3. Discrete Random Variables and Probability Distribution

3.1 Random Variables (variates):

In many situations it is useful to associate a number with each outcome of a random experiment in a sample space.

Random Variable: it is a function that assigns a real number to each outcome in the sample space of a random experiment.

Notations:

X: Name of random variable

x_i : the ith measurement of a random variable

In dealing with random variables, we associate probabilities with the values of the random variable in the sample space

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Discrete Random Variable: it is a random variable with measurements that are limited to discrete points on the real line. It has a finite (or countably infinite) range.

Continuous Random Variable: The function (out come of the measurement) assumes any value of real numbers on an interval defined between two limits (finite or infinite).

3.2 Discrete Random Variable

- These are generated by random experiments
- Focus is made on the random variable and its distribution without much regard to the details of the sample space.

Ex 1 Communication system consisting of 48 lines

X : Number of lines being used at a particular time

x_i : can assume values : 0 - 48

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Ex 2: Testing two components in sequence from a lot: assuming that components are independent

Result: Pass (f) or Fail (f)

Sample space: (pass, pass ; fail, pass ; pass, fail ; fail, fail)

X: number of components that pass

Assume the probability of pass of each component =0.8

Component		x	Prob.
Comp 1	Comp 2		
pass	pass	2	0.64
fail	pass	1	0.16
pass	fail	1	0.16
fail	fail	0	0.04

3.3 Probability Distributions and Probability Mass Functions

Probability distribution of a discrete random variable (X) is described by a function $f(x_i)$ that specifies the probability at each of the possible values of X. It is similar to loading of a beam with different masses at different points.

3. Discrete Random Variables and Probability Distribution

Definition:

For a discrete random variable (X) with possible values $x_1, x_2, \ldots x_n$ a probability mass function is a function defined such that :

1)

2)

3)

Probability mass function = probability function = probability distribution

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Cumulative Distribution Function

For any discrete random variable (X) with possible values $x_1, x_2, \ldots x_n$ the events $[X = x_1], [X = x_2], \ldots, [X = x_n]$ are mutually exclusive.

With properties:

1)

2)

3)

3. Discrete Random Variables and Probability Distribution

3.4 Mean and Variance of a discrete Random Variable

Expectation: → Long run average of a measured quantity
→ Average value of a statistic

Expected Value of a random variable E(X):

by definition E(X)= μ

For a discrete random variable:

$$\mu = E(X) = \sum_{x_i} f(x_i)\,.\,x_i$$

$=\sum mass\,.\,distance$

= First non central moment

= Centroid of the probability mass

= Weighted average of possible values of X

3. Discrete Random Variables and Probability Distribution

Variance of Random Variable (X): $\sigma^2, V(X) or Var(X)$

$$\sigma^2 = V(X) = E(X-\mu)^2 = \sum_{x_i}(x_i - \mu)^2 f(x_i)$$

$$= \sum_{x_i} x_i^2 f(x_i) - \mu^2$$

= Second central moment

Standard deviation of X is $\sigma = \sqrt{\sigma^2}$

For a specific function of X , i.e. h(X)

$$E[h(X)] = \sum_{x_i} h(x_i) f(x_i)$$

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Properties of E(X)

$E(c) = c$

$E(cX) = cE(X)$

$E(a+bX) = a + bE(X)$

$E[g(X)] \neq g[E(X)]$

Properties of VAR(X)

$V(c) = 0$

$V(a \pm bX) = b^2 V(X)$

$V(X) = E(X^2) - E(X)^2$

3.5 Discrete Uniform Probability Distribution

A discrete random variable X has a discrete uniform distribution if each of the n values in its range (x_1, x_2, …x_n) has equal probability. Then:

$$f(x_i) = 1/n$$

Suppose that the range of X is the consecutive integers

a, a+1, a+2, a+3…….b for a ≤ b ; b = a+ (n-1) and x_i = a +(i-1)

Range of X : a ---> b ; it contains b-a+1 = a + (n-1) – a +1 = n

$$f(x_i) = \frac{1}{b - a + 1}$$

$$\mu = E(X) = \frac{b+a}{2} \quad ; \quad \sigma^2 = \text{Var}(X) = \frac{(b-a+1)^2 - 1}{12}$$