



Madarju.com

Electrical

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Date: 24/9/2019

Voltage source



DC $\rightarrow$ DC circuit

1. Power supply $\rightarrow$ Delivered Power $\neq$ net Power = zero

2. Load $\rightarrow$ consumed Power no saving, no more power required

3. Connections (wires)

• DC :-

→ Constant magnitude and direction of time $\vec{F}$ مقدار واتجاه القوة ثابتان
AC :-

 Magnitude & direction change with time.

26/9/2019

- Voltage definition voltage unit $\equiv$ Voltage (V)

voltage at ∞ = zero

→ The work required to move a charge (1 coulomb) from the infinity to this point (if +ve voltage) if -ve voltage → from this point to ∞

A $V = 5V$

$$V = -3V$$
$$V_{AB} = V_A - V_B = 8 \text{ V}$$
$$C \quad V = qV$$
$$V_{AC} = V_A - V_C = 5 - 9 = -4 \text{ V}$$

- Current definition I. current unit $\equiv$ Ampere (A)

$$i = \frac{dq}{dt}$$

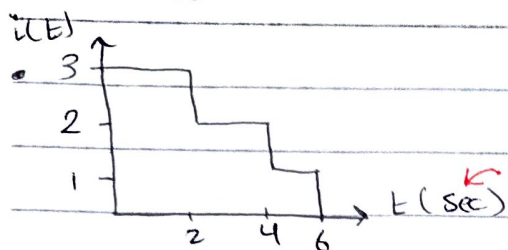
charge & charges with respect with time

In Exam Example

• $q = 4e^{-3t}$ Find $i(t) |_{t=4 \text{ sec}}$

$$i = \frac{dq}{dt} = -12e^{-3t}$$

$$|i|_{t=4} = -12e^{-3(4)} =$$

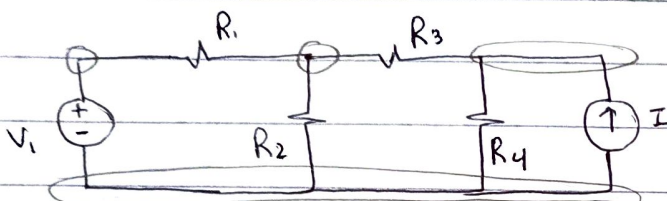


Be careful

Find q

$$q = \int i(t) dt$$

$$q = 2 \times 3 + 2 \times 2 + 2 \times 1 = 11 \text{ C}$$



6 elements

6 branches OR 5 branches

4 nodes

6 loops 3 meshes

• Terminologies

Imp mesh
elements

any closed path which doesn't has any closed path inside

branch

any part of circuit have 1 element or more

The same current passes $\rightarrow$ branch

Imp node

the connection point of two branches or more

loop

any closed path

• Power Definition

P

Power unit is Watt (W)

$\rightarrow$ Delivered power (-ve) (produced) ? $\sum P = 0$

By power supplies

$\rightarrow$ Absorbed power (+ve) (consumed)

By Load (resistances)

* Polarity is important, You always must show the sign



$$I = 2 \text{ A}$$

$$V = 5 \text{ V}$$

$$P = IV = 2 \times 5 = +10 \text{ W} \Rightarrow \text{Absorbed}$$

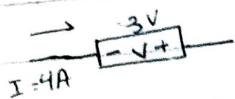
* if current passes from $+$ $\rightarrow$ +ve Power

$- \rightarrow$ -ve Power

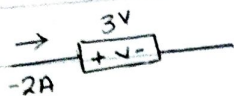
• every mesh loop but not every loop mesh.

No: _____

Date: _____



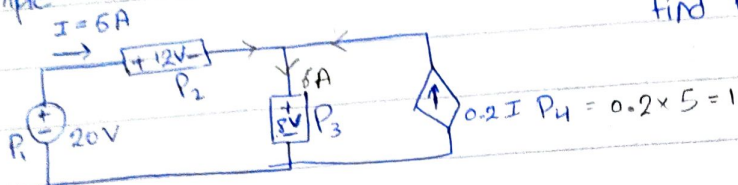
$$P = -4 \times 3 = -12 \text{ W} \rightarrow \text{Delivered power}$$



$$P = +(-2)(3) = -6 \text{ W} \rightarrow \text{Delivered}$$

Example

Find the powers?



$$P_1 = -(5)(20) = -100 \text{ W}$$

$$P_2 = +(5)(12) = +60 \text{ W}$$

$$P_3 = +(6)(8) = +48 \text{ W}$$

$$P_4 = -(8)(1) = -8 \text{ W} \quad \text{OR} \quad 0 = -100 + 60 + 48 + P_4 \quad P_4 = -8$$

* Current in mesh will not be changed, voltage is opposite (In parallel series)

* Elements in parallel have the same voltage, current is opposite.

• At any node, sum of current = 0

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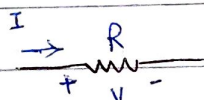
Basic Laws

1* Ohm's Law

V: voltage

I: current

R: resistance



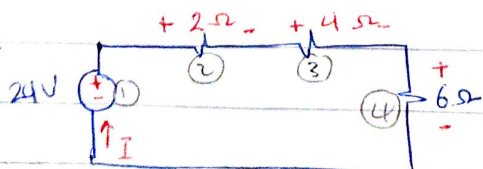
$$V = IR$$

"The voltage drop across any conductor is directly proportional with current intensity passing through the conductor." $V \propto I$ $V = IR$

• Polarity is in the enter of current $\rightarrow$ +ve polarity (In resistance)

Ex. $R = 4 \Omega$ $I = 3 \text{ A}$ $V = (3)(4) = 12 \text{ V}$

Ex.



• we can control the current direction if 1 source, current is out from the +ve polarity of the source (In sources)

$$I = \frac{V}{\sum R} = \frac{24}{2+4+6} = 2 \text{ A}$$

$$V_{6\Omega} = 12 \text{ V}$$

$$V_{4\Omega} = 8 \text{ V}$$

$$V_{2\Omega} = 4 \text{ V}$$

$$P_2 = 8 \text{ W}$$

$$P_3 = 16 \text{ W}$$

$$P_1 = (24)(2) = 48 \text{ W}$$

$$P_4 = 24 \text{ W}$$

ask & receive

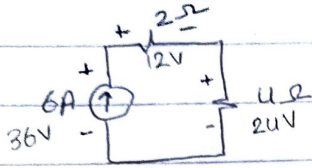
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Date: _____

$$P = V \cdot I = I^2 R = \frac{V^2}{R}$$

$$[P] = W$$

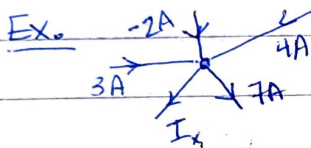
2* Kirchhoff's Laws



① Kirchhoff's current law (KCL)

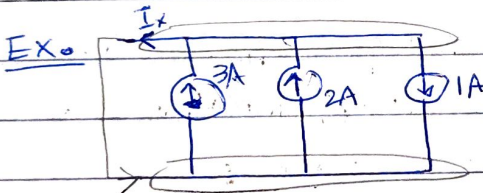
"At any node, the algebraic sum of the currents equal zero"

$$\sum I_{in} = \sum I_{out} \text{ @ node}$$



Find I_x $\sum I_{in} = \sum I_{out}$

$$(3) + (-2) + 4 = 7 + I_x \quad \boxed{I_x = -2A}$$



2 nodes

$$3 + 2 = 1 + I_x$$

$$\boxed{I_x = 4A}$$

$$I_x + 1 = 3 + 2$$

$$\boxed{I_x = 4A}$$

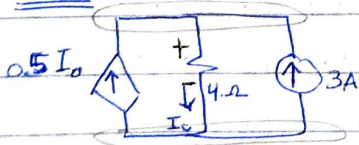
4A

- any element connected up in same node and down on the same node $\rightarrow$ connected in parallel. $(\bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5, \bar{Q}_6, \bar{Q}_7, \bar{Q}_8, \bar{Q}_9, \bar{Q}_{10})$
connected in the same node up & down

* Parallel current sources can be added together.

* Parallel elements have same voltages

EX.

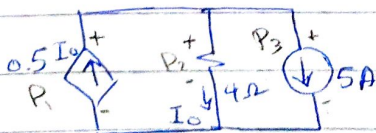


Find I_0 ? By applying KCL $\sum I = 0$ @ node

$$3 + 0.5 I_0 = I_0 \quad 0.5 I_0 = 3 \quad \boxed{I_0 = 6A}$$

$$V_{4\Omega} = (6)(4) = 24V \quad P_{4\Omega} = (6)(24)$$

* Since the answer is +ve, the assumed direction is true.



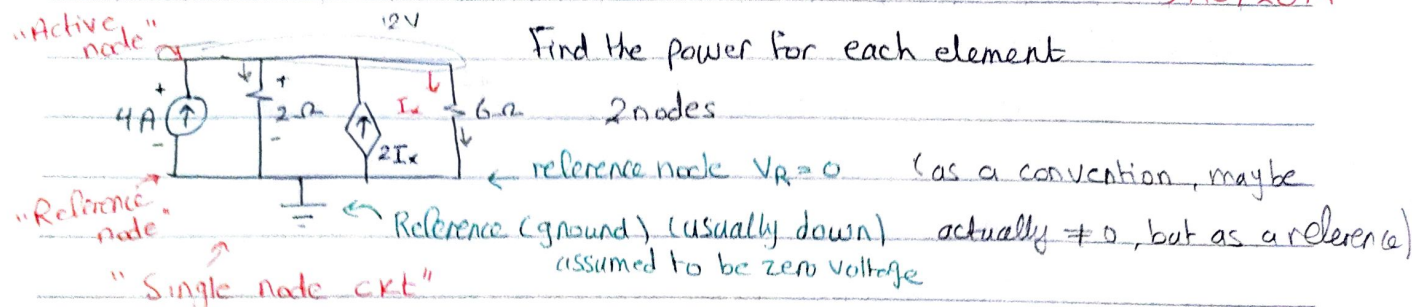
Apply KCL $\sum I = 0$ @ node

$$0.5 I_0 = I_0 + 5 \quad \boxed{I_0 = -10A}$$

$$V_{4\Omega} = (-10)(4) = -40V \quad P_1 = -(0.5 \times -10)(-40) = -200W$$

$$P_2 = P_{4\Omega} = (-10)(-40) = 400W$$

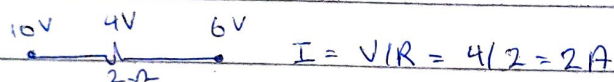
$$P_3 = +(5)(-40) = -200W$$



* Single node ckt Analysis

1. Assign a node (active & reference node)
2. Assign the current in each resistance (resistance between ref. & active node, the current is from active to ref. node, because I assumed $V_{\text{active}} > V_{\text{ref}}$, so the current is from high to low voltage)
3. Apply Kirchhoff's current law for each active node. KCL

e.g.



Solving the above ckt

$$4 + 2I_x = \frac{V}{2} + \frac{V}{6} \quad I_x = \frac{V-0}{6} \text{ since the same direction}$$

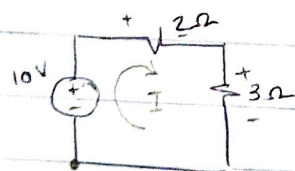


$$4 + 2\left(\frac{V}{6}\right) = \frac{V}{2} + \frac{V}{6} \quad 4 = \frac{V + 3V - 2V}{6} \quad \frac{2V}{6} = 4 \quad V = \frac{4 \times 6}{2} = 12V$$

$$I_{2\Omega} = 6V \quad I_x = 12/6 = 2A$$

$$P_{4A} = -(4)(12) = -48W \rightarrow \text{delivered power}$$

Example



$$-10 + 2I + 3I = 0 \quad I = 2A$$

our assumption of direction is true.

هذا هو التيار المرجعي والرجوع إلى نفس التيار

$$V_{2\Omega} = 4V$$

$$P_S = -(10)(2) = -20W$$

$$P_{2\Omega} = +(2)^2(2) = 8W$$

* Single mesh ckt Analysis

1. Assign current direction (Current exits from +ve pole of battery) clockwise or counter clockwise
 2. Assign the polarity of each resistance (+ve from enter of current)
 3. Apply KVL
- if more than 1 source, assume the current clock wise.

No.:

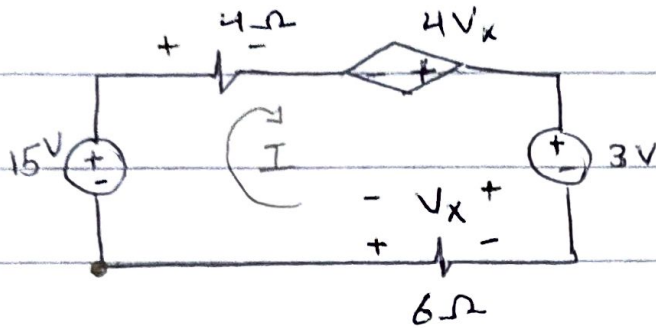
Date:

② Kirchhoff's Voltage Law (KVL)

$\sum V = 0$ For any closed path, even though in loop

Example

"single mesh"



$$-15 + 4I - 4V_x + 3 + 6I = 0$$

$$V_x = -6I$$

$$-15 + 4I + 24I + 3 + 6I = 0$$

$$\boxed{I = 12/34 \text{ A}} \text{ our assumption was correct}$$

H.W show that the total power of the ckt = 0

* Resistances Connection

6/10/2019

① Series → same current through all resistances.



$$R_T = R_1 + R_2 + \dots + R_N$$

$$[R] = \Omega$$

Conductance

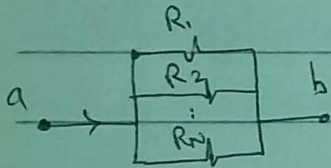
$$I_1 = I_2 = \dots = I_N$$

Conductance →

$$G = \frac{1}{R}$$

$$[G] = \text{S}$$

② Parallel → same voltages

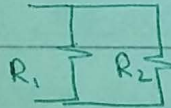


$$V_1 = V_2 = \dots = V_N$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

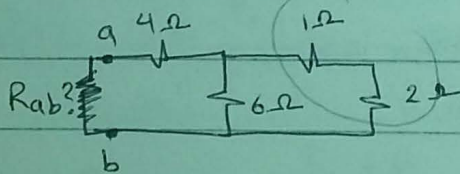
$$G_T = G_1 + G_2 + \dots + G_N$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

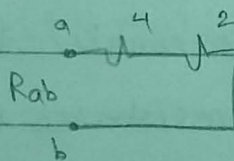
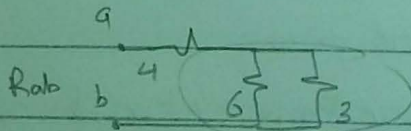


Advice

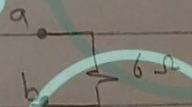
Ex.

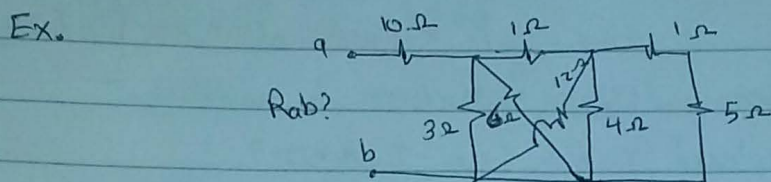
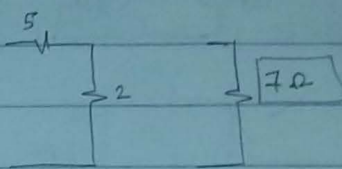
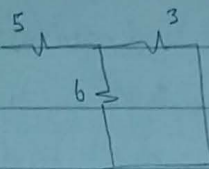
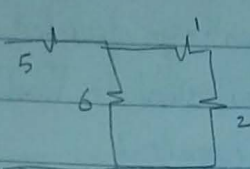
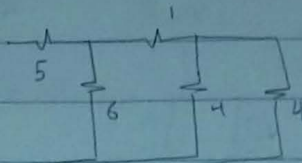
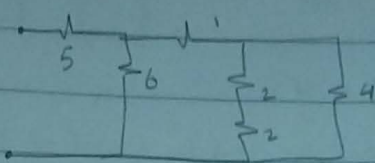
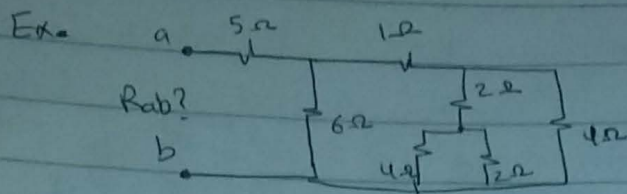


start from the end

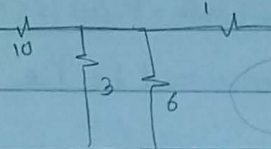
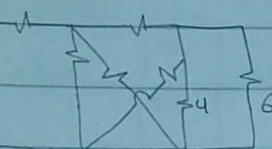


$$R_{ab} = 6 \Omega$$



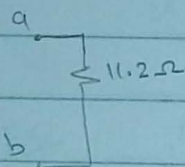
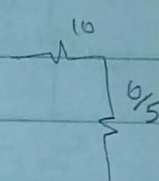
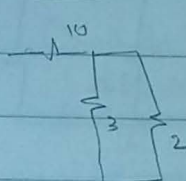
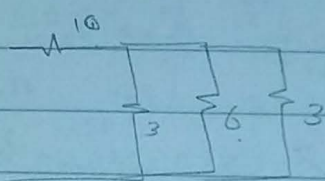
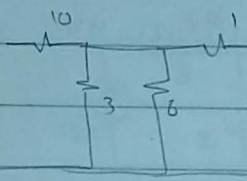


أولاً من أجل R_{ab}



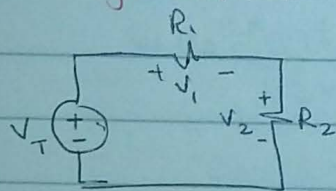
$$\frac{1}{R} = \frac{1}{12} + \frac{1}{4} + \frac{1}{6}$$

$$R = 2 \Omega$$



$$R_{ab} = 11.2 \Omega$$

* Voltage divider



This is the standard ckt (voltage source, 2R in series)

$$V_1 = \frac{V_T R_1}{R_1 + R_2}$$

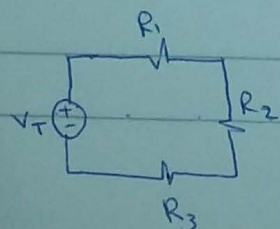
$$V_2 = \frac{V_T R_2}{R_1 + R_2} = V_T - V_1 = IR_2$$

single mesh

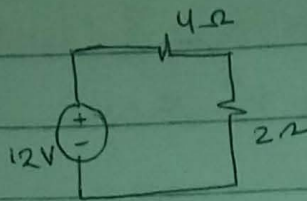
$$I = \frac{V_T}{R_1 + R_2}$$

$$V_1 = IR_1 = \frac{V_T R_1}{R_1 + R_2}$$

$$-V_T + IR_1 + IR_2 = 0 \quad I = \frac{V_T}{R_1 + R_2}$$

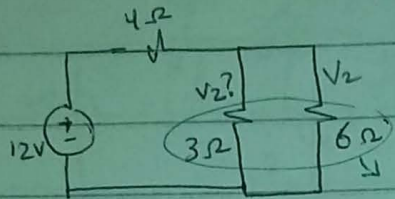


$$V_2 = \frac{V_T R_2}{R_1 + R_2 + R_3}$$

Ex.

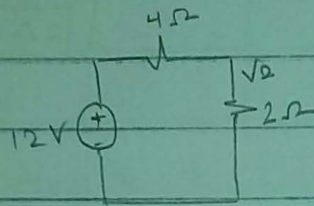
$$V_2 = \frac{12 \times 2}{4 + 2} = 4$$

$$V_1 = \frac{(12)(4)}{4 + 2} = 8$$

Ex.

$$V_2 = \frac{(12)(2)}{2 + 4} = 4 \text{ V}$$

Find equivalent in voltage divider (parallel series), but if you asked to find the current don't find the equivalent.



$$I_{6\Omega} = \frac{4}{6} \text{ A} \leftarrow \text{question will ask for } I_{6\Omega}$$

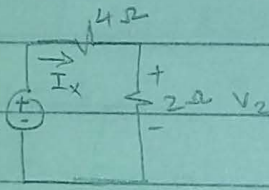
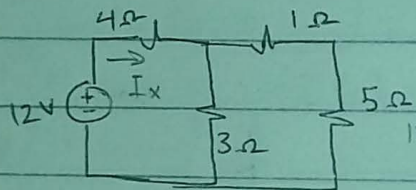
$$I_{3\Omega} = \frac{4}{3} \text{ A}$$

simplify the ckt, then

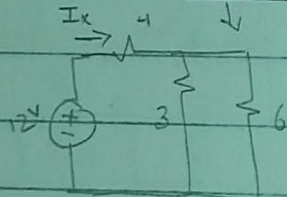
$$\text{go back to the complexed } I_{4\Omega} = I_{6\Omega} + I_{3\Omega} = 2$$

$$\text{after finding what you want } P_{12V} = -(12)(I_{4\Omega}) = -(12)(2) = -24 \text{ W}$$

Ex. Find power consumed by 1Ω resistance



$$V_2 = \frac{(12)(2)}{6} = 4 \text{ V}$$



$$V_{3\Omega} = V_{6\Omega} = 4 \text{ V}$$

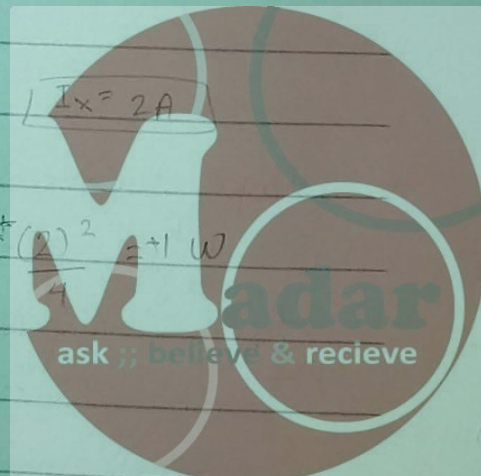
$$V_{1\Omega} = \frac{(4)(1)}{1 + 5} = \frac{4}{6} \text{ V}$$

$$V_{5\Omega} = \frac{(4)(5)}{1 + 5} = \frac{20}{6} \text{ V}$$

$$I_{1\Omega} = I_{5\Omega} = \frac{4}{6} \text{ A}$$

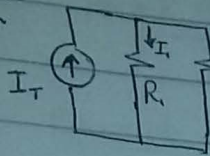
$$P_{1\Omega} = + \frac{(4)^2}{(6)} = \frac{16}{36} \text{ W absorbed}$$

$$P_{2\Omega} = + \frac{(2)^2}{4} = +1 \text{ W}$$



* Current divider rule

standard form



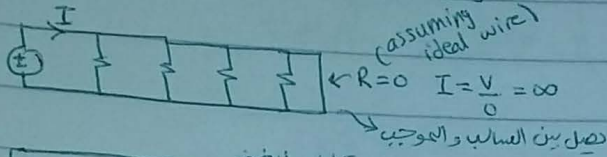
$I_1 = \frac{I_T R_2}{R_1 + R_2}$

$$I_2 = \frac{I_T R_1}{R_1 + R_2} = I_T - I_1$$

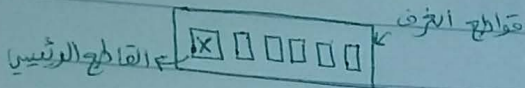
$$I \propto \frac{1}{R}$$

القانون القوي للتيار

Short in home occurs as follows



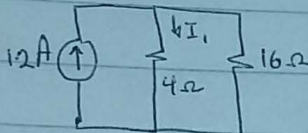
cut breakers must exist in each room



- Difference of resistances prices because of its capacity of power

$$P = I^2 R$$

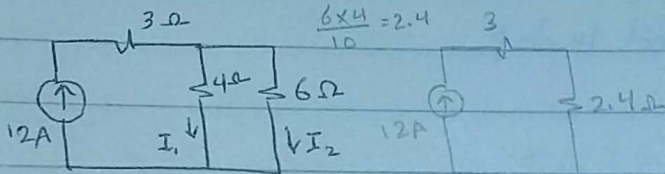
Ex.



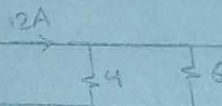
$$I_1 = \frac{(12)(16)}{20} = 9.6 A$$

$$I_2 = \frac{(12)(4)}{20} = 2.4 A = I_T - I_1$$

Ex.



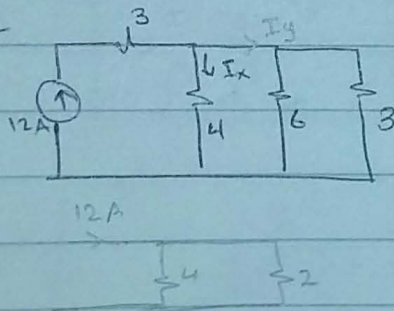
Don't do that it is current division.



$$I_1 = \frac{(12)(6)}{10} = 7.2 A$$

$$I_2 = 12 - 7.2 = 4.8 A$$

Ex.



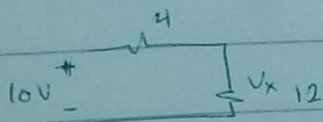
$I_x ?$

$$I_x = \frac{(12)(2)}{4+2} = \frac{24}{6} = 4 A$$

$$I_y = 12 - 4 = 8 A$$

$$I_{6\Omega} = \frac{(8)(3)}{3+6} = \frac{24}{9} A$$

$$I_{3\Omega} = \frac{(8)(6)}{3+6} = \frac{48}{9} A$$



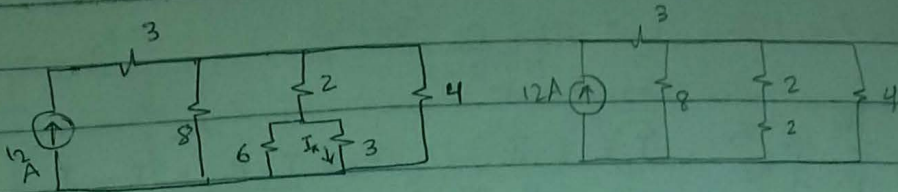
you can find V_x

$$V_x = \frac{10 \times 12}{4+12} = \frac{120}{16} V$$

No:

Date:

Ex.



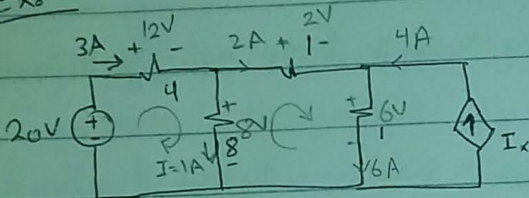
$$I_z = \frac{(12) \left(\frac{32}{12} \right)}{4 + \frac{32}{12}} = 4.8 \text{ A}$$

$$I_x = \frac{I_z (6)}{6 + 3} = 3.2 \text{ A}$$

$$I_{6\Omega} = I_z - I_x$$

In Exam

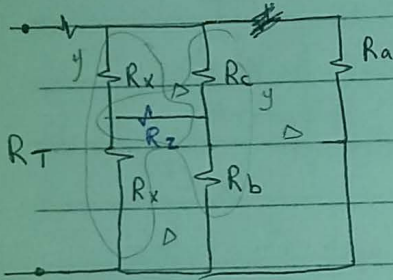
Ex.



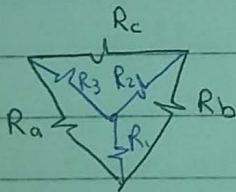
Find all unknowns

$$I_x = 4 \text{ A}$$

10/10/2019

no parallel, nor series
what to do?

* Y ↔ Δ Transformation



keep one of them, Y or Δ

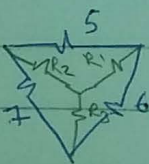
المعادلة • Δ → Y transformation

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

ex.



$$R_1 = \frac{5 \times 6}{5 + 6 + 7} = \frac{30}{18}$$

$$R_2 = \frac{5 \times 7}{18} = \frac{35}{18}$$

$$R_3 = \frac{6 \times 7}{18} = \frac{42}{18}$$

Y → Δ Transformation

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

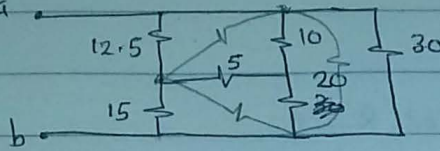
$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

Example

In exam there will be Y-Δ better to solve in Y-Δ than in loops.

Find R_{ab} ?



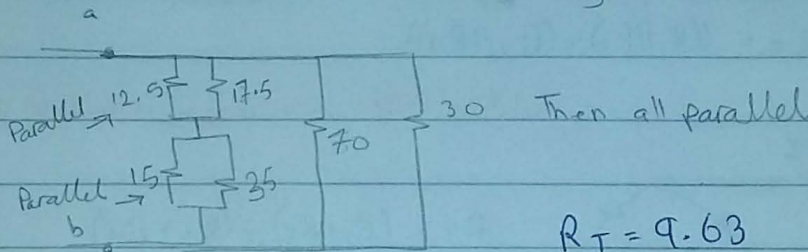
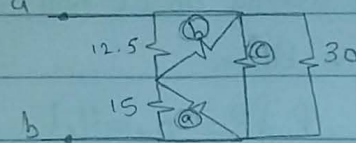
Y → Δ

$$5 \times 10 + 20 \times 5 + 5 \times 20 = 50 + 100 + 100 = 250$$

$$a) = 250 / 10 = 25 \Omega$$

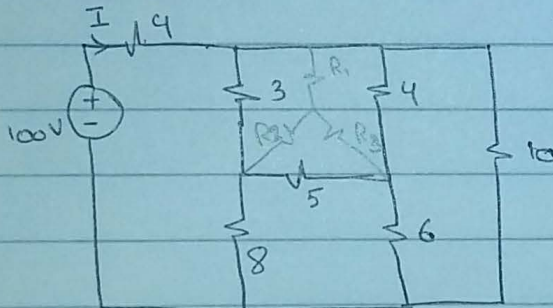
$$b) = 250 / 20 = 12.5 \Omega$$

$$c) = 250 / 5 = 50 \Omega$$



$$R_T = 9.63$$

Example



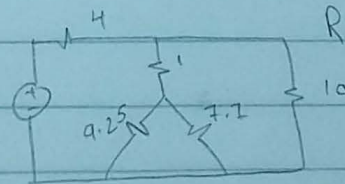
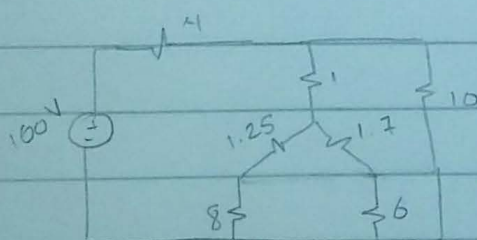
Find I ?

Δ → Y

$$R_1 = \frac{4 \times 3}{12} = 1 \Omega$$

$$R_2 = \frac{15}{12} = 1.25 \Omega$$

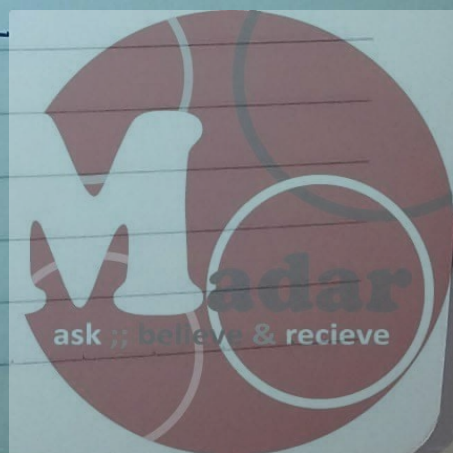
$$R_3 = \frac{20}{12} = \frac{5}{3} = 1.67 \Omega$$



$$R_T = ((1.25 / 1.67) + 1) // 10 + 4$$

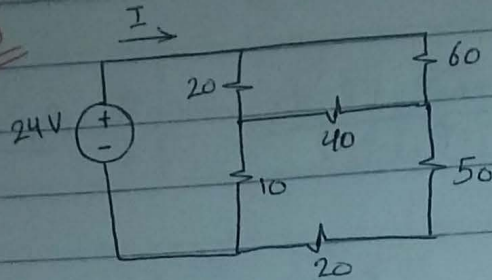
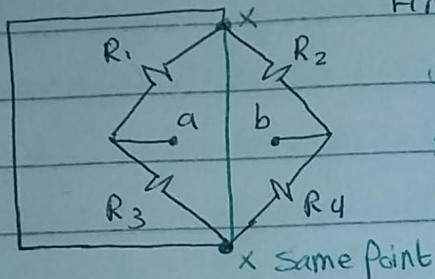
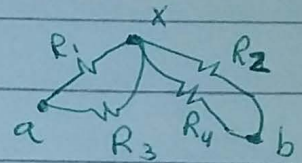
$$I = \frac{V}{R_T} = \frac{100}{R_T}$$

$$R_T = 7.42 \Omega$$



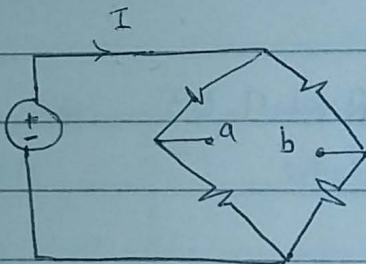
No:

Date:

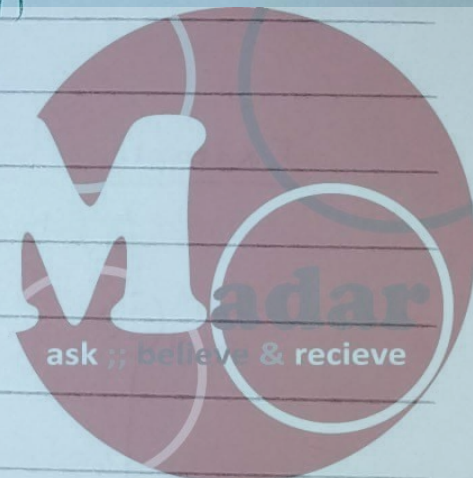
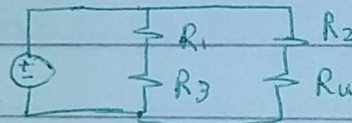
H.WFind I & Power of the source ?ExampleFind R_{ab} ?when asking about R_{ab} , it is like you put source there R_1 & R_3 parallel R_2 & R_4 parallel

$$R_T = ((R_1 // R_3) + (R_2 // R_4))$$

* لا يجوز قائلنا المقاومة ويوجد مصدر بطرف في الدارة



$$R_T = (R_1 + R_3) // (R_2 + R_4)$$

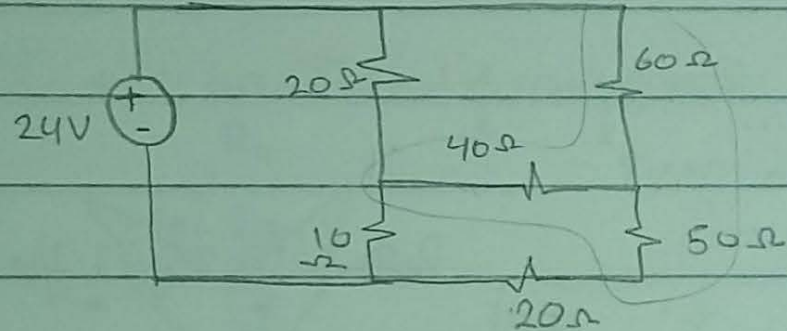


Homework solution

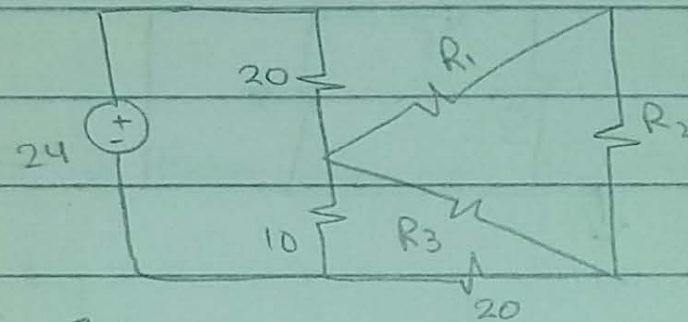
13/10/2019

2Y
1D

Y → Δ



$$40 \times 60 + 40 \times 50 + 60 \times 50 = 7400$$



$$R_1 = 148 \Omega$$

$$R_2 = 185 \Omega$$

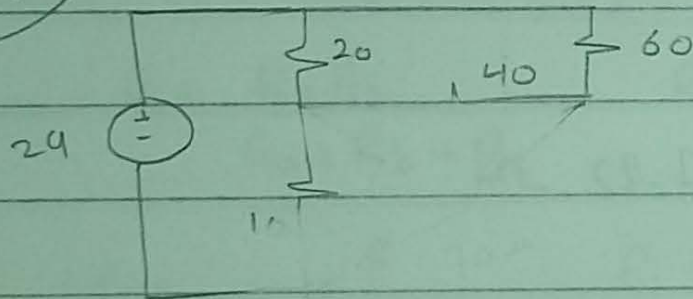
$$R_3 = 123.3 \Omega = \frac{370}{3}$$

OR

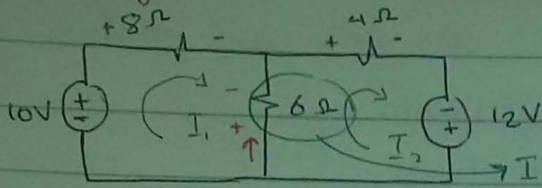
$$R_1 = 134 \Omega$$

$$R_2 = 235 \Omega$$

$$R_3 = 156.7 \Omega$$



* Mesh analysis for more than 1 mesh



→ after solving

1. Assign # mesh
2. Assign a current for each mesh (If $\neq 1$ source, it is specified)
Default: clockwise
3. Assign polarity for each resistance
4. Apply KVL for each mesh

KVL Equ.

For mesh #1 :

$$-10 + 8I_1 + 6(I_1 - I_2) = 0 \quad \text{H } I_1 - 6I_2 = 10 \rightarrow (1)$$

For mesh #2 :

$$4I_2 + -12 + 6(I_2 - I_1) = 0 \quad -6I_1 + 10I_2 = 12 \rightarrow (2)$$

* Solving by Cramer's rule :-

$$\Delta = \begin{bmatrix} 14 & -6 \\ -6 & 10 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} 10 & -6 \\ 12 & 10 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 14 & 10 \\ -6 & 12 \end{bmatrix}$$

Find the determinate

$$|\Delta| = 104$$

$$|\Delta_1| = 172$$

$$|\Delta_2| = 108$$

$$I_1 = \frac{\Delta_1}{\Delta} = 1.65$$

$$I_2 = 2.2 \text{ A}$$

$$P_{10V} = -(10)(I_1)$$

$$P_{6\Omega} = + (I_2 - I_1)^2 6$$

Example

ask ; believe & receive

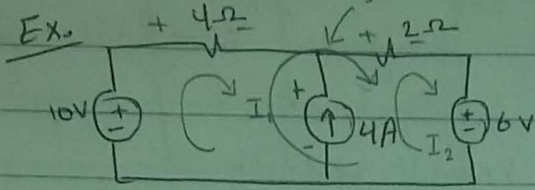
Mesh 1: $-10 + 2I_1 + 4(I_1 - I_2) = 0 \quad (1)$

Mesh 2: $6I_2 + 7(I_2 - I_1) + 4(I_2 - I_1) = 0 \quad (2)$

Mesh 3: $I_3 = 3 \quad (3)$



special case * Super mesh



But I can't write mesh equation

2 unknowns

2 equations

write

①

Supermesh

$$-10 + 4I_1 + 2I_2 + 6 = 0$$

$$4I_1 + 2I_2 = 4 \rightarrow 1$$

② From current source

$$I_2 - I_1 = 4 \rightarrow 2 \text{ (like node analysis)}$$

solve equations simultaneously

$$I_1 = -\frac{2}{3} \text{ A}$$

$$I_2 = 3.3 \text{ A}$$

$$P_{4\Omega} = (4)(-2/3) = -8/3 \text{ W}$$

$$P_{2\Omega} = (2)(I_2) = 6.67 \text{ A}$$

$$P_{10V} = -(10)(-2/3) = +20/3 \text{ W absorbed (نظير الازدواج)}$$

I can do it here because I know the current and voltage. Be careful our direction in the loop.

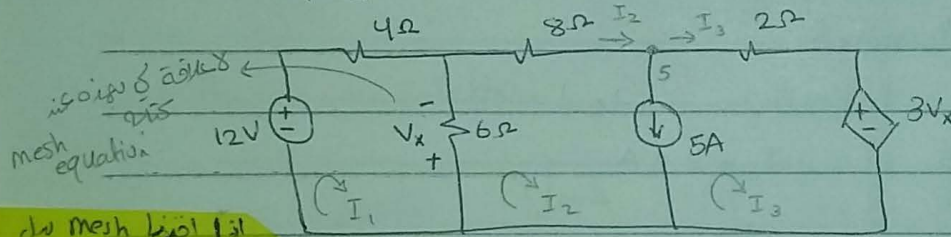
$$V_{4A} = -10 - (-8/3) + V_{4A} = 0 \quad V_{4A} = 38/3 \text{ V} = 12.6 \text{ V}$$

$$P_{4A} = -(4)(12.6) = -50.6 \text{ W according to our assumption}$$

$$P_{6V} = +(6)(3.3) = +20 \text{ W absorbed}$$

Example

$$I_2 = I_3 + 5$$



3 unknowns $\rightarrow$ 3 equations

2 supermesh 3 meshes

not necessary to go to the big mesh

do mesh biol lil ! emorzi current source

$$I_2 - I_3 = 5 \rightarrow 1$$

② mesh 1

$$-12 + 4I_1 + 6(I_1 - I_2) = 0$$

$$10I_1 - 6I_2 = 12 \rightarrow 2$$

③ Supermesh

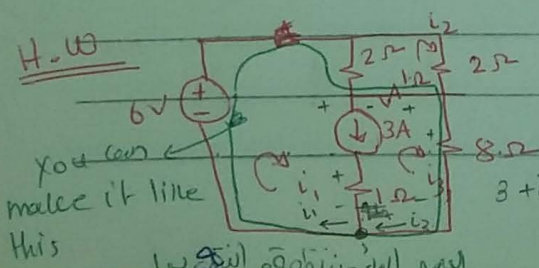
$$-12 + 4I_1 + 8I_2 + 2I_3 + 3V_x = 0 \rightarrow 3 \quad V_x = 6(I_2 - I_1) \rightarrow 4 \text{ OR}$$

KVL around mesh #1 ask, solve & review

$$-12 + 4I_1 - V_x = 0 \quad V_x = 4I_1 - 12$$

Solve these 4 equations.

H.W



Find i_1, i_2, i_3 ?

Put Point from where you start

$$3 + i_3 = i_1$$

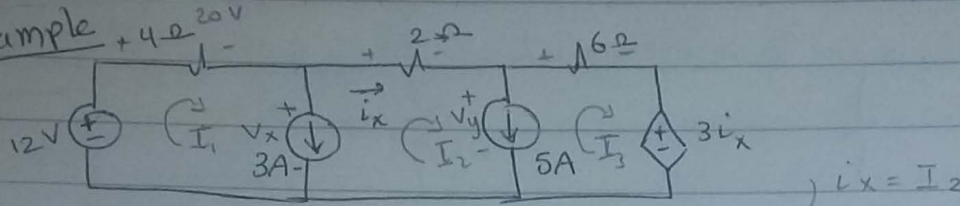
$$I_1 + I_2 = I_2 + I_3 + 1$$

$$I_1 - I_3 = 3$$

$$-3 - V - 1 + 3.2 = 0$$

YASSIN

Example



- ① $-12 + 4I_1 + 2I_2 + 6I_3 + 3i_x = 0$
- ② $I_1 - I_2 = 3$
- ③ $I_2 - I_3 = 5$

$$\begin{aligned} I_1 &= 5 \text{ A} \\ I_2 &= 2 \text{ A} \\ I_3 &= -3 \text{ A} \end{aligned}$$

$$-12 + 20 + V_x = 0 \quad V_x = -8 \text{ V}$$

$$P_{3A} = + (3)(-8) = -24 \text{ W (Delivered)}$$

$$P_{3i_x} = + (-3)(3 \times 2) = -18 \text{ W (Delivered)}$$

$$P_{12V} = - (5)(12) = -60 \text{ W (Delivered)}$$

$$P_{4\Omega} = + (5)^2(4) = 100 \text{ W (Abs.)}$$

$$P_{6\Omega} = + (-3)^2(6) = 54 \text{ W (Abs.)}$$

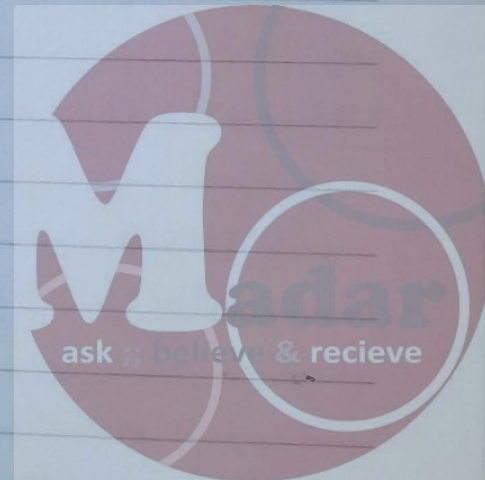
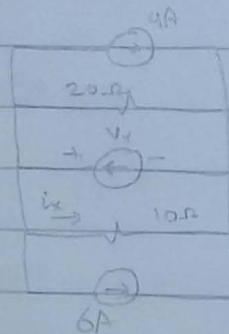
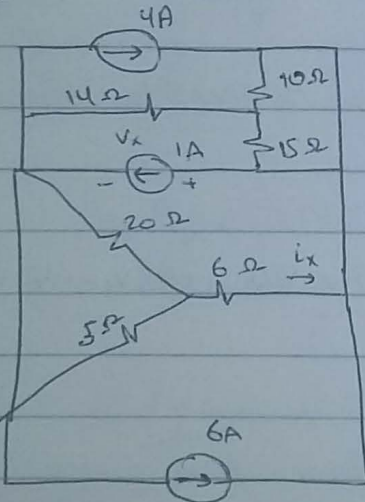
$$\Sigma P = 0 \text{ check } -24 - 18 - 60 - 60 + 100 + 54 + 8 = 0 \quad \#$$

$$-V_x + 2 \times 2 + V_y = 0 \quad V_y = -12 \text{ V}$$

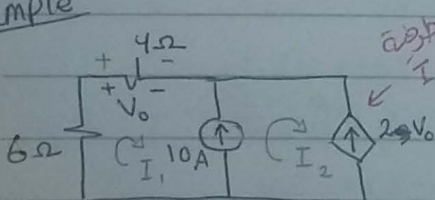
$$P_{5A} = + (5)(-12) = -60 \text{ W (Del.)}$$

$$P_{2\Omega} = + (2)^2(2) = 8 \text{ W (abs.)}$$

Example



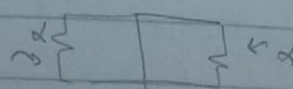
Example



$$I_2 - I_1 = 10$$

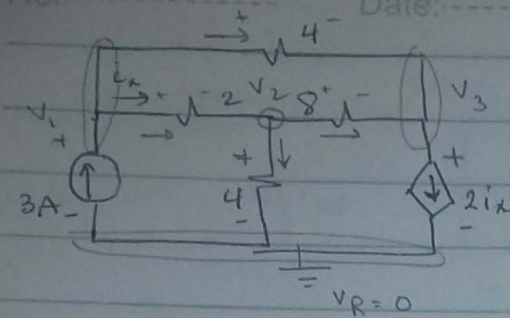
$$I_1 = 10 \text{ A}$$

$$I_2 = -2V_o = -2(4)(I_1) = -8I_1$$



* أي تقي على التوازي مع short out

No: _____ Date: _____



Power in each element?

4 nodes, 3 active nodes $\Rightarrow$ 3 equations

branches = # terms in equation

node ① $3 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} \quad \times 4 \quad 12 = 2V_1 - 2V_2 + V_1 - V_3$

$12 = 3V_1 - 2V_2 - V_3 \rightarrow \text{①}$

node ② $\frac{V_1 - V_2}{2} = \frac{V_2}{4} + \frac{V_2 - V_3}{8} \quad \times 8 \quad 4V_1 - 4V_2 = 2V_2 + V_2 - V_3$

$4V_1 - 7V_2 + V_3 = 0 \rightarrow \text{②}$

node ③

$\frac{V_2 - V_3}{8} + \frac{V_1 - V_3}{4} = 2i_x \quad \text{But } i_x = \frac{V_1 - V_2}{2}$

$\frac{V_2 - V_3}{8} + \frac{V_1 - V_3}{4} = \frac{V_1 - V_2}{2} \quad \times 8 \quad V_2 - V_3 + 2V_1 - 2V_3 = 4V_1 - 4V_2$

$6V_1 - 9V_2 + 3V_3 = 0 \rightarrow \text{③}$

$V_1 = 4.8V \quad V_2 = 2.4V \quad V_3 = -2.4V$

$P_{3A} = -3(4.8) = -14.4W$

$P_{dep} = +(-2.4)(4.8 - 2.4) \times 2 = -5.76W$

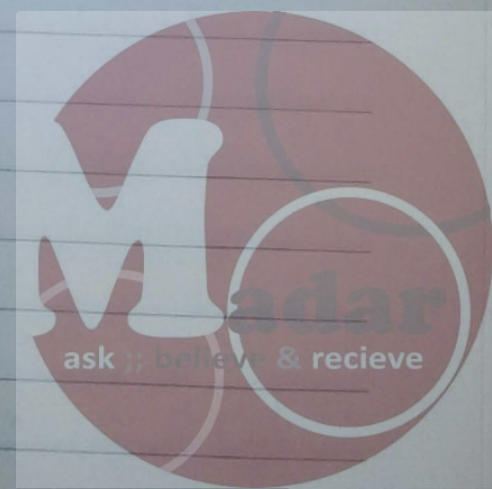
$P_{4\Omega \text{ down}} = V_2^2 / 4 = 2.4^2 / 4 = 1.44W$

$P_{2\Omega} = (V_1 - V_2)^2 / 2 = 2.88W$

$P_{8\Omega} = +(V_2 - V_3)^2 / 8 = 2.88W$

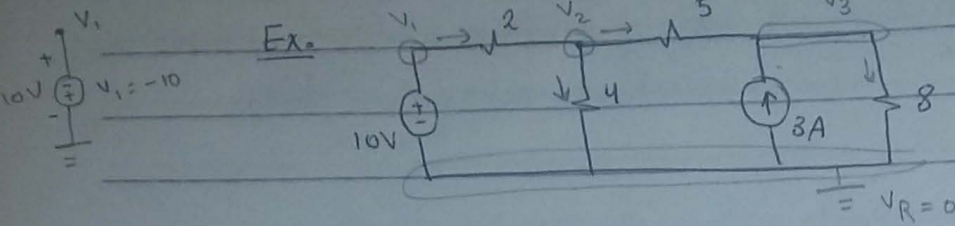
$P_{4\Omega \text{ up}} = +(V_1 - V_3)^2 / 4 = 12.96W$

Check $\Sigma P = 0 \Rightarrow \times$



* Supernode analysis

Ex.



@ node ① $V_1 = 10V \rightarrow ①$

@ node ② $\frac{V_1 - V_2}{2} = \frac{V_2}{4} + \frac{V_2 - V_3}{5}$ $\frac{V_1}{2} - \frac{1}{2}V_2 = \frac{V_2}{4} + \frac{V_2}{5} - \frac{V_3}{5} \quad | \times 20$

$-19V_2 + 4V_3 = -100 \rightarrow ②$

@ node ③ $\frac{V_2 - V_3}{5} + 3 = \frac{V_3}{8}$

$8V_2 - 8V_3 + 120 = V_3$

$8V_2 - 9V_3 = -120 \rightarrow ③$

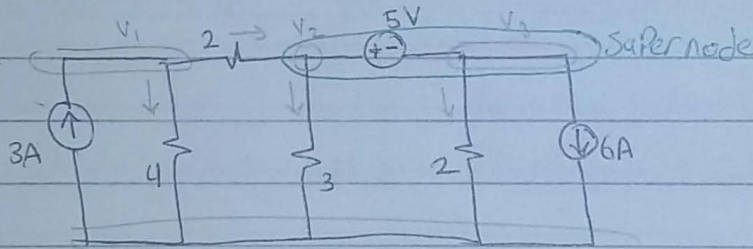
write solution in matrix form

$$\begin{bmatrix} -19 & 4 \\ 8 & -9 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -100 \\ -120 \end{bmatrix}$$

$a_1x + b_1y + c_1z = D_1$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

Ex.



write available nodes

Supernode $\rightarrow$ 1 equation

@ node ① $3 = \frac{V_1}{4} + \frac{V_1 - V_2}{2}$

$12 = 3V_1 + 2V_1 - 2V_2$

$5V_1 - 2V_2 = 12 \rightarrow ①$

@ super node $\frac{V_1 - V_2}{2} = \frac{V_2 + V_3}{3} + 6$

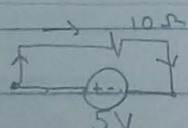
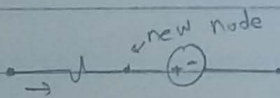
$-3V_2 + 3V_1 = 2V_2 + 3V_3 + 36$

$-3V_1 + 5V_2 - 3V_3 = +36 \rightarrow ②$

@ voltage source $V_2 - V_3 = 5$

matrix

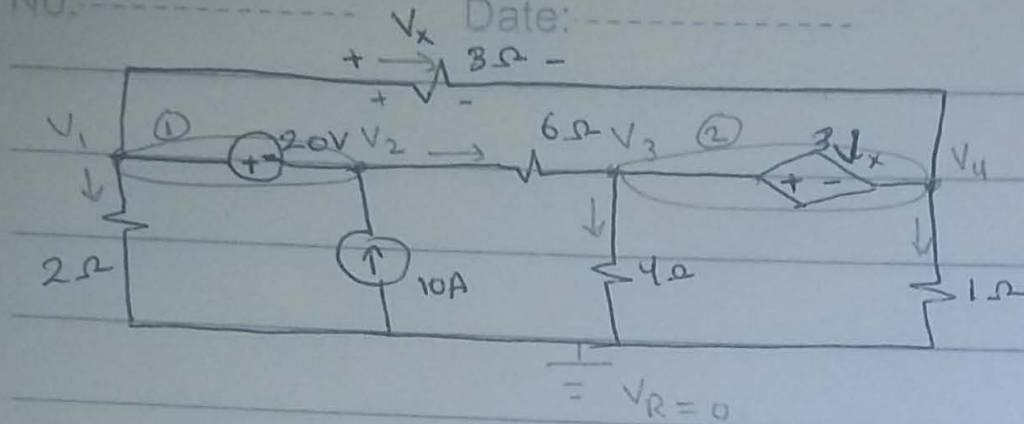
$$\begin{bmatrix} 3 & -2 & 0 \\ 3 & -5 & -3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 36 \\ 5 \end{bmatrix}$$



current $I = \frac{V_2 - V_3}{10}$
supernode

* But at end you can find current in voltage source, by applying KCL around the node.

No: _____ Date: _____



① Supernode ①

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} = 10 \quad \times 6$$

$$3V_1 + 2V_1 - 2V_4 + V_2 - V_3 = 60$$

$$5V_1 + V_2 - V_3 + 2V_4 = 60 \rightarrow \textcircled{1}$$

② Supernode ②

$$\frac{V_2 - V_3}{6} + \frac{V_1 - V_4}{3} = \frac{V_4}{1} + \frac{V_3}{4} \quad \times 12$$

$$2V_2 - 2V_3 + 4V_1 - 4V_4 = 12V_4 + 3V_3$$

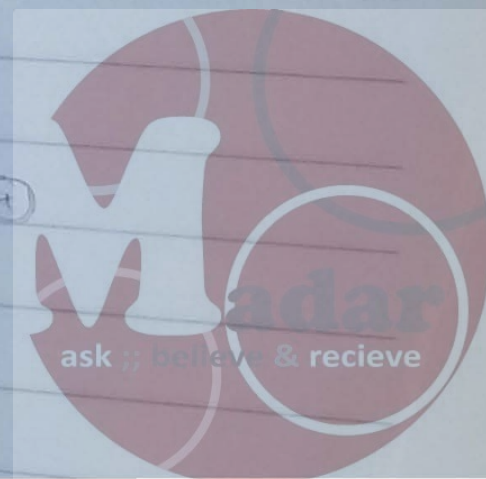
$$4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \rightarrow \textcircled{2}$$

③ $V_1 - V_2 = 20 \rightarrow \textcircled{3}$

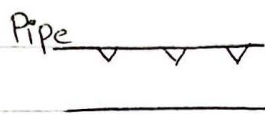
④ $V_3 - V_4 = 3V_x$ But $V_x = V_1 - V_4$

$$V_3 - V_4 = 3V_1 - 3V_4$$

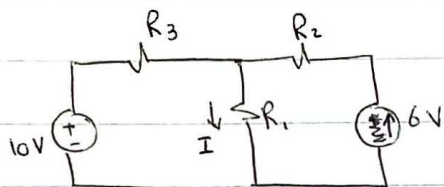
$$3V_1 - V_3 - 2V_4 = 0 \rightarrow \textcircled{4}$$



* Superposition technique طريقة التراكيب



effect of each one ~~equal~~ when added together is the same for all of them together



I is summation of the two sources every time one depends independent source

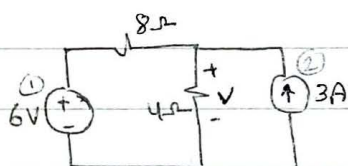
"kill all the INDEPENDENT sources except one"

kill $\rightarrow$ Voltage independent source $\rightarrow$ Short cut $\rightarrow$
 Current independent source $\rightarrow$ Open cut $\rightarrow$

NOT POWER - To calculate voltage of currents, it consists of many components

Advice: Each time you solve draw, number the sources, you have to control the direction (polarity)

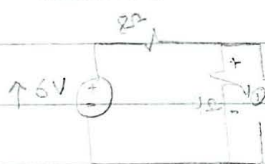
Example :



Find V using superposition (S.P)

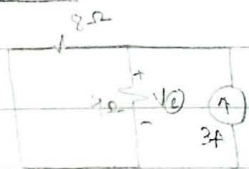
Find the current coming from source #1
 $\rightarrow$ Requires Superposition.

Source 1



$$V_1 = \frac{6 \times 4}{8 + 4} = 2V$$

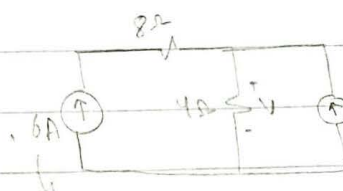
Source 2



$$V_2 = 2(4) = 8V$$

$$I_{4\Omega} = \frac{3 \times 8}{12} = 2A$$

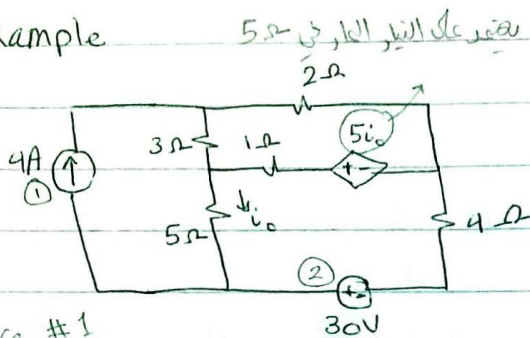
$$\rightarrow V = V_1 + V_2 = 2 + 8 = 10V$$



Source 1
 $V = 2V$



Example



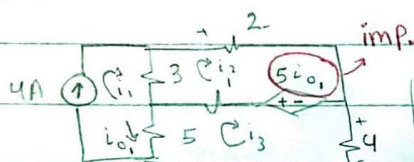
Find i_o by S.P

3 sources

2 ind. Sources

twice SP.

Source #1



Mesh analysis :-

#1 $i_1 = 4A \rightarrow (1)$

#2 $2i_2 - 5i_{o1} + (i_2 - i_3) + 3(i_2 - i_1) = 0$

$i_{o1} = i_1 - i_3$

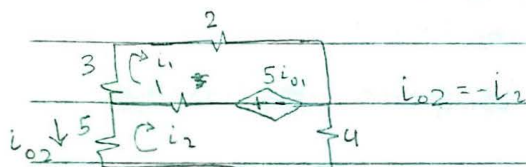
$2i_2 - 5i_1 + 5i_3 + i_2 - i_3 + 3i_2 - 3i_1 = 0$

$6i_2 + 4i_3 = 32 \rightarrow (2)$

#3 $5(i_3 - i_1) + (i_3 - i_2) + 5(i_1 - i_3) + 4i_3 = 0$

$-i_2 + 5i_3 = 0 \rightarrow (3)$

$i_{o1} = 4 - 0.94 = 3.05A$



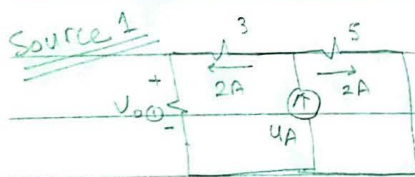
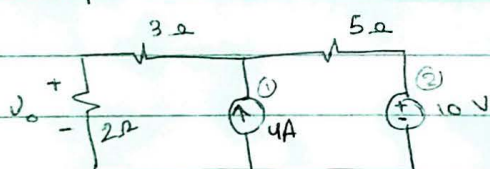
$i_{o2} = -i_2$

Source #2

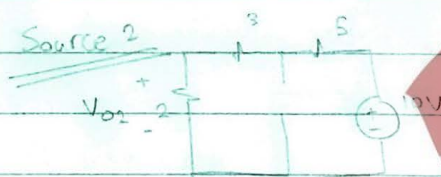
#1 $6i_1 - i_2 - 5(-i_2) = 0 \quad 6i_1 + 4i_2 = 0 \rightarrow (1)$

#2 $5i_2 + i_2 - i_1 + -5(-i_2) + 4i_2 = 30 \quad -i_1 + 5i_2 = 30 \rightarrow (2)$

Example



$V_{o1} = 2 \times 2 = 4V$



$V_{o2} = \frac{10 \times 2}{10} = 2V$

$\Rightarrow V_o = V_{o1} + V_{o2} = 4 + 2 = 6V$

IMP * why I can't apply power ?

- if opposite polarities, power will be added not subtracted ($\frac{V^2}{R} / I^2 R$ is +ve)

- we calculate it after finding the overall current/voltage

Solve H.W on Facebook

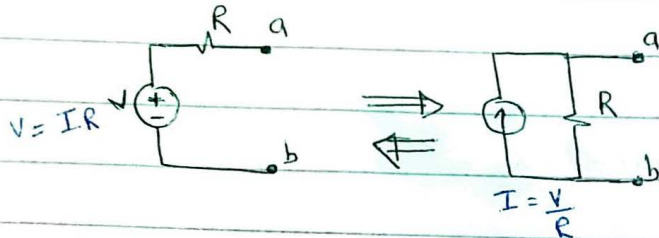
7/11/2019

H.w Answer

The second source, current division rule is easier than mesh analysis

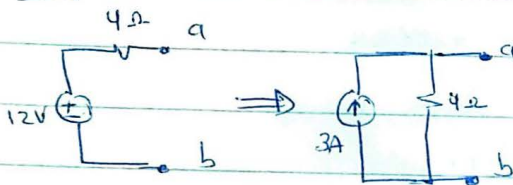
In exam The contribution of current source is — (Apply superposition)

→ Sources Transformation



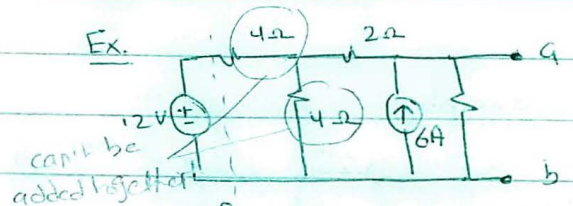
R the same value but not the same R
(not the same characteristic, like current and voltage)

Ex.

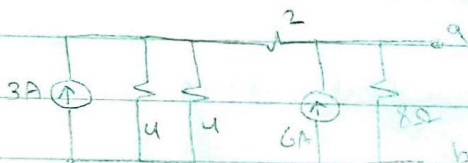


You transfer because of a reason

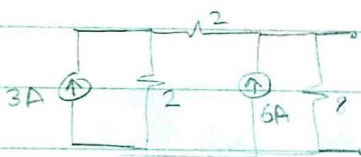
Ex.



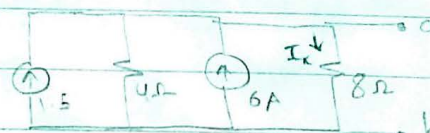
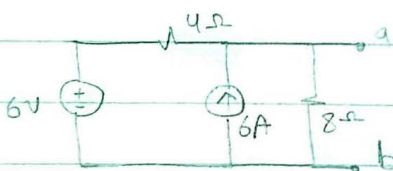
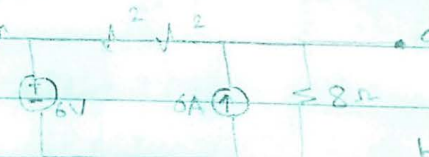
Simplify cut to 1 voltage source and one ~~set~~ resistance



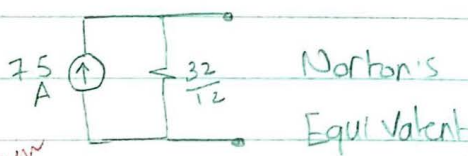
because not on the same node



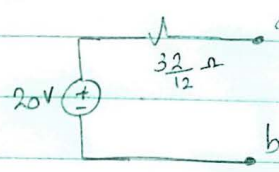
We can't add current sources



stop here if you want I_x , solve by current division. But not all source transformation lead to the final result. It is not a technique but to reduce the ckt.



Norton's Equivalent



any ckt can be converted to 1 current source and resistance in Parallel " Norton's Equivalent

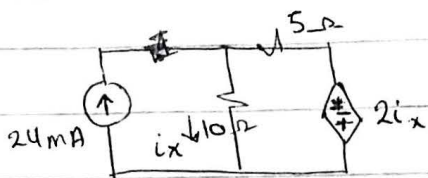
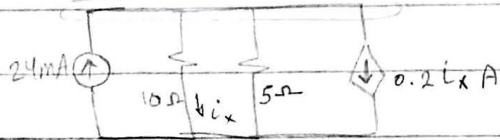
any ckt can be reduced to 1 voltage source and resistance in series " Thevenin's Equivalent

Thevenin's Equivalent

we can apply source transformation to dependent sources

Relation between source transformation

علاقة بين المصادر المتغيرة

ExampleFind i_x ?

Single node

$$10 \times 24 = \frac{V}{10} + \frac{V}{5} + 0.2 i_x \quad i_x = \frac{V}{10}$$

$$24 \times 10^{-3} = \frac{V}{10} + \frac{V}{5} + \frac{2}{50} V \quad \frac{24 \times 10^{-3} \times 50}{17} = V$$

$$V = 70.6 \text{ mV}$$

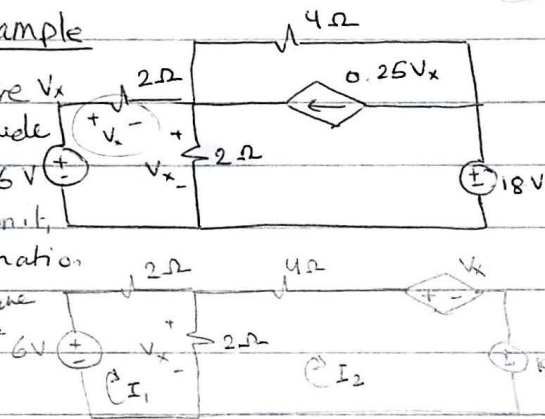
$$i_x = 7.06 \text{ mA}$$

Exampleif here V_x

You can't include it, depending 6V source depends on it.

after transformation

there will not have the same voltage

Find V_x ?

By superposition, if not you can make it by nodal analysis

By mesh analysis

$$\#1 \quad -6 + 2I_1 + 2(I_1 - I_2) = 0$$

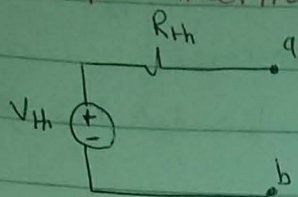
$$\#2 \quad 2(I_2 - I_1) + 4I_2 + V_x + 18 = 0 \quad V_x = 2(I_1 - I_2)$$

$$I_1 = -0.75 \text{ A} \quad I_2 = -4.5 \text{ A} \quad V_x = 2(-0.75 + 4.5) = 7.5 \text{ V}$$



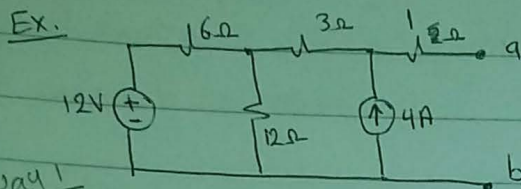
Tuesday 12/11/2019

JSJJS ckts * Thevenin's Theorem



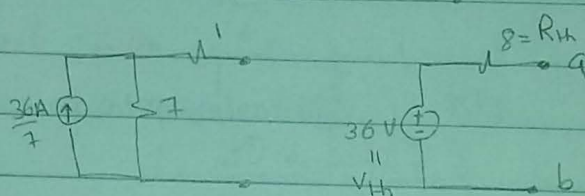
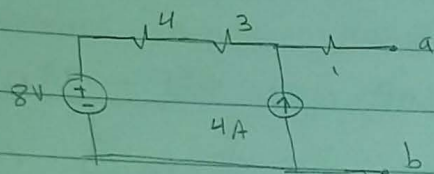
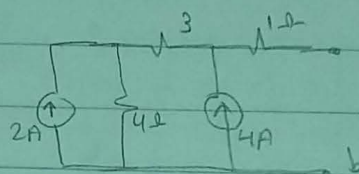
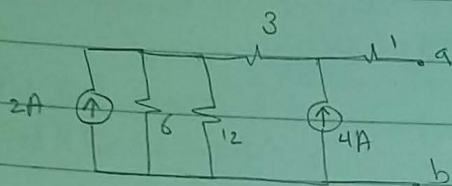
"Thevenin's equivalent"

not all source transformation can be solved



Find Thevenin's equivalent

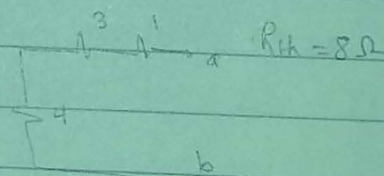
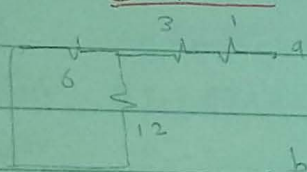
Way 1
Source Transformation



Way 2
Thevenin's equivalent

• STEPS :

1. Kill all the independent sources & Calculate R_{th}

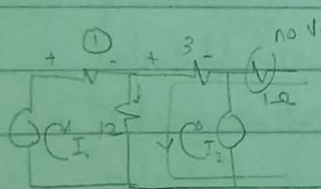


between a & b
open ckt

2. Reconnect all the independent sources and calculate (V_{th} , V_{oc} , V_{ab})

3 choices we have

- Through current source $\rightarrow$ we can't
- 1, 3, 12Ω $\oplus$
- 1, 3, 6Ω, 12V



mesh or nodal or any way

$$I_2 = -4$$

$$-12 + 6I_1 + 12(I_1 - I_2) = 0$$

$$-12 + 12 \times 4 = 18I_1 \quad I_1 = -2A$$

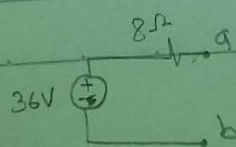
ubiosri 15p.25m

$$V_{ab} = V_{th} = V_{oc} = V_{12\Omega} + V_{3\Omega} + V_{1\Omega}$$

$$= 0 + 12 + 12(I_1 - I_2)$$

$$= 0 + 12 + 12(-2 + 4) = 36V$$

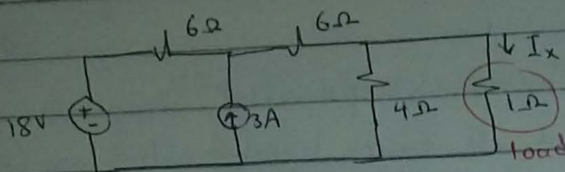
3. Draw Thevenin's equivalent



No: _____

Date: _____

Ex

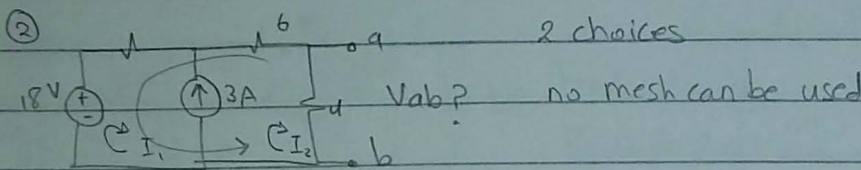
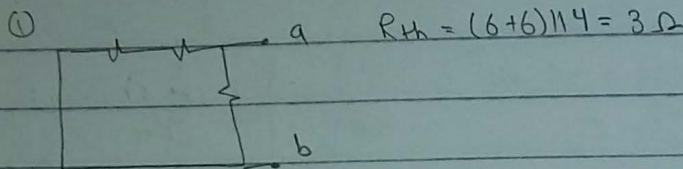
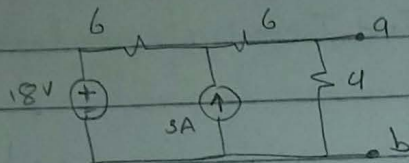


Find I_x using Thevenin's theorem

1. cut the load and put a, b

2. Find Thevenin's equivalent

Source transformation



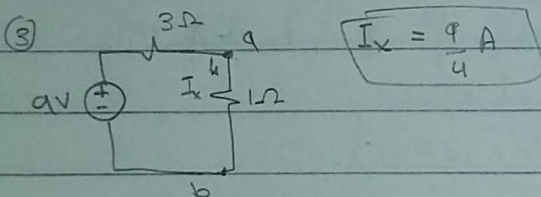
$$V_{th} = V_{oc} = V_{ab} = V_{4\Omega} = 4I_2 = 9V$$

$$-18 + 6I_1 + 6I_2 + 4I_2 = 0 \quad 6I_1 + 10I_2 = 18 \rightarrow (1)$$

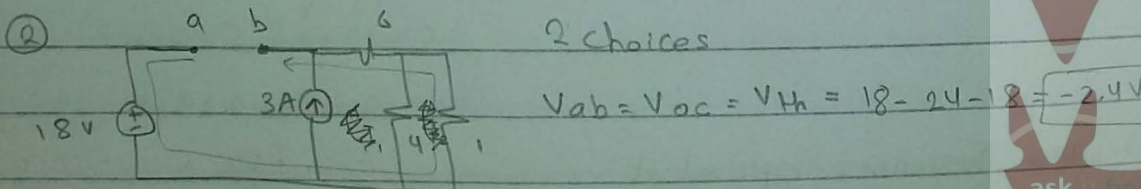
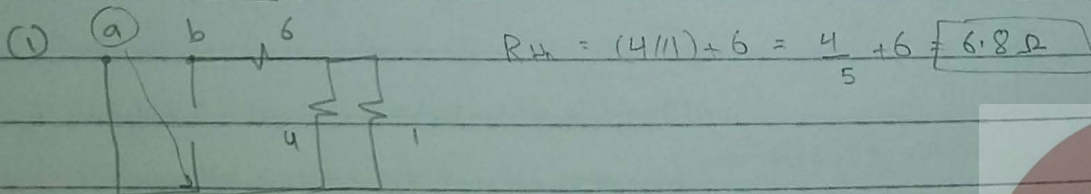
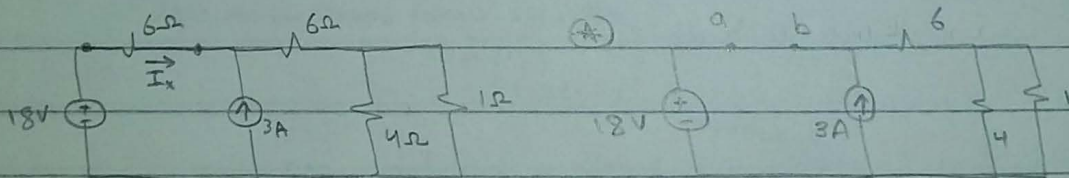
$$I_2 - I_1 = 3 \rightarrow (2)$$

$$I_1 =$$

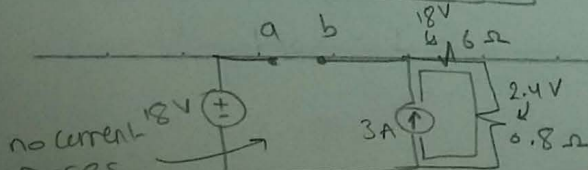
$$I_2 = \frac{9}{4} A$$



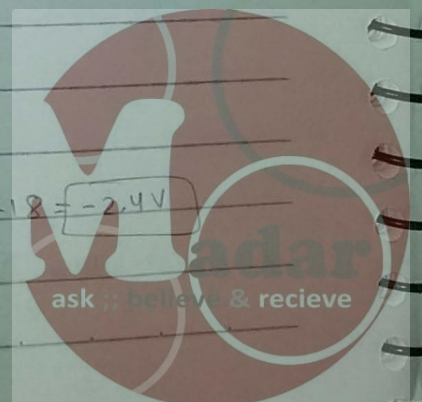
Find I_x on 6Ω (the first one)



$$V_{ab} = V_{oc} = V_{th} = 18 - 24 = -6V$$

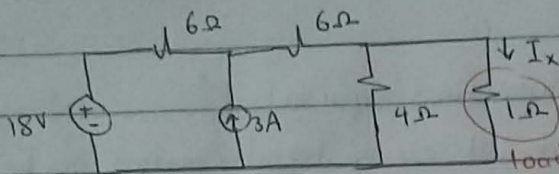


YASSIN



No: _____ Date: _____

Ex.



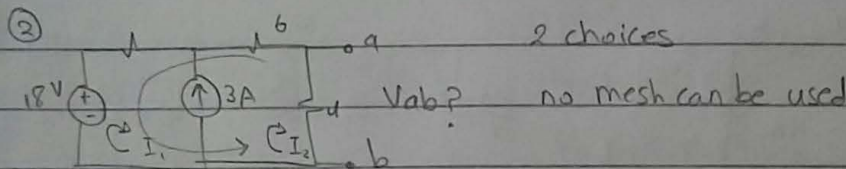
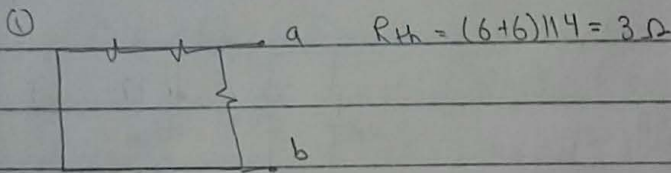
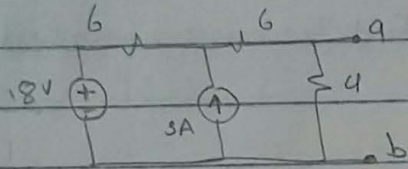
Find I_x using Thevenin's Theorem

المقاومة المعتمدة على الجهد

1. cut the load and put a, b

2. Find Thevenin's equivalent

Source transformation

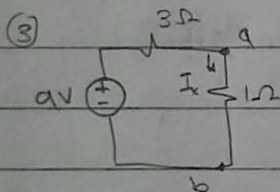


$$V_{Th} = V_{oc} = V_{ab} = V_{4\Omega} = 4I_2 = 9V$$

$$-18 + 6I_1 + 6I_2 + 4I_2 = 0 \quad 6I_1 + 10I_2 = 18 \rightarrow (1)$$

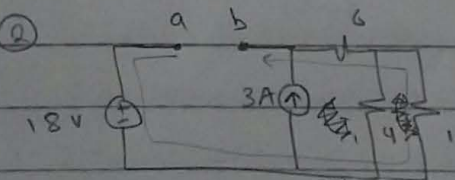
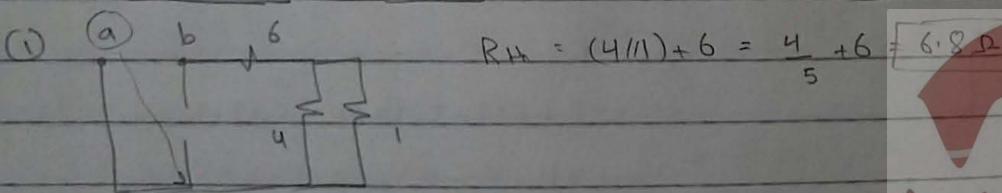
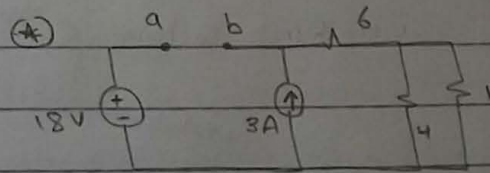
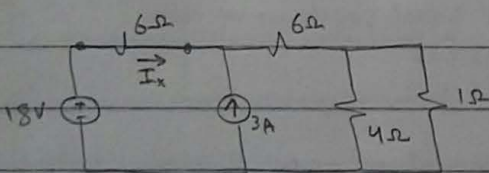
$$I_2 - I_1 = 3 \rightarrow (2)$$

$$I_2 = \frac{9}{4} A$$



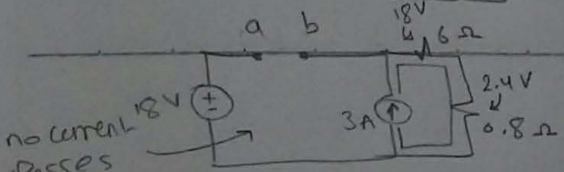
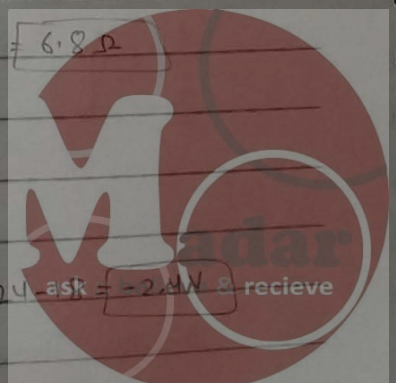
$$I_x = \frac{9}{4} A$$

Find I_x on 6Ω (the first one)



2 choices

$$V_{ab} = V_{oc} = V_{Th} = 18 - 24 = -6V$$

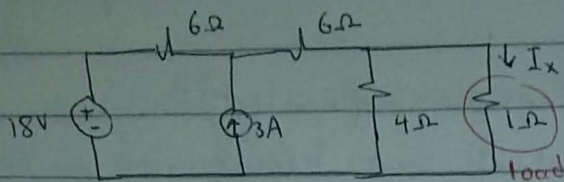


YASSIN

dependent source
مستقل

No: Date:

Ex.



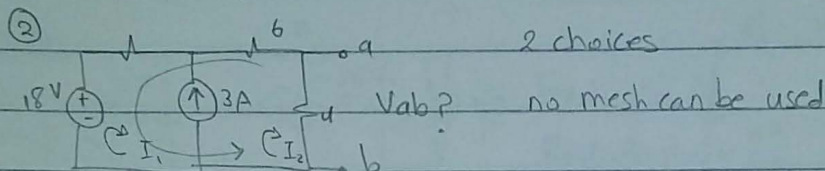
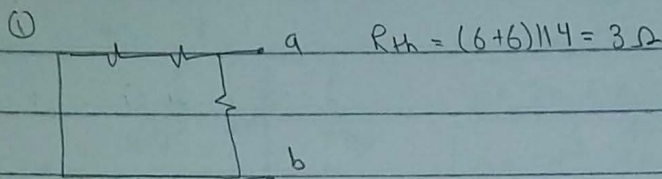
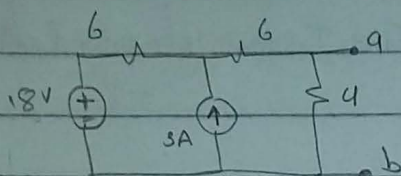
Find I_x using Thevenin's theorem

المقاومة المستقلة

1. cut the load and put a, b

2. Find Thevenin's equivalent

Source transformation



2 choices

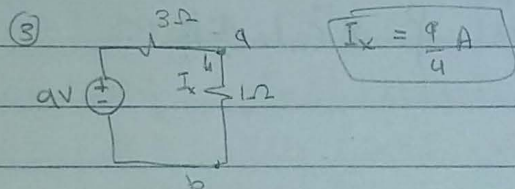
no mesh can be used

$$V_{th} = V_{oc} = V_{ab} = V_{4\Omega} = 4I_2 = 9V$$

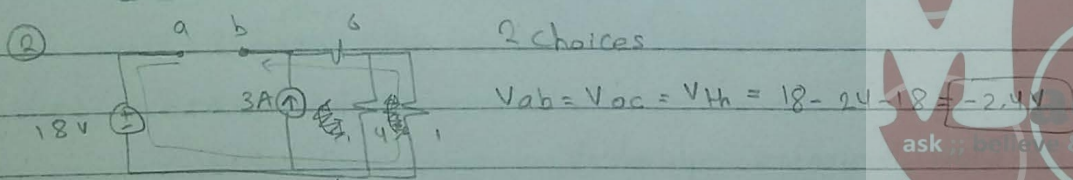
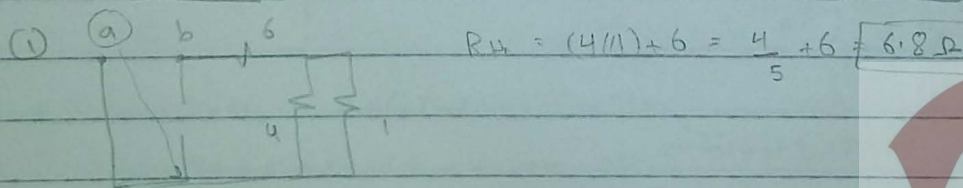
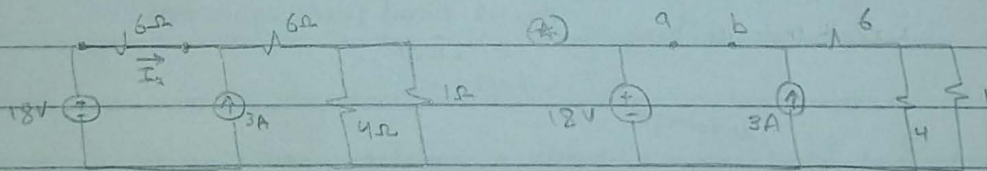
$$-18 + 6I_1 + 6I_2 + 4I_2 = 0 \quad 6I_1 + 10I_2 = 18 \rightarrow (1) \quad I_1 =$$

$$I_2 - I_1 = 3 \rightarrow (2)$$

$$I_2 = \frac{9}{4} A$$

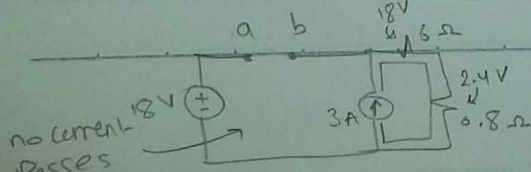


Find I_x on 6Ω (the first one)

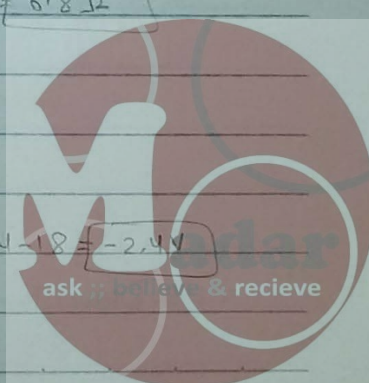


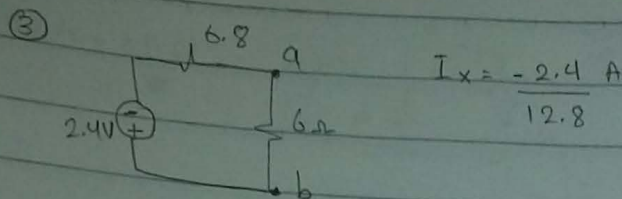
2 choices

$$V_{ab} = V_{oc} = V_{th} = 18 - 24 - 18 = -2.4V$$



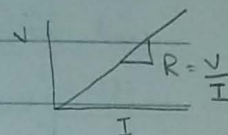
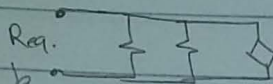
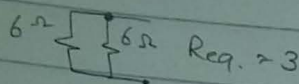
YASSIN





14/11/2019

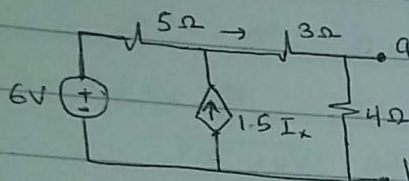
- Only Physical source is the independent voltage source
- Dependent source is a resistance



Put test source $\left\{ \begin{array}{l} \text{voltage} \rightarrow \text{find current (1A)} \\ \text{current} \rightarrow \text{find voltage (1V)} \end{array} \right.$
Find R for the black box

optimize your choice / amount is not a must / linear relation between I & $V \rightarrow$ same Req. results

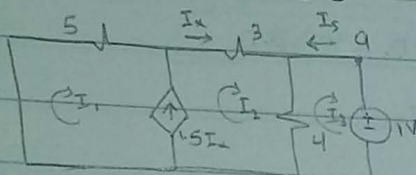
* Thevenin's Theorem with dependent source



Find Thevenin's equivalent ckt

① Find Req. $\rightarrow$ By test source

Find I_s By mesh analysis



* You can use it also if there is no dependent source.

*3

$$1 + 4I_3 - 4I_2 = 0$$

$$-4I_2 + 4I_3 = -1 \rightarrow ①$$

Super mesh

$$5I_1 + 3I_2 + 4I_2 - 4I_3 = 0$$

$$5I_1 + 7I_2 - 4I_3 = 0 \rightarrow ②$$

Branch

$$I_2 - I_1 = 1.5I_x = 1.5I_2$$

$$-I_1 - 0.5I_2 = 0 \rightarrow ③$$

$$I_1 = 1A \quad I_2 = -2A \quad I_3 = -2.25A$$

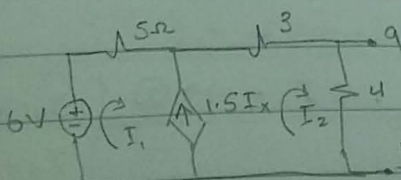
$$I_s = -I_3 = 2.25A$$

$$R_T = R_{Th} = 1/2.25 = 0.4\Omega = V_s / I_s$$

1. To calculate R_{Th} add a test source (voltage or current) then calculate R_{Th} kill the independent sources and

2. Disconnect the test source and reconnect the independent source and calculate $V_{Th} = V_{oc} = V_{ab}$

② Find V_{Th}



$$V_{ab} = V_{Th} = V_{oc} = V_{4\Omega}$$

Super mesh

$$-6 + 5I_1 + 3I_2 + 4I_2 = 0$$

By mesh analysis

$$5I_1 + 7I_2 = 6 \rightarrow ①$$

Branch

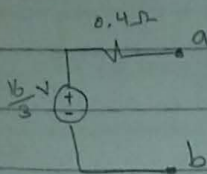
$$I_2 - I_1 = 1.5I_x = 1.5I_2$$

ask: believe & receive

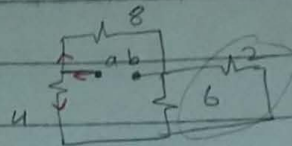
$$-I_1 - 0.5I_2 = 0 \rightarrow ②$$

$$I_1 = -\frac{2}{3}A \quad I_2 = \frac{4}{3}A \quad V_{ab} = V_{Th} = V_{oc} = \frac{4}{3} \times 4 = \frac{16}{3}V$$

③ Draw cut

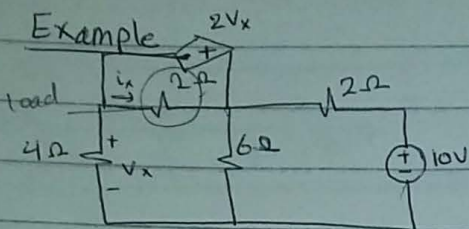


if



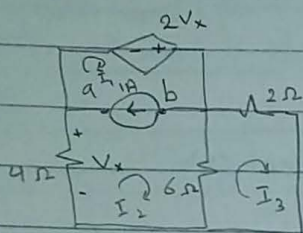
$$(2/16 + 4)/12$$

Example



Find i_x using Thevenin's Theorem

- ① Take off the load ② Find Req



$$\#3 \quad 2I_3 + 6I_3 - 6I_2 = 0 \quad -6I_2 + 8I_3 = 0 \rightarrow (1)$$

$$\text{KVL} \quad I_1 - I_2 = 1 \rightarrow (2)$$

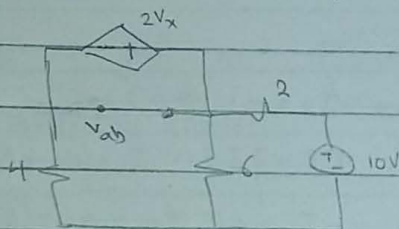
$$\text{Super mesh (external loop)} \quad -2V_x + 2I_3 + 4I_2 = 0 \quad V_x = -4I_2$$

$$8I_2 + 2I_3 + 4I_2 = 0 \quad 12I_2 + 2I_3 = 0 \rightarrow (3)$$

$$\text{Assume } I_2 = 5A \quad I_1 = 1A \quad I_3 = 0A \quad \text{Assume } I_2 = 5A \quad V_x = -20V$$

$$V_{\text{dependent}} = -40V \quad V_s = 40V \quad R_{th} = 40/1 = 40\Omega$$

- ③ Disconnect test source & connect ind. source



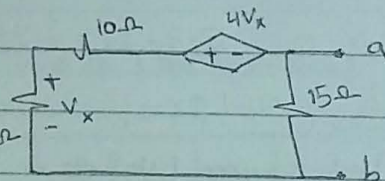
2 meshes

Imp.

In exam

Here Thevenin's Equivalent consists only of R_{th}

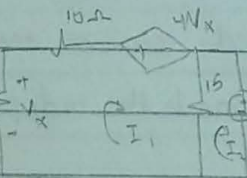
There is -ve resistance Transistor has -ve resistor



V_{th} comes from independent source

if no independent source $V_{th} = 0$

R_{th} comes from resistances and dependent sources



$$\#1 \quad 5I_1 + 10I_1 + 4V_x + 15I_1 - 15I_2 = 0$$

$$V_x = -5I_1$$

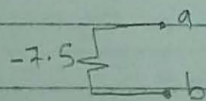
$$10I_1 - 15I_2 = 0 \rightarrow (1)$$

$$\#2 \quad 1 + 15I_2 - 15I_1 = 0$$

$$-15I_1 + 15I_2 = -1 \rightarrow (2)$$

$$I_1 = 0.2A$$

$$I_2 = 0.13A$$



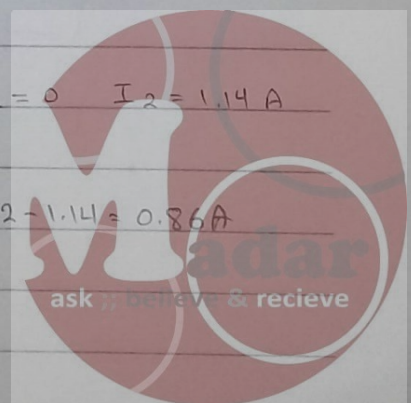
Because we don't have any independent source.

$$I_s = -0.13$$

$$R_{th} = 1$$

$$= -7.5\Omega$$

$$-0.13$$



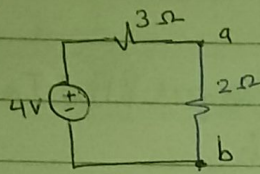
No: -----

Date: -----

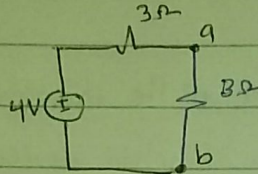
Norton's eqs
short ckt

if load is 8Ω , $R_N = 6.5\Omega$

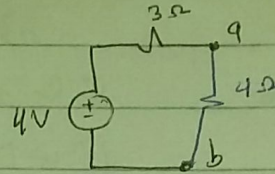
Imp Max. Power $\rightarrow$ when $R_L = R_{th}$



$$P_1 = 1.28 \text{ W}$$



$$P_2 = 1.33 \text{ W}$$



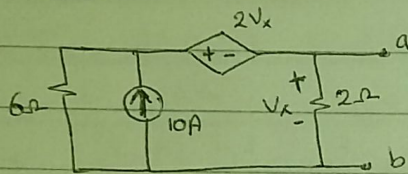
$$P_3 = 1.31 \text{ W}$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

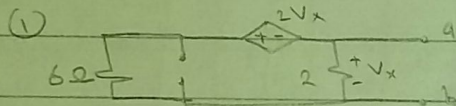
Find R which will have max. power to transfer

Example

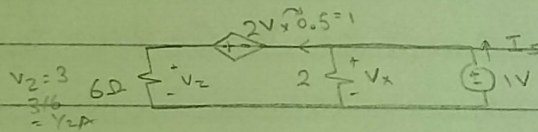
In exam



Find and sketch Norton equivalent



Put test source



$$V_x = 1 \text{ V}$$

$$-V_2 + 2V_x + 1 = 0 \quad V_2 = 3 \text{ V}$$

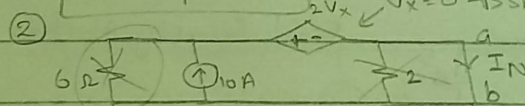
OR

$$I_{6\Omega} = 3/6 = 0.5 \text{ A}$$

$$I_x = 0.5 \text{ A} \quad I_s = 0.5 + 0.5 = 1 \text{ A}$$

$$R_{th} = \frac{1}{1} = 1\Omega$$

Imp.
Repeat

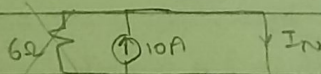


no current passes here

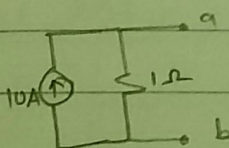
$$V_x = 0$$

$$I_N = 10 \text{ A}$$

$$I_N = 10 \text{ A}$$

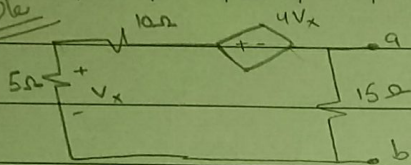


(3)



19/11/2019

Example



if $\neq$ independent $\rightarrow$ no V_{th} , no I_N

CHAPTER #6 % Energy storage elements

* Energy storage elements

$\rightarrow$ capacitor $\frac{C}{\text{F}}$ $[C] = F$ (Faraday)

$\rightarrow$ inductor $\frac{L}{\text{H}}$ $[L] = H$ (Henry)

① Capacitor $\rightarrow$ used to store charges $q = C \cdot V$

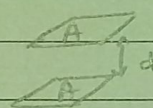
A: Area of plate

d: distance between two plates

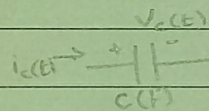
ϵ_0 : permittivity 8.8

ϵ_r : relative permittivity

C: capacitance



$$C = \frac{\epsilon_0 A}{d}$$



$$i_C(t) = C \frac{dV_C(t)}{dt}$$

$$V_C(t) = 4 \cos(10t)$$

$$C = 3 \mu F$$

$$i_C(t) = 3 \times 10^{-6} \times 4 \times 10 \sin(10t) = -120 \sin(10t)$$

Calculator in rad.

$$\omega \frac{\text{rad}}{\text{s}} \times \text{s} = \text{rad}$$

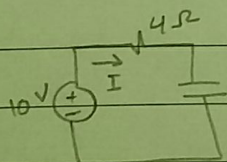
$$i_C(t) = -120 \sin(30t)$$

Imp: if voltage drop DC Not AC

Voltage DC $\rightarrow$ constant $\rightarrow$ derivative of constant = 0

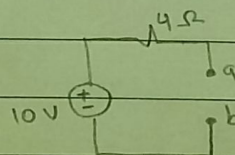
$i = 0 \rightarrow$ open circuit

In DC circuit the capacitor is open circuit BECAUSE $i = 0$
CAPACITOR works as OC in circuit.

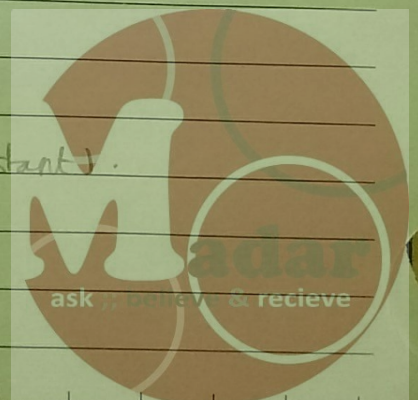


$$I_C = 0$$

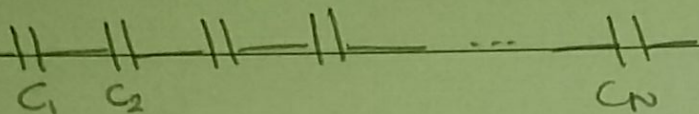
$$V_C = 10$$



* AC is a fun(time), while DC NOT (it is constant).



Series



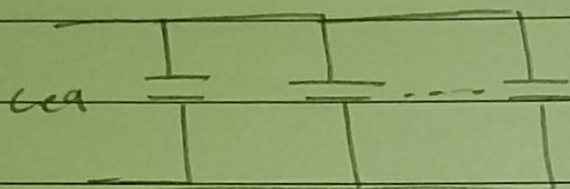
$$q_T = q_1 = q_2 = q_3 = \dots = q_N$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

$$q_T = q_1 + q_2 + \dots + q_N$$

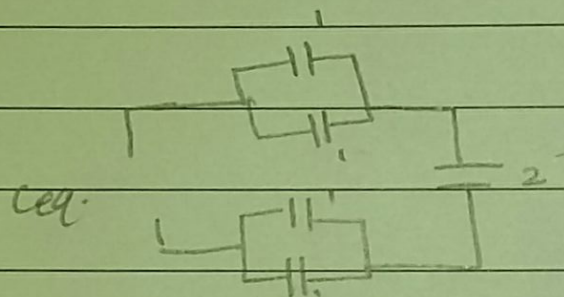
$$C_{eq} = C_1 + C_2 + \dots + C_N$$

Parallel



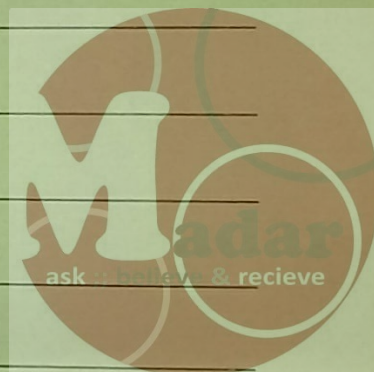
Example

Find C_{eq} .



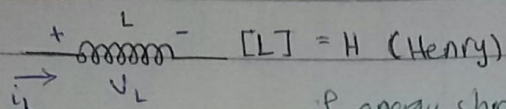
$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$C_{eq} = \frac{2}{3} F$$



The inductor

Big question in exam
 what about q?



$$[L] = H \text{ (Henry)}$$

$$V_L = L \frac{di}{dt}$$

$$I_L = \frac{1}{L} \int_0^t V_L(t) dt + I_0$$

initial current $\nearrow$

if energy stored initially $= 0 \rightarrow I_0 = 0$

$$E = W = \frac{1}{2} L I^2$$

inductors acts as

IF DC power supply $\rightarrow$ Short cut

Existence of 1 DC in huge # of AC $\rightarrow$ acts like this (SC)

Example

① $I_L = 3A$ $L = 4mH$

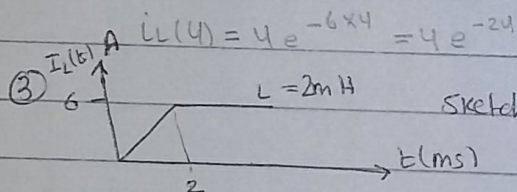
$$E = W = \frac{1}{2} \times 4 \times 3^2 \times 10^{-3} J$$

$$V_L = 0$$

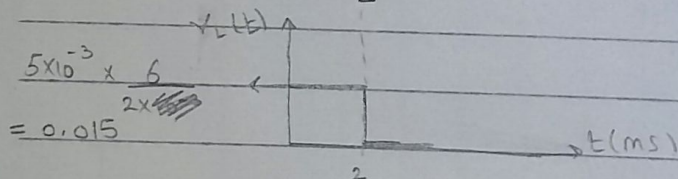
② $i_L(t) = 4e^{-6t}$ $L = 3H$

$$V_L(t) = L \frac{di_L}{dt} = 3(4)(-6)e^{-6t} = -72e^{-6t}$$

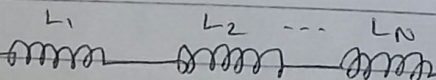
$$E|_{t=4s} = \frac{1}{2} \times 3 \times (i_L(4))^2 = \frac{1}{2} \times 3 \times (4e^{-24})^2 J$$



Sketch voltage wave form?

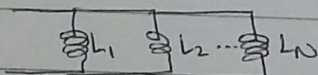


Series



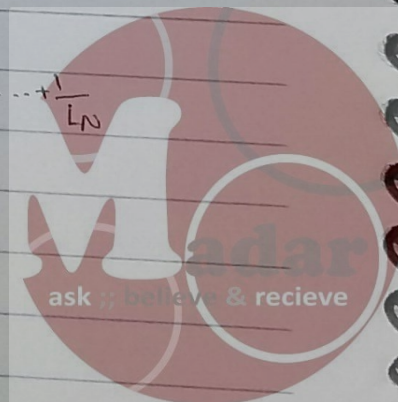
$$L_{eq} = L_1 + L_2 + \dots + L_N$$

Parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

Calculate for
 in radian
 for sin and
 cos...

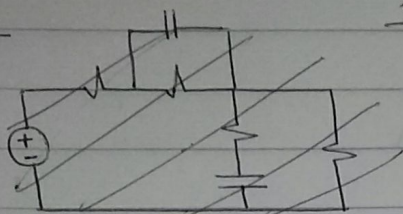


$$L = N \frac{d\phi}{dt}$$

No: _____

Date: _____

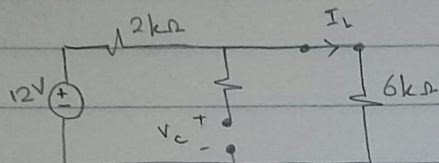
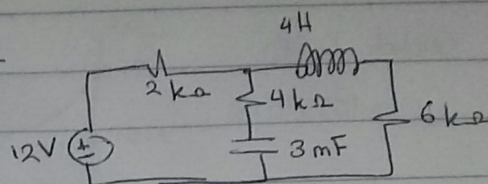
Ex.



Find energy stored

Find $I_L, V_C,$

E_L, E_C



$$I_L = \frac{12}{8 \times 10^3} = 1.5 \text{ mA}$$

$$V_C = V_{6k\Omega} = 1.5 \times 6 = 9 \text{ V or voltage division}$$

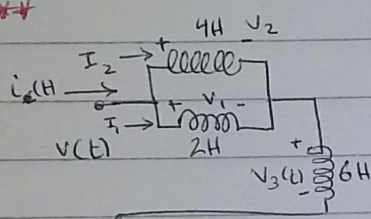
$$E_L = \frac{1}{2} \times 4 \times 1.5 \times 10^{-3} =$$

$$E_C = \frac{1}{2} \times 3 \times 10^{-3} \times 9^2 =$$

Imp

Repeat

Ex. ***



$$\text{let } i_1(t) = 0.6 e^{-4t}$$

$$i_1(0) = 1.4 \text{ A}$$

Find $i_2(0), V_1(t), V_2(t), V_3(t), i_2(t), i(t),$
energy stored in 4H inductor at $t=3 \text{ s},$
total $V(t)$

$$\rightarrow i(t) = i_1(t) + i_2(t) \quad i(0) = 1.4 \quad i_1(0) = 0.6$$

$$i_2(0) = 1.4 - 0.6 = 0.8 \text{ A}$$

$$\rightarrow i_2(t)$$

$$\text{Because Parallel } V_1(t) = V_2(t) = L \frac{di}{dt} = 2 \times -4 \times 0.6 e^{-4t} = -4.8 e^{-4t} \text{ V}$$

$$i_2(t) = C_2 \frac{dV_2(t)}{dt} + i_2(0) = 4 \times -4.8 e^{-4t} + 0.8$$

$$i_2(t) = \frac{1}{4} \times \int_0^t -4.8 e^{-4t} dt + 0.8 = \frac{+4.8 e^{-4t}}{16} \Big|_0^t = 0.3 e^{-4t} - 0.3 + 0.8$$

$$i_2(t) = 0.3 e^{-4t} + 0.5 \text{ A}$$

$$\rightarrow V_3(t)$$

$$i(t) = i_1(t) + i_2(t) = 0.6 e^{-4t} + 0.3 e^{-4t} + 0.5 = 0.9 e^{-4t} + 0.5$$

$$V_3(t) = L_3 \frac{di}{dt} = 6 \times 0.9 \times -4 e^{-4t} = -21.6 e^{-4t} \text{ V}$$

$$\rightarrow V(t) = V_1(t) + V_3(t) \text{ OR } V_2(t) + V_3(t) \text{ OR equivalent L}$$

$$L_{\text{equ}} = (4 \parallel 2) + 6 = 7.33 \text{ H}$$

$$V = L_{\text{equ}} \frac{di(t)}{dt}$$

$$V(t) = 7.33 \times \frac{di(t)}{dt} = 7.33 \times 0.9 \times -4 e^{-4t} = -26.28 e^{-4t} \text{ V}$$

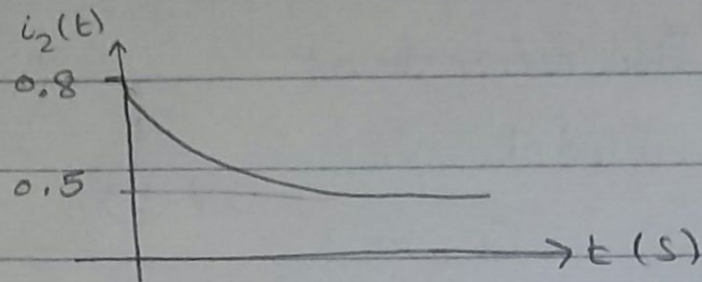
$$\rightarrow E_{4H} |_{t=3s} = \frac{1}{2} \times 4 \times (i_2(3))^2 = \frac{1}{2} \times 4 \times (0.3 e^{-4 \times 3} + 0.5)^2 = 0.5 \text{ J}$$

No: -----

Date: -----

Sketch $i_2(t)$

$$i_2(0) = 0.8$$

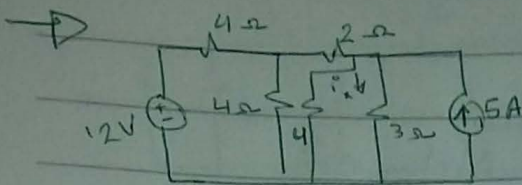


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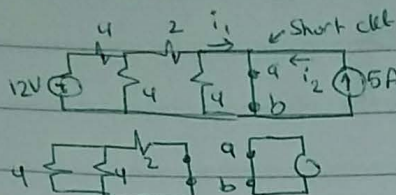
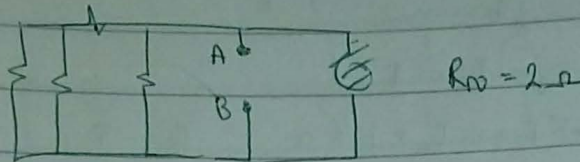
Revision Formid

24/11/2019

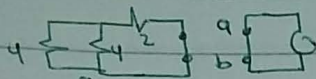


ix with Norton

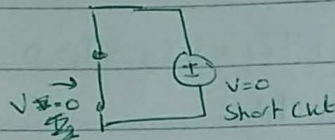
resistance → cancel
Parallel
current
voltage



→ like sum of two ckt's $I_N = i_1 + i_2$
 $= 5 + \frac{3}{2} = 6.5$
 $5 + i$

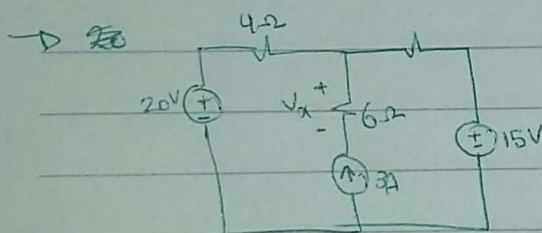


if voltage source it will be voltage source → only i_1



→ Capacitor is Parallel → same voltage
 Series → same current

→ if no independent source → max. power = 0 / Power = 0

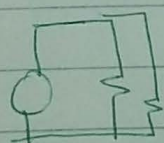


contribution of 20V source on V_x is equal to?

→ Thevenin
 $I = \frac{V}{2R_{th}}$

$$P = I^2 \cdot R_{th} = \frac{V^2}{4R_{th}} \cdot R_{th} = \frac{V^2 R_{th}}{4R_{th}}$$

Norton



$$I_L = \frac{I_N R_N}{2R_N}$$

$$V_{th} = I_N R_N$$

$$P = \left(\frac{I_N}{2} \right)^2 R_N$$

ask :: believe & recieve

$$P = \frac{I_N^2 R_N}{4}$$

No: _____

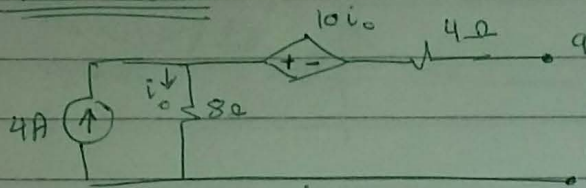
Date: _____

Capacitor

Series $\rightarrow$ same q

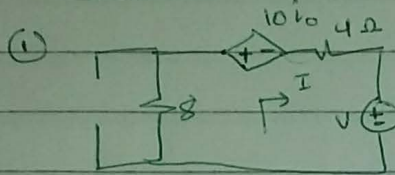
Parallel $\rightarrow$ summation

Test Source



Norton current

single
mesh
 $\downarrow$
voltage

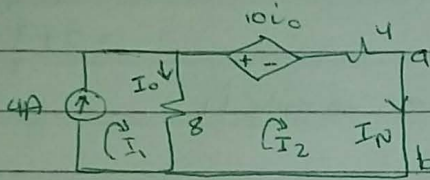


$$I_s = I_o = -I$$

$$4I + 8I + 10I_o + 1 = 0 \quad I_o = -I$$

$$I = -0.5A$$

$$R_{N} = \frac{1}{0.5} = 2 \Omega$$



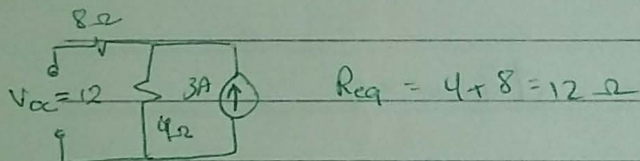
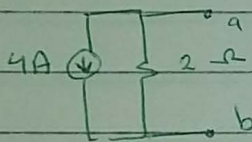
$$I_N$$

$$I_1 = 4A$$

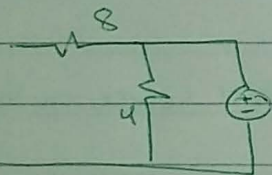
$$10I_o + 4I_2 + 8I_2 - 8I_1 = 0 \quad I_o = I_1 - I_2$$

$$I_2 = 4A$$

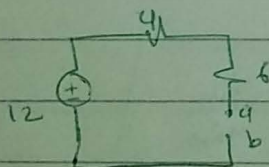
$$I_o = 4 - I_2$$



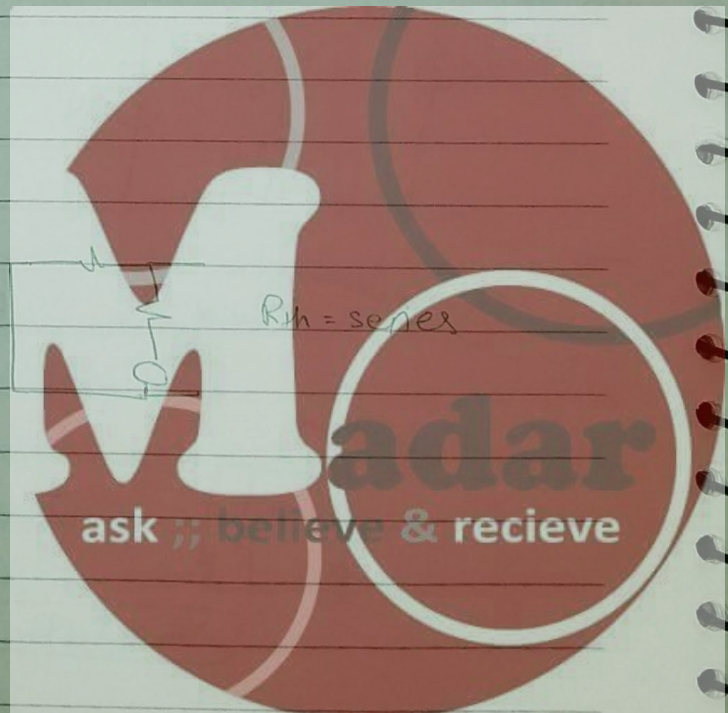
$$R_{eq} = 4 + 8 = 12 \Omega$$



$$R_{Th} = 8$$



$$V_{ab} = V_{source}$$

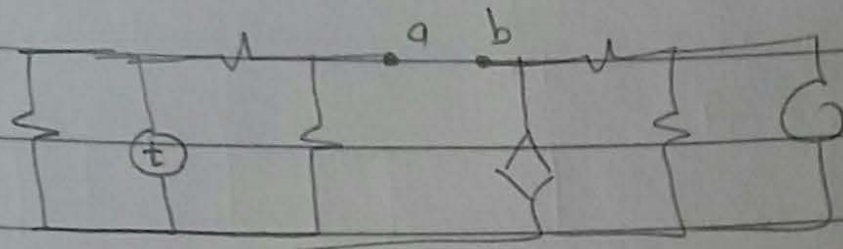
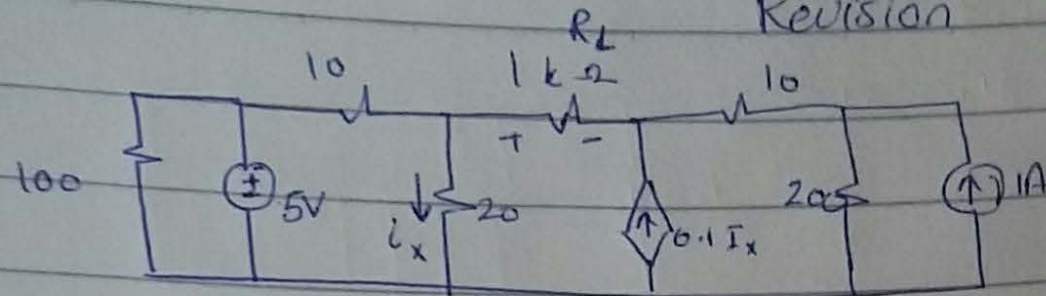


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Revision

26/11/2019



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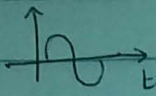
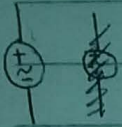
Euler Identity $r \angle \theta$
 $r e^{-j\theta} = r \cos \theta + j r \sin \theta$

11/12/2019

u(t) or v(t)

AC- ANALYSIS

(complex, polar, work)
 Phasor



changing value & direction

Source or

$$i(t) = I_m \cos(\omega t + \theta)$$

$$v(t) = V_m \cos(\omega t + \theta)$$

V_m : amplitude (max. value)

ω : angular frequency (rad/s)

θ : phase shift

even in sin. find parameters according to sin, don't convert to cos.

Time domain signal

We will use it as a vector

$$V = V_m \angle \theta$$

$$I = I_m \angle \theta$$

$f = \frac{\omega}{2\pi}$

$$\omega = 2\pi f$$

$$T = \frac{1}{f}$$

Loads

① R

② L

③ C

OR combination of them

Identity

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

Euler identity : $e^{jx} = \cos x + j \sin x$

e.g. $i(t) = 4 \cos(2\pi t + 30^\circ)$ (Time domain)

if you want to find $i(t)$, convert all to radian degree

$I_{max} = 4A$ $\omega = 2\pi$ $f = 1H$ $T = \frac{1}{f} = 1s$ $\theta = 30^\circ$

in polar coordinates $I = 4 \angle 30^\circ$

- T : Periodic time, time needs the signal to repeat itself

- ONLY polar if ~~cos~~ Time domain, if sin, convert

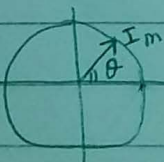
Source must be in cos

Imp. $\sin x = \cos(x - 90^\circ)$

$-\sin x = \cos(x + 90^\circ)$

$-\cos(x) = \cos(x \pm 180^\circ)$ up to you + or -

cos
or
sin

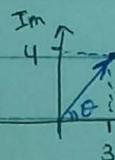


vertical $I_m \sin \theta$

horizontal $I_m \cos \theta$

cartesian
(complex)

$3 + 4j$



$\theta = \tan^{-1}(4/3)$

$3 + 4j = 5 \angle 53^\circ$

$a + jb = r \angle \theta$

Cartesian
complex

Polar
phasor

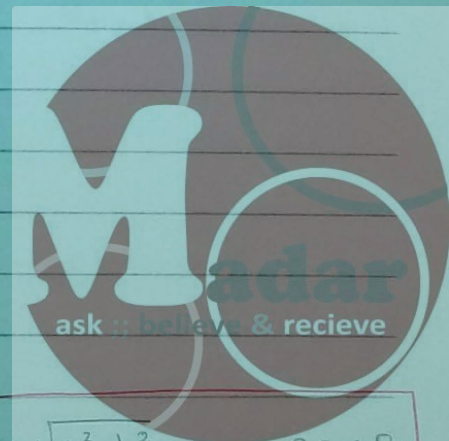
YASSIN

$r = \sqrt{a^2 + b^2}$

$a = r \cos \theta$

$\theta = \tan^{-1} \frac{b}{a}$

$b = r \sin \theta$



No: ----- Date: -----

No signals adding
in time domain

→ Adding & Subtracting & multiplication & Dividing

$$(3 \angle 60^\circ) \cdot (5 \angle 70^\circ) = 15 \angle 130^\circ$$

$$\frac{(3 \angle 60^\circ)}{(5 \angle 70^\circ)} = \frac{3}{5} \angle -10^\circ$$

• multiplication $r_1 \angle \theta_1 \cdot r_2 \angle \theta_2 = r_1 r_2 \angle \theta_1 + \theta_2$

• Division $\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$

• adding & subtracting only in cartesian form

Example $i(t) = 6 \sin(100t - 30^\circ)$

$$i(t) = 6 \cos(100t - 120^\circ) = 6 \angle -120^\circ \text{ A}$$

Imp.
In
Exam

Example $v_1(t) = 4 \cos 10t \rightarrow 4 \angle 0$

$$v_2(t) = 3 \sin 10t \rightarrow 3 \cos(10t - 90^\circ) \rightarrow 3 \angle -90^\circ$$

نقطه الزاوية، إشارة < 0

$$v_3(t) = -\cos(10t + 30^\circ) \rightarrow \cos(10t - 150^\circ) \rightarrow 1 \angle -150^\circ$$

$$\text{let } v(t) = v_1 + v_2 + v_3$$

$$\text{Find } v(t)$$

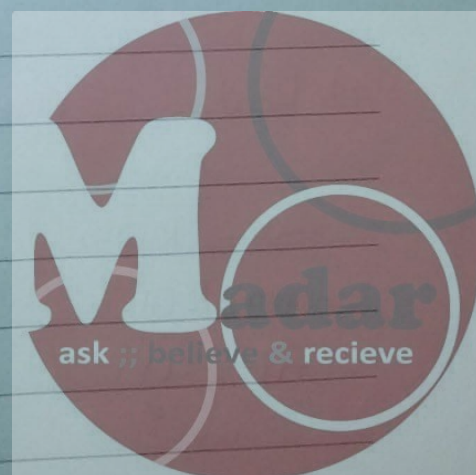
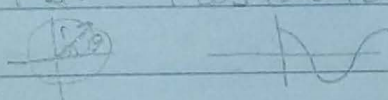
You can't add Time domain $\rightarrow$ convert to Frequency domain $\rightarrow$ return in time domain

$$V = 4 \angle 0 + 3 \angle -90^\circ + 1 \angle -150^\circ = 4.7 \angle -48^\circ = 4.7 \cos(10t - 48^\circ) \text{ V}$$

All Formulas $\vec{e}^{j\theta}$ Complex Euler Polar time domain

$$r \cos \theta + j r \sin \theta = r e^{j\theta} = r \angle \theta = r \cos(\omega t + \theta)$$

$\sin \theta$ $\downarrow$ $\frac{y}{r}$ real axis
 $\cos \theta$ $\downarrow$ $\frac{x}{r}$ Imaginary axis



Tuesday
3/12/2019

No:

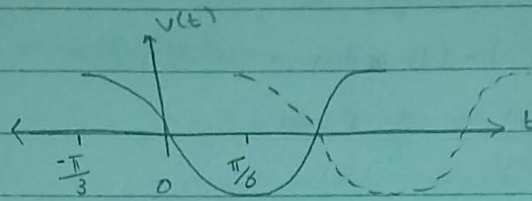
Date:

Frequency $\equiv$ Polar $\equiv$ phasor

$\Rightarrow$ Lead & Lag

--- $V_1(t) = 4 \cos(10t - 30^\circ)$

— $V_2(t) = 3 \cos(10t + 60^\circ)$



V_2 leads V_1 by $90^\circ ((60 - -30) = 90^\circ)$

V_1 lags V_2 by 90°

$V_1(t) = 4 \sin(10t + 30^\circ)$

$V_2(t) = 2 \sin(10t + 50^\circ)$

V_2 leads V_1 by 20°

• Circumstances to compare the signals

* They should have the same ω and form

f & T

both sin or cos

You can compare current and voltage together.

$i(t) = 4 \sin(10t - 30^\circ)$

$v(t) = 5 \sin(10t + 60^\circ)$

$\Rightarrow$ You can't compare

$v = 4 \sin(10t) \rightarrow 4 \cos(10t - 90^\circ)$

$i = 5 \cos(10t - 100^\circ)$

v leads i by 10°

A.W 9.9

• Exercises on calculator

$4 \angle 30^\circ + 6 \angle -50^\circ + 2 \angle 30^\circ = 9.19 \angle -10^\circ$ $6.6 \angle -32^\circ = 5.59 - 3.6j$

$\frac{(3+j4)(2-j2)}{6+j5} = 1.81 \angle -31^\circ$

$5 \angle 30^\circ \rightarrow 5 \cos(\omega t + 30^\circ)$

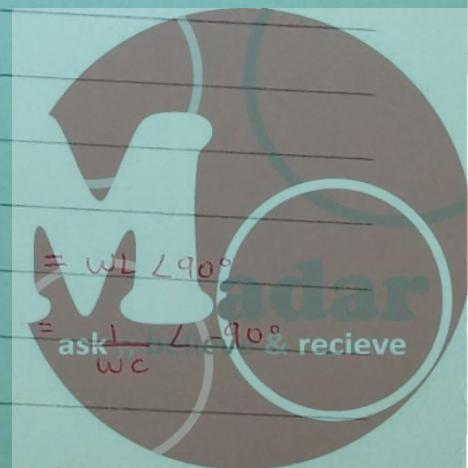
$\Rightarrow$ Loads (converting to frequency domain)

Time	Freq. / Polar. / phasor
$\sim R(\Omega)$	$R(\Omega)$
$\sim L(H)$	$Z_L = j\omega L(\Omega)$
$\sim C(F)$	$Z_L = \frac{1}{j\omega C}(\Omega)$

Impedance
ظبط
في السلسلة
والتوازي

$= \omega L \angle 90^\circ$

$= \frac{1}{\omega C} \angle -90^\circ$

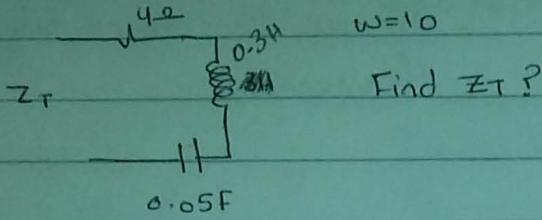


$$j^2 = -1$$

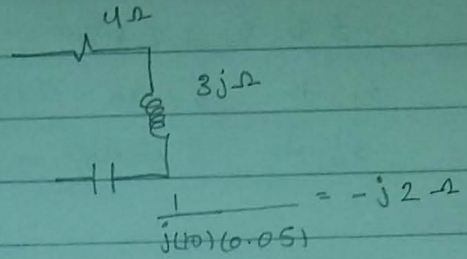
$$j = \sqrt{-1}$$

$$\frac{1}{j} = -j$$

eg ①

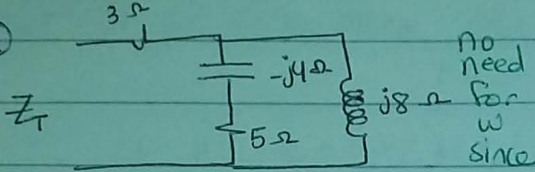


Find Z_T ?



$$Z_T = 4 + j3 + -j2 = 4 + j1 = 4.12 \angle 14^\circ \Omega$$

②



No need for ω since they are impedances

$$\left(\frac{(5 - j4)(j8)}{5 - j4 + j8} \right) + 3 = 10.94 \angle 9.2^\circ \Omega$$

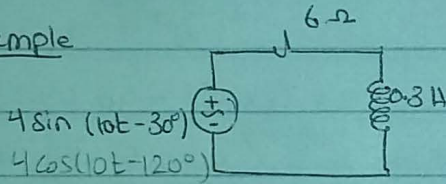
$\Rightarrow$ Admittance

Y : admittance $\sim$

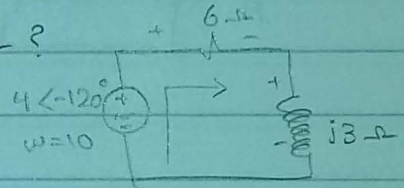
$$Y = \frac{1}{Z} \sim (\text{Siemens, } \Omega^{-1})$$

$V = IZ$ in AC
not IR

Example



Find I, V_L ?



$$Z_L = j\omega L = j(10)(0.3) = 3j \Omega$$

$$-4 \angle -120^\circ + 6I + 3jI = 0$$

$$I = \frac{4 \angle -120^\circ}{6 + j3} = 0.6 \angle -146.5^\circ \text{ A} = I_L$$

$$V_L = 0.6 \angle -146.5^\circ \times 3j = 1.8 \angle -56^\circ \text{ V}$$

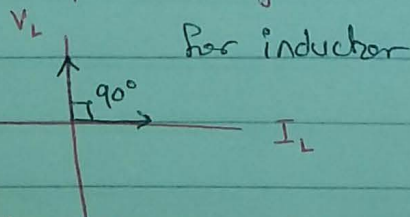
$$V_R = 6 \times 0.6 \angle -146.5^\circ = 3.6 \angle -146.5^\circ \text{ V}$$

$$I_L \text{ angle } \angle -146.5^\circ \Rightarrow V \text{ lead by } 90^\circ$$

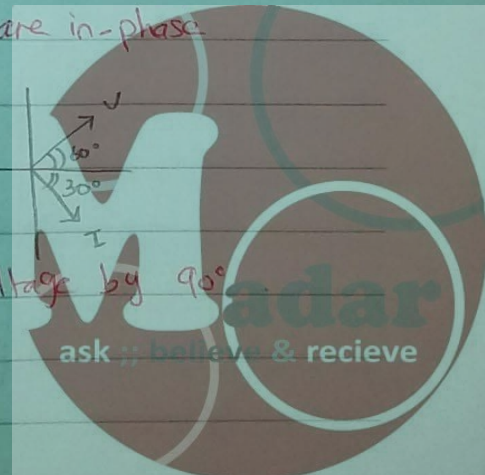
Derj

IMP

- In the inductor, the voltage leads the current by 90° .
- In the resistor, the voltage and current are in-phase



- In the capacitor, the current leads the voltage by 90°



$$105 \angle -225$$

$$P = -4W$$

OR

$$P = 4W \text{ delivered}$$

$$(3 \angle -180)^{0.5} =$$

$$\downarrow \quad \downarrow$$

$$3^{0.5} \quad \frac{180}{2}$$

lead or lag
the same difference

5/12/2019

No: -----

Date: -----

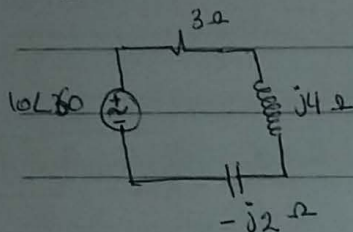
$$-3 \rightarrow 3 \angle +180^\circ$$

$$3j \rightarrow 3 \angle 90^\circ$$

$$-3j \rightarrow 3 \angle -90^\circ$$

• we must use Cramer's rule to find the unknowns

Ex.



Find i , V_L , V_R , V_C

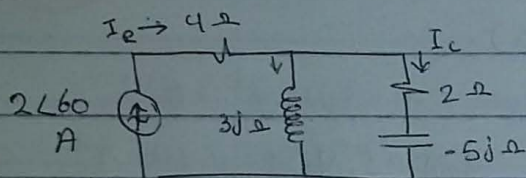
$$I = \frac{V}{Z} = \frac{10 \angle 60}{3.6 \angle 38.7} = 2.77 \angle 26.3^\circ = I_R = I_L = I_C$$

$$V_R = 3 \times 2.77 \angle 26.3^\circ = 8.3 \angle 26^\circ \text{ V}$$

$$V_L = 11.08 \angle 116.3^\circ \text{ V is leading by } 90^\circ$$

$$V_C = 5.54 \angle -68.7^\circ \text{ V is lagging by } 90^\circ \leftarrow \text{phase shift}$$

Ex.



Find I_C ?

$$I_C = \frac{2 \angle 60 \times 3j}{2 - 5j + 3j} = 2.12 \angle -16.5^\circ \text{ A}$$

$$V_C = I_C \times -j5 = 10.6 \angle -255^\circ \text{ V}$$

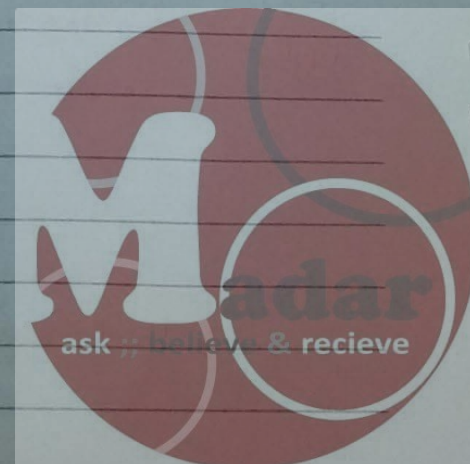
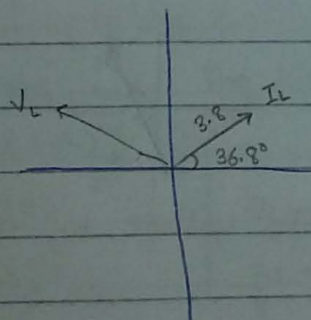
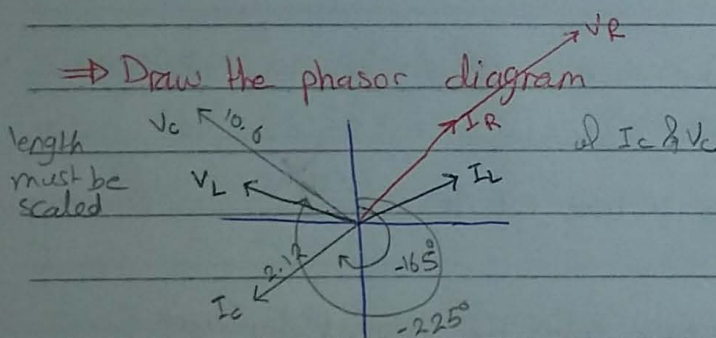
* voltage leading current by 90°

$$V_L = 3j I_L = 11.4 \angle 126^\circ \text{ V}$$

$$I_L = I_T - I_C = 3.8 \angle 36.8^\circ \text{ A}$$

* Voltage lead current by 90°

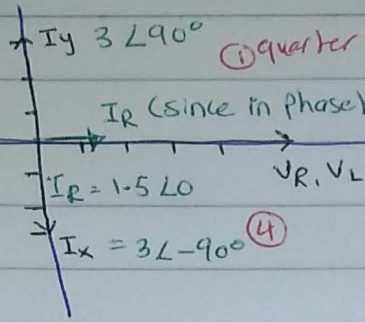
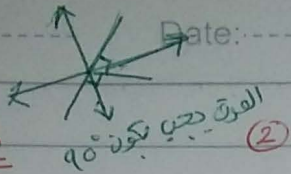
⇒ Draw the phasor diagram



no comparing between I_L and I_C

No: Date:

In exam



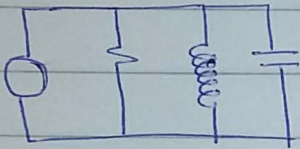
each division equals 1V

Draw ckt

Find value of

each component

So 3 components, R, L, C are in parallel (since same voltage)



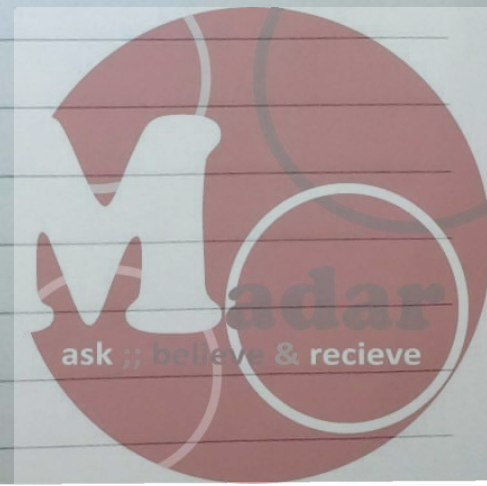
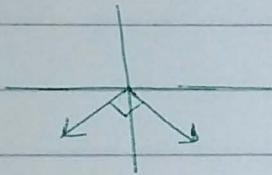
$$I_y = I_C$$

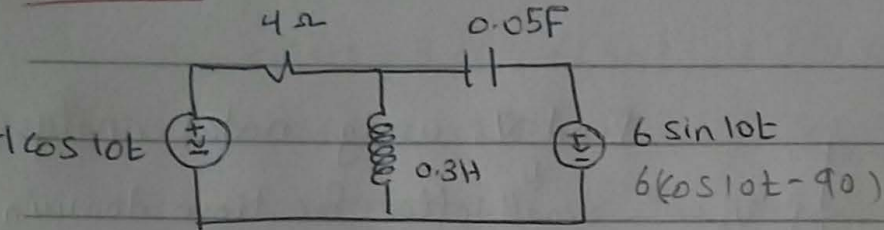
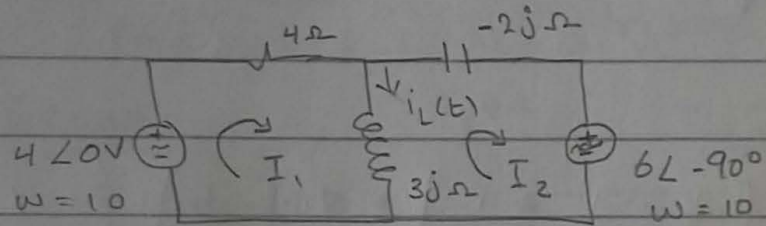
$$I_x = I_L$$

$$R = V_R / I_R$$

$$Z_C = V_C / I_C$$

$$Z_L = V_L / I_L$$



Exampleuse meshanalysis to find $i_L(t)$?mesh #1

$$-4 \angle 0 + 4I_1 + 3jI_1 - 3jI_2 = 0 \quad (4+3j)I_1 - 3jI_2 = 4 \angle 0 \rightarrow (1)$$

mesh #2(180 degrees shift for 180 degrees, i.e., same) $6 \angle 90^\circ$

$$-2jI_2 + 6 \angle -90 + 3jI_2 - 3jI_1 = 0 \quad -3jI_1 + jI_2 = -6 \angle 90^\circ \rightarrow (2)$$

Using Cramer's Rule

$$\Delta = \begin{bmatrix} 4+3j & -3j \\ -3j & j \end{bmatrix} = 6 + 4j \quad \Delta_1 = \begin{bmatrix} 4 \angle 0^\circ & -3j \\ 6 \angle 90^\circ & j \end{bmatrix} = -18 + 4j$$

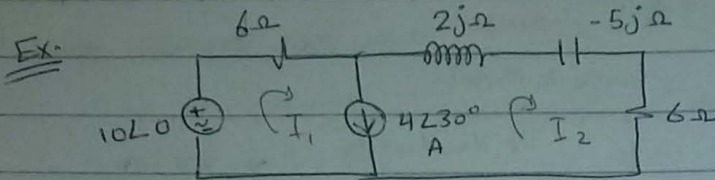
$$\Delta_2 = \begin{bmatrix} 4+3j & 4 \angle 0^\circ \\ -3j & 6 \angle 90^\circ \end{bmatrix} = -18 + 36j$$

$$I_1 = \frac{\Delta_1}{\Delta} = 2.5 \angle -21^\circ \text{ A} \quad I_2 = 5.5 \angle 82^\circ \text{ A} \quad I_L = I_1 - I_2 = 6.5 \angle 90^\circ$$

$$i_L(t) = 6.5 \cos(10t - 76^\circ) \text{ A}$$



No: Date:



Find V_L By mesh analysis

Big loop

$$-10\angle 0 + 6I_2 + 2jI_2 - 5jI_2 + 6I_2 = 0$$

$$6I_1 + (6-3j)I_2 = 10\angle 0 \rightarrow (1)$$

Branch

$$I_1 - I_2 = 4\angle 30^\circ \rightarrow (2)$$

$$\Delta = \begin{bmatrix} 6 & 6-3j \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12-37 & -184.4 \\ -12+j3 \end{bmatrix} \quad \Delta_2 = \begin{bmatrix} 6 & 10\angle 0 \\ 1 & 4\angle 30^\circ \end{bmatrix} = 16.7 + 12j$$

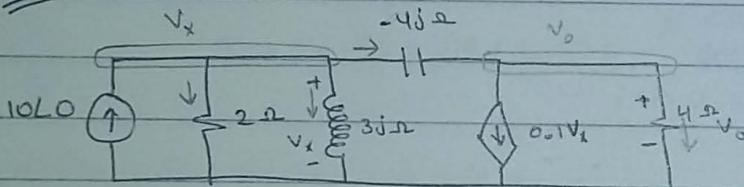
$$\Delta_1 = \begin{bmatrix} 10\angle 0 & 6-3j \\ 4\angle 30^\circ & -1 \end{bmatrix} = \begin{bmatrix} 32-36 & -184 \\ -38 & -1.6j \end{bmatrix}$$

$$I_1 = \Delta_1 / \Delta =$$

$$V_L = 2.6\angle -27.7^\circ$$

$$V_L(t) = 2.6 \cos(\omega t - 27.7^\circ)$$

Ex.



Find V_o using nodal analysis
↓
small letter for time domain

Node V_x

$$10\angle 0 = \frac{V_x}{2} + \frac{V_x}{3j} + \frac{V_x - V_o}{-j4} = V_x \left[\frac{1}{2} + \frac{1}{3j} + \frac{1}{-j4} \right] + V_o \left[\frac{1}{j4} \right] = 10\angle 0$$

$$V_x (0.51\angle -9.46^\circ) + V_o (-0.25j) = 10\angle 0 \rightarrow (1)$$

Node V_o

$$\frac{V_x - V_o}{-4j} = 0.1V_x + \frac{V_o}{4} \quad V_x \left[\frac{1}{4j} + \frac{1}{10} \right] + V_o \left[\frac{1}{4} - \frac{1}{j4} \right] = 0$$

$$V_x (0.27\angle -68.2^\circ) + (0.35\angle 45^\circ) V_o = 0 \rightarrow (2)$$

$$\Delta = \begin{bmatrix} 0.51\angle -9.46^\circ & -0.25j \\ 0.27\angle -68^\circ & 0.34\angle 45^\circ \end{bmatrix}$$

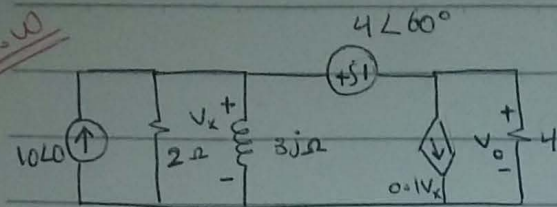
$$V_x = \frac{\Delta_1}{\Delta} \quad V_o = \frac{\Delta_2}{\Delta}$$

No: _____

Date: _____

source transformation
is the same principle in AC

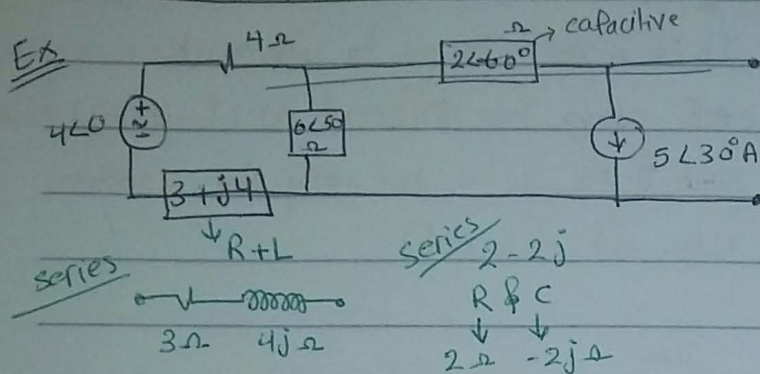
H.W



By supernode, find V_x & V_o .

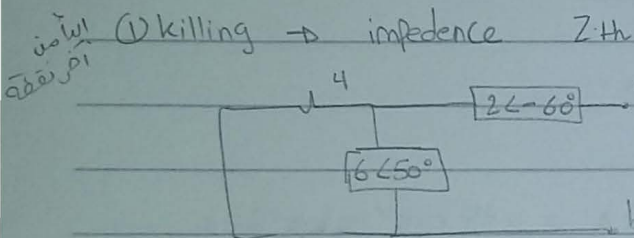
Polar form :-

+ve angle $\rightarrow$ R & L inductive load
-ve angle $\rightarrow$ R & C capacitive load



19/12/2019

Find Norton equivalent



$$Z_{th} = [(4 + 3 + j4) \parallel 6\angle 50^\circ] + 2\angle -60^\circ = 3.67\angle 9.05^\circ \Omega$$

(2) $V_{ab} = V_{th}$

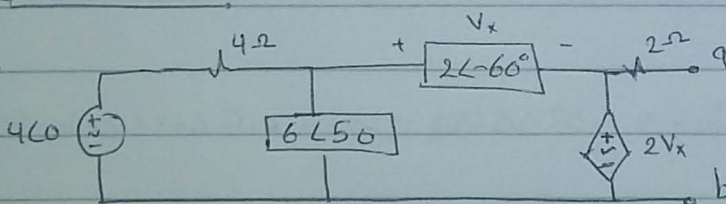
$$I_2 = 5\angle 30^\circ$$

$$-4\angle 0 + 4I_1 + 6\angle 50 I_1 - 6\angle 50 \times 5\angle 30 + 3 + j4 I_1 = 0 \quad I_1 = 0.447\angle -34.3^\circ$$

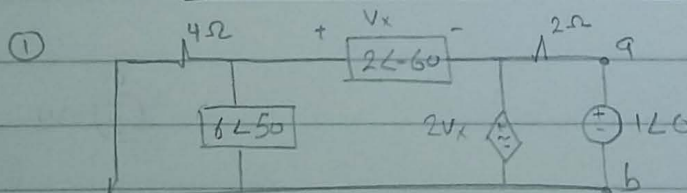
$$V_{ab} = (-2\angle -60)I_2 + 6\angle 50(I_1 - I_2) = V_{th}$$

(3) You can convert it by source transformation.

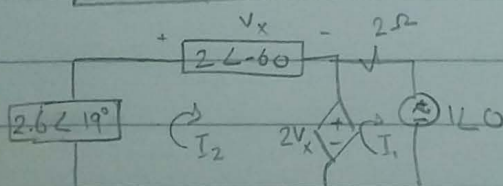
Example



Find Thevenin's equivalent



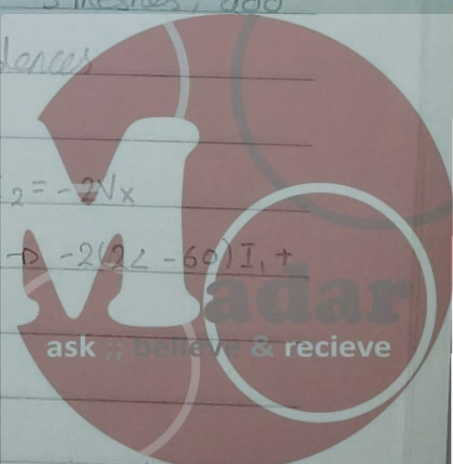
To avoid 3 meshes, add the impedances



$$V_x = 2\angle -60 I_1$$

$$(2.6\angle 19^\circ + 2\angle -60)I_2 = -2V_x$$

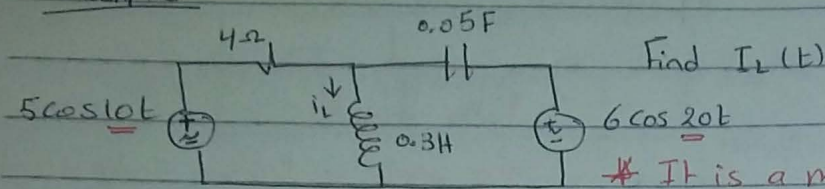
$$-2V_x + 2\angle 120 = 0 \rightarrow -2(2\angle -60)I_1 +$$



No: -----

Date: -----

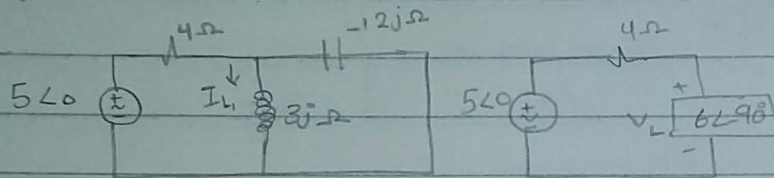
when one source, mesh is long

Imp Example

* It is a must to go to superposition, since each source has different ω (frequency) value even if DC, $\omega = 0$, but apply the constraints (open and close cut)

Apply superposition

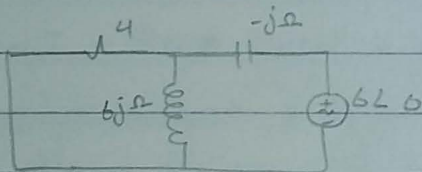
1 Source 1



$$V_{L1} = \frac{5 \angle 0 (6 \angle -90^\circ)}{4 + 6 \angle -90^\circ} = 4.1 \angle -33^\circ$$

$$I_{L1} = \frac{4.1 \angle -33^\circ}{3j} = 1.38 \angle -123^\circ \text{ A} = 1.38 \cos(10t - 123^\circ)$$

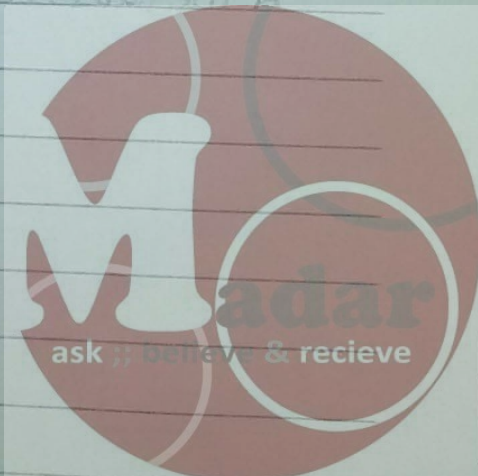
2



$$V_{L2} = 3.3 \angle 33.7^\circ \quad I_{L2} = 0.5 \angle -56^\circ \text{ A} = 0.5 \cos(20t - 56^\circ)$$

Superposition

$$I_L = I_{L1} + I_{L2} = 1.38 \cos(10t - 123^\circ) + 0.5 \cos(20t - 56^\circ) \text{ A}$$



AC - Power

22/12/2019

In DC $P = IV = I^2 R = V^2 / R$ $[P] = W$

In AC $\rightarrow$ Instantaneous power $P(t) = i(t) \cdot v(t)$ $[P(t)] = W$

$\rightarrow$ Average power (Real / mean / Active power) $P = \frac{1}{2} |I| |V| \cos(\theta_v - \theta_i)$ $[P] = W$

can't be negative because $\cos(\theta) = \cos(-\theta)$

Example $I = 4 \angle 30^\circ$

$V = 6 \angle -15^\circ$

$P = \frac{1}{2} \times 4 \times 6 \times \cos(-15 - 30) = 12 \cos(-45) = 12 / \sqrt{2} W = 8.4 W$

Reactive power $Q = \frac{1}{2} |I| |V| \sin(\theta_v - \theta_i)$ $[Q] = VAR$ Volt Ampere Reactive

e.g. $Q = \frac{1}{2} \times 4 \times 6 \times \sin(-45) = -12 / \sqrt{2} VAR \rightarrow$ capacitive load $= -8.4 VAR$

Q in resistance, the phase shift $= 0$ ($\theta_v = \theta_i$)

		P	Q	
R		$\frac{1}{2} IV$	0	$\left\{ \begin{array}{l} \theta_v - \theta_i = 0 \quad \cos(0) = 1 \\ \theta_v - \theta_i = 0 \quad \sin(0) = 0 \end{array} \right.$
L		0	+ve	$\left\{ \begin{array}{l} \theta_v - \theta_i = 90 \quad \cos(90) = 0 \\ \theta_v - \theta_i = +90 \quad \sin(\theta_v - \theta_i) +ve \end{array} \right.$
C		0	-ve	$\left\{ \begin{array}{l} \cos(-90) = 0 \\ \sin(-90) -ve \end{array} \right.$

Impedance Z is the

complex power

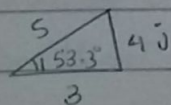
Complex Power $S = P + jQ$ $[S] = VA$ Volt Ampere

in the previous example $S = 8.4 - j8.4 VA = 11.87 \angle -45^\circ VA$

$|S|$: Apparent Power $|S| = 11.87 VA$

Power Triangle

$3 + j4$



Also, $S = \frac{1}{2} V I^*$ change the sign of angle i

As vectors $S = \frac{1}{2} \times 6 \angle -15^\circ \times 4 \angle 30^\circ = 12 \angle -45^\circ VA$

$S = \frac{1}{2} \times V \times \frac{V^*}{Z} = \frac{1}{2} V \times \frac{V^*}{Z}$

$I = 4 \angle 30^\circ$

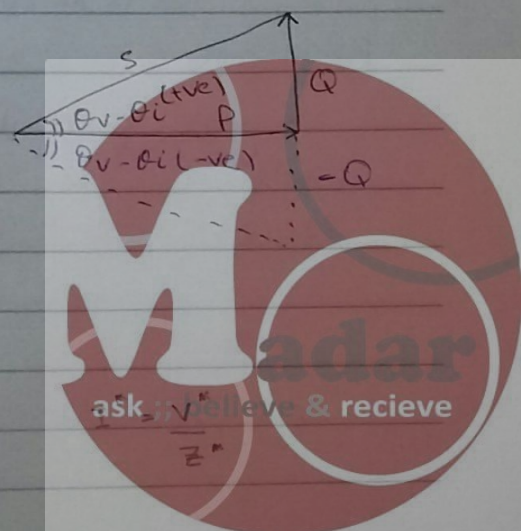
$V = 6 \angle -15^\circ$

$Z = 2 \angle 30^\circ$

$Z = 1.5 \angle -45^\circ$

$S = \frac{1}{2} V I^* = \frac{1}{2} V \times V^* / Z^*$

$S = 12 \angle -45^\circ = 12 \angle -45^\circ$

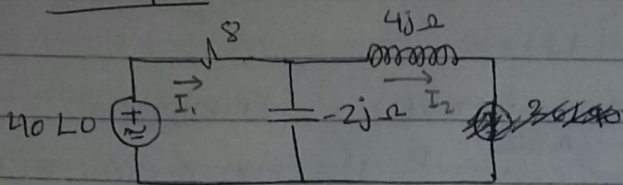


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$$\frac{1}{2} |I| |V|$$

$$= \frac{1}{2} \times |I| |I| |Z|$$

ExampleFind P, Q, S for each element

$$\frac{(j4)(-j2)}{2j} = 4\angle -90^\circ \quad Z_T = 8 - j4$$

$$I_1 = \frac{40\angle 0}{8 - j4} = 4.47\angle 26.5^\circ$$

$$I_2 = \frac{40\angle 0 \cdot I_1(-j2)}{4j - 2j} = 4.47\angle -153.4^\circ \text{ A}$$

$$P_{4\Omega} = 0$$

$$P_{-2j\Omega} = 0$$

$$Q_R = 0$$

$$V_2 = 17.88\angle -63.4^\circ$$

$$Q_L = \frac{1}{2} \times 4.47 \times 17.88 \sin(-63.4^\circ - 153.4^\circ) = 39.97 \text{ VAR}$$

$$S_L = 0 + 39.9j \text{ VA}$$

$$S_L = \frac{1}{2} \times 17.88 \angle -63.4^\circ \times 4.47 \angle +153.4^\circ = 39.9 \angle 90^\circ \text{ VA}$$

$$S_R = P_R = \frac{1}{2} \times |V_R| \times |I_R|$$

→ In the source, two ways

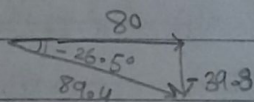
$$(1) P_{\text{source}} = \sum P_{\text{loads}}$$

$$Q_{\text{source}} = \sum Q$$

$$(2) S = \frac{1}{2} V I^* = \frac{1}{2} \times 40\angle 0^\circ \times 4.47\angle -26.5^\circ = 89.4\angle -26.5^\circ = 80 + -39.9j \text{ VA}$$

$$P_R \quad Q_R = Q_S - Q_L$$

Source Power Triangle



$$\Rightarrow \cos(\theta_v - \theta_i) \Rightarrow \text{Power Factor (PF)} \quad 0 < PF < 1$$

$\cos$ is an even function.

$$P.F = \cos(-26.5) = 0.89 \text{ leads}$$

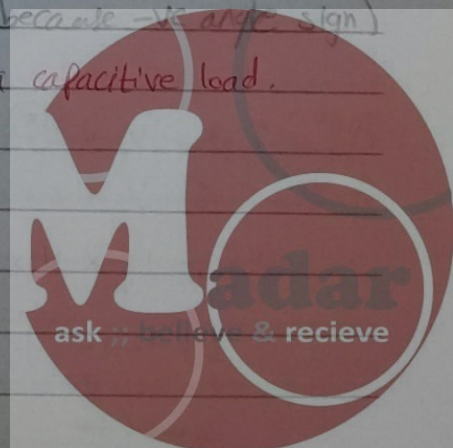
Generally, the dominant in the ckt is the

S angle → +ve P.F lags

capacitive load (because -ve angle sign)

w.r.t

-ve P.F leads → ckt behaves like a capacitive load.



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usefull $\rightarrow P$

24/12/1

$$S = P + jQ$$

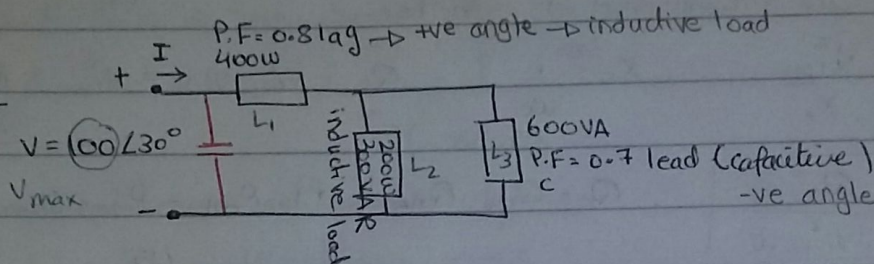
من أجل توصيل S لا فرق إذا loads على التوازي أو على التوالي

$$S = \frac{1}{2} \vec{V} \vec{I}^*$$

$$S_{Total} = \sum S_i$$

Imp.

Example



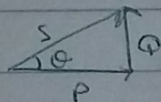
Every load, given 2 informations
Find P, Q, S, θ
from P.F.

$$P = S \cos \theta$$

Find the total S (S_{source} , $P.F_{source}$, I)

$$S \cos \theta = \frac{P}{\sin \theta}$$

$$S = \frac{P}{P.F}$$



$$P = S \cos \theta = S \times P.F \quad S = P / P.F$$

Load #1

$$S_1 = \frac{400}{0.8} = 500$$

$$S_1 = 500 \angle 36.86^\circ \text{ VA}$$

$$\cos^{-1}(0.8) = 36.86^\circ$$

Load #2

$$P = 200 \text{ W}$$

$$S_2 = P + jQ$$

$$Q = 300 \text{ VAR}$$

$$= 200 + j300 = 360.5 \angle 56.3^\circ \text{ VA}$$

Load #3

$$S = 600 \text{ VA}$$

$$S = 600 \angle -45.6^\circ \text{ VA}$$

$\Rightarrow S_{Total} = S_1 + S_2 + S_3$ regardless of type of connection.

$$\Rightarrow S_{source} = S_{total} = 850 \angle 40.9^\circ \text{ VA} = 642.5 + j556.5 \text{ VA}$$

$$\Rightarrow P.F_{source} = \cos(40.9) = 0.76 \text{ lags}$$

$$\Rightarrow P_{total} = 642.5 \text{ W}$$

$$\Rightarrow S = \frac{1}{2} V I^* \quad 850 \angle 40.9 = \frac{1}{2} \times 100 \angle 30^\circ I^*$$

$$Q_{total} = 556.5 \text{ VAR}$$

$$I^* = 17 \angle 10.9 \quad I = 17 \angle -10.9 \text{ A}$$

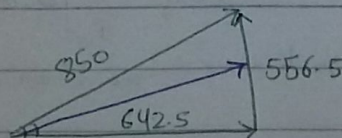
Imp.

$\downarrow P_{avg} \quad \downarrow S$ by $\downarrow Q$ & keep P value

$\downarrow \theta \quad \uparrow P.F. (\uparrow \cos \theta)$

In exam

How to correct the P.F.?



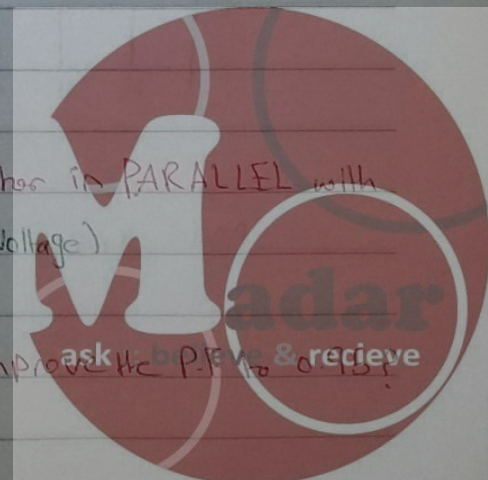
* To improve the P.F. we have to add a capacitor in PARALLEL with the POWER SUPPLY. (in order to know its voltage)

Imp.

Ideal case $\rightarrow P.F. = 1 \quad \theta = 0$

In exam
don't forget

What is the value of the capacitor to be added to improve the P.F. to 0.95 assuming $f = 60 \text{ Hz}$.



in final there is $Y \rightarrow \Delta$

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1. Find $\theta_{\text{new}} = \cos^{-1}(P.F. \text{ new}) = 18.2^\circ$

2. $Q_{\text{new}} = P \times \tan \theta = 211.2 \text{ VAR}$

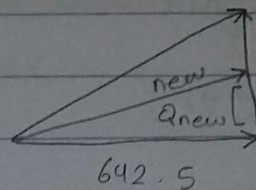
3. $Q_{\text{capacitor}} = Q_{\text{old}} - Q_{\text{new}} = 345.3 \text{ VAR}$

it is the voltage drop of capacitor

4. $Q_c = \frac{V_{\text{rms}}^2}{X_c}$

$X_c = \frac{1}{\omega C}$

$Q_c = V_{\text{rms}}^2 \omega C = V_{\text{rms}}^2 2\pi f C$



$C = \frac{Q_c}{V_{\text{rms}}^2 \omega}$

$Q_c = Q_{\text{old}} - Q_{\text{new}} = P \tan \theta_{\text{old}} - P \tan \theta_{\text{new}}$

$V_{\text{rms}} = V_{\text{max}} / \sqrt{2}$

$C = \frac{P(\tan \theta_{\text{old}} - \tan \theta_{\text{new}})}{V_{\text{rms}}^2 2\pi f}$

Find C to make the P.F unity $\rightarrow Q_c = Q_{\text{old}}$

$C = 1.8 \times 10^{-4} \text{ F} = 180 \text{ nF}$

$\rightarrow V_{\text{rms}}$ root mean squared value - rms - effective

$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$

physically, it means if replaced with DC device, it gives the same power that was putted for the same AC device.

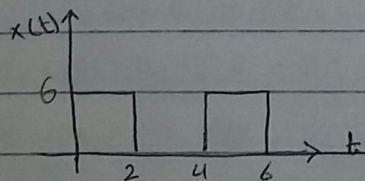
For sinusoidal sign $\leftarrow \frac{\sin}{\cos}$, $V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$

e.g $I(t) = 6 \sin(10t)$

$I_{\text{rms}} = 6 / \sqrt{2}$

$v(t) = 10 \cos(\omega t)$

$V_{\text{rms}} = 10 / \sqrt{2}$



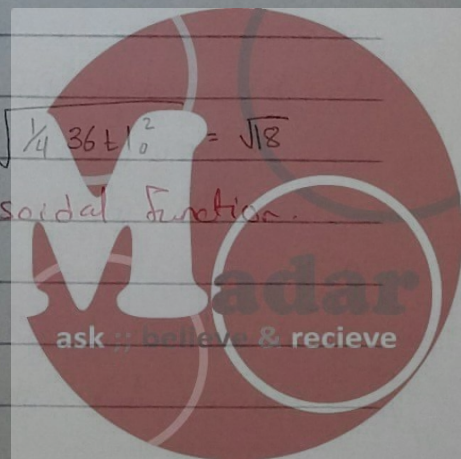
$T = 4$

$X_{\text{avg}} = \frac{1}{4} \int_0^4 6 \cdot dt$

$2 \rightarrow 4 = 0$

$X_{\text{rms}} = \sqrt{\frac{1}{4} \int_0^4 6^2 \cdot dt} = \sqrt{\frac{1}{4} 36 t \Big|_0^4} = \sqrt{18}$

Not $X_{\text{rms}} = X_{\text{max}} / \sqrt{2}$ BECAUSE Not sinusoidal function.



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26/12/2019

Source

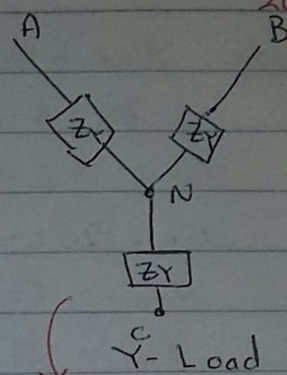
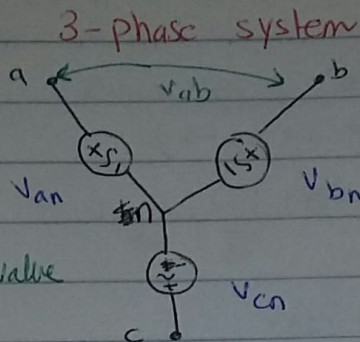
Load

Y

Y

Y

Δ



Impossible to find a point at zero value

-120 with clock $\rightarrow$ clock wise $\rightarrow$ +ve sequence

θ difference = 120°

$$\begin{aligned} V_{an} &= 10 \angle 0^\circ \\ V_{bn} &= 10 \angle -120^\circ \\ V_{cn} &= 10 \angle -240^\circ \end{aligned}$$

"Phase Voltage"

$$\begin{aligned} V_{ab} &= 10\sqrt{3} \angle 30^\circ \\ V_{bc} &= \sqrt{3} \times 10 \angle -90^\circ \\ V_{ca} &= \sqrt{3} \times 10 \angle -210^\circ \end{aligned}$$

"line voltage" $\rightarrow V_{ab} = \sqrt{3} V_{an} \angle \theta + 30^\circ$

Ex. $V_{an} = 100 \angle 60^\circ$

$V_{ab} = 100\sqrt{3} \angle 90^\circ$

if V_{ba} (- vector)

$V_{bn} = 100 \angle -60^\circ$

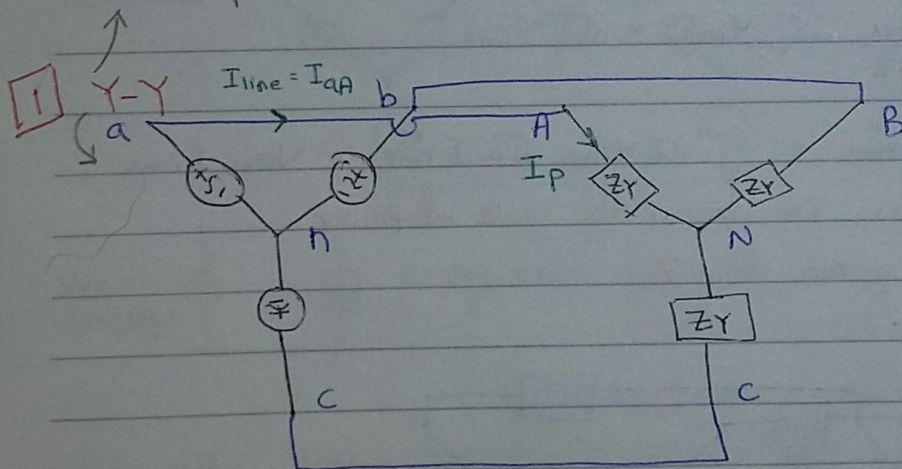
$V_{bc} = 100\sqrt{3} \angle -30^\circ$

$V_{cn} = 100 \angle -180^\circ$

$V_{ca} = 100\sqrt{3} \angle -150^\circ$

V_p "Phase voltage"

"Line voltage" V_L

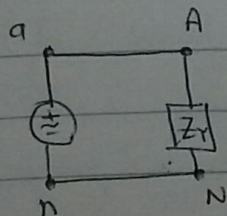


in Y-Y

$I_{line} = I_{phase}$

$V_{line} = \sqrt{3} V_{phase} = \frac{V_{an}}{Z_Y} \angle \theta + 30^\circ$

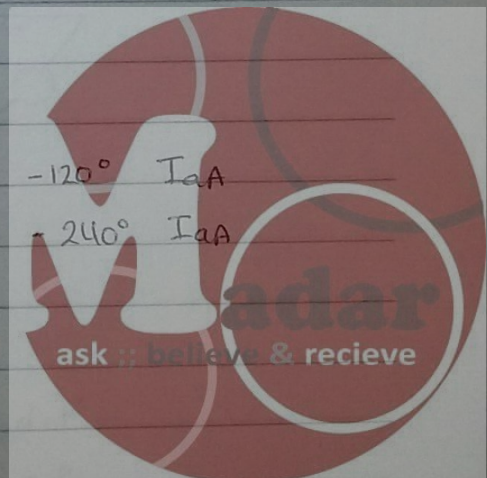
since balanced load



$I_{aA} = \frac{V_{an}}{Z_Y}$

I_{bB} Just -120° I_{aA}

I_{cC} Just 240° I_{aA}



Example $V_{bn} = 120 \angle 70^\circ$

$$Z_Y = 4 \angle 10^\circ$$

$$V_{an} = 120 \angle 190^\circ = V_{AN}$$

$$V_{cn} = 120 \angle -50^\circ$$

$$V_{ab} = \sqrt{3} 120 \angle 220^\circ$$

$$V_{bc} = \sqrt{3} 120 \angle 100^\circ$$

$$V_{ca} = \sqrt{3} 120 \angle -20^\circ$$

$$I_{aA} = \frac{V_{an}}{Z_Y} = \frac{120 \angle 190^\circ}{4 \angle 10^\circ} = 30 \angle 180^\circ$$

$$I_{bB} = 30 \angle 60^\circ$$

$$I_{cC} = 30 \angle -60^\circ$$

→ $I_{aA} + I_{bB} + I_{cC} = 0 = I_{nn}$ ONLY in balanced load

Find the complex power in load

$$S = \frac{1}{2} V I^* \leftarrow \text{For 1 phase}$$

$$\text{For 3 phase} \rightarrow S = 3 \times \frac{1}{2} \times \overset{\text{max}}{V_{an}} \overset{\text{max}}{I_{aA}^*}$$

$$S = 1.5 \times (120 \angle 190^\circ) (30 \angle -180^\circ) = 5400 \angle 10^\circ \text{ VA}$$

→ With rms

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$V_{max} = \sqrt{2} V_{rms}$$

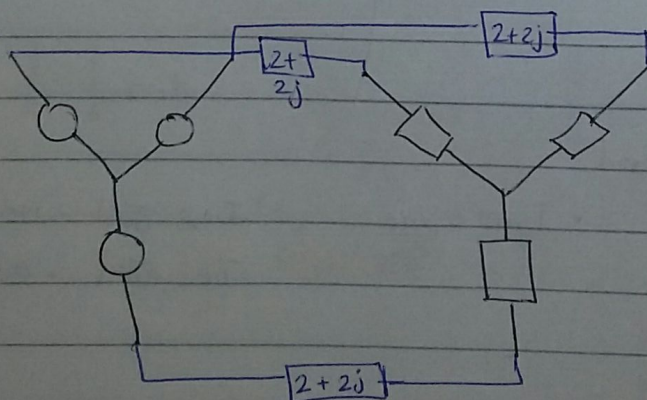
$$\frac{1}{2} V_{max} I_{max}^* = \frac{1}{2} \sqrt{2} V_{rms} \sqrt{2} I_{rms}^* = V_{rms} I_{rms}^*$$

if with rms, no $\frac{1}{2}$ in the laws of power

→ if complex power given, Find Z_L

Example

impedance on line

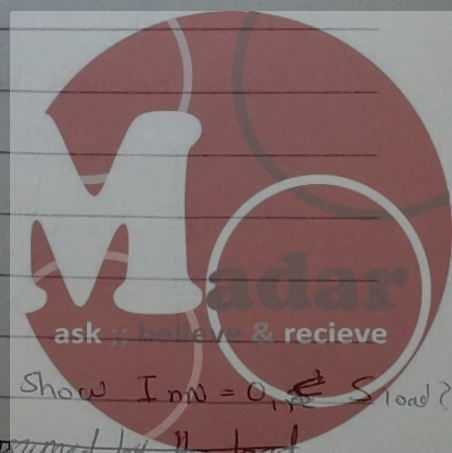


$$V_{ab_{rms}} = 173 \angle 30^\circ \text{ V}$$

$$Z_Y = 4 \angle 10^\circ$$

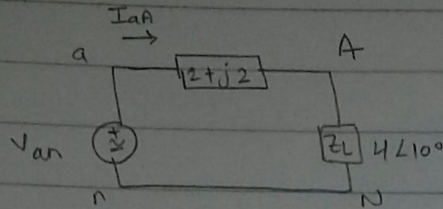
$$S = 210 \angle 75^\circ \text{ VA}$$

Complex power consumed by the load.



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collective rms 50 to 100
max 100

$$I_{aA} = \frac{V_{an}}{(2+j2) + 4\angle 10^\circ} = 15\angle -24^\circ \text{ A}$$

$$V_{an} = \frac{173\angle 0^\circ}{\sqrt{3}} = 100\angle 0^\circ$$

$$I_{bB} = 15\angle -144^\circ$$

$$I_{cC} = 15\angle -264^\circ$$

Complex Power consumed by load $\theta_s = \theta_z$

$$S = 3 \times V_{AN}^{\text{load}} I_{aA}^*$$

NOT V_{an} , there is voltage consuming

$$V_{AN} = I_{aA} Z_Y = 15\angle -24^\circ \cdot 4\angle 10^\circ = 60\angle -14^\circ \text{ V}$$

$$S = 3 \times 60\angle -14^\circ \times 15\angle 24^\circ = 2700\angle 10^\circ \text{ VA}$$

$$\text{P.F. load} = \cos(10^\circ) \text{ lag}$$

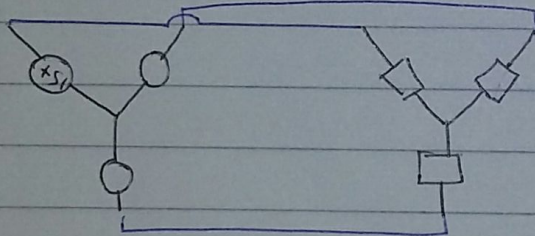
→ Complex Power ~~consumed~~ delivered by the source

$$S_{\text{source}} = 3 V_{an} I_{aA}^* = 3 \times (100\angle 0^\circ) \times (15\angle 24^\circ) = 4500\angle 24^\circ$$

$$\text{P.F. source} = \cos(24^\circ) \text{ lag}$$

→ Complex power consumed by line $S_{\text{line}}^{\text{transmission}} = S_{\text{source}} - S_{\text{load}}$

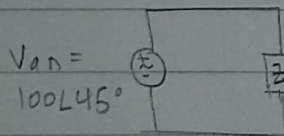
Example



$$V_{an} = 100\angle 45^\circ \text{ V}$$

$$S_{\text{load}} = 200\angle 60^\circ \text{ VA}$$

Find Z , I_{aA} , P.F. source



$$S = \frac{200}{3} \angle 60^\circ$$

$$S = 3 V_{an} I_{aA}^*$$

$$200\angle 60^\circ = 3 (100\angle 45^\circ) I_{aA}^*$$

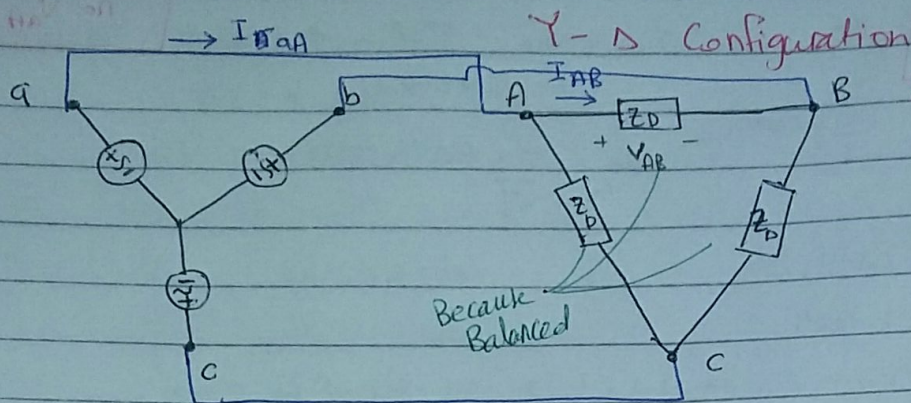
$$I_{aA} = 0.67\angle -15^\circ \text{ A}$$

$$Z_Y = \frac{V_{AN}}{I_{aA}} = \frac{100\angle 45^\circ}{0.67\angle -15^\circ} = 150\angle 60^\circ \Omega$$

$$\text{P.F.}_L = \cos(60^\circ) \text{ lag}$$

$S_{\text{load}} = S_{\text{source}}$ ONLY for ideal transmission





You can convert Δ to Y

$$Z_Y = Z_{\Delta} / 3$$

$$I_{aA} = \sqrt{3} I_{AB} \angle -30^\circ$$

$$V_{an} \rightarrow V_{ab} \rightarrow V_{AB} \rightarrow I_{AB} = \frac{V_{AB}}{Z_{\Delta}} \rightarrow I_{aA}$$

$$V_{ab} = V_{AB}$$

Z_{Δ} given $\rightarrow$ Find I_{aA}

Example

$$Z_{\Delta} = 4 \angle 15^\circ$$

$$V_{an} = 100 \angle 60^\circ$$

Complex power consumed by load? delivered by source?

$$V_{an} \rightarrow V_{ab}$$

$$V_{ab} = \sqrt{3} V_{an} \angle +30^\circ \rightarrow V_{ab} = 173 \angle 90^\circ \text{ V}$$

$$V_{ab} = V_{AB}$$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{173 \angle 90^\circ}{4 \angle 15^\circ} = 43.25 \angle 75^\circ \text{ A}$$

$$S_{\text{load}} = \frac{1}{2} \times 3 \times V_{AB} \times I_{AB}^* = 11223.4 \angle 15^\circ \text{ VA}$$

consumed by phase AB, for 1 load, without 3

$$\theta_{\text{load}} = \theta_{Z_{\text{load}}}$$

$$S_{\text{source}} = \frac{1}{2} \times 3 \times V_{an} \times I_{aA}^* = S_{\text{load}} = S_{\text{delivered}} = 11223.4 \angle 15^\circ \text{ VA}$$

$$I_{aA} = \sqrt{3} \times 43.25 \angle 45^\circ = 74.9 \angle 45^\circ \text{ A}$$

$$I_{cc} = 74.9 \angle 45 - 240$$

$$V_{bc} = V_{AB} \angle -120 + \theta$$

what angle to θ
 S_{load} is vector

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Y-Y

$$I_L = I_P \quad I_{aA} = I_{AN} \\ V_L = \sqrt{3} V_P \quad V_{ab} = \sqrt{3} V_{an}$$

}

Y-Δ

$$I_L = \sqrt{3} I_P \quad I_{aA} = \sqrt{3} I_{AB} \\ V_L = V_P \quad V_{ab} = V_{AB}$$

Imp.
Example

Find line current $\rightarrow I_{aA}, I_{bB}, I_{cC}$, same source phase current
load Δ phase current $\rightarrow I_{AB}, I_{BC}, I_{CA}$

Phase voltage $\rightarrow V_{AB}$ line voltage $\rightarrow V_{ab}$ Avg. Power $\rightarrow P$ Given

$$V_{an \text{ rms}} = 220 \angle 0$$

$$\tilde{V}_{bn} = 220 \angle -120$$

$$\tilde{V}_{cn} = 220 \angle 120 \text{ (or } -240)$$

$$Z_{\Delta} = 9 - j6$$

$$\tilde{V}_{ab} = 381 \angle 30 = \tilde{V}_{AB} \quad \tilde{V}_{bc} = 381 \angle -90 = \tilde{V}_{BC} \quad \tilde{V}_{ca} = 381 \angle -210 = \tilde{V}_{CA}$$

$$I_{AB} = \frac{V_{AB}}{Z_A} = 35.2 \angle 63.7 \quad I_{BC} = 35.2 \angle -56.4 \quad I_{CA} = 35.2 \angle -176.4 \text{ A}$$

$$I_{aA} = \sqrt{3} I_{AB} \angle -30$$

$$I_{aA} = 60.9 \angle 33.6$$

$$I_{bB} = 60.9 \angle -86.4$$

$$I_{cC} = 60.9 \angle -206.4 \text{ A}$$

$$S_{\text{source}} = 3 \tilde{V} \tilde{I}^* = 3 \tilde{V}_{an} I_{aA}^* = 3(220 \angle 0)(60.9 \angle -33.6) = 40194 \angle -33.6 \text{ VA}$$

$$P.F._{\text{source}} = \cos(-33.6) = 0.83 \text{ leading}$$

$$S_{\text{load}} = S_{\text{source}}$$

$$S_{\text{1 phase}} = S_{\text{source}} / 3$$

$\rightarrow$ If the power delivered by load = $40194 \angle -33.6$, given V_{an} , find Z_{Δ} ?

$$S_{\text{1 phase}} = S / 3 \rightarrow S = \tilde{V} \tilde{I}^*$$

$$\checkmark \quad \checkmark \quad \downarrow \quad \text{find } I \rightarrow \frac{V}{I} = Z_{\Delta}$$

