Solid Particulates

Chap. 1: Particulate Solids

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Particle Characteristics

The three most important characteristics of an individual particle are its:

- **Particle Composition:** which determines such properties as density and conductivity, provided that the particle is completely uniform. In many cases, however, the particle is porous or it may consist of a continuous matrix in which small particles of a second material are distributed.

- **Particle Size:** which affects properties such as the surface per unit volume and the rate at which a particle will settle in a fluid

- **Particle Shape:** which may be regular, such as spherical or cubic, or it may be irregular as, for example, with a piece of broken glass. Regular shapes are capable of precise definition by mathematical equations.

- Operations systems of particles include storage in hoppers, flow through orifices and pipes, and metering of flows.
- It is frequently necessary to reduce the size of particles, or alternatively to form them into aggregates or sinters.
- Sometimes it may be necessary to mix two or more solids, and there may be a requirement to separate a mixture into its components or according to the sizes of the particles.

Single particles

- The simplest shape of a particle is the sphere (because of its symmetry)
- Sphere particle looks exactly the same from whatever direction it is viewed and behaves in the same manner in a fluid, irrespective of its orientation.
- Frequently, the size of a particle of irregular shape is defined in terms of the size of an **equivalent sphere** although the particle is represented by a sphere of different size according to the property selected.
- Some of the important sizes of equivalent spheres are:

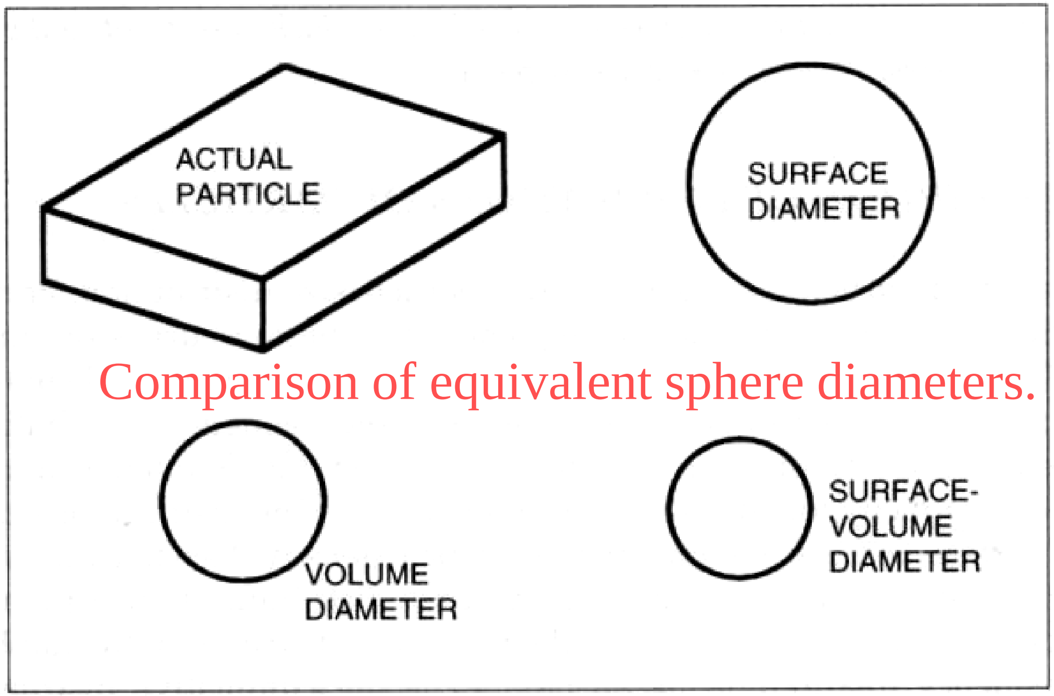
  (a) The sphere of the same volume as the particle.
  (b) The sphere of the same surface area as the particle.
  (c) The sphere of the same surface area per unit volume as the particle.
  (d) The sphere of the same area as the particle when projected on to a plane perpendicular to its direction of motion.
  (e) The sphere of the same projected area as the particle, as viewed from above, when lying in its position of maximum stability such as on a microscope slide for example.
  (f) The sphere which will just pass through the same size of square aperture as the particle, such as on a screen for example.
  (g) The sphere with the same settling velocity as the particle in a specified fluid.

Describing the size of a single particle

Regular-shaped particles

Shape	Sphere	Cube	Cylinder	Cuboid	Cone
Dimensions	Radius	Side length	Radius and height	Three side lengths	Radius and height

- The orientation of the particle on the microscope slide will affect the projected image and consequently the measured equivalent sphere diameter.
- Sieve measurement: Diameter of a sphere passing through the same sieve aperture.
- Sedimentation measurement: Diameter of a sphere having the same sedimentation velocity under the same conditions.



Comparison of equivalent sphere diameters.

Feret's statistical diameter

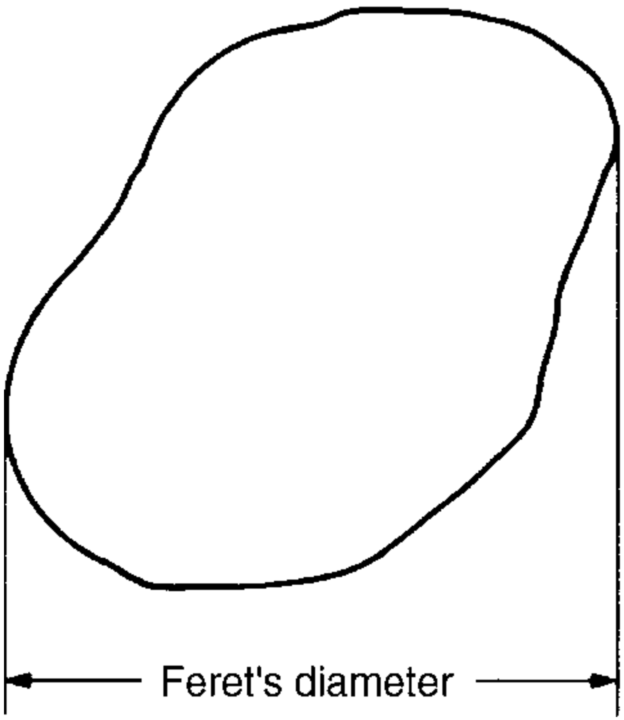
- **Feret's statistical diameter** is the mean distance apart of two parallel lines which are tangential to the particle in an arbitrarily fixed direction, irrespective of the orientation of each particle coming up for inspection.

- A measure of particle shape which is frequently used is the **sphericity** (which is a measure of how spherical (round) an object is):

$$\psi = \frac{\text{surface area of sphere of same volume as particle}}{\text{surface area of particle}}$$

$$\Psi = \frac{\pi^{\frac{1}{3}}(6V_p)^{\frac{2}{3}}}{A_p}$$

Other properties of the particle which may be of importance are whether it is **crystalline or amorphous**, whether it is **porous**, and the **properties of its surface**, including **roughness and presence of adsorbed films**.



Calculation of Sphericity

Sphericity Φ_s is defined as $\Phi_s = \frac{6V_p}{D_p A_p}$ where V_p is the volume of the object, A_p is its surface area, and D_p is the diameter of a sphere with the same volume ($\pi D_p^3/6$).

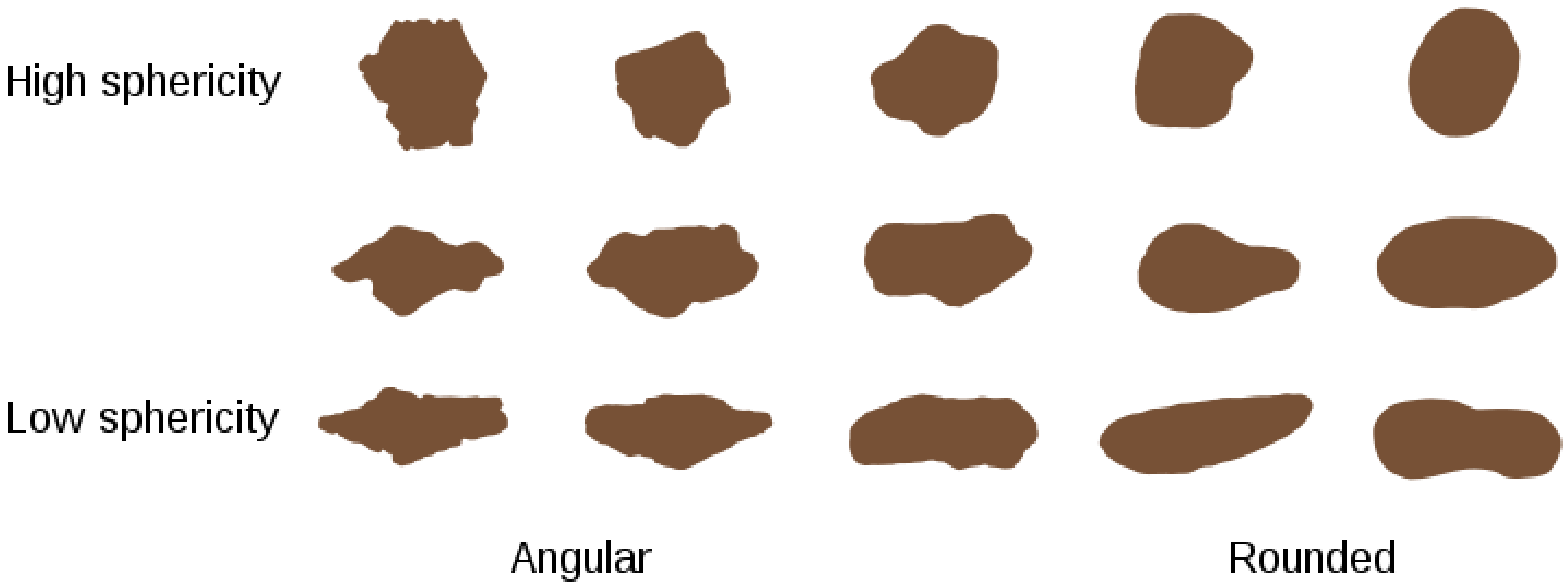
For a sphere of diameter d, $V_p = \pi d^3/6$, $A_p = \pi d^2$, $D_p = d$, and so $\Phi_s = 1$.

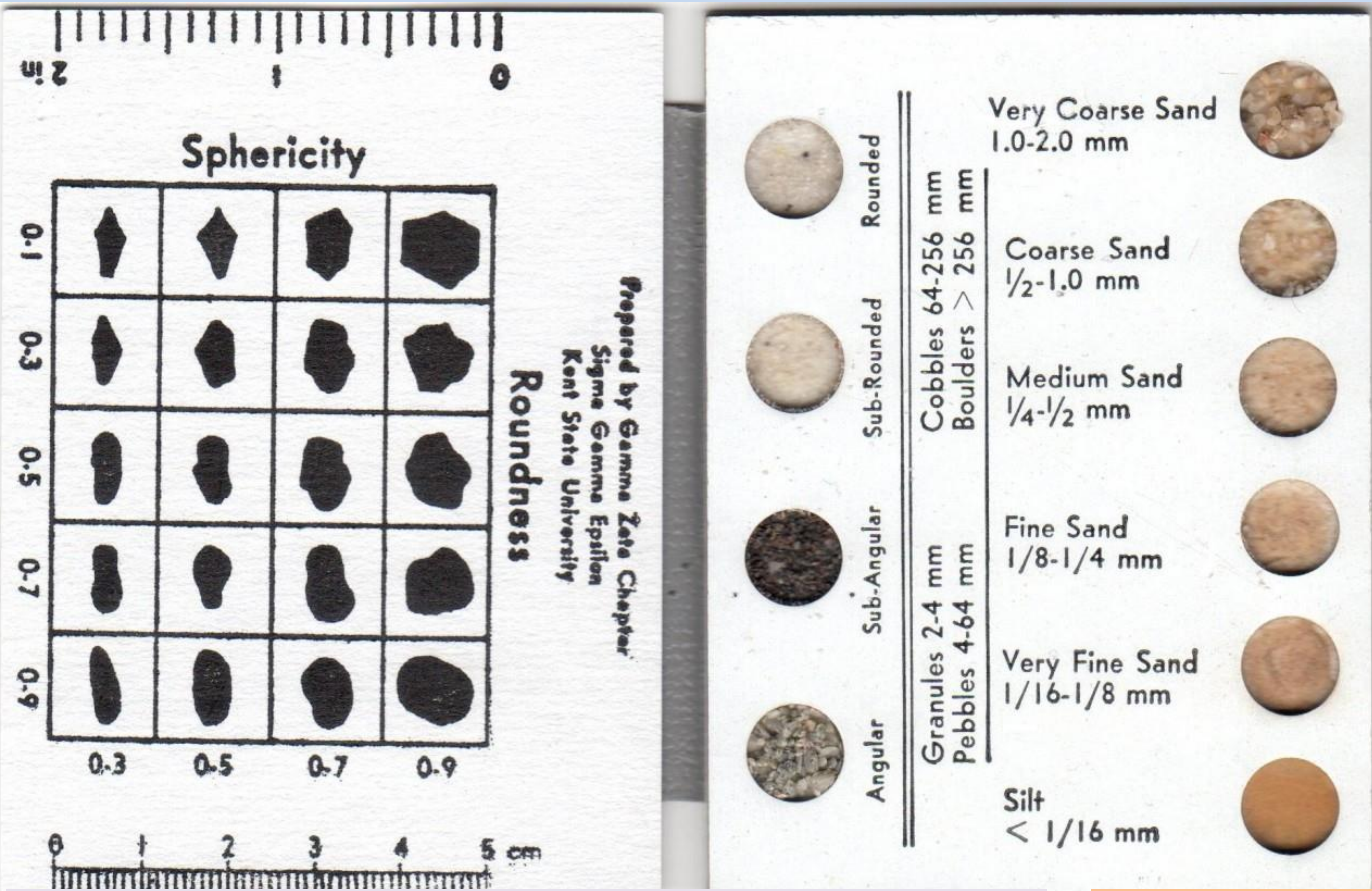
For a cylinder of diameter d and length L, $V_p = \pi d^2 L/4$, $A_p = \pi dL + \pi d^2/2$, $D_p = (6d^2L/4)^{1/3}$,

For a cube of width a, $V_p = a^3$, $A_p = 6a^2$, and $c = a^3$, and so

$$\Phi_s = \frac{6a^3}{(6/\pi)^{(1/3)} a \left(6a^2\right)} = \left(\frac{\pi}{6}\right)^{(1/3)} = 0.806$$

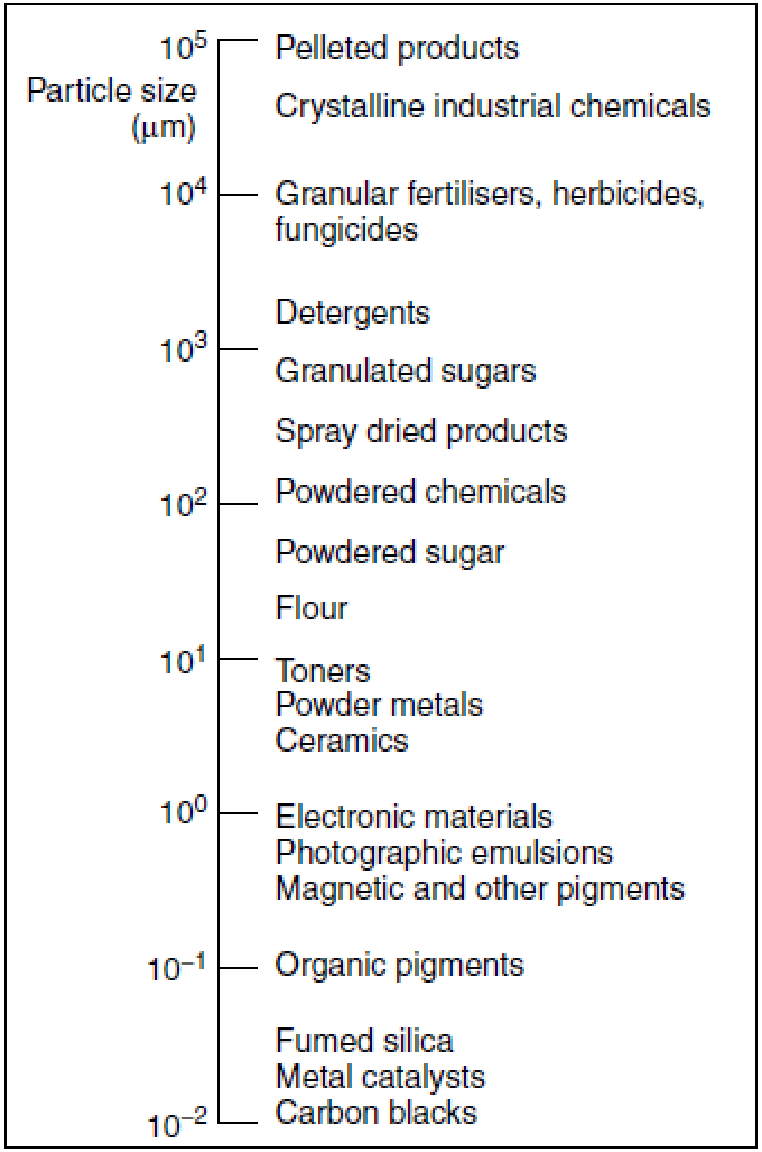
http://en.wikipedia.org/wiki/Sphericity


High sphericity
Low sphericity
Angular
Rounded

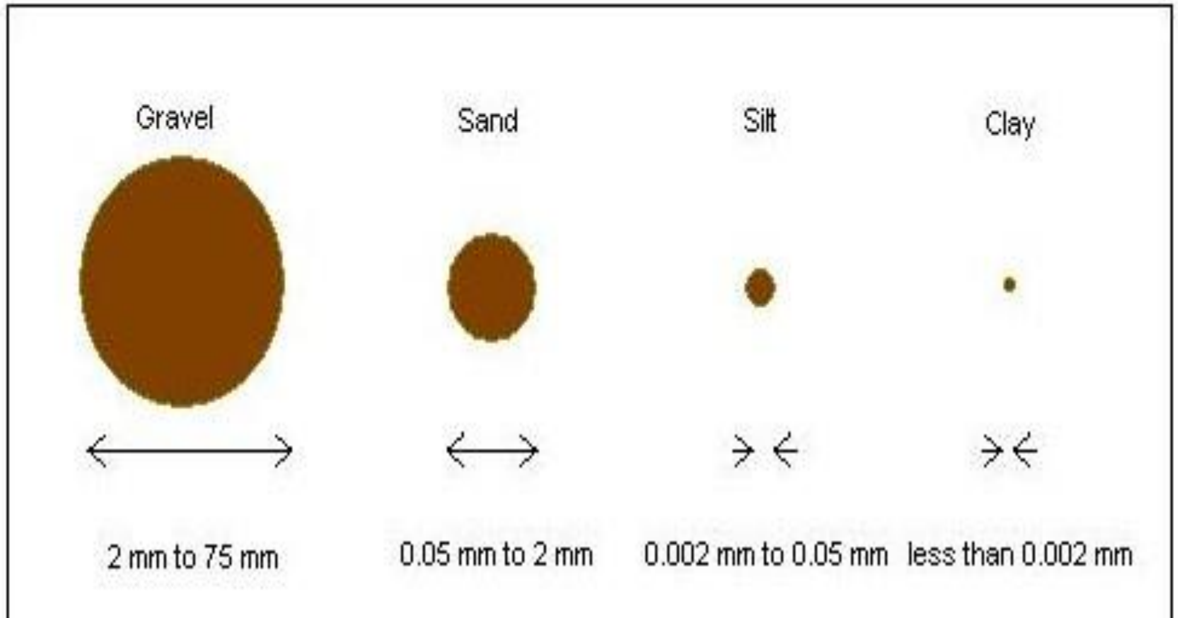

2 in
1
0
Sphericity
0.1
0.3
0.5
0.7
0.9
0.3
0.5
0.7
0.9
Roundness
Prepared by Gamma Zeta Chapter
Sigma Gamma Epsilon
Kent State University
0 1 2 3 4 5 cm
Rounded
Sub-Rounded
Sub-Angular
Angular
Granules 2-4 mm
Pebbles 4-64 mm
Cobbles 64-256 mm
Boulders > 256 mm
Very Coarse Sand
1.0-2.0 mm
Coarse Sand
½-1.0 mm
Medium Sand
¼-½ mm
Fine Sand
1/8-1/4 mm
Very Fine Sand
1/16-1/8 mm
Silt
< 1/16 mm

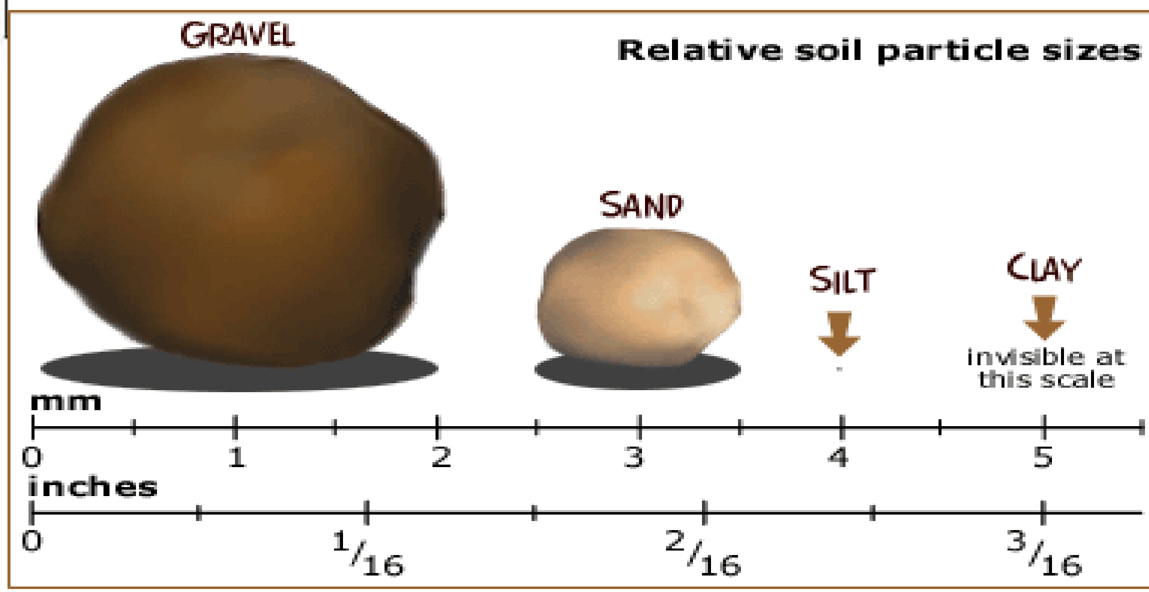
Measurement of particle size

- A wide range of measuring techniques is available both for single particles and for systems of particles.
- Before a size analysis can be carried out, it is necessary to collect a representative sample of the solids, and then to reduce this to the quantity which is required for the chosen method of analysis.



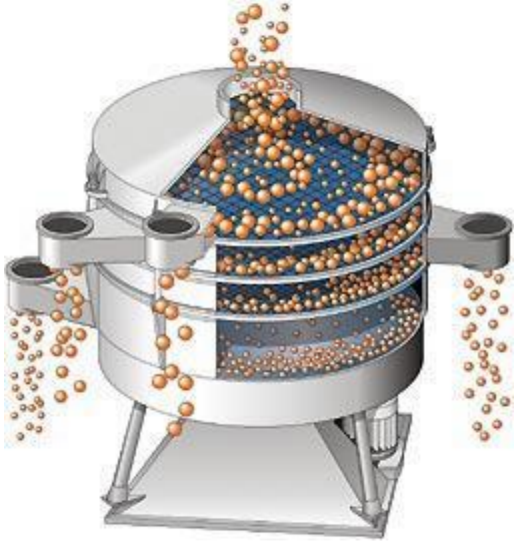
Particle Size





Sieving (>50 µm)

- Sieve analysis may be carried out using a nest of sieves, each lower sieve being of smaller aperture size.
- Generally, sieve series are arranged so that the ratio of aperture sizes on consecutive sieves is 2, $2^{1/2}$ or $2^{1/4}$ according to the closeness of sizing that is required. The are of openings in any one screen in the series is twice that of the openings in the next smaller screen.
- Standard screens size range between 3 and 0.0015in. (76 mmm and 38 micrometer).
- The sieves may either be mounted on a vibrator, which should be designed to give a degree of vertical movement in addition to the horizontal vibration, or may be hand shaken.
- Passing of particle through an aperture depends not only upon its size, but also on the probability that it will be presented at the required orientation at the surface of the screen.



- Since particles on any one screen are passed by the screen immediately ahead of it, then two number are needed to specify the size range of an increment, one for the screen through which the fraction passes and the other on which it is retained, thus the notation 14/20 mean “though 14 mesh and on 20 mesh”

Screen analysis

Mesh	Screen opening D_{pi}, mm
4	4.699
6	3.327
8	2.362
10	1.651
14	1.168
20	0.833
28	0.589
35	0.417
48	0.295
65	0.208
100	0.147
150	0.104
200	0.074
Pan	—

Screening

- The **efficiency of screening** is defined as the ratio of the mass of material which passes the screen to that which is capable of passing.
- This will differ according to the size of the material.
- It may be assumed that the rate of passage of particles of a given size through the screen is proportional to the number or mass of particles of that size on the screen at any instant
- Thus, if ***w*** is the mass of particles of a particular size on the screen at a time ***t*** , then:

$$\frac{dw}{dt} = -kw$$

where k is a constant for a given size and shape of particle and for a given screen. Thus, the mass of particles $(w_1 - w_2)$ passing the screen in time t is given by:

$$\ln \frac{w_2}{w_1} = -kt$$

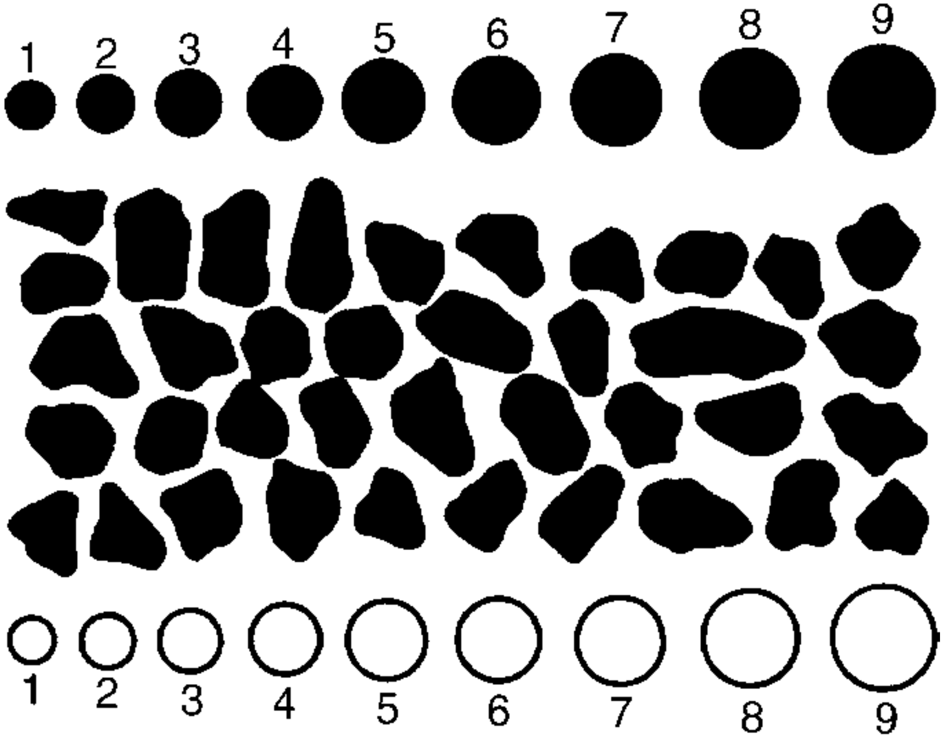
or:

$$w_2 = w_1 e^{-kt}$$

Microscopic analysis (1–100 μm)

- Microscopic examination permits measurement of the projected area of the particle and also enables an assessment to be made of its two-dimensional shape.

- Automatic methods of scanning have been developed. By using the electron microscope, the lower limit of size can be reduced to about 0.001 μm.



Electronic particle counters

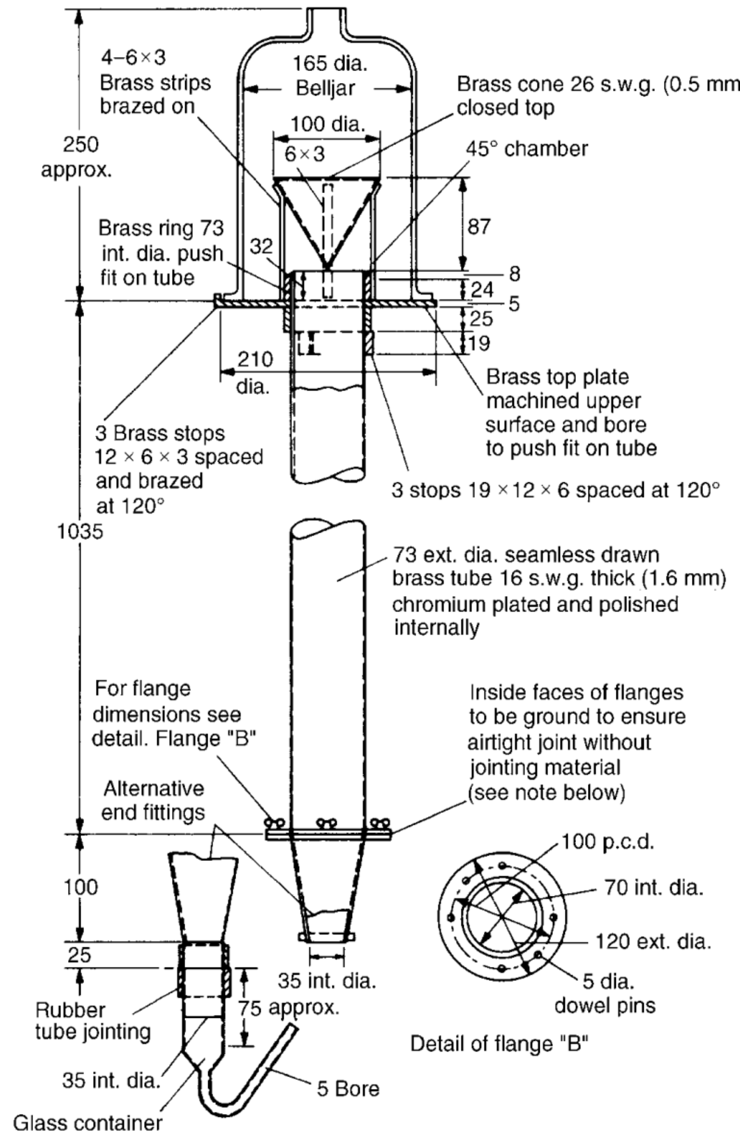
A suspension of particles in an electrolyte is drawn through a small orifice on either side of which is positioned an electrode.

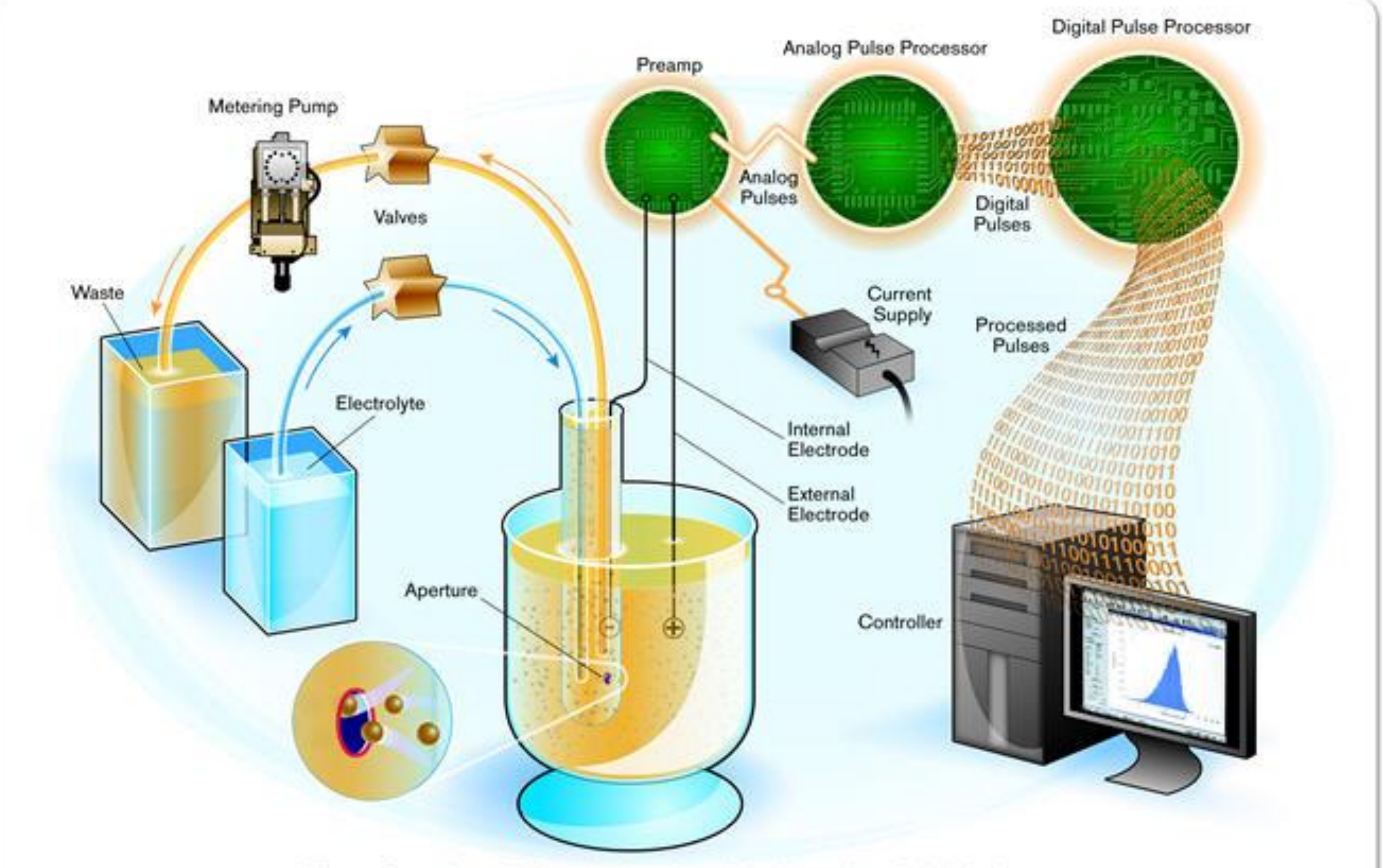
A constant electrical current supply is connected to the electrodes and the electrolyte within the orifice constitutes the main resistive component of the circuit.

As particles enter the orifice they displace an equivalent volume of electrolyte, thereby producing a change in the electrical resistance of the circuit, the magnitude of which is related to the displaced volume.

The consequent voltage pulse across the electrodes is fed to a multi-channel analyser. The distribution of pulses arising from the passage of many thousands of particles is then processed to provide a particle (volume) size distribution.

By using orifices of various diameters, different particle size ranges may be examined and the resulting data may then be combined to provide size distributions extending over a large proportion of the sub-millimetre size range.





The Coulter Principle Applied in the Multisizer 4

Particle size distribution

- Particulate systems consist of particles of a wide range of sizes
- A quantitative indication of the mean size and of the spread of sizes of particulate systems should be given and represented by means of:

  - ✓ a *cumulative mass fraction curve*,

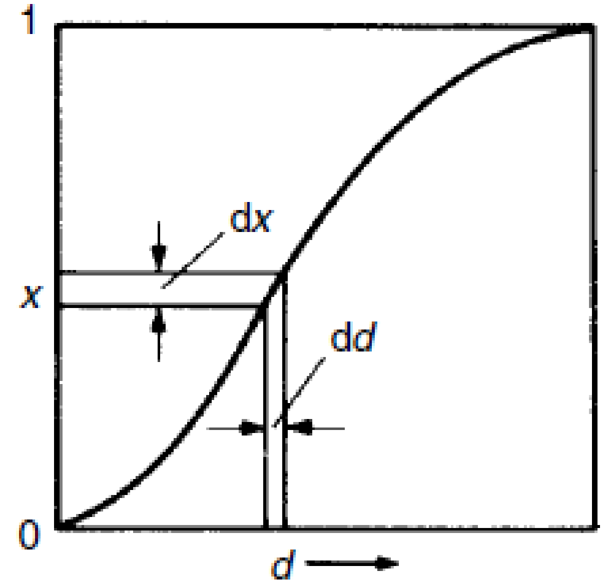
  in which the proportion of particles *(x)* smaller than a certain size *(d)* is plotted against that size *(d)*.

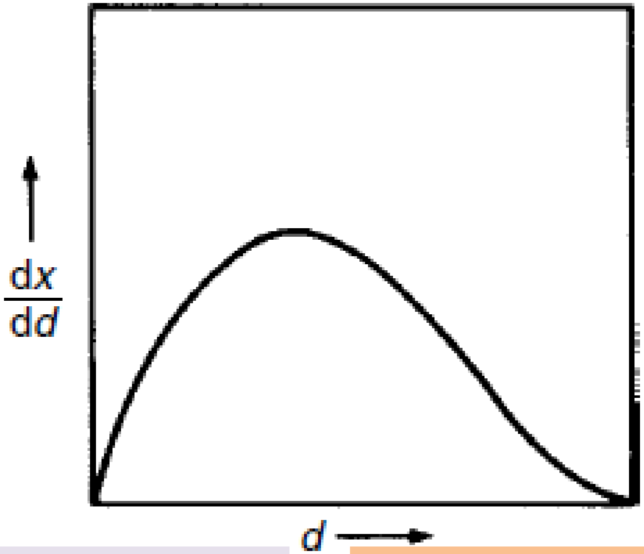
- In most practical determinations of particle size, the size analysis will be obtained as a series of steps, each step representing the proportion of particles lying within a certain small range of size.

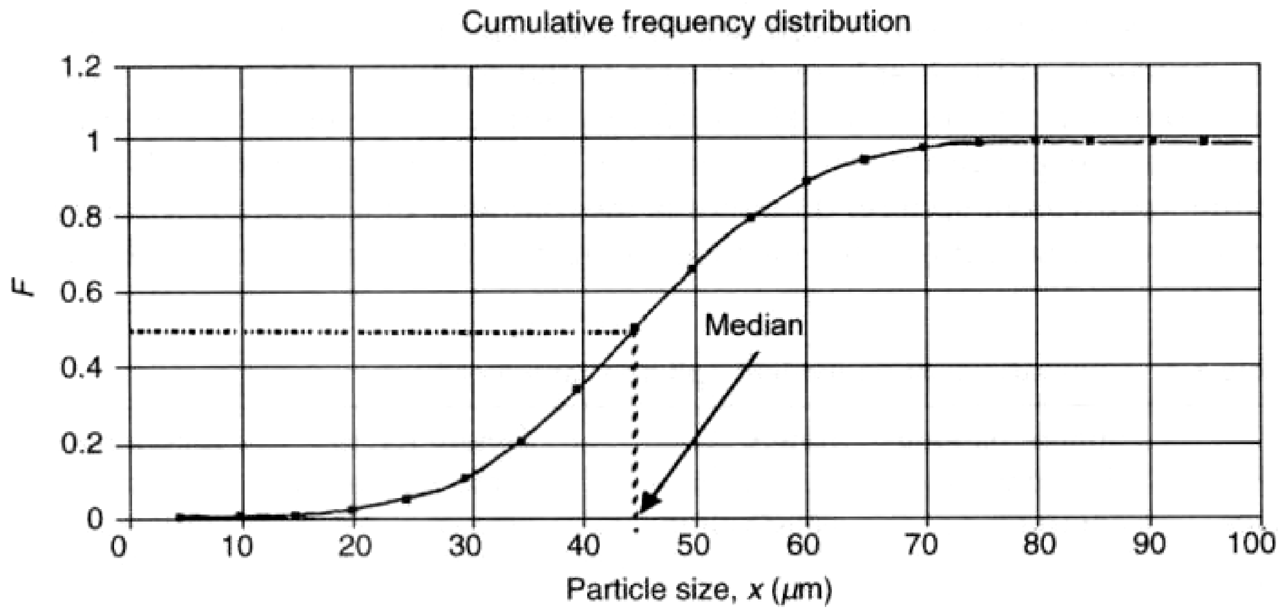
- From these results a cumulative size distribution can be built up and this can then be approximated by a smooth curve provided that the size intervals are sufficiently small.

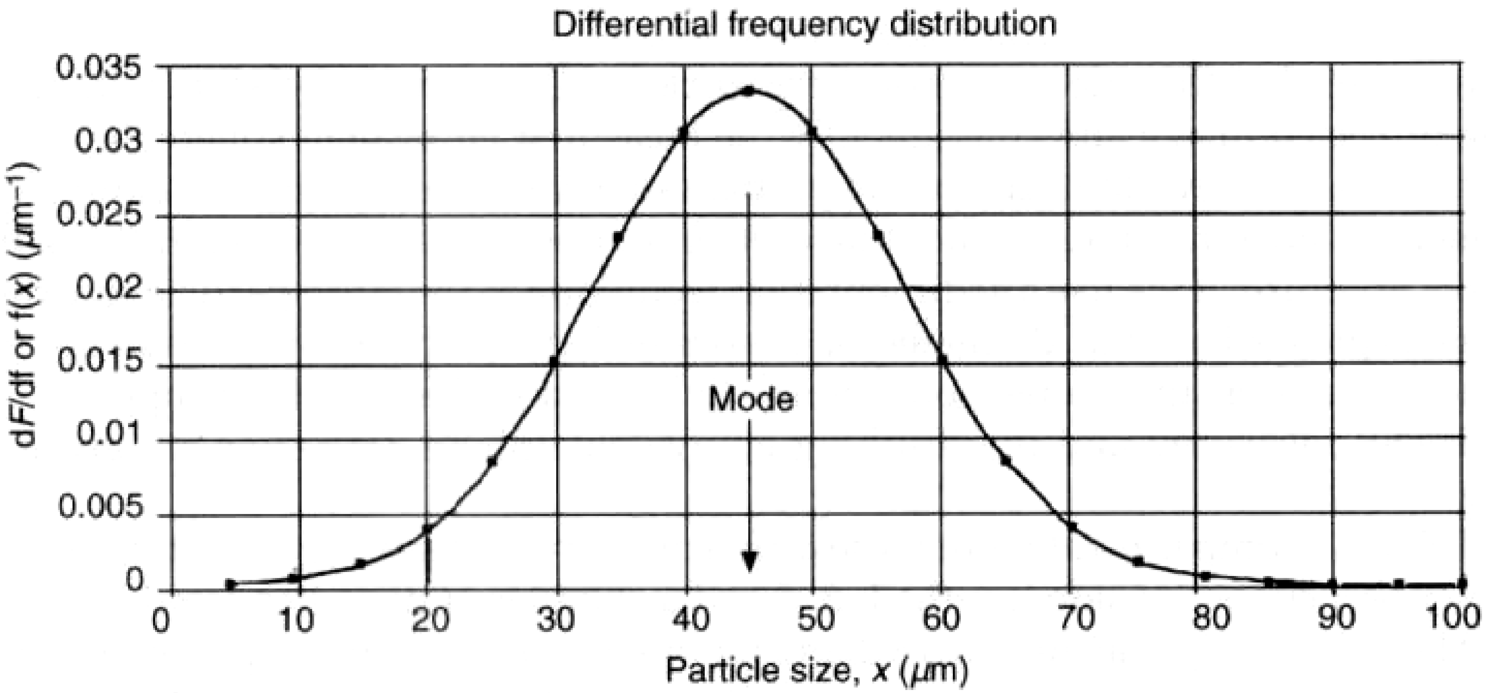
Size distribution curve — cumulative basis

- The distribution of particle sizes can be seen more readily by plotting a *size frequency curve*, in which the slope (dx/dd) of the cumulative curve is plotted against particle size (d).
- The most frequently occurring size is then shown by the maximum of the curve.
- For naturally occurring materials the curve will generally have a single peak.
- For mixtures of particles, there may be as many peaks as components in the mixture.
- If the particles are formed by crushing larger particles, the curve may have two peaks, one characteristic of the material and the other characteristic of the equipment.

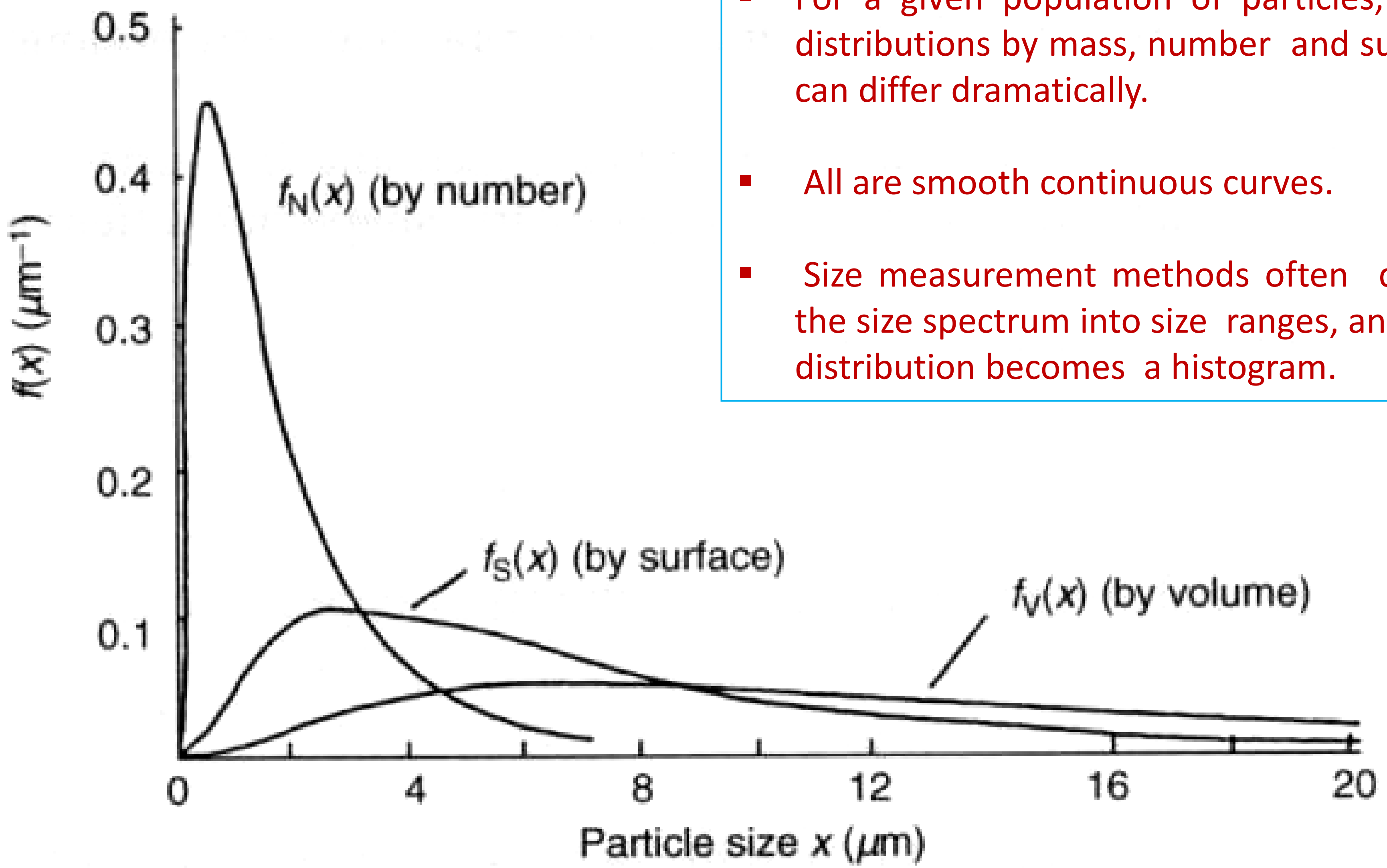





Cumulative frequency distribution
1.2
1
0.8
0.6
0.4
0.2
0
F
Median
0
10
20
30
40
50
60
70
80
90
100
Particle size, x (μm)


Differential frequency distribution
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
dF/df or f(x) (μm⁻¹)
Mode
0
10
20
30
40
50
60
70
80
90
100
Particle size, x (μm)

Comparison between distributions



- For a given population of particles, the distributions by mass, number and surface can differ dramatically.
- All are smooth continuous curves.
- Size measurement methods often divide the size spectrum into size ranges, and size distribution becomes a histogram.

Mixed particle size and size analysis

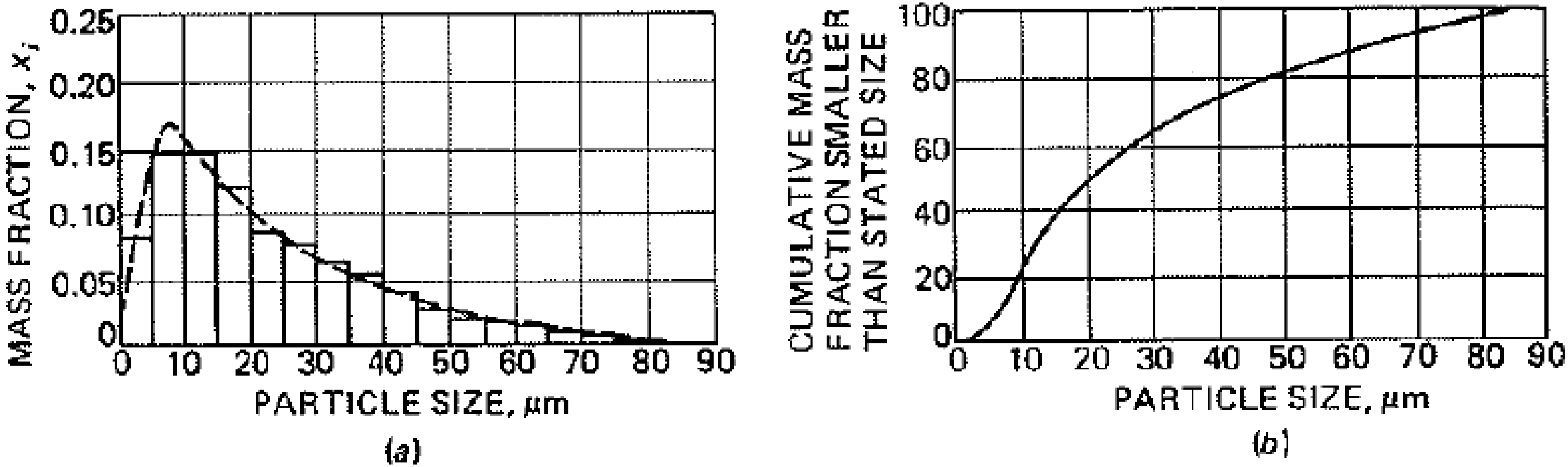
- In a sample of uniform particles of diameter D_p the total volume of the particles is m/ρ_p
- The volume of one particle is v_p, the number of particles in the sample *N* is

$$N = \frac{m}{\rho_p v_p}$$

- The total surface area of particles is:

$$A = N s_p = \frac{6m}{\Phi_s \rho_p D_p}$$

- Now to apply these equations to mixtures of particles having various sizes and densities, the mixtures should be screened and sorted into fractions (each of constant density and constant size).



Particle-size distribution for powder: (*a*) differential analysis; (*b*) cumulative analysis.

✓ **Cumulative analysis is more precise**

Specific Surface Area of Mixture

- If particle density and sphericity are known in each fraction, the surface area of particles is calculated by:

$$A_w = \frac{6x_1}{\Phi_s \rho_d \bar{D}_{p1}} + \frac{6x_2}{\Phi_s \rho_p \bar{D}_{p2}} + \cdots + \frac{6x_n}{\Phi_s \rho_p \bar{D}_{pn}}$$

$$= \frac{6}{\Phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\bar{D}_{pi}}$$

where subscripts = individual increments

x_i = mass fraction in a given increment

n = number of increments

$\bar{D}_{pi}$ = average particle diameter, taken as arithmetic average of smallest and largest particle diameters in increment

Average Particle Size

- The average particle size of mixture particles is most probably expressed by volume-surface mean diameter

$$\bar{D}_s \equiv \frac{6}{\Phi_s A_w \rho_p}$$

- Substituting the equation of surface area in the above equation, then

$$\bar{D}_s = \frac{1}{\sum_{i=1}^{n} (x_i/\bar{D}_{pi})}$$

- If the number of particles in each fraction is known (N_i) , instead of mass fraction, then the arithmetic mean diameter is used:

$$\bar{D}_N = \frac{\sum_{i=1}^{n} (N_i \bar{D}_{pi})}{\sum_{i=1}^{n} N_i} = \frac{\sum_{i=1}^{n} (N_i \bar{D}_{pi})}{N_T}$$

Where the (N_T) is the number of particles in the entire sample

- The mass mean diameter is calculated by:

$$\bar{D}_w = \sum_{i=1}^{n} x_i \bar{D}_{pi}$$

- Dividing the total volume of the sample by the number of particles in the mixture gives **the average volume of a particle**. The diameter of such a particle is **the volume mean diameter** which is calculated by:

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{n} (x_i / \bar{D}_{pi}^3)} \right]^{1/3}$$

Number of Particles in the Mixture

- The volume of one particle is v_p , the number of particles in the sample *N* is

$$N = \frac{m}{\rho_p v_p}$$

- For a given particle shape, the **volume** of any particle is proportional to its **(diameter)**$^{\mathbf{3}}$

$$v_p = aD_p^3$$

*Where **a** is the volume shape factor (assuming it is independent on size))*

- Based on that, the number of particles in the mixture is:

$$N_w = \frac{1}{a\rho_p} \sum_{i=1}^{n} \frac{x_i}{\bar{D}_{pi}^3} = \frac{1}{a\rho_p \bar{D}_V^3}$$

HW1: Full assessment

Mesh	Screen opening D_{pi}, mm	Mass fraction retained, x_i,	Average particle diameter in increment, $\bar{D}_{pi}$, mm	Cumulative fraction smaller than D_{pi}
4	4.699	0.0000	—	1.0000
6	3.327	0.0251	4.013	0.9749
8	2.362	0.1250	2.845	0.8499
10	1.651	0.3207	2.007	0.5292
14	1.168	0.2570	1.409	0.2722
20	0.833	0.1590	1.001	0.1132
28	0.589	0.0538	0.711	0.0594
35	0.417	0.0210	0.503	0.0384
48	0.295	0.0102	0.356	0.0282
65	0.208	0.0077	0.252	0.0205
100	0.147	0.0058	0.178	0.0147
150	0.104	0.0041	0.126	0.0106
200	0.074	0.0031	0.089	0.0075
Pan	—	0.0075	0.037	0.0000

Using the table in the previous slide

a sample of crushed quartz. The density of the particles is 2650 kg/m^3 (0.00265 g/mm^3), and the shape factors are $a = 2$ and $\Phi_s = 0.571$. For the material between 4-mesh and 200-mesh in particle size, calculate (*a*) A_w in square millimeters per gram and N_w in particles per gram, (*b*) $\bar{D}_V$, (*c*) $\bar{D}_s$, (*d*) $\bar{D}_w$, and (*e*) N_i for the 150/200-mesh increment. (*f*) What fraction of the total number of particles is in the 150/200-mesh increment?