Solid Particulates

Flow Through Granular Beds and Packed Column

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Introduction

- Most of technical process, liquid or gases flow through beds of solid particles.

  Example:

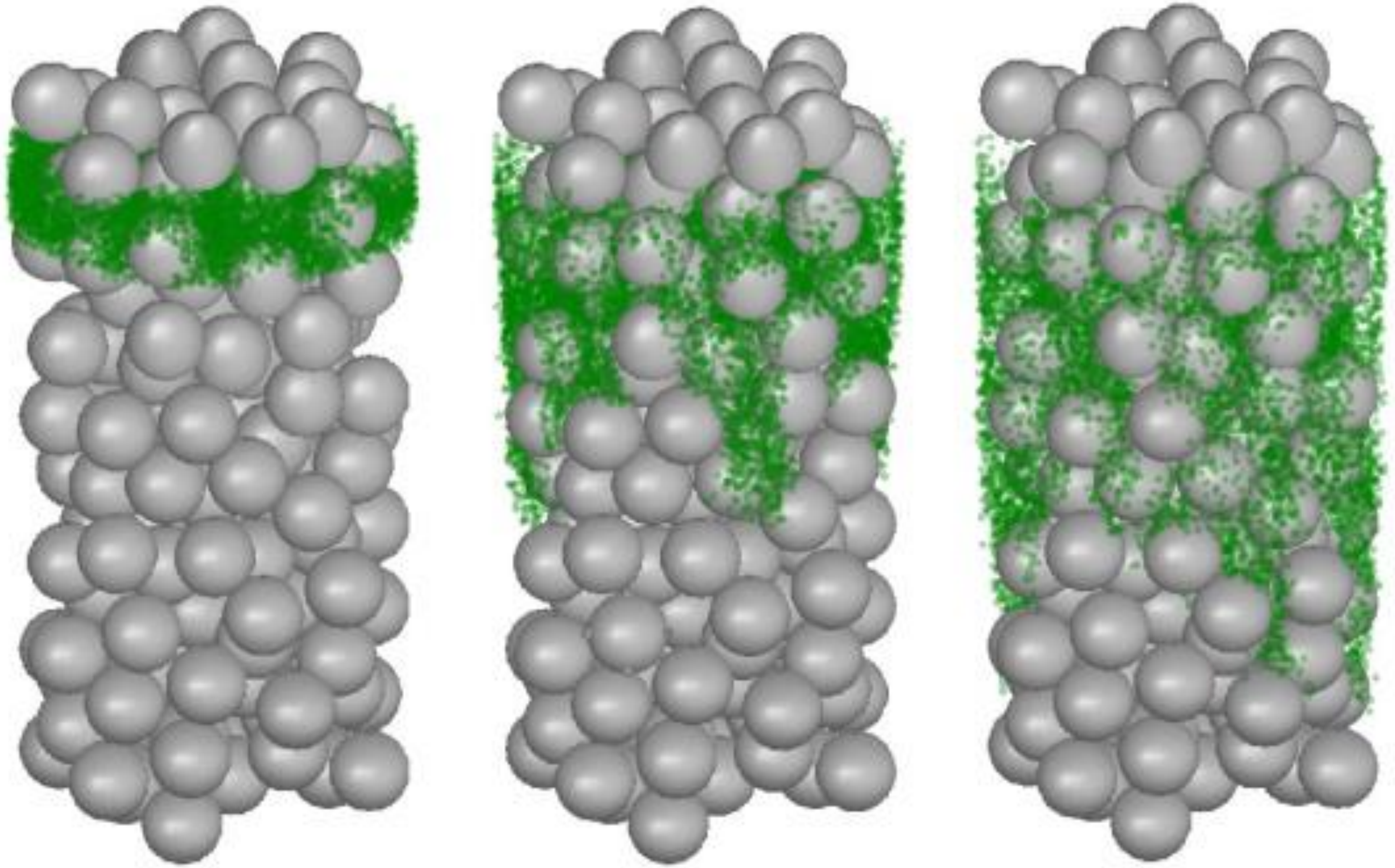
  i. A single fluid flow through a bed of granular solid

  ii. Two phase countercurrent flow of liquid and gas through packed columns.

- Single fluid flow through a granular bed or porous medium involves in;

  - ✓ fixed bed reactor
  - ✓ filtration
  - ✓ adsorption
  - ✓ seepage of underground water or petroleum

Porous packed bed



Darcy Law and Permeability

- Permeability measurement is conducted to determine the surface power

- **Darcy's Law:** the average velocity, as measured over the whole area of the bed, is directly proportional to the driving pressure and inversely proportional to the thickness of the bed.

$$u_c = \frac{K(-\Delta P)}{l} = B\frac{(-\Delta P)}{\mu l}$$

where $-\Delta P$ is the pressure drop across the bed,
l is the thickness of the bed,
u_c is the average velocity of flow of the fluid, defined as $(1/A)(dV/dt)$,
A is the total cross sectional area of the bed,
V is the volume of fluid flowing in time t, and
K is a constant depending on the physical properties of the bed and fluid.

where μ is the viscosity of the fluid and B is termed the permeability coefficient for the bed, and depends only on the properties of the bed.

Specific surface and Voidage

The general structure of a bed of particles can often be characterised by the specific surface area of the bed S_B and the fractional voidage of the bed e.

S_B is the surface area presented to the fluid per unit volume of bed when the particles are packed in a bed. Its units are $(\text{length})^{-1}$.

- Voidage/porosity (ε) -The fraction of the volume of the bed not occupied by solid material. It is dimensionless and given by;

$$\boldsymbol{\varepsilon} = \frac{\text{volume of the bed} - \text{volume of particles}}{\text{volume of bed}}$$

$$= 1 - \frac{\text{volume of particles}}{\text{volume of bed}}$$

Specific surface area of the particles (S or a_V)

- The Specific surface area of the particles equals to the surface area of a particle divided by its volume. Its units are (length)^-1

$$a_v = \frac{S_p}{v_p}$$

S_p : surface area of a particle in m^2
v_p: volume of particle in m^3

- For a spherical particle,

$$a_v = \frac{6}{D_p}$$

D_p : diameter in m

$$S = \frac{\pi d^2}{\pi (d^3/6)} = \frac{6}{d}$$

- For a packed bed of non-spherical particle, the effective particle diameter D_p is defined as

$$D_p = \frac{6}{a_v}$$

Since $(1-\varepsilon)$ is the volume fraction of particles in the bed,

$$a = a_v(1-\varepsilon) = \frac{6}{D_p}(1-\varepsilon)$$

where a is the ratio of total surface area in the bed to total volume of bed (void volume plus particle volume) in m^{-1}.

Table 4.1. Properties of beds of some regular-shaped materials (R&C page 193)

Example

A packed bed is composed of cylinders having a diameter $D = 0.02$ m and a length $h = D$. The bulk density of the overall packed bed is 962 kg/m^3 and the density of the solid cylinders is 1600 kg/m^3.

(a) Calculate the void fraction ε.

(b) Calculate the effective diameter D_p of the particles.

(c) Calculate the value of a

Solution: For part (a), taking 1.00 m^3 of packed bed as a basis, the total mass of the bed is (962 kg/m^3)(1.00 m^3) = 962 kg. This mass of 962 kg is also the mass of the solid cylinders. Hence, volume of cylinders = 962 kg/(1600 kg/m^3) = 0.601 m^3. Using Eq. (3.1-6),

$$\varepsilon = \frac{\text{volume of voids in bed}}{\text{total volume of bed}} = \frac{1.000 - 0.601}{1.000} = 0.399$$

For the effective particle diameter D_p in part (b), for a cylinder where $h = D$, the surface area of a particle is

$$S_p = (2)\frac{\pi D^2}{4}\text{ (ends)} + \pi D(D)\text{ (sides)} = \tfrac{3}{2}\pi D^2$$

The volume v_p of a particle is

$$v_p = \frac{\pi}{4}D^2(D) = \frac{\pi D^3}{4}$$

Then,

$$a_v = \frac{S_p}{v_p} = \frac{\frac{3}{2}\pi D^2}{\frac{1}{4}\pi D^3} = \frac{6}{D}$$

Finally,

$$D_p = \frac{6}{a_v} = \frac{6}{6/D} = D = 0.02 \text{ m}$$

Hence, the effective diameter to use is $D_p = D = 0.02$ m. For part (c),

$$a = \frac{6}{D_p}(1 - \varepsilon) = \frac{6}{0.02}(1 - 0.399) = 180.3 \text{ m}^{-1}$$

Fluid flow through beds (Carman–Kozeny equations)

Streamline flow—Carman–Kozeny equation

- The pore space in the bed is assumed to be a tube with equivalent diameter which satisfies the following assumptions:
  - The internal surface area is equal to the surface area of particles
  - The free space is equal to that in granular bed.

- If the free space in the bed is assumed to consist of a series of tortuous channels, the equation for streamline flow through a circular tube is:

$$u_1 = \frac{d_m'^2}{K'\mu} \frac{(-\Delta P)}{l'}$$

where: d_m' is some equivalent diameter of the pore channels,
K' is a dimensionless constant whose value depends on the structure of the bed,
l' is the length of channel, and
u_1 is the average velocity through the pore channels.
μ is the viscosity of the fluid,

$$d'_m = \frac{e}{S_B} = \frac{e}{S(1-e)}$$

$$\frac{e}{S_B} = \frac{\text{volume of voids filled with fluid}}{\text{wetted surface area of the bed}}$$

$$= \frac{\text{cross-sectional area normal to flow}}{\text{wetted perimeter}}$$

$$\frac{e}{S_B} = \frac{1}{4}\ (\text{hydraulic mean diameter})$$

$$S_B = S(1-e)$$

e is the fractional voidage

Then taking $u_1 = u_c/e$ and $l' \propto l$,

$$u_c = \frac{1}{K''}\frac{e^3}{S_B^2}\frac{1}{\mu}\frac{(-\Delta P)}{l}$$

$$= \frac{1}{K''}\frac{e^3}{S^2(1-e)^2}\frac{1}{\mu}\frac{(-\Delta P)}{l}$$

K'' is generally known as Kozeny's constant and a commonly accepted value for K'' **is 5**

K'' is dependent on porosity, particle shape, and other factors.

The permeability coefficient is given by:

$$B = \frac{1}{K''}\frac{e^3}{S^2(1-e)^2}$$

Inserting a value of 5 for K''

$$u_c = \frac{1}{5}\frac{e^3}{(1-e)^2}\frac{-\Delta P}{S^2 \mu l}$$

For spheres: $S = 6/d$ and:

$$u_c = \frac{1}{180}\frac{e^3}{(1-e)^2}\frac{-\Delta P d^2}{\mu l}$$

$$= 0.0055\frac{e^3}{(1-e)^2}\frac{-\Delta P d^2}{\mu l}$$

For non-spherical particles, the Sauter mean diameter d_s should be used in place of d.

Streamline and turbulent flow

Since equation applies to streamline flow conditions, though CARMAN and others have extended the analogy with pipe flow to cover both streamline and turbulent flow conditions through packed beds.

In this treatment a modified friction factor $R_1/\rho u^2_1$ is plotted against a modified Reynolds number Re_1. This is analogous to plotting $R/\rho u^2$ against Re for flow through a pipe

The modified Reynolds number Re_1 is obtained by taking the same velocity and characteristic linear dimension d'_m as were used in deriving equation 4.9. Thus:

$$Re_1 = \frac{u_c}{e} \frac{e}{S(1-e)} \frac{\rho}{\mu}$$

$$= \frac{u_c \rho}{S(1-e)\mu}$$

*R*1 can be related to the properties of the bed and pressure gradient as follows:

Considering the forces acting on the fluid in a bed of unit cross-sectional area and thickness *l*, the volume of particles in the bed is $l(1-e)$ and therefore the total surface is $S\,l(1-e)$.

Thus the resistance force is $R_1 S\,l(1-e)$ This force on the fluid must be equal to that produced by a pressure difference of ΔP across the bed. Then, since the free cross-section of fluid is equal to ***e***:

$$(-\Delta P)e = R_1 S l(1-e)$$

and

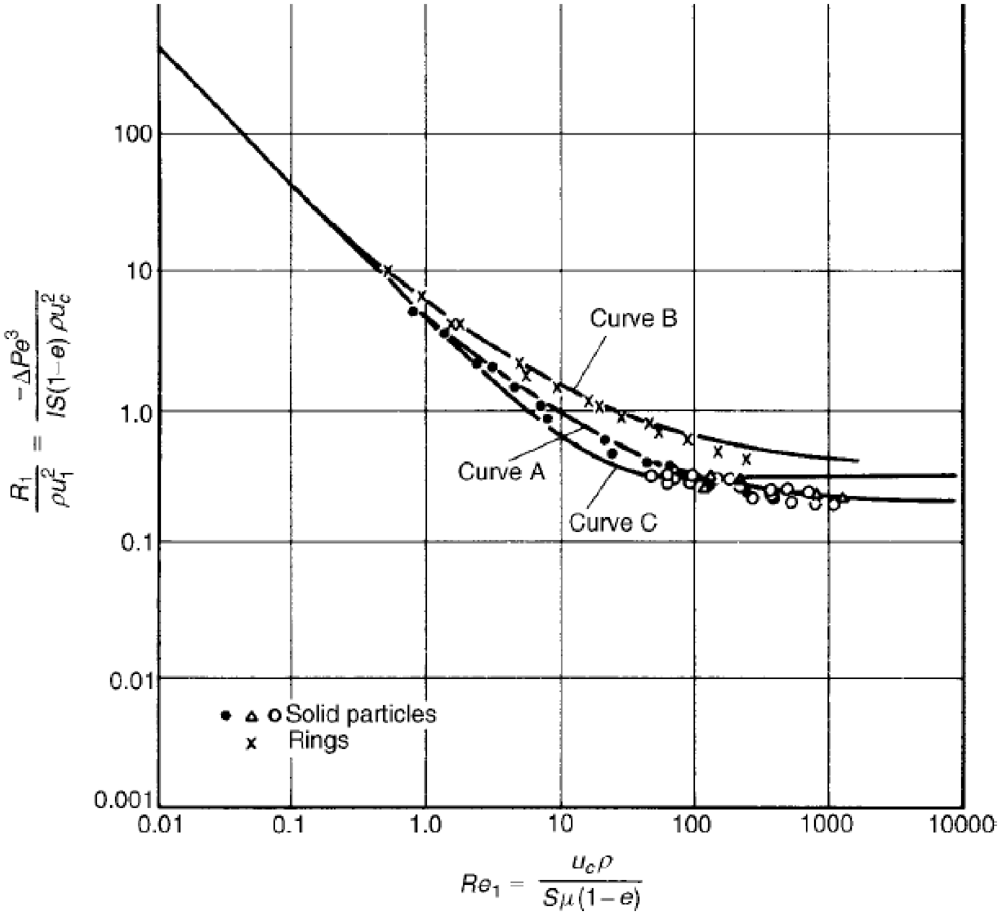
$$R_1 = \frac{e}{S(1-e)}\frac{(-\Delta P)}{l}$$

Thus

$$\frac{R_1}{\rho u_1^2} = \frac{e^3}{S(1-e)}\frac{(-\Delta P)}{l}\frac{1}{\rho u_c^2}$$

Carman found that when $R_1/\rho u_1^2$ was plotted against Re_1 using logarithmic coordinates, his data for the flow through randomly packed beds of solid particles could be correlated approximately by a single curve (curve A, Figure 4.1), whose general equation is:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + 0.4Re_1^{-0.1} \tag{4.16}$$



From equation 4.16 it can be seen that for values of Re_1 less than about 2, the second term is small and, approximately:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} \tag{4.18}$$

$$u_c = \frac{1}{5}\left(\frac{1}{1-e}\right)\left(\frac{\rho u_c^2}{S\mu}\right)\left(\frac{R_1}{\rho u_1^2}\right)$$

$$\frac{R_1}{\rho u_1^2} = 5\left(\frac{S(1-e)\mu}{u_c\rho}\right)$$

$$= 5Re_1^{-1}$$

As the value of Re_1 increases from about 2 to 100, the second term in equation 4.16 becomes more significant and the slope of the plot gradually changes from -1.0 to about $-\frac{1}{4}$. Above Re_1 of 100 the plot is approximately linear. The change from complete streamline flow to complete turbulent flow is very gradual because flow conditions are not the same in all the pores. Thus, the flow starts to become turbulent in the larger pores, and subsequently in successively smaller pores as the value of Re_1 increases.

Rings, which as described later are often used in industrial packed columns, tend to deviate from the generalised curve A on Figure 4.1 particularly at high values of Re_1.

Sawistowski[9] compared the results obtained for flow of fluids through beds of hollow packings (discussed later) and has noted that equation 4.16 gives a consistently low result for these materials. He proposed:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + Re_1^{-0.1} \tag{4.19}$$

This equation is plotted as curve B in Figure 4.1.

Ergun semi-empirical correlation

For flow through ring packings which as described later are often used in industrial packed columns, ERGUN[10] obtained a good semi-empirical correlation for pressure drop as follows:

$$\frac{-\Delta P}{l} = 150\frac{(1-e)^2}{e^3}\frac{\mu u_c}{d^2} + 1.75\frac{(1-e)}{e^3}\frac{\rho u_c^2}{d} \tag{4.20}$$

Writing $d = 6/S$ (from equation 4.3):

$$\frac{-\Delta P}{Sl\rho u_c^2}\frac{e^3}{1-e} = 4.17\frac{\mu S(1-e)}{\rho u_c} + 0.29$$

or:

$$\frac{R_1}{\rho u_1^2} = 4.17 Re_1^{-1} + 0.29 \tag{4.21}$$

This equation is plotted as curve C in Figure 4.1. The form of equation 4.21 is somewhat similar to that of equations 4.16 and 4.17, in that the first term represents viscous losses which are most significant at low velocities and the second term represents kinetic energy losses which become more significant at high velocities. The equation is thus applicable over a wide range of velocities and was found by Ergun to correlate experimental data well for values of $Re_1/(1-e)$ from 1 to over 2000.

Dependence of K`` on bed structure

Tortuosity

Carman[1] has shown that:

$$K'' = \left(\frac{l'}{l}\right)^2 \times K_0 \qquad (4.22)$$

where (l'/l) is the tortuosity and is a measure of the fluid path length through the bed compared with the actual depth of the bed,

K_0 is a factor which depends on the shape of the cross-section of a channel through which fluid is passing.

Non-spherical particles

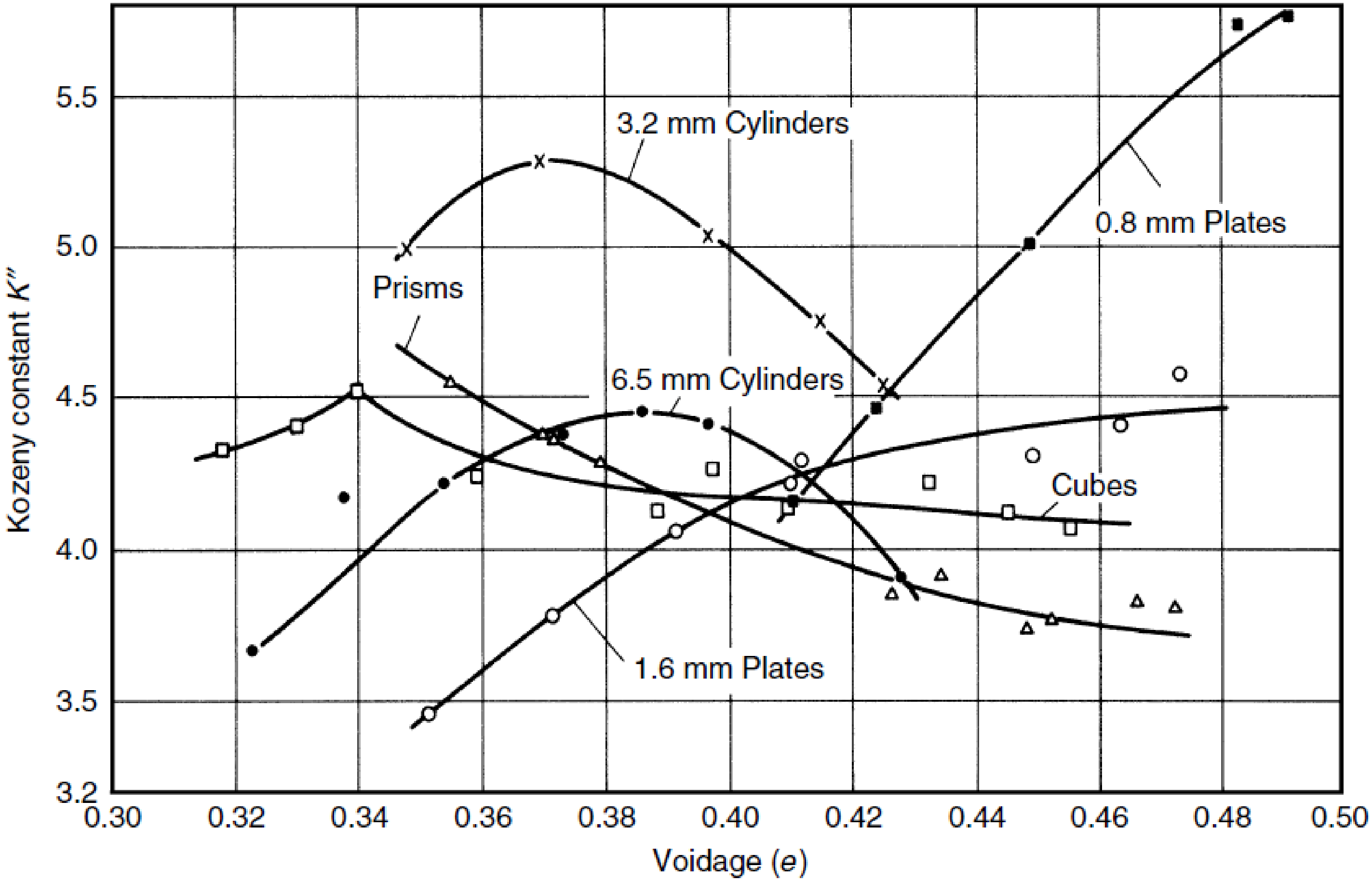


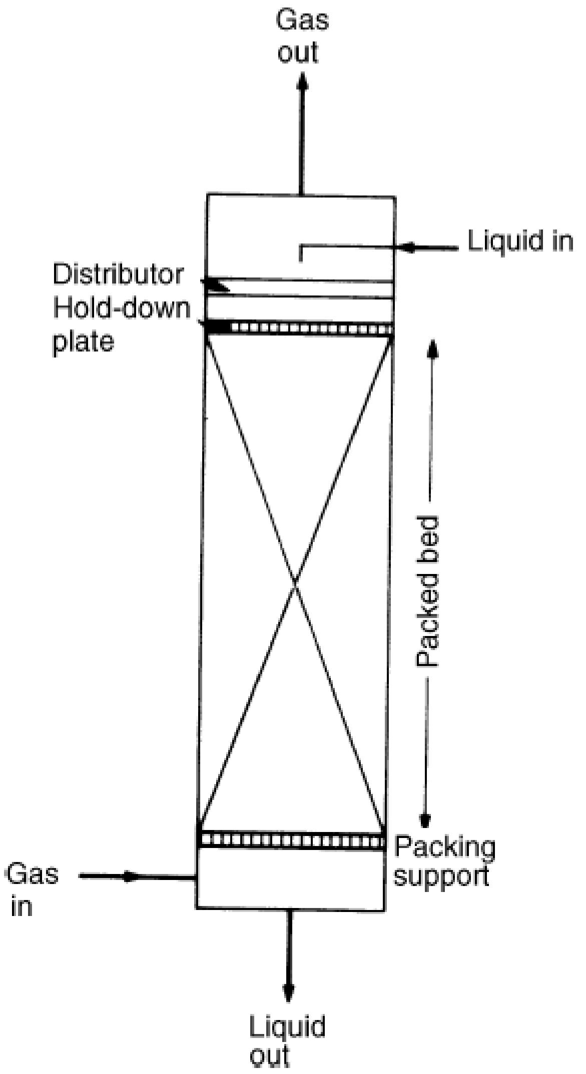
Figure 4.2. Variation of Kozeny's constant K'' with voidage for various shapes

Table 4.2. Experimental values of K'' for beds of high porosity

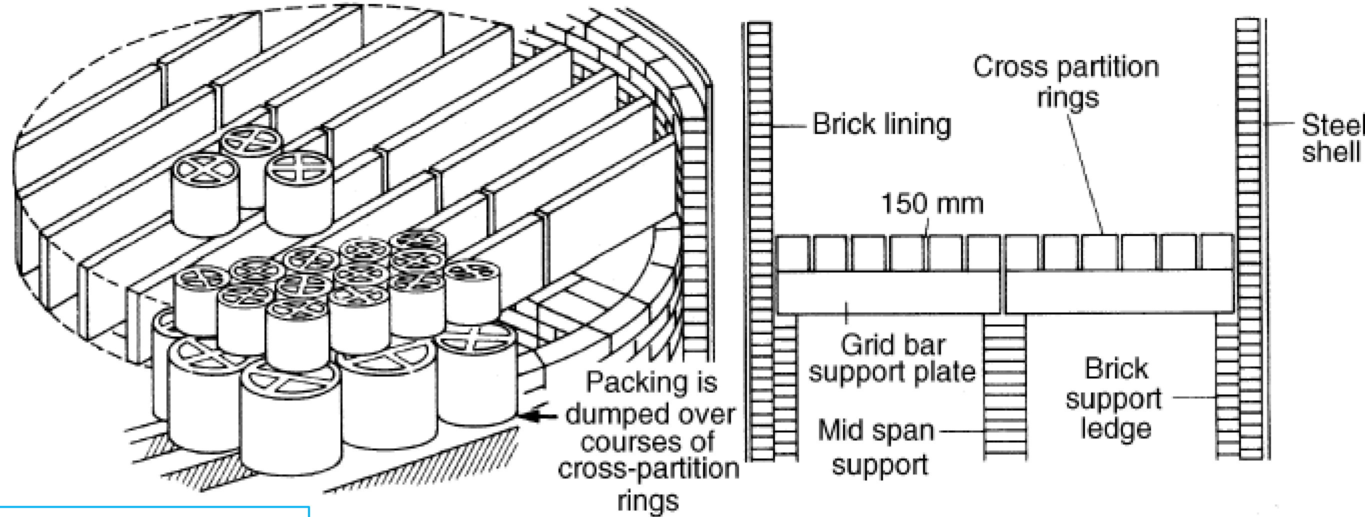
Voidage e	Experimental value of K''		
	BRINKMAN[3]	DAVIES[21]	Silk fibres LORD[20]
0.5	5.5		
0.6	4.3		
0.8	5.4	6.7	5.35
0.9	8.8	9.7	6.8
0.95	15.2	15.3	9.2
0.98	32.8	27.6	15.3

Packed Columns

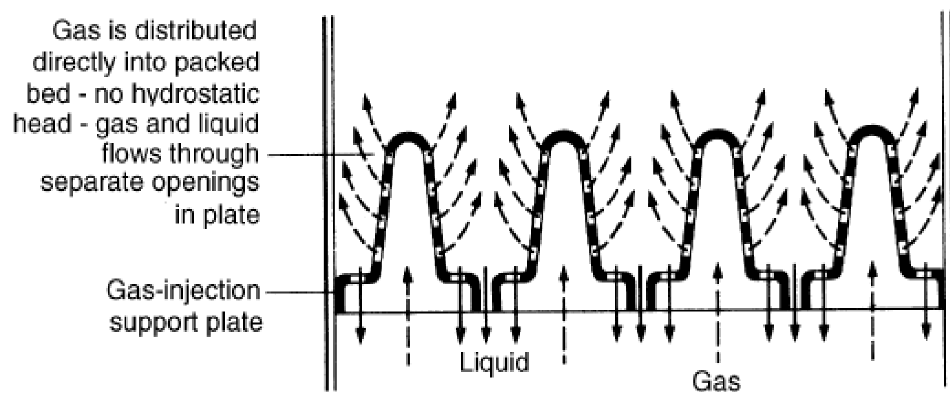
- Packed columns consist of shaped particles contained within a column, their behavior will in many ways be similar to that of packed beds which have already been considered.
- Packed towers are used for bringing two phases in contact with one another and there will be strong interaction between the fluids.
- Normally one of the fluids (liquid) will preferentially wet the packing and will flow as a film over its surface; the second fluid (gas) then passes through the remaining volume of the column.
- An example of the liquid–gas system is an absorption process where a soluble gas is scrubbed from a mixture of gases by means of a liquid.
- In the construction of packed towers, the shell of the column may be constructed from metal, ceramics, glass, or plastics material. The column should be mounted truly vertically to help uniform liquid distribution.



- The bed of packing rests on a support plate which should be designed to have at least 75 % free area for the passage of the gas so as to offer as low a resistance as possible.

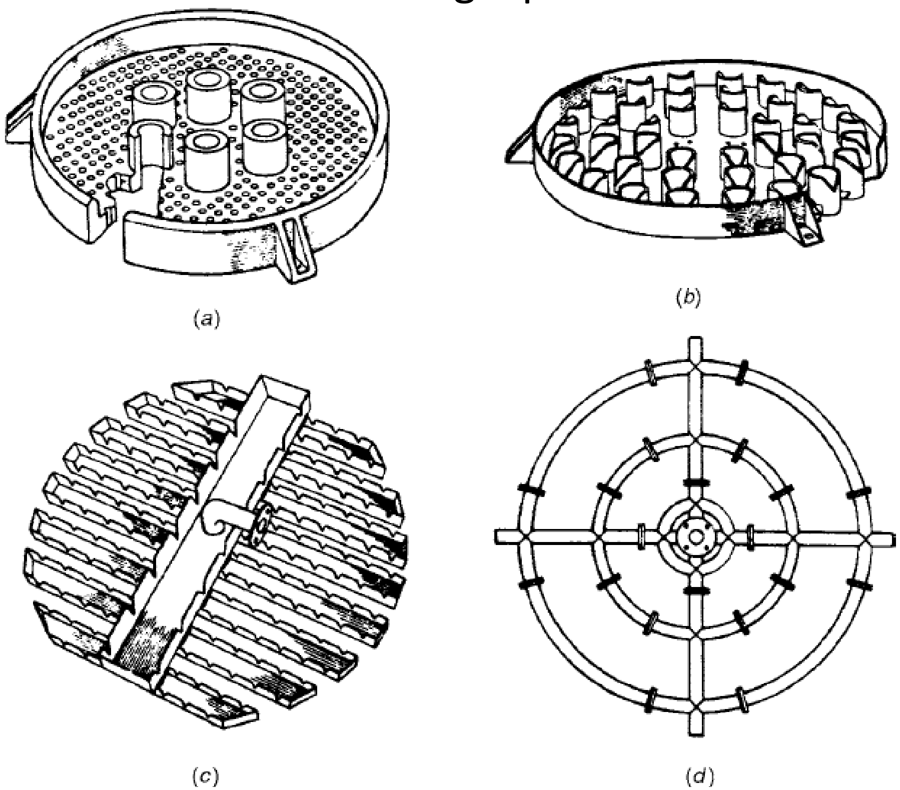


The gas injection plate is designed to provide separate passageways for gas and liquid so that they need not vie for passage through the same opening. This is achieved by providing the gas inlets to the bed at a point above the level at which liquid leaves the bed.



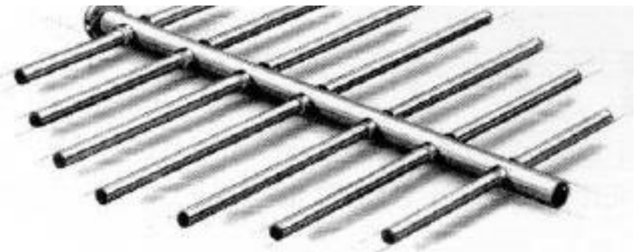
Types of liquid distributor

- At the top of the packed bed a liquid distributor of suitable design provides for the uniform irrigation of the packing



- ✓ A “hold-down” plate is often placed at the top of a packed column to minimize movement and breakage of the packing caused by surges (sudden increase) in flowrates.

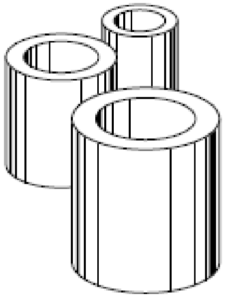
- ✓ The gas inlet should also be designed for uniform flow over the cross-section and the gas exit should be separate from the liquid inlet



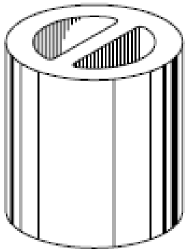
- ➢ Columns for both absorption and distillation vary in diameter from about 25 mm for small laboratory purposes to over 4.5 m for large industrial operations; these industrial columns may be 30 m or more in height.

Packings (Read through R&C page 216 - 221)

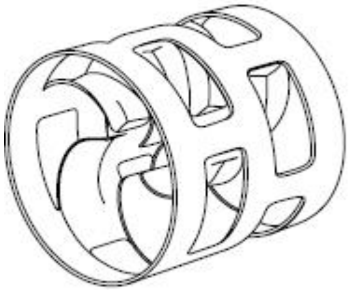
- Packings can be divided into four main classes—broken solids, shaped packings, grids, and structured packings.
- Most of these packings are available in a wide range of materials such as ceramics, metals, glass, plastics, carbon, and sometimes rubber.

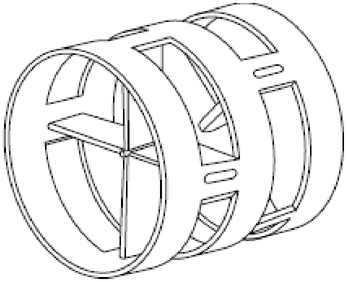
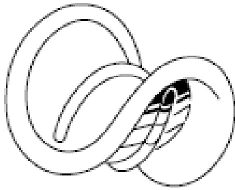


- Non-porous solid should be used if there is any risk of crystal formation in the pores when the packing dries, as this can give rise to serious damage to the packing elements.
- Channeling, that is non-uniform distribution of liquid across the column cross-section, is much less marked with shaped packings, and their resistance to flow is much less.



- Shaped packings also give a more effective surface per unit volume because surface contacts are reduced to a minimum and the film flow is much improved compared with broken solids.

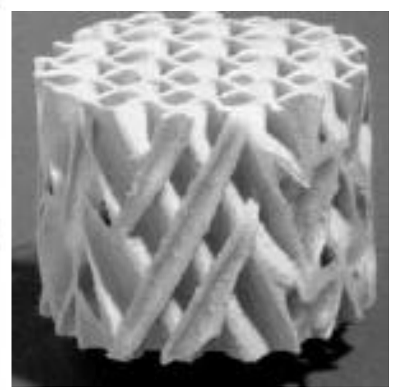



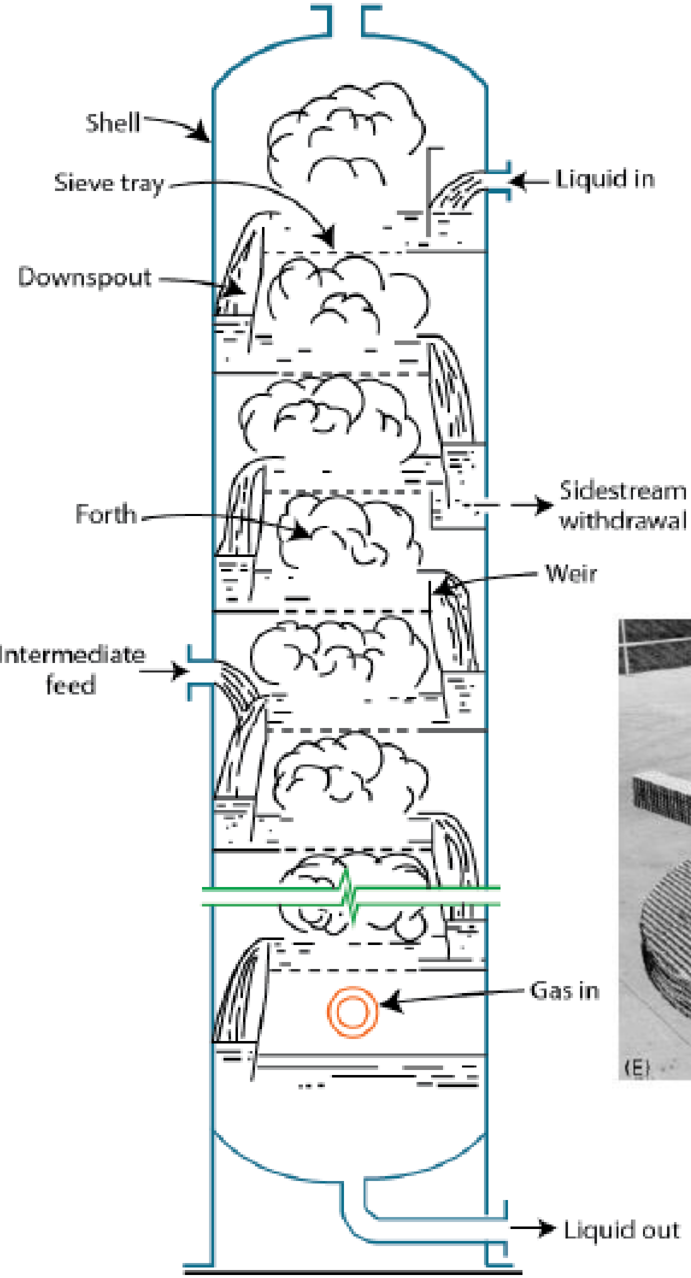
Packings

Table 4.3. Design data for various packings

- The voidage obtainable with these packings varies from about 0.45 to 0.95
- To obtain high and uniform voidage and to prevent breakage, it is often found better to dump the packings into a tower full of liquid.
- Grid packings, which are relatively easy to fabricate, are usually used in columns of square section, and frequently in cooling towers.
- They may be made from wood, plastics, carbon, or ceramic materials, and, because of the relatively large spaces between the individual grids, they give low pressure drops.



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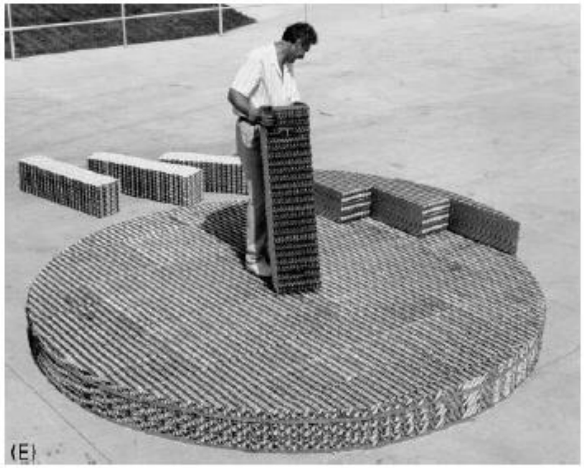
Sieve tray
Liquid in
Downspout
Forth
Sidestream
withdrawal
Weir
Intermediate
feed
Gas in
Liquid out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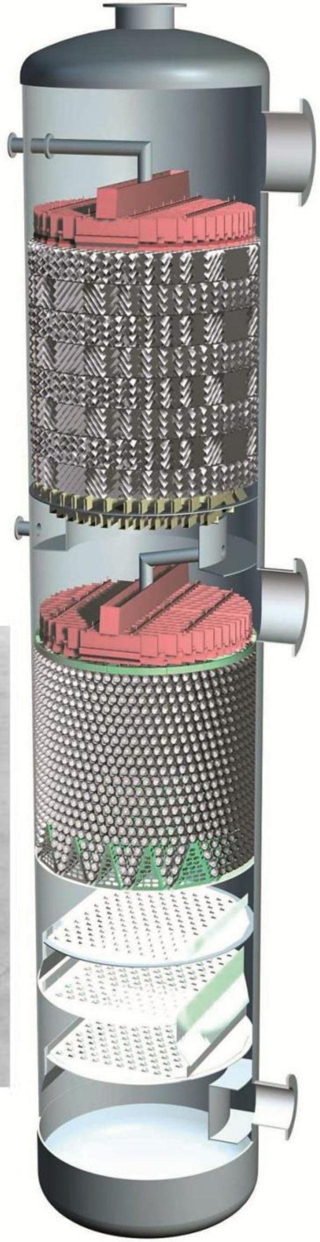
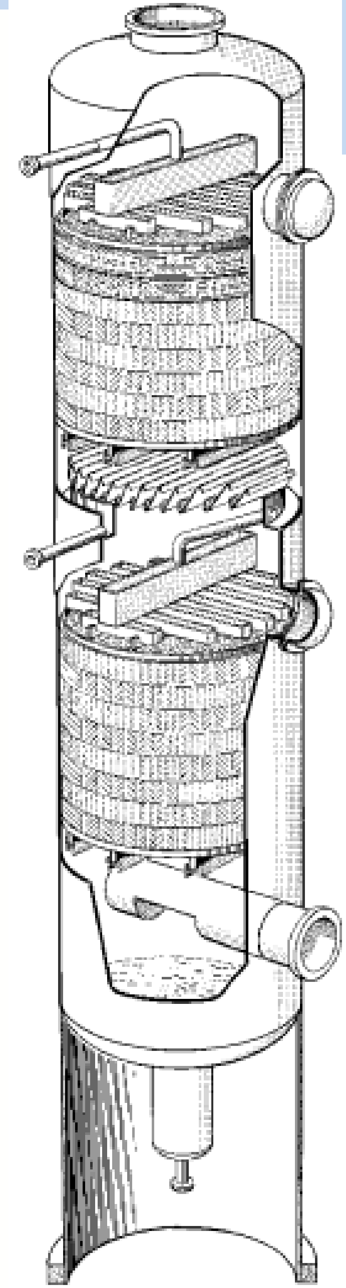




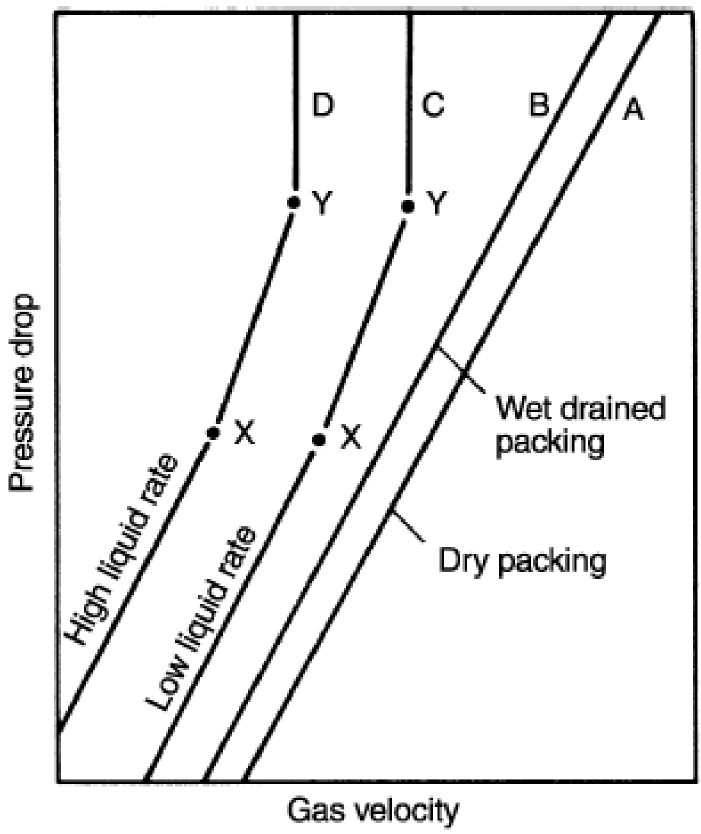
Fluid flow in packed columns

Pressure drop

- The drop in pressure arising from the flow of a single phase through granular beds is considered and the same general form of approach is usefully adopted for the flow of two fluids through packed columns.

Loading (X) and flooding (Y) points

Hold-up



Read through R&C pages 222-228