

Electrical Engineering



2022 - 2023

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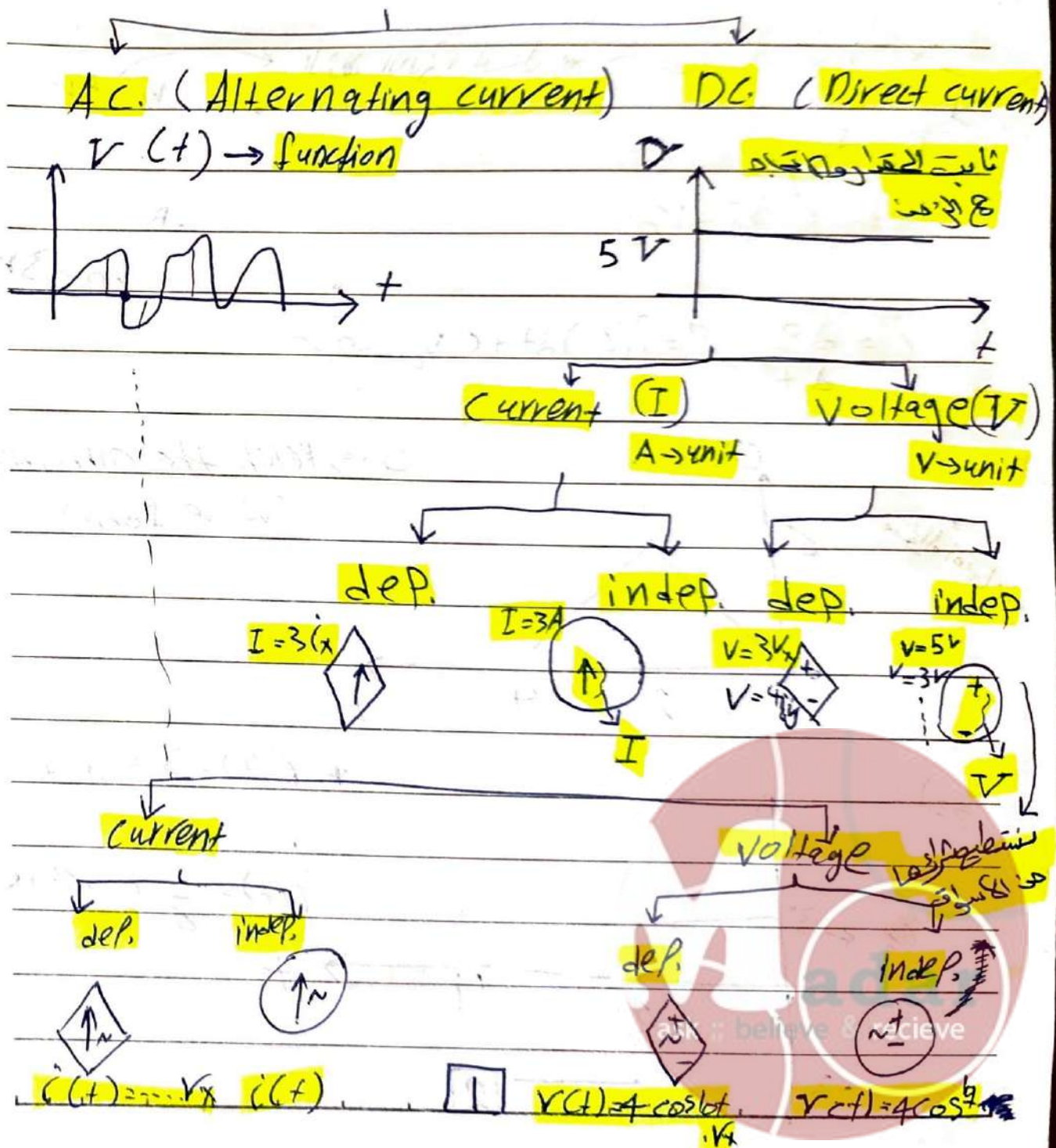
By. Omar Al-Hunaiti



Electrical engineering

- ① power supply → produce energy
- ② connections
- ③ load → consume the energy.

Power supplies (P.S) (source)



* load : $R (\Omega)$



$$R = \frac{P \cdot L}{A}$$

التي A الى ∞ ← الشغل الذي لنقل شحنة من A الى ∞

$V = \frac{dq}{dq}$

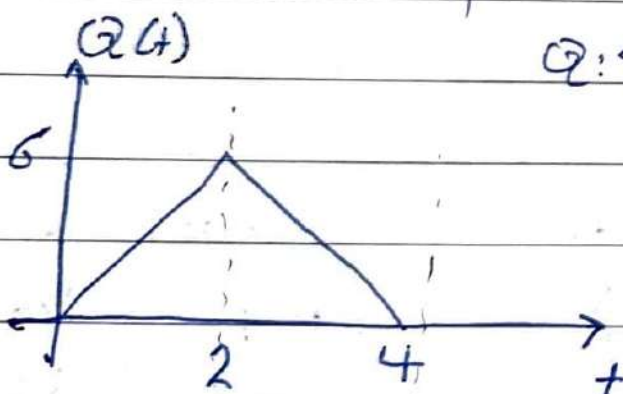
$V_{AB} = V_A - V_B$

$V_A = 5V$

$V_B = 3V$

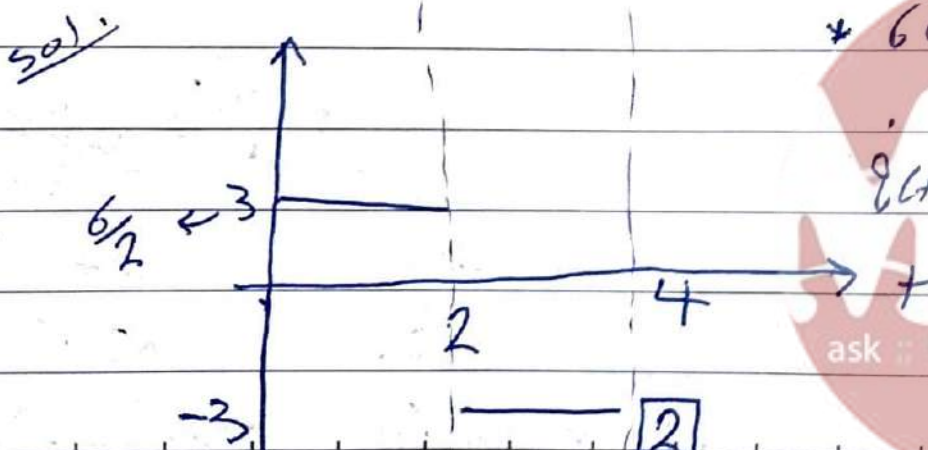
$$i = \frac{dq}{dt}, q = \int i(t) dt + C \quad V_C = -2V$$

Q: Sketch the current??
(wave form)

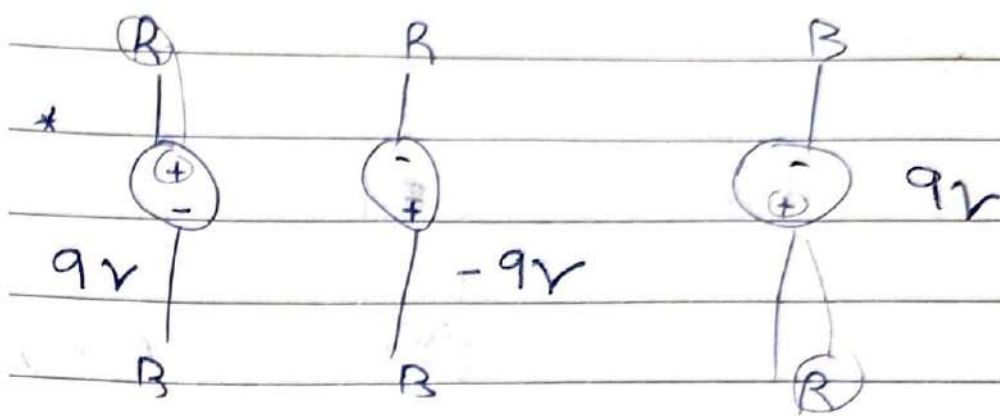


$$i(t) = 3 \sin 6t$$

$$q(t) = -\frac{3}{6} \cos 6t + C$$



ask :: believe & recieve

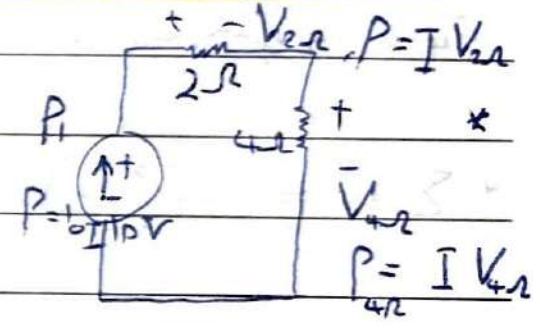


* $V = I \cdot R$, $V \propto I$ (ohm's law)

* التيار يخرج من القطب الموجب
 $\hookrightarrow$

* Power (P) $\rightarrow$ watt (W)

* Energy (W, E) $\rightarrow$ Joule (J)



* $P = \frac{\Delta E}{\Delta t}$, $E = \int P(t) \cdot dt$

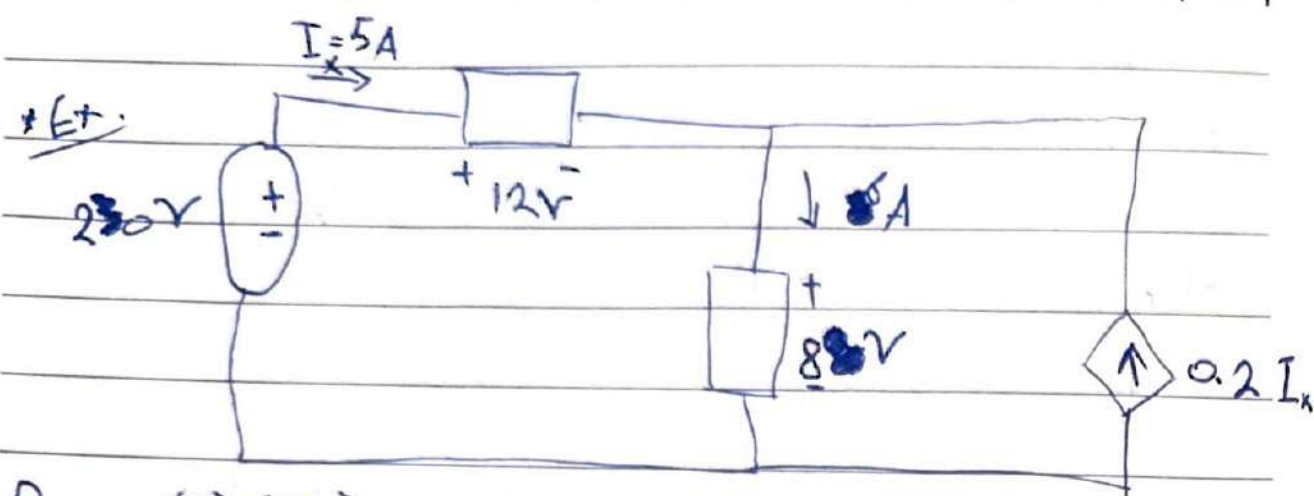
* $P = I \cdot V = \frac{V^2}{R} = I^2 R$ (W)

* P(-) produce, deliver

* P(+) consumed, absorbed

إنتاج
استهلاك

* إذا التيار يخرج من الـ + $\Rightarrow$ $P_{\text{consumed}} + P_{\text{produced}} = 0$ (من جوار)
 * إذا التيار من الـ - $\Rightarrow$ $P_{\text{consumed}} - P_{\text{produced}} = 0$



$$P_1 = -(5)(230) = -1150 \text{ W (del.)}$$

$$P_2 = +(5)(12) = +60 \text{ W (cons.)}$$

$$P_3 = +(5)(8) = +40 \text{ W (cons.)}$$

$$P_4 = -2 \text{ W (del.)}$$

$$\sum P = 0 \text{ (P.E.)}$$

* Parallel element have same Voltage same Polarity.

$$* \text{ 700 kWh} = 12 + 16 + 20 + 24 = 72 \$$$

$$12 \$$$

$$0-100, 0.16 \$$$

$$101-300, 0.1 \$$$

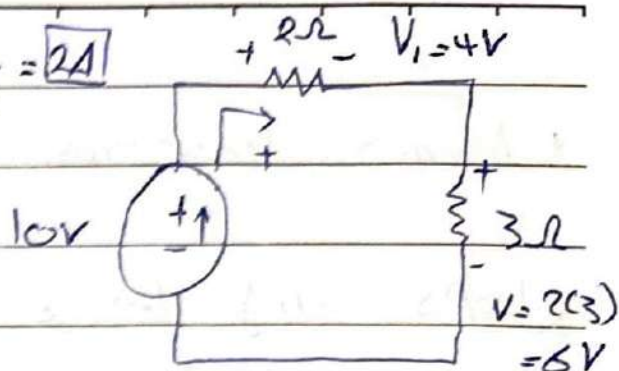
$$* 301 \rightarrow 700, 0.06 \$$$

$$I = \frac{10}{5} = 2A$$

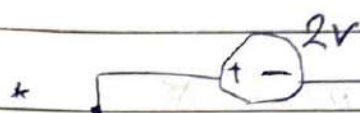
$$* P_1 = -(10)(2) = -20 \text{ wdet.}$$

$$* P_2 = + (4)(2) = +8 \text{ w cons.}$$

$$* P_3 = + (6)(2) = +12 \text{ w cons.}$$



* $R(\Omega)$, G : conductance = $1/R$ (S) (أهم)

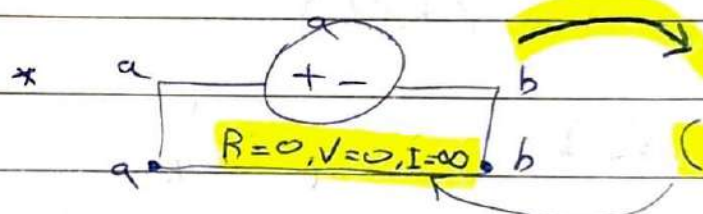


open ckt (O.C), $V = \checkmark$

$$I = \frac{V}{R} \quad \leftarrow \quad \begin{matrix} I=0 \\ R=\infty \end{matrix} \text{ (مقاومة لا نهائية)}$$

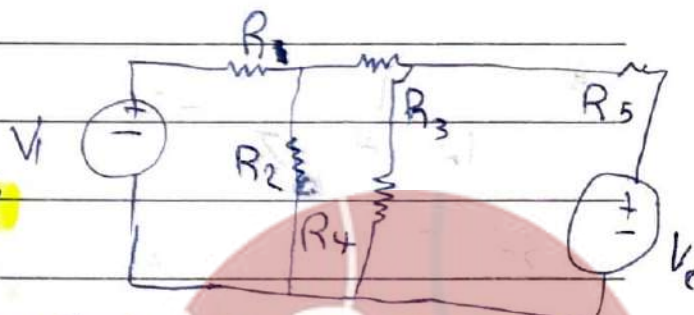
* الظهير يوصل على التوازي مع البطارية

* الأميتر يوصل على التوالي مع البطارية



(S.C) short ckt

* Branch: Any part of circle have 1 element or more



* short.c ; any element on parallel with S.C is erase



* Nodes: connection of two branches or more.
(Voltage) (current)

* loops: any closed circle.

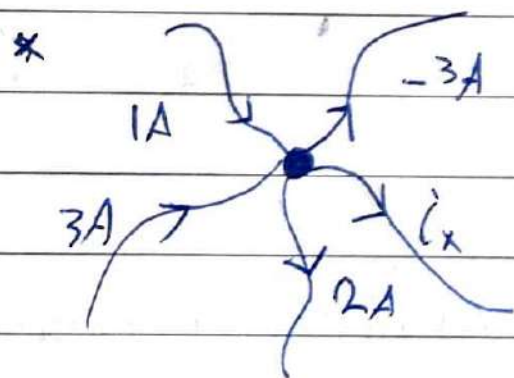
* mesh: current (loop) (junction)

* mesh and loop are same and loop and mesh are

* Reference node: 1P & node voltage is zero

* kirchoff's current law (KCL)
voltage law (KVL)

KCL: at any node $\rightarrow \sum I = 0$, $\sum I_{in} = \sum I_{out}$



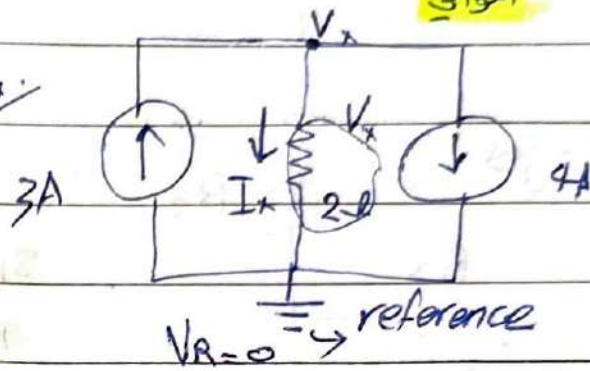
$$\begin{aligned}\sum I_{in} &= \sum I_{out} \\ 3 + 1 &= (3) + 2 + I_x \\ \therefore I_x &= 5A\end{aligned}$$

* current sources in parallel can be added.

المعادي

* مصادر التيار ~~المعادي~~ تجمع و الجهد متساوي

* مصادر الجهد ~~المعادي~~ تجمع و التيار متساوي
التوالي



Find V_x ??

2 Noods

(Single Noods Ckt)

1 active nood

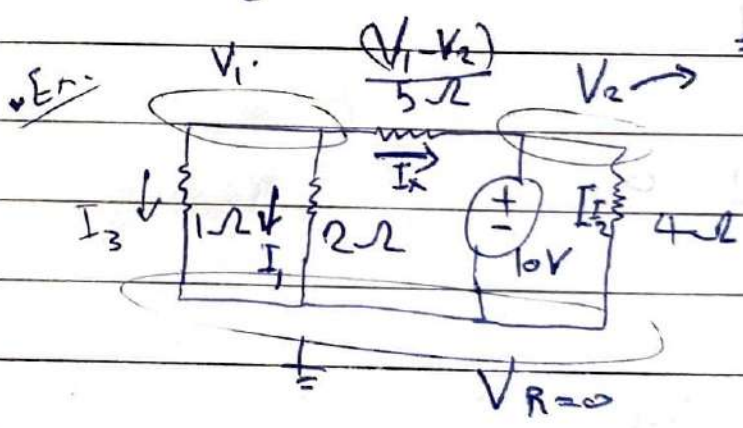
sol.

المعادي و ~~المعادي~~ active nood ~~المعادي~~ reference

يكون لازم من (active nood) (reference)

$$3 = I_x + 4$$

$$3 = \frac{V_x}{2} + 4 \Rightarrow V_x = -2V$$



3 Noods

$$\frac{V_1}{1} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_2}{2} = 0$$

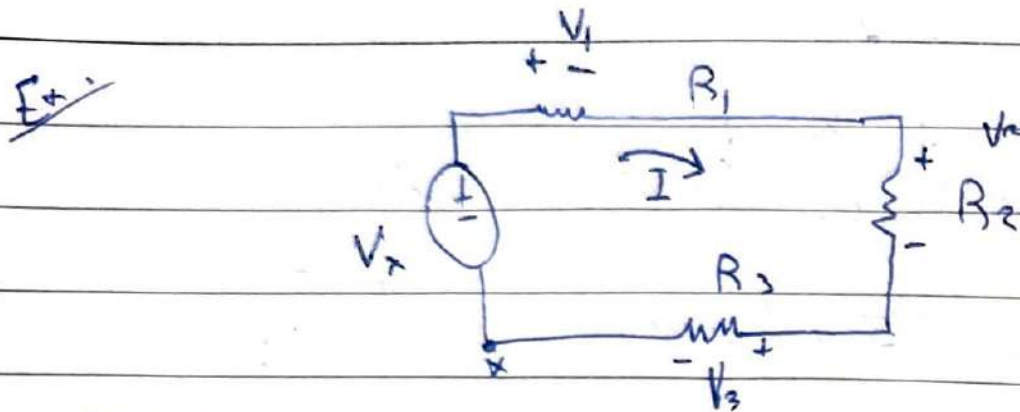
$$10V_1 + 5V_1 + 2V_1 - 2V_2 = 0$$

$$V_1 = 20/17$$

المعادي و ~~المعادي~~ active noods ~~المعادي~~ reference



VKL: for any closed path the SUM of voltages equal zero.



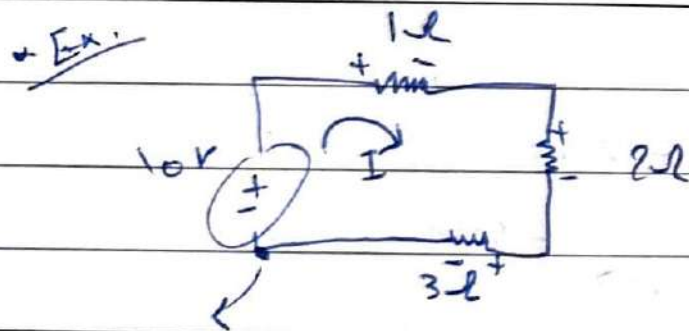
single mesh

G: x line

$$-V_x + V_1 + V_2 + V_3 = 0$$

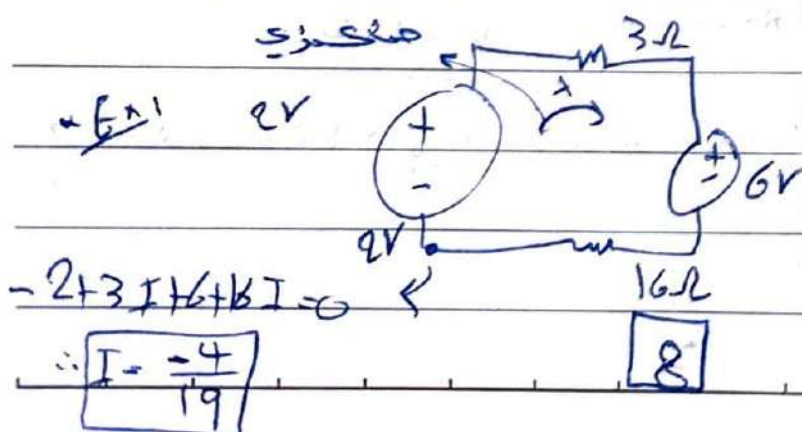
$$-V_x + IR_1 + IR_2 + IR_3 = 0$$

solve for I?



$$-10 + 1I + 2I + 3I = 0$$

$$I = \frac{10}{6} \text{ A}$$

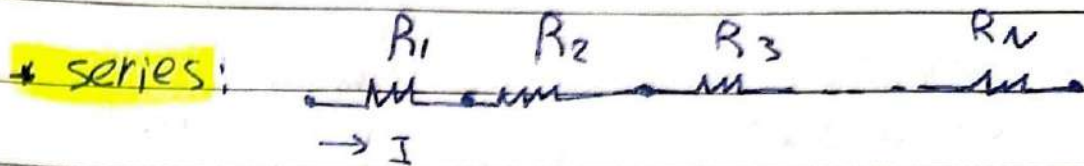


$$-2 + 3I + 6 + 16I = 0$$

$$\therefore I = \frac{-4}{19}$$

8

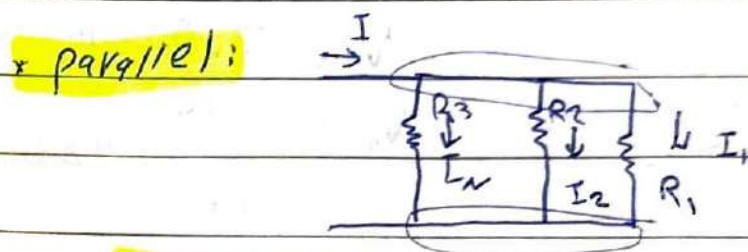
ask :: believe & recieve



$$I = I_1 = I_2 = \dots = I_N$$

$$V_T = V_1 + V_2 + \dots + V_N$$

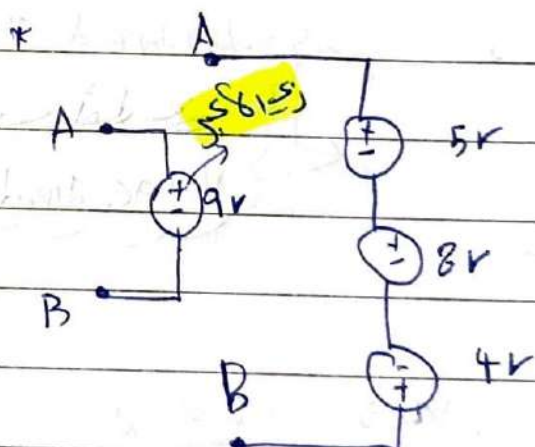
$$R_T = R_1 + R_2 + \dots + R_N$$



$$I_T = I_1 + I_2 + \dots + I_N$$

$$V_1 = V_2 = \dots = V_N$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}, \quad R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



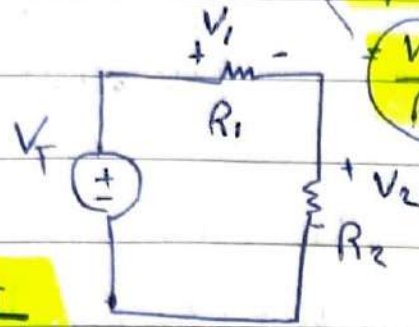
Voltage divider

* Voltage divider:

(جهد تقاسم)

(الحال يكون مفتوح)

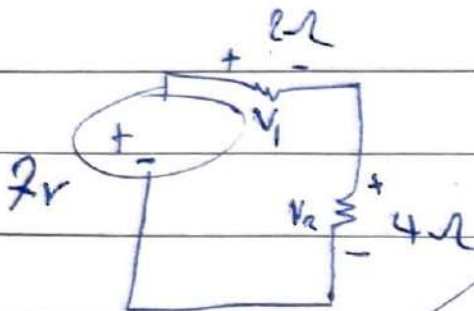
$$I = \frac{V_T}{R_1 + R_2}$$



$$V_1 = I \cdot R_1$$

$$V_1 = \frac{V_T \cdot R_1}{R_1 + R_2}$$

Ex:

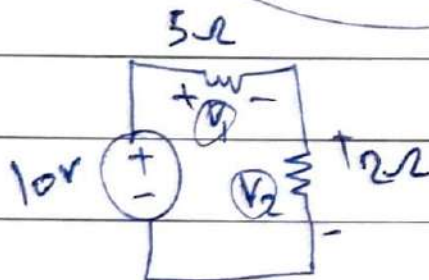
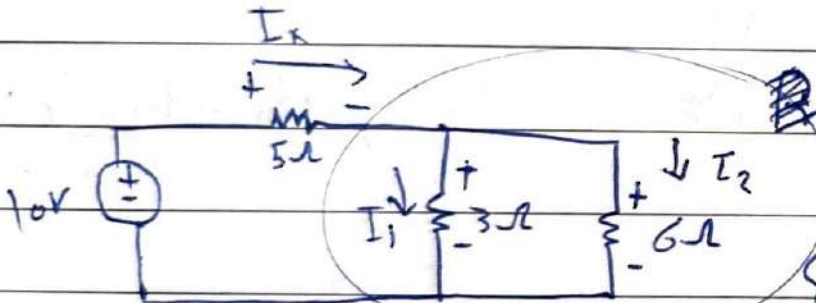


(standard CKT)

$$V_1 = \frac{7 \cdot 2}{2 + 4} = \frac{14}{6}$$

$$V_2 = \frac{7 \cdot 4}{2 + 4} = \frac{28}{6}$$

$$V_2 = V_T - V_1$$



standard CKT
voltage divider

$$V_1 = \frac{(10)(5)}{2+5} = 50/7 \text{ V}$$

$$I_1 = \frac{V_1}{3} = \frac{(50/7)}{3} \text{ A}$$

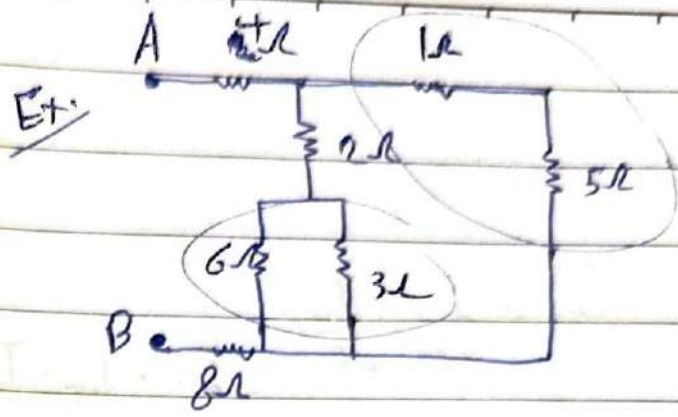
$$I_2 = \frac{V_2}{6} = \frac{(20/7)}{6} \text{ A}$$

$$V_2 = \frac{(10)(2)}{2+5} = 20/7 \text{ V}$$

$$I_x = I_1 + I_2 \text{ A}$$

$$I_x = \frac{V_1}{5} \text{ A}$$

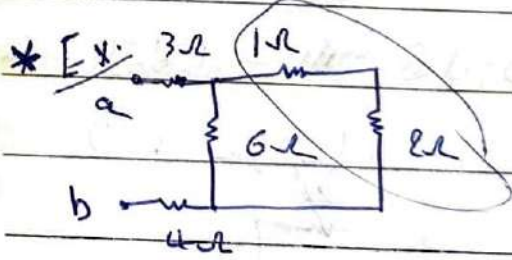
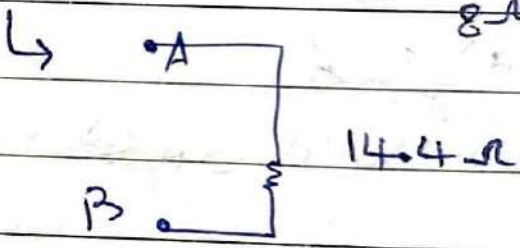
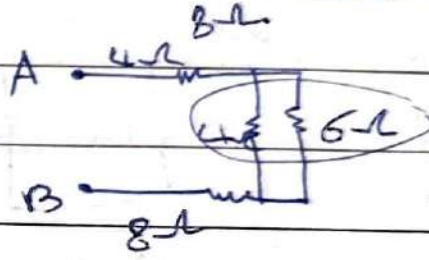
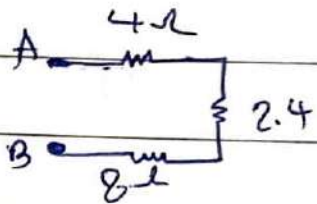
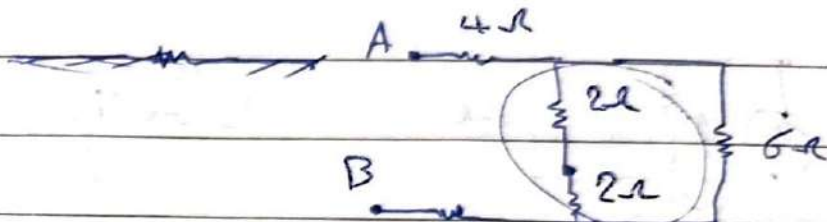
10



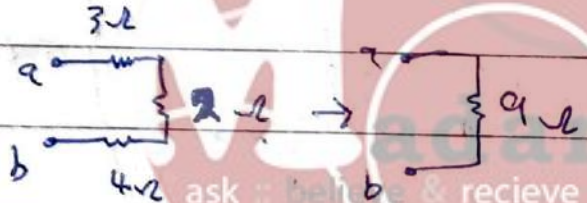
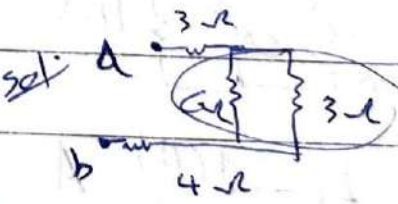
Req
 R_T
 R_{AB}

sol:

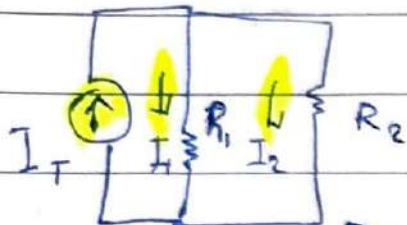
دائماً ابتدا من الجهد فقط



R_{ab}
 R_{eq}
 R_T



* current divider:

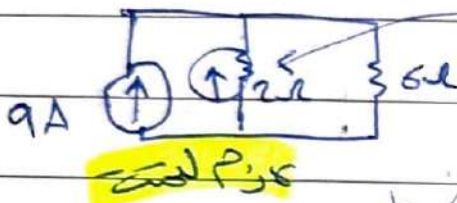


(Short circuit)

$$I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}, \quad I_2 = I_T - I_1$$

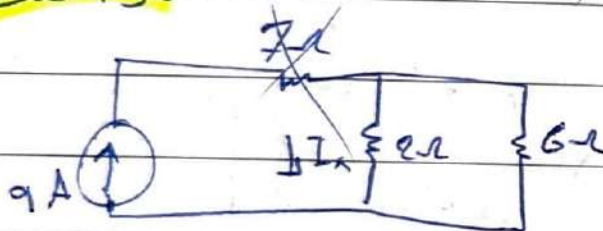
$$I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}$$

Ex1:



$$I_1 = 9 \cdot \frac{6}{8}$$

Ex2:

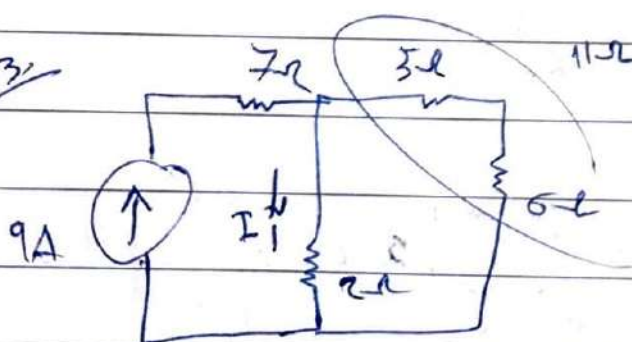


$$I_x = \frac{9(6)}{2+6} \text{ A}$$

* المقادير التي يجب استخدامها هي 9A و 2 ohm و 6 ohm
تواري

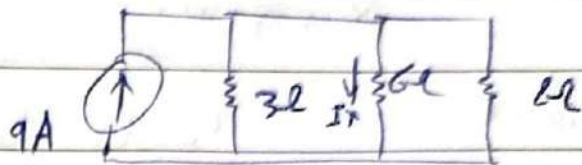
* الخطوات التي قبلها needs ما قبل في هذا التيار الكارص

Ex3:



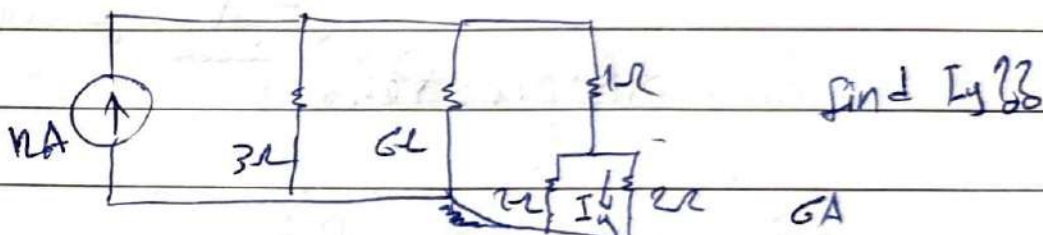
$$I_1 = \frac{9(11)}{11+2} \text{ A}$$

Ex

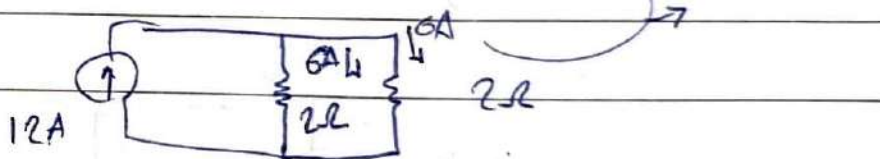


$$I_x = \frac{9 \left(\frac{6}{5} \right)}{\frac{6}{5} + 6} \text{ A}$$

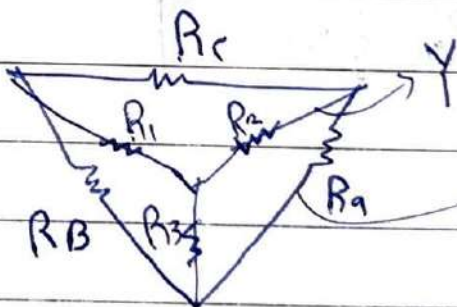
Ex.



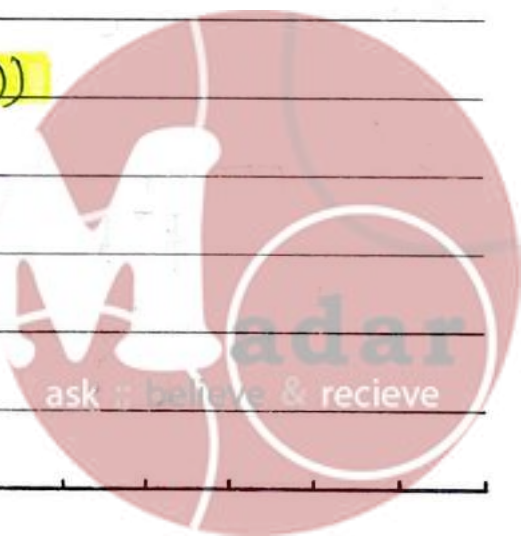
sol:



9



10/10(11)



* $D \rightarrow Y$: $R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \rightarrow$ مقاومة بين $R_1 \rightarrow R_B R_C$
(الحال ثابت)

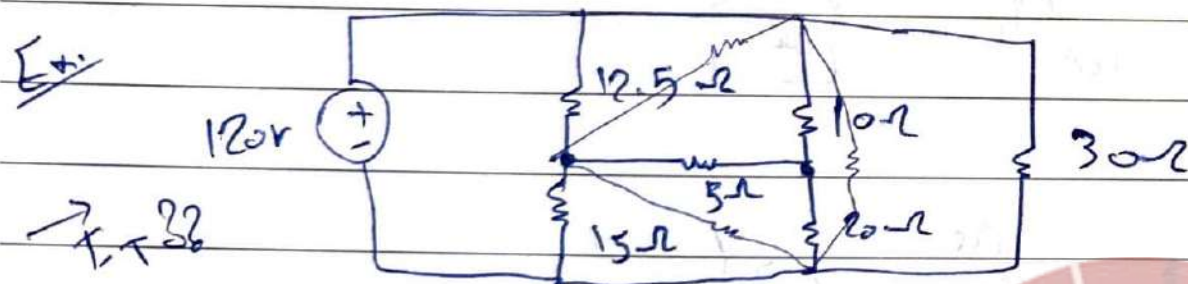
$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$

$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$

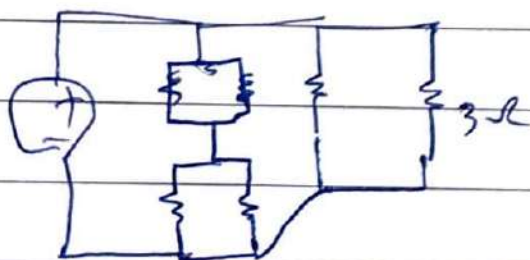
* $Y \rightarrow D$: $R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$ الكبر عتفا
(الحال ثابت)

$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$

$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$



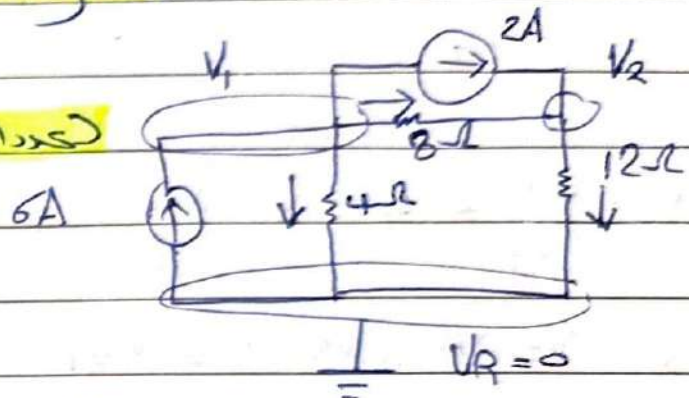
sol:



* لنأخذ قوانين التيار الكهربائي ~~في~~ في حسابات ← المبدأ الأساسي
 حسابي السهل

* chp #3: Nodal analysis

(Active nodes) = عدد العقد = عدد العقد



$$\text{N1: } 2 + \frac{V_1 - V_2}{8} + \frac{V_1}{4} = 6$$

$$4 = \frac{V_1 - V_2}{8} + \frac{V_1}{4} \rightarrow 32 = V_1 - V_2 + 2V_1 \rightarrow \boxed{3V_1 - V_2 = 32} \quad \text{--- (1)}$$

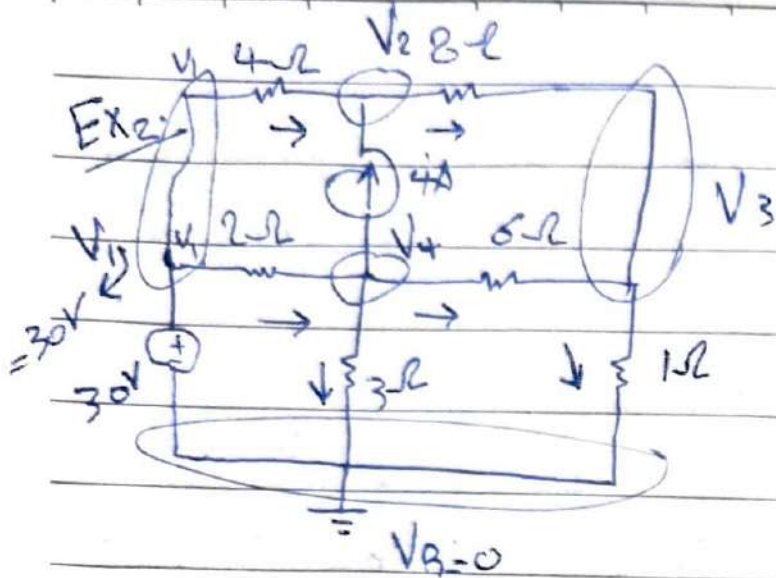
$$\text{N2: } I_5 + I_3 = I_4 \rightarrow 2 + \frac{V_1 - V_2}{8} = \frac{V_2}{12}$$

$$\boxed{3V_1 - 5V_2 = -48} \leftarrow 48 + 3V_1, \frac{3}{2}V_2 = 2V_2 \quad \text{--- (2)}$$

$$\text{(1), (2)} \rightarrow \begin{array}{l} 3V_1 - V_2 = 32 \\ 3V_1 - 5V_2 = -48 \end{array}$$

$$4V_2 = 80 \rightarrow \boxed{V_2 = 20 \text{ V}}$$

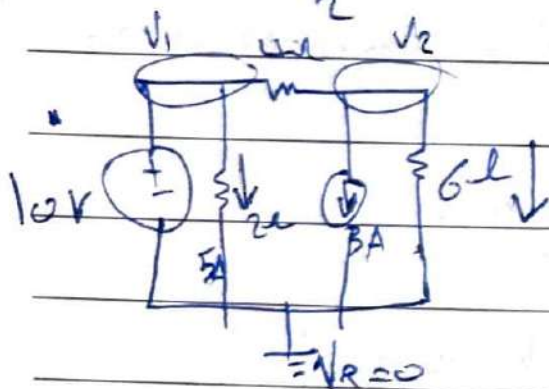
$$3V_1 - 20 = 32 \rightarrow \boxed{V_1 = 52 \text{ V}}$$



$$\text{KVL: } \frac{V_1 - V_2}{4} + 4 = \frac{V_2 - V_3}{8} \quad \text{--- (1)}$$

$$\text{KVL: } \frac{V_2 - V_3}{8} + \frac{V_4 - V_3}{6} = V_3/1 \quad \text{--- (2)}$$

$$\text{KVL: } \frac{30 - V_4}{2} = 4 + \frac{V_4 - V_3}{6} + \frac{V_4}{3} \quad \text{--- (3)}$$

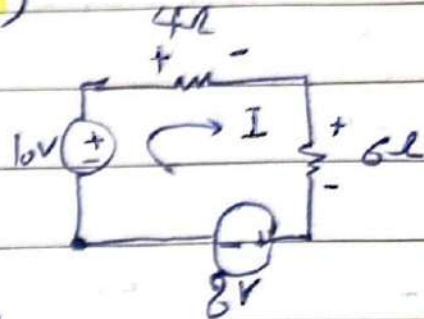


$$V_1 = 10 \quad \text{--- (1)}$$

$$\text{KVL: } \frac{V_1 - V_2}{4} = 3 + \frac{V_2}{6} \quad \text{--- (2)}$$

* Mesh analysis: (KVL)

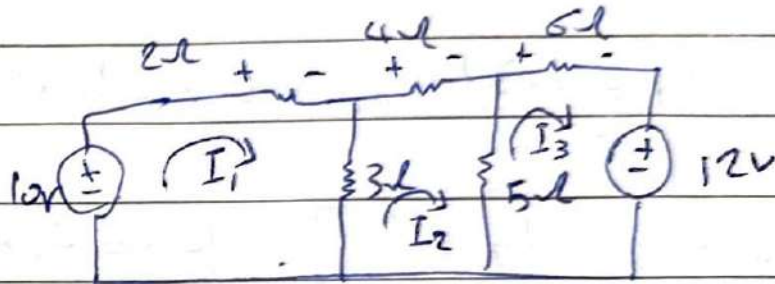
Single mesh $\rightarrow$



$$-10 + 4I + 6I + 8 = 0$$

solve for I

Ex.



M₁: $-10 + 2I_1 + 3(I_1 - I_2) = 0$ — (1)

↓
mesh 1
rule

$$5I_1 - 3I_2 = 10$$

M₂: $+4I_2 + 5(I_2 - I_3) + 3(I_2 - I_1) = 0$

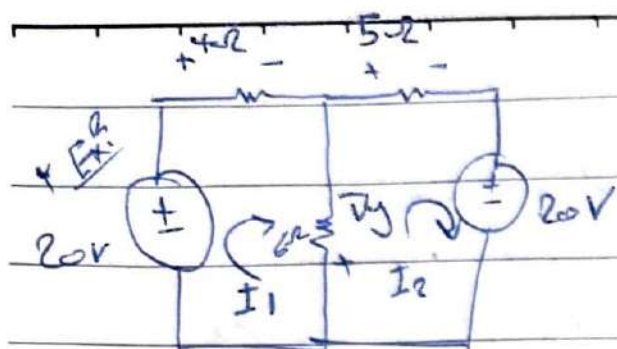
$$-3I_1 + 12I_2 - 5I_3 = 0$$
 — (2)

M₃: $6I_3 + 5(I_3 - I_2) = 0$

$$-5I_2 + 11I_3 = -12$$
 — (3)

$$\therefore I_1 = 2.04 \text{ A}, I_2 = 0.068 \text{ A}, I_3 = -1.06 \text{ A}$$





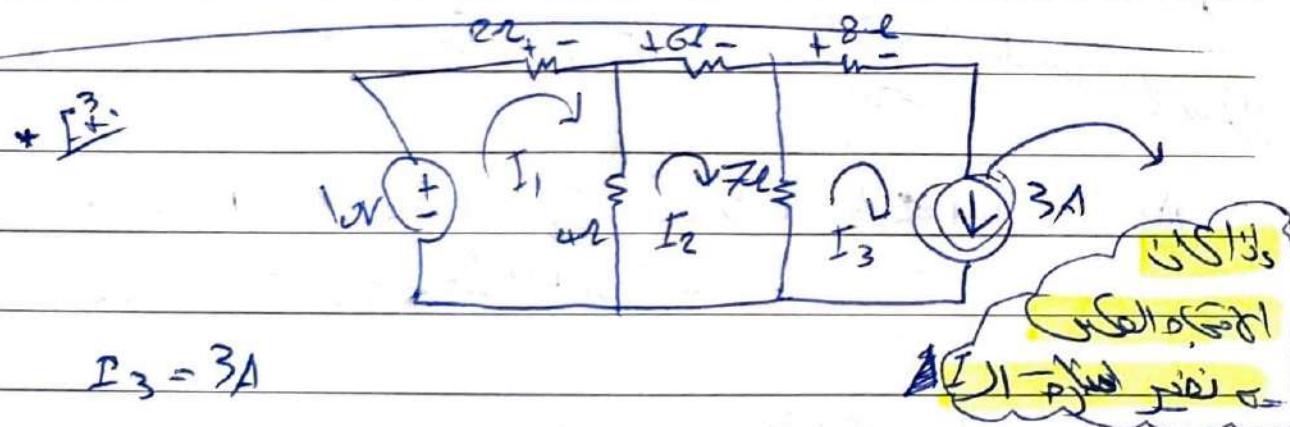
١٥٦٦

$$18I_1 - 6I_2 = 20 \quad \text{--- (1)}$$

$$-6I_1 + 11I_2 = -20 \quad \text{--- (2)}$$

$$I_1 = 1.35A, \quad I_2 = -1.08A$$

$$V_y = 6[I_2 - I_1] \quad \left[\begin{array}{l} \text{بطلع الـ } I \text{ بعدين بحسب} \\ \text{القطيب} \end{array} \right]$$



$$I_3 = 3A$$

١٥٦٦
التيه القوي
٥ زفير القوي الـ I

M1

$$10 + 2I_1 + 4(I_1 - I_2) = 0 \quad \text{--- (1)}$$

$$M2: 6I_2 + 7(I_2 - I_3) + 4(I_2 - I_1) = 0 \quad \text{--- (2)}$$

$$I_1, I_2 \quad \checkmark$$

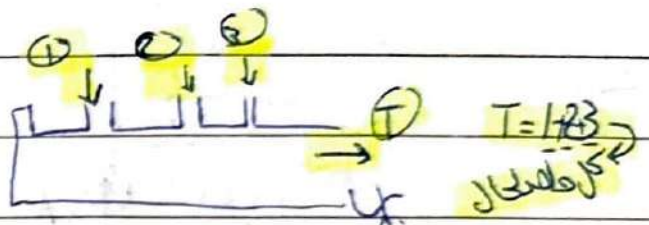
* current source in mesh is

SS Loop KVL

ثُمَّ لَا تَسْأَلُ عَنِ الْإِبَادَةِ لِلْفُؤَادِ (مِنْ مَطْلُوبٍ)

* CHP4 :

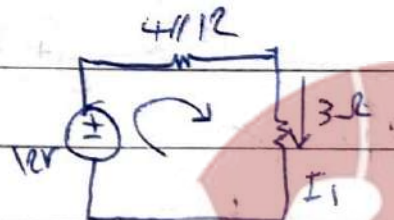
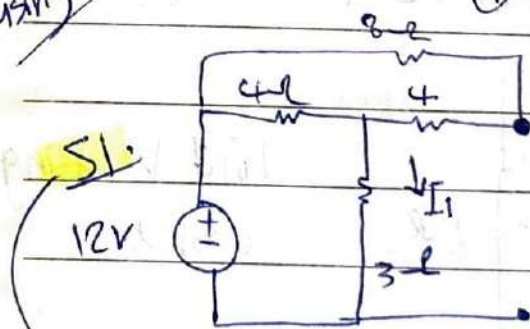
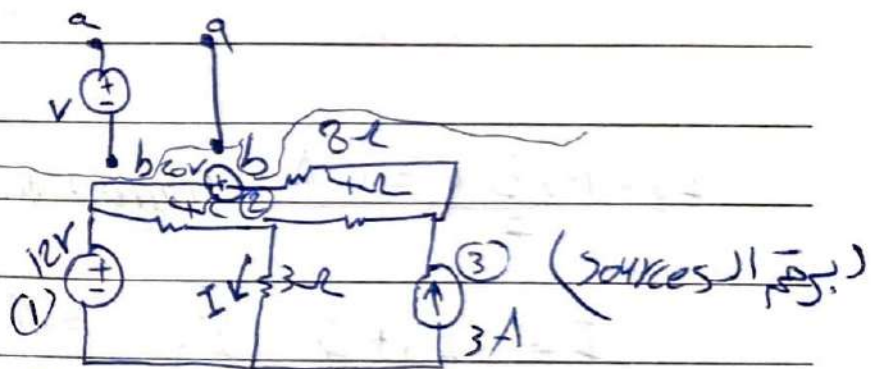
Super position →



Ex: Kill all source except one

$$V \rightarrow \boxed{Sc}$$

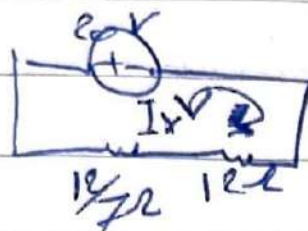
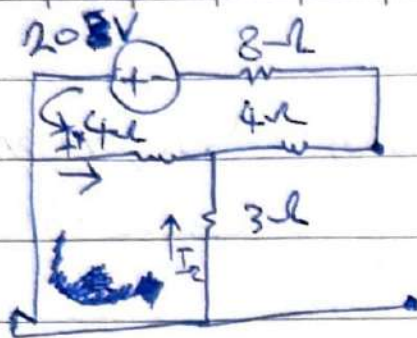
Ex:
Find I
using SP.

$$I \rightarrow 0.5$$


$$I_1 = 12/6 = 2A$$

بجملہ اشیاء (۲، ۳) میں سے

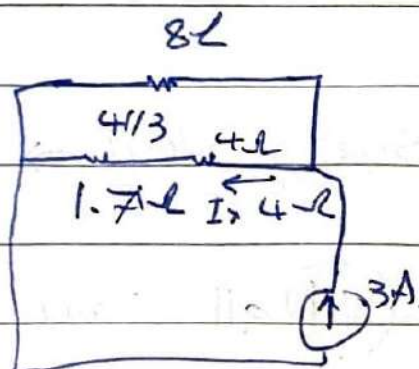
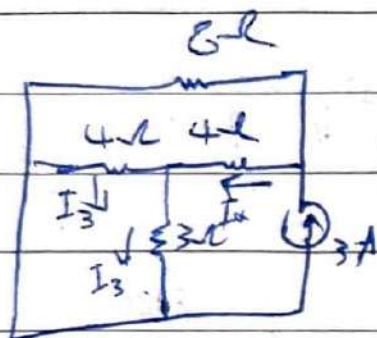
S2:



$$I_x = 1.45 \text{ A}$$

$$I_2 = \frac{-1.45 \times 4}{4+2} = -0.83 \text{ A} \quad (\text{Current divider})$$

S3:

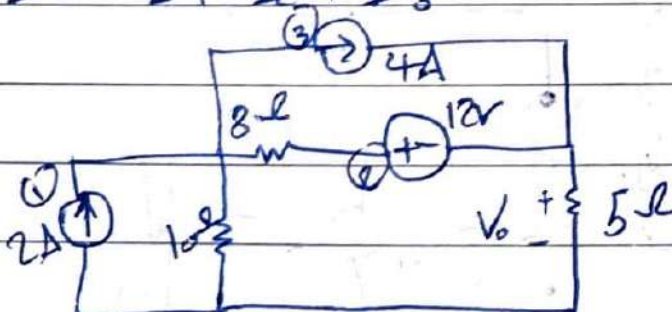


$$I_x = 3 \times \frac{8}{8+4+1.7} = 1.75 \text{ A}$$

$$I_3 = \frac{1.75 \times 4}{7} = 1 \text{ A} \quad (\text{current divider})$$

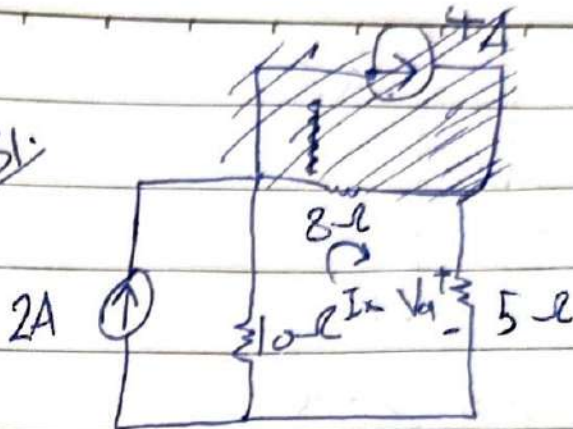
$$I = I_1 + I_2 + I_3 = 2 + 0.83 + 1.2 = 4.03 \text{ A}$$

Ex2:



Find V_0 using (SP).

S1:

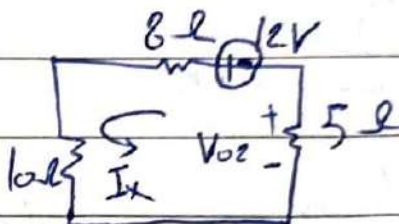


مجموع توان در شاخه 5Ω
Total 5Ω شاخه

$$I_x = \frac{2(10)}{23} = \frac{20}{23} \text{ A}$$

$$V_{01} = I_x (5) = \frac{100}{23} \text{ V}$$

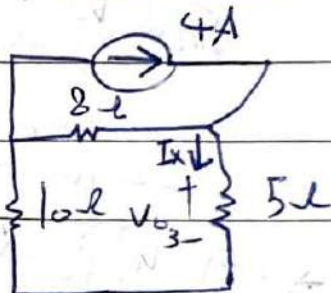
S2:



$$I_x = \frac{12}{23} \text{ A}$$

$$V_{02} = \frac{12}{23} \times 5 = \frac{60}{23} \text{ V}$$

S3:



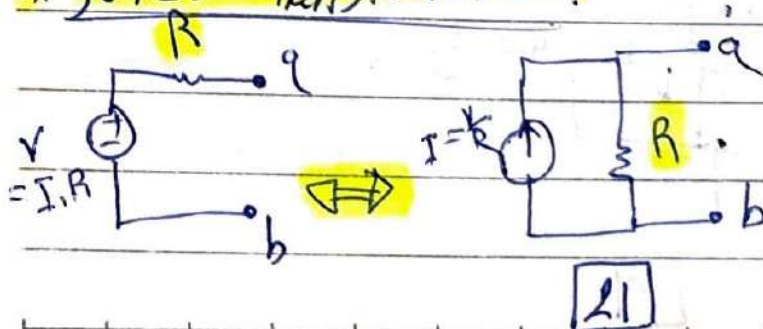
$$I_x = \frac{4(8)}{23} = \frac{32}{23}$$

$$V_{03} = I_x (5) = \frac{32}{23} (5) = \frac{160}{23} \text{ V}$$

$$V_0 = \frac{100}{23} + \frac{160}{23} - \frac{60}{23} = 8.7 \text{ V}$$

2) and S1, individual branch power & power at
Power in each branch Total [I or V]

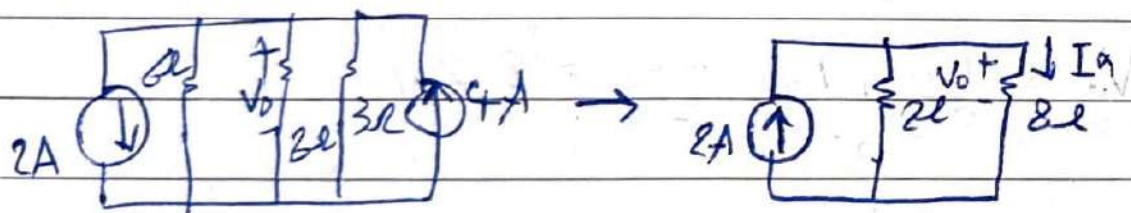
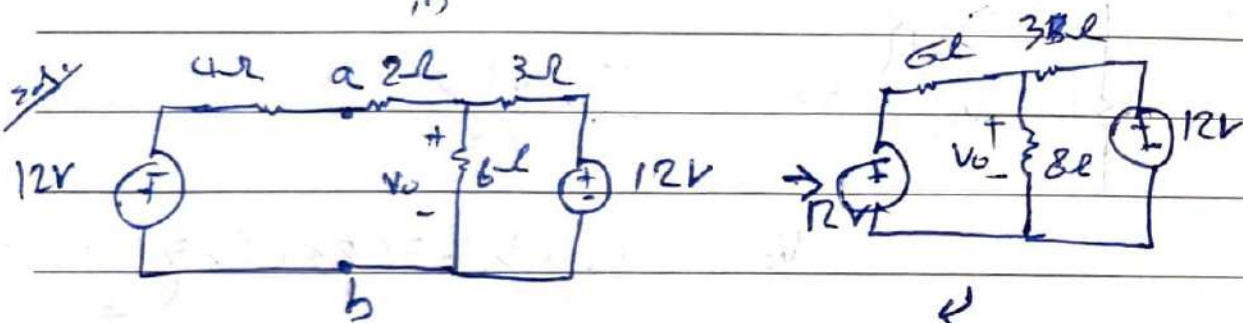
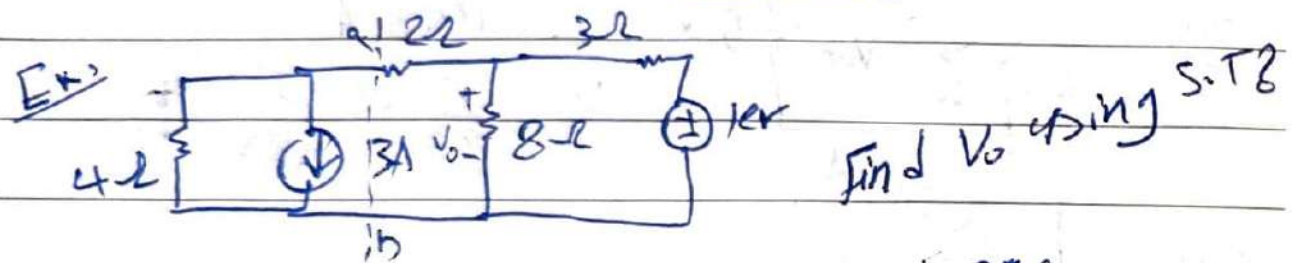
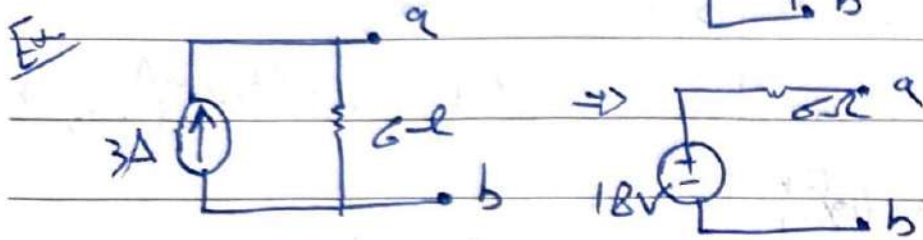
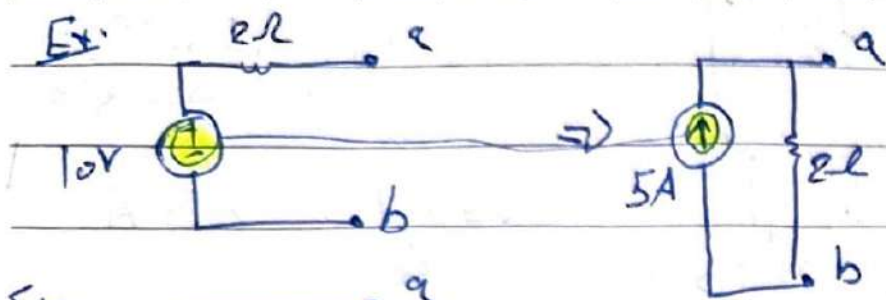
* Sources Transformation



Standard
for transform.

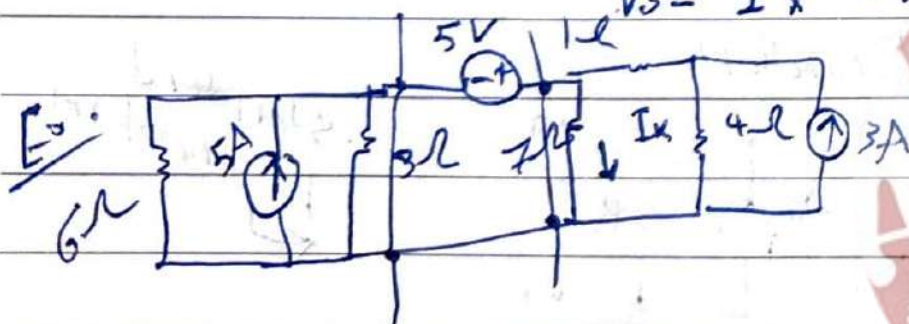
ask : believe & receive

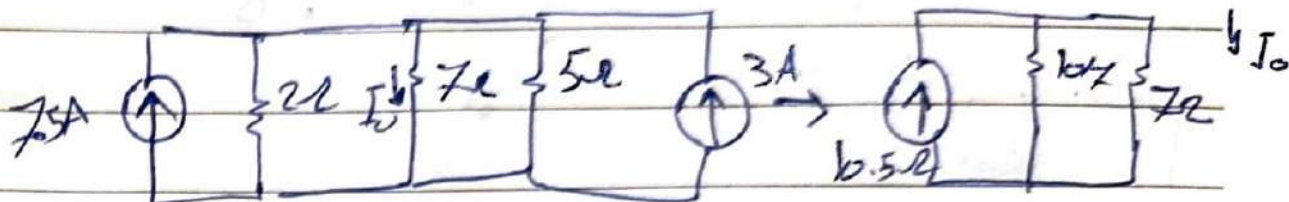
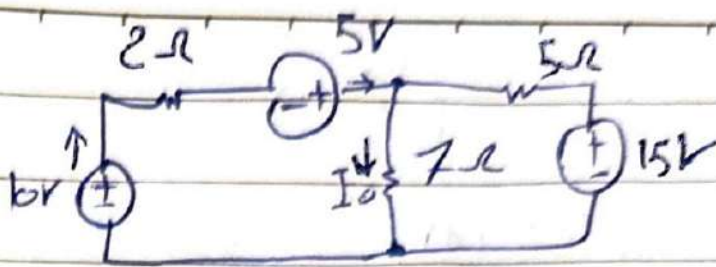
21



$$I_x = \frac{2(2)}{10} = 0.4A$$

$$V_o = I_x (8) = 0.4(8) = 3.2V$$

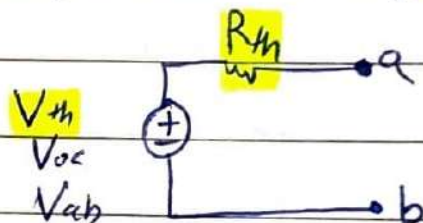




$$I_0 = \frac{10.5 \times \left(\frac{10}{4}\right) A}{7 + 10/4}$$

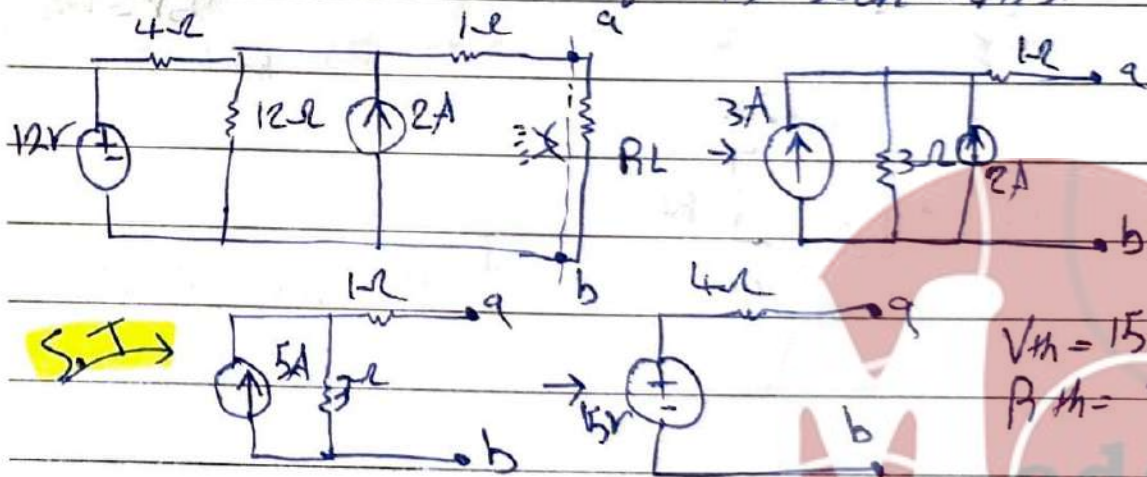
* Thevenin's Theorem.

any network can be reduced to



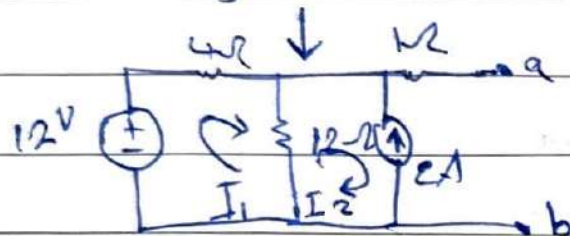
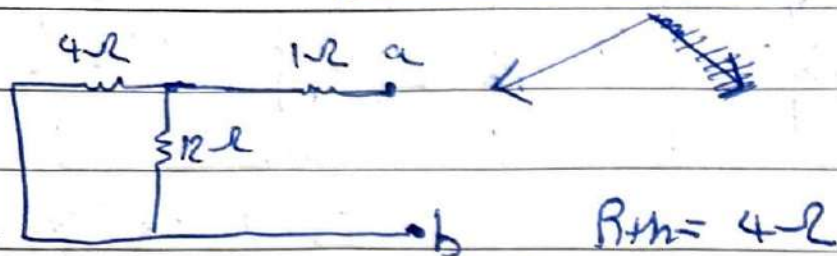
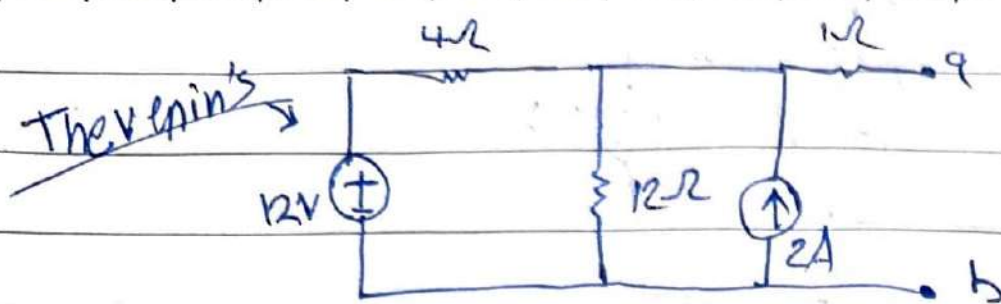
Thevenin's eqn.

Ex: Draw thevenin's eq. as seen (a,b).



$$V_{th} = 15V$$

$$R_{th} = 4\Omega$$



$$I_2 = 2A$$

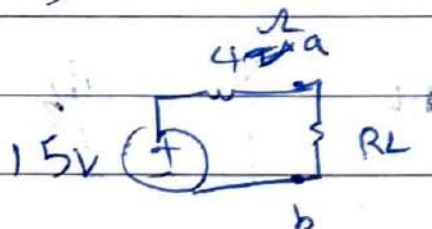
$$-12 + 4I_1 + 12(I_1 - I_2) = 0$$

$$-12 + 16I_1 + 24 = 0$$

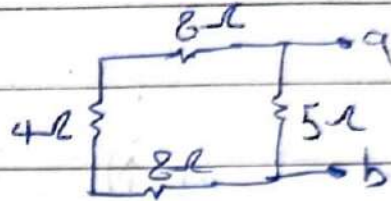
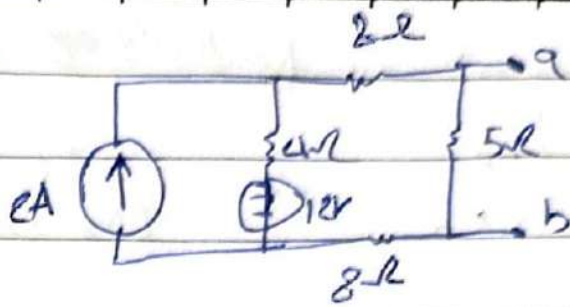
$$I_1 = \frac{-12}{16} = -0.75A$$

$$V_{ab} = V_{1\Omega} + V_{4\Omega} + 12 = 0 + (-)(-0.75)(4) + 12 = 15V = V_{th}$$

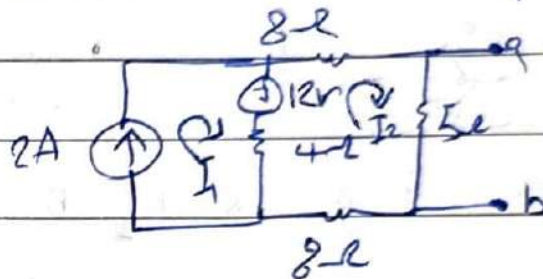
load



Ex.



$$R_{th} = (8 + 4) \parallel 5 = 4\Omega$$

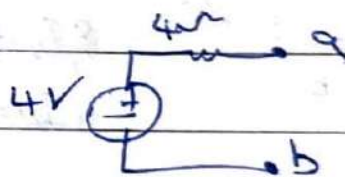


$$V_b = V_{5\Omega} = 5I_2$$

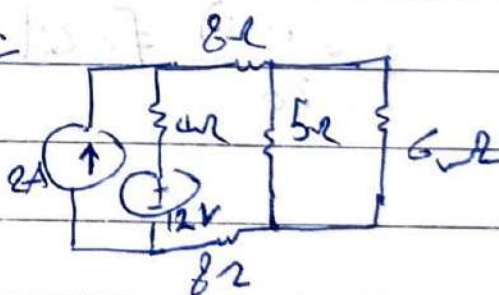
$$8I_2 + 5I_2 + 8I_2 + 4\left(\frac{I_2}{2}\right)12 = 0$$

$$25I_2 = 12 + 8 \Rightarrow I_2 = \frac{20}{25} = 0.8A$$

$$\therefore V_{oc} = V_{5\Omega} = V_{th} = V_{ab} = 0.8(5) = \boxed{4V}$$



Ex.



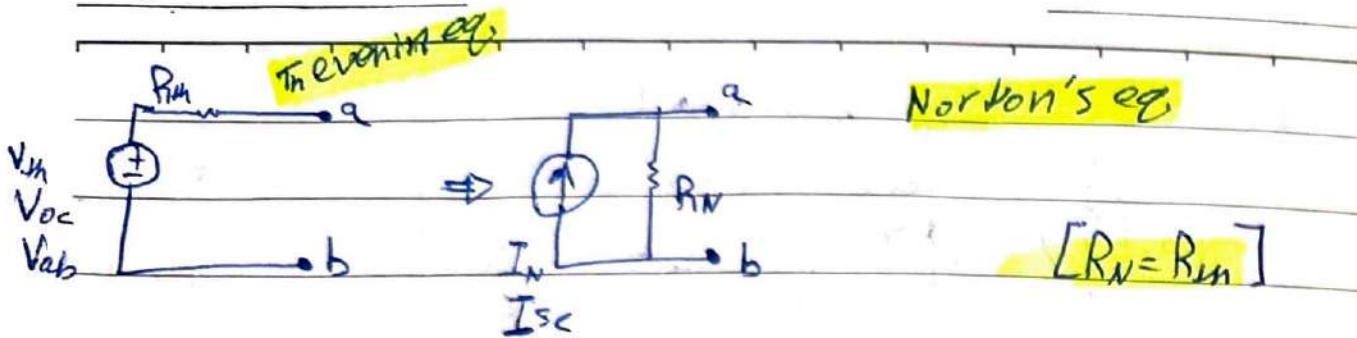
Find I in 4Ω ??

$$R_{th} = 18.7\Omega$$

$$V_{ab} = V_{th} = 25.5V$$

25.

$$I = (25.5 / 22.7) A$$



Ex.

1) cut the load

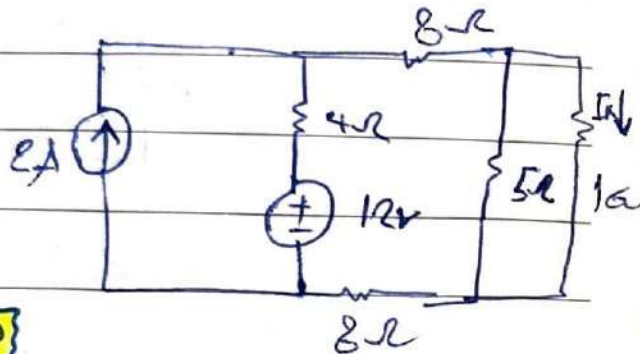
2) kill all sources

3) find R_N

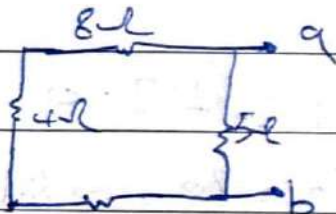
4) restore all sources

5) find I_N (99b ~~circuits~~)

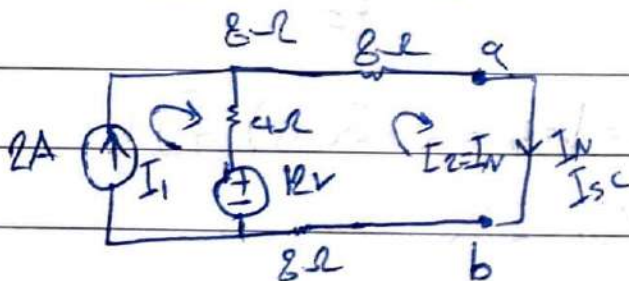
6) Draw norton's eq. find I_x by norton's th.



sol:



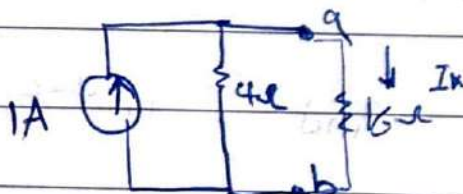
$$R_N = (4 + 8 + 8) \parallel 5 = 4\Omega$$



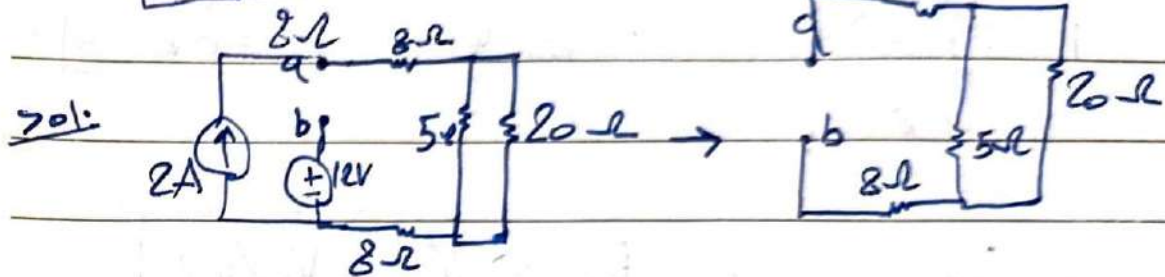
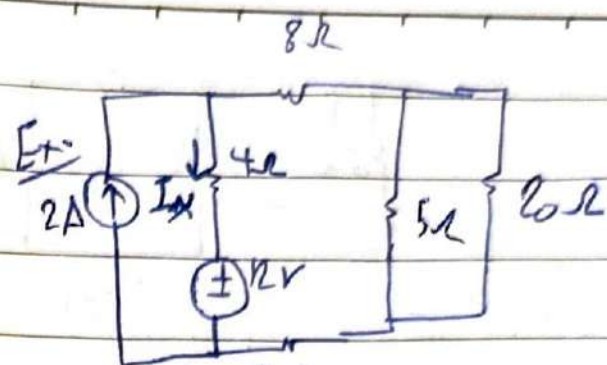
$$I_1 = 2A$$

$$8I_N + 8I_N - 12 + 4(I_N - 2) = 0$$

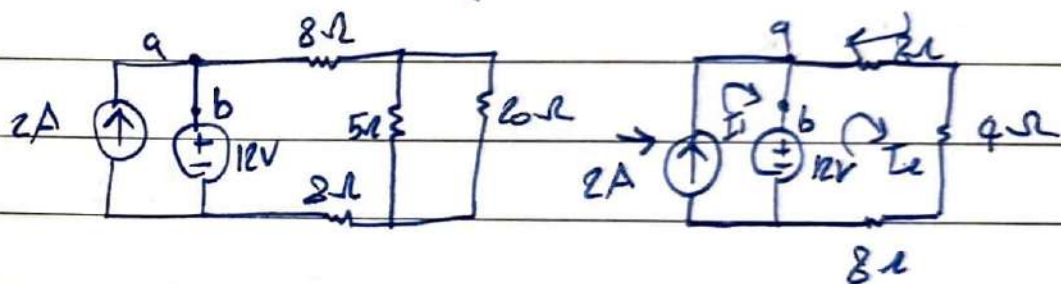
$$I_N = 1A$$



$$I_x = \frac{1(4)}{20} = 0.2A$$



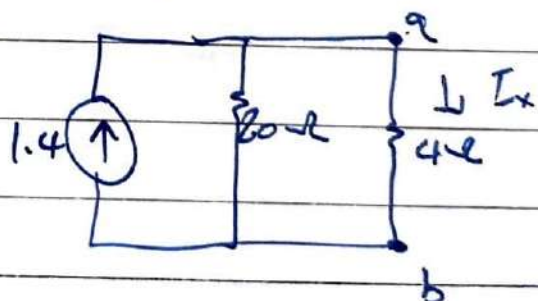
$$R_N = (20 \parallel 5) + 8 + 8 = 20\Omega$$



$$I_1 = 2A, -12V + 8I_2 + 4I_2 + 8I_2 = 0$$

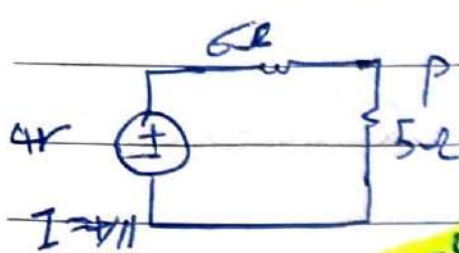
$$20I_2 = 12 \Rightarrow I_2 = 0.6A$$

$$I_{N_x} = I_1 - I_2 = 2 - 0.6 = 1.4A$$



$$I_x = \frac{(1.4)(20)}{24} A$$

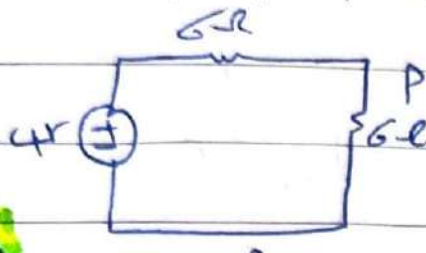
• Max. power at $R_{th} = R_L$



$$P = I^2 R$$

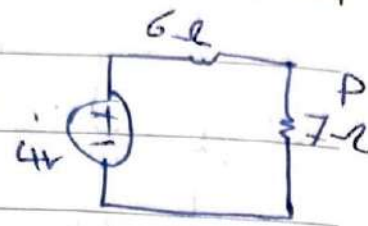
$$P = \left(\frac{4}{11}\right)^2 (5) =$$

$$\boxed{0.668 \text{ W}}$$



$$P = \left(\frac{4}{12}\right)^2 (6)$$

$$\boxed{= 0.666 \text{ W}}$$



$$P = \left(\frac{4}{13}\right)^2 (7)$$

$$\boxed{P = 0.662 \text{ W}}$$

$$* P_{max} = I^2 R_{th} = \frac{V_{th}^2}{4R_{th}^2} \cdot R_{th} \Rightarrow \boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

$$I = V_{th} / 2R_{th}$$



* إذا كان mesh لا يحتوي على current source لا KVL

(إذا كانت المقاومة متساوية) $R_y = \frac{R}{3}$

* Energy storage elements:
(loads)

Capacitor , Inductor 

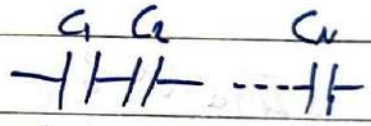
* Capacitor  Two plates $\pm$

capacity C (farad) F
MF, nF
PF, mF

$$C = \epsilon_0 \frac{A}{d}, \quad \epsilon_0 : \text{permittivity}$$

A: Area of the plate

d: distance between two plates.

series { 
 $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$


 $C_T = C_1 + C_2 + \dots + C_N$

charge stored in the capacitor : $q = C \cdot v$

$$q = C \cdot v$$

$$\frac{dq}{dt} = C \frac{dv}{dt}$$

$$\Rightarrow I_c = C \frac{dv_c}{dt}$$

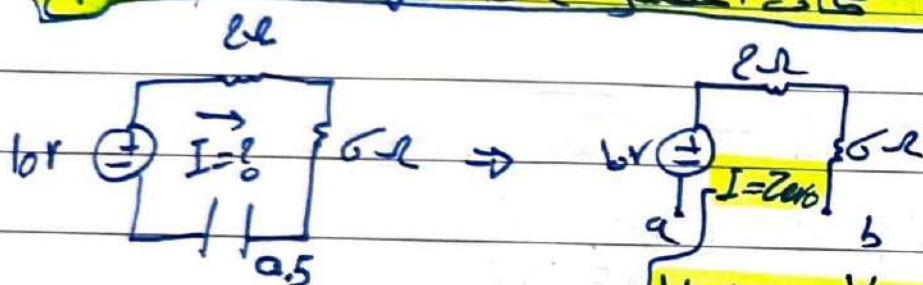


$$\Rightarrow V_c = \frac{1}{C} \int I_c(t) dt + V_0$$

* The energy stored in the capacitor :

$$W = E = \frac{1}{2} C V^2 \text{ (J)}$$

* D.C capacitor $\Rightarrow$ D.C voltage



* Ex 1: $V = 20V$

$C = 3 \mu F$

$\Rightarrow Q = C \cdot V = 60 \mu C$

$$\begin{cases} V_{ab} = V_c = 10V \\ E = \frac{1}{2} C V^2 = 25 \mu J \\ Q = C \cdot V = 5 \mu C \end{cases}$$

* Ex 2: $C = 5 \mu F$

Find i_c ?

sol: $V_c(t) = 10 \cos(6000t)$

$i_c = C \frac{dv}{dt} = 5 \times 10^{-6} (\sin(6000t) \times -10 \times 6000)$

* Ex 3: $C = 2 \mu F$

$i(t) = 6e^{-3000t} \text{ mA}$

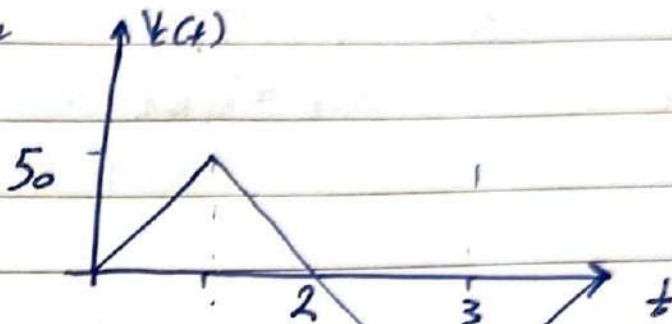
$V_0 = 0$

V_c ?

sol: $V_c = \frac{1}{C} \int I_c(t) dt + V_0 = \frac{1}{2 \times 10^{-6}} \int 6e^{-3000t} dt + 0$

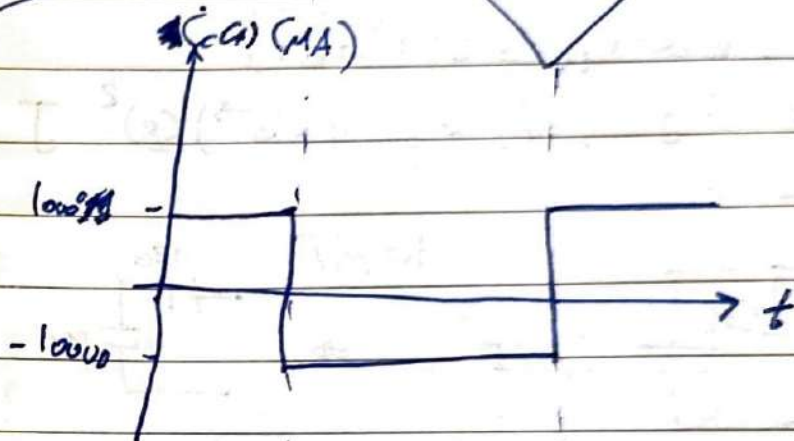
$$= \frac{6}{2 \times 10^{-6}} \left(\frac{e^{-3000t}}{-3000} \right) + 0$$

* Ex 1



Sketch the current wave form?
 $C = 200 \mu F$

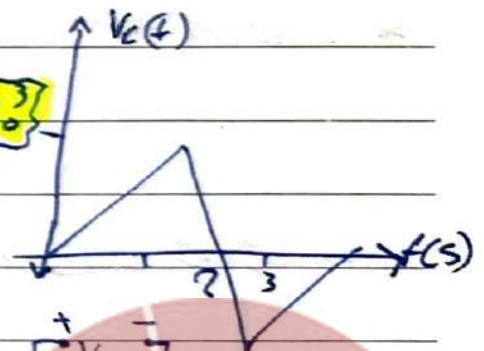
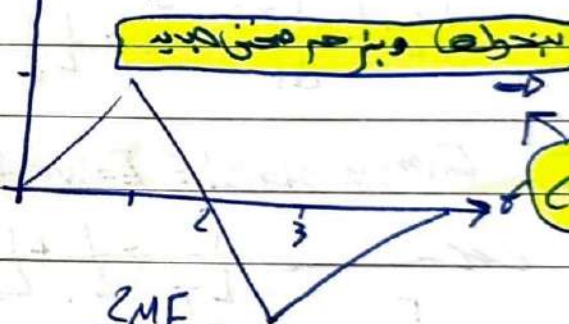
sol:



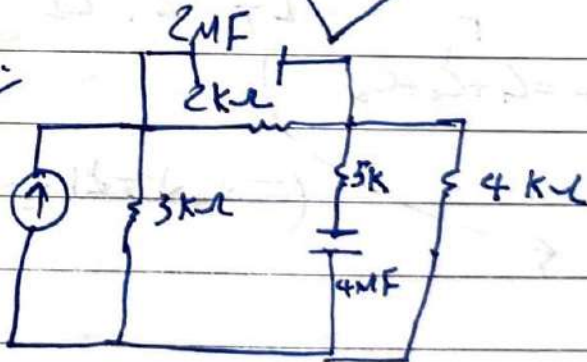
$$E_{f=1} = \frac{1}{2} C V^2 = 0.7 \text{ J}, \quad E_{f=1} = (50 \text{ V})^2 \cdot \frac{1}{2} \cdot 200 \times 10^{-6}$$

* Ex 5

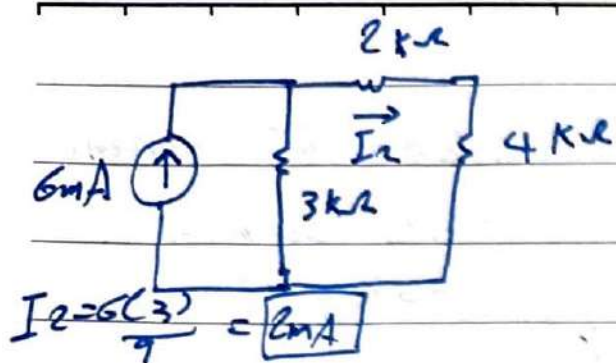
$V_c(t)$



* Ex 6



ask :: believe & recieve



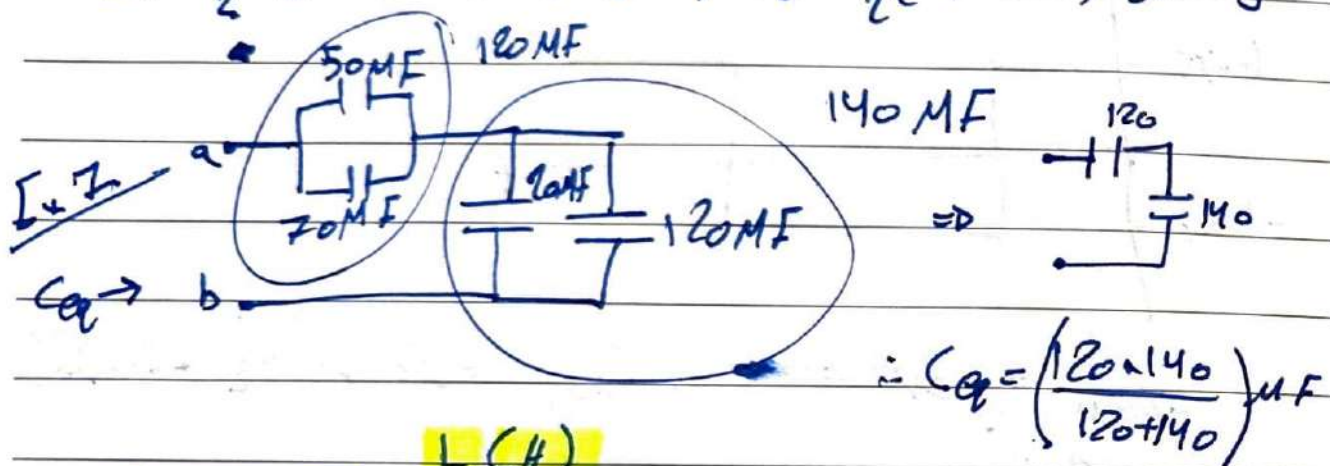
$$V_{C1} = V_{2\text{k}\Omega}$$

$$V_{C2} = V_{4\text{k}\Omega}$$

$$I_2 = \frac{6(3)}{9} = 2\text{mA}$$

$$V_{2\text{k}\Omega} = V_{C1} = 4\text{V}, \quad V_{4\text{k}\Omega} = V_{C2} = 2(4) = 8\text{V}$$

$$E_1 = \frac{1}{2} (2 \times 10^{-3} \times 4)^2 \text{ J}, \quad E_2 = \frac{1}{2} (4 \times 10^{-3}) (8)^2 \text{ J}$$



$$C_{eq} = \left(\frac{120 \times 140}{120 + 140} \right) \mu\text{F}$$

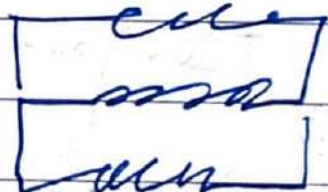
L (#)

* Inductor $V_L = L \frac{dI}{dt}$, $I_L = \frac{1}{L} \int V_L(t) dt + I_0$

Energy stored: $E = W = \frac{1}{2} L I^2$

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$L_T = L_1 + L_2 + L_3$$



(رسي الحلقه)

* The inductor act as a short CKT with D.C power supply.

* Ex1: $i(t) = 10t e^{-5t}$, $L = 0.1 \text{ H}$, V_{30} , $E = \frac{1}{2} \int_{t=0}^{\infty} i^2 dt$

$$V_L(t) = L \frac{di}{dt} = 0.1 [10 e^{-5t} - 50t e^{-5t}]$$

$$E = \frac{1}{2} L I^2 = \frac{1}{2} (0.1) [10(5) \cdot 0^{-25}] \int$$

Ex2: $L = 5 \text{ H}$, $V(t) = \begin{cases} 30t^2 & t > 0 \\ 0 & t < 0 \end{cases}$
 $I_0 = \text{Zero}$

$$i(t) = \frac{1}{L} \int_0^t V_L(t) dt + I_0$$

$$= \frac{1}{5} \int_0^t 30 \lambda^2 d\lambda = 6 \left[\frac{\lambda^3}{3} \right]_0^t = 2t^3$$

$$\int_0^t 2t^3 \quad t > 0, \quad \text{at } t = 5 = 2(125) = 250 \text{ A}$$

$$W|_{t=5} = \frac{1}{2} L I^2 = \frac{1}{2} (5) (250)^2 \int$$

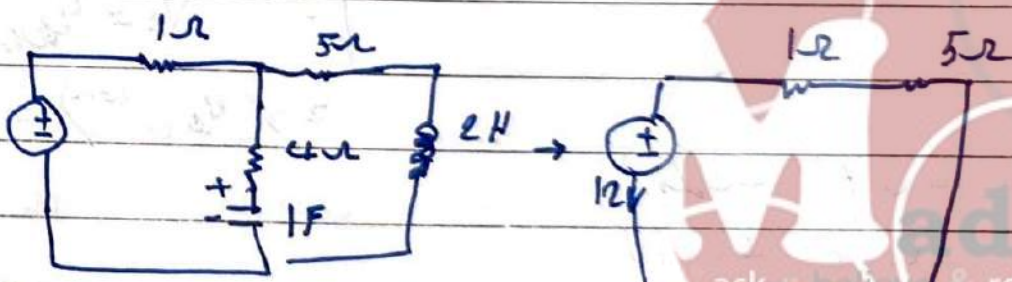
Ex3:

PP

12V

30

30

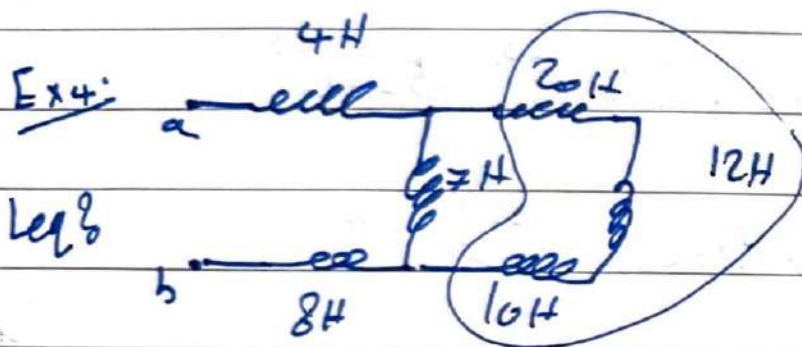


$$I = I_L = \frac{12V}{6\Omega} = 2A, \quad E_{ind} = \frac{1}{2} LI^2$$

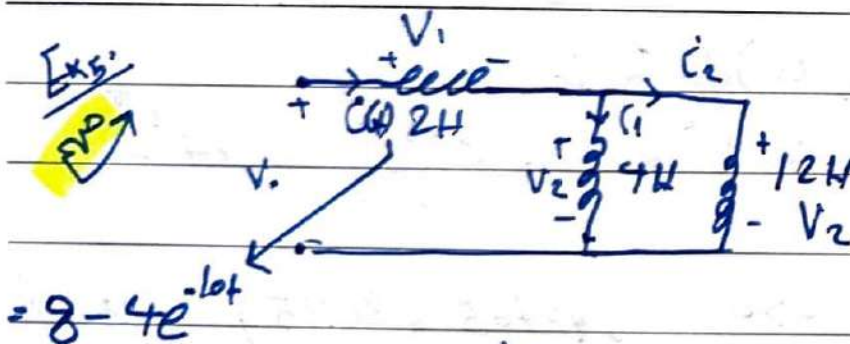
$$= \frac{1}{2} (2)(2)^2 = 4J$$

$$V_C = V_{5\Omega} = I(5) = \boxed{10V}$$

$$\therefore E_C = \frac{1}{2} CV^2 = \frac{1}{2} \times 1 \times 100 = \boxed{50J}$$

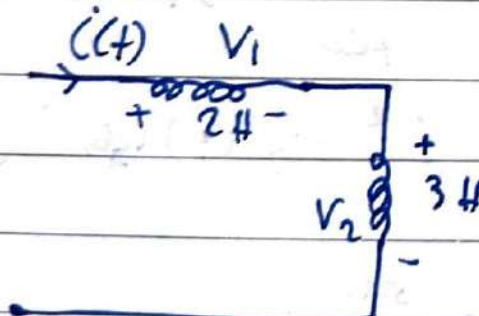


$$L_{eq} = (4 \parallel 7) + 4 + 8 = \boxed{18H}$$



$$= 8 - 4e^{-10t}$$

$$i_2(0) = -1mA$$



$$V_1 = 2 \frac{di_1}{dt} = 8e^{-10t}$$

$$V_2 = L \frac{di_2}{dt}$$

$$= 3(40e^{-10t}) = 120e^{-10t}$$

$$i(t) = i_1(t) + i_2(t)$$

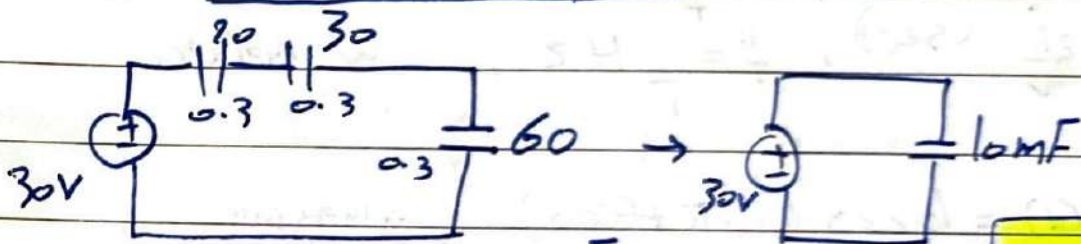
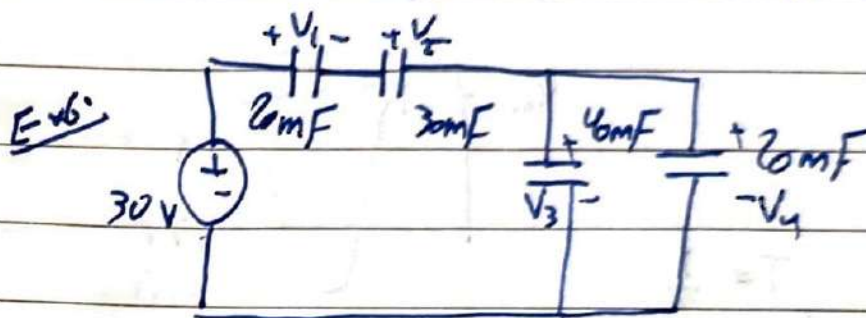
$$i(0) = i_1(0) + i_2(0)$$

$$4 = i_1(0) + (-1)$$

$$\boxed{i_1(0) = 5mA}$$

$$\boxed{34}$$

$$\begin{aligned}
 i(4) &= \frac{1}{4} \int V_2(t) dt + i(0) = \frac{1}{4} \int_0^t 120 e^{-10K} dK + 5 \\
 &= \frac{30}{-10} e^{-10K} \Big|_0^t + 5 = \left[\frac{300}{-10} e^{-10t} - \frac{300}{-10} \right] + 5 \\
 &= 3 - 30 e^{-10t} + 5 = 8 - 30 e^{-10t}
 \end{aligned}$$



$$Q = q = C \cdot V = 30 \times 10 \times 10^{-3} = 0.3 \text{ C}$$

حالت اول
 $Q = Q_1 + Q_2 + Q_3$
 $Q = 20\mu\text{F} \cdot 30\text{V} + 30\mu\text{F} \cdot 30\text{V} + 40\mu\text{F} \cdot 30\text{V}$

$V_3 = V_4$

$$V_1 = \frac{Q}{C_1} = \frac{0.3}{20 \times 10^{-3}} \text{ V}, \quad V_2 = \frac{Q}{C_2} = \frac{0.3}{30 \times 10^{-3}} \text{ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}, \quad Q_4 = 40 \times 10^{-3} \times 5 = 0.2 \text{ C}$$

$$Q_6 = 0.1 \text{ C}$$

برای اعداد مختلف و مقادیر مختلف و مقادیر مختلف و مقادیر مختلف

*CH 9. Sinusoidal & phasor

cos sin

$$V(t) = V_m \cos[\omega t + \phi_1]$$

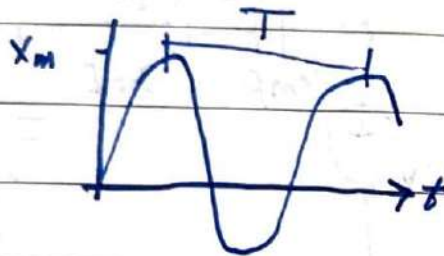
$$i(t) = I_m \sin[\omega t + \phi_2]$$

$$x(t) = X_m \cos(\omega t + \phi)$$

peak value

radial freq. (omega)

or max-value



$$\omega = \frac{2\pi}{T}$$

$$T = \frac{1}{f}$$

$$T = \frac{2\pi}{\omega} \text{ (sec)}$$

$$f = \frac{1}{T} \text{ Hz}$$

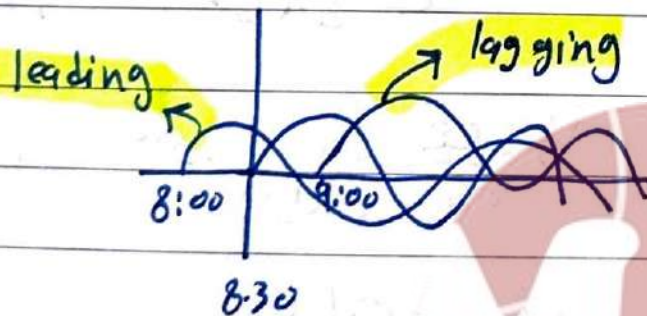
ω rad/sec

Ex: $i(t) = 10 \cos(100\pi t - 60^\circ)$ $\therefore$ lagging

sol:

$$I_{\max} = 10, \omega = 100\pi, f = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

$$T = \frac{1}{50} \text{ sec}, \phi: \text{phase shift} = -60^\circ \text{ \&}$$



* علاقات افلاك بين 2-signals

1) ω 2) $\sin$ or $\cos$

تحويل / تحويل
(من $\sin$ الى $\cos$ و العكس)
بقيت

* $\sin(\omega t - 30^\circ) \Rightarrow \cos(\omega t - 30^\circ - 90^\circ) = \cos(\omega t - 120^\circ)$

* $-\cos(x) = \cos(x \pm 180^\circ)$

Ex: $V_1(t) = -10 \cos(\omega t + 50^\circ)$

$V_2(t) = 12 \sin(\omega t - 10^\circ)$

sol: $V_1(t) = 10 \cos(\omega t - 130^\circ)$

$V_2(t) = 12 \cos(\omega t - 100^\circ)$

$\therefore V_2$ leads V_1 by 30°

or V_1 lags by 30°

Phasor * $V(t) = V_m \cos(\omega t + \theta)$

$\vec{V} = V_m \angle \theta$ (phasor polar) (vector)

Ex: $i(t) = 5 \cos(\omega t - 30^\circ)$

sol: $\vec{I} = 5 \angle -30^\circ \text{ A}$

Ex: $i(t) = 6 \sin(20\pi t + 16^\circ) \Rightarrow 6 \cos(20\pi t - 74^\circ)$

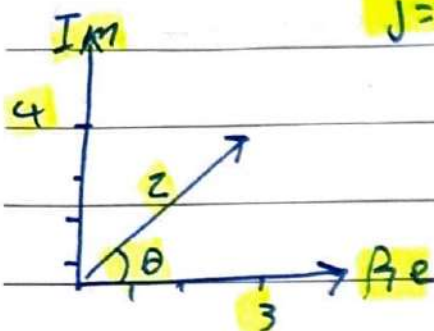
sol: $\vec{I} = 6 \angle -74^\circ \text{ A}$



* complex (Phasor / Polar)

$$a + jb \rightarrow \begin{matrix} \text{cartizi} \\ 3 + j4 \end{matrix} = \begin{matrix} \text{Polar, Phasor} \\ 5 \angle 53.13^\circ \end{matrix}$$

$$j = \sqrt{-1}$$



$$z = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1} \frac{a}{b}$$

* Sin vsoidal $\rightarrow$ $\cos$, $V(t) = 60 \times (20\pi t - 60^\circ) \Rightarrow$
 $V = 6 \angle -60$

no * $V_1(t) = 6 \cos(20t + 30^\circ)$

$$V_2(t) = 4 \sin(20t + 50^\circ)$$

$$\text{find } V_4(t) = V_1(t) + V_2(t)$$

$$V_3(t) = 12 \cos(20t)$$

$$+ V_3(t) + V_4(t)$$

sol: $V_1 = 6 \angle 30^\circ$

$$V_2 = 4 \angle -40^\circ$$

$$V_3 = 12 \angle 0$$

دوال و جیبی
Phasor

$$\therefore V_4 = 20 \angle 1.2 \Rightarrow V_4(t) = 20 \cos(20t + 1.2^\circ)$$

* AC ckt loads :

① resistors ② inductor

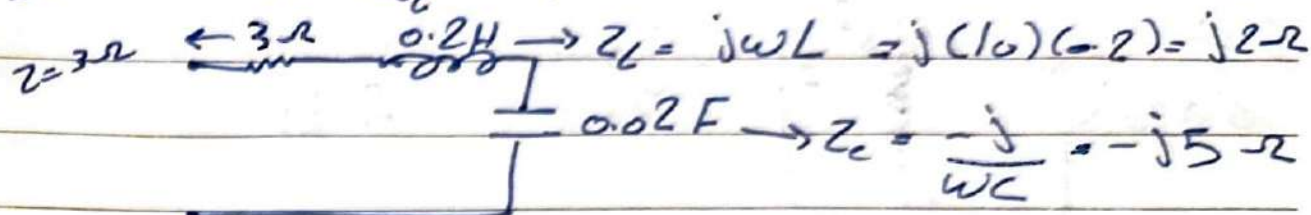
② capacitor

[Impedance Z]

unit: (Ω)

$R \rightarrow Z = R$, $L \rightarrow Z_L = j\omega L$, $C \rightarrow Z_C = -\frac{j}{\omega C}$

Ex: Find Z_T if $\omega = 10 \text{ rad/sec}$.



$$Z_T = 3 + j2 - j5$$

$$= 3 - j3 = 4.24 \angle -45^\circ$$

① $3 - j3$

② OPTN

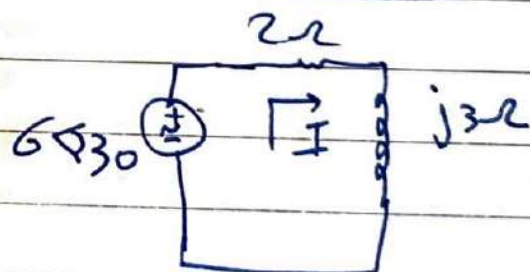
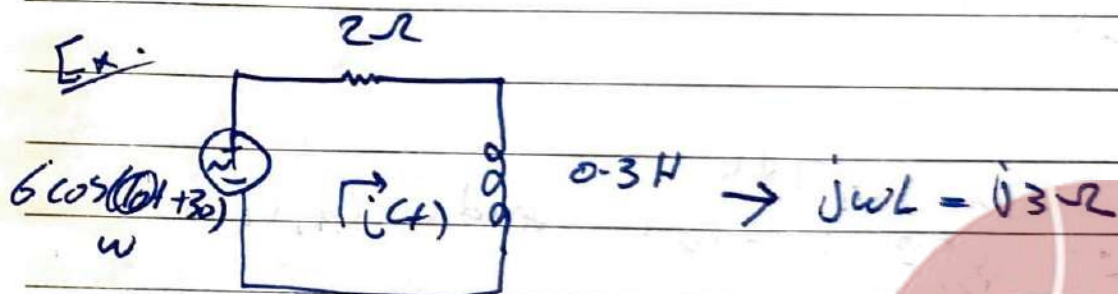
③

④ 1

⑤ =

on calculator

Ex:

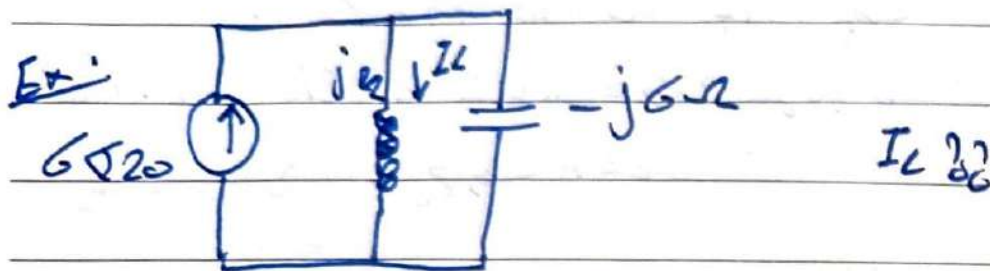


$$I = \frac{V}{Z_T} = \frac{6 \angle 30^\circ}{2 + j3} = 1.66 \angle -26.34^\circ$$

$$i(t) = 1.66 \cos(10t - 26.34^\circ) A$$

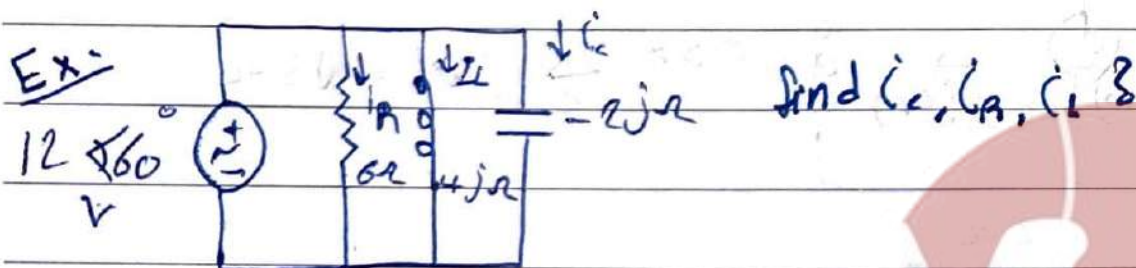
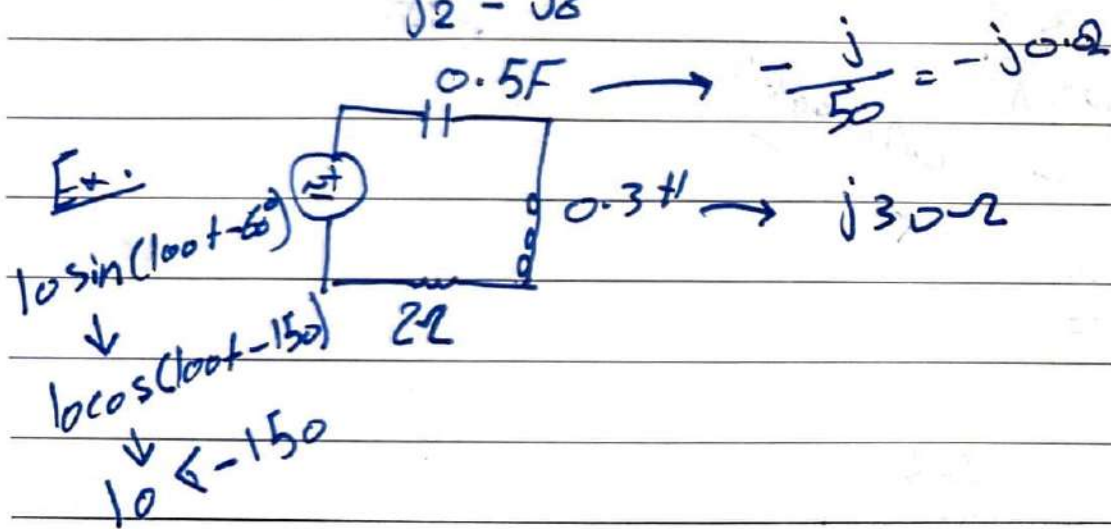
$$V_{Z_L} I \cdot Z_R = (1.66 \angle -26.3^\circ)(2) \quad \checkmark$$

$$V_{j3\Omega} = I Z_L = (1.66 \angle -26.3^\circ)(j3) \quad \checkmark$$



sol:

$$I_L = \frac{[6 \angle 20^\circ] [-j6]}{j2 - j6} = 9 \angle 20^\circ$$



$V_R = I_R \cdot Z_R$ $\Rightarrow I_R = 12 \angle 60 / 6 = 2 \angle 60 \text{ A}$
 $V_L = I_L \cdot Z_L$ $\Rightarrow I_L = 12 \angle 60 / 4j = 3 \angle -30 \text{ A}$
 $V_C = I_C \cdot Z_C$ $\Rightarrow I_C = 12 \angle 60 / -2j = 6 \angle 150 \text{ A}$

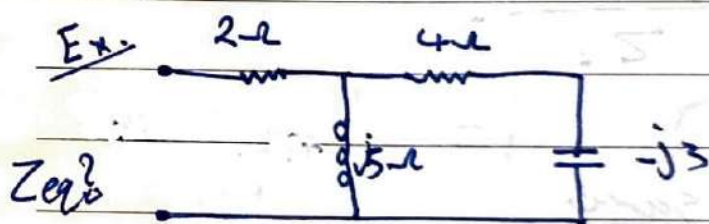
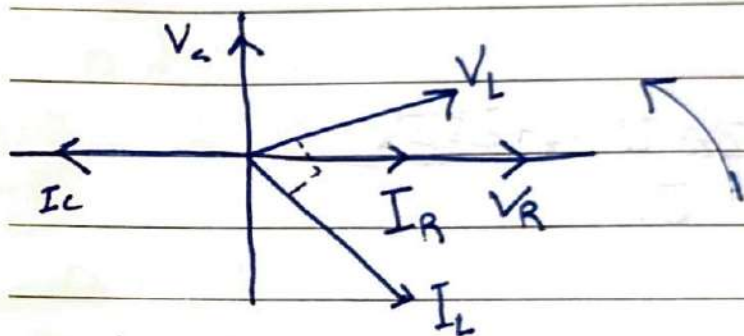
R: V_R, I_R in phase ($\theta_v = \theta_i$)

L: V_L leads I_L by 90°

C: I_C leads V_C by 90°

phase Angle =

$\theta_v - \theta_i$



$$Z_{eq} = [(4 - 3j) \parallel 5j] + 2 = 7.43 \angle 19.65^\circ = 7 + 2.5j$$

* $3 - j2 \Rightarrow 3 \Omega \parallel -j2$

[capacitive]

* $3 + j2 \Rightarrow 3 \Omega \parallel j2$

[inductive]

41

D $\rightarrow$ Y:

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

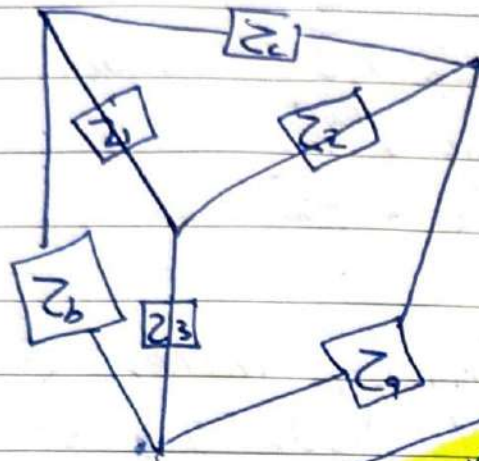
$$Z_2 = \frac{Z_a Z_c}{\text{same.}}$$

$$Z_3 = \frac{Z_a Z_b}{\text{same.}}$$

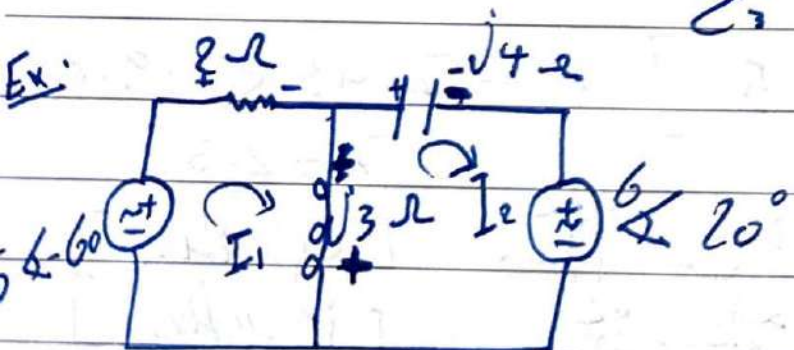
Y $\rightarrow$ D: $Z_a = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}$

$$Z_b = \frac{\text{same.}}{Z_2}$$

$$Z_c = \frac{\text{same.}}{Z_3}$$



same the resistance



Find Z_1, Z_2, Z_3

$$m_1: -5 \angle -60^\circ + 2I_1 + 3j(I_1 - I_2) = 0 \quad \dots \textcircled{1}$$

$$m_2: -4jI_2 + 6 \angle 20^\circ + 3j(I_1 - I_2) = 0$$

← تبدیل

$$\left\{ \begin{array}{l} j3 = 3 \angle 90^\circ \quad \leftarrow \text{ضرب} \quad -3 = 3 \angle 180^\circ \\ -j3 = 3 \angle 90^\circ \quad \leftarrow \text{ضرب} \quad 3 = 3 \angle 0^\circ \end{array} \right\}$$

$$\rightarrow -4jI_2 + 6 \angle 20^\circ + 3jI_2 - 3jI_1 = 0$$

$$-jI_2 - 3jI_1 + 6 \angle 20^\circ = 0 \Rightarrow -3jI_1 - jI_2 = -6 \angle 20^\circ$$

--- \textcircled{2}

$$D = \begin{bmatrix} (2+3j) & -3j \\ -3j & -j \end{bmatrix} = 12.2 \angle -9.5^\circ$$

$$D_1 = \begin{bmatrix} 5 \angle -60^\circ & -3j \\ 6 \angle -160^\circ & -j \end{bmatrix} = 19.5 \angle -24.6^\circ$$

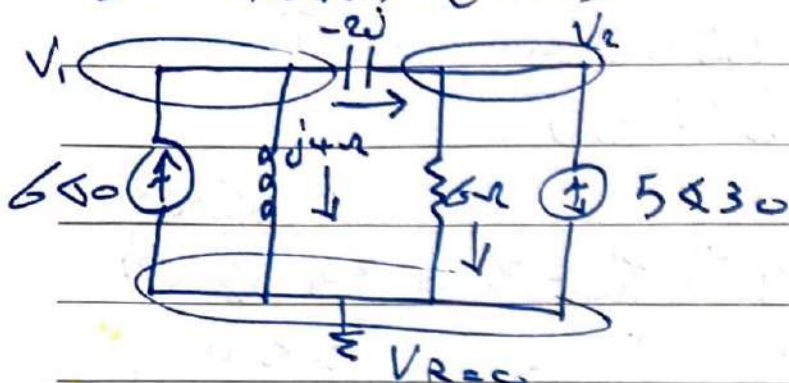
Apply
Cramer
rule

$$D_2 = \begin{bmatrix} 2+3j & 5 \angle -60^\circ \\ -3j & 6 \angle -160^\circ \end{bmatrix} = 15.6 \angle -59.8^\circ$$

$$I_1 = \frac{D_1}{D} = 1.6 \angle -75.1^\circ, \quad I_2 = \frac{D_2}{D} = 1.28 \angle -50.3^\circ$$

$$V_C = [I_2 \cdot (-j4)] \angle, \quad V_L = [(I_1 - I_2) (-j3)] \angle$$

Ex: (Nodal), (KVL)



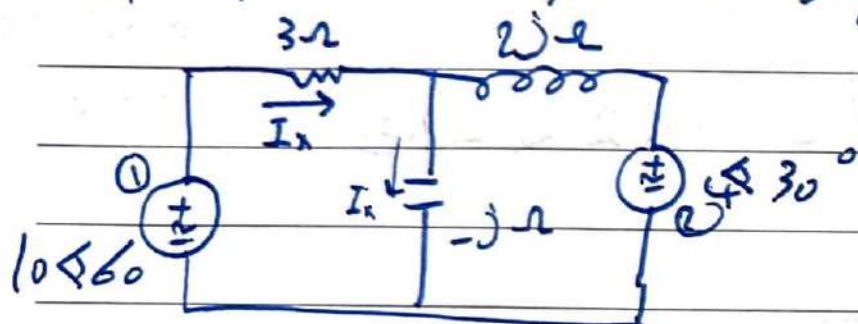
N1:

$$6 \angle 0 = \frac{V_1 - V_2}{-2j} + \frac{V_1}{j4}$$

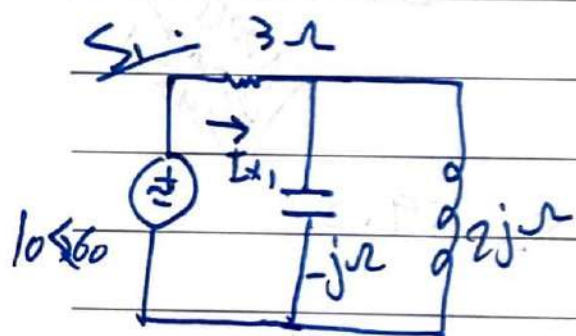
N2:

$$5 \angle 30 + \frac{V_2}{6} = \frac{V_1 - V_2}{-2j}$$

* super position: Find I_x using SP?



SP1:



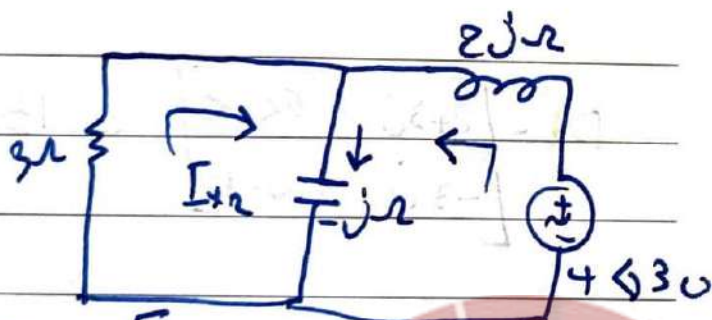
$$Z_{eq} = \left[\frac{-j1 \parallel j2}{1} \right] + 3 \Omega = 3.6533 \Omega$$

$$I_{x1} = \frac{10 \angle 60}{3.6533} = 2.78593 A$$

$$\therefore I_x = I_{x1} + I_{x2} = 2.4 \angle 8.2 A = 2.38 + 0.34j A$$

4.4

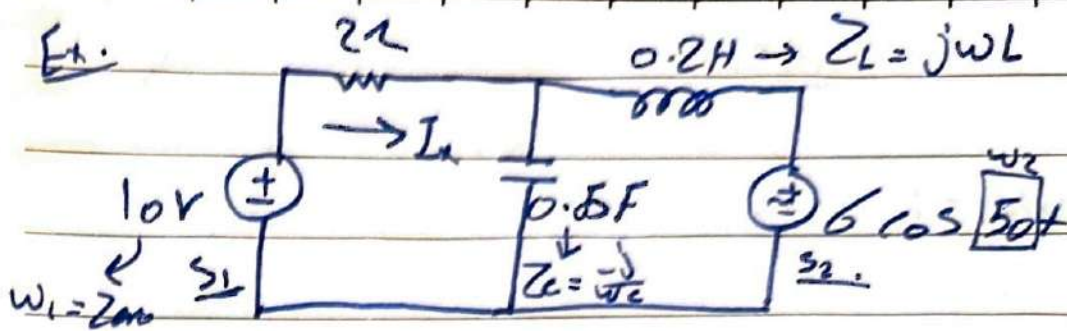
SP2:



$$Z_{eq} = \left[\left(\frac{3 \parallel -j1}{1} \right) + 2j \right] \Omega = 1.14574 \Omega$$

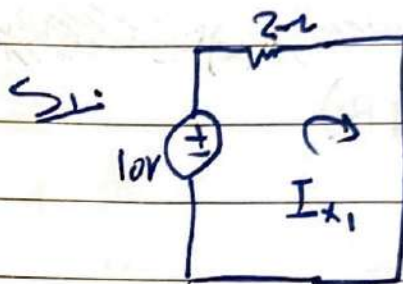
$$I_{x2} = \frac{4 \angle 30}{1.14574} = 3.5 \angle -44.7 A$$

ask : believe & recieve

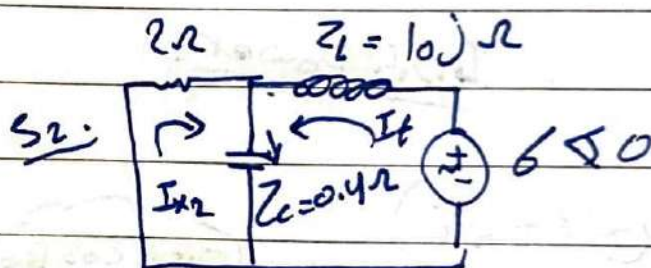


* اذا كانت ω مختلفة $\Rightarrow$ لا يمكن دمجها في س.ب

* ال D.C $\Rightarrow$ العناصر فيها ال ω يساوي Zero



$$I_{x1} = \frac{10}{2} = 5A$$



$$I_{x2} = \frac{I_1 (-0.4j)}{Z_T}$$

$$I_T = \frac{6 \cos 50t}{Z_T}$$



CH II.

EMG

AC Power:

4 types of AC - power:

- ① Instantaneous power, $P(t)$ (W)
- ② Average [mean, real] power, P (W)
- ③ Reactive power, Q (VAR)
- ④ complex power, S (VA)
- ⑤ Apparent power, S (VA)

① $P(t) = i(t) * v(t)$

Inst. Power

let $i(t) = I_m \cos(\omega t + \theta_i)$

$v(t) = V_m \cos(\omega t + \theta_v)$

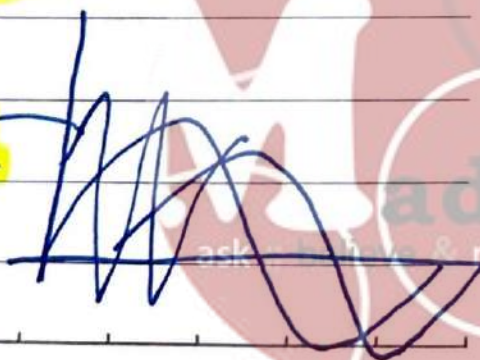
$\cos A \cos B = \frac{1}{2} \cos(A+B) + \frac{1}{2} \cos(A-B)$

$\therefore P(t) = i(t) * v(t) = I_m \cos(\omega t + \theta_i) * V_m \cos(\omega t + \theta_v)$

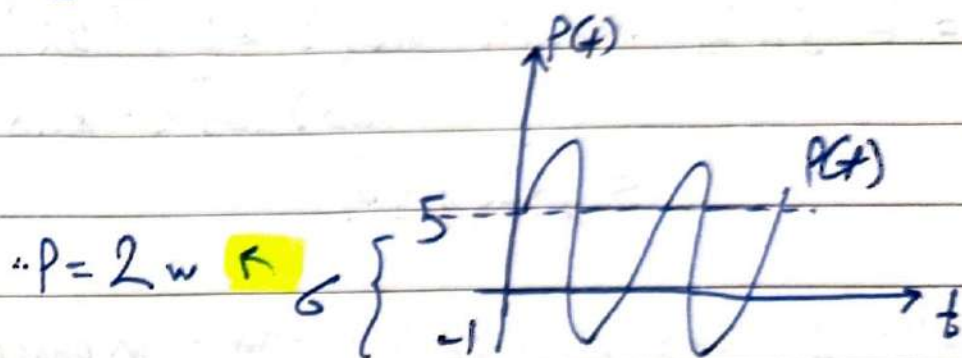
$I_m V_m \left[\frac{1}{2} \cos(\omega t + \theta_i + \theta_v) + \frac{1}{2} \cos(\omega t + \theta_i - \theta_v) \right]$

constant (Avg. Power)

Inst. Power



$$\therefore P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) (\omega)$$



$$* \vec{v}(t) = 120 \cos[377t + 45^\circ] \text{ V}$$

$$I(t) = 10 \cos[377t - 10^\circ]$$

find $P(t)$, P_{avg}

sol: ① $P_{avg} = \frac{1}{2} [120] [10] \cos[45 - (-10)] = 344.2 \text{ W}$

② $P(t) = \frac{1}{2} [120] [10] \cos(754t + 35^\circ) + 344.2$

* $I = I_m \angle \theta_i$

$V = V_m \angle \theta_v$

$\rightarrow P = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i)$

$P = \frac{1}{2} \vec{V} \vec{I}^*$ → conjugate

$I = 5 \angle 60^\circ \rightarrow I^* = 5 \angle -60^\circ$

$I = 3 - j4 \rightarrow I^* = 3 + j4$

Ex: $Z = 30 - j70 \Omega$

$V = 120 \angle 0^\circ$

find $P(t)$

$$\text{sol: } I = \frac{V}{Z} = \frac{120 \angle 0}{30 - j70} = 1.58 \angle 66.8$$

$$\therefore P = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i) = \frac{1}{2} (1.58) (120) \cos(0 - 66.8)$$

$$= \boxed{37.24 \text{ W}}$$

$$\begin{aligned} * P &= \frac{1}{2} |V_m| |I_m| \cos(\theta_v - \theta_i) \\ Q &= \frac{1}{2} |V_m| |I_m| \sin(\theta_v - \theta_i) \end{aligned}$$

$\text{---} \text{---} \text{---}$: in phase $\theta_v = \theta_i$
 $\text{---} \text{---} \text{---}$: I lead V by 90° ($\theta_v - \theta_i = -90^\circ$)
 $\text{---} \text{---} \text{---}$: V lead I by 90° ($\theta_v - \theta_i = +90^\circ$)

$$Q_R = 0$$

$$P_C = 0$$

$$Q_C = \text{---} (-)$$

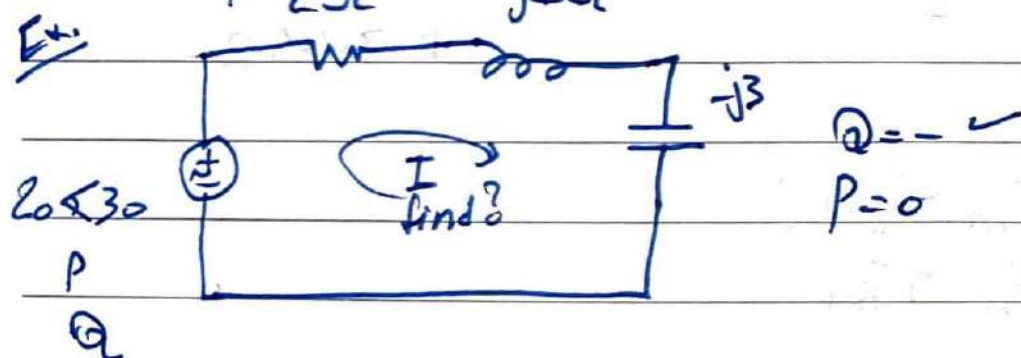
$$P_C = 0$$

$$P_R \text{---}$$

$$Q_L = \text{---} (+)$$

$$P = 0$$

$$Q = + \text{---}$$



$$I = \frac{20 \angle 30}{2 + j4 - j3} = 8.9 \angle 3.43 \text{ A}$$

$$P_R = \frac{1}{2} I_m \bar{V}_m = \frac{1}{2} I_m^2 R = \frac{1}{2} (8.9)^2 (2) = \underline{79.6W}$$

$$P_s = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} (20)(8.9) \cos(30 - 3.43) = \underline{79.6W}$$

$$Q_s = \frac{1}{2} (20)(8.9) \sin(30 - 3.43) = \underline{39.8VAR}$$

$$Q_L = \frac{1}{2} |I_L|^2 |Z_L| = \frac{1}{2} (8.9)^2 (4) = \underline{158.42 VAR}$$

$$Q_C = \frac{1}{2} |I_C|^2 |Z_C| = \frac{1}{2} (8.9)^2 (3) = \underline{-118.8VAR}$$

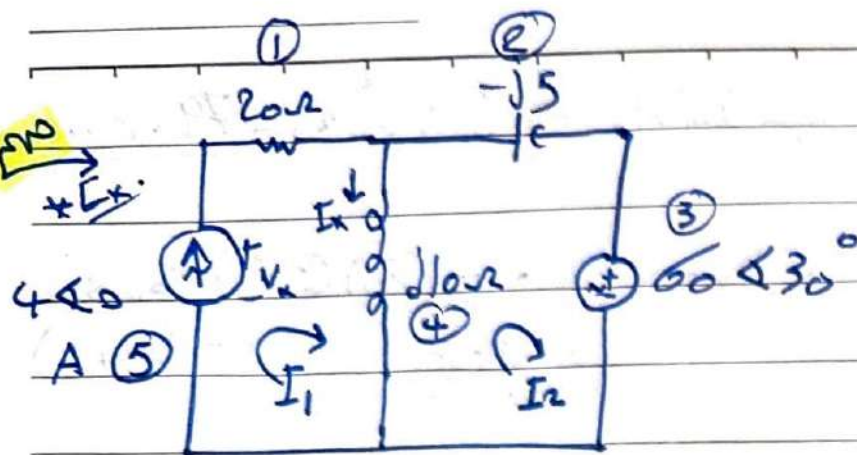
$$\frac{1}{2} |I_m| |V_m| \cos(\theta_v - \theta_i)$$

$$Q = \frac{1}{2} |I_m| |V_m| \sin(\theta_v - \theta_i)$$

* Conclusion:

	P	Q
R	✓	X
L	X	+Q
C	X	-Q





$$I_1 = 4 \angle 0^\circ \text{ A}$$

$$-j5 I_2 + 60 \angle 30^\circ + j10 [I_2 - I_1] = 0$$

$$P_1 = \frac{1}{2} |I_1|^2 R = \frac{1}{2} (4)^2 (20) = 160 \text{ W}$$

$$Q_1 = 0$$

$$P_2 = 0$$

$$Q_2 = \frac{1}{2} |I_2| |Z_c| = -\left(\frac{1}{2}\right) (10.58)^2 (5) = -279.8 \text{ VAR}$$

$$P_3 = \frac{1}{2} |I_2| |V| \cos (\theta_v - \theta_i)$$

$$= \frac{1}{2} (10.58) (60) \cos [30 - 79.1] = 207.8 \text{ W}$$

$$Q_3 = \frac{1}{2} |I_2| |V_s| \sin (30 - 79.1) = -239.9 \text{ VAR}$$

$$P_4 = 0$$

$$Q_4 = \frac{1}{2} (I_r)^2 |Z_l| = \frac{1}{2} (10.58)^2 (10) = 559.08 \text{ VAR}$$

$$= DI_x = 4 \angle 0 - 10.52 \angle 79.1 = 10.58 \angle -79.1^\circ A$$

$$-V_x + 20 I_1 + I_x (j10) = 0 \quad V_x \checkmark$$

$$V_x = 184.98 \angle 6.2^\circ$$

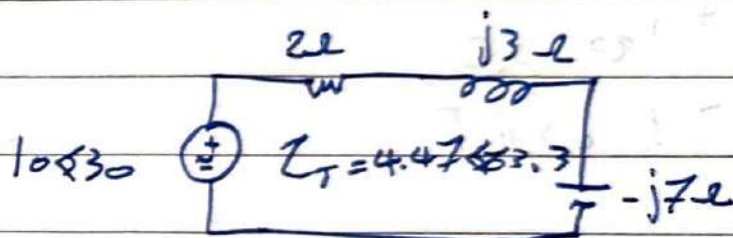
$$\therefore P_s = \frac{1}{2} |I_1| |V_x| \cos(\theta_v - \theta_i) = 367.8 \text{ W}$$

$$Q_s = \frac{1}{2} |I_1| |V_x| \sin(\theta_v - \theta_i) = 40 \text{ VAR}$$

$$* \vec{S} = P + jQ, R: \quad, L: \quad, C: \quad$$

* Ex.

$$P_R = \frac{1}{2} |2.23|^2 (2) = 4.97 \text{ W}$$



$$P_L = 0, P_C = 0, P_s = \frac{1}{2} |10| |2.23| \cos |30 - 93.4| = 4.99 \text{ W}$$

$$Q_s = \frac{1}{2} |2.23| |10| \sin [30 - 93.4] = -j 9.96 \text{ VAR}$$

$$P_L = 0, P_C = 0$$

$$Q = \frac{1}{2} [2.23]^2 \times 3 = 7.48 \text{ VAR}$$

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$$Q_c = \frac{1}{2} (2.23)^2 (-7) = -17.4 \text{ VAR}$$

$$S_L = 0 + j7.46 \text{ VAR}$$

$$S_C = -j17.4 \text{ VAR}$$

$$S_R = P + jQ = 5$$

$$S_S = 5 - j9.96 \text{ VAR}$$

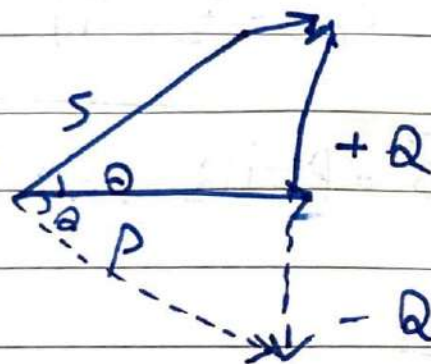
* power triangle :

$$\cos \theta = \text{Power factor (P.F.)}$$

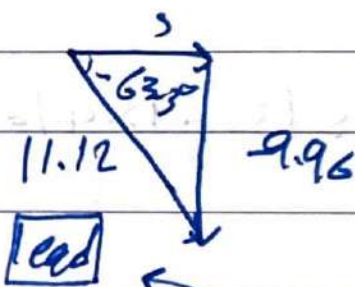
$$0 \leq \text{P.F.} \leq 1$$

$$Q^+ \text{ lag P.F.}$$

$$Q^- \text{ leads P.F.}$$



$$S_S = 5 - j9.96 = 11.12 \angle -63.3^\circ$$



$$\text{P.F.} = \cos \theta = 0.449$$

* To correct the P.F. we have to decrease the (Q) and keep (P) as it is,

[So we have to add a capacitor in parallel with the voltage source]

$$*Z_{eq} = S_S$$

* Root mean squared value (RMS):

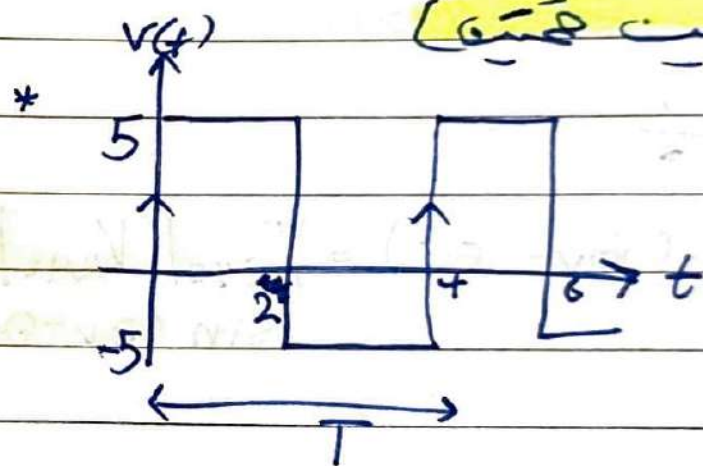
$$X_{\max} = \sqrt{\frac{1}{T} \int x^2(t) dt}, \quad x(t) < \begin{matrix} v(t) \\ i(t) \end{matrix}$$

* special case: $v(t) = V_m \cos(\omega t + \theta)$

$$V_{\text{rms}} = V_m / \sqrt{2}$$

Ex: $i(t) = 6 \sin(2\pi t)$, $I_m = 6$, $I_{\text{rms}} = \frac{6}{\sqrt{2}} = 4.24$

* T : الزمن لإتمام دورة واحدة (period time)
(الحساب = بيت قتيبة)



$$V_{\text{rms}} = \sqrt{\frac{1}{4} \left[\int_0^2 (5)^2 dt + \int_2^4 (-5)^2 dt \right]}$$

$$= \sqrt{\frac{1}{4} \left[25t \Big|_0^2 + 25t \Big|_2^4 \right]} = \sqrt{\frac{1}{4} [50 + 50]}$$

$$= \sqrt{25} = \boxed{5V}$$

$$P = \frac{1}{2} |V_m| |I_m| \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \sqrt{2} V_{rms} \cdot \sqrt{2} I_{rms} \cos(\theta_v - \theta_i)$$

$$= V_{rms} \cdot I_{rms} \cos(\theta_v - \theta_i)$$

* Thevenin's equivalent in AC:

← Analyse

$$\begin{aligned} * [V_m, I_m] &\rightarrow P = \frac{1}{2} |I_m| |V_m| \cos(\theta_v - \theta_i) \\ * [V_{rms}, I_{rms}] &\rightarrow P = |I_{rms}| |V_{rms}| \cos(\theta_v - \theta_i) \end{aligned}$$

$$* \text{Sinusoidal} \rightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$* Q = \frac{1}{2} |I_m| |V_m| \sin(\theta_v - \theta_i) = |I_{rms}| |V_{rms}| \sin(\theta_v - \theta_i)$$

$$* \vec{S} = \frac{1}{2} V \vec{I}^*$$

* In DC:

$$R_L = R_{th}$$

to have
 P_{max}
↔

* in AC:

$$Z_L = Z_{th}^*$$

$$P_{max} = \frac{V_{th}^2}{4 R_{th}}$$

$$P_{max} = \frac{|V_{th}|^2}{8 R_{th}}$$

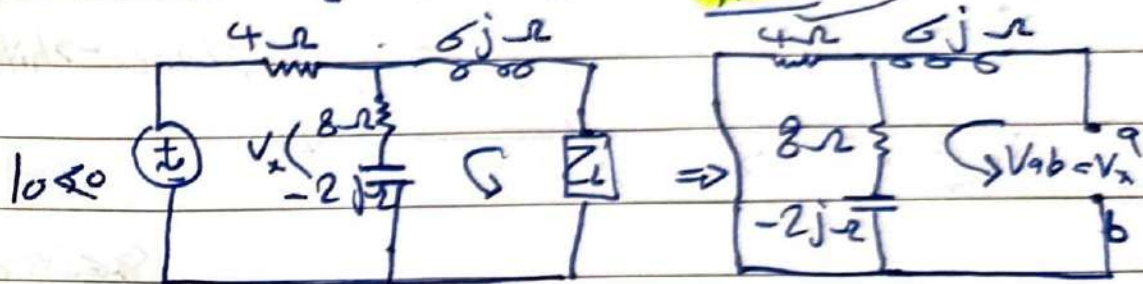
* Ex. if $Z = 6 \angle 30^\circ$, $\omega = 10$, find R, L ?

sol.

$$Z = 5.2 + j3 \therefore R = 5.2$$

$$\omega L = 3 \Rightarrow L = \frac{3}{\omega}$$

* Ex. find Z_L to have P_{max} Thevenin (Circuit):

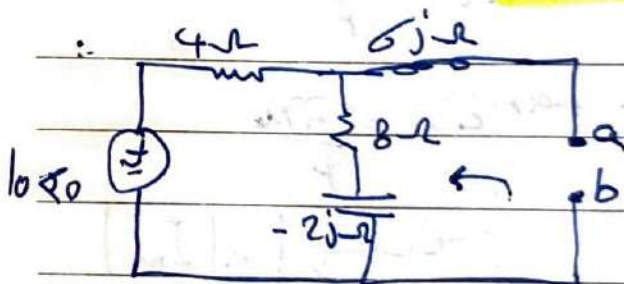


(Kartesian) الجواب بال (Cartesian)

$$Z_{eq} = 2.7 + 5.7j$$

R_{th}

$$\therefore Z_L = Z_{th}^* = 2.7 - 5.7j$$



$$V_{th} = V_{ab} = V_x = \frac{(10 \angle 0^\circ)(8 - 2j)}{4 + 8 - 2j} = 6.7 \angle -4.5^\circ$$

$$\therefore P_{max} = \frac{|V_{th}|^2}{8 R_{th}} = \frac{|6.7 \angle -4.5^\circ|^2}{8(2.7)} \text{ W}$$

(10-12 marks
maximal)

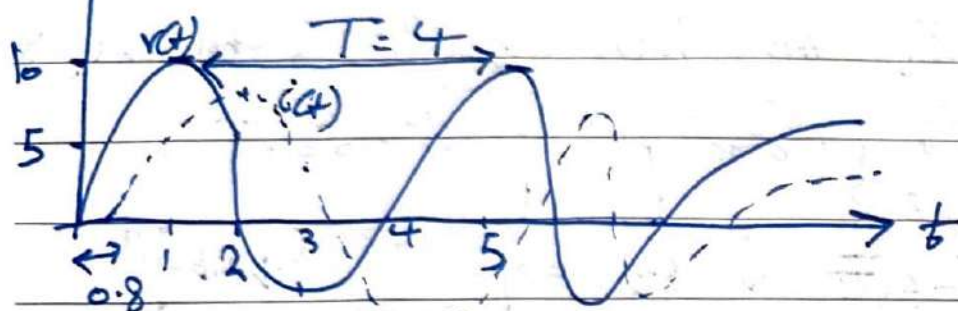
* how to calculate phase shift??

→

$$v(t) = V_m \sin(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$v(t) = 10 \sin\left(\frac{\pi}{2} t\right)$$



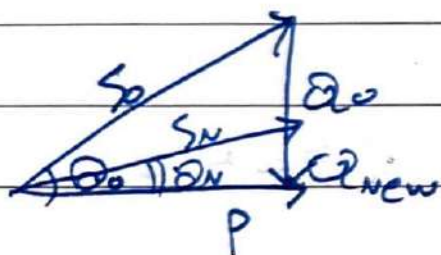
phase-shift

$$i(t) = 5 \sin\left(\frac{\pi}{2} t - 72^\circ\right)$$

(360/0.8) → T = 72

(angle = 72)

* 5



$$\cos \theta = P.F., 0 \leq P.F. \leq 1$$

$$\tan \theta_0 = \frac{Q_0}{P}$$

$$\tan \theta_N = \frac{Q_N}{P}$$

$$Q_c = Q_0 - Q_N$$

$$Q_c = P \tan \theta_0 - P \tan \theta_N$$

$$= P [\tan \theta_0 - \tan \theta_N]$$

$$Q_c = \frac{1}{2} |V_m| |I_m|$$

$$= \frac{1}{2} \frac{|V_m|^2}{|Z_c|}$$

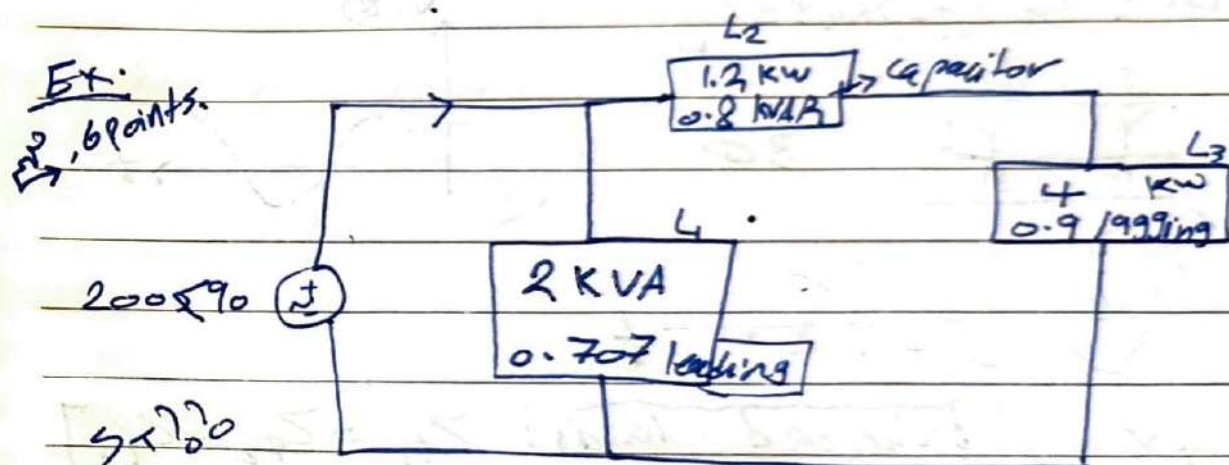
$$= \frac{1}{2} \frac{|V_m|^2}{\omega C}$$

ask & recieve

$$\therefore C = \frac{P [\tan \theta_0 - \tan \theta_N]}{\omega V_{rms}^2}$$

Ex: $V_{rms} = 120 \text{ V}$, $f = 60 \text{ Hz}$, $P = 4 \text{ kW}$
 $P.F. = 0.8$ lags , $P.F_N = 0.95$, $C \text{ ?}$

sol: $C = \frac{P [\tan \theta_0 - \tan \theta_N]}{\omega V_{rms}^2}$
 $= \frac{4000 [\tan(36.8^\circ) - \tan(18.5^\circ)]}{(2\pi(60)) (120)^2}$
 $= 305 \times 10^{-6} \text{ F} = \underline{305 \mu\text{F}}$



$$S_T = S_{L1} + S_{L2} + S_{L3} \Rightarrow$$

$$S_{L1} = 2000 \angle -45^\circ \text{ VA}$$

$$S_{L2} = 1200 - j800 \text{ VA}$$

$$S_{L3} = 4444.4 \angle -25.8^\circ \text{ VA}$$

$$S_T = S_{L1} + S_{L2} + S_{L3} = \underline{57} \text{ VA}$$

6621.5
 $\angle -12.5^\circ \text{ VA}$

$$S = \frac{P}{\cos \theta}$$

$$= \frac{4000}{0.9}$$

$$= 4444.44$$

$$S = \frac{1}{2} \vec{V} \vec{I}^* \Rightarrow 21.5 \angle -2.4 = \frac{1}{2} (200 \angle 90) \vec{I}^*$$

$$\therefore \vec{I}^* = 66.2 \angle -92.4^\circ$$

$$\therefore \vec{I} = 66.2 \angle 92.4^\circ$$

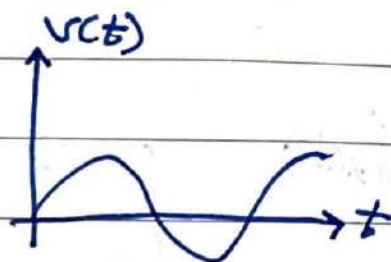
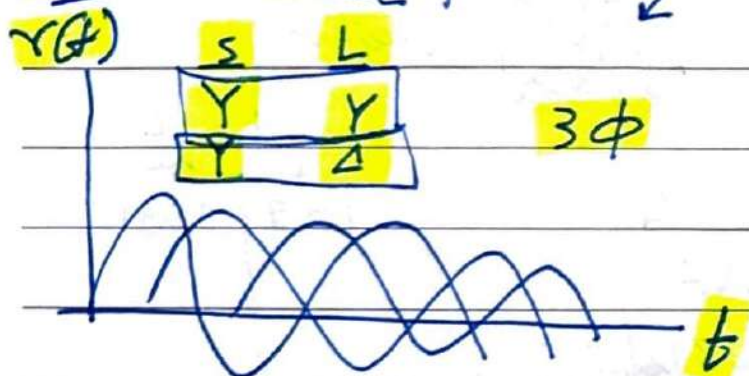
* 11.27, 11.12, 11.63

لحساب

التيار

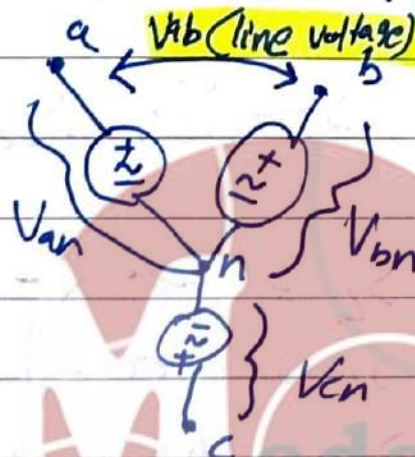
* P.F for system $\Rightarrow Z_{Tot}$ or S_{Tot}

* CH 12: Three-phase:



* Y $\rightarrow$ Y [Balanced load: $Z_{\phi_1} = Z_{\phi_2} = Z_{\phi_3}$]

+ve: $|V_{an}| = |V_{bn}| = |V_{cn}|$



Phase Voltage

$$V_{an} = V \angle 0^\circ$$

$$V_{bn} = V \angle -120^\circ$$

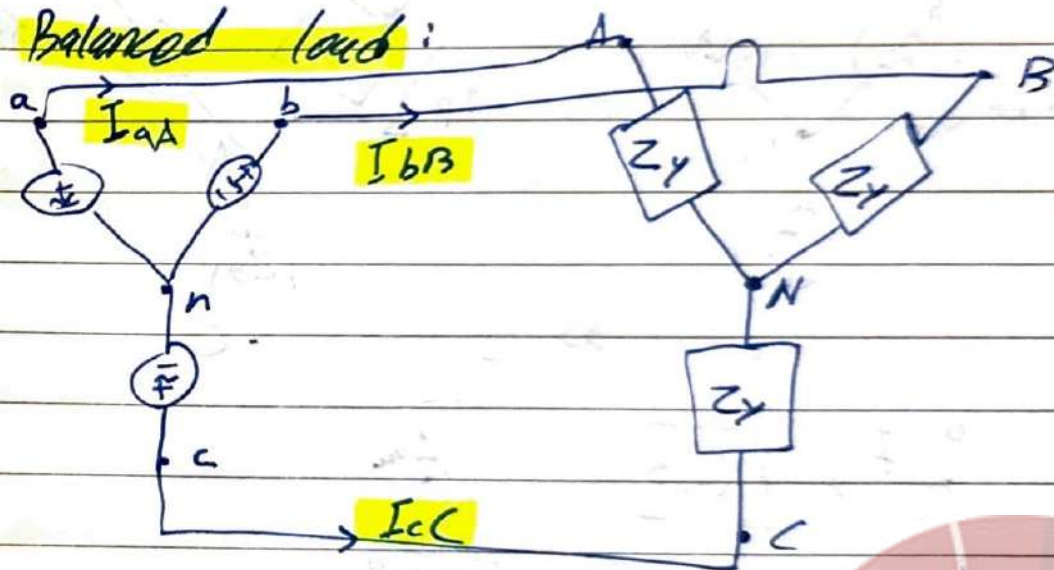
$$V_{cn} = V \angle -240^\circ$$

$-Ve$: $V_{an} = V \angle 0^\circ$
 $V_{bn} = V \angle +120^\circ$
 $V_{cn} = V \angle +240^\circ$

$V_{ab} = V_{an} - V_{bn} = \sqrt{3} |V_{an}| \angle +30^\circ$

Ex: $V_{an} = 5 \angle 60^\circ \Rightarrow V_{ab} = (\sqrt{3})(5) \angle 90^\circ$
 $V_{bn} = 5 \angle -60^\circ \Rightarrow V_{bc} = (\sqrt{3})(5) \angle -30^\circ$
 $V_{cn} = 5 \angle -180^\circ \Rightarrow V_{ca} = (\sqrt{3})(5) \angle 150^\circ$
 line voltage

* **Balanced load:**

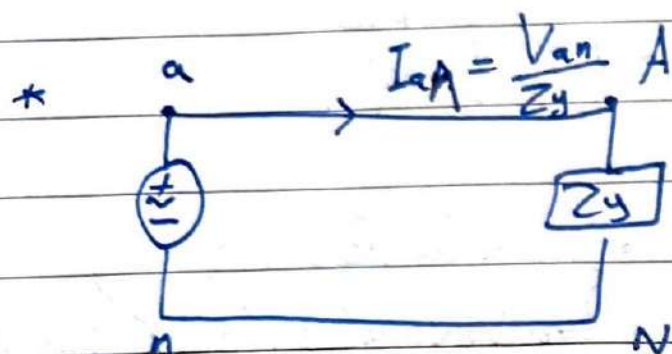


* **Y-Y:** $I_L = I_P$, $V_L \neq V_P$ (إذا كان الحمل متوازن)
 $V_L = \sqrt{3} V_P \angle +30^\circ$

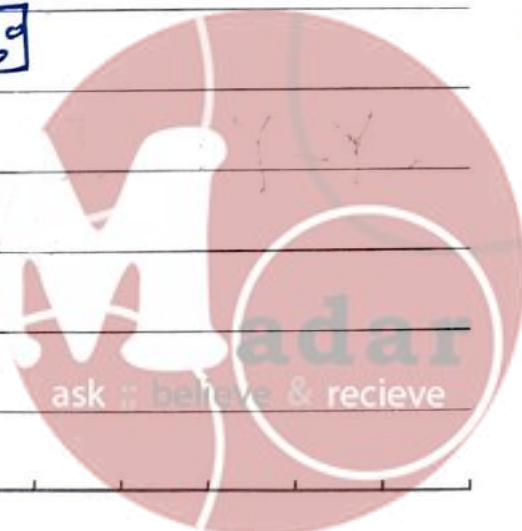
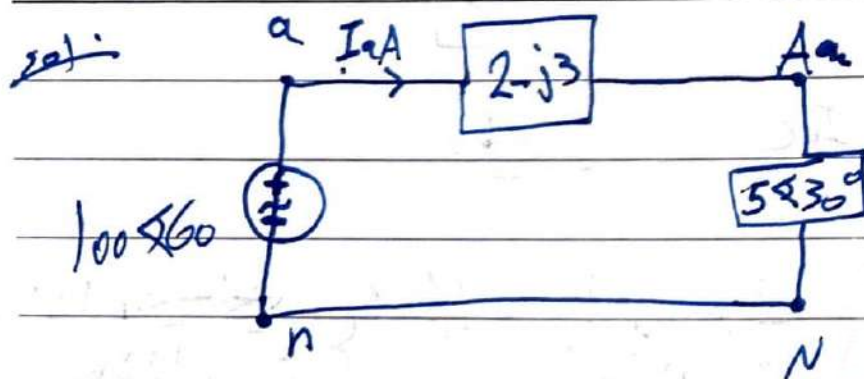
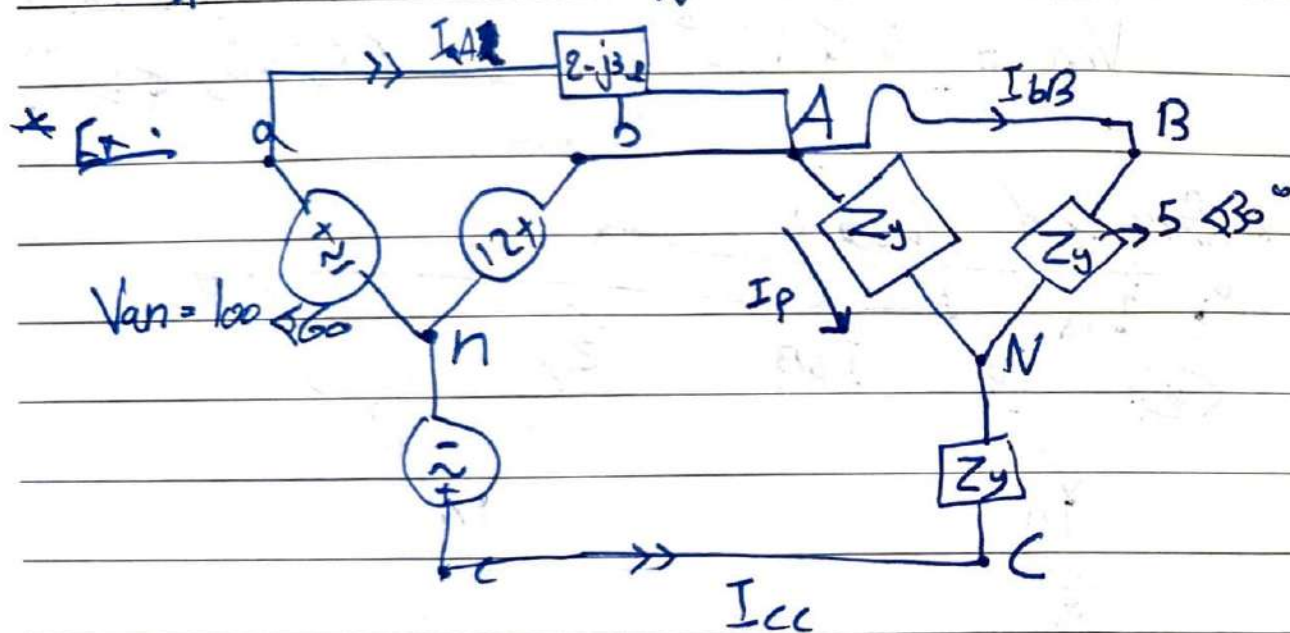
$$I_{aA} = I$$

$$I_{bB} = I \angle -120^\circ$$

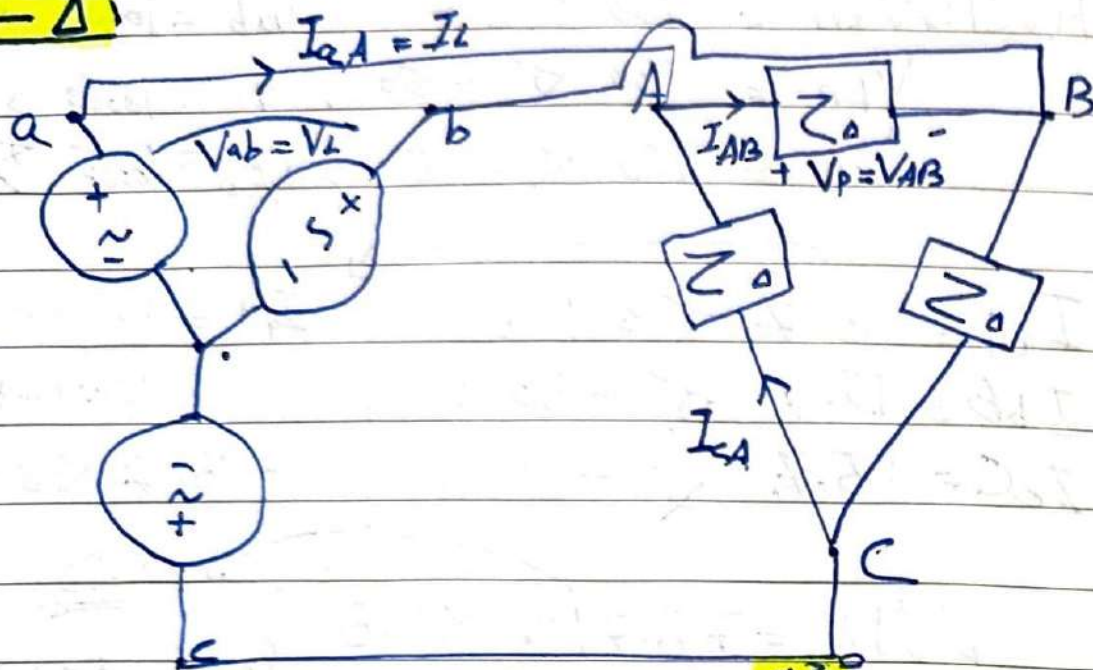
$$I_{cC} = I \angle -240^\circ$$



$\angle 120^\circ$



* Y-Δ



$$Z_Y = \frac{Z_{\Delta}}{3}, \quad I_{LA} = \sqrt{3} I_{AB} \angle +30^\circ$$

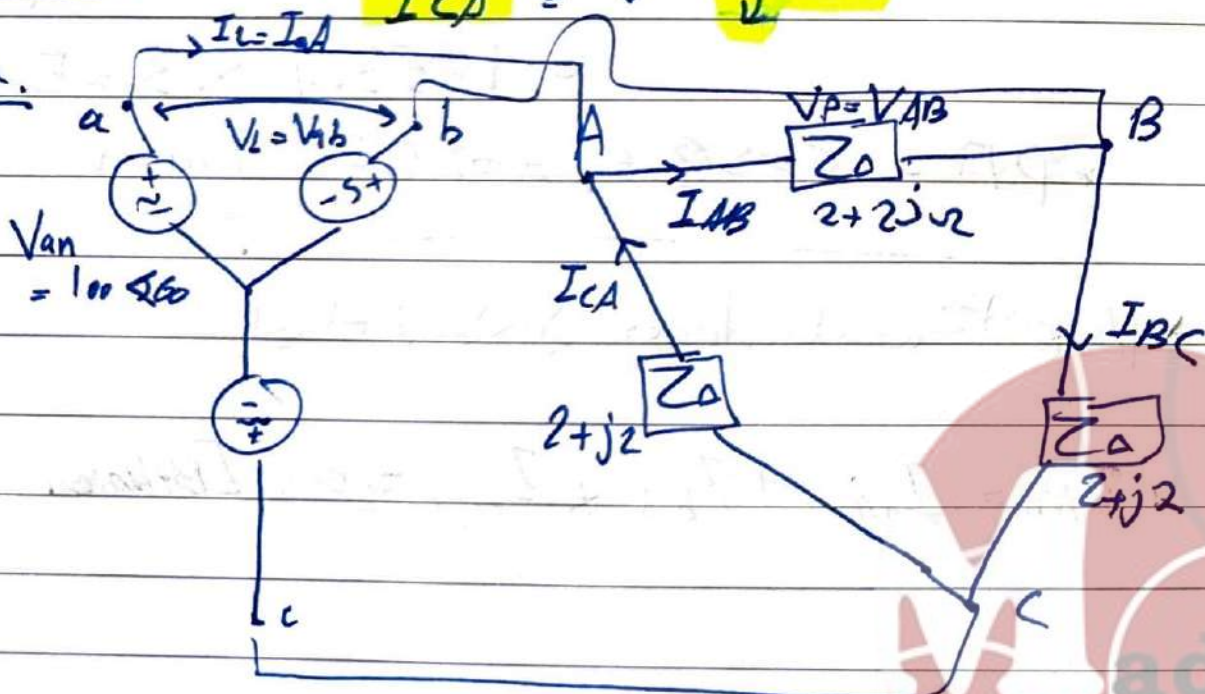
$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$$

$V_L \neq V_P$
 $I_L \neq I_P$

$$I_{BC} = \downarrow \angle -120^\circ$$

$$I_{CA} = \downarrow \angle -120^\circ$$

* Ex.



$$* V_{an} = 100 \angle 60^\circ \text{ V}$$

$$V_{bn} = 100 \angle -60^\circ \text{ V}$$

$$V_{cn} = 100 \angle -180^\circ \text{ V}$$

$$* V_{ab} = V_{AB} = 100\sqrt{3} \angle 90^\circ \text{ V}$$

$$V_{bc} = V_{BC} = 100\sqrt{3} \angle -30^\circ \text{ V}$$

$$V_{ca} = V_{CA} = 100\sqrt{3} \angle -150^\circ \text{ V}$$

$$* I_{AB} = \frac{V_{AB}}{Z_0} = \frac{100\sqrt{3} \angle 90^\circ}{2+2j} = 61.2 \angle 45^\circ \text{ A}$$

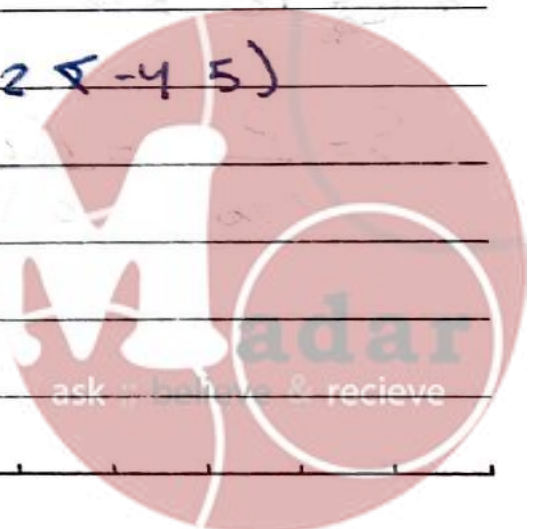
$$\text{phase } \left\{ \begin{array}{l} I_{BC} = 61.2 \angle -75^\circ \text{ A} \\ I_{CA} = 61.2 \angle -195^\circ \text{ A} \end{array} \right.$$

$$* I_{aA} = \sqrt{3}(61.2) \angle 15^\circ$$

$$I_{bB} = 106 \angle -105^\circ$$

$$I_{cC} = 106 \angle -225^\circ$$

$$\begin{aligned} * S_L &= 3 * \frac{1}{2} * V_{AB}^* I_{AB}^* \\ &= \frac{3}{2} * (100\sqrt{3} \angle 90^\circ)(61.2 \angle -45^\circ) \\ &= 15900.2 \angle 45^\circ \text{ VA} \end{aligned}$$



E_{ph}

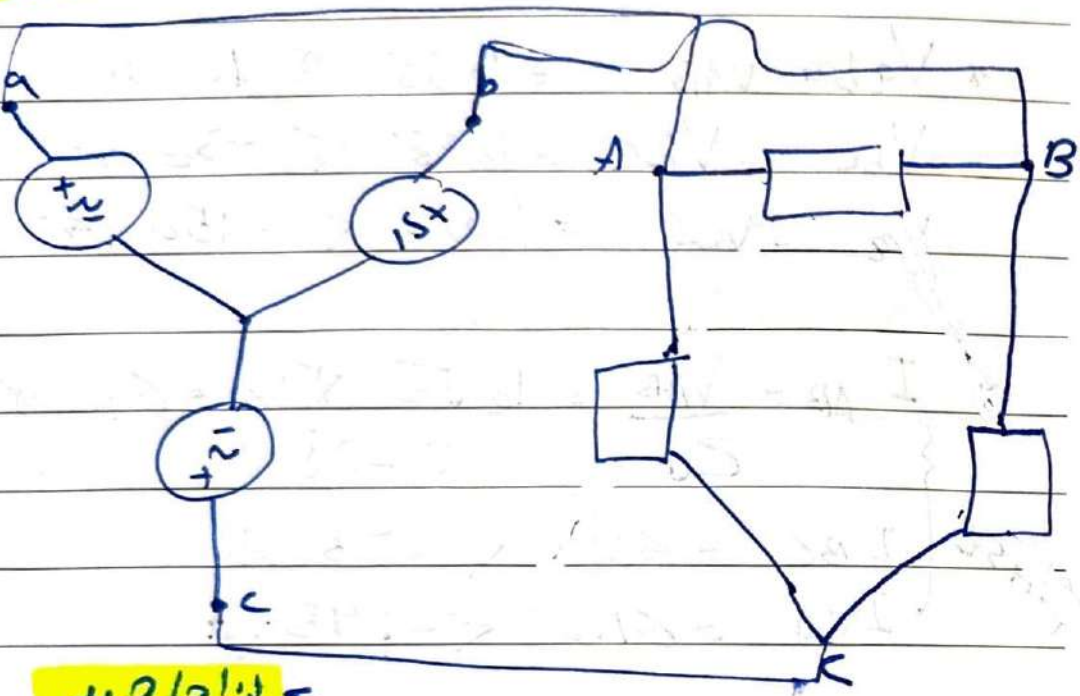
Y- Δ Balanced, $V_{ab} = V_{\text{line}} = 204$
(source)

اذا طالع

(3 phase) source

load draws 18 kW, P.F. = 0.8 lag

sol:



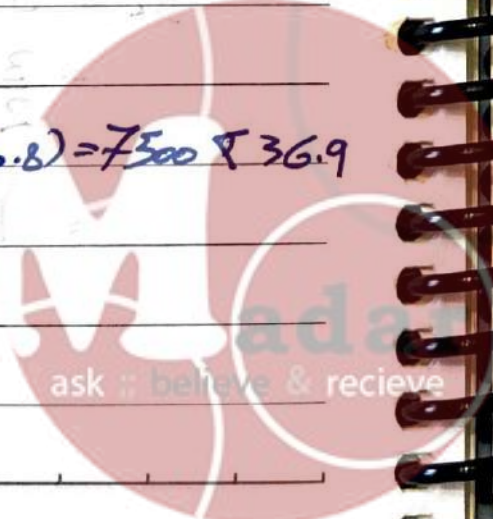
اذا طالع

$$V_{ab} = V_{AB} = 204 \angle 0$$

$$* V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30 = \frac{204}{\sqrt{3}} \angle -30 = 118 \angle -30$$

$$* P_{\text{Load}} = 18 \text{ kW}, P_{\text{phase}} = 18/3 = 6 \text{ kW}$$

$$* S = \frac{P}{\cos \theta} = \frac{6000}{0.8} = 7500 \angle \cos^{-1}(0.8) = 7500 \angle 36.9$$



$$* S = 7500 \angle 36.9^\circ = \frac{1}{2} V_{AB} I_{AB}^*$$

$$\therefore I_{AB}^* = 73.5 \angle 36.9^\circ$$

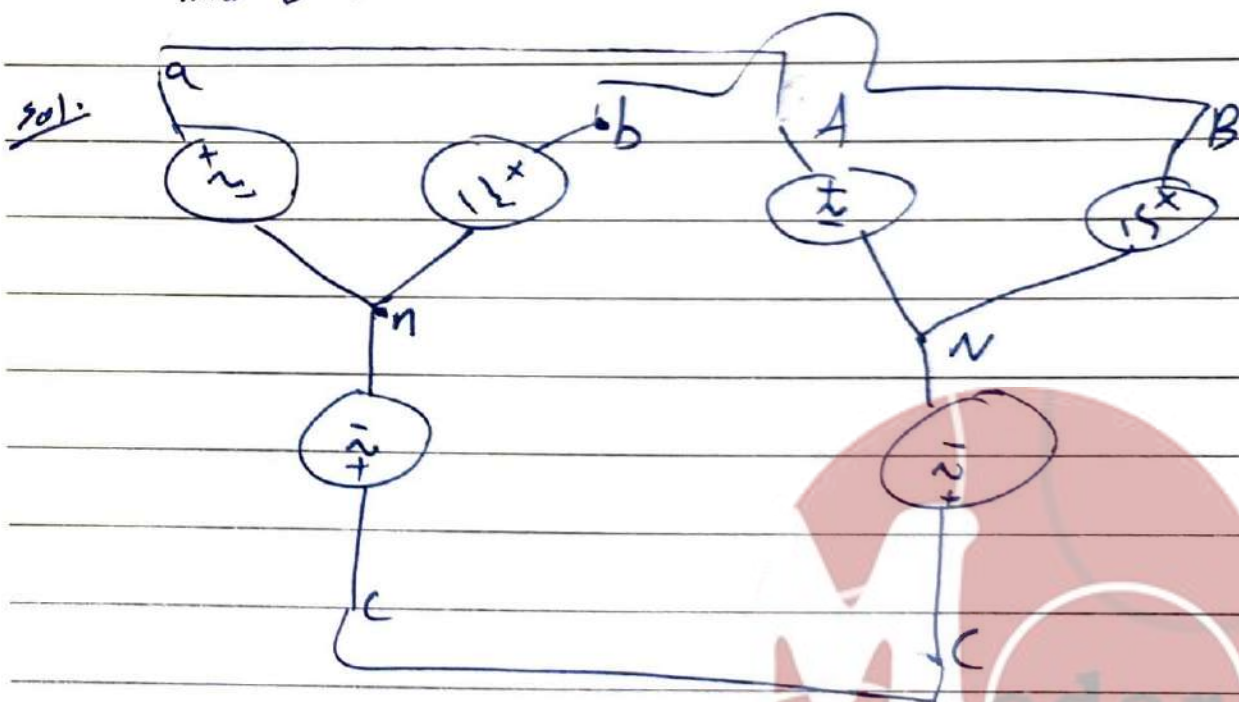
$$I_{AB} = 73.5 \angle -36.9^\circ$$

$$\therefore Z_0 = V_{AB} / I_{AB} = 2.22 - 1.67j$$

$$* S_s = 3 \times \frac{1}{2} \times V_{an} I_{aA}^* = 22533 \angle -96.5^\circ$$

$$I_{aA} = \sqrt{3} \times 73.5 \angle -66.5^\circ$$

**
 $\text{P.F.} = 0.8 \text{ lags}$, $Y-Y$, $S_L = 1 \text{ kVA}$
 $V_{\text{line}} = 204 \text{ V}$



$$V_{ab} = 204 \angle 0$$

$$* V_{an} = \frac{204}{\sqrt{3}} \angle -30 = 118 \angle -30$$

$$* S_p = \frac{4.8}{3} = 1.6 \text{ kVA}$$

$$* S_p = 1600 \angle 36.8^\circ$$

$$* S_p = \frac{1}{2} * V_{an} I_{aA}^* \rightarrow I_{aA} = 27.1 \angle -66.8$$

$$* Z = \frac{V_{an}}{I_{aA}} = 2.49 + j2.6 \Omega$$

