Solid Particulates

Motion of Particles in Fluid

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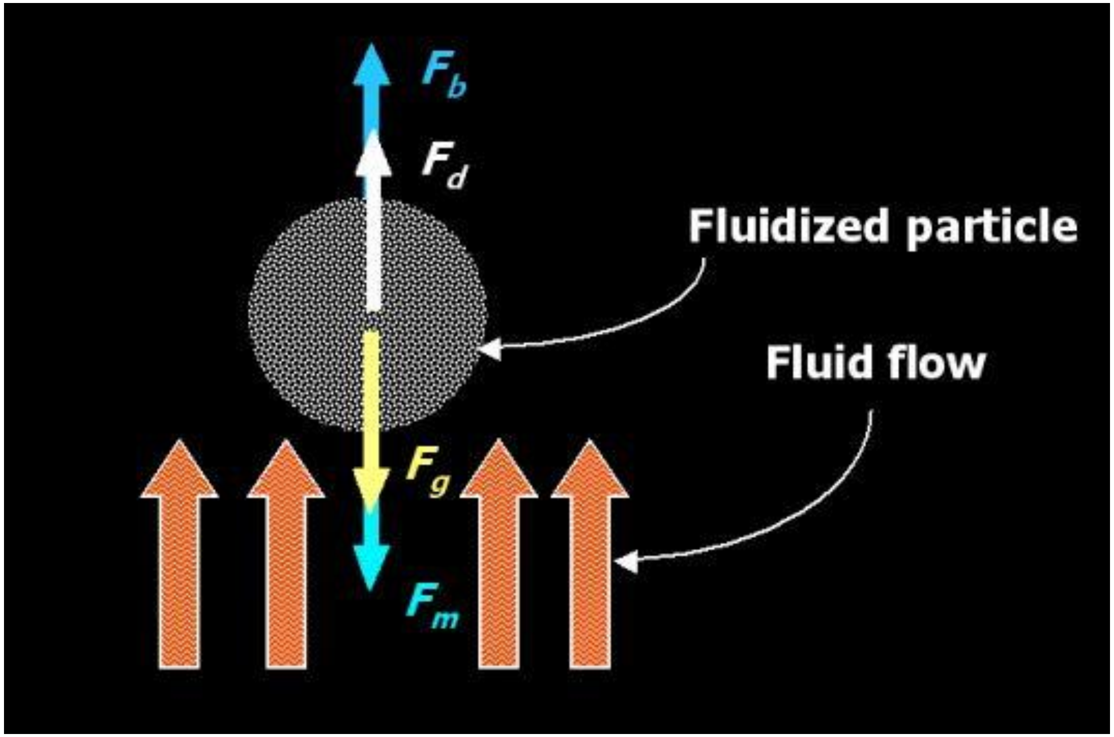
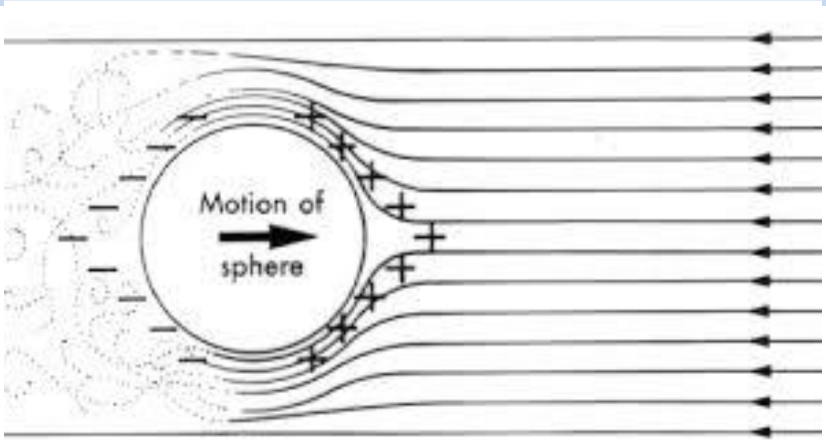
Mechanics of Particle Motion

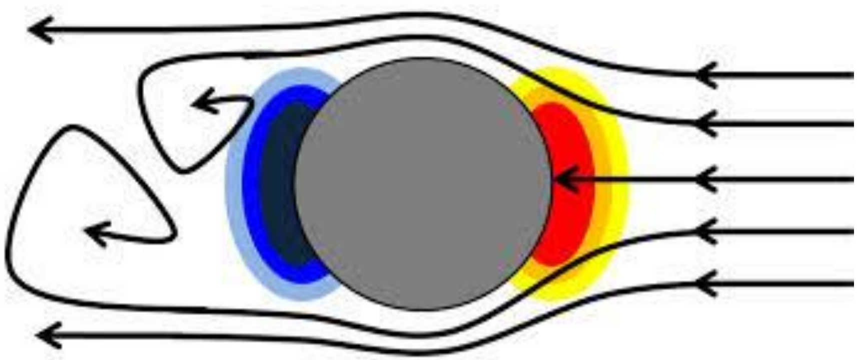
- For a rigid particle moving through a fluid, there are 3 forces acting on the body

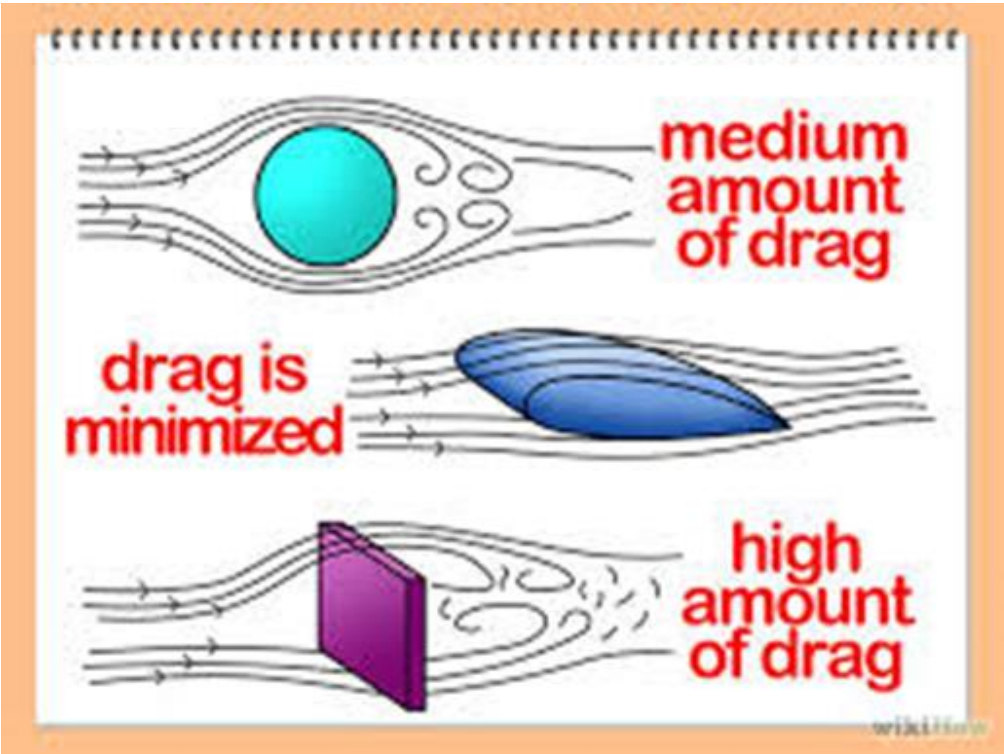
  - The **external force** (gravitational or centrifugal force)
  - The **buoyant force** (opposite but parallel direction to external force)
  - The **drag force** (opposite direction to the particle motion)

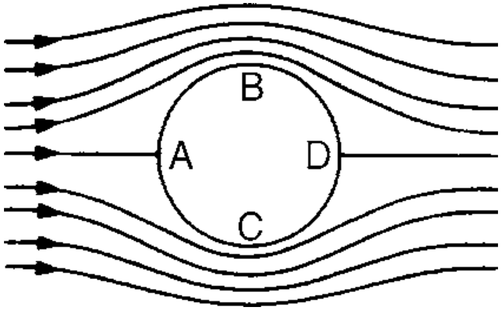


Drag Force


Motion of
sphere




medium amount of drag
drag is minimized
high amount of drag


B
A
D
C

One-dimensional Motion of Particle through Fluid

- Consider a particle of mass **m** moving through a fluid under the action of an external force F_e. Let the velocity of the particle relative to the fluid be u, let the buoyant force on the particle be F_b and let the drag be F_D, then

$$m\frac{du}{dt} = F_e - F_b - F_D \qquad (1)$$

- The external force (F_e) - Expressed as a product of the mass (***m***) and the acceleration (a_e) of the particle from this force

$$F_e = ma_e \qquad (2)$$

The buoyant force (F_b) – Based on Archimedes' law, the product of the mass of the fluid displaced by the particle and the acceleration from the external force.

- The volume of the particle is $$Vp = \frac{m}{\rho_p}$$
- The mass of fluid displaced is $$m = \frac{m}{\rho_p}\rho$$

where ρ is the density of the fluid. The buoyant force is given by

$$F_b = \frac{m\rho a_e}{\rho_p} \qquad (3)$$

The drag force (F_D)

$$F_D = \frac{C_D u^2 \rho A_p}{2} \qquad (4)$$

where C_D is the drag coefficient, A_p is the projected area of the particle in the plane perpendicular to the flow direction.

- By substituting all the forces in the Eq. (1)

$$\frac{du}{dt} = a_e - \frac{\rho a_e}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = a_e \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \qquad (5)$$

Case 1 : Motion from gravitational force

$$\frac{du}{dt} = g \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \qquad (6)$$

Case 2 : Motion in a centrifugal field

$$a_e = r\omega^2$$

$$\frac{du}{dt} = r\omega^2 \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \qquad (7)$$

r = radius of path of particles

ω = angular velocity, rad/s

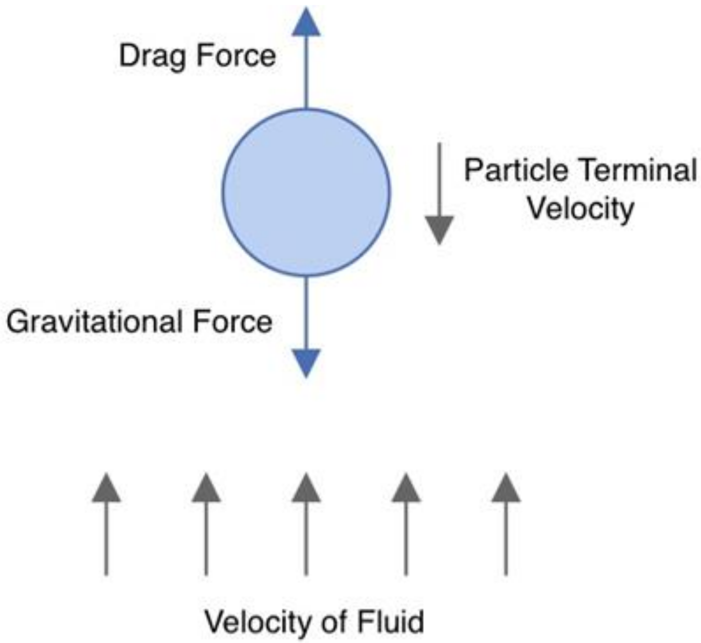
In this equation, ***u*** is the velocity of the particle relative to the fluid and is directed outwardly along a radius.

Terminal Velocity

- **In gravitational settling**, *g* is constant (9.81m/s2)
- The acceleration (a) decreases with time and approaches zero.
- The particle quickly reaches a constant velocity which is the maximum attainable under the circumstances.
- This maximum settling velocity is called **terminal velocity.**

$$\frac{du}{dt} = g\,\frac{\rho_{p-}\rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = 0 \qquad (8)$$

$$u_t = \sqrt{\frac{2g(\rho_{p-}\rho\ )m}{A_p \rho_p C_D \rho}} \qquad (9$$



- For **spherical particle** of diameter D_p moving through the fluid, the terminal velocity is given by

$$m = \frac{1}{6}\pi D_p^3 \rho_p \qquad A_p = \frac{1}{4}\pi D_p^2$$

- Substitution of $\boldsymbol{m}$ and $\boldsymbol{A_p}$ into the equation for $\boldsymbol{u_t}$ gives the equation for **gravity settling** of spheres

$$u_t = \sqrt{\frac{4 g D_p (\rho_p - \rho)}{3 C_D \rho}} \qquad (12)$$

Frequently used

- **In motion from a centrifugal force**, the velocity depends on the radius
- The acceleration is not constant if the particle is in motion with respect to the fluid.
- In many practical use of centrifugal force, is small ($\frac{du}{dt} = \sim 0$) thus, it can be neglected to give

$$\frac{du}{dt} = r\omega^2 \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = 0 \qquad (10)$$

$$u_t = \omega \sqrt{\frac{2r(\rho_p - \rho)m}{A_p \rho_p C_D \rho}} \qquad (11)$$

Reynolds Number

- Particle Reynolds Number

$$Re = \frac{uD_p\rho}{\mu}$$

u : velocity of fluid stream
D_p : diameter of the particle
ρ : density of fluid
μ : viscosity of fluid

- For the case of creeping flow, that is flow at very low velocities relative to the sphere, the Navier–Stokes equations, give:

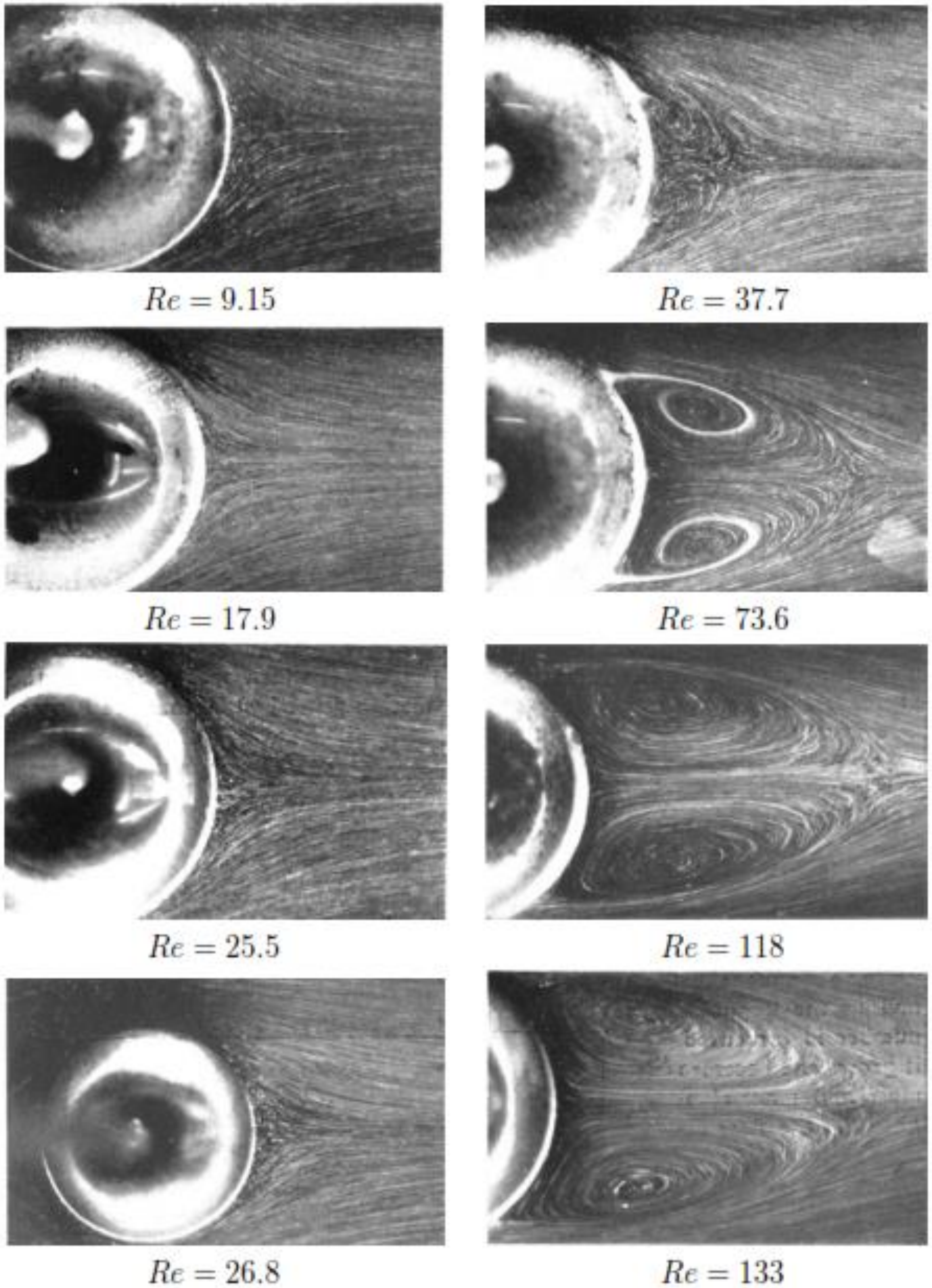
$$F = 3\pi\mu d u$$

$$\left.\begin{array}{ll}\text{(i) skin friction:} & 2\pi\mu d u \\ \text{(ii) form drag:} & \pi\mu d u\end{array}\right\} \text{ total } 3\pi\mu d u$$

- This equation is known as **Stokes' law** and it is applicable only at very low values of the particle Reynolds number and deviations become progressively greater as ***Re*** increases.

- As R_e increases, skin friction becomes proportionately less and, at values greater than about 20, flow separation occurs with the formation of vortices in the wake of the sphere.

- At high Reynolds numbers, the size of the vortices progressively increases until, at values of between 100 and 200, instabilities in the flow give rise to vortex shedding. The effect of these changes in the nature of the flow on the force exerted on the particle is now considered.



Drag Coefficient

- **Drag coefficient** is a function of Reynolds number (N_{RE}).
- The drag curve applies only under restricted conditions:
  i. The particle must be a solid sphere;
  ii. The particle must be far from other particles and the vessel wall so that the flow pattern around the particle is not distorted;
  iii. It must be moving at its terminal velocity with respect to the fluid.
- The most satisfactory way of representing the relation between drag force and velocity involves the use of two dimensionless groups:

The first group is the particle Reynolds number $Re'(= ud\rho/\mu)$.

The second is the group $R'/\rho u^2$, in which R' is the force per unit projected area of particle in a plane perpendicular to the direction of motion.

- For a sphere, the projected area is that of a circle of the same diameter as the sphere.

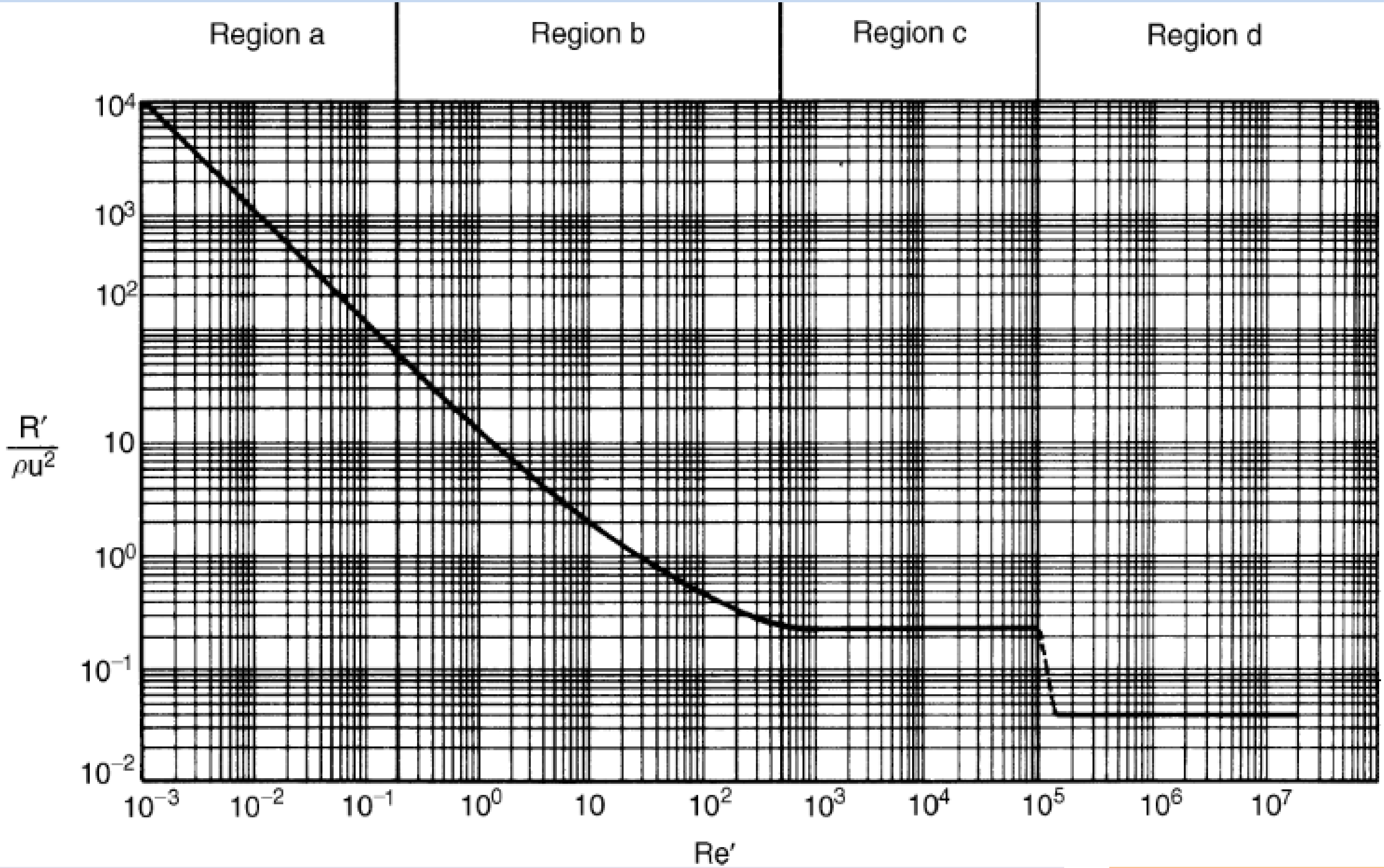
Thus:
$$R' = \frac{F}{(\pi d^2/4)}$$

and
$$\frac{R'}{\rho u^2} = \frac{4F}{\pi d^2 \rho u^2}$$

$R'/\rho u^2$ is a form of *drag coefficient*, often denoted by the symbol C'_D. Frequently, a drag coefficient C_D is defined as the ratio of R' to $\frac{1}{2}\rho u^2$.

Thus:
$$C_D = 2C'_D = \frac{2R'}{\rho u^2}$$

It is seen that C'_D is analogous to the friction factor $\phi(= R/\rho u^2)$ for pipe flow, and C_D is analogous to the Fanning friction factor f.


Region a
Region b
Region c
Region d
$\frac{R'}{\rho u^2}$
10^4
10^3
10^2
10
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10
10^2
10^3
10^4
10^5
10^6
10^7
Re′

Region (a) ($10^{-4} < Re' < 0.2$)

At very **low values of the Reynolds number**, the force ***F*** is given by **Stokes' law**:

$$\frac{R'}{\rho u^2} = 12\frac{\mu}{ud\rho} = 12Re'^{-1}$$

In this region, the relationship between $\frac{R'}{\rho u^2}$ and Re' is a straight line of slope -1

The limit of 10^{-4} is imposed because reliable experimental measurements have not been made at lower values of Re'.

$$R' = 12\rho u^2 \left(\frac{\mu}{ud\rho}\right) = \frac{12u\mu}{d}$$

$$F = \frac{12u\mu}{d}\frac{1}{4}\pi d^2 = 3\pi\mu du$$

Region (b) ($0.2 < Re' < 500$–1000)

In this region, the slope of the curve changes progressively from -1 to 0 as Re' increases.

Thus:

$$\frac{R'}{\rho u^2} = 12Re'^{-1} + 0.22$$

Stokes' law

Additional non-viscous effects

- A reasonable approximation for values of Re' up to about 1000:

$$\frac{R'}{\rho u^2} = 12Re'^{-1}(1 + 0.15Re'^{0.687})$$

$$R' = \frac{12u\mu}{d}(1 + 0.15Re'^{0.687})$$

$$F = 3\pi\mu du(1 + 0.15Re'^{0.687})$$

Region (c) ($500\text{–}1000 < Re' <$ ca 2×10^5)

In this region, *Newton's law* is applicable and the value of $R'/\rho u^2$ is approximately constant giving:

$$\frac{R'}{\rho u^2} = 0.22$$

$$R' = 0.22\rho u^2$$

$$F = 0.22\rho u^2 \tfrac{1}{4}\pi d^2 = 0.055\pi d^2 \rho u^2$$

Region (d) ($Re' >$ ca 2×10^5)

When Re' exceeds about 2×10^5, the flow in the boundary layer changes from streamline to turbulent and the separation takes place nearer to the rear of the sphere. The drag force is decreased considerably and:

$$\frac{R'}{\rho u^2} = 0.05$$

$$R' = 0.05\rho u^2$$

$$F = 0.0125\pi d^2 \rho u^2$$

Terminal falling velocities

- If a spherical particle is allowed to settle in a fluid under gravity, its velocity will increase until the accelerating force is exactly balanced by the resistance force.

- The accelerating force due to gravity is given by:

$$= (\tfrac{1}{6}\pi d^3)(\rho_s - \rho)g$$

where ρ_s is the density of the solid.

The terminal falling velocity u_0 corresponding to region (a) is given by:

Or if the particle has started from rest, the drag force is given by :

$$(\tfrac{1}{6}\pi d^3)(\rho_s - \rho)g = 3\pi \mu d u_0$$

and:

$$u_0 = \frac{d^2 g}{18\mu}(\rho_s - \rho)$$

The terminal falling velocity corresponding to region (c) is given by:

$$(\tfrac{1}{6}\pi d^3)(\rho_s - \rho)g = 0.055\pi d^2 \rho u_0^2$$

or:

$$u_0^2 = 3dg\frac{(\rho_s - \rho)}{\rho}$$

Under terminal falling conditions, velocities are rarely high enough for ***Re'*** to approach 10^5, with the small particles generally used in industry.

Assumptions

In the expressions given for the drag force and the terminal falling velocity, the following assumptions have been made:

A. That the settling is not affected by the presence of other particles in the fluid. This condition is known as “**free settling**”. When the interference of other particles is appreciable, the process is known as “**hindered settling**”.

B. That the walls of the containing vessel do not exert an appreciable retarding effect.

C. That the fluid can be considered as a continuous medium, that is the particle is large compared with the mean free path of the molecules of the fluid, otherwise the particles may occasionally “slip” between the molecules and thus attain a velocity higher than that calculated.

Terminal Velocity for two materials

- If for a particle of material **A** of diameter $\boldsymbol{d_A}$ and density $\boldsymbol{\rho_A}$, Stokes' law is applicable, then the terminal falling velocity $\boldsymbol{u_{0A}}$ is given by equation

$$u_{0A} = \frac{d_A^2 g}{18\mu}(\rho_A - \rho)$$

Similarly, for a particle of material B:

$$u_{0B} = \frac{d_B^2 g}{18\mu}(\rho_B - \rho)$$

The condition for the two terminal velocities to be equal is then:

$$\frac{d_B}{d_A} = \left(\frac{\rho_A - \rho}{\rho_B - \rho}\right)^{1/2}$$

If Newton's law is applicable, equation 3.25 holds and:

$$u_{0A}^2 = \frac{3 d_A g(\rho_A - \rho)}{\rho}$$

and

$$u_{0B}^2 = \frac{3 d_B g(\rho_B - \rho)}{\rho}$$

For equal settling velocities:

$$\frac{d_B}{d_A} = \left(\frac{\rho_A - \rho}{\rho_B - \rho}\right)$$

In general, the relationship for equal settling velocities is:

$$\frac{d_B}{d_A} = \left(\frac{\rho_A - \rho}{\rho_B - \rho}\right)^S$$

where $S = \frac{1}{2}$ for the Stokes' law region, $S = 1$ for Newton's law and, as an approximation, $\frac{1}{2} < S < 1$ for the intermediate region.

Galileo number (Ga)

The dimensionless group $(R_0'/\rho u_0^2)Re_0'^2$ does not involve u_0 since:

$$\frac{R_0'}{\rho u_0^2}\frac{u_0^2 d^2 \rho^2}{\mu^2} = \frac{2dg(\rho_s - \rho)}{3\rho u_0^2}\frac{u_0^2 d^2 \rho^2}{\mu^2}$$

$$= \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$

The group $\dfrac{d^3\rho(\rho_s - \rho)g}{\mu^2}$ is known as the Galileo number Ga

$$\frac{R_0'}{\rho u_0^2} Re_0'^2 = \tfrac{2}{3} Ga$$

Table 3.4. Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{2}$								$\bar{3}.620$	$\bar{3}.720$	$\bar{3}.819$
$\bar{1}$	$\bar{3}.919$	$\bar{2}.018$	$\bar{2}.117$	$\bar{2}.216$	$\bar{2}.315$	$\bar{2}.414$	$\bar{2}.513$	$\bar{2}.612$	$\bar{2}.711$	$\bar{2}.810$
0	$\bar{2}.908$	$\bar{1}.007$	$\bar{1}.105$	$\bar{1}.203$	$\bar{1}.301$	$\bar{1}.398$	$\bar{1}.495$	$\bar{1}.591$	$\bar{1}.686$	$\bar{1}.781$
1	$\bar{1}.874$	$\bar{1}.967$	0.008	0.148	0.236	0.324	0.410	0.495	0.577	0.659
2	0.738	0.817	0.895	0.972	1.048	1.124	1.199	1.273	1.346	1.419
3	1.491	1.562	1.632	1.702	1.771	1.839	1.907	1.974	2.040	2.106
4	2.171	2.236	2.300	2.363	2.425	2.487	2.548	2.608	2.667	2.725
5	2.783	2.841	2.899	2.956	3.013	3.070	3.127	3.183	3.239	3.295

Example

What is the terminal velocity of a spherical steel particle, 0.40 mm in diameter, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of steel is 7870 kg/m^3.

Solution

For a sphere:

$$\frac{R_0'}{\rho u_0^2} Re_0'^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$

$$= \frac{2 \times 0.0004^3 \times 820(7870 - 820)9.81}{3(10 \times 10^{-3})^2}$$

$$= 24.2$$

$$\log_{10} 24.2 = 1.384$$

From Table 3.4: $\log_{10} Re_0' = 0.222$

Thus: $Re_0' = 1.667$

and: $$u_0 = \frac{1.667 \times 10 \times 10^{-3}}{820 \times 0.0004}$$

$$= 0.051 \text{ m/s or } \underline{\underline{51 \text{ mm/s}}}$$

Non-Spherical Particles

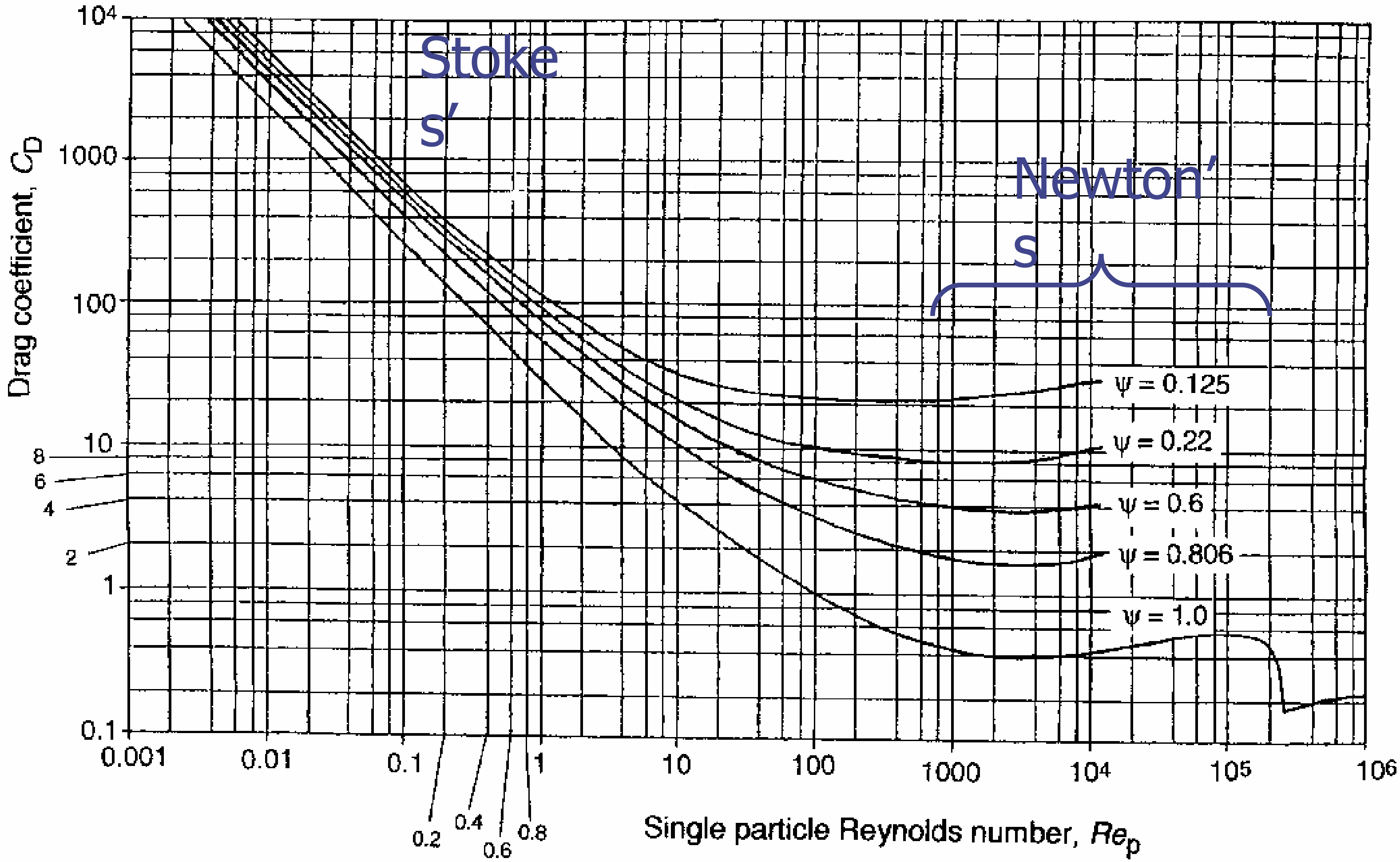


Figure 1.3 Drag coefficient C_D versus Reynolds number Re_p for particles of sphericity ψ ranging form 0.125 to 1.0 (note Re_p uses the equivalent volume diameter)

Terminal falling velocities of non-spherical particles

For a non-spherical particle:

$$\text{total drag force, } F = R'_0 \tfrac{1}{4}\pi d_p^2 = (\rho_s - \rho) g k' d_p^3$$

Thus:

$$\frac{R'_0}{\rho u_0^2} = \frac{4k' d_p g}{\pi \rho u_0^2}(\rho_s - \rho)$$

$$\frac{R'_0}{\rho u_0^2} Re_0'^2 = \frac{4k' \rho d_p^3 g}{\mu^2 \pi}(\rho_s - \rho)$$

and:

$$\frac{R'_0}{\rho u_0^2} Re_0'^{-1} = \frac{4k' \mu g}{\pi \rho^2 u_0^3}(\rho_s - \rho)$$

Heywood Approach

- A mean projected diameter of the particle ***dp*** is defined as the diameter of a circle having the same area as the particle when viewed from above and lying in its most stable position.

- Volume= $K' d_p^3$

If d_p is the mean projected diameter, the mean projected area is $\pi d_p^2/4$ and the volume is $k' d_p^3$, where k' is a constant whose value depends on the shape of the particle. For a spherical particle, k' is equal to $\pi/6$. For rounded isometric particles, that is particles in which the dimension in three mutually perpendicular directions is approximately the same, k' is about 0.5, and for angular particles k' is about 0.4. For most minerals k' lies between 0.2 and 0.5.

Table 3.7. Corrections to $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{2}$	−0.022	−0.002	+0.032	+0.131
$\bar{1}$	−0.023	−0.003	+0.030	+0.131
0	−0.025	−0.005	+0.026	+0.129
1	−0.027	−0.010	+0.021	+0.122
2	−0.031	−0.016	+0.012	+0.111
2.5	−0.033	−0.020	0.000	+0.080
3	−0.038	−0.032	−0.022	+0.025
3.5	−0.051	−0.052	−0.056	−0.040
4	−0.068	−0.074	−0.089	−0.098
4.5	−0.083	−0.093	−0.114	−0.146
5	−0.097	−0.110	−0.135	−0.186
5.5	−0.109	−0.125	−0.154	−0.224
6	−0.120	−0.134	−0.172	−0.255

Table 3.8. Corrections to $\log Re'$ as a function of $\{\log(R'/\rho u^2)Re'^{-1}\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^{-1}\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{4}$	+0.185	+0.217	+0.289	
$\bar{4}.5$	+0.149	+0.175	+0.231	
$\bar{3}$	+0.114	+0.133	+0.173	+0.282
$\bar{3}.5$	+0.082	+0.095	+0.119	+0.170
$\bar{2}$	+0.056	+0.061	+0.072	+0.062
$\bar{2}.5$	+0.038	+0.034	+0.033	−0.018
$\bar{1}$	+0.028	+0.018	+0.007	−0.053
$\bar{1}.5$	+0.024	+0.013	−0.003	−0.061
0	+0.022	+0.011	−0.007	−0.062
1	+0.019	+0.009	−0.008	−0.063
2	+0.017	+0.007	−0.010	−0.064
3	+0.015	+0.005	−0.012	−0.065
4	+0.013	+0.003	−0.013	−0.066
5	+0.012	+0.002	−0.014	−0.066

Example

What will be the terminal velocities of mica plates, 1 mm thick and ranging in area from 6 to 600 mm^2, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of mica is 3000 kg/m^3.

Solution

	smallest particles	largest particles
A'	6×10^{-6} m^2	6×10^{-4} m^2
d_p	$\sqrt{(4 \times 6 \times 10^{-6}/\pi)} = 2.76 \times 10^{-3}$ m	$\sqrt{(4 \times 6 \times 10^{-4}/\pi)} = 2.76 \times 10^{-2}$ m
d_p^3	2.103×10^{-8} m^3	2.103×10^{-5} m^3
volume	6×10^{-9} m^3	6×10^{-7} m^3
k'	0.285	0.0285

$$\left(\frac{R'_0}{\rho u^2}\right) Re_0'^2 = \frac{4k'}{\mu^2 \pi}(\rho_s - \rho)\rho d_p^3 g \qquad \text{(equation 3.52)}$$

$$= (4 \times 0.285/\pi \times 0.01^2)(3000 - 820)(820 \times 2.103 \times 10^{-8} \times 9.81)$$

$= 1340$ for the smallest particles and, similarly, 134,000 for the largest particles.

Thus:

	smallest particles	largest particles
$\log\left(\frac{R'_0}{\rho u_0^2} Re_0'^2\right)$	3.127	5.127
$\log Re'_0$	1.581	2.857 (from Table 3.4)
Correction from Table 3.6	−0.038	−0.300 (estimated)
Corrected $\log Re'_0$	1.543	2.557
Re'_0	34.9	361
u_0	0.154 m/s	0.159 m/s

Thus it is seen that all the mica particles settle at approximately the same velocity.