

3. Discrete Random Variables and Probability Distribution

3.1 Random Variables (variates):

In many situations it is useful to associate a number with each outcome of a random experiment in a sample space.

Random Variable: it is a function that assigns a real number to each outcome in the sample space of a random experiment.

Notations:

X : Name of random variable

x_i : the i th measurement of a random variable

In dealing with random variables, we associate probabilities with the values of the random variable in the sample space

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Discrete Random Variable: it is a random variable with measurements that are limited to discrete points on the real line. It has a finite (or countably infinite) range.

Continuous Random Variable: The function (outcome of the measurement) assumes any value of real numbers on an interval defined between two limits (finite or infinite).

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3.2 Discrete Random Variable

- These are generated by random experiments
- Focus is made on the random variable and its distribution without much regard to the details of the sample space.

Ex 1 Communication system consisting of 48 lines

X : Number of lines being used at a particular time

x_i : can assume values : 0 - 48

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Ex 2: Testing two components in sequence from a lot:
assuming that components are independent

Result: Pass (p) or Fail (f)

Sample space: (pass, pass ; fail, pass ; pass, fail ; fail, fail)

X: number of components that pass

Assume the probability of pass of each component = 0.8

Component		x	Prob.
Comp 1	Comp 2		
pass	pass	2	0.64
fail	pass	1	0.16
pass	fail	1	0.16
fail	fail	0	0.04

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3.3 Probability Distributions and Probability Mass Functions

Probability distribution of a discrete random variable (X) is described by a function $f(x_i)$ that specifies the probability at each of the possible values of X . It is similar to loading of a beam with different masses at different points.

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Definition:

For a discrete random variable (X) with possible values x_1, x_2, \dots, x_n a probability mass function is a function defined such that :

1)

2)

3)

Probability mass function = probability function =
probability distribution

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Cumulative Distribution Function

For any discrete random variable (X) with possible values x_1, x_2, \dots, x_n the events $[X = x_1], [X = x_2], \dots, [X = x_n]$ are mutually exclusive.

With properties:

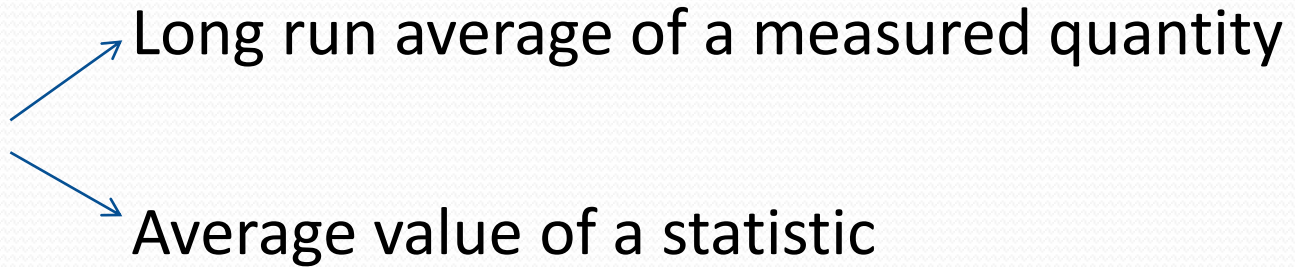
1)

2)

3)

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3.4 Mean and Variance of a discrete Random Variable

Expectation: 

- Long run average of a measured quantity
- Average value of a statistic

Expected Value of a random variable $E(X)$:

by definition $E(X) = \mu$

For a discrete random variable:

$$\mu = E(X) = \sum_{x_i} f(x_i) \cdot x_i$$

$= \sum \text{mass} \cdot \text{distance}$

= First non central moment

= Centroid of the probability mass

= Weighted average of possible values of X

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Variance of Random Variable (X): σ^2 , $V(X)$ or $Var(X)$

$$\begin{aligned}\sigma^2 &= V(X) = E(X - \mu)^2 = \sum_{x_i} (x_i - \mu)^2 f(x_i) \\ &= \sum_{x_i} x_i^2 f(x_i) - \mu^2\end{aligned}$$

= Second central moment

Standard deviation of X is $\sigma = \sqrt{\sigma^2}$

For a specific function of X, i.e. $h(X)$

$$E[h(X)] = \sum_{x_i} h(x_i) f(x_i)$$

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Properties of $E(X)$

$$E(c) = c$$

$$E(cX) = cE(X)$$

$$E(a+bX) = a + bE(X)$$

$$E[g(X)] \neq g[E(X)]$$

Properties of $VAR(X)$

$$V(c) = 0$$

$$V(a \pm bX) = b^2 V(X)$$

$$V(X) = E(X^2) - E(X)^2$$

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3.5 Discrete Uniform Probability Distribution

A discrete random variable X has a discrete uniform distribution if each of the n values in its range (x_1, x_2, \dots, x_n) has equal probability. Then:

$$f(x_i) = 1/n$$

Suppose that the range of X is the consecutive integers $a, a+1, a+2, a+3, \dots, b$ for $a \leq b$; $b = a + (n-1)$ and $x_i = a + (i-1)$
Range of X : $a \rightarrow b$; it contains $b-a+1 = a + (n-1) - a + 1 = n$

$$f(x_i) = \frac{1}{b - a + 1}$$

$$\mu = E(X) = \frac{b+a}{2} \quad ; \quad \sigma^2 = \text{Var}(X) = \frac{(b-a+1)^2 - 1}{12}$$