$2.8: Random Variables: Xrandom variable: function that assigns a real number to each outcome in the sample space of a random experiment – X: random variable – x: measured value of the random variable after the experiment

continuous random variable: range of the random variable includes all values ole in an interval of a real numbers, the range can be thought of as a continuum. Ex: current lengthe pressurez temperature stime voltage, weighten **discrete random variable**:_random variable with a finite Icountably infinite rangeEx: #of scratches on surfaces proportion of defective parts among otesteds
#of transmitted bits received in error

Ch 3: Discrete Random Variables & Probability Distributions:

–

$ 3.1: Discrete Random variables:

Ex: test to 3 cell-phone cameras. The probability that a camera passes the test is 0.8, the camera performs independently, Complete the table

Camera 1 _ Camera 2 camera 3 Solution R X

p
a
s
s

p
a
s
s

pass 0.8*0.8*0.8 0.512 3 Fail

p
a
s
s

passe . 0.20.8*0. 0.28 & Fail

F
ai
l

pass 0.2*0.240.8 0.032 1 Fail

f
a

il

Fail 0.2*0,2*0.2 0.008 0. X: pass

PC Fai) : 1-0-8=0-2 p(pass) - 0.8 P(x=3) = P(pass)*

P(pass)x P(pass) > independentlyy

. . 08*0-8*0.8 0.512 PCX-3) =

$3.2: Probability distributions $ Probability Mass Functions:

probability distribution: description of the *probabilities* *associated* with

the possible

values of X.

Probability mass

Flu*nction:*

(= PX - هلاک

(0) دسمبر (f(x (2)

و رعایا

ه اند

ا

ببلجونم بهر عنض لها

ملاهي فقط ولم P ان ميلاد

1

Ex 3-8: Let ***the random variable x denote the***

number of _Semiconductor wafers that need to be *analyzed in order to detect a large particle* of contamination. Assume that the probability that a **wafer contains** a large_particle is 0.01 the wafers are independet. Determine the probability distribution of X.

لتطماني بدون قلله ابو اياها على شكل معادلة

جا انه طالبها

distributi
on)

= (ا=)P

= (2-PX

– ل3

=PX

الحاليقلعولقطة عوامل

مولازل و امه

د إجمالي الاول صعية و ه ة كبيرة

0.9
9

= 0

0.004
9

0.04 = احماله اول گفتن بخرة و ليلة كق

+

0.0
4

0.01
+
0.0078
0 =
/
چسب بلیط بيطلع مو عروق و
P(X=x) - (0.99)*-! (0.01) >
probability
distributi
on
. . .
ر2ار =
X
ساطه
3-
2
(
-
11.2
5
ک
.
2/ |
ک.
2.25 :ماده | انما
009 | 002 (

=

(2

P (X =

1.25) P

= 5) P1 ،45) 2PX

(2

=

) م (0)

= 0.2*0.4+

0.

002

=

0.

9

=)P – ل=

OR

2.

2

=

(

–

001

–

0.

9

3: Cumulative Distribution Functions:

7W

0.048
6

0.003
6

0.6561 P(X<3) = ?? 0.2916

P(x53) - 1-P(x=4) 2

= 1 –

0.000)

=0.9999 4 0 .0001 P(X=3) = 0.0036

*always: for any *discrete random variables* with *possible values* to **the events** X is *mutually exclusive* Cumulative ..

F(x) = P(x4x) = *f*(xi) * Beste *properties*: (1) F68)

P(x6x)= *f(xi)* (22 oS FOST

(3) If xsy → F®<ECY

Xix

Ex 3-73 Determine *the probability mass function* of x *from the following Cumulative* *distribution function:*

a so XX-2 P(XK-2) = 0 Jure, Suer. Fx 10.2 -

2<x<0 *P(-25*X50*)* = 0.2

is 10.7 Os x42

Cumulative

distributi

on

F(x)=Luzsu ex

pas

ك

2

ف

ي

كبرتي يصل

اليه

دل إذا كانت

f(-2) = P(X=-2) - P(-2«≤0) - P(x>2)

- 0.2-0-0

foP(x=0) = 0.7 -0.2=0.5 F(2) -

P(x=2) = |-.0.75 0.3

03.4: Mean *V*ariance of a Discrete *Rand*om *variables*: *

mean: measure of the center for ***meddle* point** of *the*

p*robability*

distribution (M) or *Expected value (E* (x) variance:

measure of the *dispersion* or variabili*ty in the distribu*tion o

* M= E) - 3 xfQ

*VG): E(X-M)?- Zlx-M]zfQ)

o ro > *Standar*d *deviation*

xlo 12 13 14

PG) 10.6561 10.2916

0.0486 0.0036

10.000) M 1-0.4

0.6 11.6 2.6 3.6 and (X-M) 2 10.160.36

2.56 6.76 22.96L

M-22 o 2 ?? 6=??

Ms Exf(x) = Ox 0.*6561*+ 1*0.*2916*+ 2* *0.0486*+ 3*0.0036+ 450.0001

M = 0.4 O Z (X-*M) 26%) = 0.16*0.6561*+ 0-360*.2916 +*

*2.56**0.048*6* +6.*76** 0.0036

*+12.96*0.000* o2 =

0.36

III liiiiii

6

=

2 = 10.36 =

0.6

_ *Expected value of a Function of discrete random variables:

E(ha)) - E ha) f(x) *EQ* -

E*FCC)*

hx) y lo lis, & lib. EX 3-

12 h(x) = x2

Xlo 12 13 14 PGXD0.6561

0.2916 0.0436 0.0036 0.0001

Elhal=? E(h(x)) = E(x2) =

2x2fG)

z Ox0.6561 + 1*

0.2916+2*0.0486+330.0036

+12+0.0001_ Elha)=

0.52)

Discrete *Uni*form

Distrib*ution*:

fW= P(x) = e qual pos*ibilit*ies Ex: X=20,1;

2,3,4,5,6,7,8,9}

X = {0, at), at2, --- obku f = L

= 2xfx) = bta

-

o (6-a+1) 2 - 1 Ex 3-

14: X = 0-48 aso b=482

M=48+0 = 24 6-48-0 +12) 22400 -

200

6=562 = 1200 =

14.14

Exi

X= 5,10, B...,30

X=3 (1

M= 5(6+1) = 5* 35 = 175 o *sf*66-1*2)=1) B(36-1)

=25335 = H 72.92

9999999991111

oor

s

=

8.5

4

$ 3.6: Binomial Distibution: Bernoulli tr*ail: tr*a*il* with only *then p*ossib*l*e *out*comes *(success / Failure)*

is used so f*requently* as a b*uilding block of a rand*om experiment * independent Erails with constant probability of a *success* on each *trail*

Ex 3-16: Rerror) =
0.1

X = # of bits i*n excer* in the next four *bits*
transmitted on PCX=2) ??

() success amelijk cinay Bernou*l*li oslo centi
PCfailure) or P(success) - Ju *a ny* experiment ass

Solution ? P(X=*x*) = *6* px (1-P)**
x=2 noh - od 1-P=1-01 = 0.9 → P(X=23-
6) Coul (0-9)* 2

41 Cau2 (o.gl2 = 0.0486
21 (4-2)! * Binomial: has
parameter *(pen).*

Xe #*of traits that result in a*
success

foo= P(x=x) = A) (1-P)
mean = *E*(X) - Ma
np *va*riance - G?=
npci-p)

for the last
example:

*M*enp=*4**011 Eoint 6 npci-
p) - *4**01*09 = 0,*36*

savore = 10.36 = 0.6

P3

$3.6: Geometric Negative Binomial distributions: .

. Geometric Distribution. Parameter: P: Probability x=trial to Ist succes

X: # of trails until the first success

$F(x) = PCX=x) = (1-P))$!

M1

l

e Ex3.22:

The probability that a wafer contains a large partide of contamination is 0.01 . If there it is assumed that the waters are independents what is the probability that exactly 125 wafers need to be analyzed before alarge partide is detected ? _solution: __

tv p =0:01 X=125 € Geom. [Success og bopel mo kails Lexperiment de ses sibe) 151

$f(125) = P(x= 125) =$

(1 – P)

=(-0.01)*(0.01) | _ =

0.99l24 * 0.01

= 2,846

X103

-

2 Negative Binomial distribution: Parameters: P: Probability

V: #

Success X: # of trails until r successes occur

itrials

fa)= P(X=x) = (44)o-phie pa

1111111111

EX 3-24: PCerror): 0.1

X: # of *bits* trans*mitted until the 4th*

error. P(X=10);? X=10 r=4 P=0.1 .

f(10)= (-1) (+-0.1)

(0.1)*

-(I) (0g)* Cuj* = 4, 4sxie M = r e coon Geom.

Jed)

1.9: Poisson

Distribution:

interval subintervals Ex: (1) particles of contamination de in semiconductor manufacturing. (2) flaws in rells of textiles (3) calls to a telephone exchange (H) Power outages _ (s) atomic partides et – : Poisson v il's – I Probability of more than one event in a subinterval tends to zero .

ف

حالة حرث لكم موحد في جزء من الفترة =

2- Probability of one event in a subminterval tend to

ast_

ل

ا

احالة روث مرة واحد في جو من الفترة =

3- The event in each

subinterval is independent of other subintervals.

الالوان متعلق خيل اجزاء متساوية من الفترة .

Poisson random variable:

£x) = et (AT*

Ti interval of real numbers (time, length, area , volumes.) At = subinterval of small length. 2 = average number of events in interval.

n = number of trails – I

P - Success probability

= a

= 21

EX 3-31: thin copper wire, suppose that the number of flaws a Poisson distribution with a mean of 2.3 flows per millimeter Determine the probability of exactly 2 flows in 1 millimeter wire.

T= 1

mm

2=2.3 flows / min

x=2

AT–23 How am

9.3

elow.

f(2)

=

ē

"

1111111111

FG)= e 25 (23)2 =

0.265

2! @ Determine the probability of lo

flows in sum of wire

X = # of flows in 5 mm of wire T = 5mm

AT = 2.3 flow snom = 11.5
flow

ē
"
(11.5) 10!

=

0.113

3 Determine the probability of at least one flow in 2 mm.
of wires

X: # of flaws in 2mm wire.

AT= 2.3 flow/2mm = 4.6
flow

mm

P(X20-1-P(X=0)

1 ē6 (4.68 -0.0101 +1 = 0.9899

(MET

=6?

19999

Ch 4: Continuous Random Variables 3) Probability Distribution.

$4.1: Continuous random variables: random
variable with an interval of real numbers for
its
range

$4.2: Probability distributions $ Probability Density Functions:

"Integ

ration Probability density function:

(1) f47o (2) *ff&id*x = 1 3) Plasxx bla S*fande* _

(4) PCXXX<x) = P(x*4* X

<

- *P(*XX<X*)* = *P(X*<XC*X)*

Example 4-1: X= current measured in a thin copper

wire in *(*MA)

- X=[4.925*.1]*mA

f(x) = 5 for 4.98x< 5.1 @ What is the *probability*

that a current measurement is less than

SmA? s - P(x(s) = 5 dx = 5(3-4.9) =

5*0.1=0.5

4.9 Sil 2 P (4.95<x<5:1) = f 5 dx = 5(Jil-4.95) =

0.75

x 4.95

ووادلكعيكال بالنسبة للحدود لعظة بالول = (x) ا لواعظى

Ex H-2 :

f(x) = 20 € *20*(X=12.57 ex

> 12:

5 1 PC

*712.6) -70 p(x>12.6)= k02 20lxndx
= 20e-2018-12.5)
12.
6
20
Ji2.6
-
rolo
ok
t
12.5 2
20/12:6-
1204
=
é - =
0.135
e
Sunda
Distribuition
Function: $4.3: Cumulative

FO) = P(xxx) =) flus
de

ك= (لاکا رد , 9.4
X
0- (لاگ دوبالا : 3- ة Ex
في عمليصلك Zanston 22-hutto تعبر عودلا بمتسكباج

– جدا من الف

0 ، والا

م

من التكمل سے

x-2.5 ، 167X Sك

S (، داره نملة عنترة بنے

، ا)

10عددددددددد

2(للكها = كل dx= ك = دايك = لاك2 را

بائل اعطفال وال ها وبدو لم ر

xo و 2

(f(x = ج م

*

= (Ex 4-3:

F(

x

dx

o

c

x

leooo

x

مم
ن
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23-
(200>PCY
e
d
fal= f'w = 0. ole oix
a
F(x) = 1-
ebote
ف
2
0.
01
3
لهم)F (=
(هم)- (ک هم 0.36
= 2 −
ا=
P(
x
Me
an

Varia
nce

of a continuous Random
Variab*l*e

M = x fx dx

2- ((x-42 fx dx . Gora Ex4-65, faltas

*x[*4:9,5:1]

M = fsx dx = 5x270.1 = 5(5.12 - 4.92) = 5 o

J*M e foo*dx = S(x-3*)?dx*

Ş.(4-3)*)* – $ (615) +- (19-38) = 3 (1040)

-

2

J

u

g

.-

4.9

=

3,33*x10*

2

§ 4.5: *Continu*ous Uniform Distribution:

f(x) = 16e asksb

Ex4-9; X = ran*dom variable h*as a co*ntinuou*s uniform distri*bution* on [4.965.1]

fox= [4.965.17 P(4.95<x<5.0) = f S d x = 56.5-4.95) = 0,25 h a

4.9
5

Ms 4.9+5.-100-S

82 (s.1-4.92 3.33x10

1
2

$4-6: Normal distribution: *(G*aussian *distribution)*

Dean

(M)

Probability density function of normal distribution:

-X-M2

V2015

M=M

0262

A)?>ek2mA

EXCHAO2Monte o

Per

POM-6 <xM o . 0.6827.

PCM-

26 <x< M+

26) =

0.9545 P(M-36 SXSM435),

0.9973

variables

(Z)

Standard normal random

M=0 o21

Z=X-

M

P

(2) → from table

Ex 4-13: M = 10 mA 64 (MA)2 →0=19-2 mA

P(X)13) = ?? » P(Z > ??) P(ZM) 13-
M) - P(X=10 > 13-10)

- P(Z5 1,5) = FR 0.*0668* Đ from table.

*Standardizing. Z

PX <<) = (x H = = P(2<) Ex 44-
14: M=10 0 24 0:2 PC9<<<11) - PC 9—10
< xxx0 < 110)

= P(-0.5<750.5) - P(Z <0.5)=*P(Z<-015]*

= 0.6915-
0.3085

| | | | |
|

10:382
92

| | |
|
|
| |
| | |