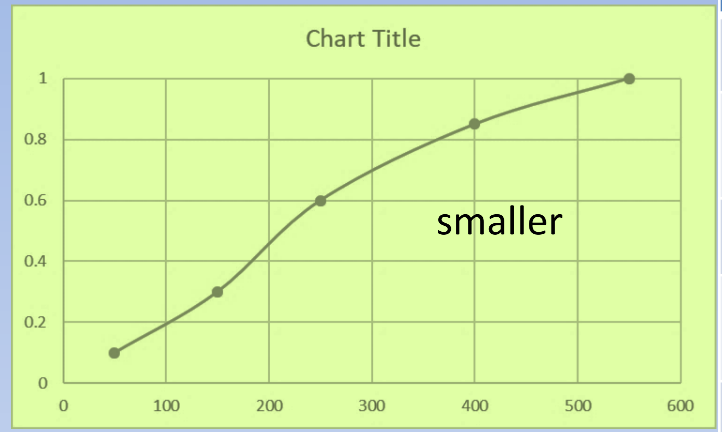
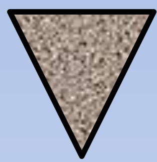
Mean Particle Size

Example for Discussion

Size range	x	d	x_{cum}
0 - 100	0.1	50	0.1
100 – 200	0.2	150	0.3
200 – 300	0.3	250	0.6
300 – 500	0.25	400	0.85
500 - 600	0.15	550	1.00
total	1.00		



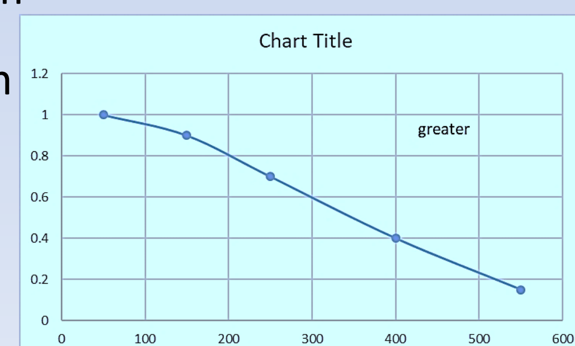


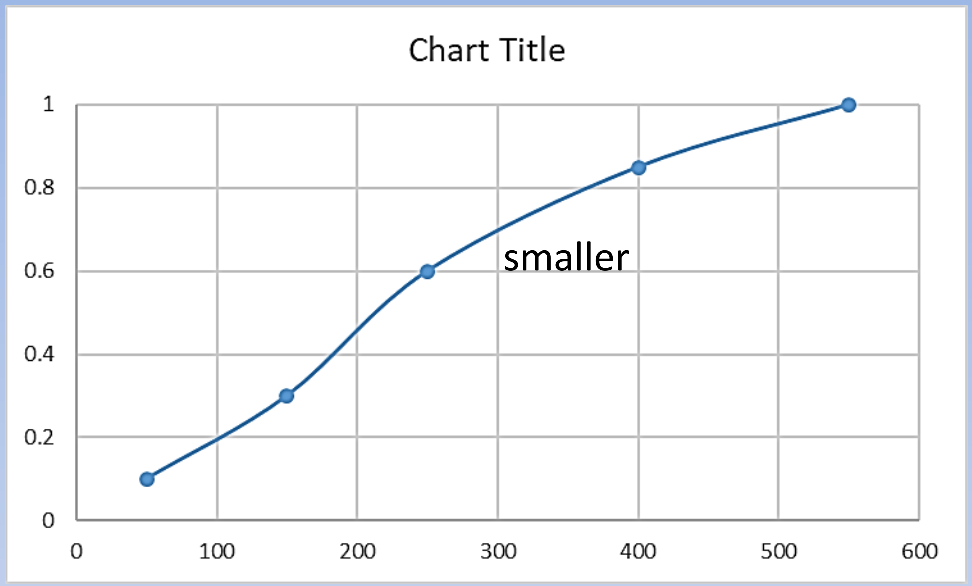
Sample
100 g

Arbitrary values

	Wt on sieve	Fraction, x	x_{cum}	davg
600	0 g	0.00 0r 0%		
500	15 g	0.15 or 15%	15%	550 µm
300	25 g	0.25 or 25%	40%	400 µm
200	30 g	0.30 or 30%	70%	250 µm
100	20 g	0.20 0r 20%	90%	150 µm
Pan	10 g	0.10 0r 10%	100%	50 µm

Total 100 g Total 1.00 100%

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1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
smaller

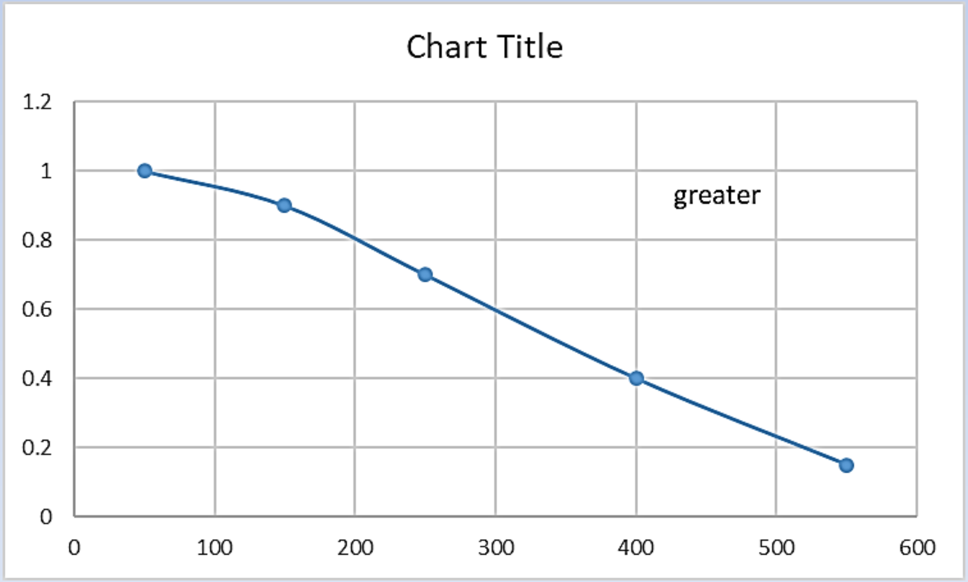
Chart Title
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
greater

Particle size distribution

cumulative mass fraction curve, in which the proportion of particles *(x) smaller* than a certain size *(d) is plotted against that size (d)*

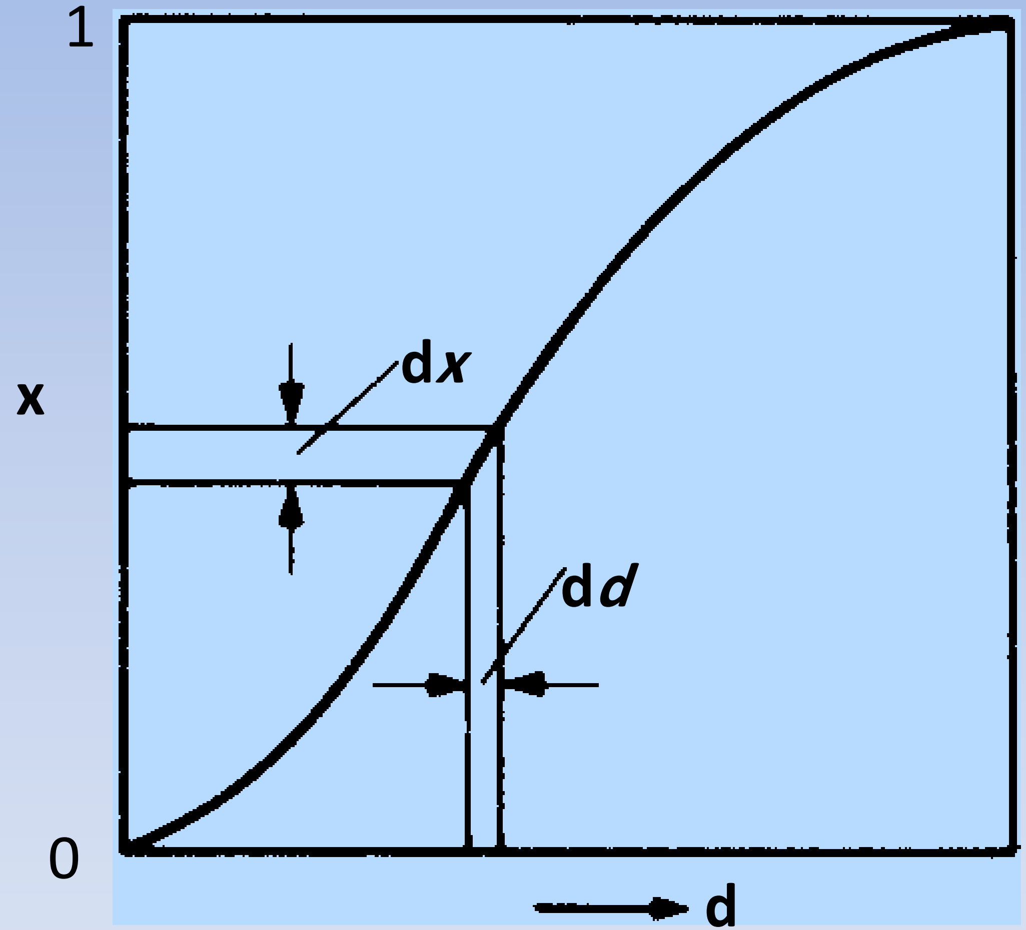

Figure 1.5. Size distribution curve—cumulative basis

Particle size distribution

the slope *(dx/dd) of the cumulative* curve (Figure 1.5) is plotted against particle size *(d).*

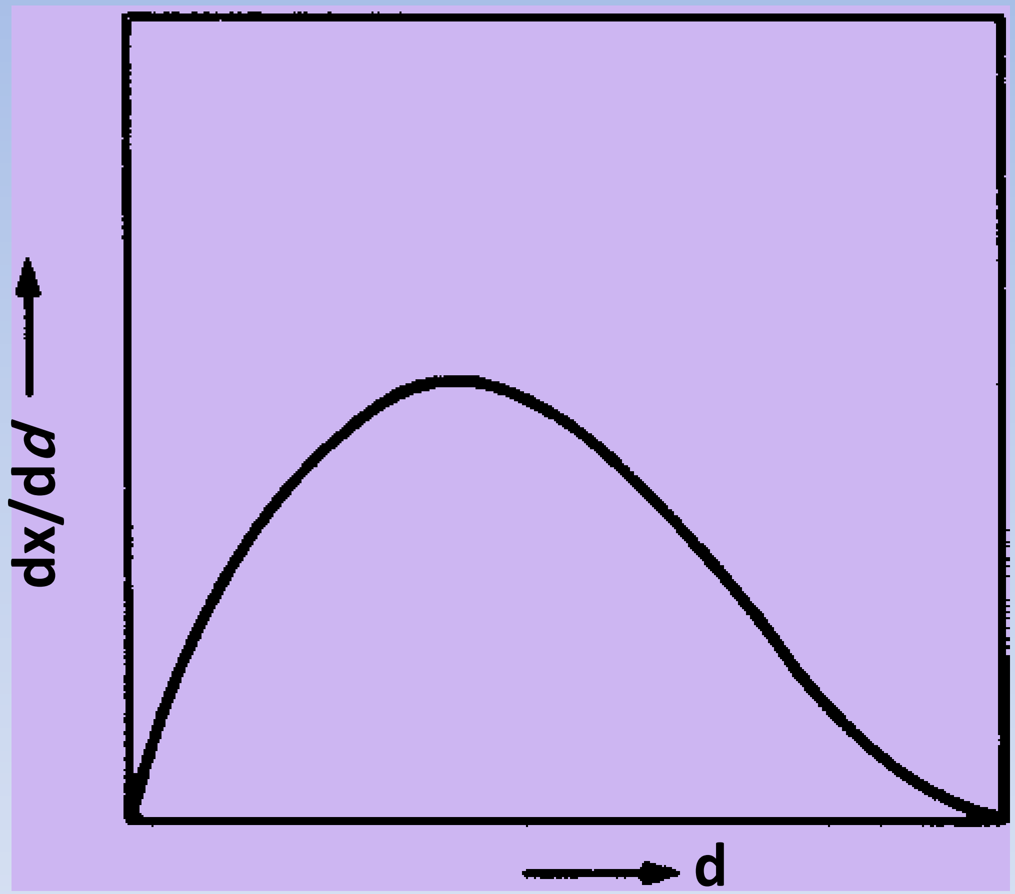

Figure 1.6. Size distribution curve—frequency basis

Consider a unit mass of n_1 particles of characteristic length d_1 with mass fraction x_1 and so on

$$\left\{ \begin{array}{ccc} n1 & d1 & x1 \\ n2 & d2 & x2 \\ . & . & . \\ . & . & . \\ nx & d_x & x_x \end{array} \right\} \textbf{unit mass}$$

Note:

Aggregate length $n_i d_i$

Aggregate surface $n_i d_i^2$

Aggregate volume $n_i d_i^3$

Mean particle size

- Considering unit mass of particles consisting of *n1 particles of characteristic dimension d1, constituting a mass fraction x1, n2 particles of size d2, and so on, then:*

$$x_1 = n_1 k_1 d_1^3 \ \rho_s \qquad (1.4)$$

and: $$\Sigma x_1 = 1 = \rho_s k_1 \Sigma (n_1 d_1^3) \qquad (1.5)$$

Thus: $$n_1 = (1 / \rho_s k_1)\ (x_1 / d_1^3) \qquad (1.6)$$

- If the size distribution can be represented by a continuous function, then:

$dx = \rho_s k_1 d^3 \, dn$

or:

$$\frac{dx}{dn} = \rho_s k_1 d^3 \qquad (1.7)$$

And:

$$\int_0^1 dx = 1 = \rho_s k_1 \int d^3 \, dn \qquad (1.8)$$

where ρ_s *is the density of the particles, and*

k_1 *is a constant whose value depends on the shape of the particle.*

Summary
Means based on volume

- Volume mean diameter, dv

$$d_v = \frac{\sum (n_i d_i) v_i}{\sum n_i v_i} = \frac{\sum n_i d_i^4}{\sum n_i d_i^3}$$

in terms of x_i :

$$d_v = \frac{\sum d_i x_i}{\sum x_i} = \sum d_i x_i$$

- Mean volume diameter dv'

$$d_{v'}^3 \sum n_i = \sum n_i d_i^3$$

$$d_{v'} = \sqrt[3]{\frac{\sum n_i d_i^3}{\sum n_i}} \quad \text{, since } n_i = x_i \big/ \rho_s k_i d_i^3$$

$$\therefore \qquad d_{v'} = \sqrt[3]{\frac{\sum x_i}{\sum (x_i / d_i^3)}}$$

Means based on surface

- Surface mean diam, ds

$$d_s = \frac{\sum (n_i d_i) s_i}{\sum n_i s_i} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2}$$

in terms of x_i :

$$d_s = \frac{\sum x_i}{\sum (x_i / d_i)} = \frac{1}{\sum (x_i / d_i)}$$

Sauter mean diameter

- Mean surface diam, ds'

$$d_{s'}^2 \sum n_i = \sum n_i s_i = \sum n_i d_i^2$$

$$d_{s'} = \sqrt{\frac{\sum n_i d_i^2}{\sum n_i}} \quad \text{, since} \quad n_i = x_i \Big/ \rho_s k_i d_i^3$$

$$\therefore \qquad d_{s'} = \sqrt{\frac{\sum (x_i / d_i)}{\sum (x_i / d_i^3)}}$$

Means based on length

- Length mean diam, d_l

$$d_l = \frac{\sum (n_i d_i) d_i}{\sum n_i d_i} = \frac{\sum n_i d_i^2}{\sum n_i d_i}$$

in terms of x_i :

$$d_l = \frac{\sum (x_i / d_i)}{\sum (x_i / d_i^2)}$$

- Mean length diam, d_l'

$$d_{l'} \sum n_i = \sum n_i d_i$$

$$d_{l'} = \frac{\sum n_i d_i}{\sum n_i}$$

in terms x_i

$$\therefore \qquad d_{l'} = \frac{\sum (x_i / d_i^2)}{\sum (x_i / d_i^3)}$$