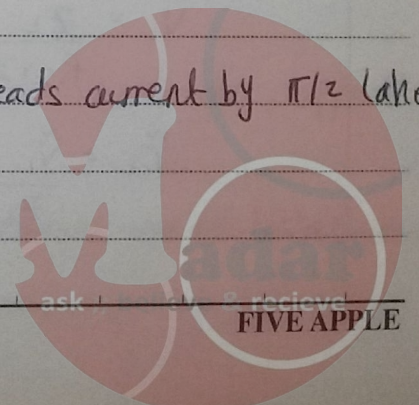
-ve angle $\rightarrow$ voltage

lags current by $\pi/2$

-ve

$$Z_L = \frac{V_L(j\omega)}{I_L(j\omega)}$$

+ve angle $\rightarrow$ voltage leads current by $\pi/2$ (ahead of)



* Derivative with current conjugate $\rightarrow Z_{TH}^* = Z_L$

* Sources can't be written as rectangular, but if add/subtract the conversion we use the rectangular representation. THEN, return to polar.
NO. **Complex power (S)**

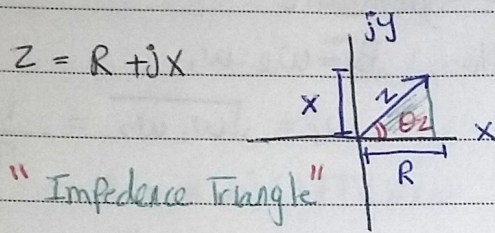
* Definition

$S \triangleq \tilde{V} \tilde{I}^*$ if in phasor it is easy to multiply.

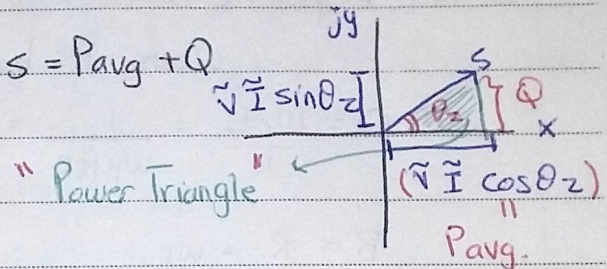
$$\tilde{V} = \frac{V}{\sqrt{2}} = V_{rms} \quad \tilde{I} = \frac{I}{\sqrt{2}} = I_{rms}$$

$$S = \tilde{V} \tilde{I} \cos \theta_z + j \tilde{V} \tilde{I} \sin \theta_z$$

$$Z = R + jX$$



$$S = P_{avg} + Q$$



• Both triangles are similar (same angles)

$$Q = \tilde{V} \tilde{I} \sin \theta_z \leftarrow \text{"reactive power"} \quad [Q] = \text{VAR} : \text{Volt-amp reactive}$$

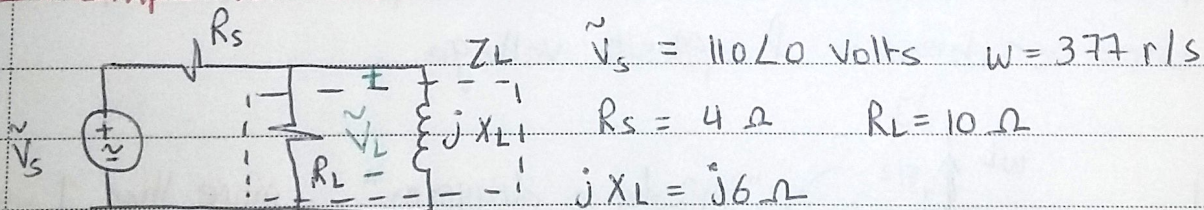
$$P_{avg} = \tilde{V} \tilde{I} \cos \theta_z \leftarrow \text{avg. power} \quad [P_{avg}] = \text{Watts}$$

$$S = P_{avg} + Q$$

$$[S] = \text{VA} : \text{Volt-Amp}$$

• Also, we conclude that P_{avg} is consumed by R , and Q is consumed by the complex part (as losses)

Example from slides



• Find complex, real, and reactive power dissipated across the load

$$Z_L = R_L \parallel jX_L = \frac{R_L(jX_L)}{R_L + jX_L} = \frac{(10)(j6)}{10 + j6} = 5.145 \angle 1.03 \text{ ohms}$$

$$\tilde{V}_L = \frac{Z_L}{R_s + Z_L} \tilde{V}_s = 110 \frac{5.145 \angle 1.03}{5.145 \angle 1.03 + 4} = 70.9 \angle 0.444 \text{ V}$$

$$\tilde{I}_L = \frac{\tilde{V}_L}{Z_L} = \frac{70.9 \angle 0.444}{5.145 \angle 1.03} = 13.8 \angle -0.586 \text{ A}$$

$$\begin{aligned}
 S &= V I^* \\
 &= I I^* R \\
 &= \frac{V V^*}{R}
 \end{aligned}$$

تو پاور استورجیو
503 دیر پی
the other part is losses

NO.

$$S = \tilde{V}_L I_L^*$$

$$\begin{aligned}
 &= (70.9 \angle 0.444)(13.8 \angle 0.586) = 978 \angle 1.03 \text{ VA} \\
 &= 503 + j 839 \text{ VA}
 \end{aligned}$$

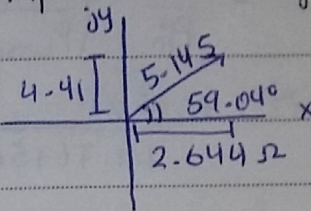
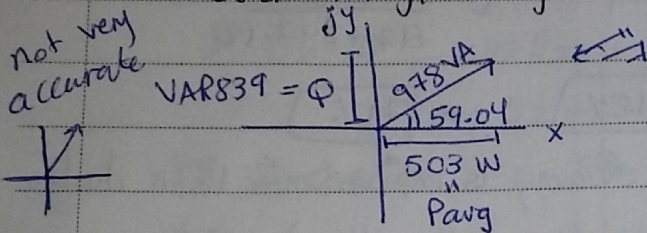
$$P_L = 503 \text{ Watts} \quad Q = 839 \text{ VAR}$$

We conclude :-

• angle in complex power = angle in the impedance

"Power Triangle diagram":

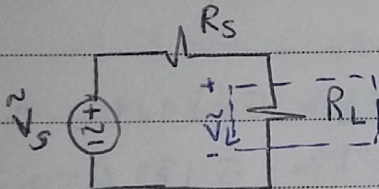
Impedence Triangle



503 Watts on 2.644Ω

839 Watts $5.145 \times 4.41 \Omega$

If inductor is removed from the circuit (like if $s x_L = j\infty$)



$$Z_L = R_L \angle 0 = 10 \Omega$$

$$\tilde{V}_L = \frac{R_L}{R_s + R_L} \tilde{V}_s = \frac{10}{14} \times 110 = 78.6 \angle 0 \text{ V}$$

$$\tilde{I}_L = \frac{\tilde{V}_L}{R_L} = \frac{78.6 \angle 0}{10} = 7.86 \angle 0 \text{ A}$$

$$S_L = \tilde{V}_L \tilde{I}_L^* = (78.6 \angle 0)(7.86 \angle 0) = 617 \angle 0 = 617 + j0 \text{ VA}$$

$$P_{avg} = 617 \text{ W} \quad Q = 0 \text{ VAR}$$

zero
عادی الرقم و صریح

* in the presence of inductor, it is better as a customer, opposite to government

• devices are inductive load has angle $\rightarrow$ this angle make losses $\rightarrow$ adjusting before using electricity

ask, believe & receive

FIVE APPLE

long wires and magnetic field leads to having inductor

NO.

Comparing the efficiencies in the previous example

Case 1 (with inductor)

delivered to the load $\rightarrow P_{avg} = 503 \text{ W}$

$\tilde{I}_s = \tilde{I}_L = 13.8 \angle -0.586 \text{ A}$

loss $\rightarrow P_{Rs} = \tilde{I}_s^2 R_s = (13.8)^2 (4) = 761.76 \text{ W}$

$\eta = \frac{P_{out}}{P_{in}} = \left(\frac{P_{avg} (\tilde{P}_{load})}{P_{losses} + P_{load}} \right) \times 100\%$ (same way)

$= \frac{503}{761.76 + 503} \times 100\% = 39.8\%$

Case 2 (without inductor)

$P_{L, avg} = 617 \text{ W} (\theta = 0)$

$\tilde{I}_s = \tilde{I}_L = 7.86 \angle 0 \text{ A}$

$P_{Rs} = \tilde{I}_s^2 R_s = (7.86)^2 (4) = 247.12 \text{ W}$

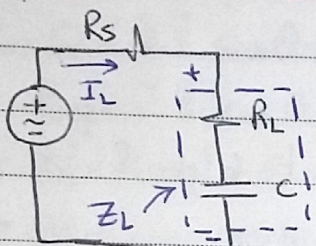
$\eta = \frac{617}{617 + 247.12} \times 100\%$

$= 71\%$

efficiency is improved more than double

Wednesday 10/14/2019

Example from slides (there is mistakes in the slides).



$\tilde{V}_s = 110 \angle 0^\circ \text{ V}$ $\omega = 377 \text{ rad/s}$

$R_s = 2 \Omega$ $R_L = 5 \Omega$ $C = 2000 \mu\text{F}$

Find the complex, real & reactive power delivered to the load.

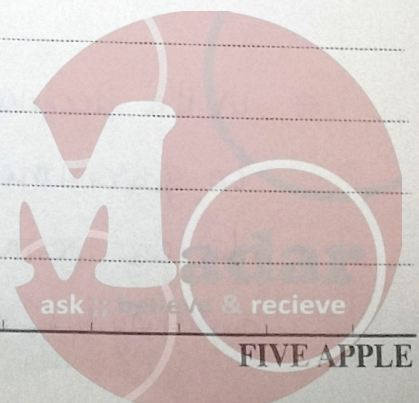
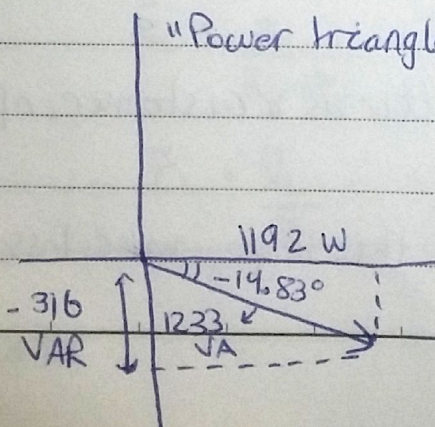
$\tilde{Z}_L = R_L + \frac{1}{j\omega C} = 5 + \frac{1}{j(377)(2000 \times 10^{-6})} = 5 - j1.326 \Omega = 5.173 \angle -0.259 \text{ rad}$

$\tilde{V}_L = \frac{\tilde{Z}_L}{\tilde{Z}_L + R_s} \tilde{V}_s = \frac{5.173 \angle -0.259}{2 + 5.173 \angle -0.259} (110) = 79.867 \angle -0.072 \text{ V}$

$\tilde{I}_L = \frac{\tilde{V}_L}{\tilde{Z}_L} = \frac{79.867 \angle -0.072}{5.173 \angle -0.259} = 15.44 \angle 0.187 \text{ A}$

$S_L = \tilde{V}_L \tilde{I}_L^* = (79.867 \angle -0.072) (15.44 \angle -0.187) = 1233 \angle -0.259 \text{ VA}$ # same angle $= 1192 - j316 \text{ VA}$

"Power triangle"



NO. Power Factor

- When comparing the two previous examples:-

if you want to cancel the inductor in a circuit, add capacitor to cancel this effect. Since capacitor power is the negative of inductor power. (making $\theta_2 = 0$)

Power Factor

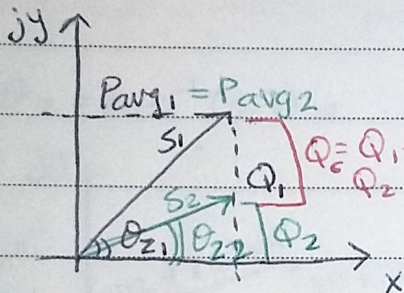
$$S = \tilde{V} \tilde{I}^* = \tilde{V} \tilde{I} \cos \theta_2 + j \tilde{V} \tilde{I} \sin \theta_2$$

if $\theta_2 = 0$ $\begin{cases} \cos \theta_2 = 1 \\ \sin \theta_2 = 0 \end{cases}$

$$\Rightarrow S = \tilde{V} \tilde{I}$$

$$\theta_2 \rightarrow 0 \quad PF \rightarrow 1$$

$$PF \triangleq \cos \theta_2$$



$$Q_2 < Q_1$$

$$P_{avg1} = P_{avg2}$$

$$\theta_{22} < \theta_{21}$$

$$S_2 < S_1$$

Q_c is the effect of adding capacitor

in slides $P = P_1 = P_2 \quad P_1 = P_{avg1} \quad P_2 = P_{avg2}$

$$P = S_1 \cos \theta_1 = S_2 \cos \theta_2$$

$$Q = S_1 \sin \theta_1 = \frac{P}{\cos \theta_1} \sin \theta_1 = P \tan \theta_1$$

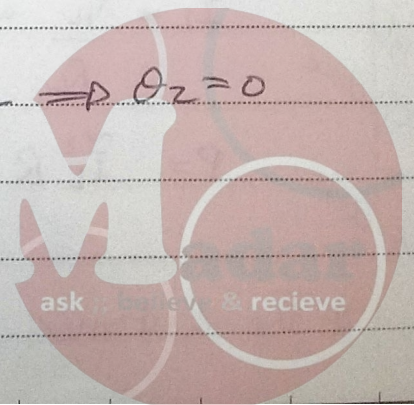
it is power calculations

$$Q_c = Q_1 - Q_2 = P (\tan \theta_1 - \tan \theta_2) = \frac{\tilde{V}^2}{X_c} \quad \left(X_c = \frac{1}{\omega C} \right)$$

$$= \omega C \tilde{V}^2$$

$$C = \frac{(\tan \theta_1 - \tan \theta_2) P}{\omega \tilde{V}^2}$$

capacitor reactance = inductor reactance $\Rightarrow \theta_2 = 0$



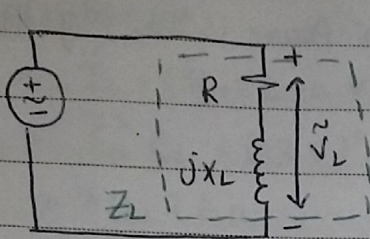
Power Factor $\left\{ \begin{array}{l} \text{inductor} \rightarrow \text{lagging} \\ \text{capacitor} \rightarrow \text{leading} \end{array} \right. *$ with respect to current

NO.

Power Factor correction

The previous equation is correction

Example from slides



$$\tilde{V}_s = 117 \angle 0^\circ \quad \omega = 377 \text{ rad/s}$$

$$R_L = 50 \Omega \quad \text{and} \quad jX_L = j86.7 \Omega$$

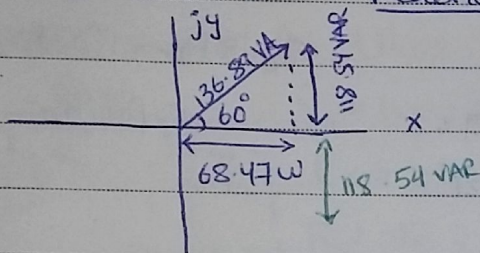
* Find the complex, real, and reactive powers delivered to the load.

$$Z_L = 50 + j86.7 \Omega = 100 \angle 1.047 \text{ rad}$$

Complex: $S_L = \tilde{V}_s^2 = (117)^2 = 136.89 \angle 1.047 \text{ VA}$

comes from $\tilde{I}^* \leftarrow \tilde{Z}_L^*$ $100 \angle -1.047 = 68.47 + j118.54 \text{ VA}$

Average: $P_L = 68.47 \text{ W}$ Reactive: $Q_L = 118.54 \text{ VAR}$

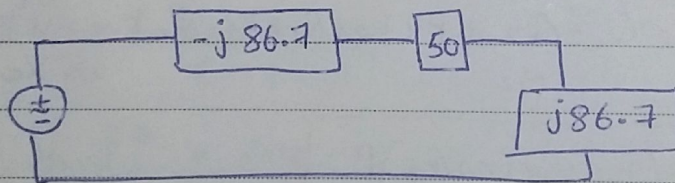


* Find the value of reactance connecting in series to the load to correct the power factor to unity ($\text{PF} = 1$) $\rightarrow (\theta_z = 0)$

I want to add capacitor with $Q = -118.54 \text{ VAR}$ $jX_C = -jX_L$

$$C = \frac{1}{\omega |X_C|} = \frac{1}{(377)(86.7)} = 30.6 \mu\text{F}$$

we know that it is
-ve, but there is no
-ve capacitor



$$\tilde{I}_L = \frac{\tilde{V}_L}{R_L} = \frac{117}{50} = 2.34 \text{ A}$$

$$P = \tilde{I}_L^2 R = (2.34)^2 (50) = 273.78 \text{ W}$$

It is not
at resonance
It is a
different case

* The customer is consuming more electricity (when making θ_z)

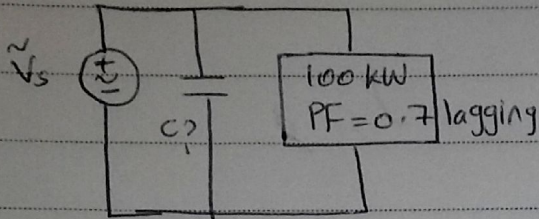
ask & believe & receive

FIVE APPLE

* Putting the capacitor in series or in parallel leads to the same adjusting (correcting) but different values of capacitance will be required

NO.

Example from slides from slides (there is mistake)

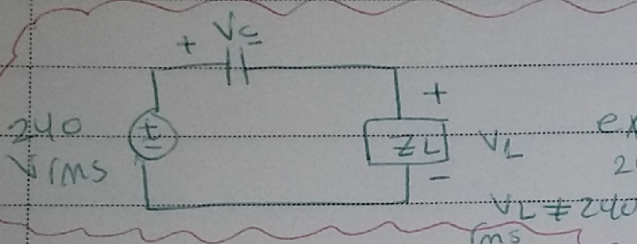


$$\tilde{V}_s = 480 \angle 0^\circ \text{ V} \quad \omega = 377 \text{ rad/s}$$

$$P_L = 100 \text{ kW}$$

$$PF = 0.7 \text{ lagging} \rightarrow \text{inductor}$$

What value capacitance would need to be shunted across the load to correct the P.F. to 0.95?



moving in parallel will not affect the voltage

$$\theta_2 = \cos^{-1}(0.7) =$$

$$|S_L| = \frac{P}{\cos \theta_L} = \frac{P}{P.F.} = \frac{100 \times 10^3}{0.7} = 142.9 \text{ kVA}$$

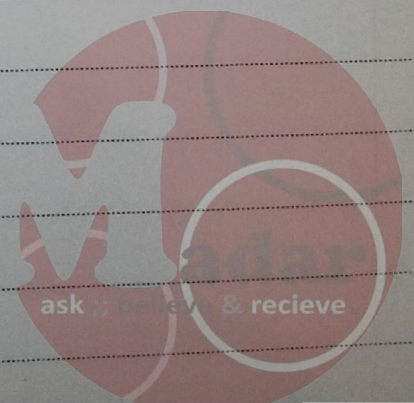
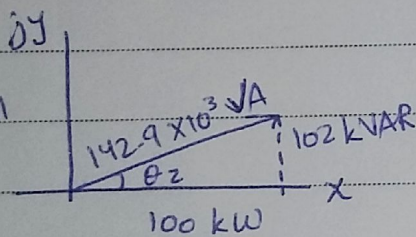
$$\theta_L = \cos^{-1}(P.F.) = \cos^{-1}(0.7) = 0.795 = 45.6^\circ$$

$$Q_L = |S_L| \sin \theta_L = 142.9 \times 10^3 \sin(0.795) = 102 \text{ kVAR}$$

$$\theta_2 = \cos^{-1}(0.95) = 0.318 = 18.2^\circ$$

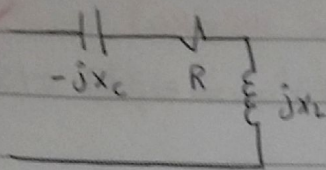
$$C = \frac{P(\tan \theta_1 - \tan \theta_2)}{\omega V^2} = \frac{100 \times 10^3 (\tan 45.6^\circ - \tan 18.2^\circ)}{377 \times 480^2}$$

$$C = 797 \mu\text{F}$$



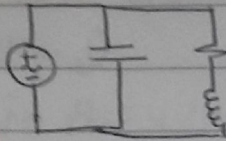
capacitive loading $\rightarrow$ like solar system (generating electricity)
 DC energy stored in capacitors, then using inverter to convert it to AC to use.
 stored in batteries but not used because unproven & it requires cooling. Monday 15/4

Review



$$jX_c = -jX_L$$

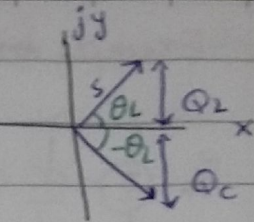
PF = 1 "perfect PF correction"



$$Q_c = -Q_L$$

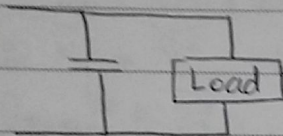
"perfect PF correction"

PF = 1



$$X_c = \frac{|\tilde{V}|^2}{P}$$

$$C = \frac{1}{\omega X_c}$$



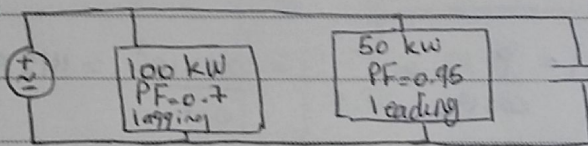
$$C = \frac{P_{avg} [\tan \theta_1 - \tan \theta_2]}{\omega \tilde{V}^2}$$

Partial PF correction

* complex power $\rightarrow$ individual complex power (for each element)

Example

These have the same frequency, can be plotted on the same diagram.



combined Perfect
Find C? to have PF correction

$$\tilde{V}_s = 480 \angle 0^\circ \text{ V} \quad \omega = 377 \text{ rad/s}$$

PF₁ = 0.7 lagging

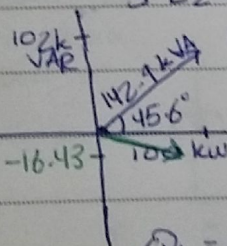
PF₂ = 0.95 leading

like a house with solar system

$$S_{L1} = \frac{P_1}{\cos \theta_1} = \frac{100 \times 10^3}{0.7} = 142.9 \text{ kVA} \quad \theta_1 = \cos^{-1}(0.7) = 0.795 = 45.6^\circ$$

$$Q_{L1} = |S_{L1}| \sin \theta_1 = 142.9 \times 10^3 \times \sin(45.6^\circ) = 102 \text{ kVAR}$$

$$S_{L2} = \frac{P_2}{\cos \theta_2} = \frac{50 \times 10^3}{0.95} = 52.632 \text{ kVA} \quad \theta_2 = \cos^{-1}(0.95) = 0.318 = 18.2^\circ$$



$$Q_{L2} = |S_{L2}| \sin \theta_2 = 52.632 \times 10^3 \times \sin(-18.2^\circ) = -16.43 \text{ kVAR}$$

$$Q_L = Q_{L1} + Q_{L2} = 102 - 16.43 = 85.6 \text{ kVAR}$$

$\rightarrow Q_c = -Q_L = -85.6 \text{ kVAR}$ capacitor brings the -ve sign

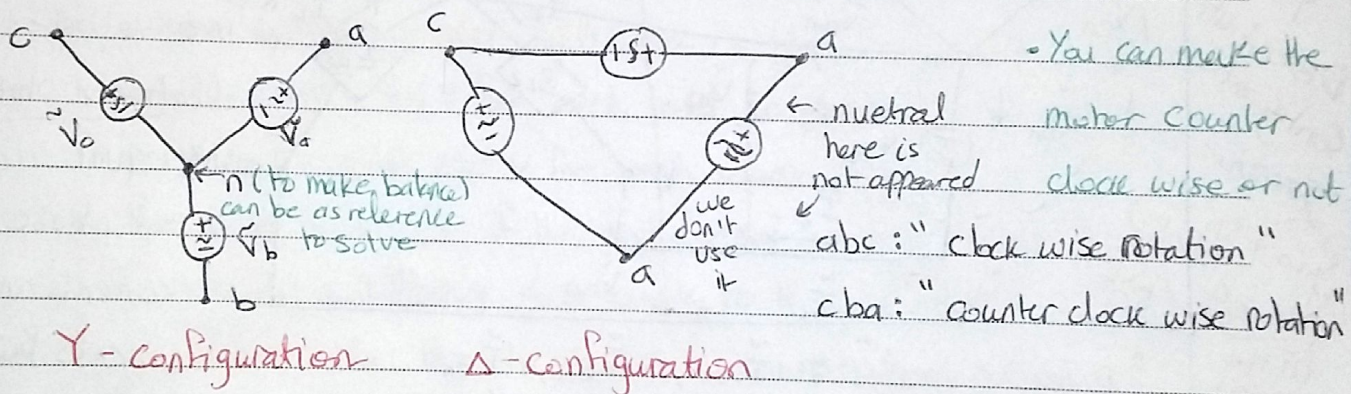
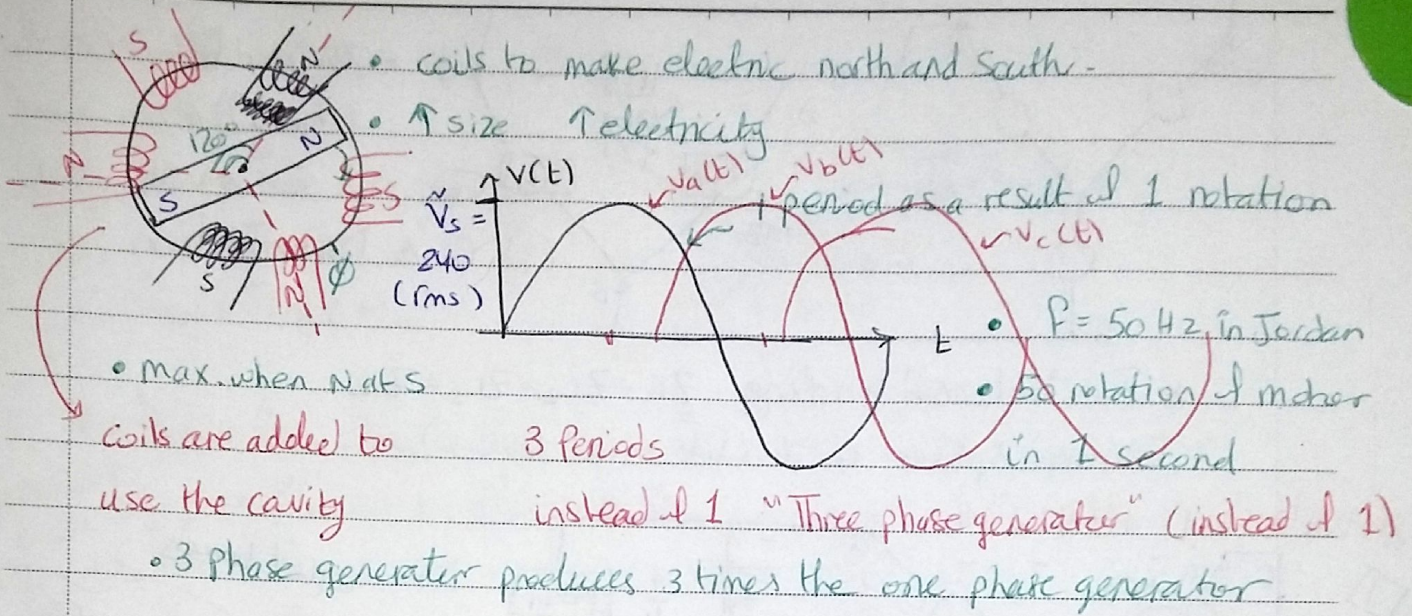
$$X_c = \frac{|\tilde{V}_L|^2}{Q_c} = \frac{(480)^2}{85.6} = 2.692 \Omega$$

$$C = \frac{1}{\omega X_c} = \frac{1}{377 \times 2.692} = 985 \mu\text{F}$$

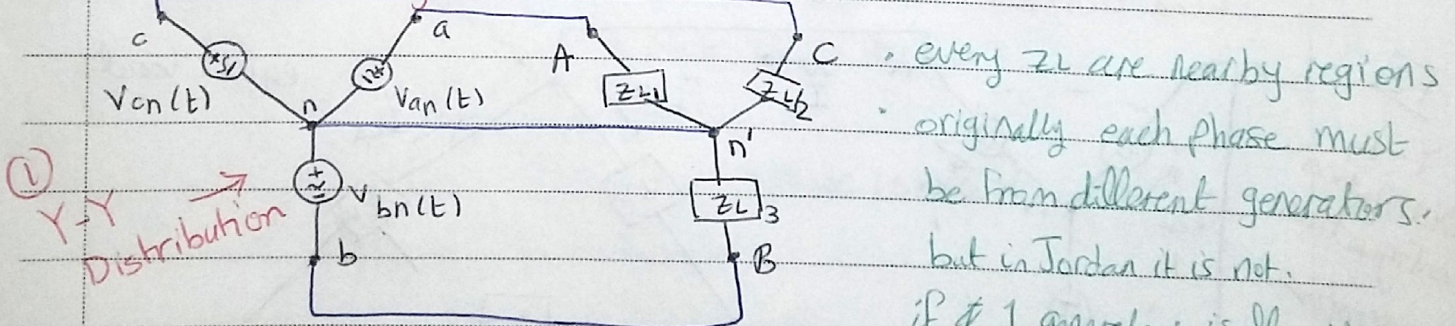
magnitude

100 Ampere, cots in streets there is 5 wires $\rightarrow$ 3 phase
in wires in Jordan $\downarrow$ neutral
to light the street

NO. Three phase systems



To connect the generator to the load

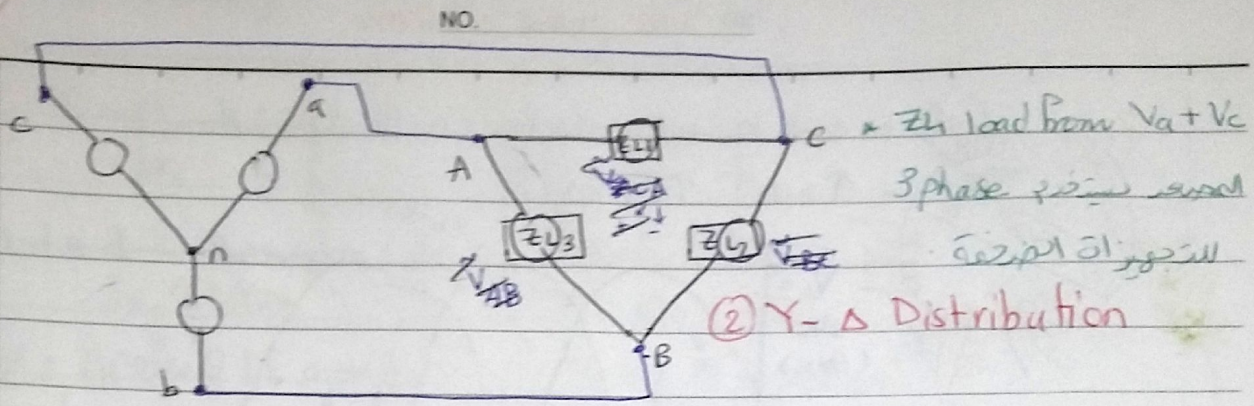


For balanced loading for the

generator $Z_{L1} = Z_{L2} = Z_{L3} = Z_L$ (it must be)

but actually it is not exactly equal, for a certain percentage if not, the rotor will be shifted leading to vibration and the generator will be broken.

$$\vec{I}_a + \vec{I}_b + \vec{I}_c = \frac{1}{Z_L} [\vec{V}_{cn} + \vec{V}_{bn} + \vec{V}_{an}] = \vec{I}_n \text{ Ideally } I_n = 0$$

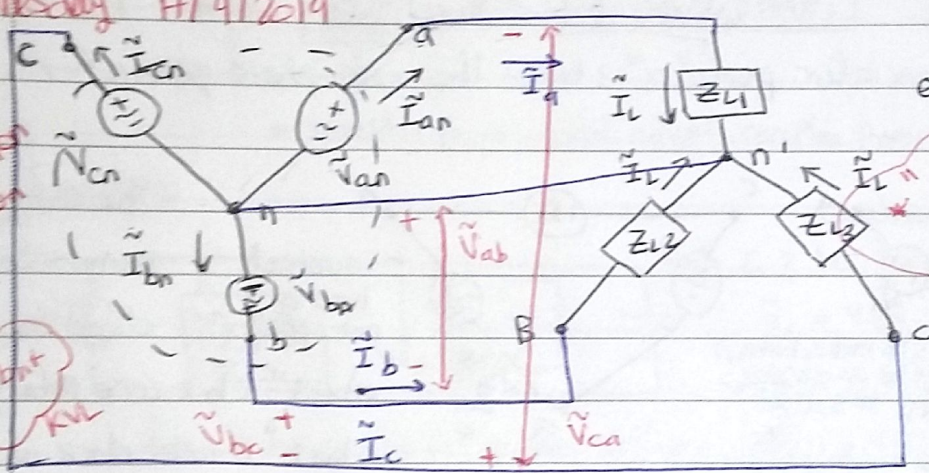


② Y-Δ Distribution

* For balanced loading $Z_{L1} = Z_{L2} = Z_{L3} = Z_L$
 To make 3phase to Y (for small devices).

Wednesday 11/4/2019

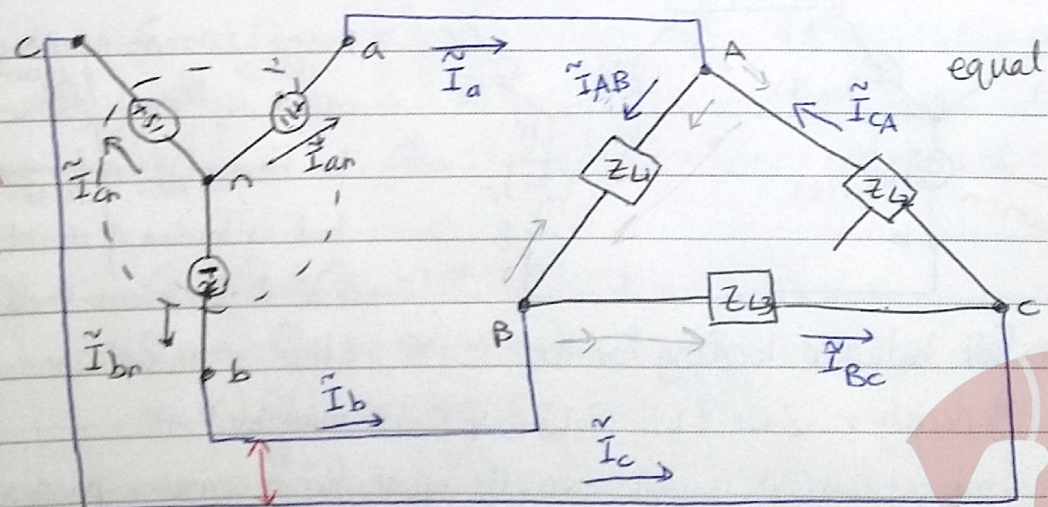
Y-Y Distribution Configuration
 1 generator
 $V_{an} + V_{bn} + V_{cn} = 0$
 in the panel



equal loads
 same $n(2)$
 * line voltages (between phases)
 * equal currents of phase currents and line currents But NOT equal to load currents

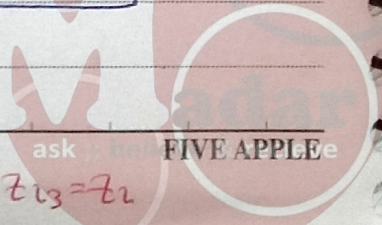
"Phase currents" : $\vec{I}_{an}, \vec{I}_{bn}, \vec{I}_{cn}$
 "Line currents" : $\vec{I}_a, \vec{I}_b, \vec{I}_c$
 "Load currents" : $\vec{I}_L$

Y-Δ Distribution Configuration
 1 generator



* line currents are equal to phase currents
 But $\vec{I}_a$ is bigger than $\vec{I}_b$

$$\vec{V}_{an} = \vec{V}_{bn} = \vec{V}_{cn} = \vec{V}_p \text{ For ①+② } Z_{L1} = Z_{L2} = Z_{L3} = Z_L$$



$$-120^\circ = 240^\circ$$

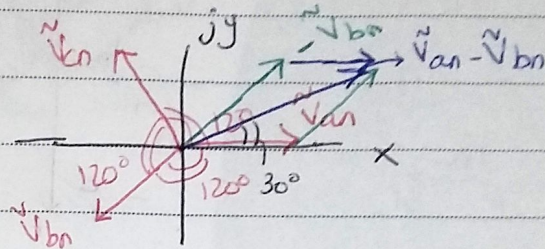
NO.

* if the voltages are in phase (the voltage will be doubled compared with (1))

$$\tilde{V}_{an} = \tilde{V}_{an} \angle 0^\circ$$

$$\tilde{V}_{bn} = \tilde{V}_{bn} \angle -120^\circ$$

$$\tilde{V}_{cn} = \tilde{V}_{cn} \angle +120^\circ$$



$$\tilde{V}_{ab} = \tilde{V}_{an} - \tilde{V}_{bn} = \tilde{V}_{an} \angle 0^\circ - \tilde{V}_{bn} \angle -120^\circ = \sqrt{3} \tilde{V}_p \angle +30^\circ$$

$$= \tilde{V}_{an} \cos 0 + j \tilde{V}_{an} \sin 0 - \tilde{V}_{bn} \cos(-120^\circ) - j \tilde{V}_{bn} \sin(-120^\circ)$$

$$\tilde{V}_{bc} = \tilde{V}_{bn} \angle -120^\circ - \tilde{V}_{cn} \angle +120^\circ = \sqrt{3} \tilde{V}_p \angle -90^\circ$$

$$\tilde{V}_{ca} = \tilde{V}_{cn} \angle +120^\circ - \tilde{V}_{an} \angle 0^\circ = \sqrt{3} \tilde{V}_p \angle +150^\circ$$

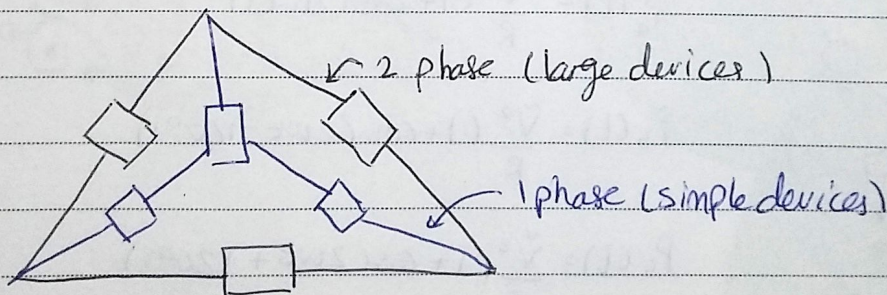
• line voltages angles difference is 120° , but starts from 30°

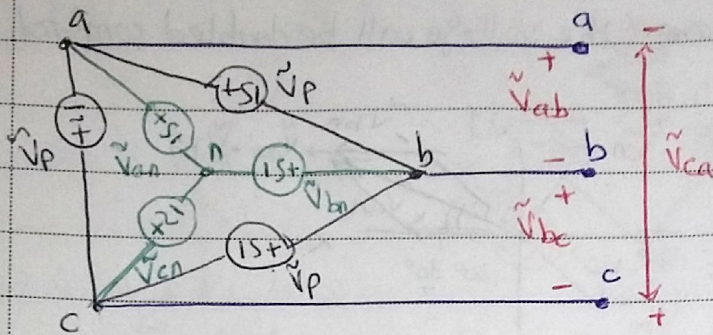
$\sqrt{3} \times 240 = 415$ V for each load in phase (2 phases side)
generator gives 240° V per each source phase

* why there is wire in (1), if the impedances are not equal, there is a difference, this difference goes back to the generator (safety reason)

* a b c $\rightarrow$ counter clock Va then Vb then Vc, counter vice versa

* Configuration 2, For plants, high voltage, but for simple devices there is





line voltage here $\sqrt{3} \tilde{V}_p$, before, line voltage = $\tilde{V}_p$

• $\tilde{V}_{ab} - \tilde{V}_{bc} = \tilde{V}_{ca} = \tilde{V}_p = \tilde{V}_L$ Here (phase to phase $\rightarrow$ load must be Δ)

• $\tilde{V}_{ab} + \tilde{V}_{bc} + \tilde{V}_{ca} = 0$ (at each period of generator)

$\rightarrow$ The configuration in green is smaller than the black one.

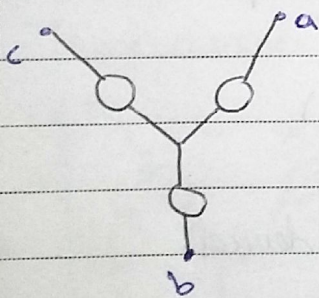
• $\tilde{V}_{an} = \frac{\tilde{V}_{bn}}{\sqrt{3}} \angle -30^\circ$ • $\tilde{V}_{bn} = \frac{\tilde{V}_p}{\sqrt{3}} \angle -180^\circ$ • $\tilde{V}_{cn} = \frac{\tilde{V}_p}{\sqrt{3}} \angle +90^\circ$

less voltage phase $\rightarrow$ start from -30° , angle between = 120°

Δ generator $\rightarrow$ Δ load Δ generator is more effective than Δ load

$\Rightarrow$ Power for Δ or Δ configuration -

$P(t) = i(t)v(t)$ assuming there is no reactance (for analysis)



$$P_a(t) = \frac{\tilde{V}^2}{R} (1 + \cos(2\omega t))$$

$$P_b(t) = \frac{\tilde{V}^2}{R} (1 + \cos(2\omega t - 120^\circ))$$

$$P_c(t) = \frac{\tilde{V}^2}{R} (1 + \cos(2\omega t + 120^\circ))$$

$$P(t) = P_a(t) + P_b(t) + P_c(t)$$

$$= \frac{3\tilde{V}^2}{R} + \frac{\tilde{V}^2}{R} (\cos(2\omega t) + \cos(2\omega t - 120^\circ) + \cos(2\omega t + 120^\circ))$$

P_{avg}

3x Power

Pulsating power

like if we multiply by 3

ask

review

FIVE APPLE

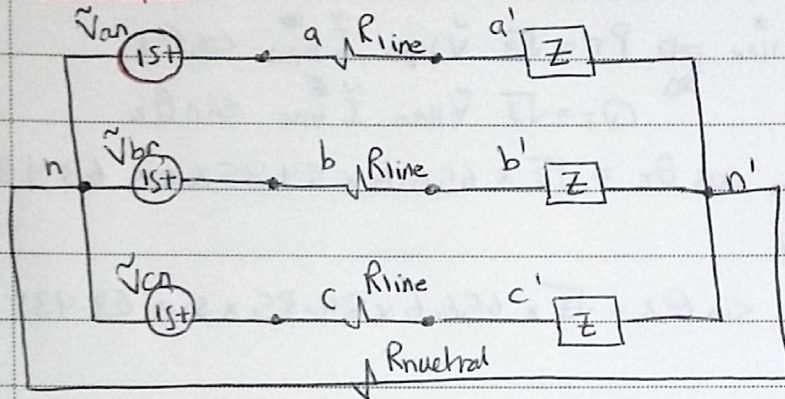
NO. 3- Phase Complex Power

1.73

- $S_T = \sqrt{3} \tilde{V}_{line} \tilde{I}_{line}^*$
- avg. $P_T = \sqrt{3} \tilde{V}_{line} \tilde{I}_{line} \cos \theta_{load}$
- $Q_T = \sqrt{3} \tilde{V}_{line} \tilde{I}_{line} \sin \theta_L$

*Example (from slides)

Y-Y configuration



• resistance are wire resistance (usually inductor)

$$\tilde{V}_{an} = 480 \angle 0^\circ \text{ V}$$

$$\tilde{V}_{bn} = 480 \angle -120^\circ \text{ V}$$

$$\tilde{V}_{cn} = 480 \angle +120^\circ \text{ V}$$

$$\omega = 377 \text{ rad/s}$$

Find the power delivered to the load by

$$Z_Y = 2 + j4 \Omega$$

3-phase generator?

$$R_{line} = 2 \Omega \quad R_{neutral} = 10 \Omega$$

way 1 $\tilde{I} = \frac{\tilde{V}_a}{R_{line} + Z_Y} = \frac{480 \angle 0^\circ}{2 + 2 + j4} = \frac{480 \angle 0^\circ}{5.66 \angle 45^\circ} = 84.85 \angle -45^\circ \text{ A}$

$$P_a = |\tilde{I}|^2 R_L = (84.85)^2 \times 2 = 14.4 \text{ kW}$$

$$P_L = 3 P_a = 3 \times 14.4 \times 10^3 = 43.2 \text{ kW} \quad \text{Avg. Power}$$

way 2 Alternately, same phase voltage

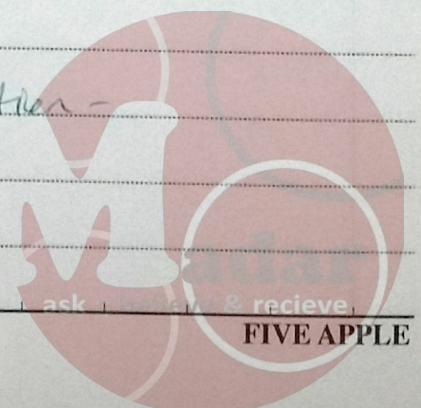
$$S_L = 3 \tilde{V}_{p-line} \tilde{I}_{line}^*$$

$$= 3 (480 \angle 0^\circ) (4.47 \angle 63.43^\circ) = 96.495 \angle 63.43^\circ \text{ kVA}$$

$$= 43.2 + j 86.3 \text{ kVA}$$

$P_L = 43.2 \text{ kW} \quad Q_L = 86.3 \text{ kVAR}$

• This system requires a power factor correction -



Way 3

$$Z_L = Z_Y = 2 + j4 = 4.47 \angle 63.43^\circ$$

$$|\tilde{V}_{L-P}| = 480 \angle 0 \quad \frac{4.47 \angle 63.43}{5.66 \angle 45} = 379.1 \text{ V}$$

$$|\tilde{V}_{L-L}| = \sqrt{3} |\tilde{V}_{L-P}| = 656.6 \text{ V}$$

$$\tilde{I}_L = |\tilde{I}_P| = 84.85 \text{ A}$$

$$S_T = \sqrt{3} \tilde{V}_{\text{line}} \tilde{I}_{\text{line}}^* \Rightarrow P_T = \sqrt{3} \tilde{V}_{\text{line}} \tilde{I}_{\text{line}}^* \cos \theta_Z$$

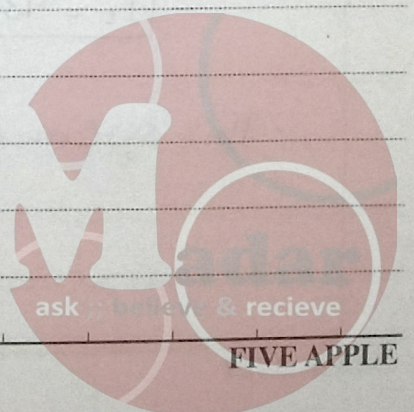
$$Q_T = \sqrt{3} \tilde{V}_{\text{line}} \tilde{I}_{\text{line}}^* \sin \theta_Z$$

$$P_L = \sqrt{3} \tilde{V}_{\text{line}} \tilde{I}_{\text{line}} \cos \theta_Z = \sqrt{3} \times 656.6 \times 84.85 \times \cos 63.43^\circ$$

$$= 43.2 \text{ kW}$$

$$Q_L = \sqrt{3} \tilde{V}_{\text{line}} \tilde{I}_{\text{line}} \sin \theta_Z = \sqrt{3} \times 656.6 \times 84.85 \times \sin 63.43^\circ$$

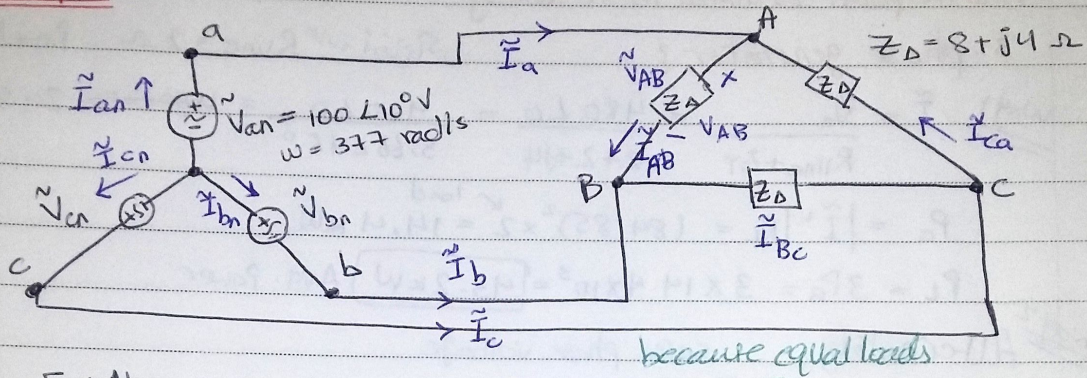
$$= 86.3 \text{ kVAR}$$



$$= 86.3 \text{ kVAR}$$

22/4 Example

Plant distribution



Find the phase and line currents in this balanced 3 phase system

$$I_{ab} \text{ since } \tilde{I}_{an} = \tilde{I}_a$$

$$Z_D = 8 + j4 = 8.944 \angle 26.57^\circ$$

$$\tilde{V}_{ab} = \tilde{V}_{AB} = \sqrt{3} \tilde{V}_p \angle 30^\circ = 173.2 \angle 40^\circ \leftarrow 30^\circ + 10^\circ$$

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{Z_D} = \frac{173.2 \angle 40^\circ}{8.944 \angle 26.57^\circ} = 19.36 \angle 13.43^\circ \text{ A}$$

$$\tilde{I}_{BC} = \tilde{I}_{AB} \angle -120^\circ = 19.36 \angle -106.57^\circ \text{ A}$$

$$\tilde{I}_{CA} = \tilde{I}_{AB} \angle 120^\circ = 19.36 \angle 133.43^\circ \text{ A} \quad \text{+ve (includes -ve) (wrong)}$$

$$\tilde{I}_a = \sqrt{3} \tilde{I}_{AB} \angle -30^\circ = 33.53 \angle -16.57^\circ \text{ A}$$

$$\tilde{I}_b = \tilde{I}_a \angle -120^\circ = 33.53 \angle -136.57^\circ \text{ A}$$

$$\tilde{I}_c = \tilde{I}_a \angle +120^\circ = 33.53 \angle 103.43^\circ \text{ A} \quad \text{+ve not -ve}$$

FIVE APPLE



- read slides for general knowledge.

NO.

$$Z_Y = \frac{Z_{\Delta}}{3} \Rightarrow Z_{\Delta} = 8.944 \angle 26.57^\circ \Omega$$

* ما غير Δ و Y موجودين مع

نفس نفس الوقت

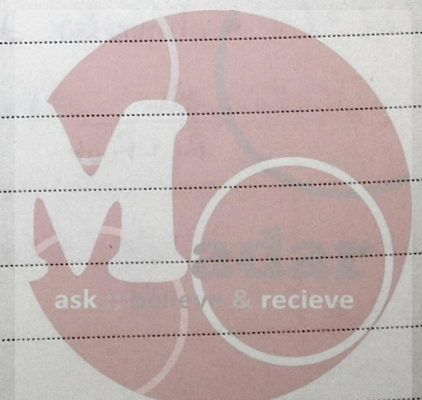
$$Z_Y = \frac{8.944}{3} \angle 26.57 = 2.981 \angle 26.57 \Omega$$

لا نقدر حصة Δ الى Y

بل ان موجود

$$\downarrow \tilde{I}_a = \frac{\tilde{V}_{an}}{Z_Y} = \frac{100 \angle 10^\circ}{2.981 \angle 26.57^\circ} = 33.53 \angle -16.57^\circ A$$

* Δ - Y conversion From the same line ~~equivalent~~ current, you can use it to check your work.

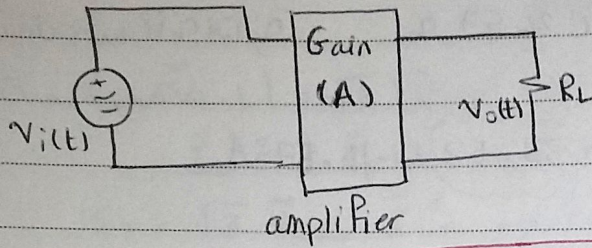


amplifiers & applications, page 1 by 2020
gain > 1

NO. Introduction to electronics

⇒ Ideal amplifier ^{pin}

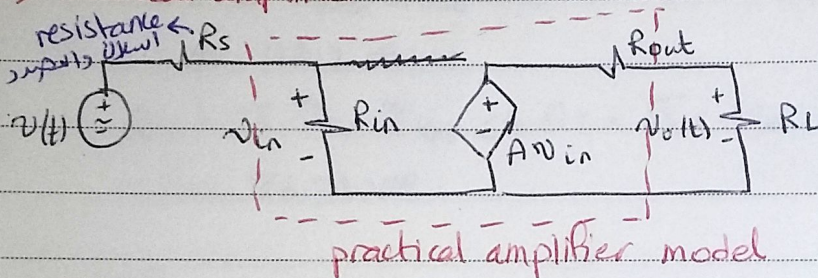
not necessary the source is sin function (AC)



Ideal amplifier $v_o(t) \approx A v_i(t)$

A: unitless number $A \gg 1$
↑ A better

⇒ Practical amplifier



$$v_o(t) = \frac{R_L}{R_L + R_{out}} A v_{in}(t) \quad v_{in}(t) = \frac{R_{in}}{R_s + R_{in}} v_s(t) \quad \text{substitute}$$

$$v_o(t) = A \frac{R_L}{R_L + R_{out}} \left(\frac{R_{in}}{R_{in} + R_s} \right) v_s(t)$$

* In order to have a good amplifier :- (To have the "best" amplifier possible)

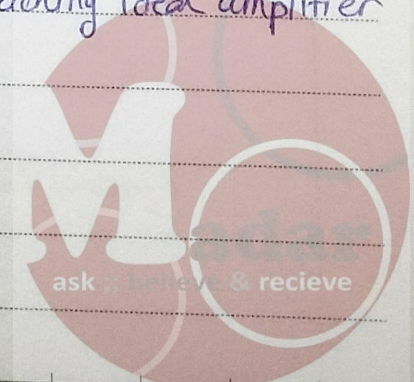
(1) high input impedance ^{Pick} $R_{in} \rightarrow \infty$ (in mega or giga ohms)

$$\frac{R_{in}}{R_{in} + R_s} = 1 \rightarrow v_o(t) = A \left(\frac{R_L}{R_L + R_{out}} \right) v_s(t)$$

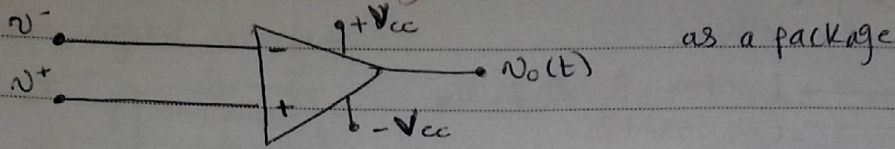
(2) Pick R_{out} small $R_{out} \rightarrow 0$

• ohmmeter of amplifier output when off without $R_L \Rightarrow R_{out}$ measured

$$\frac{R_L}{R_L + R_{out}} = 1 \rightarrow v_o(t) \approx A v_s(t) \quad \therefore \text{approaching ideal amplifier}$$

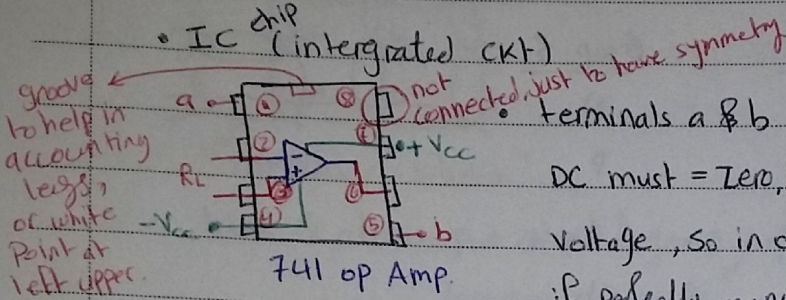


⇒ Operational amplifier (a type of amplifiers)



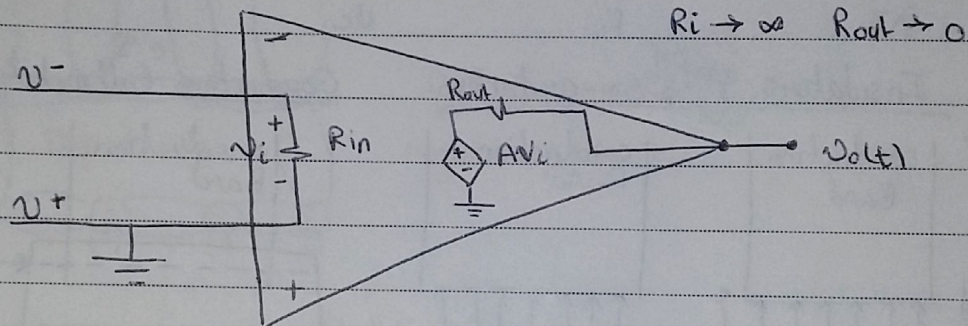
as a package

• IC chip (integrated ckt)



741 op Amp.

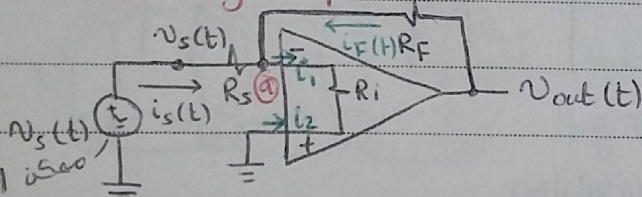
DC must = Zero, because of Fabrication extra small voltage, so in order to make it = zero if perfectly manufactured, no need of a & b

 $R_i \rightarrow \infty$ $R_o \rightarrow 0$

$$v_o(t) = A_{ol} v_i(t) = A_{ol} (v^- - v^+)$$

A_{ol} - open loop gain
 no feedback

∴ Inverting amplifier



$i_1 = i_2 = 0$ if big R_i , no voltage drop
 virtual ground $v^- = v^+ = 0$

→ KCL @ node a $i_F(t) + i_s(t) + -i_1 = 0$ $i_1 = 0$ $i_F(t) = i_s(t)$

$$i_s(t) = \frac{v_s(t) - v^-}{R_s} \quad i_F(t) = \frac{v_{out}(t) - v^-}{R_F}$$

$$\frac{v_{out}(t)}{R_F} = - \frac{v_s(t)}{R_s}$$

$$v_{out}(t) = - \frac{R_F}{R_s} v_s$$

closed loop
 gain (of
 amplifier)

$$A_{cl} = - R_F / R_s \gg 1$$

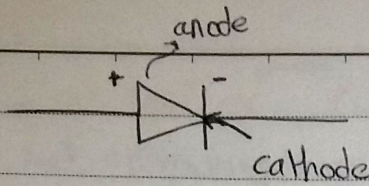
solve example in slides

output is inversion of input (to amplify it)

$$\begin{array}{ll} \cos & -\cos \\ \sin & -\sin \end{array}$$

graphene carbon can be made as semiconductor or metal
 1 layer of graphite

NO. Electrical diode

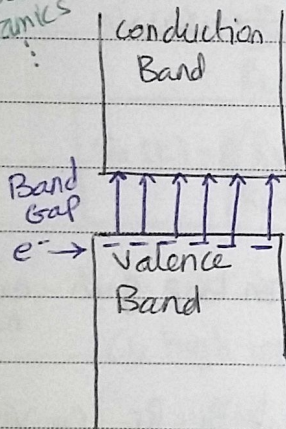


→ i ✓ Permit Passing
 ← $-i$ ✗ doesn't pass

Group 3	Group 4	Group 5
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb
Ti	Pb	Bi
Nh	Fi	Mc

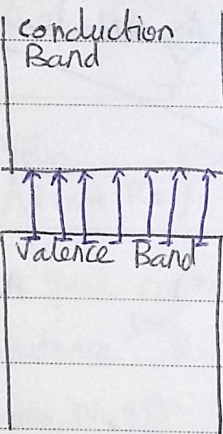
Plastic
wood
ceramics

Insulators

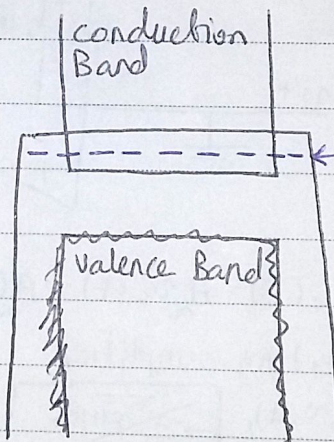


group 4 elements

Semi-conductors



Conductors (all metals are conductors)



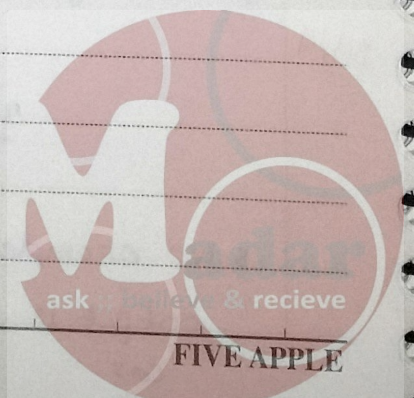
already in the conduction band.

- Big distance for band gap, high energy will lead to destroy the material.
- smaller gap than insulators

- Gap unit [electron volt]

- giving energy — light, with specific wavelength heat, may destroy

ceramic used to make capacitor

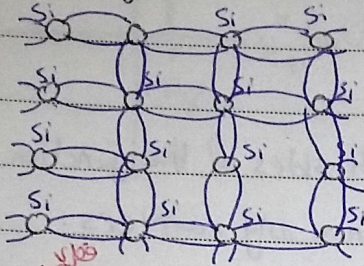


ionic bond is stronger than metallic bond and stronger than covalent bond.

NO.

* We are interested in Si and Ge

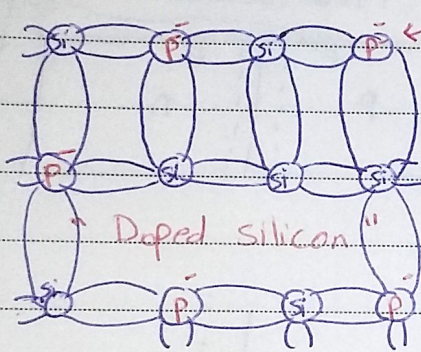
lattice crystal structure



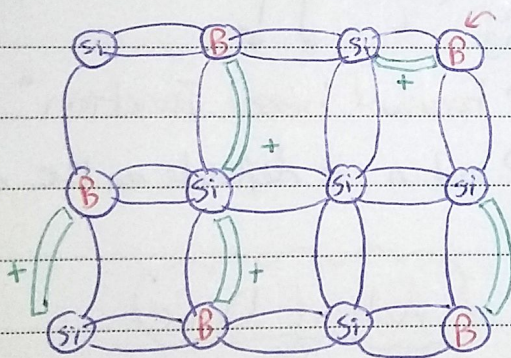
all electrons are bonded, covalent bonds
metallic bonds $\rightarrow$ free electrons

"intrinsic silicon"

- take Si atoms, put atom of group 3 or 5, ~~at~~ small amounts
common to use B usually P



"n-type material"
negatively charged



"impurity atoms"

"Doped silicon"

"p-type material"

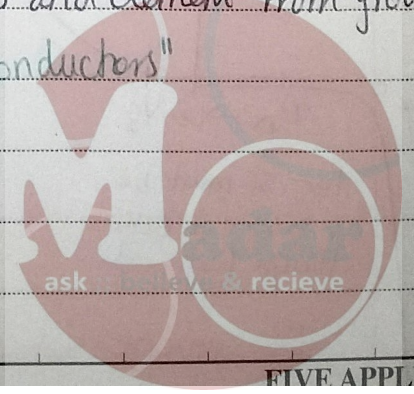
"holes" +vely charged

* not necessary
excess electrons to
make current, lack
make current also!

$\rightarrow$ Semi-conductor may not be pure (Hybrid material)

it can be mixture of element from group 3 and element from group 5

e.g. GaAs InP "Hybrid semiconductors"



FIVE APPLE

* B is introduced also chemically

NO.

* Diode - structure

* p-type & n-type attached to gether mechanically
chemical interaction

2) V_{bi} because of charge attraction
e.g. Cl^- Na^+

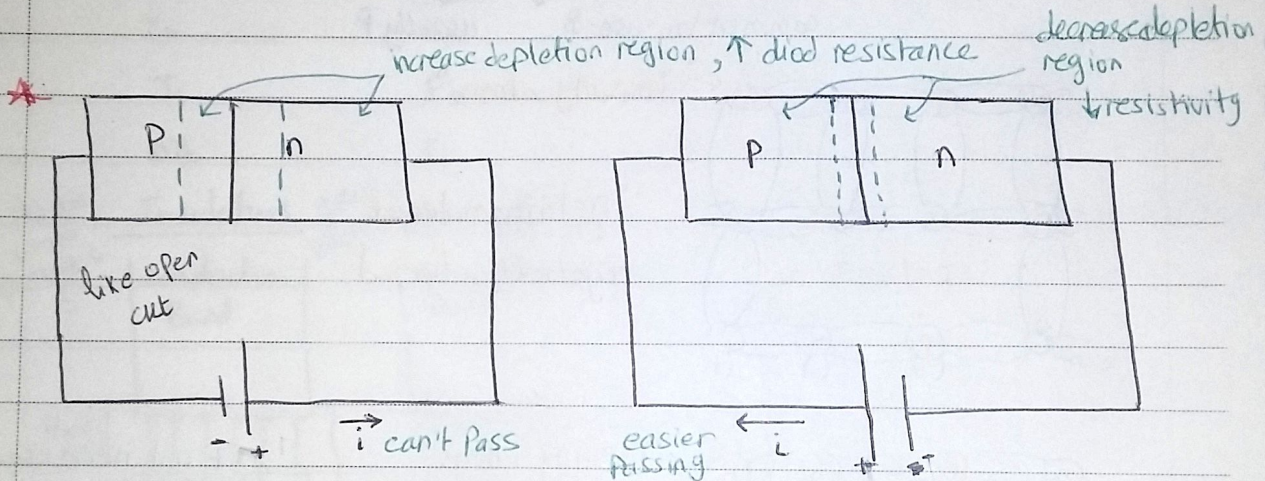


Diffusion Process

Depletion region : a region across both sides of the junction that is totally depleted from any carriers.

V_{bi} : internal junction potential, For Si $V_{bi} = 0.6 \sim 0.7$ volts

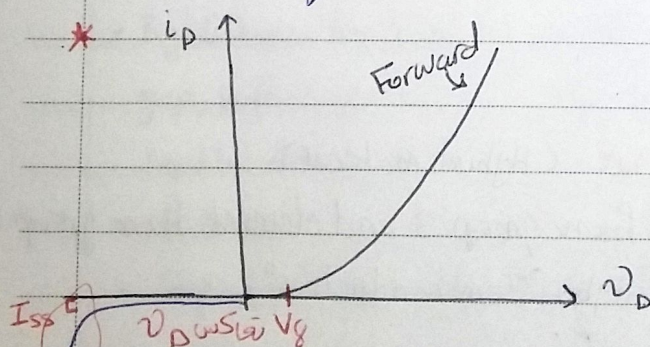
more expensive, better thermal char. For Ge $V_{bi} = 0.2 \sim 0.3$ volts



"Reverse biased Junction"

"Forward biased Junction"

st * depletion region not necessary equal in P and n, it depends on the amount of impurity



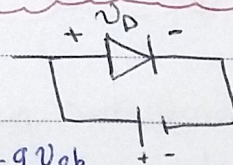
For Si must be $> 0.6 \sim 0.7$ even though forward.

I_{ss} : reverse saturation current

$\downarrow I_{ss} \rightarrow$ better

reverse "Break down region" (Avalanche break down) (destroying material)

Actual Diode



$$w = qV_{ab}$$

$$i_p = I_{ss} \left[e^{\frac{qV_D}{KT}} - 1 \right]$$

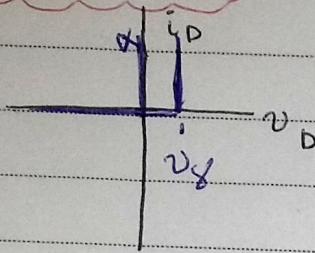
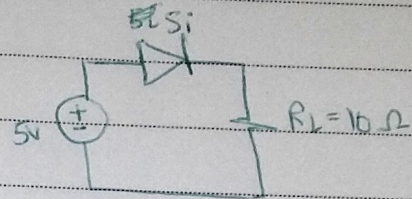
V_D : voltage across diode

k : Boltzman's constant (J/K)

T : absolute temperature (K)

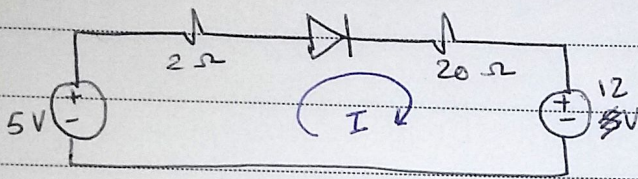
ask FIVE APPLE recieve

Ideal Diode


 i_D
 $i = 0$ 

$$i = \frac{(5 - 0.7)}{10} = \frac{4.3}{10} = 0.43 \text{ A}$$

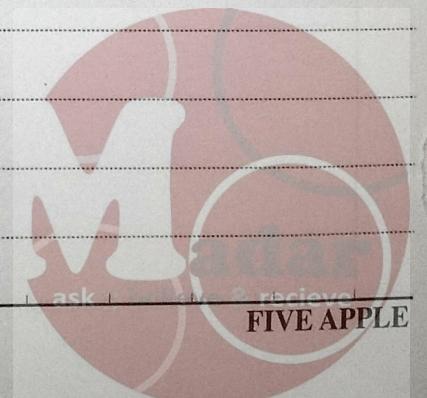
★ Example



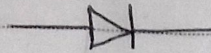
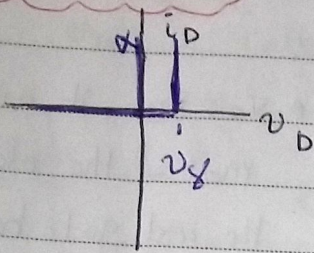
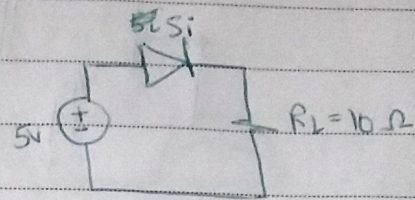
assume no diode

$$-5V + 2I + 20I + 12 = 0 \quad \boxed{I = -0.318 \text{ A}}$$

this means opposite direction

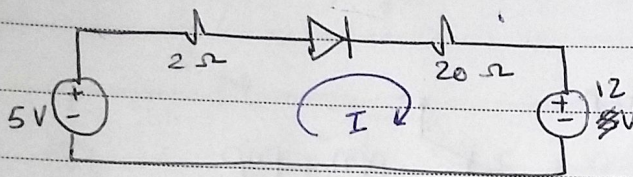
Diode is off, if I +ve $\rightarrow$ Diode on

Ideal Diode


 i_D
 $i = 0$ 

$$i = \frac{(5 - 0.7)}{10} = \frac{4.3}{10} = 0.43 \text{ A}$$

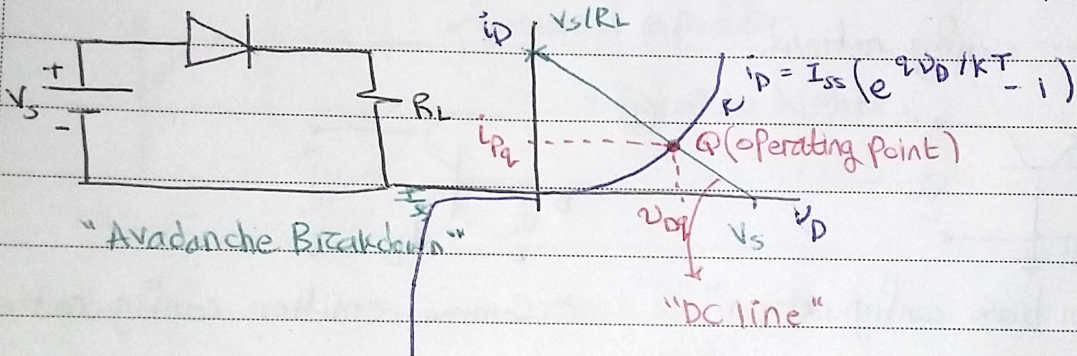
* Example



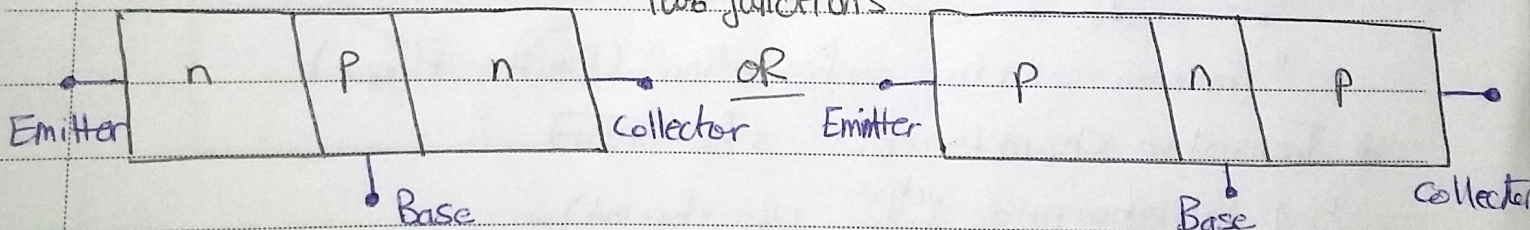
assume no diode

$$-5V + 2I + 20I + 12 = 0 \quad \boxed{I = -0.318 \text{ A}}$$

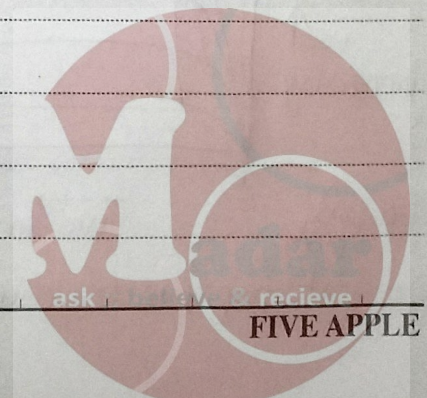
this means opposite direction

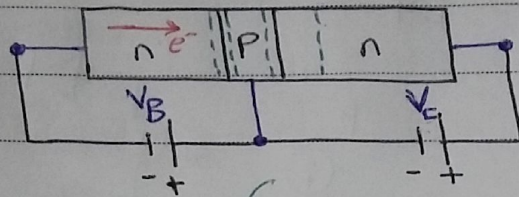
Diode is off, if I +ve $\rightarrow$ Diode on29/4/2019
MondayTransistor (Bipolar junction transistor) $\rightarrow$ BJT

Two junctions



- Base region width is the smallest
- npn is more common than pnp





* $V_C > V_B$, it overcome most of the electrons

direct bias

reverse bias

The rest go to base

smaller junction

bigger junction

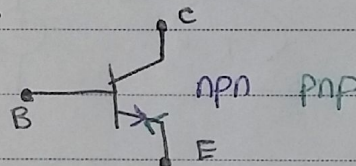
(small amount)

internal resistance: $r_i \rightarrow 0$ (few Ω)

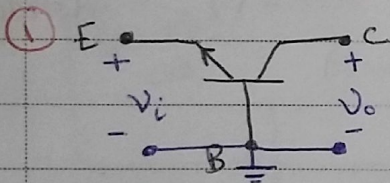
$r_i \rightarrow \infty$ (big)

* Transfer resistor $\Rightarrow$ transistor

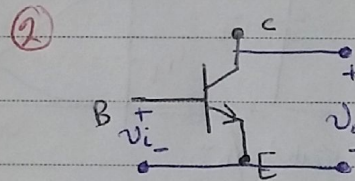
* Transistor electrical symbol



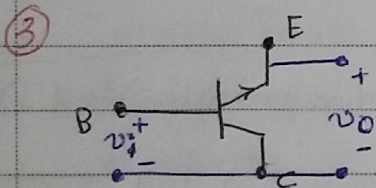
* Transistor configurations



"Common base configuration"

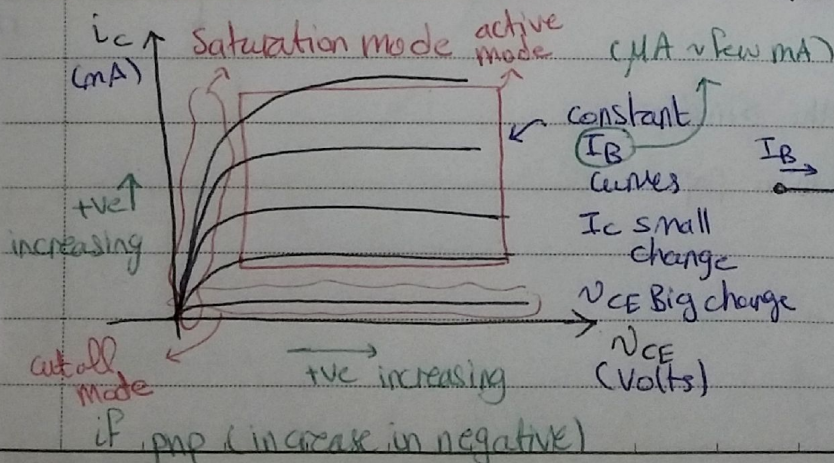


"Common emitter configuration"

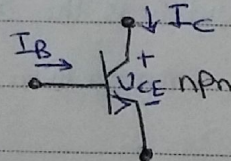


"Common collector configuration" (Emitter Follower)

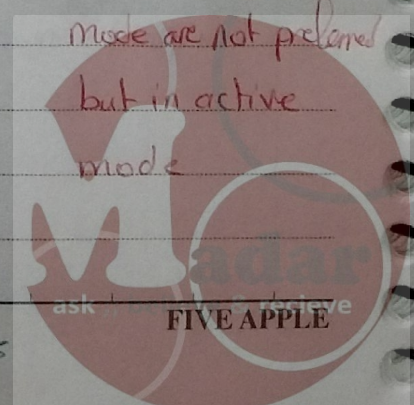
* Transistor Characteristics (for npn)

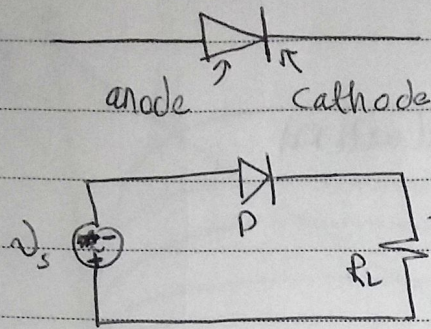


* Saturation & cutoff mode are not preferred but in active mode

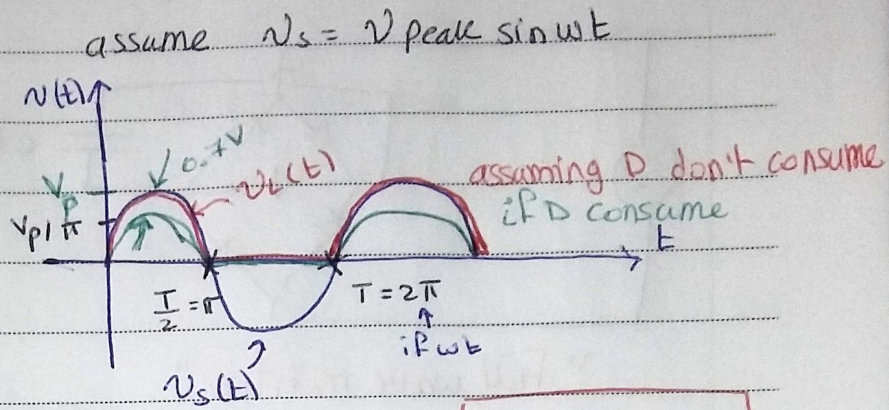


if I_B is 0 (cutoff) or too high (saturation)

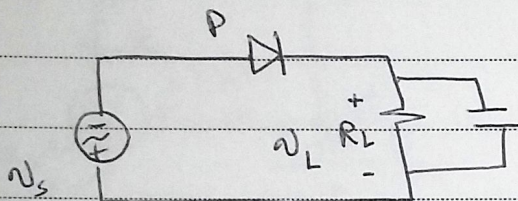




"Half wave rectifier"

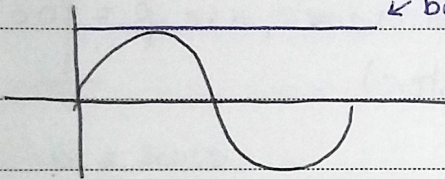


$$\langle v_L(t) \rangle = \frac{1}{T} \int_0^{T/2} V_p \sin \omega \tau d\tau + \frac{1}{T} \int_{T/2}^T 0 d\tau = \frac{V_p}{\pi} = \frac{\sqrt{2} V_{rms}}{\pi} V_{DC}$$



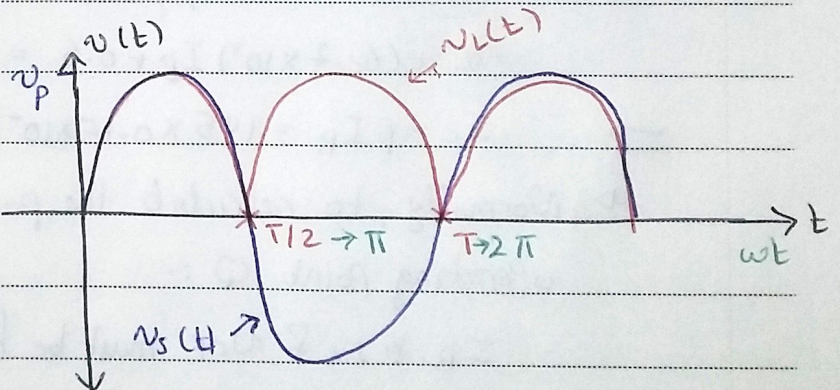
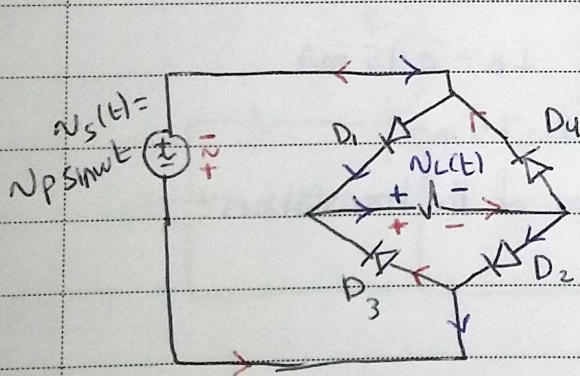
capacitor will have voltage, then when D is off, the charge will be $i_F \omega t$

← because of capacitor

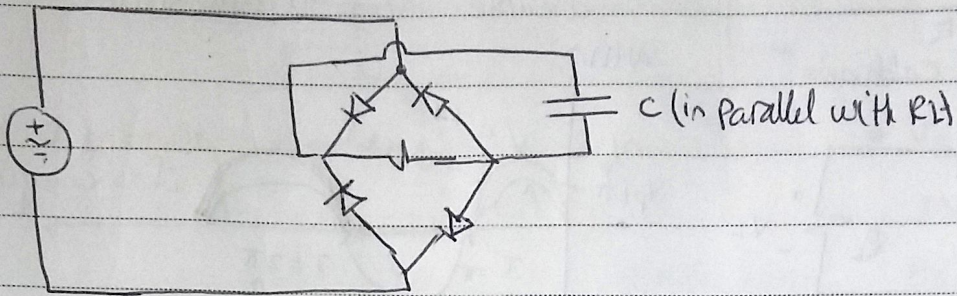


DC will be higher

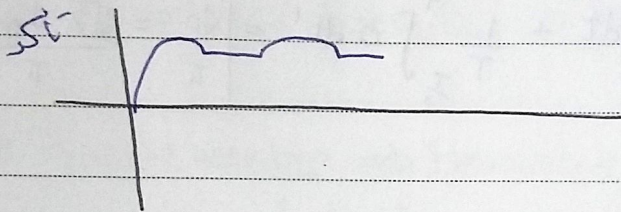
"Half wave rectifier"



$$\langle v_L(t) \rangle = \left(\frac{1}{T} \int_0^{T/2} V_p \sin \omega \tau d\tau \right) \times 2 = \frac{2V_p}{\pi} = \frac{2\sqrt{2} V_{rms}}{\pi} V_{DC}$$



"Full wave rectifier"



if diodes consume 0.7 volts

المستوى سيكون 0.7 فولت

Example from slides #158

- β is a gain (unitless), if not given in exam, use $\beta = 100$
Gain Parameter (current gain for transistor)

$\beta = 145$ electronics in 351

$$I_C = \beta I_B$$

→ KVL around base loop (left side)

$$-10 + (62.7 \times 10^3) I_B + 0.6 = 0 \quad I_B = 0.15 \text{ mA}$$

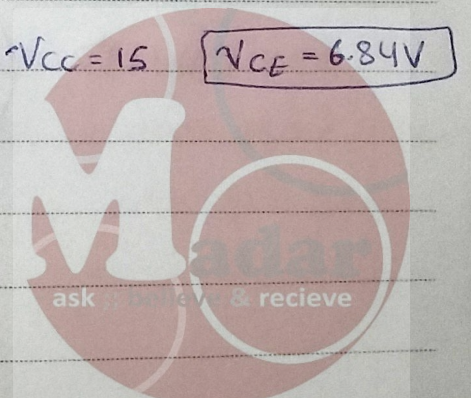
$$\rightarrow I_C = \beta I_B = 145 \times 0.15 \times 10^{-3} = 21.75 \text{ mA}$$

→ Normally, to calculate the position of the transistors' operating point Q :-

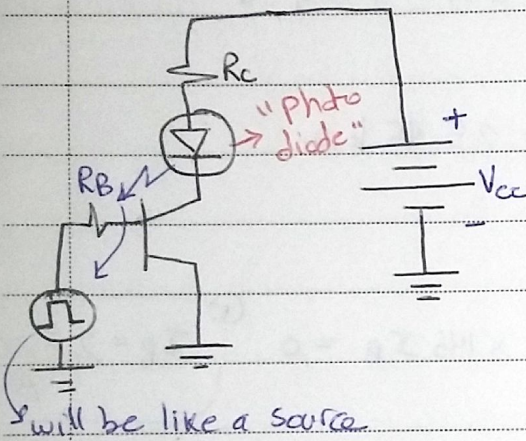
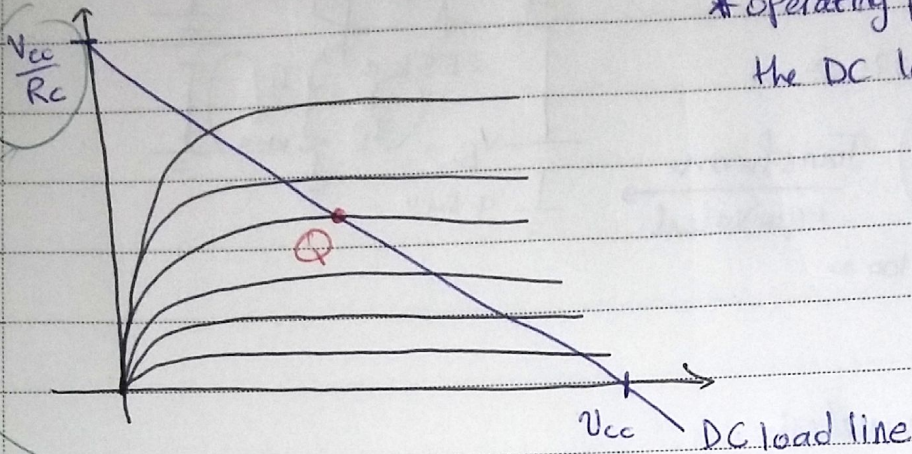
I_B & I_C & V_{CE} must be found

→ V_{CC} from loop on the right loop

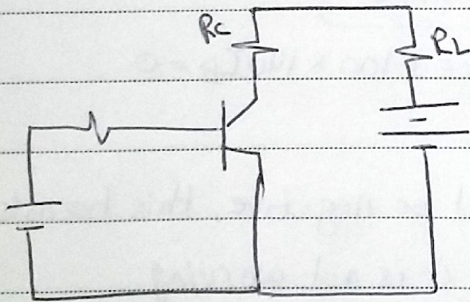
$$-V_{CC} + 375(21.75 \times 10^{-3}) + V_{CE} = 0 \quad V_{CC} = 15 \quad V_{CE} = 6.84 \text{ V}$$



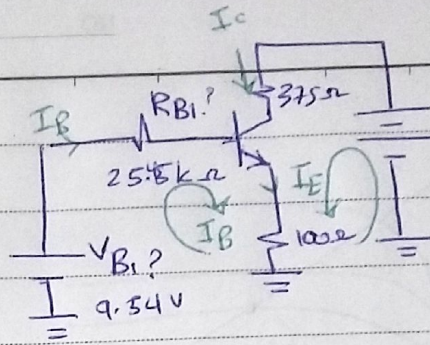
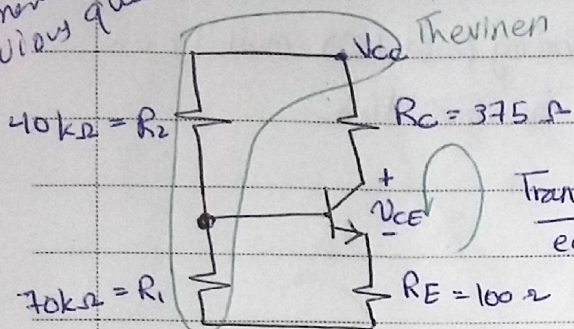
* operating point (Q) must be on the DC load line



the point will be $\frac{V_{CC}}{R_C + R_L}$



From previous question



Find Q?

Thevenin equivalent

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{40 \times 10^3}{(70 + 40) \times 10^3} (15) = \frac{40}{110} (15) = 9.54 \text{ V}$$

$$R_{B1} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{70 \times 40 \times 10^6}{(70 + 40) \times 10^3} = 25.45 \text{ k}\Omega$$

writing KVL around the base loop

$$-9.54 + 25.45 \times 10^3 I_B + 0.6 + 100 \times 146 I_B = 0$$

$$I_E = I_C + I_B$$

$$= \beta I_B + I_B = I_B (\beta + 1)$$

$$\textcircled{2} I_C = \beta I_B = 0.0322 \text{ A} = 32.2 \text{ mA}$$

$$\textcircled{3} V_{CE} - 15 + 375 (32.2 \times 10^{-3}) + V_{CE} + 100 \times 146 I_B = 0$$

writing KVL around collector loop

$$V_{CE} = -0.275, \text{ it must not be negative, this transistor means that it is not working.}$$

$$\textcircled{1} I_B = 2.22 \times 10^{-4} \text{ A} = 0.222 \text{ mA}$$

